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Measurements of the mass difference
 $m(B^0) - m(B^+)$ and the energy dependence of
 $\sigma(B^0\bar{B}^0)/\sigma(B^+B^-)$ at Belle and Belle II

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Motivation: $m(B^0) - m(B^+)$

$$\Delta m = m(B^0) - m(B^+)$$

Isospin violation arises from:

- Light-quark mass difference $m_d - m_u$.
- Electromagnetic interaction between constituent quarks.

Why measure Δm precisely?

- Sensitive to the light-quark mass difference.
- Provides constraints on theoretical models and the B -meson wave function.

Previous measurements of Δm

Direct measurements of B -meson masses

$$\begin{aligned}m(B^0) &= (5279.63 \pm 0.20) \text{ MeV}/c^2, \\m(B^+) &= (5279.42 \pm 0.08) \text{ MeV}/c^2\end{aligned}$$

Dominated by LHCb measurements, notably $B^0 \rightarrow J/\psi p \bar{p}$ and $B^- \rightarrow J/\psi \Lambda \bar{p}$.

$$\Rightarrow \Delta m = (0.21 \pm 0.22) \text{ MeV}/c^2$$

Measurement of B -meson momenta in $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$

$p_B \simeq 320 \text{ MeV}/c$ – favorable near-threshold kinematics:

$$\delta(\Delta m) \simeq \frac{p_B}{m_B} \delta(p_B) \simeq \frac{1}{16} \delta(p_B)$$

$$\text{BaBar: } \Delta m = (0.33 \pm 0.05 \pm 0.03) \text{ MeV}/c^2$$

BaBar, Phys. Rev. D 78, 011103 (2008).

Difficulty: B^0 and B^+ energies can differ

B mesons produced at $\Upsilon(4S)$ are not monochromatic:

- e^+e^- energy spread due to stochastic emission of synchrotron radiation:
 $\sigma_{E_{\text{cm}}} \simeq 5 \text{ MeV}$ (FWHM $\simeq 12 \text{ MeV}$); cf. $\Gamma_{\Upsilon(4S)} \simeq 20 \text{ MeV}$.
- Initial-state radiation (ISR).

Energy distribution of produced B^+B^- pairs:

$$f_{B^+B^-} \propto [G(E_{\text{cm}}) \otimes f_{\text{ISR}}] \sigma(e^+e^- \rightarrow B^+B^-)$$

If the charged and neutral $B\bar{B}$ cross sections vary differently with energy, the B^+ and B^0 mesons have **different energy distributions**.

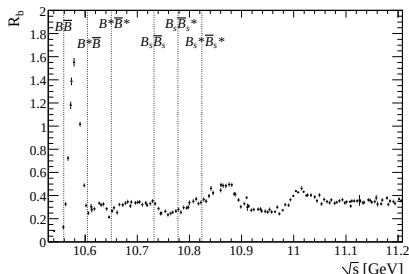
**$\implies$ Need the energy dependences of
 $\sigma(e^+e^- \rightarrow B^+B^-)$ and $\sigma(e^+e^- \rightarrow B^0\bar{B}^0)$.**

Available information on cross sections

Total $b\bar{b}$ cross section

$$R_b = \frac{\sigma(e^+e^- \rightarrow b\bar{b})}{\sigma_0}, \quad \sigma_0 = \frac{4\pi\alpha^2}{3s}$$

BaBar, PRL **102**, 012001 (2009)



- Count hadronic events.
- Subtract continuum using data below the $B\bar{B}$ threshold.

Cross-section ratio

$$\mathcal{R}(E_{\text{cm}}) = \frac{\sigma(e^+e^- \rightarrow B^0\bar{B}^0)}{\sigma(e^+e^- \rightarrow B^+B^-)}$$

Measured at the $\Upsilon(4S)$ peak:

$$f^{00}/f^{+-} = 0.951 \pm 0.028$$

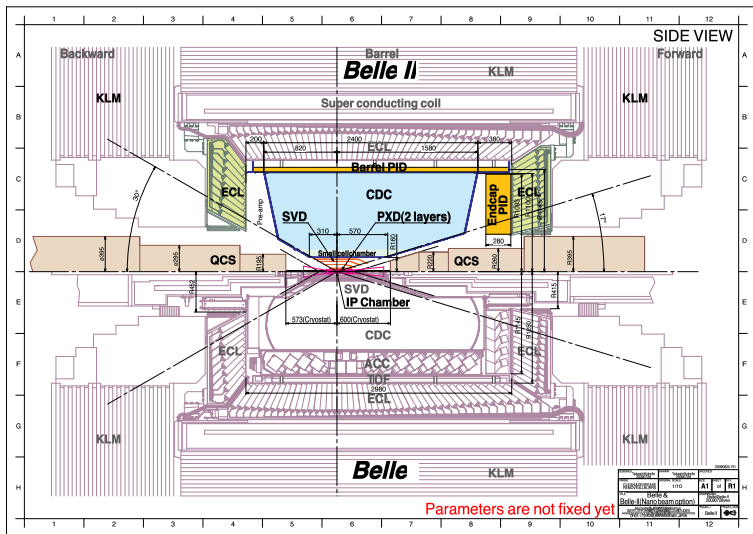
(HFLAV average)

Our idea

Measure $\mathcal{R}(E_{\text{cm}})$ using time variations of the average collision energy during data taking.

Belle and Belle II Collaborations,
JHEP **05** (2026) 172.

Belle 1999–2010, **Belle II** 2019–now at KEK, Tsukuba.



At $\Upsilon(4S)$, Belle: 711 fb^{-1} , Belle II: 808 fb^{-1} , BaBar: 433 fb^{-1} .

Event selection

Reconstruct B mesons in about **1000 hadronic final states**.

$$B^+ \quad \bar{D}^{(*)0}\pi^+(\pi^+\pi^-), D_s^{(*)+}\bar{D}^{(*)0}, J/\psi K^+(\pi^+\pi^-), J/\psi K_S^0\pi^+, D^{(*)-}\pi^+\pi^+$$

$$B^0 \quad D^{(*)-}\pi^+(\pi^+\pi^-), D_s^{(*)+}D^{(*)-}, J/\psi K_S^0, J/\psi K^+\pi^-, D^{*-}K^+K^-\pi^+$$

$$D^0 \quad K^-\pi^+, K^-\pi^+\pi^0, K^-\pi^+\pi^+\pi^-, K_S^0\pi^+\pi^-, K_S^0\pi^+\pi^-\pi^0, K^+K^-, K^+K^-K_S^0$$

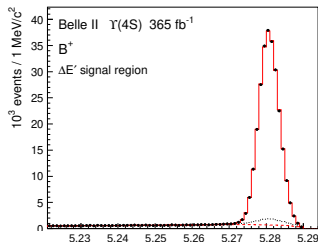
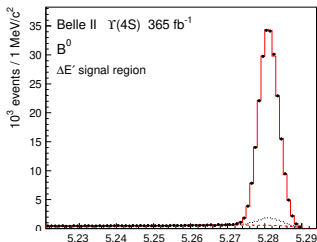
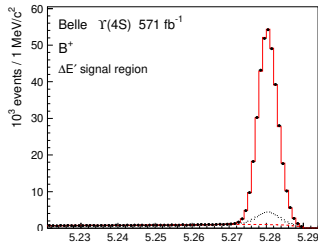
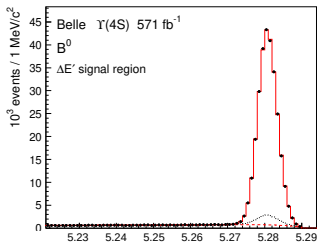
$$D^+ \quad K^-\pi^+\pi^+, K^-\pi^+\pi^+\pi^0, K_S^0\pi^+, K_S^0\pi^+\pi^0, K_S^0\pi^+\pi^+\pi^-, K^+K^-\pi^+, \\ K^+K^-\pi^+\pi^+\pi^-, K^+\pi^+\pi^-, \pi^+\pi^+\pi^-$$

$$D_s^+ \quad K^+K^-\pi^+, K^+K^-\pi^+\pi^0, K^+K_S^0, K^+K_S^0\pi^+\pi^-, K^-K_S^0\pi^+\pi^+$$

Automated reconstruction uses **multivariate classifiers** to suppress combinatorial background; inputs include intermediate-particle masses and vertex-fit quality.

M_{bc} distributions

$$M_{bc} = \sqrt{\left(\frac{m_{\Upsilon(4S)}}{2}\right)^2 - p_B^2}$$



M_{bc} (GeV/c²)

M_{bc} (GeV/c²)

Very high signal-to-background ratio.

M_{bc} fit function

B^+ candidates:

$$G(E_{\text{cm}}) \otimes f_{\text{ISR}} \times \sigma(e^+e^- \rightarrow B^+B^-) \xrightarrow{m_{B^+}} f(p_B), \otimes f_{\text{resolution}}^{B^+} \mapsto f(M_{bc})$$

B^0 candidates:

$$G(E_{\text{cm}}) \otimes f_{\text{ISR}} \times \sigma(e^+e^- \rightarrow B^0\bar{B}^0) \xrightarrow{m_{B^0}} f(p_B), \otimes f_{\text{resolution}}^{B^0} \mapsto f(M_{bc})$$

Calculations are performed numerically at each fit iteration.

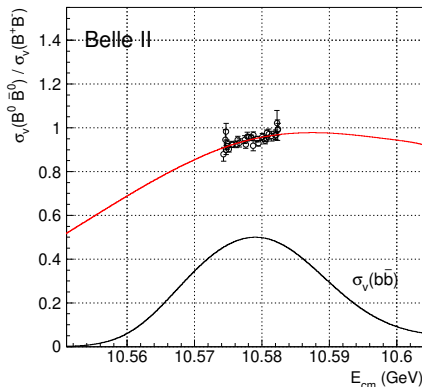
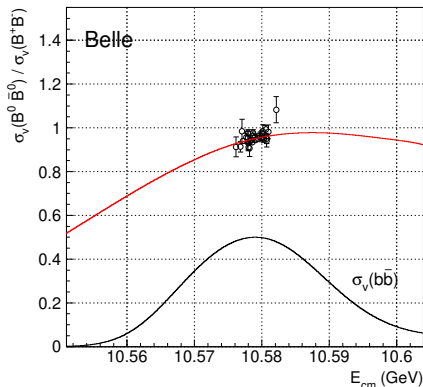
Fit-function parameters have clear physical meaning:

B -meson masses, average collision energy, energy spread.

Measurement of $\mathcal{R}(E_{\text{cm}})$

- Divide data into bins of **calibrated** collision energy E_{cm} .
- In each E_{cm} bin, fit the M_{bc} spectra for B^0 and B^+ .

$$\mathcal{R}(E_{\text{cm}}) = \frac{\sigma(e^+e^- \rightarrow B^0\bar{B}^0)}{\sigma(e^+e^- \rightarrow B^+B^-)}$$



Cross-section parameterization and combined fit

Cross-section energy dependences:

$$\begin{aligned}\sigma(e^+e^- \rightarrow B^+B^-) &= p_{B^+}^3 P_{11}(E_{\text{cm}}), \\ \sigma(e^+e^- \rightarrow B^0\bar{B}^0) &= p_{B^0}^3 P_{11}(E_{\text{cm}}) P_2(E_{\text{cm}}).\end{aligned}$$

p_B^3 : threshold factor; P_{11} : common shape; P_2 : ratio shape (Chebyshev polynomials).

Combined fit to:

- M_{bc} spectra for B^+ and B^0 at Belle and Belle II;
- $\mathcal{R}(E_{\text{cm}})$ measurements and the BaBar R_b energy scan.

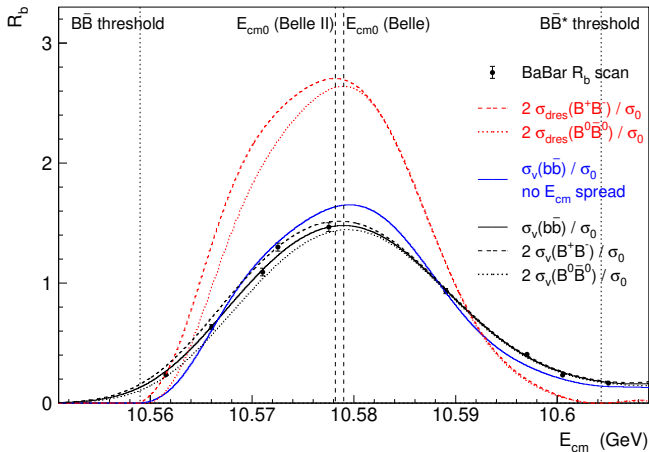
Why high-order polynomial? – A reliable physics-motivated description of the cross sections is not available.

Why a combined fit? – M_{bc} spectra provide additional constraints on the cross-section shapes.

$m(B^+)$ is fixed; the mass difference Δm is determined directly from the fit.

R_b fit result

$$R_b = \frac{\sigma_v(e^+e^- \rightarrow b\bar{b})}{\sigma_0}$$



The combined fit describes all data distributions well.

$$m(B^0) - m(B^+)$$

$$\Delta m = (0.495 \pm 0.024 \pm 0.005) \text{ MeV}/c^2$$

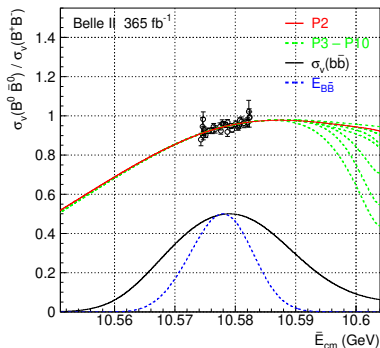
- cf. WA of direct measurements: $(0.21 \pm 0.22) \text{ MeV}/c^2$.
- cf. BaBar $\Delta m = (0.33 \pm 0.05 \pm 0.03) \text{ MeV}/c^2$ PRD 78, 011103 (2008).
BaBar used PHSP hypothesis: $\mathcal{R}(E_{\text{cm}}) = (p_{B^0}/p_{B^+})^3$.
- Repeat analysis with PHSP hypothesis:
 - $\Delta m = (0.386 \pm 0.006) \text{ MeV}/c^2$ – consistent with BaBar.
 - Very poor fit quality (M_{bc} spectra). The PHSP hypothesis is excluded at 10σ level.
- Difference w.r.t. BaBar is understood – model assumption in BaBar analysis.

Belle and Belle II, JHEP 05 (2026) 172.

Measurement of $\mathcal{R} = \sigma(B^0\bar{B}^0)/\sigma(B^+B^-)$ over a wide energy range

$$\sigma(e^+e^- \rightarrow B^+B^-) = p_{B^+}^3 P_{11}(E_{\text{cm}})$$

$$\sigma(e^+e^- \rightarrow B^0\bar{B}^0) = p_{B^0}^3 P_{11}(E_{\text{cm}}) P_2(E_{\text{cm}})$$

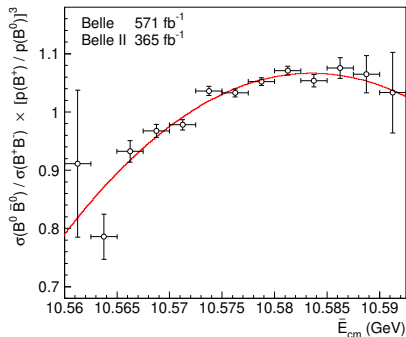
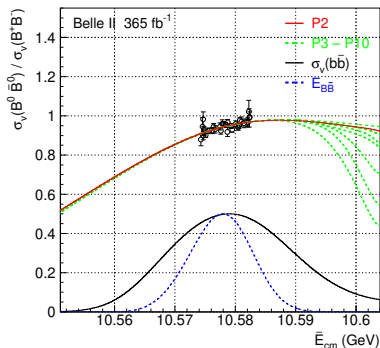


- Replace P_2 by $P_3, P_4, \dots, P_{10}$: stable results.
- M_{bc} spectra of B^0 and B^+ provide additional constraints on $\mathcal{R}(E_{\text{cm}})$.

Measurement of $\mathcal{R} = \sigma(B^0 \bar{B}^0) / \sigma(B^+ B^-)$ over a wide energy range

$$\sigma(e^+ e^- \rightarrow B^+ B^-) = p_{B^+}^3 P_{11}(E_{\text{cm}})$$

$$\sigma(e^+ e^- \rightarrow B^0 \bar{B}^0) = p_{B^0}^3 P_{11}(E_{\text{cm}}) P_2(E_{\text{cm}})$$

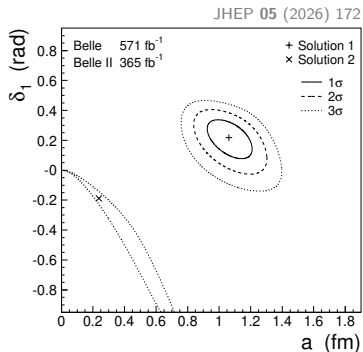


- Replace P_2 by $P_3, P_4, \dots, P_{10}$: stable results.
- M_{bc} spectra of B^0 and B^+ provide additional constraints on $\mathcal{R}(E_{\text{cm}})$.
- Replace P_2 by a histogram function $\Rightarrow$ measurement of $\mathcal{R}(E_{\text{cm}})$ over a wide energy range.

Phenomenological analysis

- Strong rectangular potential + electromagnetic interaction.
Parameters: a – interaction range; δ_1 – isovector scattering phase.

Voloshin, PRD **97**, 114023 (2018).



- $\Rightarrow$ Isovector potential is attractive ($\delta_1 > 0$) at 2σ level.
- In the $m_b \rightarrow \infty$ limit, $B\bar{B}$, $B\bar{B}^*$ and $B^*\bar{B}^*$ potentials are the same. Relevant for understanding $Z_b(10610)$ and $Z_b(10650)$.

Conclusions

- Precise measurement of $m(B^0) - m(B^+)$; the $\sim 3\sigma$ discrepancy with BaBar is traced to its phase-space assumption.
- Measurement of $\mathcal{R}(E_{\text{cm}})$ over a broad energy range.
- Isovector $B\bar{B}$ potential favors attraction at the 2σ level.

Outlook

- Significant isospin violation (tens of percent) predicted near $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds. Salnikov and Milstein, Nucl. Phys. A **1073** (2026) 123443.
- Further energy scans $\Rightarrow$ information on $B\bar{B}$, $B\bar{B}^*$, and $B^*\bar{B}^*$ scattering.

Spin-off

- Improved collision-energy calibration, critical for the τ -mass measurement. Updated Belle II analysis ongoing.