Electroweak precision predictions in the Standard Model

GHENT UNIVERSITY
FACULTY OF SCIENCES
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Simone Devoto

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- Introduction
  - Motivations;
  - Precision physics @LHC.
- A phenomenological example
  - Relevance of EW corrections;
  - Mixed QCDxEW corrections to neutral-current Drell-Yan.
- The challenges
  - Treatment of γ_5;
  - Multiscale loop integrals;
  - Development of the framework OceaNN.
- Towards NNLO EW
  - QED corrections to Drell-Yan;
  - Fermionic contribution to NNLO EW for the Drell-Yan.



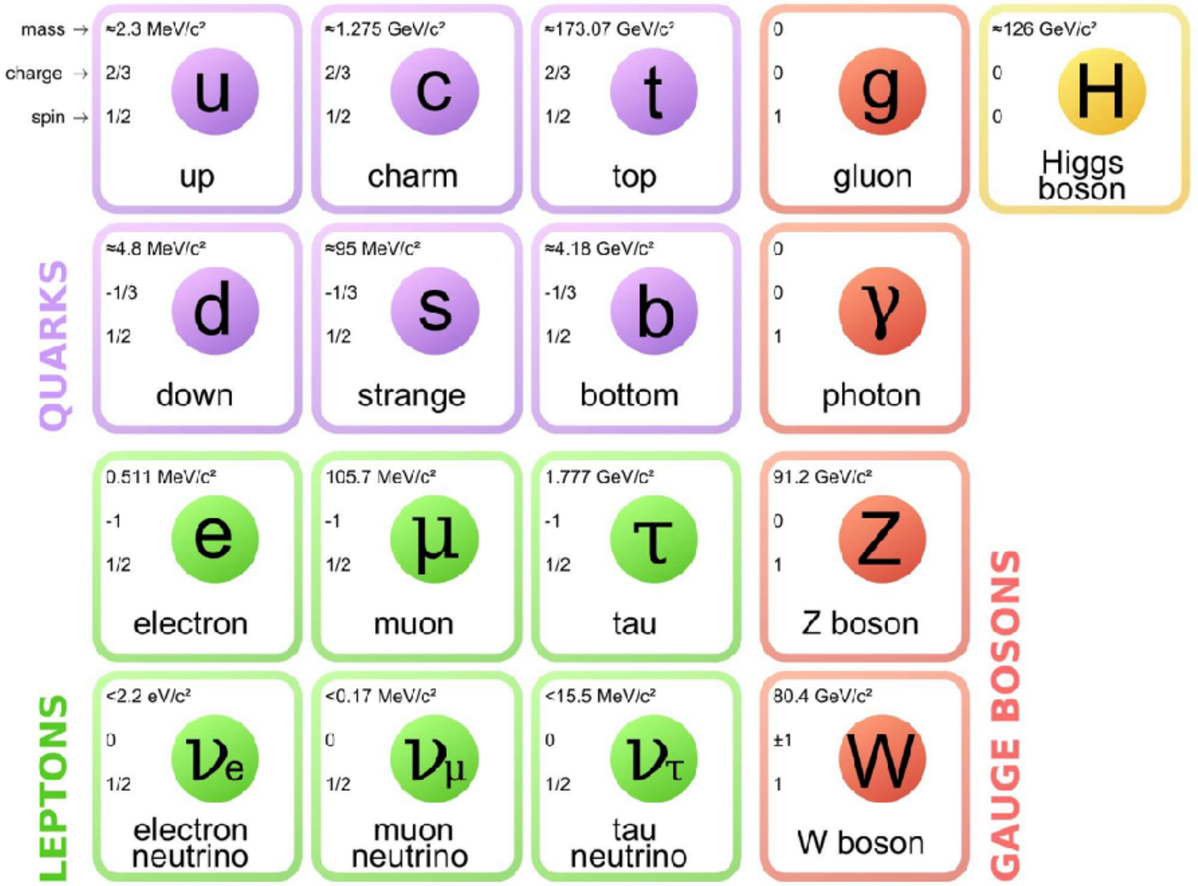
Introduction



The Standard Model

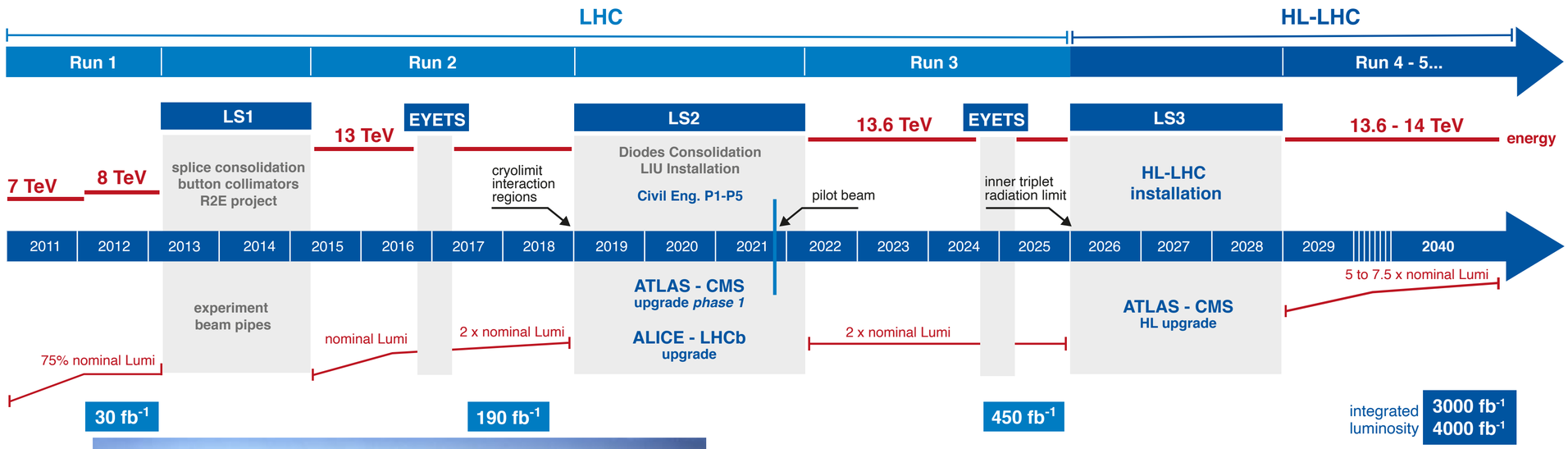
Our (incomplete) understanding of the particle world

- Open questions:
  - How can we accommodate Gravity?
  - How can we accomodate Neutrinos masses?
  - What is the Dark Matter?
  - What is the Dark Energy?
  - Where is all the antimatter (origin of the matter-antimatter asymmetry)?
  - ...
- The EW sector comes from a broken symmetry: can the answer lie in its rich structure?



The Large Hadron Collider

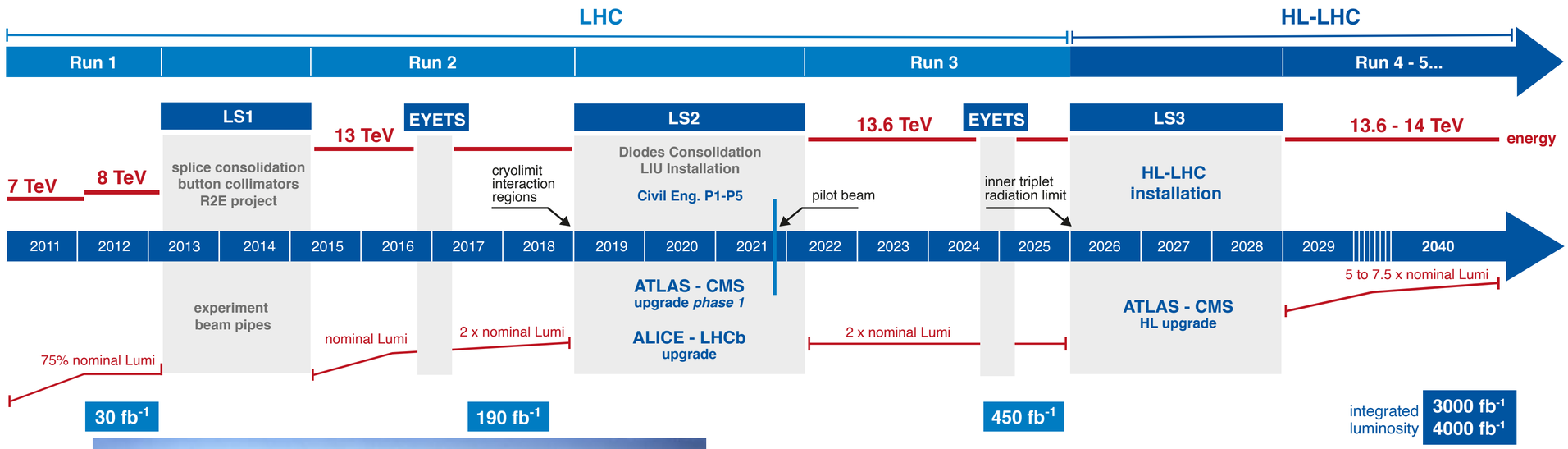
Probing the Standard Model



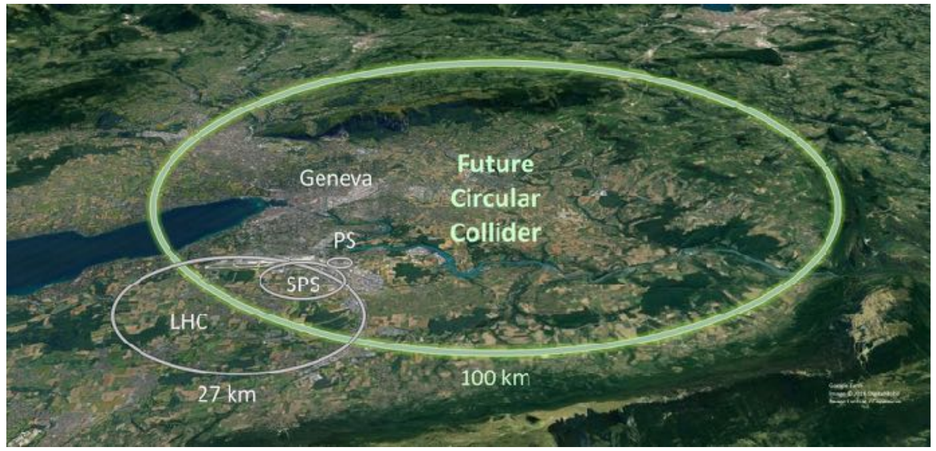


The Large Hadron Collider

Probing the Standard Model



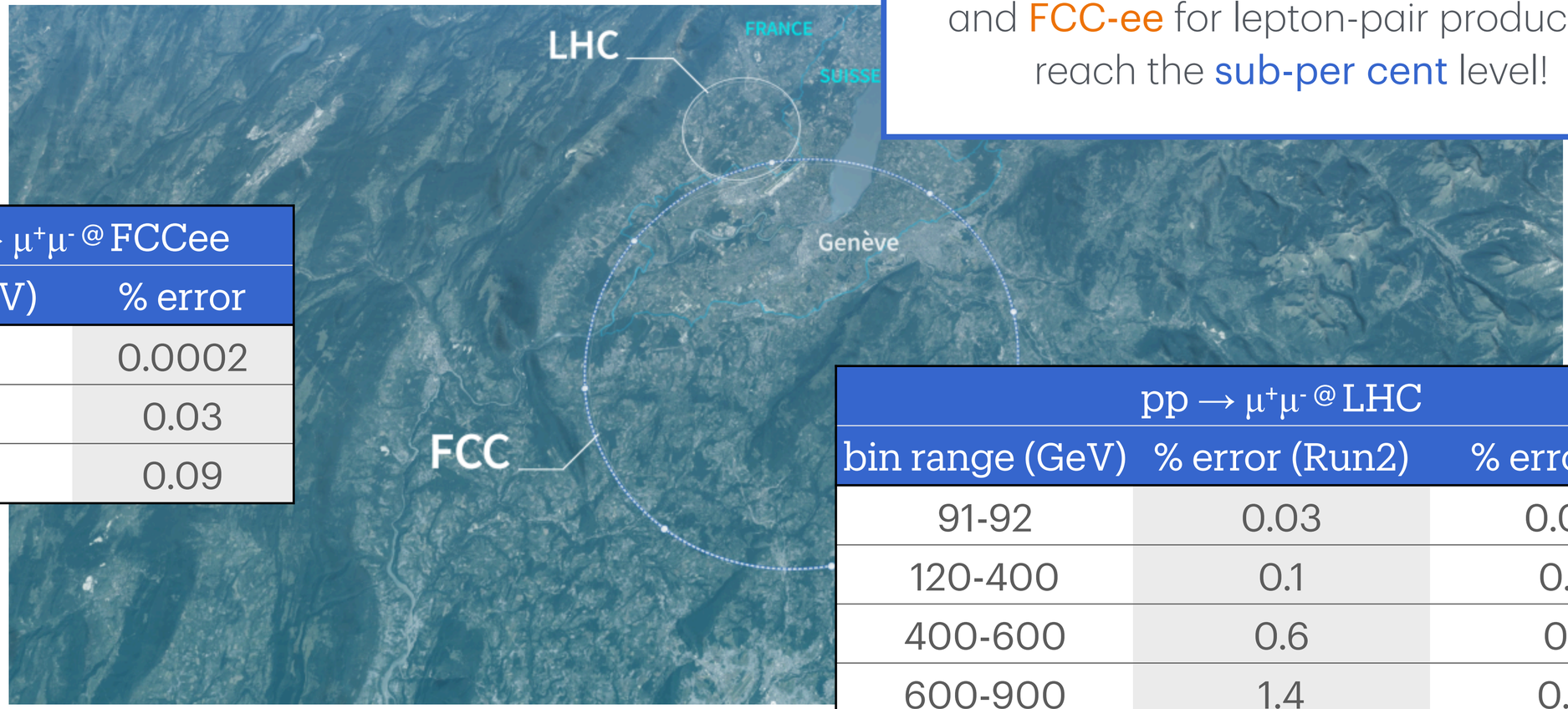




The Large Hadron Collider

Precision

The projected statistical errors at HL-LHC and FCC-ee for lepton-pair production reach the sub-per cent level!



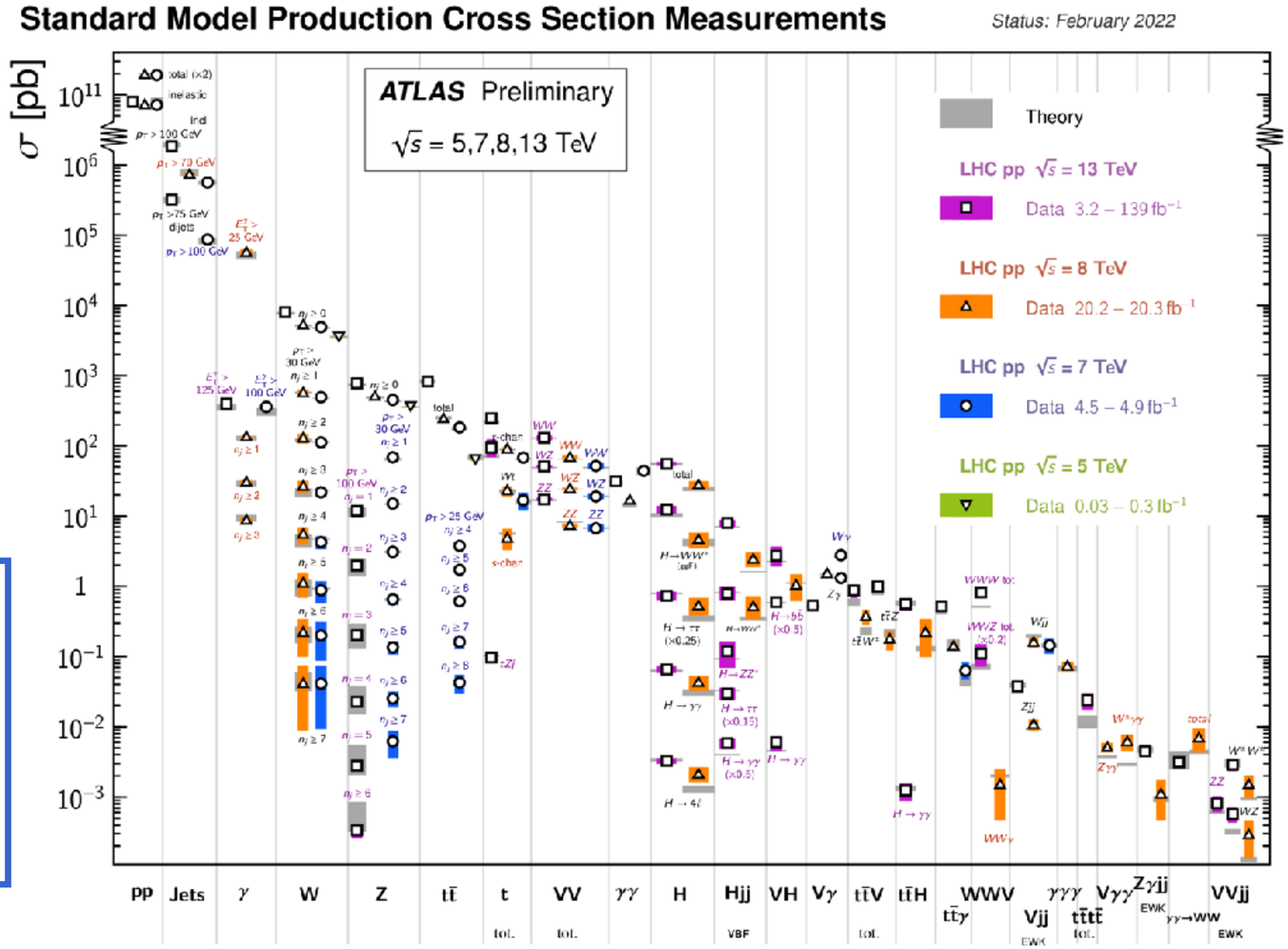
$ee \to \mu^+\mu^-$ @ FCCee	
$\sqrt{s}$ (GeV)	% error
91	0.0002
240	0.03
365	0.09

$pp \to \mu^+\mu^-$ @ LHC		
bin range (GeV)	% error (Run2)	% error (HL)
91-92	0.03	0.006
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

The Large Hadron Collider

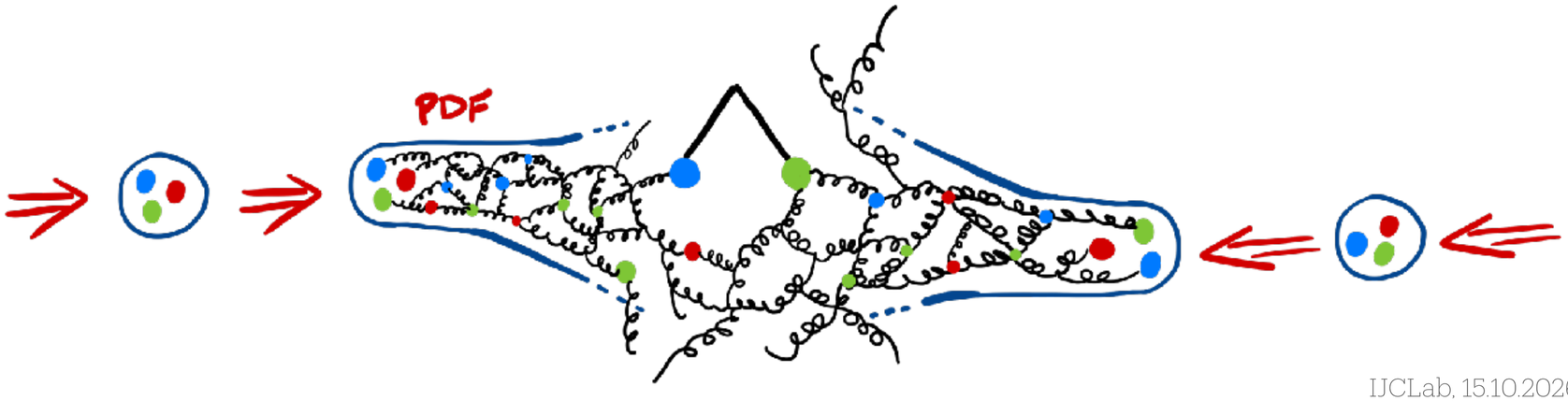
What are we learning?

- LHC has proven itself to be:
  - A discovery machine: Higgs boson discovery;
  - A precision machine: precise measurement of SM parameters.
- Model Building (BSM): Create new models that go beyond the Standard Model (BSM) trying to explain the open questions.
- Precision Studies: Search for small deviations from the predictions of the Standard Model by systematically comparing the best theoretical predictions with data.



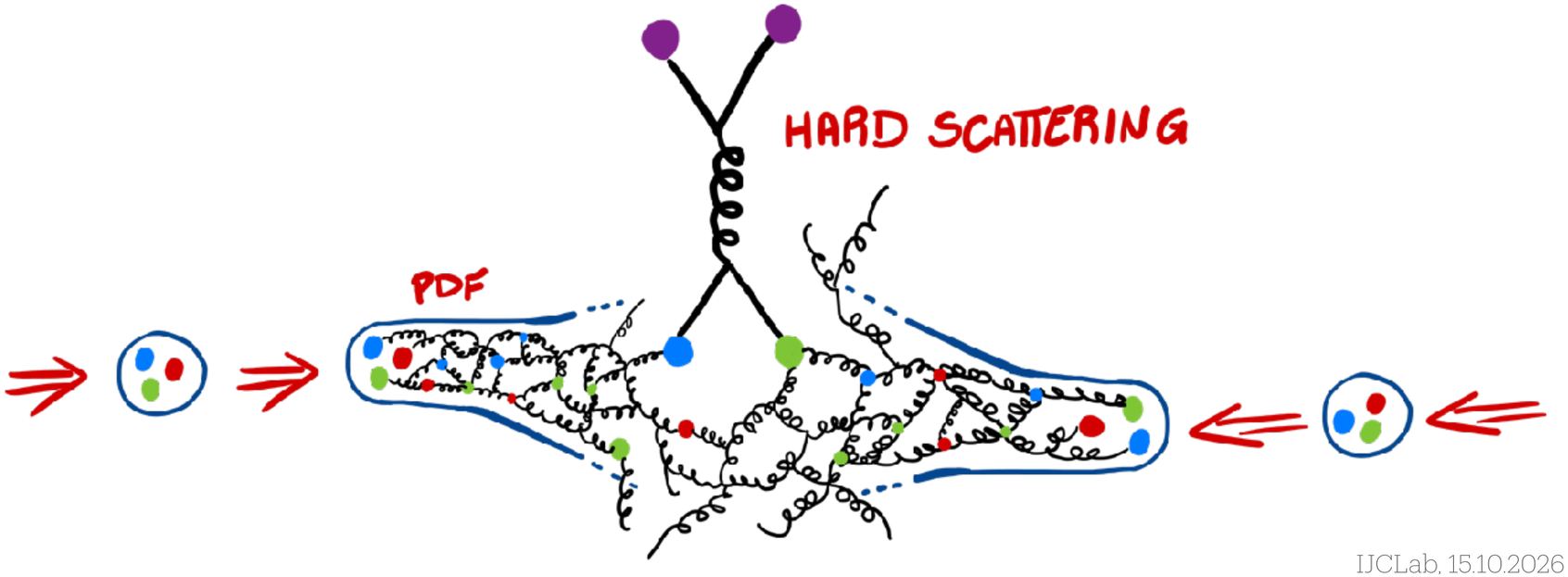
A hadronic collision

The theoretical ingredients



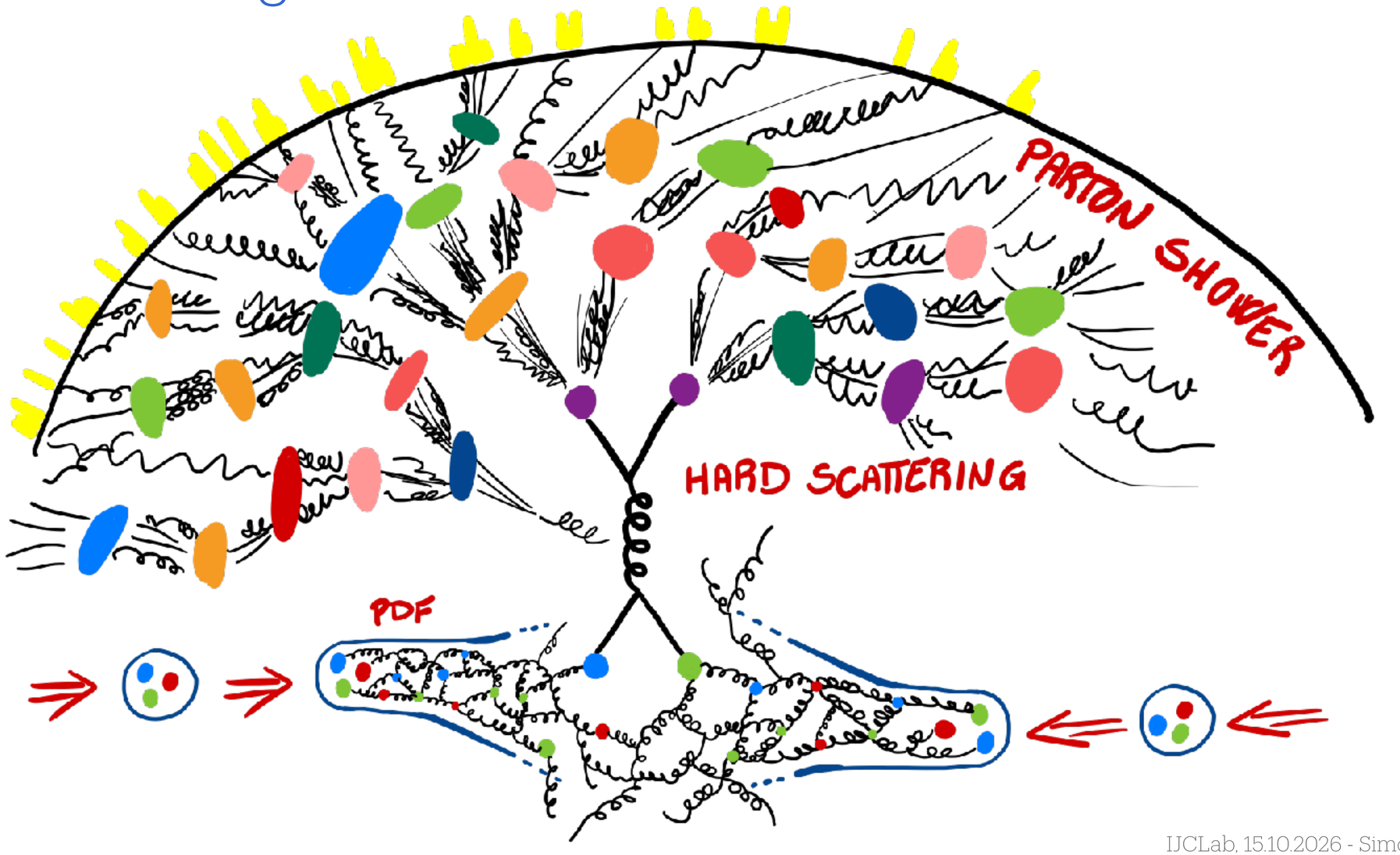
A hadronic collision

The theoretical ingredients



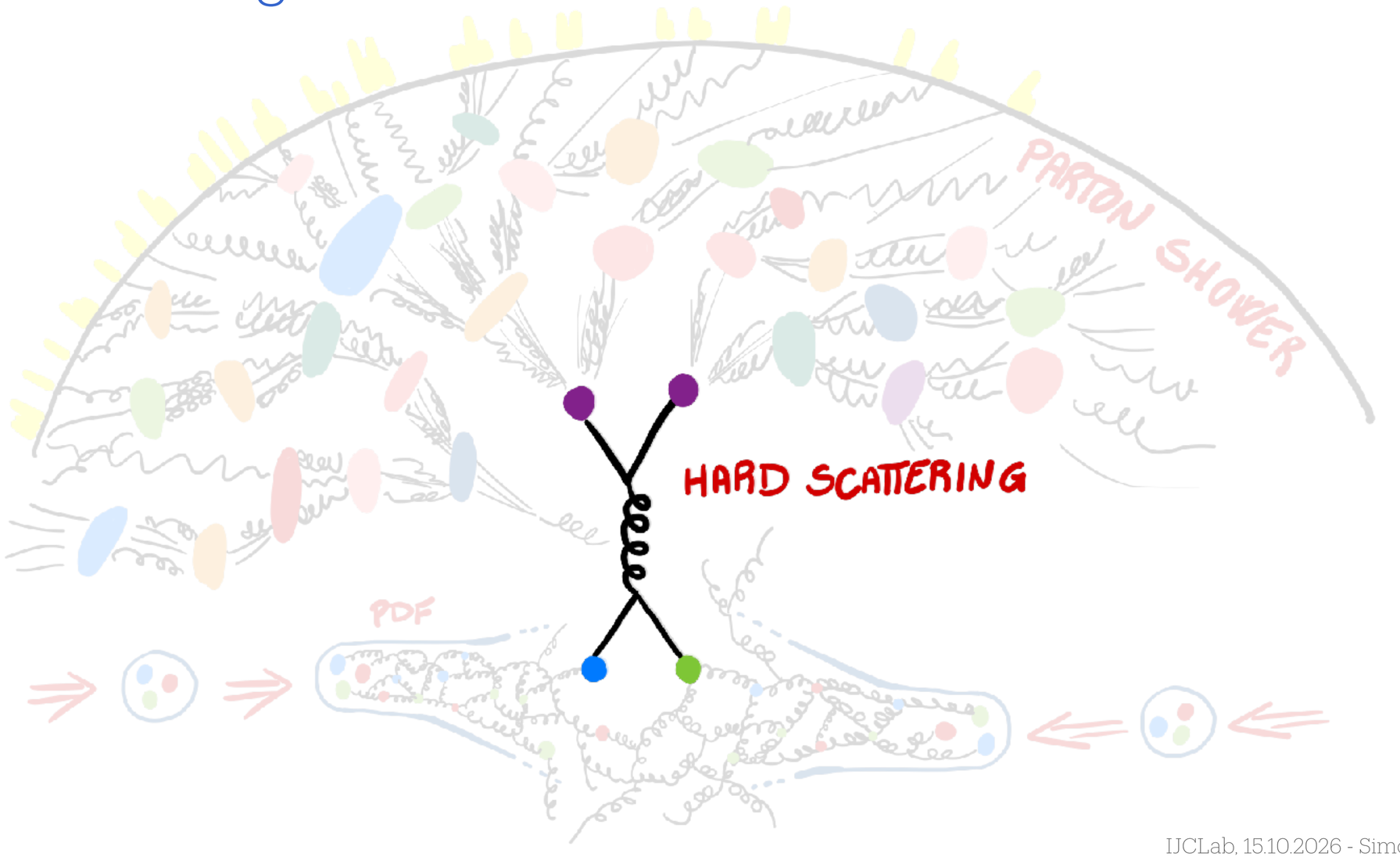
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The theoretical ingredients



A hadronic collision

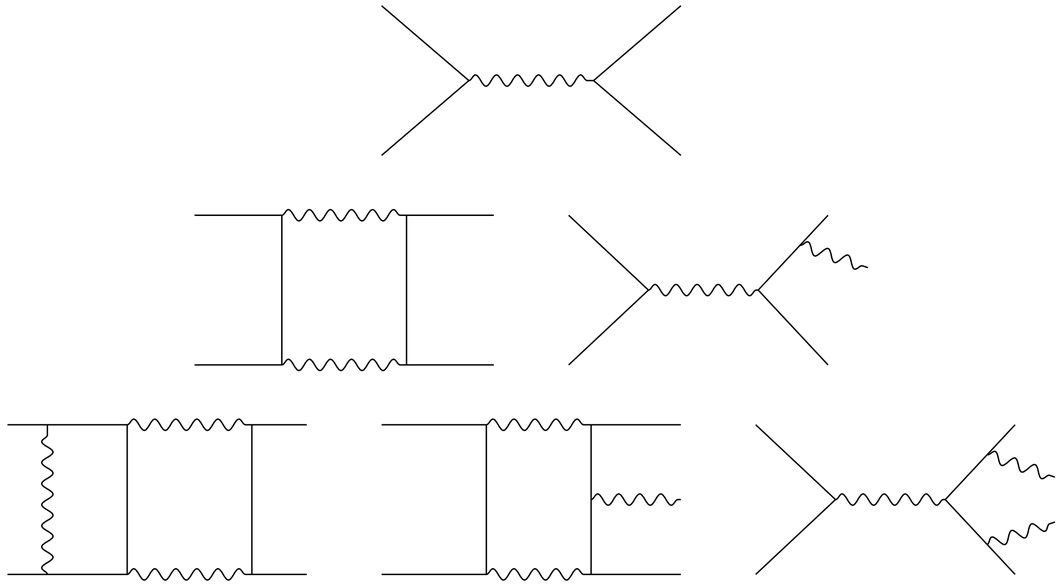
The theoretical ingredients



The hard scattering

The theory bottlenecks

- Leading Order (LO)
- Next-to-Leading Order (NLO)
- Next-to-Next-to-Leading Order (NNLO)



What makes higher-order corrections challenging?

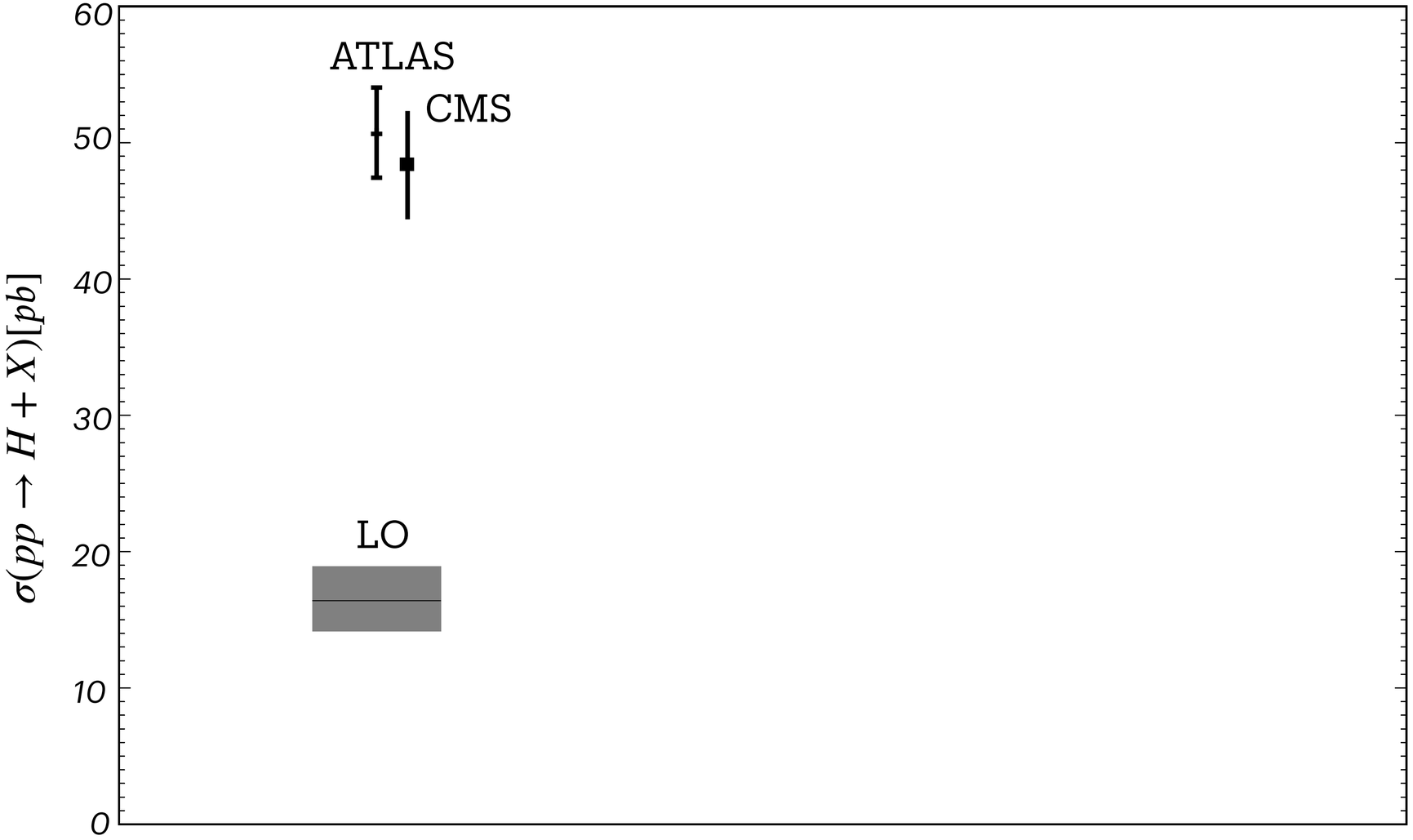
Multi loop amplitudes

Subtraction procedure

The hard scattering

Slide from M.Grazzini

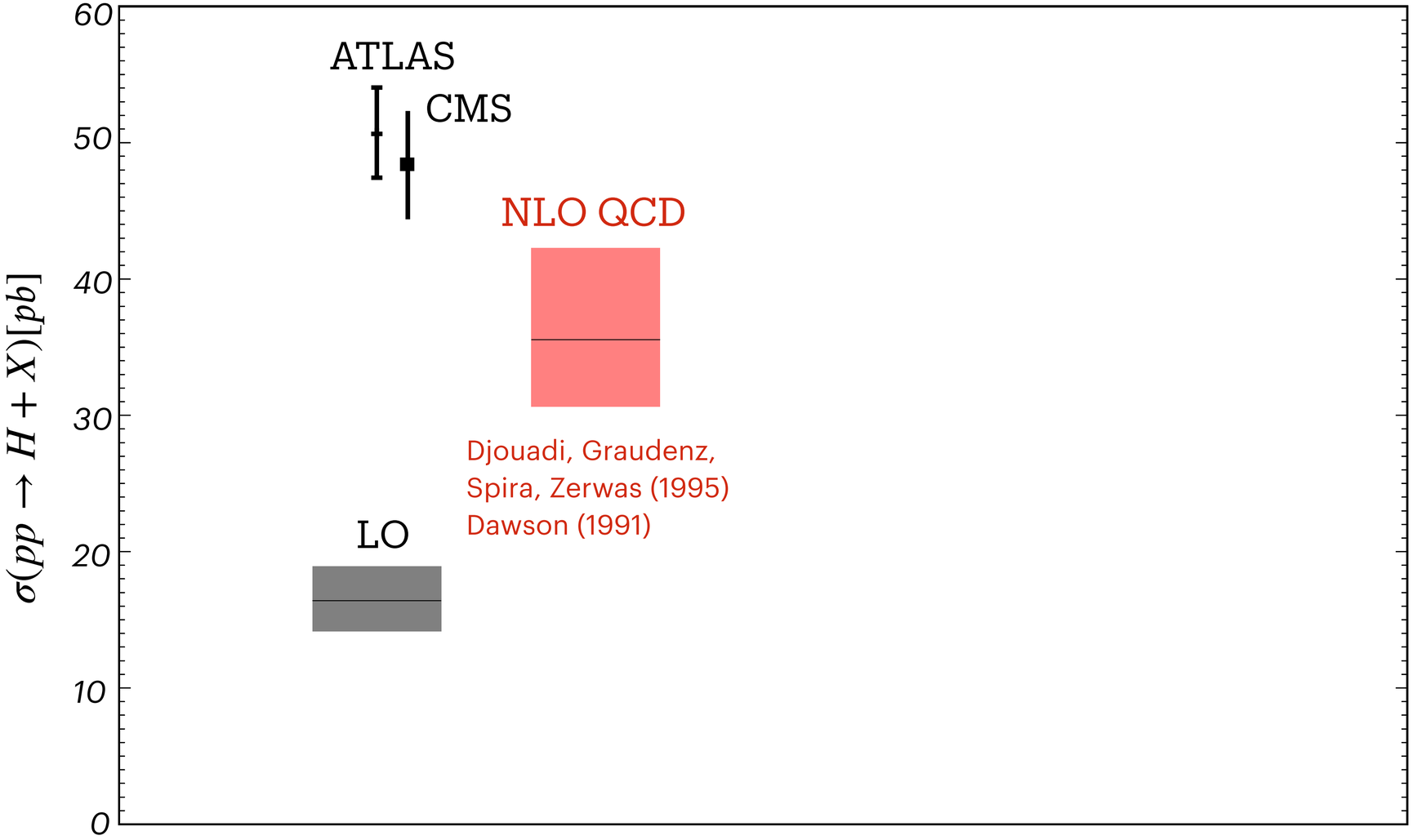
The importance of precision



The hard scattering

Slide from M.Grazzini

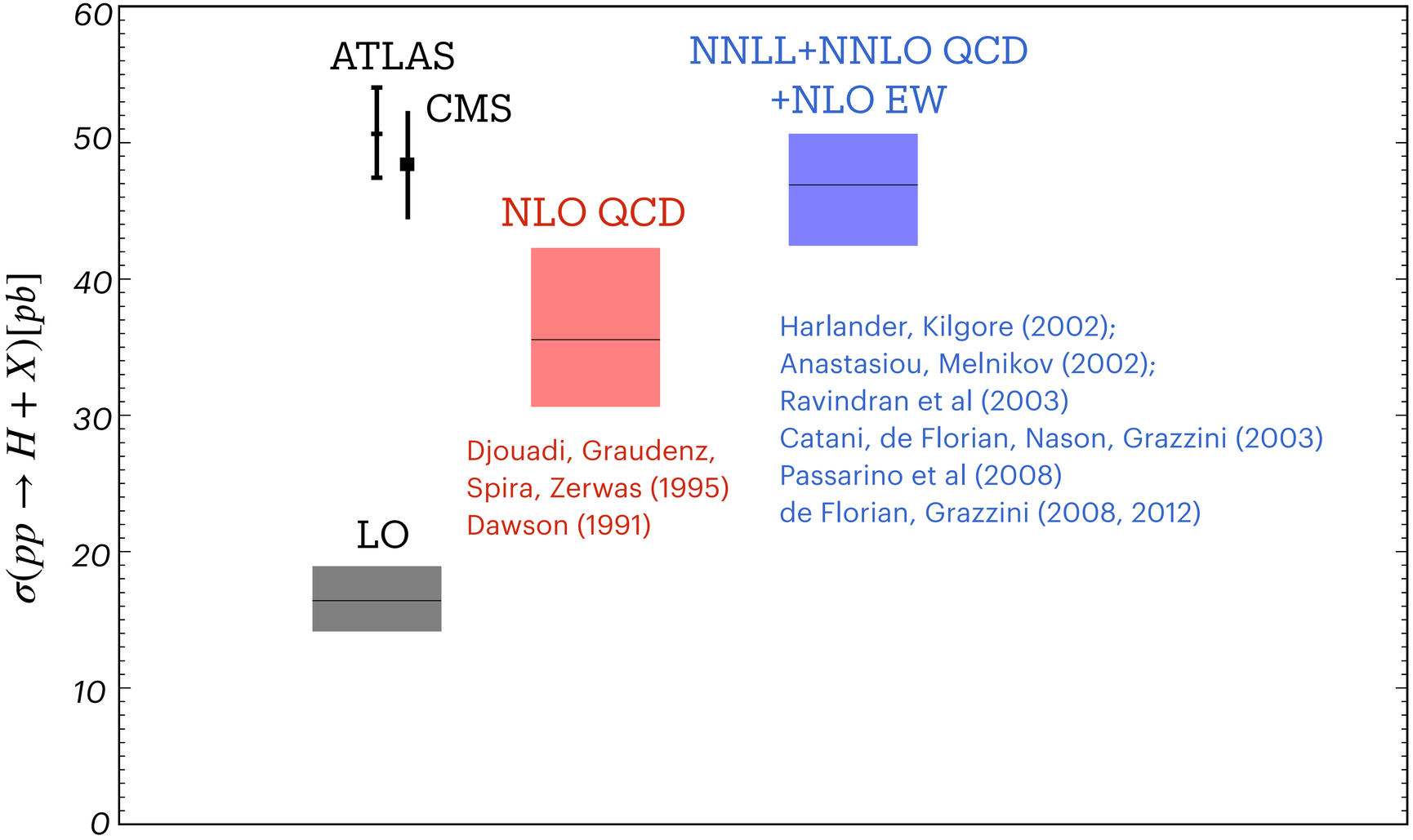
The importance of precision



The hard scattering

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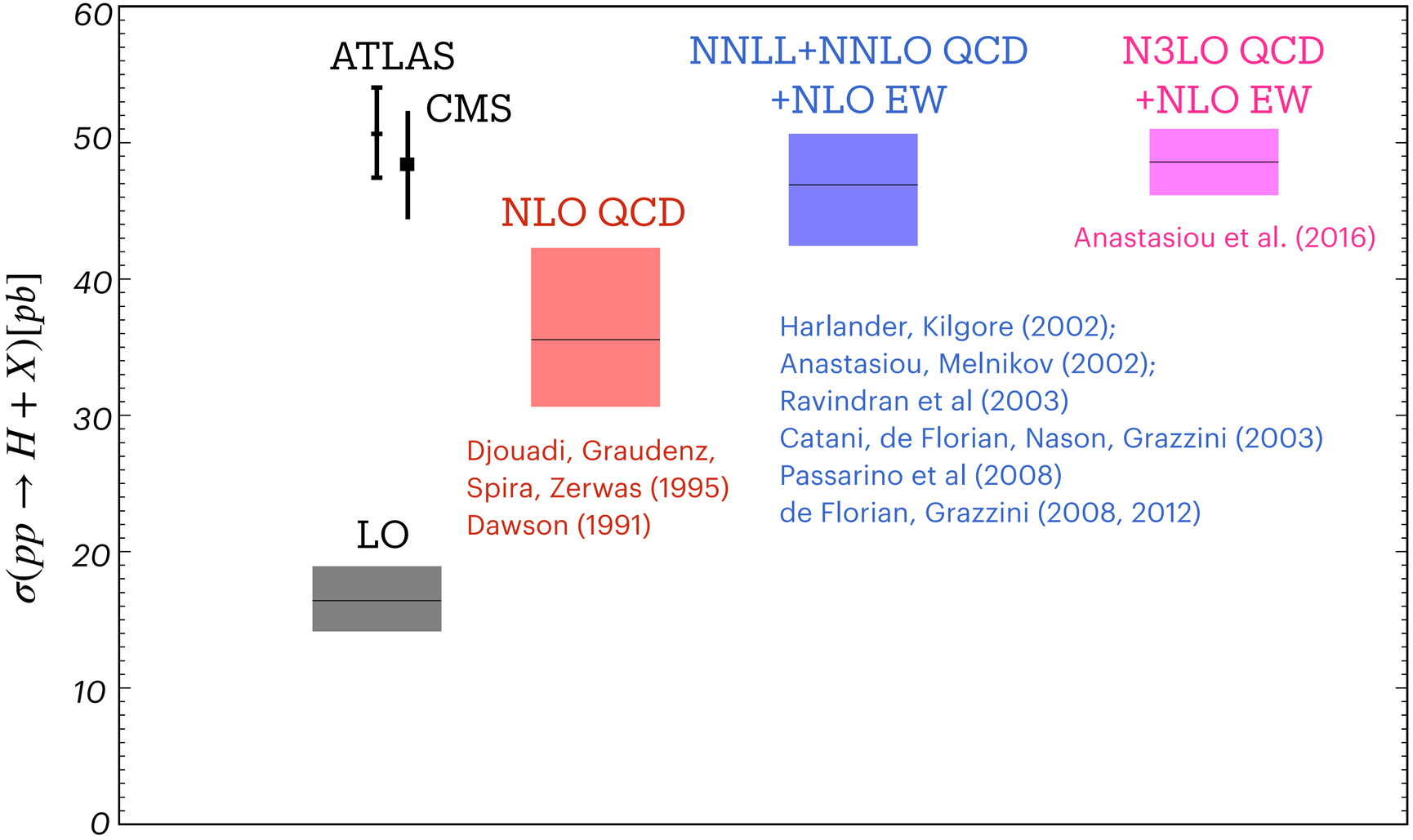
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The importance of precision

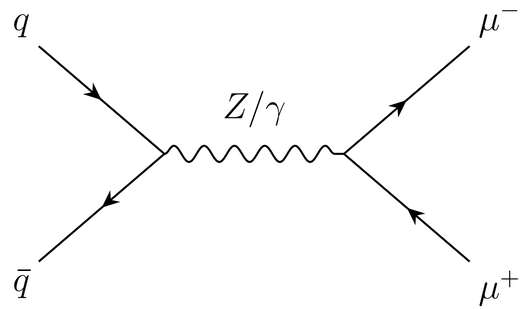


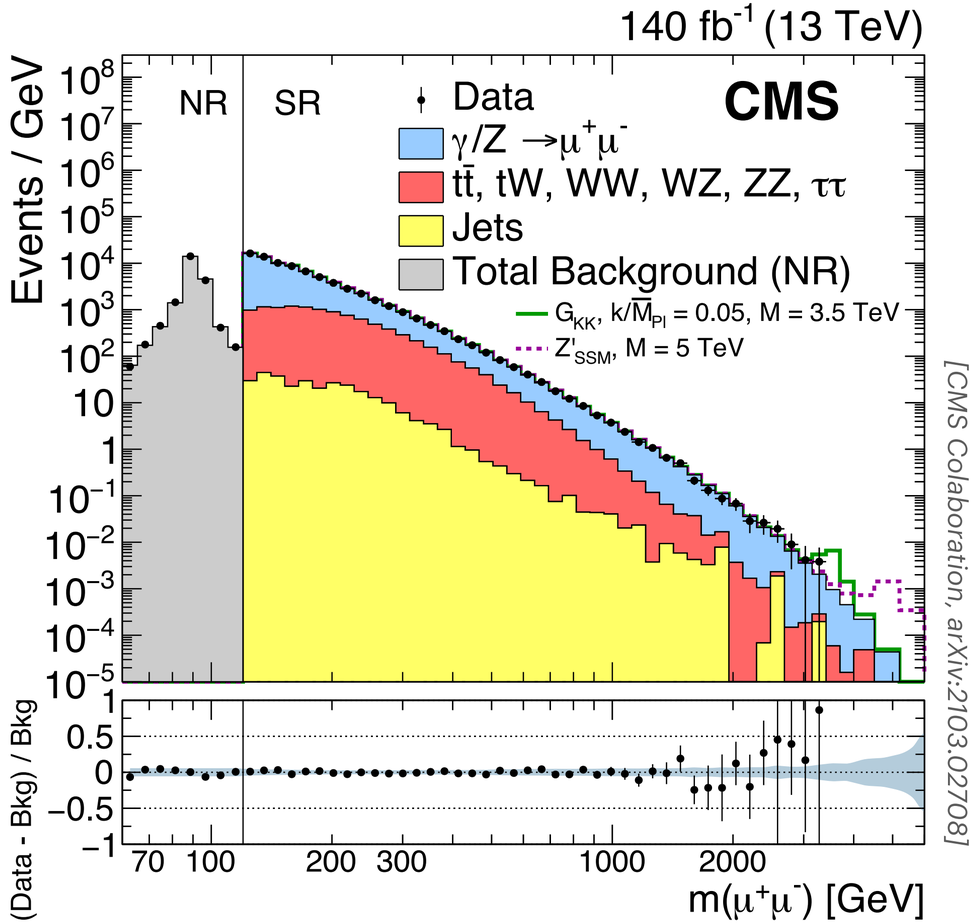
Phenomenology



The Drell-Yan Process

A standard candle





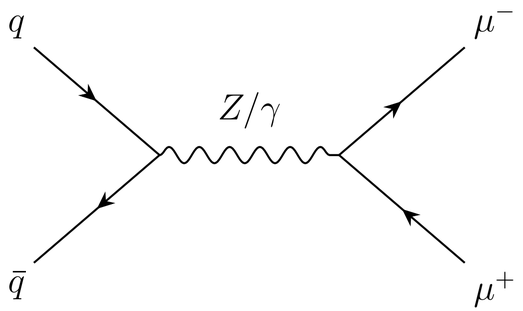
[CMS Collaboration, arXiv:2103.02708]

- **Lepton-pair invariant mass:** at Z resonance: Z properties; AFB: $\sin^2\theta^{\ell}_{eff}$;
- **Lepton-pair transverse mass:** W resonance: m_W;
- **Charged lepton transverse momentum:** m_Z, m_W;
- **Lepton-pair (pseudo-)rapidity:** PDF constraints;
- **Lepton-pair transverse momentum:** low transverse momentum - α_S; large transverse momentum - PDF constraints;
- **Large invariant mass tail:** BSM Signal, PDF constraint.

Higher order corrections

Expansion in the coupling

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$



$$+\ \alpha_S\,\sigma^{(1,0)} \quad + \ \alpha\,\sigma^{(0,1)}$$

$$+\ \alpha_S^2\,\sigma^{(2,0)} \quad + \ \alpha\,\alpha_S\,\sigma^{(1,1)} \quad + \ \alpha^2\,\sigma^{(0,2)}$$

$$+\ \alpha_S^3\,\sigma^{(3,0)} \quad + \ \ldots$$

QCD MIXED EW

Higher order corrections

QCD corrections

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$

$$+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)}$$

$$+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)}$$

$$+ \alpha_S^3 \sigma^{(3,0)} + \ldots$$

- Dominant effect;
- Known up to N3LO.

NLO:
[G.Altarelli, R.Ellis, G.Martinelli Nucl.Phys.B 157 (1979)];

NNLO:
[R.Hamberg, T.Matsuura, W.van Nerveen, Nucl. Phys. B 359 (1991)];
[C.Anastasiou, L.J.Dixon, K.Melnikov, F.Petriello, hep-ph:0306192];
[S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini arXiv:0903.2120];

N3LO:
[C.Duhr, F.Dulat, B.Mistlberger arXiv:2007.13313];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, T.Yang, and H.Zhu arXiv:2107.09085];
[S.Camarda, L.Cieri, G.Ferrera arXiv:2103.04974];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli arXiv:2203.01565];.

Higher order corrections

EW corrections

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$

$$+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)}$$

$$+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)}$$

$$+ \alpha_S^3 \sigma^{(3,0)} + \ldots$$

- Subdominant with respect to QCD: physical counting $\alpha_S^2 \simeq \alpha$;

NLO:

[U.Baur, O.Brein, W.Hollik, C.Schappacher, D.Wackeroth, hep-ph:0108274]; [S.Dittmaier, M.Kramer, hep-ph:0109062];[U.Baur, D.Wackeroth, hep-ph:0405191];

Ongoing work towards NNLO:

[B. Jantzen, J.H.Kühn. A.A.Penin, V.A.Smirnov, hep-ph:0509157]; [R. Bonciani, A. Broggio, S. Di Vita, A. Ferroglia, M. K. Mandal, P. Mastrolia, L. Mattiazzi, A. Primo, J. Ronca, U. Schubert, W. J. Torres Bobadilla, F. Tramontano, arXiv:2106.13179]; [A. Freitas, E.J.Wallace, arXiv:2512.15700]; [T. Armadillo, SD, M. Dradi, A.Vicini, arXiv:2511.20365].

Higher order corrections

Mixed corrections

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$

$$+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)}$$

$$+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)}$$

$$+ \alpha_S^3 \sigma^{(3,0)} + \ldots$$

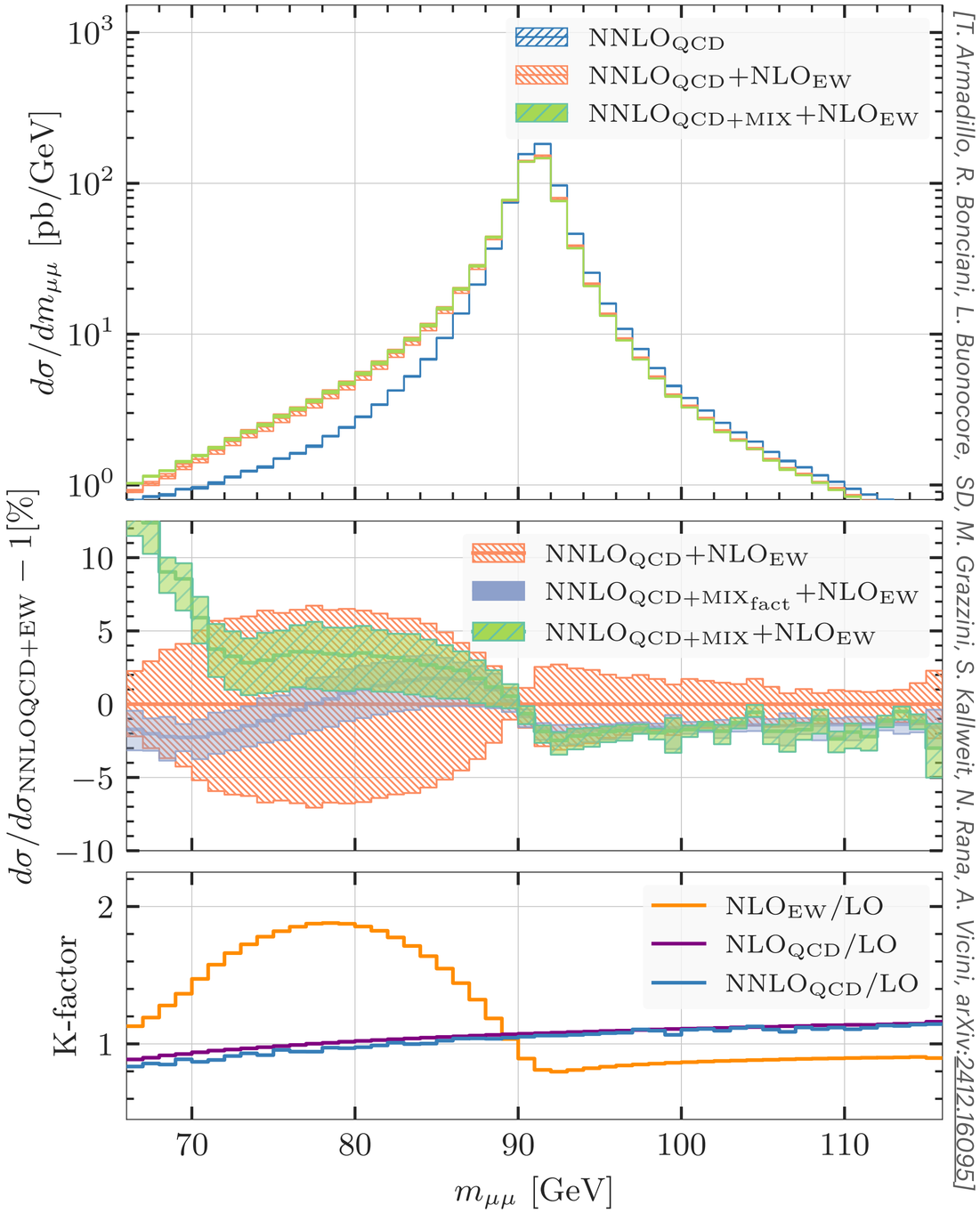
- By physical counting, the expected size is comparable to N^{3}LO QCD.

[L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315]; [L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539]; [R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A.Vicini, arXiv:2106.11953]; [T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A.Vicini, arXiv:2412.16095]; [F. Buccioni, F. Caola, H. Chawdhry, F. Devoto, M.Heller, A. von Manteuffel, K. Melnikov, R. Röntsch, C. Signorile-Signorile, arXiv:2203.11237]

Differential results

Invariant mass distribution

- Mild shape distortion from QCD corrections;
- Strong shape distortion from EW corrections, which generate a radiative tail before the peak;
- The mixed corrections provide a non-trivial shape distortion;
- The factorised approximation does not give a good description of the resonant region.

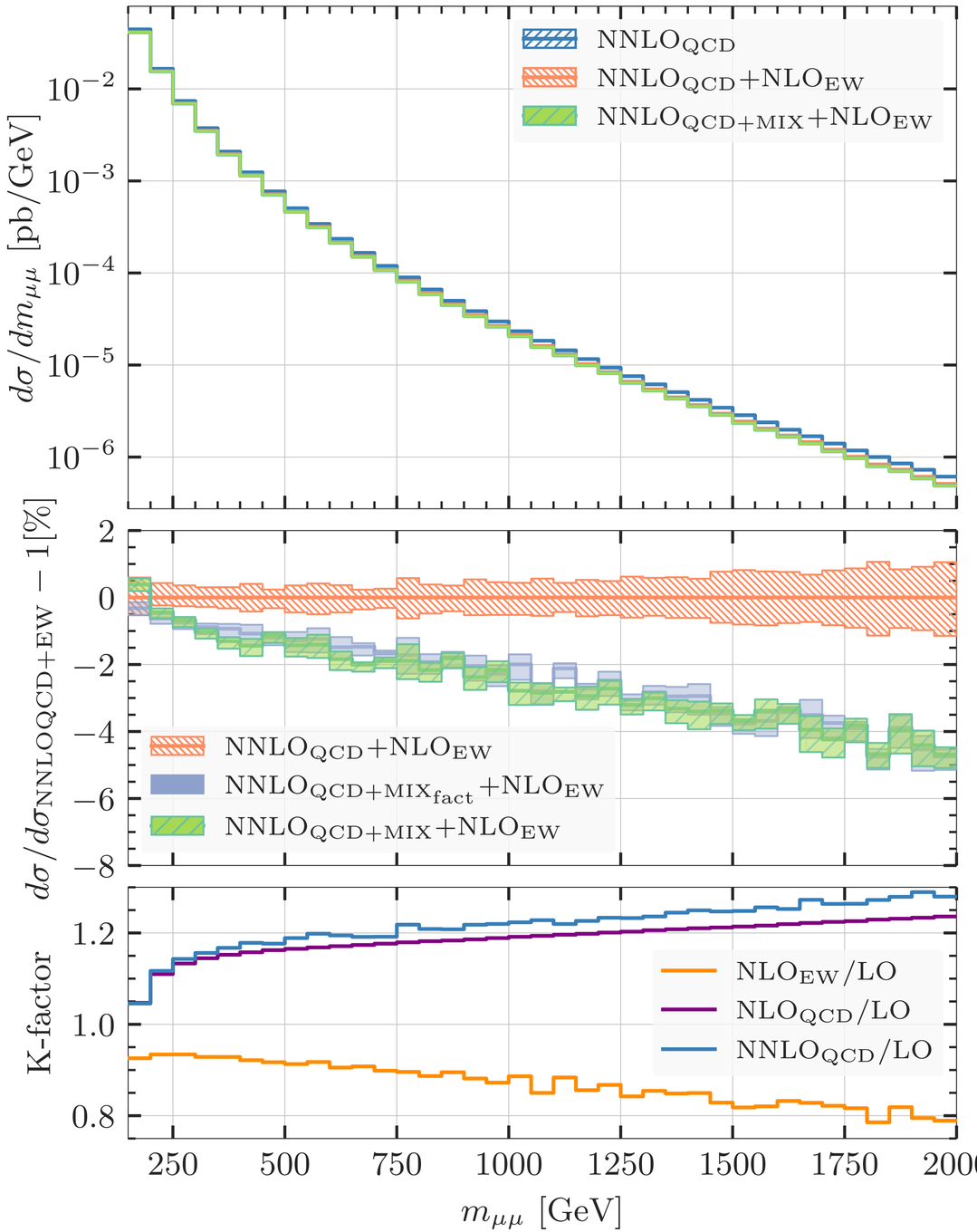


[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

Differential results

Invariant mass distribution

- We examine the high invariant-mass region, following cuts from *[CMS collaboration, arXiv:2103.02708]*: $p_{T,\mu^\pm} > 53\,\text{GeV},\ |y_\mu| < 2.4,\ m_{\mu\mu} > 150\,\text{GeV}$
- The mixed corrections provide a relevant contribution, increasing at higher invariant mass and larger than the statistical error expected by the end of HL-LHC;
- The factorised approximation gives a good description of the high-invariant mass region.



[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

Forward-backward asymmetry

Measuring the weak mixing angle

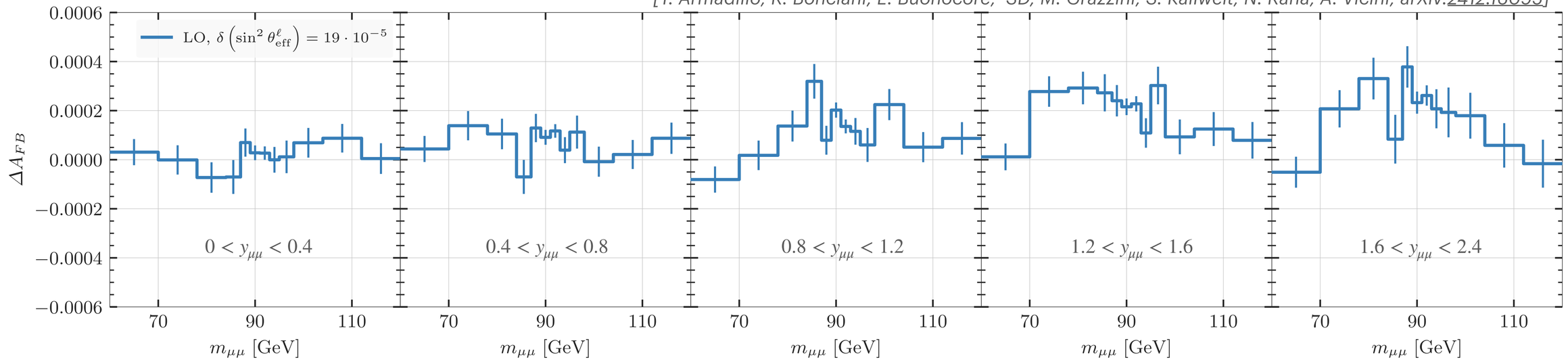
$$A_{FB}(m_{\mu\mu}) = \frac{F(m_{\mu\mu}) - B(m_{\mu\mu})}{F(m_{\mu\mu}) + B(m_{\mu\mu})}$$

$$B(m_{\ell\ell}) = \int_{-1}^{0} d\cos\theta^* \frac{d\sigma}{d\cos\theta^* dm_{\ell\ell}}(m_{\ell\ell})$$

Around the Z resonance, **sensitivity** to $\sin^2\theta^{\ell}_{eff}$!

$$F(m_{\ell\ell}) = \int_{0}^{1} d\cos\theta^* \frac{d\sigma}{d\cos\theta^* dm_{\ell\ell}}(m_{\ell\ell})$$

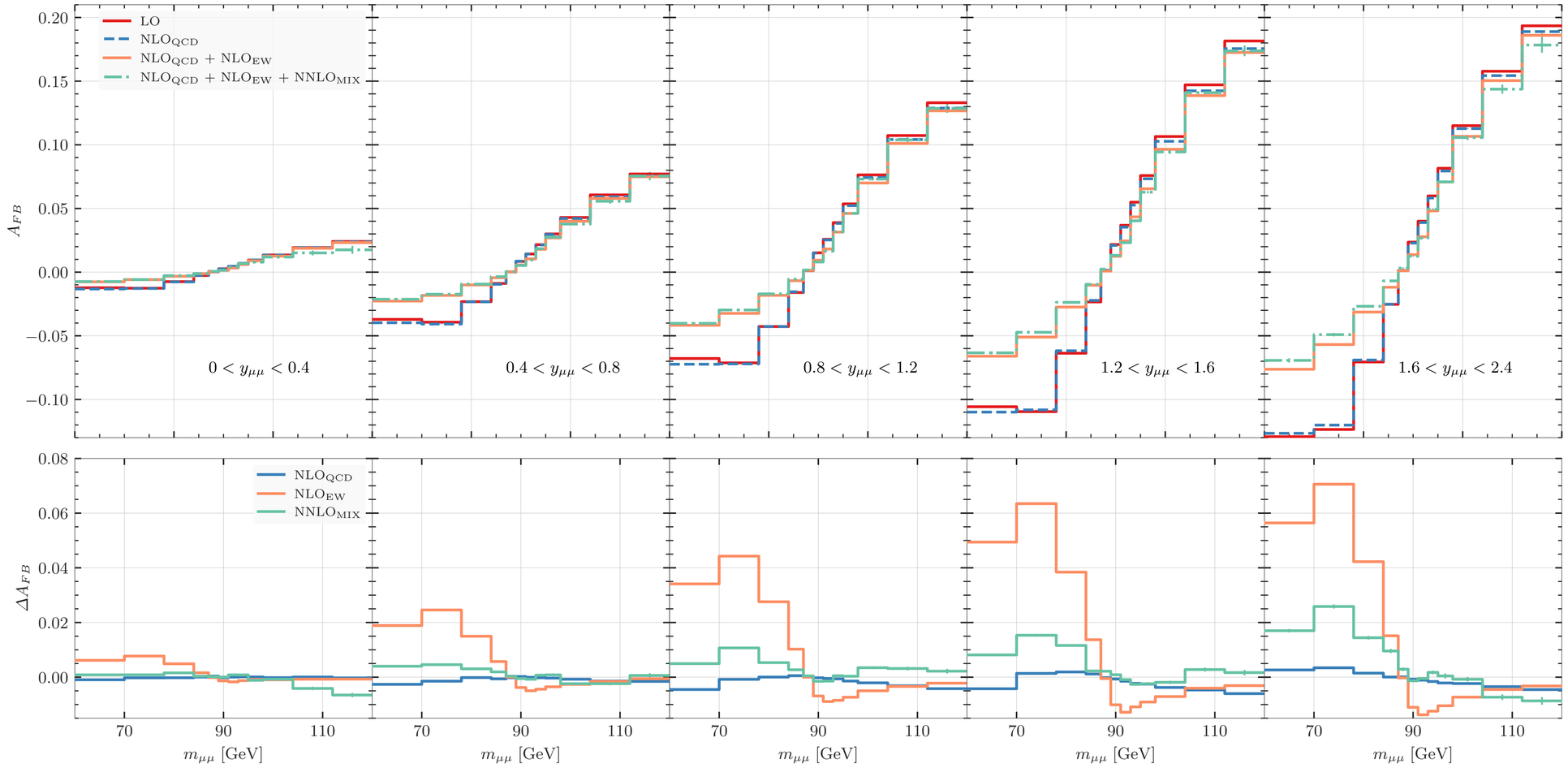
[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]



Forward-backward asymmetry

The effect of mixed corrections

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]



The challenges



Computing an amplitude

Coefficients and integrals

What makes higher-order corrections challenging?

Multi loop amplitudes

Subtraction procedure

Computing an amplitude

Coefficients and integrals

What makes higher-order corrections challenging?

Multi loop amplitudes

Subtraction procedure

- The amplitude can be cast in terms of a sum of algebraic coefficients and integrals:
  - Scalar integrals;
  - Master integrals.

$$\sum_{i=1}^{N} c^{(S,i)} I_{S,\vec{\beta}^{(i)}} \qquad I_{S,\vec{\beta}^{(i)}} = \int \prod_{j=1}^{L} d^n q_j \, \frac{1}{\mathscr{P}_1^{\beta_1^{(i)}} \dots \mathscr{P}_t^{\beta_t^{(i)}} \mathscr{P}_{t+1}^{\beta_{t+1}^{(i)}} \dots \mathscr{P}_N^{\beta_N^{(i)}}}$$

$$\sum_{i=1}^{n} c^{(M,i)} I_{M,\vec{\beta}^{(i)}} \qquad n \ll N$$

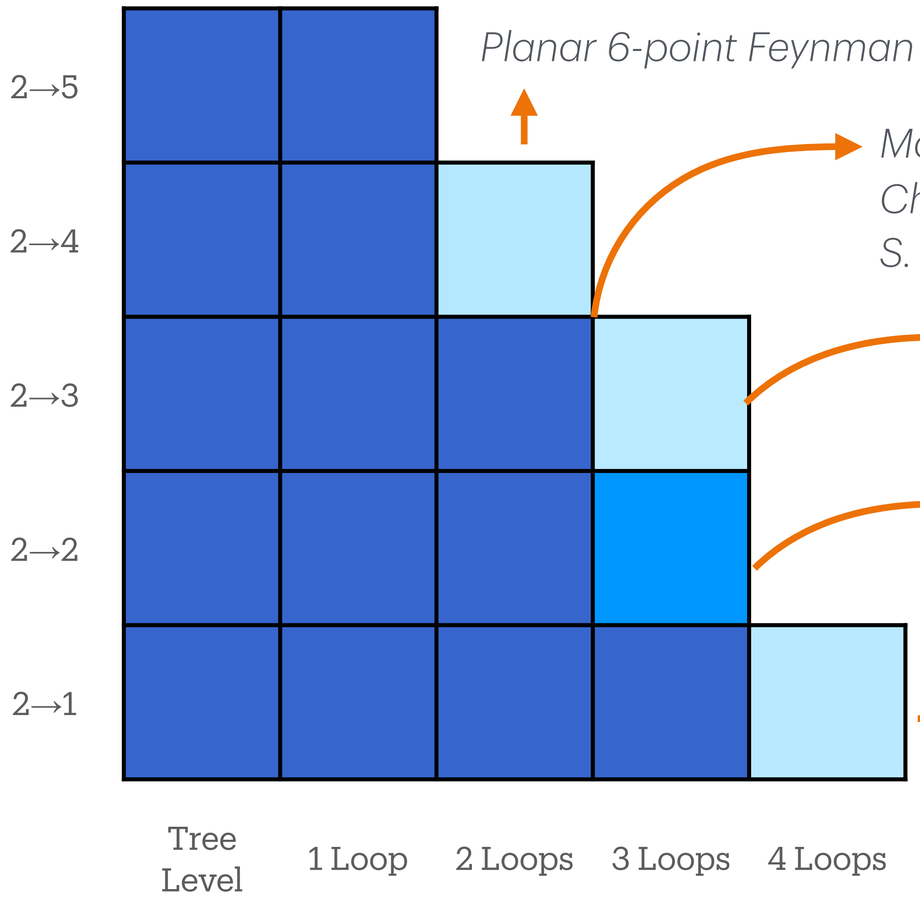
$$\mathscr{A} = \sum_k c_k I_k$$

- Master Integrals are closed under differentiation: deriving with respect to one of the kinematic variables one of the master integrals, we obtain a linear combination of master integrals;
- We can write a system of differential equations satisfied by the master integrals!

$$dI_k = A_{kj} I_j$$

State of the Art

QCD Corrections



Planar 6-point Feynman Integrals: e.g. [S. Abreu, P.F. Monni, B. Page, J. Usovitsch: 2412.19884]; …

Massless available, massive ongoing: e.g. $t\bar{t}W$ *[M. Becchetti, D. Canko, X. Cheng, V. Chestnov, M. Delto, S. Ditsch, T. Peraro, M. Pozzoli, C. Savoini, S. Zoia: 2608.03746]; …*

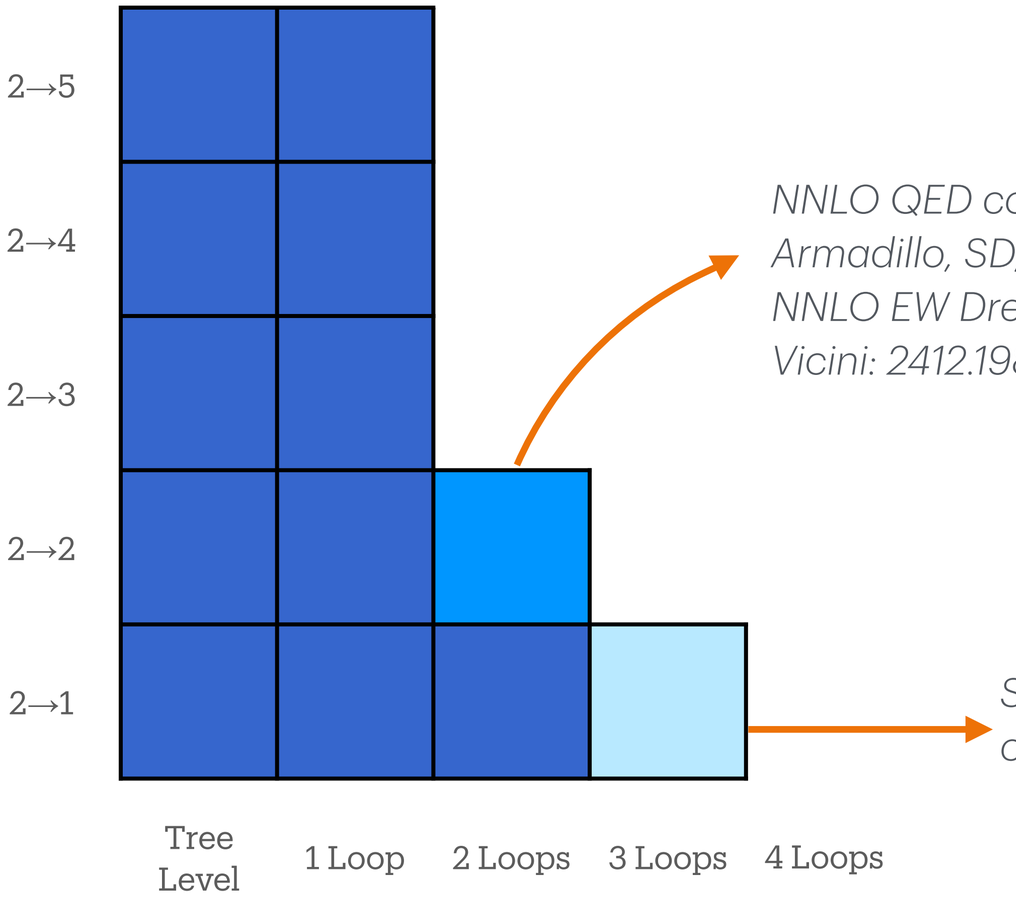
First results for Feynman Integrals [Y. Liu , A. Matijašić, J. Miczajka, Y. Xu, Y. Xu, Y. Zhang: 2411.18697]; …

Selected amplitudes in Leading Colour approximation: e.g. $H+$ *jet [X. Chen, X. Guan, B. Mistlberger: 2504.06490]*

RVVxV contribution to Higgs production [B. Mistlberger, A. Suresh: 2504.10574]

State of the Art

EW Corrections



NNLO QED corrections to Drell-Yan [R. Bonciani et al., arXiv:2106.13179], [T. Armadillo, SD, M. Dradi, A.Vicini, arXiv:2511.20365] ; Fermionic contributions to NNLO EW Drell-Yan [A. Freitas, J. Wallace: 2512.15700],[T. Armadillo, SD, M. Dradi, A. Vicini: 2412.19884].

Selected contributions from the leading fermionic contributions [L. Chen, A. Freitas: 2002.05845]

The Challenges

What makes EW corrections challenging?

- Large computations ($\mathcal{O}(10^4)$ diagrams);
  systematic bookeeping is a priority;
- Treatment of γ_5;

  how can γ_5 be consistently used in dimensional regularisation?
- Additional internal massive lines;
  additional scales in the problem (m_Z, m_W, m_H...) bring additional complications, in particular elliptic integrals;
- Need for the complex mass scheme;
  requires to analytically continue the master integrals on the complex plane!

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OceaNN

Organised **C**alculation of **E**lectroweak **A**mplitudes at **NN**LO

Process definition

$$|\mathscr{M}|^2 = \sum_k c_k I_k$$

c_k=coefficients; I_k=Master Integrals

Numerical grid

FeynArts
Public Mathematica package

Abiss
Private Mathematica package

Kira
With C++ Firefly library

Evaluation of I_k

Differential Equations

Boundaries

Evolution

LITERED
Public Mathematica package

AMFlow
Public Mathematica package

Seasyde
Public Mathematica package

ABISS

Evaluating c_k



- ABISS is a MATHEMATICA package to perform Dirac and Lorentz algebra;



- Output of ABISS:
  - Unpolarized virtual amplitude expressed as a sum of scalar integrals.
- Main Strengths of ABISS:
  - Automation, keeping bookkeeping manageable;
  - Consistent handling of γ_5, allowing for checking independence from the conventions.

Public release coming soon!
STAY TUNED!

Handling γ_5

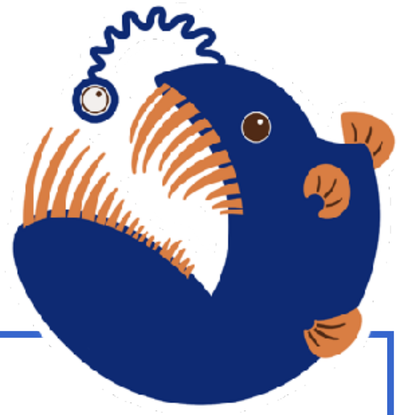
Prescriptions in dimensional regularisation



	't Hooft and Veltmann Nucl. Phys. B 44 (1972) 189–213	Kreimer et al. Phys. Lett. B 237 (1990) 59–62
Anticommutation	✘	✔
Cyclicity	✔	✘
Implementation	• $\bar{\gamma}_\mu \rightarrow \mu = 0,\ldots,3$ • $\hat{\gamma}_\mu \rightarrow (-2\epsilon)$ dimensional • $\{\bar{\gamma}_\mu, \gamma_5\} = 0$; $[\hat{\gamma}_\mu, \gamma_5] = 0$;	• Define a consistent reading point as the "start" of each dirac trace: fixed point prescription;
Comments	• The most mathematically sound approach; • Increased computational load; • Chiral symmetry needs to be restored with ad hoc counterterms; • Non-trivial implementation at higher orders.	• More streamlined computational implementation; • Single traces give different results for different choices of the fixed point; • Sanity check: verify that the final result does not depend on the choice of the fixed point.

HANDLING γ_5

Prescriptions in dimensional regularisation



- For neutral-current Drell-Yan, proven *[M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, arXiv:hep-ph/2012.05918]* that at two loops the two prescriptions yield:
  - different bare scattering amplitudes;
  - same finite UV-renormalised and IR-subtracted amplitudes.

- ABISS automates Kreimer's prescription;
- A consistent implementation of Kreimer's scheme requires:
  - treating $\epsilon_{\mu\nu\rho\sigma}$ as a pure 4-dimensional object;
  - computing higher-dimensional integrals.

$$k^\mu = \bar{k}^\mu + \hat{k}^\mu \qquad g^{\mu\nu} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu} \qquad \bar{g}^{\mu\nu}\bar{g}_{\mu\nu} = 4$$

- Successful automated treatment of 2→2 processes;
- Next Steps: Generalisation to arbitrary processes? Implementation of 't Hooft-Veltmann prescription?

Computing an amplitude

Evaluating the integrals

- The integrals can be computed by following different strategies:

Analytic methods

- Canonical basis: differential equation in dlog form, solution in terms of special functions;
- Internal masses complicate the picture: elliptic functions, Calabi-Yau, …

Numerical methods

- Numerical integration via sector decomposition: pySecDec *[G. Heinrich et al. 2305.19768]* ;
- Numerical integration of a parametric representation (Feynman parameter, Baikov variables, …);
- Numerical solution of the differential equation via series expansion.

How to compute integrals

Strengths and weaknesses of the methods

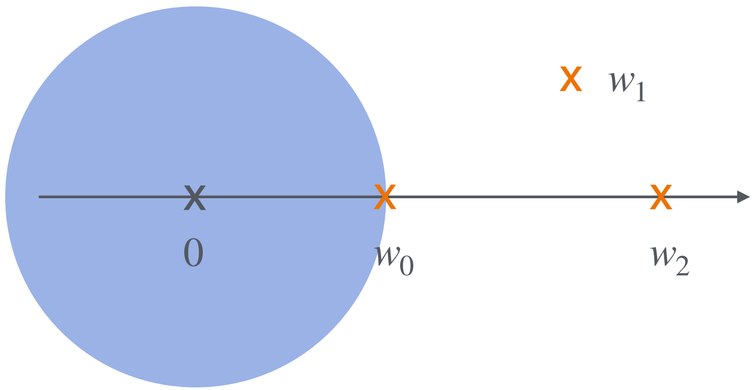
	Analytical Methods	Numerical Methods	Semi-Analytical methods
Functional relations	✔	✘	✘
Analytic properties *(poles location)*	✔ *Mathematical study of function properties.*	✘ *In pySecDec, the pole structure is exploited at the integrand level.*	✔ *Pole structure in the coefficients of the differential equations.*
Numerical evaluation	*Series expansion of analytical functions.*	• *Monte Carlo integration;* • *Numerical quadrature of parametric representations (Feynman, Baikov, ...).*	*Result expressed as series expansion.*
Error esitmation	Estimation through the truncation of the series.	• Monte Carlo: $1/\sqrt{n}$; • Numerical quadrature: comparison between different number of nodes.	Estimation through the truncation of the series.

Series Expansion

Taylor and logarithmic expansion

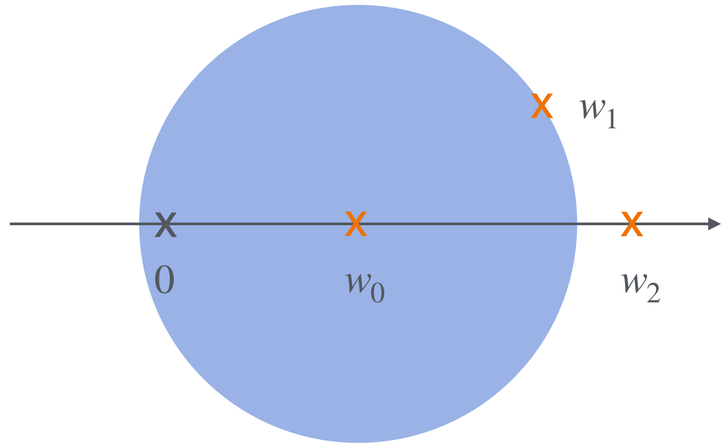
Taylor Ansatz:

$$f(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$



Logarithmic ansatz:

$$f(x) = x^r \sum_{k,j=0}^{\infty} c_{k,j} (x - w_0)^k \log^j(x - w_0)$$

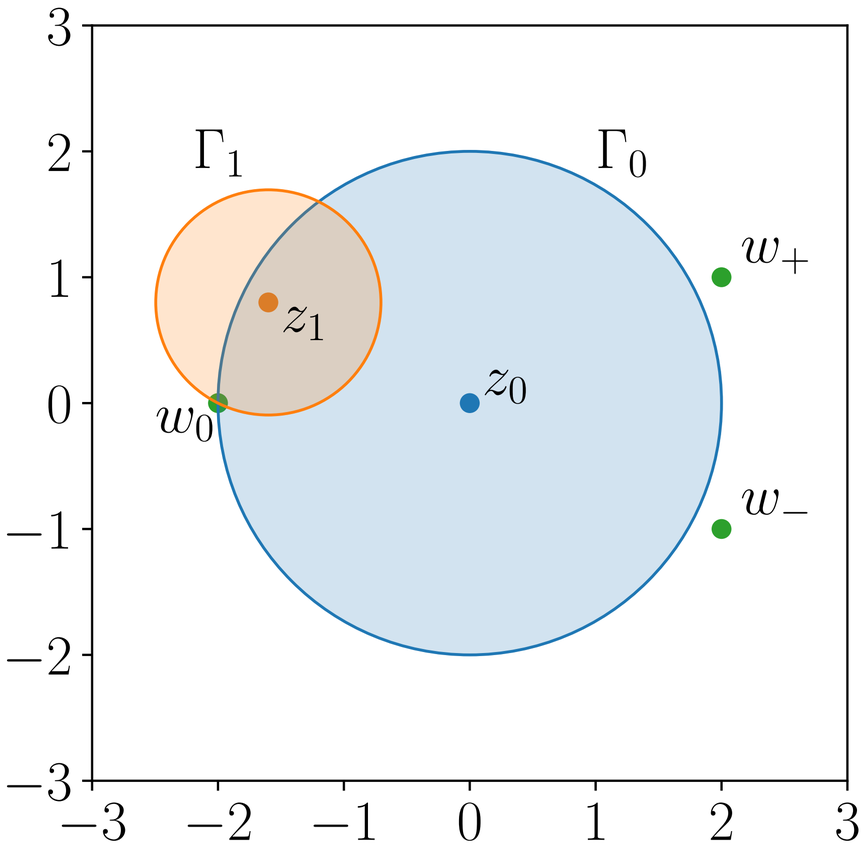


- Original implementation *[F.Moriello, arXiv:1907.13234]* in DIFFEXP, *[M.Hidding, arXiv:2006.05510]*;
- Now, multiple codes are available (SEASYDE *[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345]*, AMFLOW *[R. Huang, X. Liu, Y. Ma , arXiv: 2607.08477]*, LINE *[R. Prisco, J. Ronca, F. Tramontano, arXiv: 2501.01943]*, ...).

SeaSyde

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345]

Moving into the complex plane

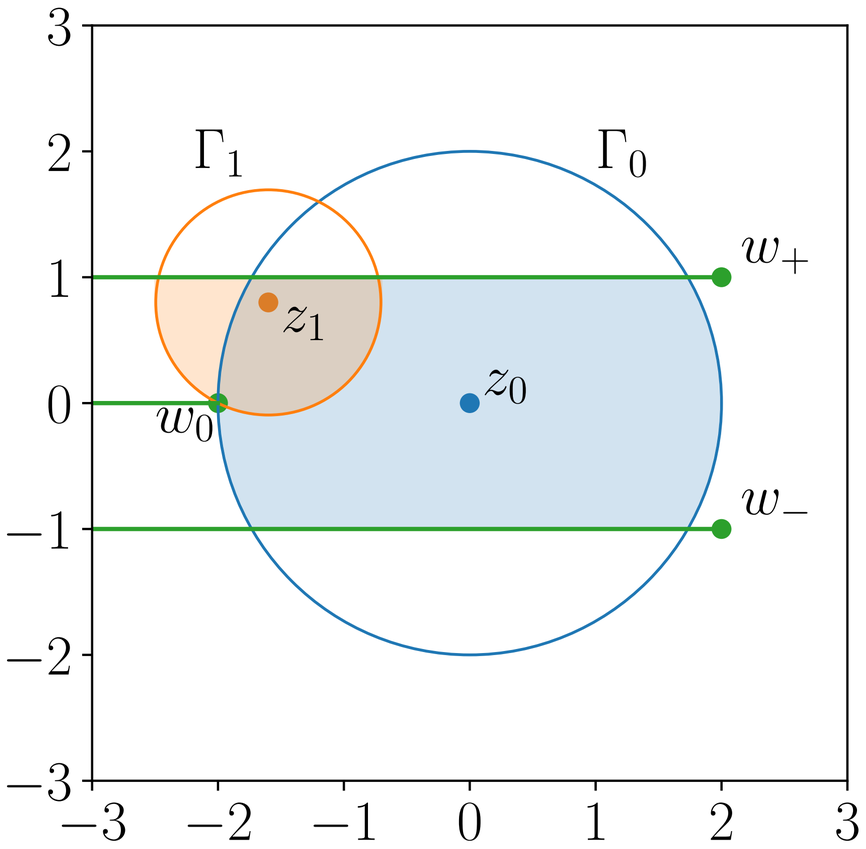


- A **gauge-invariant** definition of the mass can be obtained through the **complex mass scheme**;
- Complex kinematical variables require analytically continuing the solution over the complex plane;
- SeaSyde is a Mathematica package implementing the series expansion approach on **the complex plane**.
- The radius of convergence of the series is limited by the presence of **poles**;
- "Transport" of the boundary conditions needs to consider **branch cuts**.

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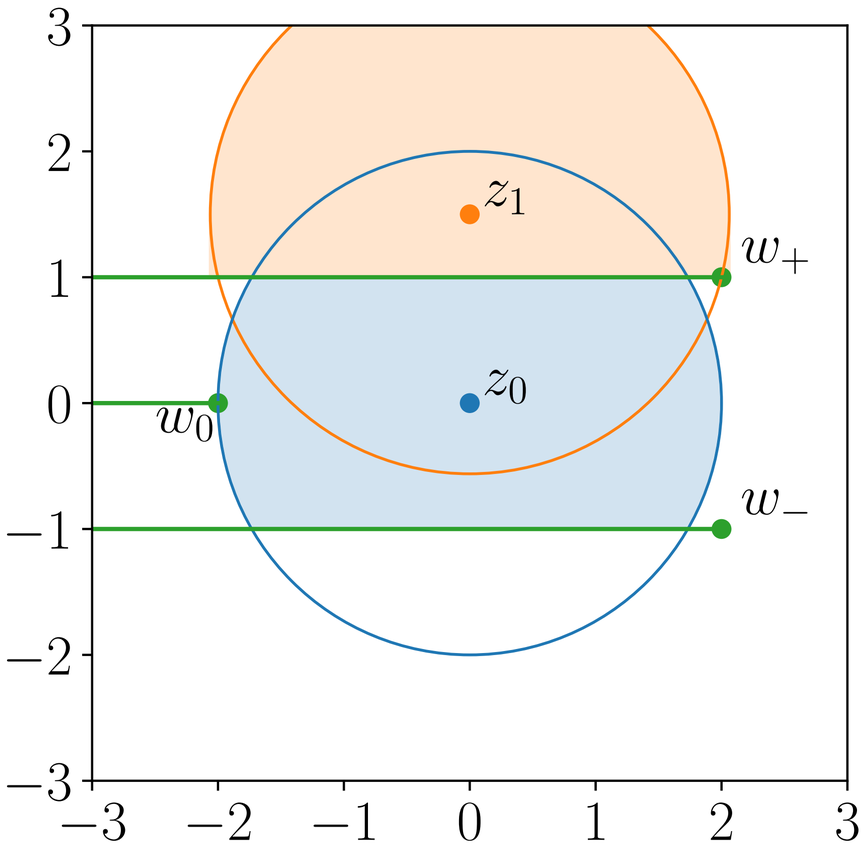


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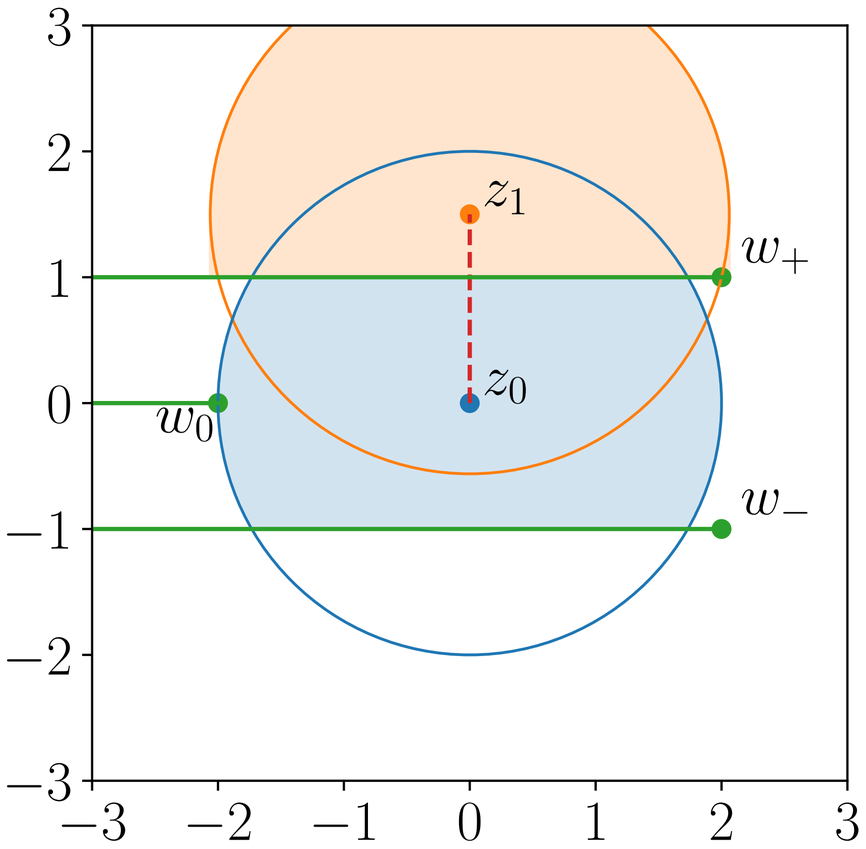


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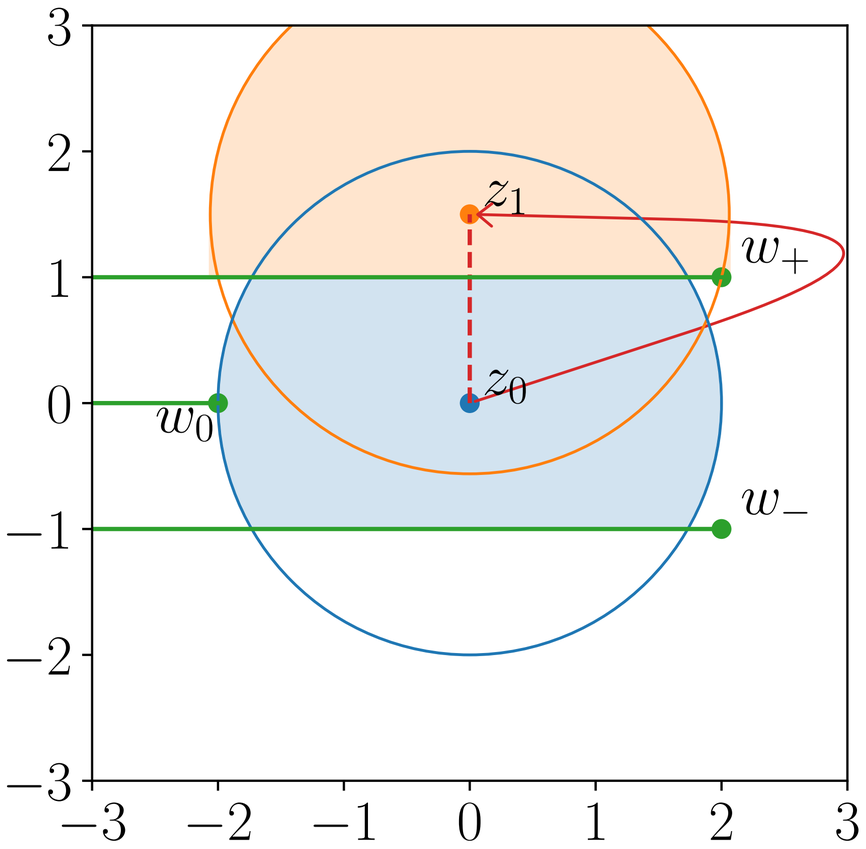


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Towards NNLO EW

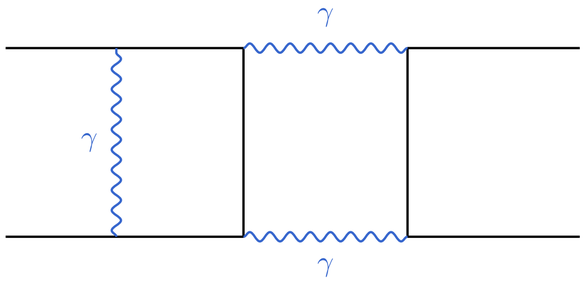


Towards NNLO EW

[R. Bonciani et al., arXiv:2106.13179]

NNLO QED

- QED: easiest subset of EW corrections: no γ_5, no massive gauge bosons;
- Analytic results have been computed in terms of special functions;
- IR structure shared with the full EW calculation;
- IR subtraction operator can be obtained via abelianisation of the corresponding QCD operator:



$$C_F \to Q_{q/e}^2 \qquad C_A \to 0 \qquad T_R n_{l/h} \to \sum_{l/h} N_{C,i} Q_i^2$$

Expansion in the couplings:

$$|\hat{\mathcal{M}}\rangle = 4\pi\hat{\alpha} \sum_{i,j} \left(\frac{\hat{\alpha}_S}{4\pi}\right)^i \left(\frac{\hat{\alpha}}{4\pi}\right)^j |\hat{\mathcal{M}}^{(i,j)}\rangle$$

$$|\mathcal{M}^{(0,0),fin}\rangle = |\mathcal{M}^{(0,0)}\rangle$$

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{I}^{(0,1)} |\mathcal{M}^{(0,0)}\rangle$$

$$|\mathcal{M}^{(0,2),fin}\rangle = |\mathcal{M}^{(0,2)}\rangle - \mathcal{I}^{(0,2)} |\mathcal{M}^{(0,0)}\rangle - \mathcal{I}^{(0,1)} |\mathcal{M}^{(0,1),fin}\rangle$$

Towards NNLO EW

[T. Armadillo, SD, M. Dradi, A.Vicini, arXiv:2511.20365]

NNLO QED

$s=10000\ \text{GeV}^2$; $t=-2500\ \text{GeV}^2$; $\mu_R=m_Z$.		
ε^{-4}	0.43895747601285225040892043215096	0
	-0.43895747601285225040892043215096	
	0	
ε^{-3}	-50.507936715422813794673578353212	0
	50.507936715422813794673578353212	
	0	
ε^{-2}	729.67339746440682380540184060883	0
	-1334.1794138782263523717542814259 2	
	604.50601641381952856635244081709	
ε^{-1}	26528.662344660489566812058589673 5	0
	-463.09023418725937944723920907369	
	-26065.572110473230187364819380599 86	
ε^{0}	401847.44734513081272965403938263 52	97243.871577620920355894313041307 0
	4376.81276799416282425402258394 02	
	-308980.38853550405519801374892526 8	

- Semi-analytic calculation to prove automation OceaNN;
- Main methodological choice: arbitrary numerical precision at each step of the calculation;
- Control of the cancellation of the poles with 35-digit precision;
- Successful comparison with analytic results.

$$\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,2)} \rangle - \mathcal{F}^{(0,1)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,1)} \rangle - \mathcal{F}^{(0,2)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,0)} \rangle$$



Towards NNLO EW

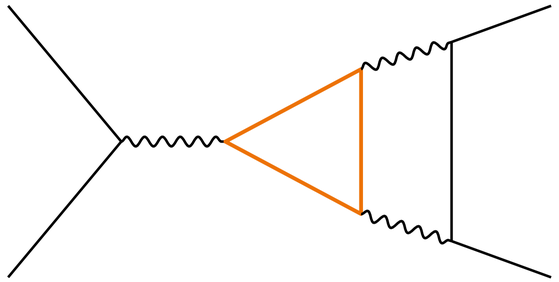
[T. Armadillo, SD, M. Dradi, A. Vicini: 2412.19884]

NNLO EW: Fermionic Contribution



s=10000 GeV2; t=-2500 GeV2; μR=mZ.		
ε^{-3}	-35.4668	0
	35.4672	
	0	
ε^{-2}	1856.03 + 374.962 i	0
	-1207.81 + 241.417 i	
	-648.21 - 616.378 i	
ε^{-1}	-35083.52 - 4774.08	0
	9463.56 - 5495.62 i	
	25620.07 + 10269.72 i	
ε^{0}	-523472.8 + 82010.2 i	-454654.8 + 118286.9 i
	32550.2 - 16686.4 i	
	36267.7 + 52963.0 i	

- Fermionic contribution: gauge-invariant subset with at least one closed fermion sub-loop;



- Recently computed with numerical methods *[A. Freitas, J. Wallace: 2512.15700]*
- Alternative calculation within OceaNN;
- Calculation performed with the muon mass regulating collinear divergences;
- Cancellation of the poles up to m_μ^2 power corrections

$$\langle \mathcal{M}^{(0,0)} \,|\, \mathcal{M}^{(0,2)} \rangle - \mathcal{F}^{(0,1)} \langle \mathcal{M}^{(0,0)} \,|\, \mathcal{M}^{(0,1)} \rangle - \mathcal{F}^{(0,2)} \langle \mathcal{M}^{(0,0)} \,|\, \mathcal{M}^{(0,0)} \rangle$$

Summary

- Higher-order EW corrections are needed to fully exploit the scientific potential of HL-LHC and FCC-ee;
- Multiloop EW corrections are the main challenge;
- We developed OceaNN, a framework which relies on Abiss for the evaluation of rational coefficients and SeaSyde for the evaluation of the Master Integrals;
- We successfully applied our framework to the computation of QED corrections and the fermionic contribution to the Drell-Yan process.
- On our way to meeting the requirements of HL-LHC?



Summary

- Higher-order EW corrections are needed to fully exploit the scientific potential of HL-LHC and FCC-ee;
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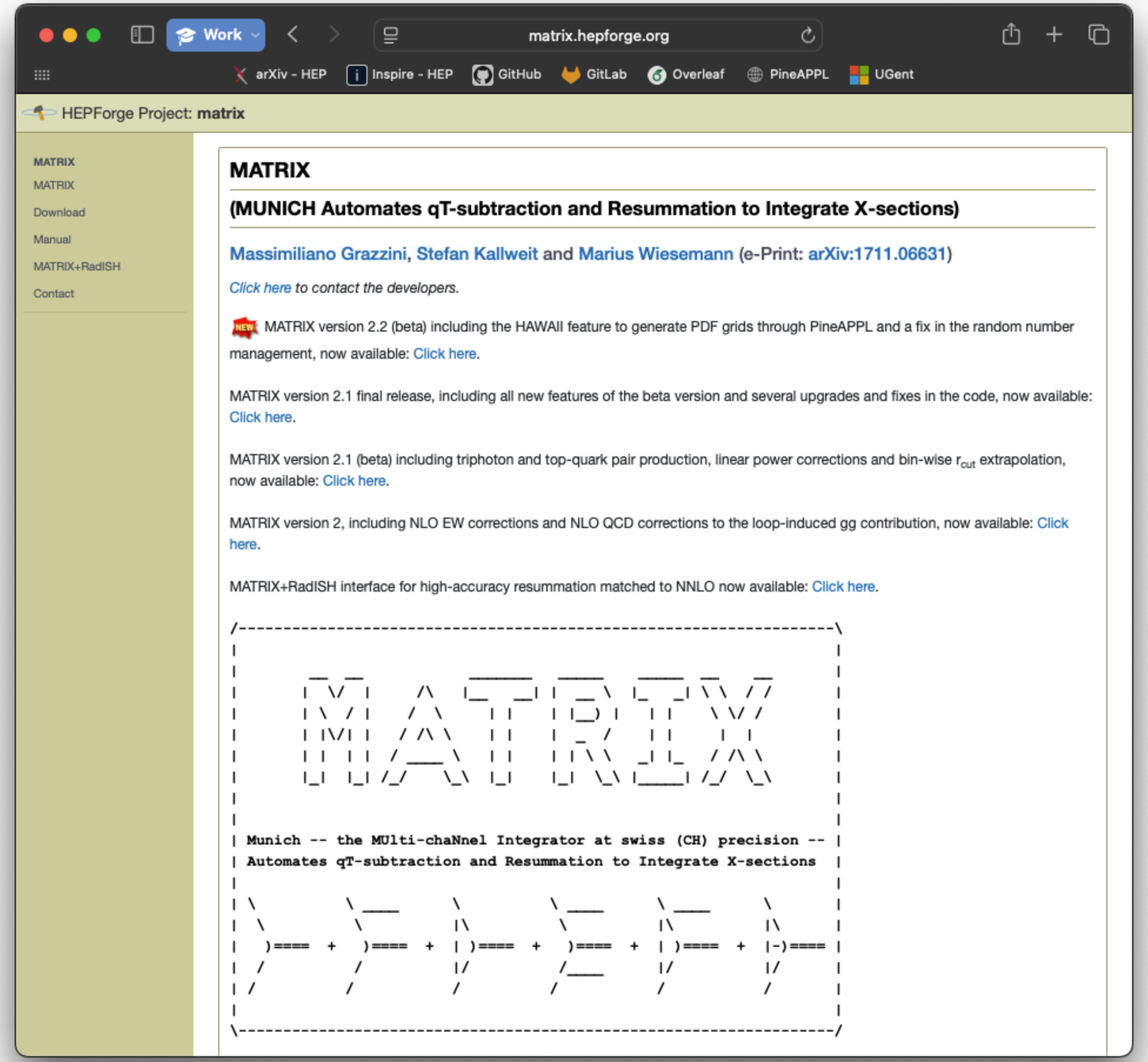
Thank you!



Backup slides

MATRIX

- MATRIX *[M. Grazzini, S. Kallweit, M. Wiesemann: 1711.06631]* allows the user to evaluate fully differential cross sections for a wide class of processes at hadron colliders in NNLO QCD and NLO EW;
- New: support for the generation of interpolation grids for the processes via PINEAPPL (MATRIX HAWAII) *[SD, T. Ježo, S. Kallweit, C. Schwan: 2506.14486]*



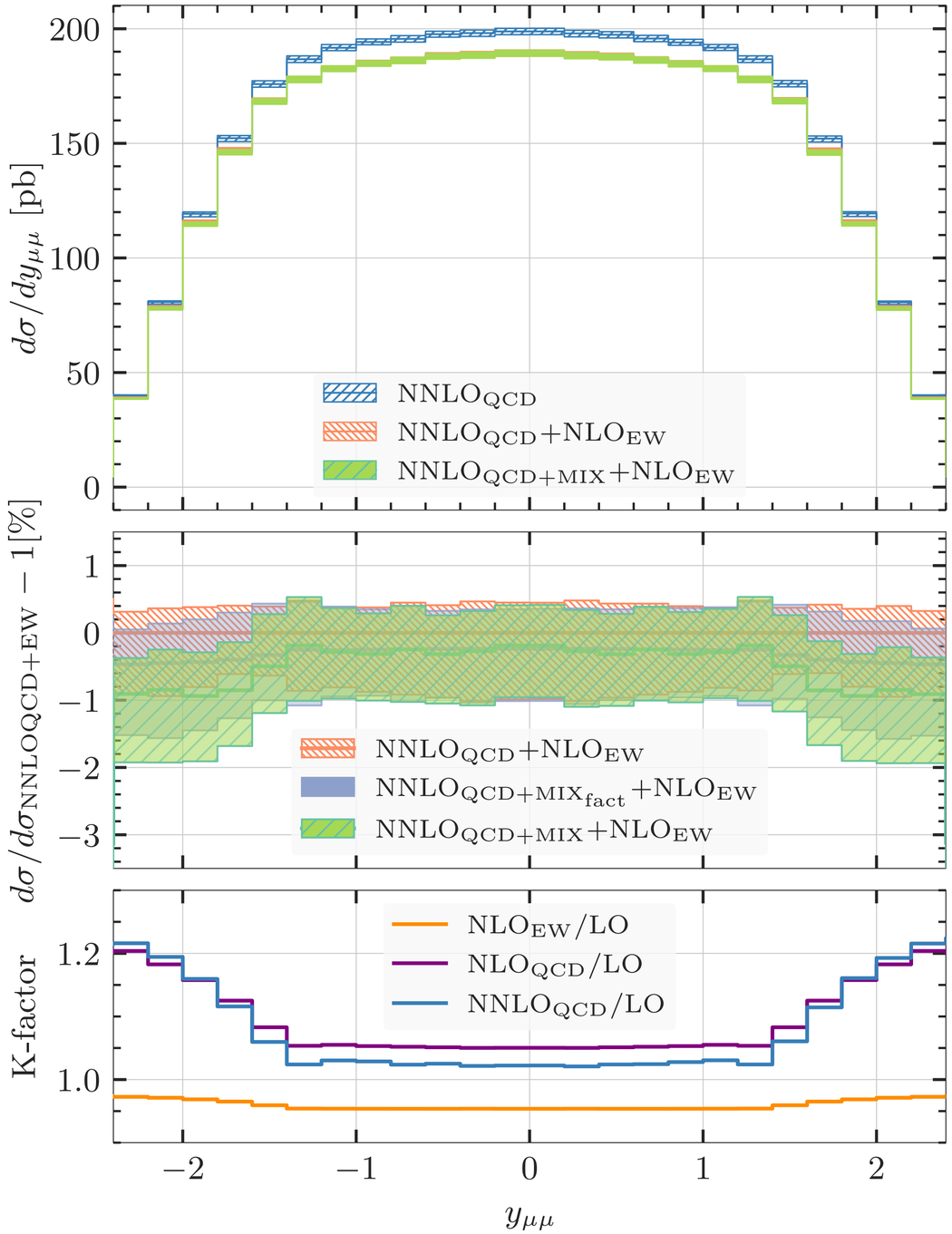
Differential results

Rapidity distribution

- We compare exact results MIX with the factorised approximation $\mathrm{MIX}_{\mathrm{fact}}$

$$\frac{d\sigma^{(1,1)}_{\mathrm{fact}}}{dX} = \left(\frac{d\sigma^{(1,0)}}{dX}\right) \times \left(\frac{d\sigma^{(0,1)}_{q\bar{q}}}{dX}\right) \times \left(\frac{d\sigma_{\mathrm{LO}}}{dX}\right)^{-1} ;$$

- We observe a shape distortion: the strongest effect comes from NNLO QCD, but it is emphasised by the mixed corrections;
- Effect of the mixed corrections up to the $\mathcal{O}(1\%)$ level;
- Shape distortions are relevant for PDF determinations.



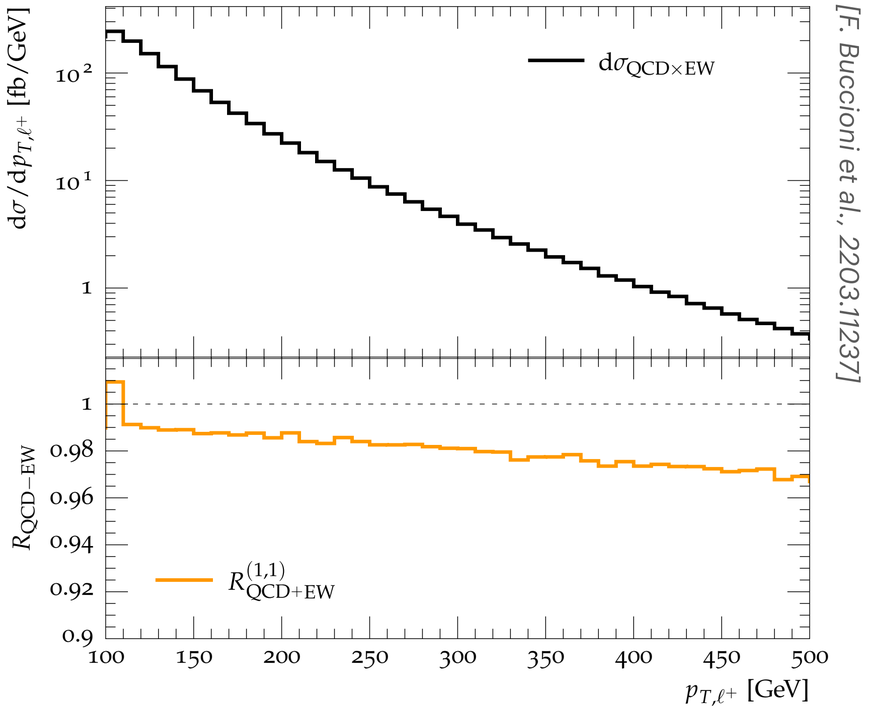
Mixed QCD-EW Corrections

[T. Armadillo, et al, arXiv:2412.16095]
[F. Buccioni et al., arXiv:2203.11237]

Neutral Current Drell-Yan

- Two fully independent calculations: massive bare leptons and semi-analytic amplitudes *[T. Armadillo, et al., arXiv:2412.16095]*; massless dressed leptons and analytic amplitudes *[F. Buccioni et al., arXiv:2203.11237]*.
- Dressing the massive leptons allowed for comparison, finding excellent agreement: differences are due to the lepton mass and are negligible for fiducial cross sections.

	T. Armadillo et al.	F. Buccioni et al.
Final State Leptons	Massive	Massless
Subtraction Method	q_T subtraction	Nested Subtraction
2-loop Amplitudes	Semi-Analytic	Analytic
$\sigma^{(1,1)}(qq)$ [pb]	-15.7(7)	-16.80
$\sigma^{(1,1)}(qg)$ [pb]	2.15(7)	2.07
$\sigma^{(1,1)}(q\gamma)$ [pb]	-0.231(9)	-0.207
$\sigma^{(1,1)}(g\gamma)$ [pb]	0.266(1)	0.2668

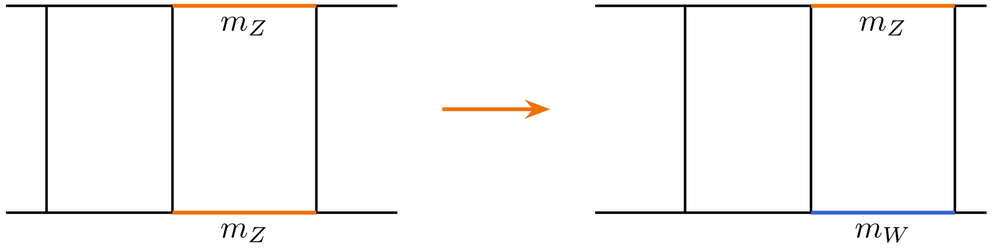


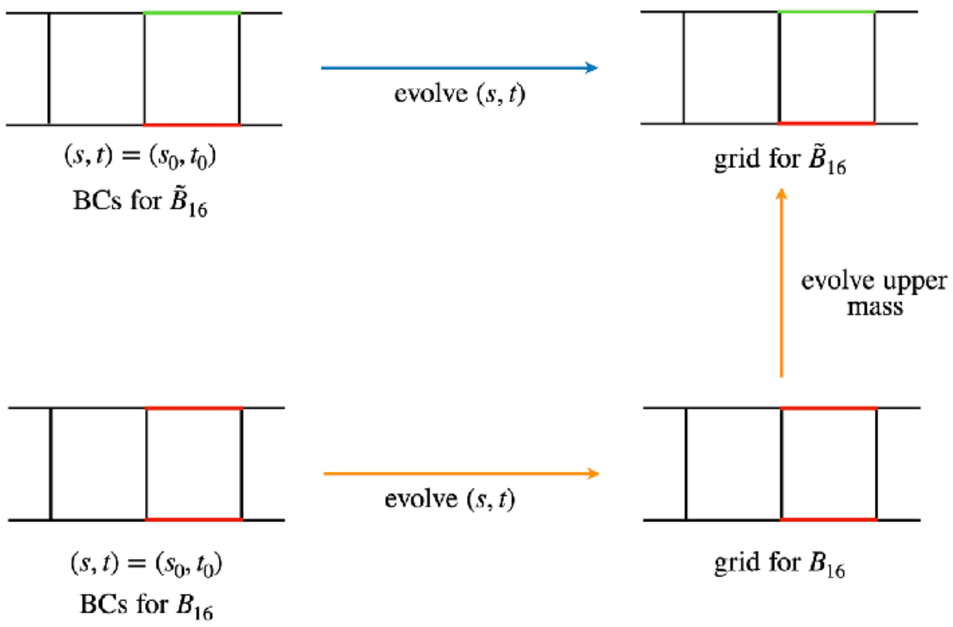
Mixed QCD-EW Corrections

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2405.00612]

Charged Current Drell-Yan

- Computationally similar to neutral-current Drell-Yan;
- Extra complexity comes from new diagrams with two different internal massive lines;
- At the moment, only semi-analytical results are available.





- Numerical grid for the amplitude computed with SEASYDE;
- Possible approaches for the evolution:
  - Compute the boundary conditions with AMFLOW; use the differential equations in s and t to build the grid;
  - Use the neutral-current Drell-Yan grid as a boundary condition; use the differential equation in one of the masses;
- The differential equation in the mass allows keeping a dependence on the W boson mass.

HANDLING γ_5

Evaluation of (-2ϵ)-dimensional integrals



$$k^\mu = \bar{k}^\mu + \hat{k}^\mu \qquad g^{\mu\nu} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu} \qquad \bar{g}^{\mu\nu}\bar{g}_{\mu\nu} = 4$$

- The introduction in the numerator of a (-2ϵ)-dimensional scalar product can be obtained by introducing the Schwinger parametrisation and differentiating with respect to the appropriate Schwinger parameter.

  [M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, arXiv:hep-ph/2012.05918]

$$J_N^{(L)} = (\mu^{D-4})^L \int \prod_{r=1}^{L} d^D q_r \prod_{j=1}^{N} \frac{1}{(k_j^2 - m_j^2)^{\nu_j}}$$

$$\frac{1}{A^\nu} = \frac{1}{\Gamma(\nu)} \int_0^\infty d\alpha\, \alpha^{\nu-1} e^{-\alpha A}$$

$$= \frac{(i\pi^{D/2}\mu^{D-4})^L}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_{\alpha_j \geq 0} d^N\vec{\alpha} \left(\prod_{j=1}^{N} (-1)^{\nu_j} \alpha_j^{\nu_j - 1} \right) [\mathcal{U}(\vec{\alpha})]^{-D/2} \exp - \frac{\mathcal{F}(\vec{\alpha})}{\mathcal{U}(\vec{\alpha})}$$

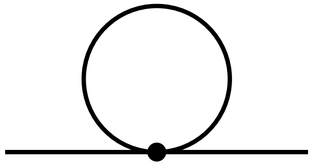
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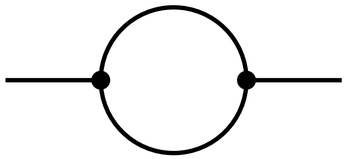
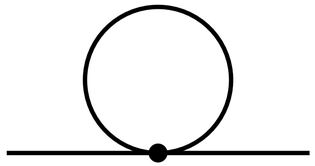
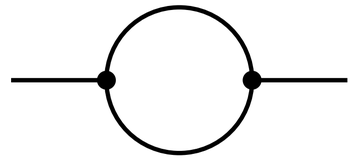
Integration through differentiation

- Master Integrals are **closed under differentiation**: by deriving with respect to one of the kinematic variables one of the master integrals, we obtain alinear combination of master integrals;
- We can write a **system of differential equations** satisfied by the master integrals!

An example: 

; Masters: {  ,  } .

$$\frac{d}{dp_1^2}$$  $$= 0$$

$$\frac{d}{dp_1^2}$$  $$= \frac{d-2}{p_1^2(4m^2-p_1^2)}$$  $$-\frac{(d-4)p_1^2+4m^2}{2p_1^2(4m^2-p_1^2)}$$ 

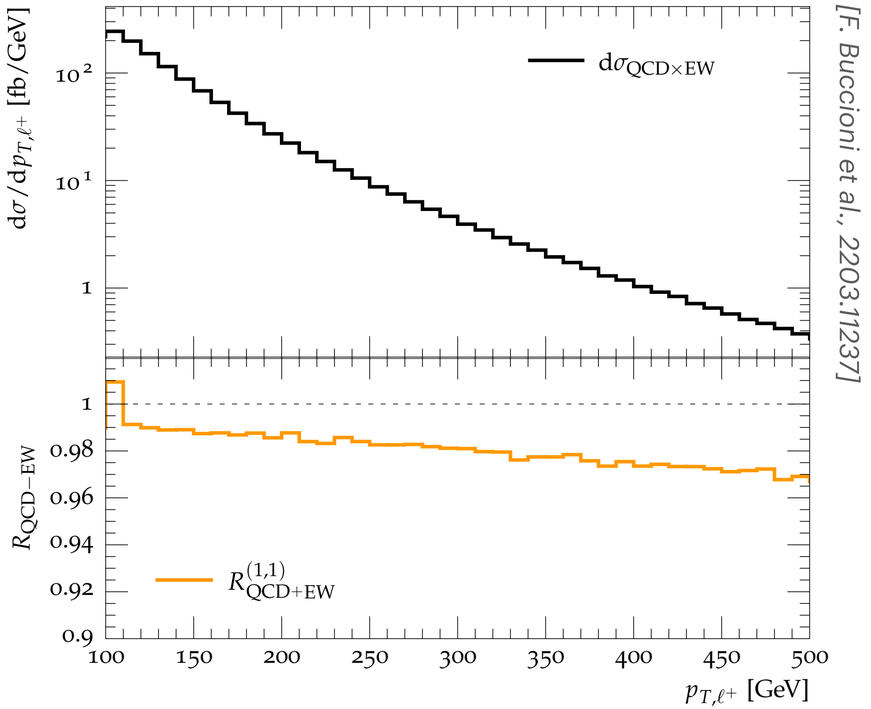
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AMFlow

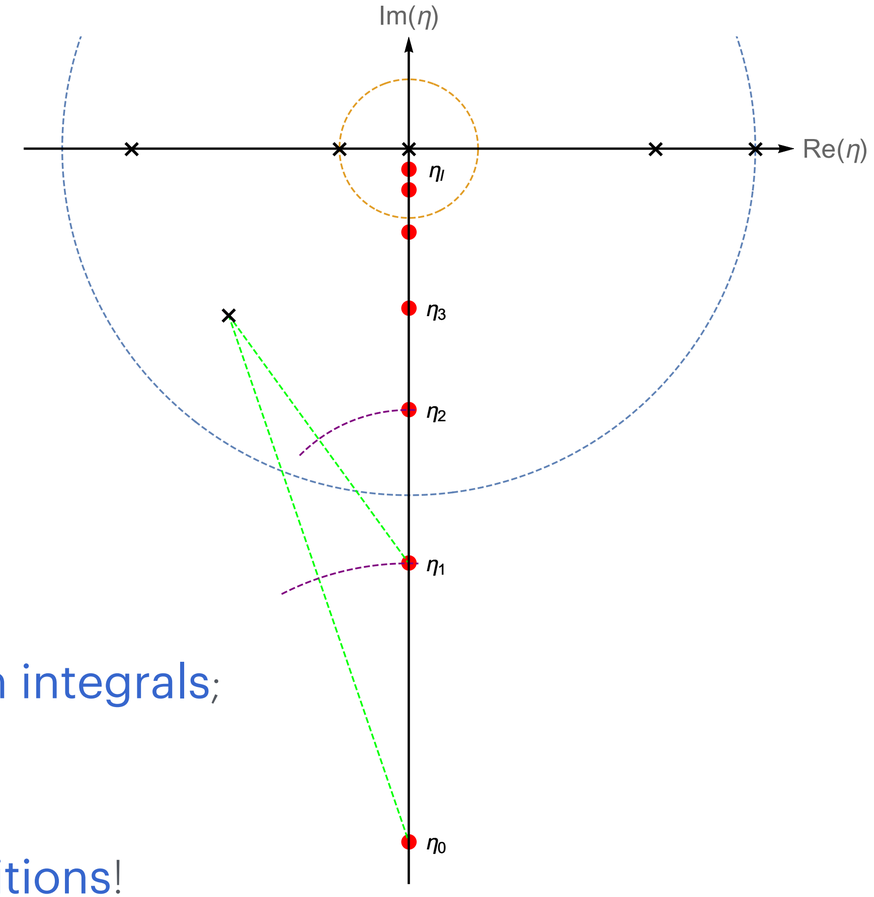
[R. Huang, X. Liu, Y. Ma , arXiv: 2607.08477]
[X. Liu and Y.-Q. Ma, arXiv: 2201.11669]

No need to compute boundaries

- Introduce an auxiliary mass parameter η:

$$I(\vec{\nu},\vec{s},\epsilon)=\int\prod_{i=1}^{L}\frac{\mathrm{d}^D\ell_i}{\mathrm{i}\pi^{D/2}}\frac{\mathscr{D}_{K+1}^{-\nu_{K+1}}\cdots\mathscr{D}_N^{-\nu_N}}{(\mathscr{D}_1+\mathrm{i}0^+)^{\nu_1}\cdots(\mathscr{D}_K+\mathrm{i}0^+)^{\nu_K}}$$

$$I_{\text{aux}}(\vec{\nu},\vec{s},\epsilon,\eta)=\int\prod_{i=1}^{L}\frac{\mathrm{d}^D\ell_i}{\mathrm{i}\pi^{D/2}}\frac{\mathscr{D}_{K+1}^{-\nu_{K+1}}\cdots\mathscr{D}_N^{-\nu_N}}{(\mathscr{D}_1-\eta)^{\nu_1}\cdots(\mathscr{D}_K-\eta)^{\nu_K}}$$



- The limit $\eta\to\infty$ can be computed analytically in terms of vacuum integrals;
- The limit $\eta\to 0$ provides the target integrals;
- The auxiliary mass flow approach does not require boundary conditions!

NEW: AMFlow2

Alternate recursion mode, High-performance differential equation solver, improved IBP interfaces, ...

Series Expansion

A simple example

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x+2} \\ f(0) = 1 \end{cases}$$

Ansatz:

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k+r)\, c_k\, x^{(k+r-1)}$$

$$\begin{cases} r c_0 = 0 \\ \frac{1}{5} c_0 + c_1 (r+1) = 0 \\ \frac{4}{25} c_0 + \frac{1}{5} c_1 + c_2 (2+r) = 0 \\ \dots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10} x^2 + \frac{11}{150} x^3 + \dots$$

Expanded around $x' = 0$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \frac{1}{(x'+2)} f_{hom}^{-1}(x')$$

$$= \frac{1}{2} x - \frac{7}{40} x^2 + \frac{2}{75} x^3 + \dots$$

$$f(x) = f_{part}(x) + C f_{hom}(x)$$

$$f(0) = 1 \rightarrow C = \frac{1}{5}$$

Towards NNLO EW

[A. Freitas, J. Wallace: 2512.15700]

NNLO EW: Fermionic Contribution

- Fermionic contribution: at least one closed fermion sub-loop;
- Gauge-invariant subset which is expected to be relatively large due to the top quark mass;
- Direct numerical integration of each contribution (no IBP):
  - Vertex and box diagrams with a sub-loop fermion self-energy: dispersion relation for the self-energy (numerical integration over σ);
  - Vertex diagrams with a triangle fermion sub-loop: Feynman parametrisation followed by dispersion relation (numerical integration over x and σ)
- Numerical integration performed in C++ (GNU Scientific Library, QUADPACK)

