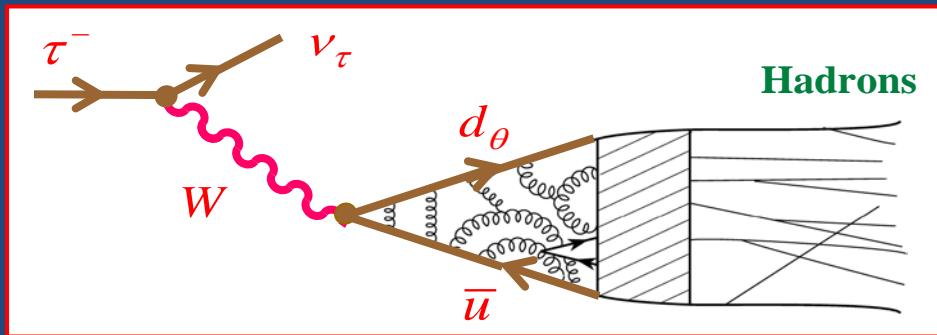


# $\tau$ Physics & QCD

## A successful scientific challenge

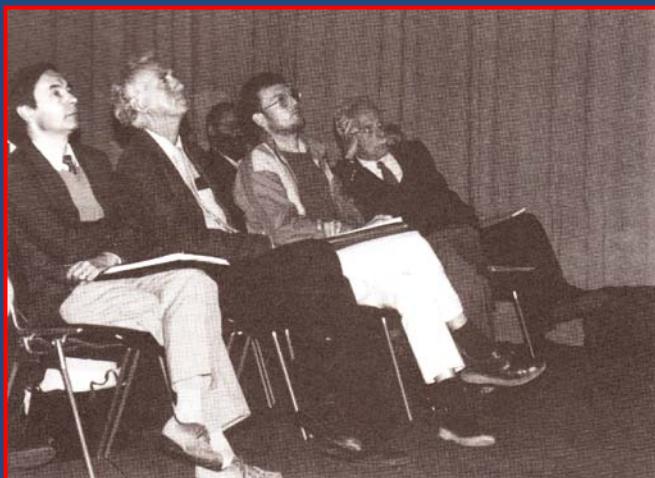
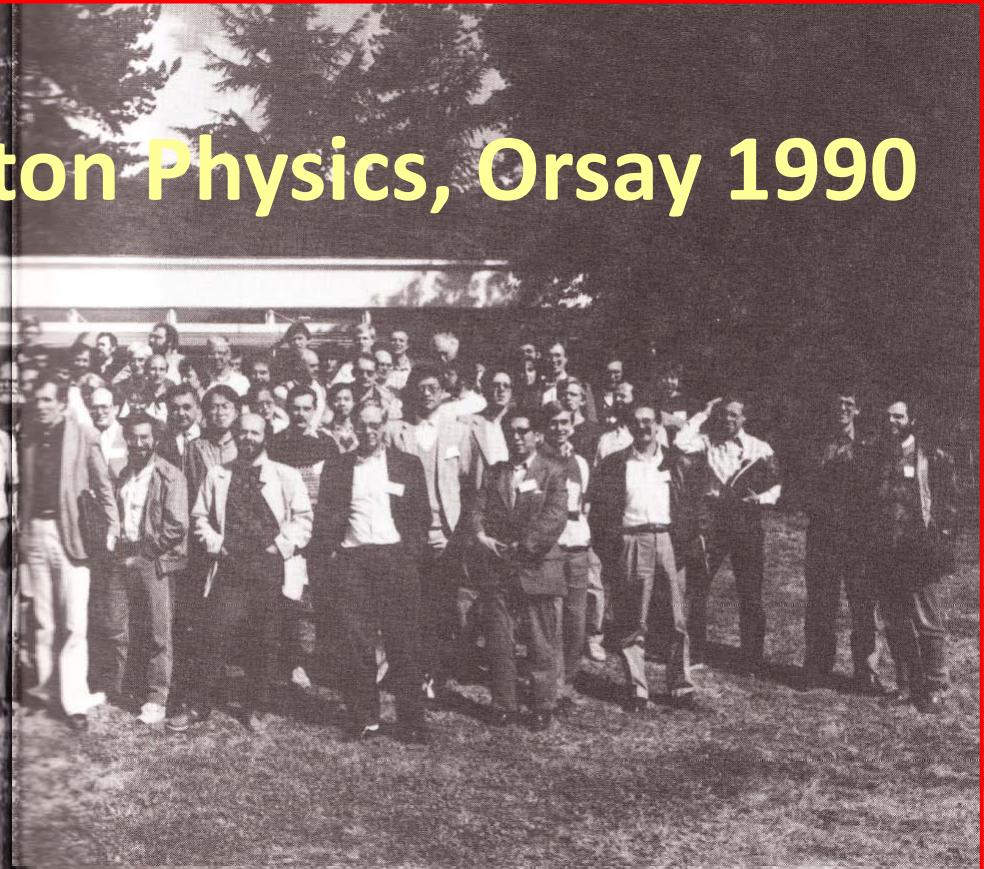
A. Pich

IFIC, Valencia



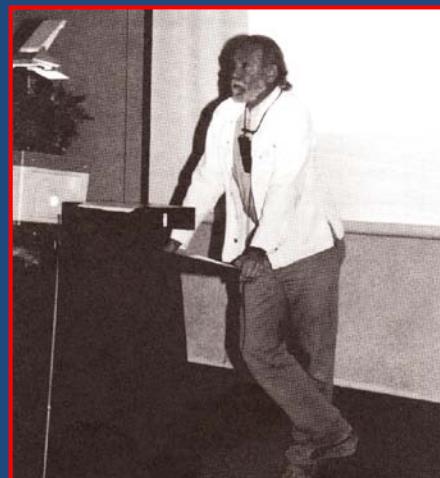
Ceremony in honour of Michel Davier,  
Awarded the Prix Lagarrigue 2010, Orsay 26 April 2011

# Workshop on Tau Lepton Physics, Orsay 1990

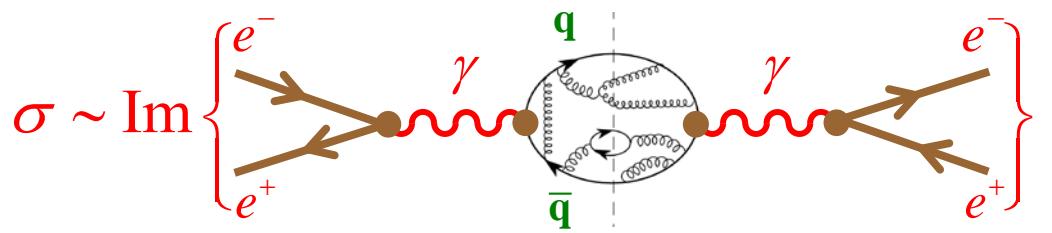


Michel Davier &  $\tau$  Physics

Prix Lagarrigue 2010, Orsay

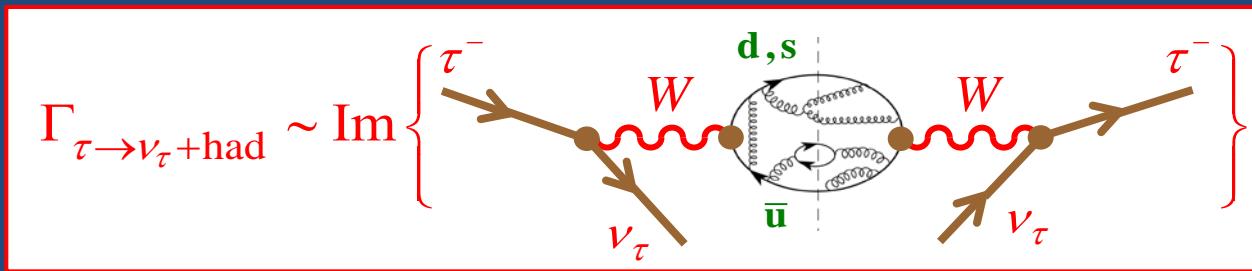


A. Pich



$$\frac{\sigma(e^+e^- \rightarrow \text{had})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 12\pi \text{ Im} \Pi_{\text{em}}(s)$$

$$\Pi_{\text{em}}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T[J_{\text{em}}^\mu(x) J_{\text{em}}^\nu(0)] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{\text{em}}(q^2)$$



$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^{m_\tau^2} dx \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[ \left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im} \Pi^{(1)}(s) + \text{Im} \Pi^{(0)}(s) \right]$$

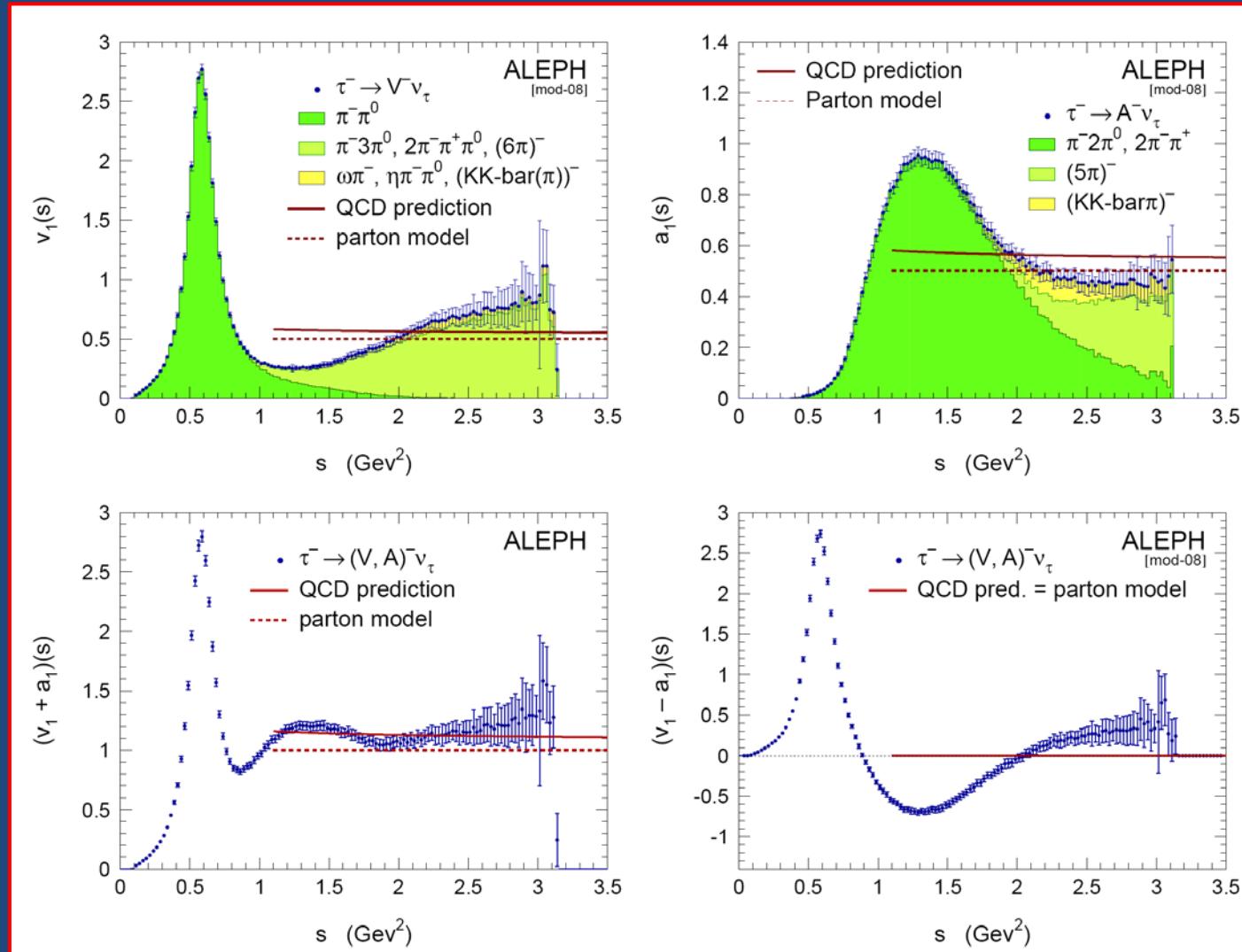
$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[ \Pi_{ud,V}^{(J)}(s) + \Pi_{ud,A}^{(J)}(s) \right] + |V_{us}|^2 \left[ \Pi_{us,V}^{(J)}(s) + \Pi_{us,A}^{(J)}(s) \right]$$

$$\Pi_{ij,J}^{\mu\nu}(q) \equiv i \int d^4x e^{iqx} \langle 0 | T[J_{ij}^\mu(x) J_{ij}^\nu(0)^\dagger] | 0 \rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \Pi_{ij,J}^{(1)}(q^2) + q^\mu q^\nu \Pi_{ij,J}^{(0)}(q^2)$$

# SPECTRAL FUNCTIONS

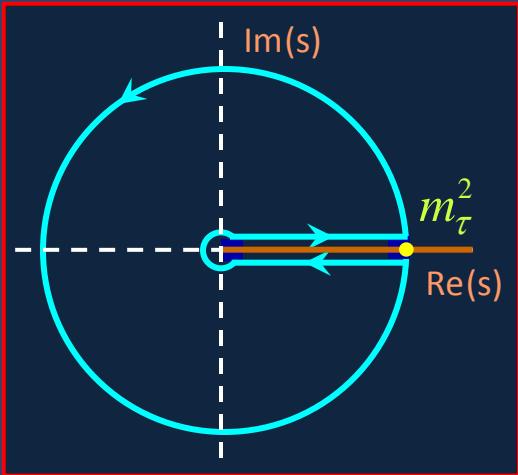
$$v_1(s) = 2\pi \operatorname{Im} \Pi_{ud,V}^{(0+1)}(s)$$

$$a_1(s) = 2\pi \operatorname{Im} \Pi_{ud,A}^{(0+1)}(s)$$



M. Davier, S. Descotes-Genon, A. Höcker, B. Malaescu, Z. Zhang 2008

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{had})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx (1-x)^2 \left[ (1+2x) \text{Im} \Pi^{(1)}(xm_\tau^2) + \text{Im} \Pi^{(0)}(xm_\tau^2) \right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx (1-x)^2 \left[ (1+2x) \Pi^{(0+1)}(xm_\tau^2) - 2x \Pi^{(0)}(xm_\tau^2) \right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s, \mu) \langle O_D(\mu) \rangle}{(-s)^{D/2}} \quad \text{OPE}$$

$$R_\tau = N_C S_{\text{EW}} (1 + \delta_{\text{P}} + \delta_{\text{NP}}) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$S_{\text{EW}} = 1.0201 (3)$$

Marciano-Sirlin, Braaten-Li, Erler

$$\delta_{\text{NP}} = -0.0059 \pm 0.0014$$

Fitted from data (Davier – Höcker-Zhang 2006)

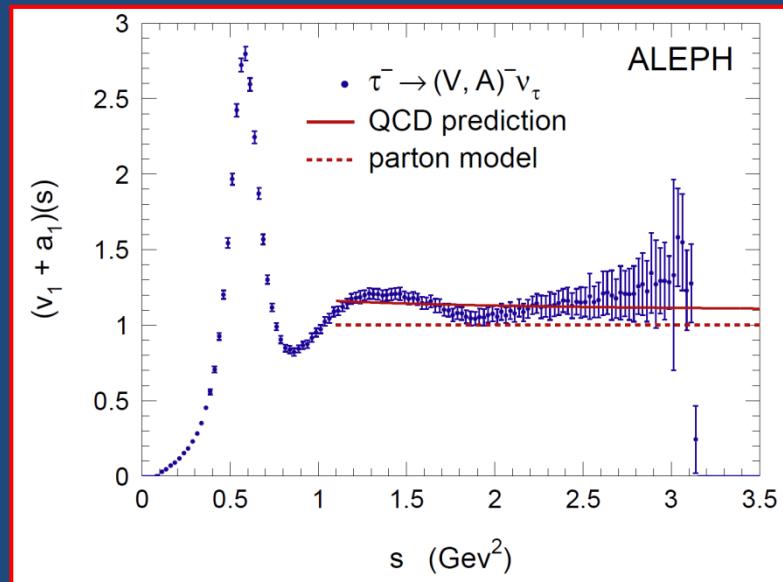
$$\delta_{\text{P}} = a_\tau + 5.20 a_\tau^2 + 26 a_\tau^3 + 127 a_\tau^4 + \dots \approx 20\% \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Similar predictions for  $R_{\tau,V}$  ,  $R_{\tau,A}$  ,  $R_{\tau,S}$  and the moments

$$R_{\tau}^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_{\tau}^2}\right)^l \frac{dR_{\tau}}{ds}$$

Sensitivity to power corrections through  $k, l$

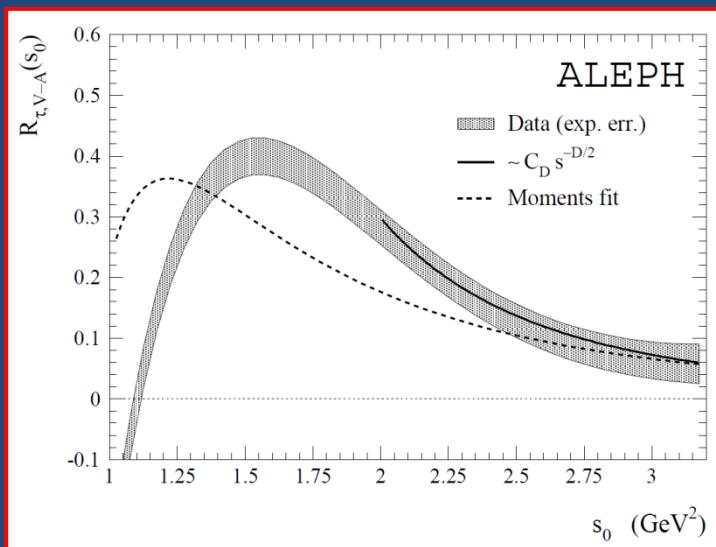
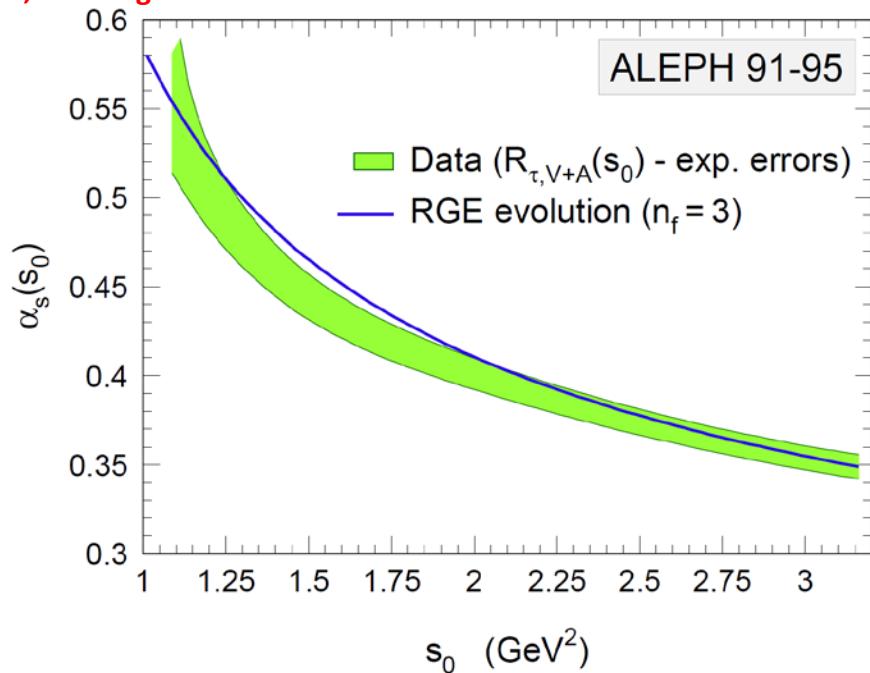
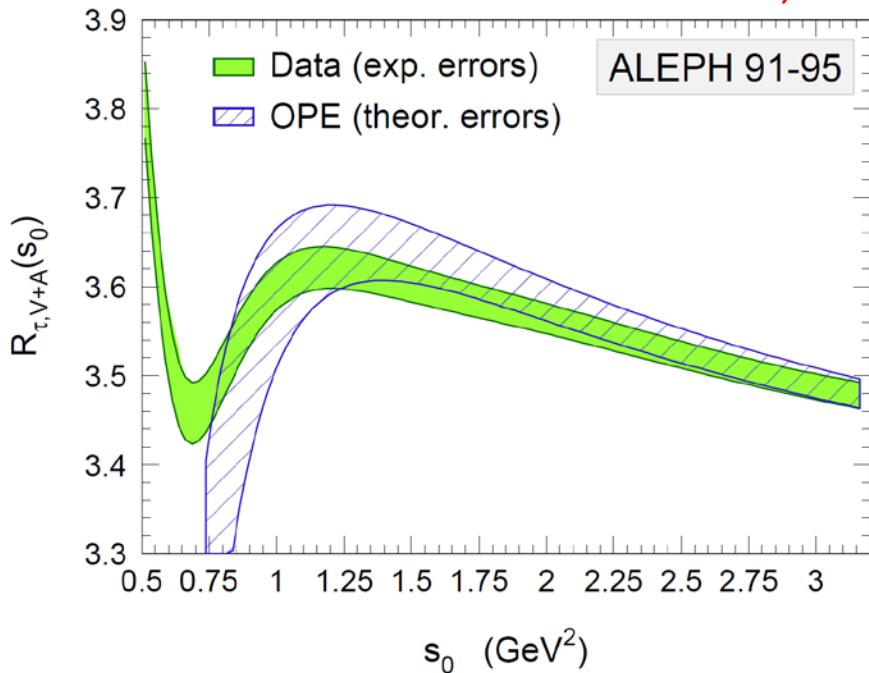
F. Le Diberder, A. Pich 1992



The non-perturbative contribution to  $R_{\tau}$  can be obtained from the invariant-mass distribution of the final hadrons:

$$\delta_{NP} = -0.0059 \pm 0.0014$$

M. Davier, A. Höcker, Z. Zhang 2006



ALEPH 1998

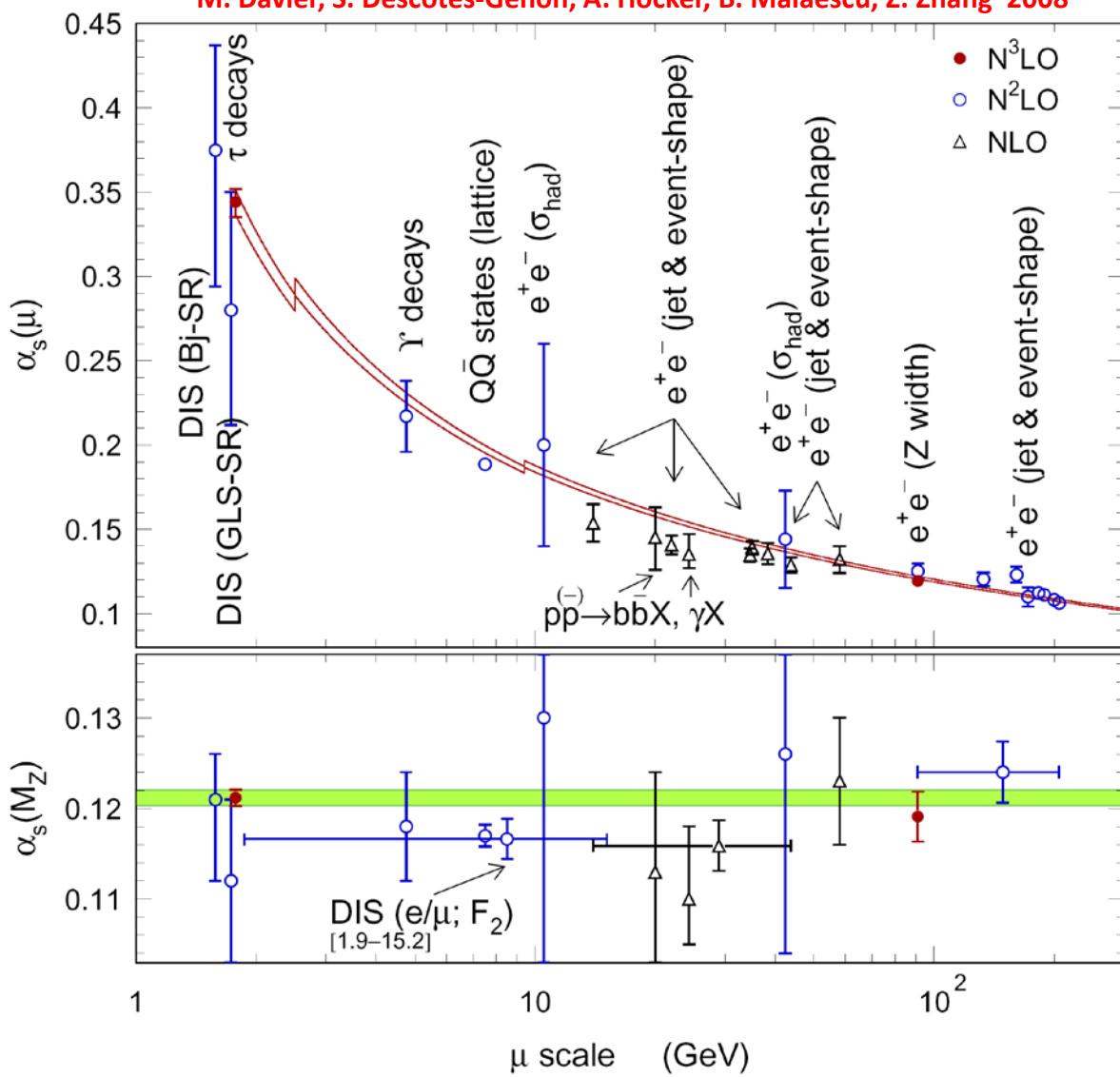
$$R_{\tau,V-A}(s_0) = \frac{C}{s_0^{D/2}} \quad \rightarrow \quad D = 6.9 \pm 0.9$$

In agreement with OPE expectation       $D = 6$

$$R_{\tau,V} = 1.783 \pm 0.011 \quad ; \quad R_{\tau,A} = 1.695 \pm 0.011 \quad ; \quad R_{\tau,V+A} = 3.478 \pm 0.010$$

ALEPH

M. Davier, S. Descotes-Genon, A. Höcker, B. Malaescu, Z. Zhang 2008



$$\alpha_s(m_\tau^2) = 0.344 \pm 0.009$$



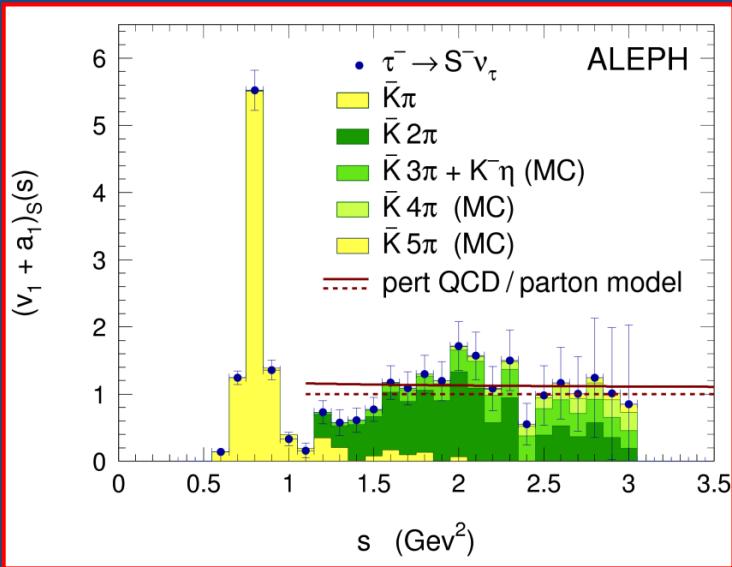
$$\alpha_s(M_Z^2) = 0.1212 \pm 0.0011$$

$$\alpha_s(M_Z^2)_{Z\text{ width}} = 0.1193 \pm 0.0028$$

The most precise test of  
Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0019 \pm 0.0011_\tau \pm 0.0028_Z$$

# Strange Spectral Function: SU(3) Breaking

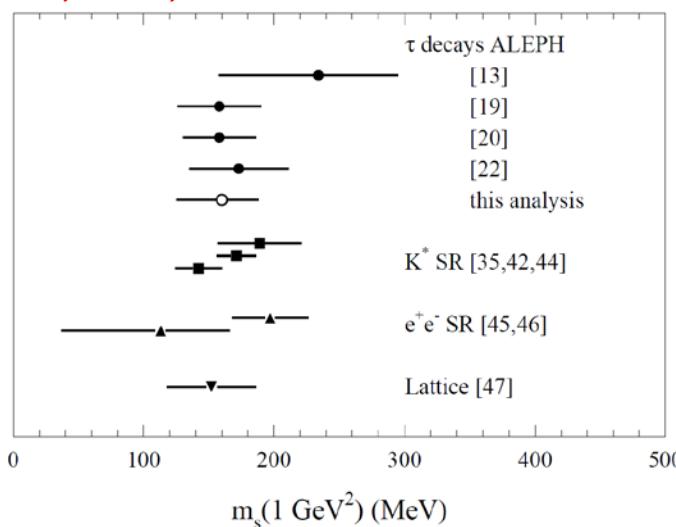
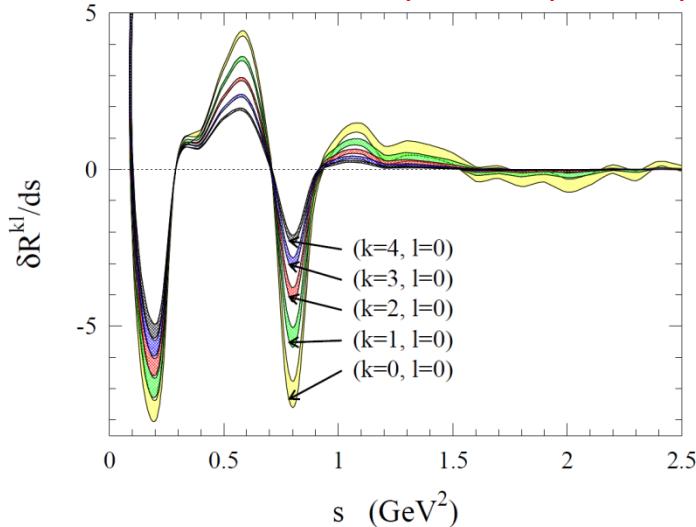


$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta_{kl}(\alpha_s)$$



$m_s$  and/or  $V_{us}$

S. Chen, M. Davier, E. Gámiz, A. Höcker, A. Pich, J. Prades 2001



Ximo Prades  
(1963-2010)

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,\text{th}}^{00}}$$

**τ data:**  $R_{\tau,S}^{00} = 0.1615$  (40)

$$R_{\tau,V+A}^{00} = 3.479 \quad (11)$$

**PDG 10:**  $|V_{ud}| = 0.97425$  (22)

$$\delta R_{\tau,\text{th}}^{00} = 0 \quad \rightarrow \quad |V_{us}| = 0.210 (3)$$

Taking as input (from non τ sources)

$$m_s(m_\tau) = 100 \pm 10 \text{ MeV}$$

$$\delta R_{\tau,\text{th}}^{00} = 0.216 \quad (16)$$



$$|V_{us}| = 0.2164 \pm 0.0027_{\text{exp}} \pm 0.0005_{\text{th}}$$

**K13:**  $|V_{us}| = 0.2241 \pm 0.0024$

$$[f_+(0) = 0.965 \pm 0.010]$$

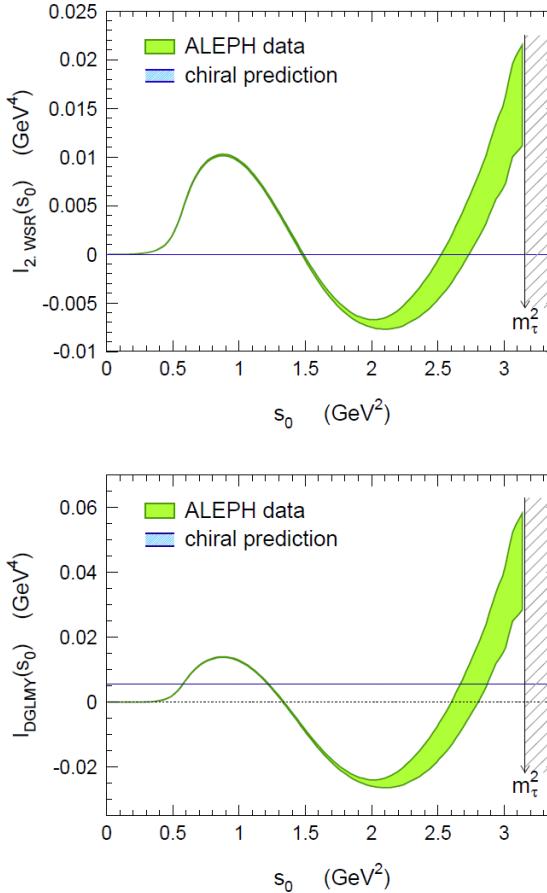
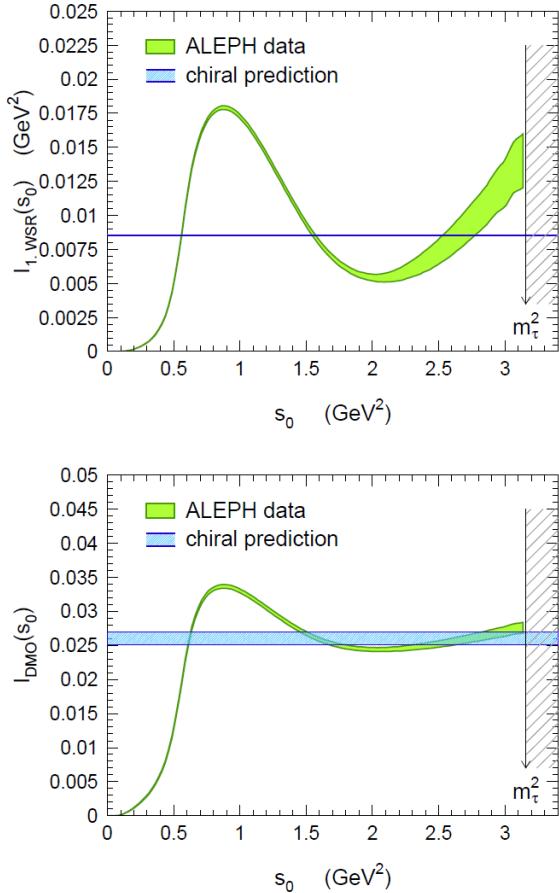
The τ could give the most precise  $V_{us}$  determination

# Chiral Sum Rules

$$\lim_{s \rightarrow \infty} s^2 \left[ \Pi_{ud,V}^{(0+1)}(s) - \Pi_{ud,A}^{(0+1)}(s) \right] = 0$$

$$\Pi_{ud,J}^{(0+1)}(s) = \frac{1}{\pi} \int_0^\infty \frac{\text{Im} \Pi_{ud,J}^{(0+1)}(t)}{t - s - i\varepsilon}$$

M. Davier, A. Höcker, Z. Zhang 2006



When  $s_0 \rightarrow \infty$

$$I_1 = \int_0^{s_0} ds \hat{\rho}(s) = f_\pi^2$$

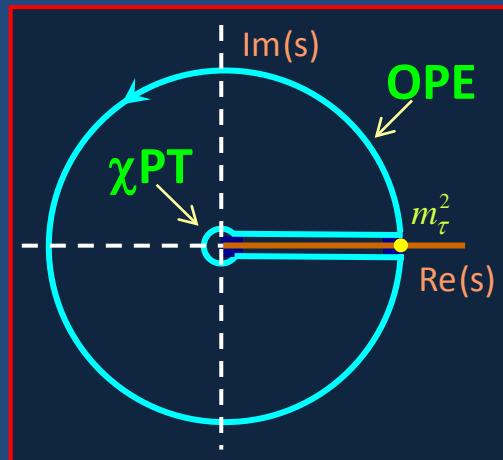
$$I_2 = \int_0^{s_0} ds s \hat{\rho}(s) = 0$$

$$I_3 = \int_0^{s_0} \frac{ds}{s} \hat{\rho}(s) = f_\pi^2 \frac{\langle r_\pi^2 \rangle}{3} - F_A$$

$$I_4 = \int_0^{s_0} ds s \ln s \hat{\rho}(s)$$

$$= \frac{4\pi f_\pi^2}{3\alpha} \left( m_{\pi^0}^2 - m_{\pi^\pm}^2 \right)$$

$$\hat{\rho} = \frac{1}{2} \rho \equiv \frac{1}{2\pi} \text{Im} \left[ \Pi_{ud,V}^{(0+1)} - \Pi_{ud,A}^{(0+1)} \right]$$



$$\lim_{s \rightarrow \infty} s^2 \Pi(s) = 0 \quad \rightarrow \quad \Pi^{\text{OPE}}(s) = -\frac{O_6}{s^3} + \frac{O_8}{s^4} - \dots$$

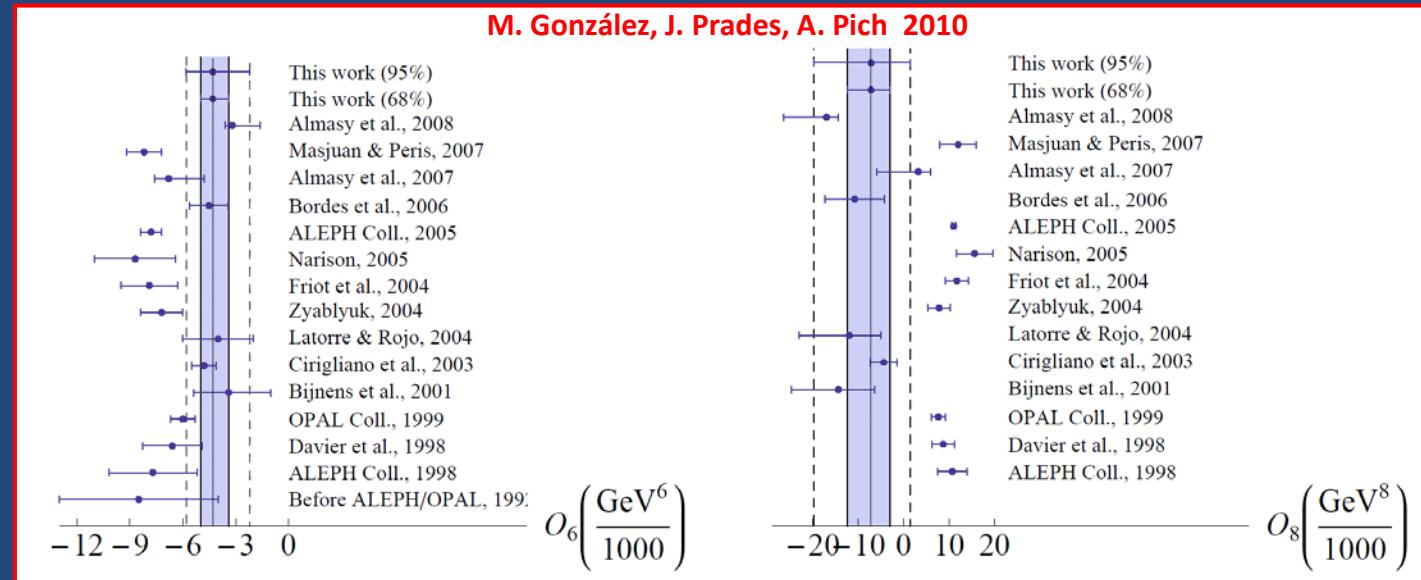
 **$\chi\text{PT}$  :**

$$\lim_{s \rightarrow 0} \Pi(s) = \frac{2F^2}{s} - 8L_{10}(\mu) - \frac{\Gamma_{10}}{4\pi^2} \left( \frac{5}{3} - \ln \frac{-s}{\mu^2} \right) + \dots$$

$$\Pi(s) \equiv \Pi_{\text{VV}}(s) - \Pi_{\text{AA}}(s)$$

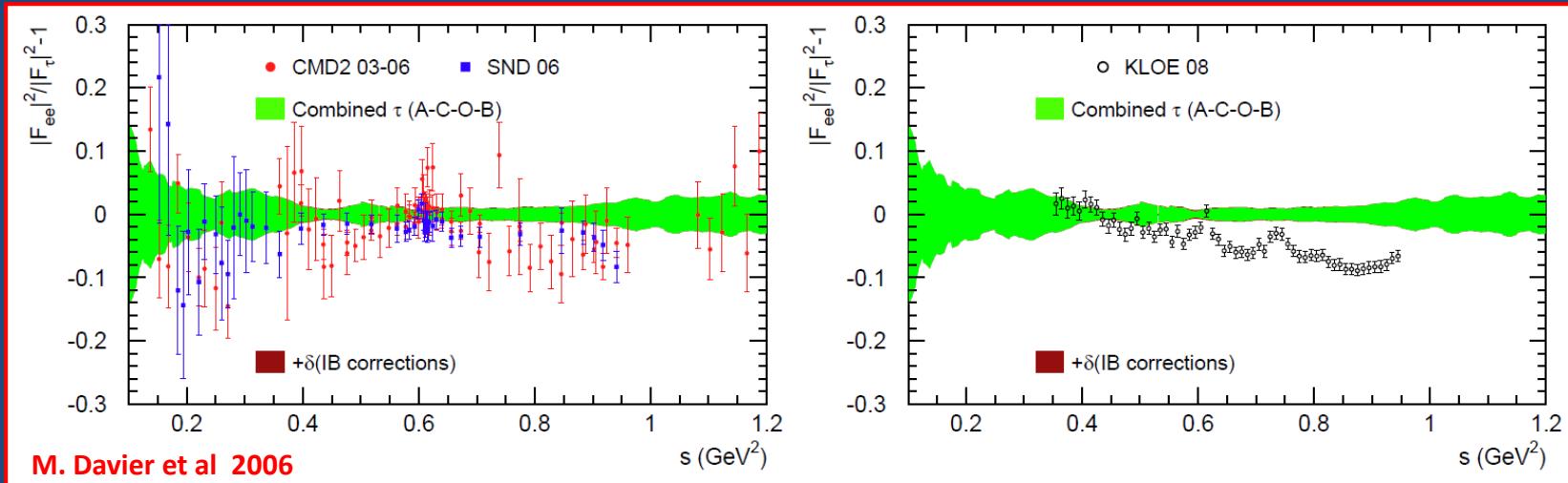


Jan Stern

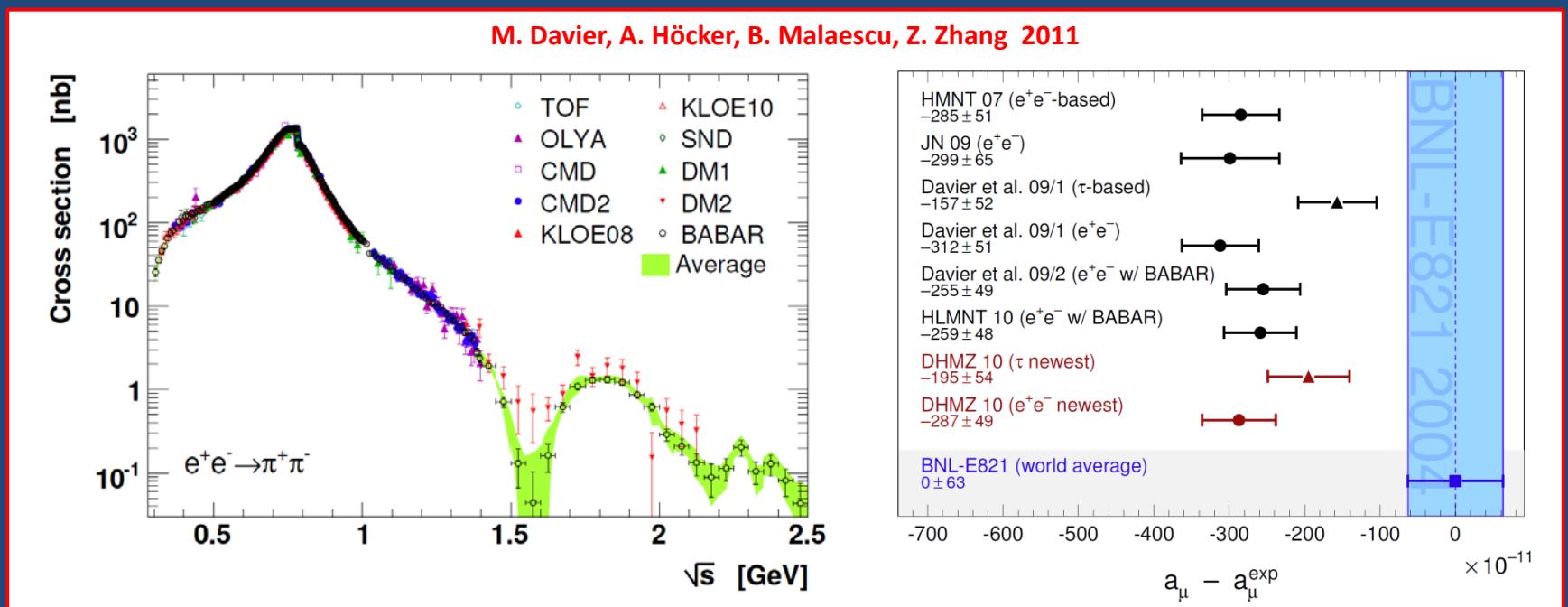


Implications  
for  $\epsilon'/\epsilon$   
(e.m. Penguin)

# Spectral Functions & $g-2$ ( $e^+e^-$ versus $\tau$ )

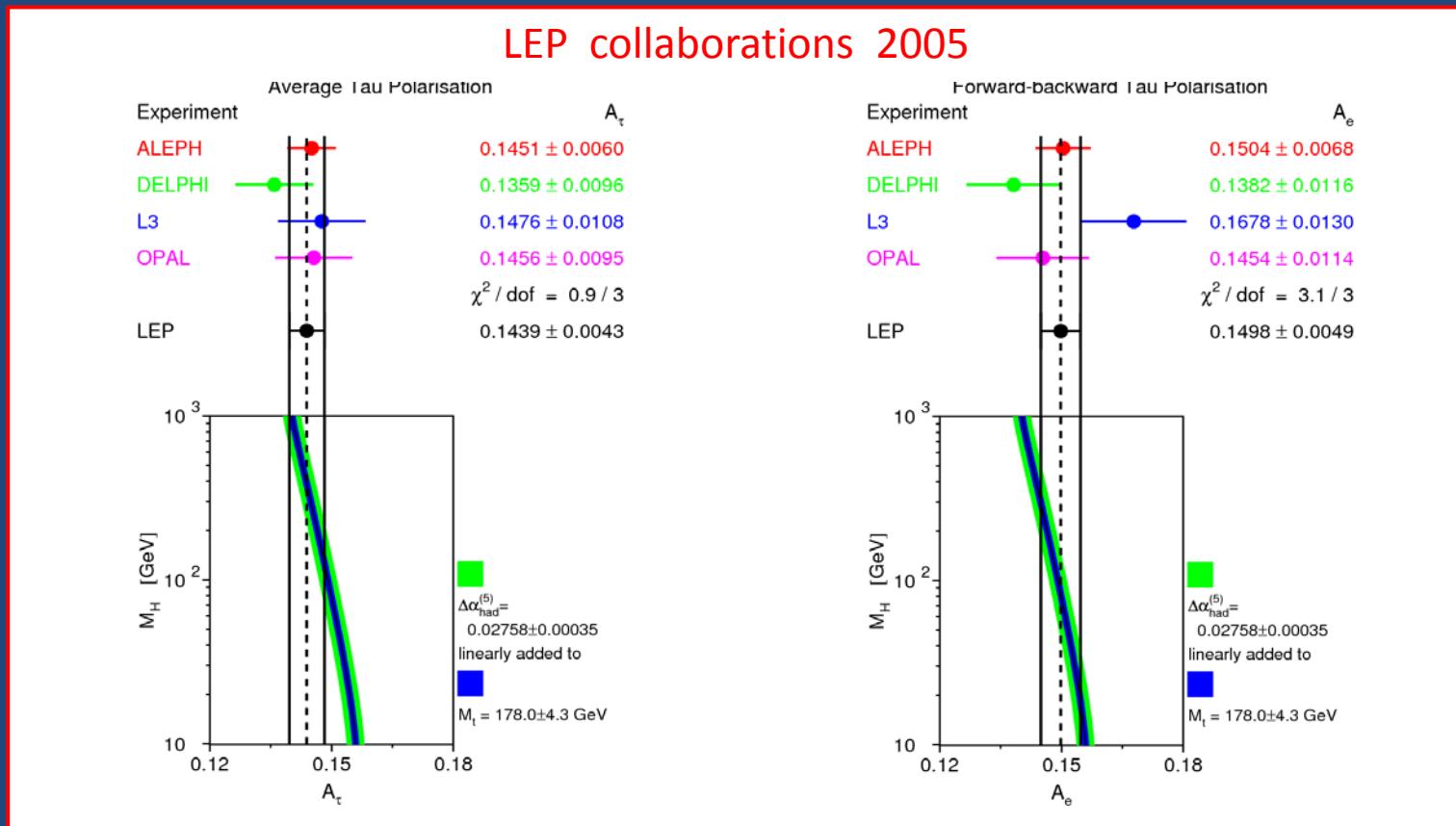


M. Davier, A. Höcker, B. Malaescu, Z. Zhang 2011



# $\tau$ Polarization in $Z \rightarrow \tau^+ \tau^-$

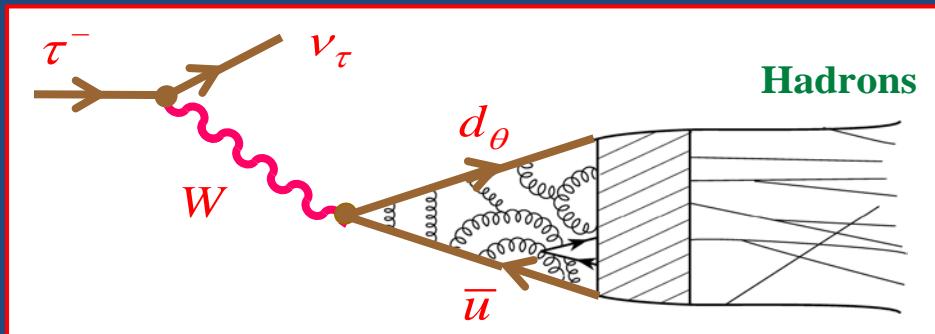
- “The optimal method to measure the  $\tau$  polarization”  
(M. Davier, L. Duflot, F. Le Diberder, A. Rouge 1993)
- “Tau polarization at the Z peak from the acollinearity between both  $\tau$  decay products”  
(R. Alemany, N. Rius, J. Bernabéu, J.J. Gómez, A. Pich 1991)



# $\tau$ Physics & QCD

## A successful scientific challenge

Congratulations!



Ceremony in honour of Michel Davier,  
Awarded the Prix Lagarrigue 2010, Orsay 26 April 2011