

# Stampede of the Wild Gluons



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This colloquium concerns physics at the Large Hadron Collider or LHC, located at the laboratory CERN in Geneva, Switzerland.

The LHC is the world's largest and most expensive scientific instrument. Its goal is to discover new particles and forces at distances less than  $10^{-18}$  cm.

You have heard about this goal -- the discovery of the Higgs boson, dark matter, supersymmetry, new space dimensions, etc.

Taking the next step is not so easy, however.

To reach high energies, we accelerate protons. However, a proton is not an elementary particle. It is a bound state, with a large size.

The cross section for proton proton collisions at high energy is about

$$0.1 \text{ barn} = 10^{-25} \text{ cm}^2$$

the area of a typical atomic nucleus.

Cross sections for more exceptional processes are much smaller

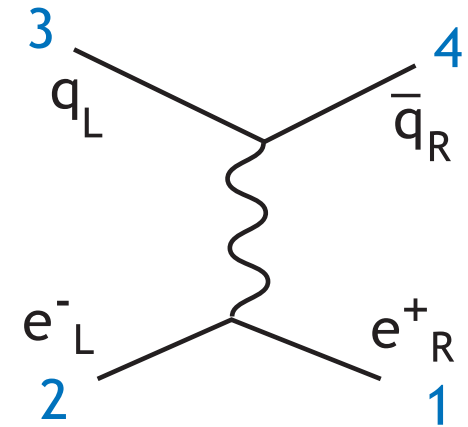
quark-quark hard scattering	$1 \mu\text{b}$
$W(\rightarrow \ell\nu)$	$10 \text{ nb}$
$Z(\rightarrow \ell^+\ell^-)$	$1 \text{ nb}$
$t\bar{t}(\rightarrow \ell\nu jjj)$	$0.3 \text{ nb}$
new particles	$1 - 10 \text{ pb}$

I'll begin by discussing QCD in a simpler environment, that of

$$e^+e^- \rightarrow q\bar{q}$$

Here is the idealized process:

In reality, quarks are bound up in strongly interacting particles - meson and baryons.

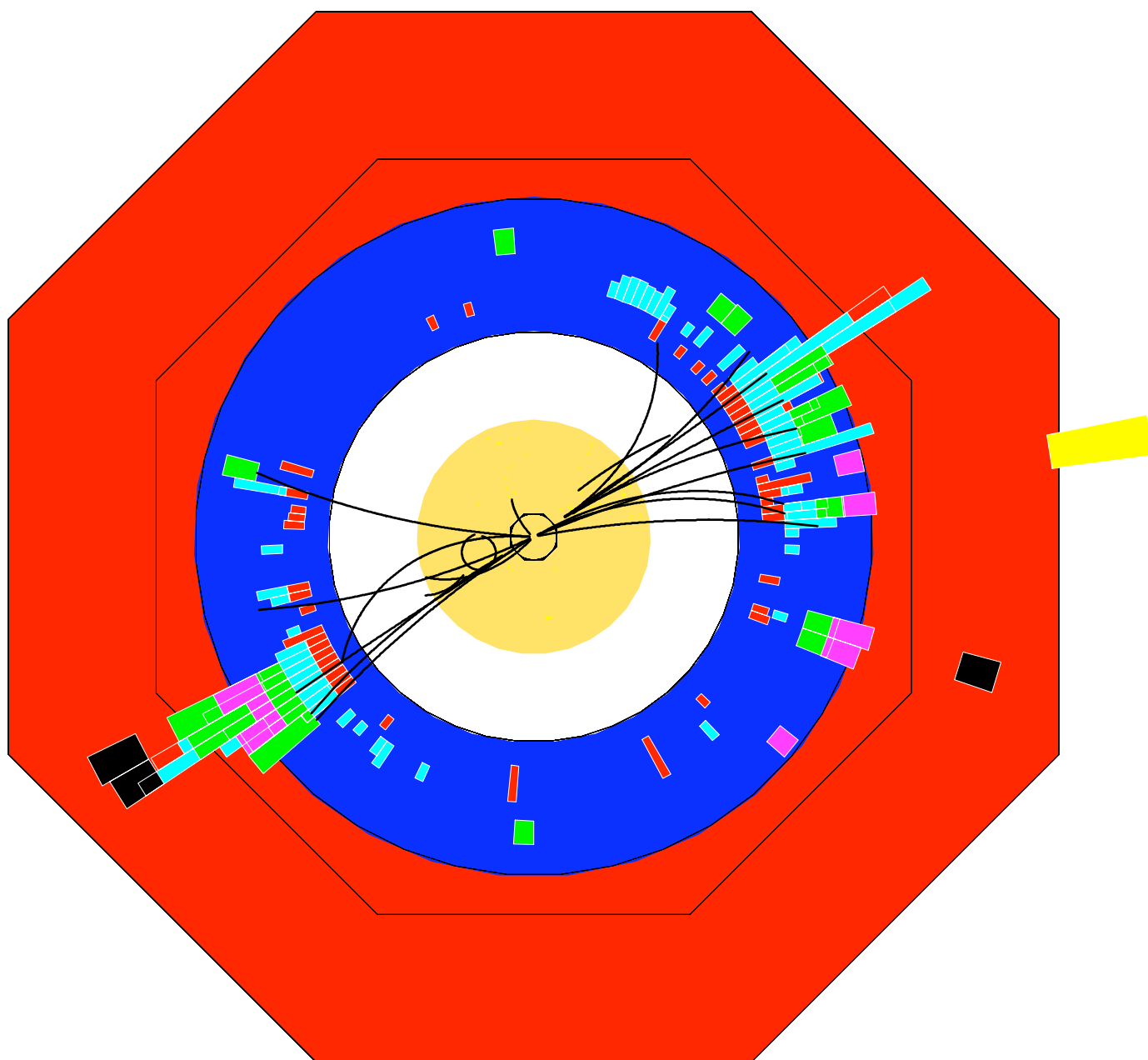


So we need to think about

$$e^+e^- \rightarrow \text{hadrons}$$

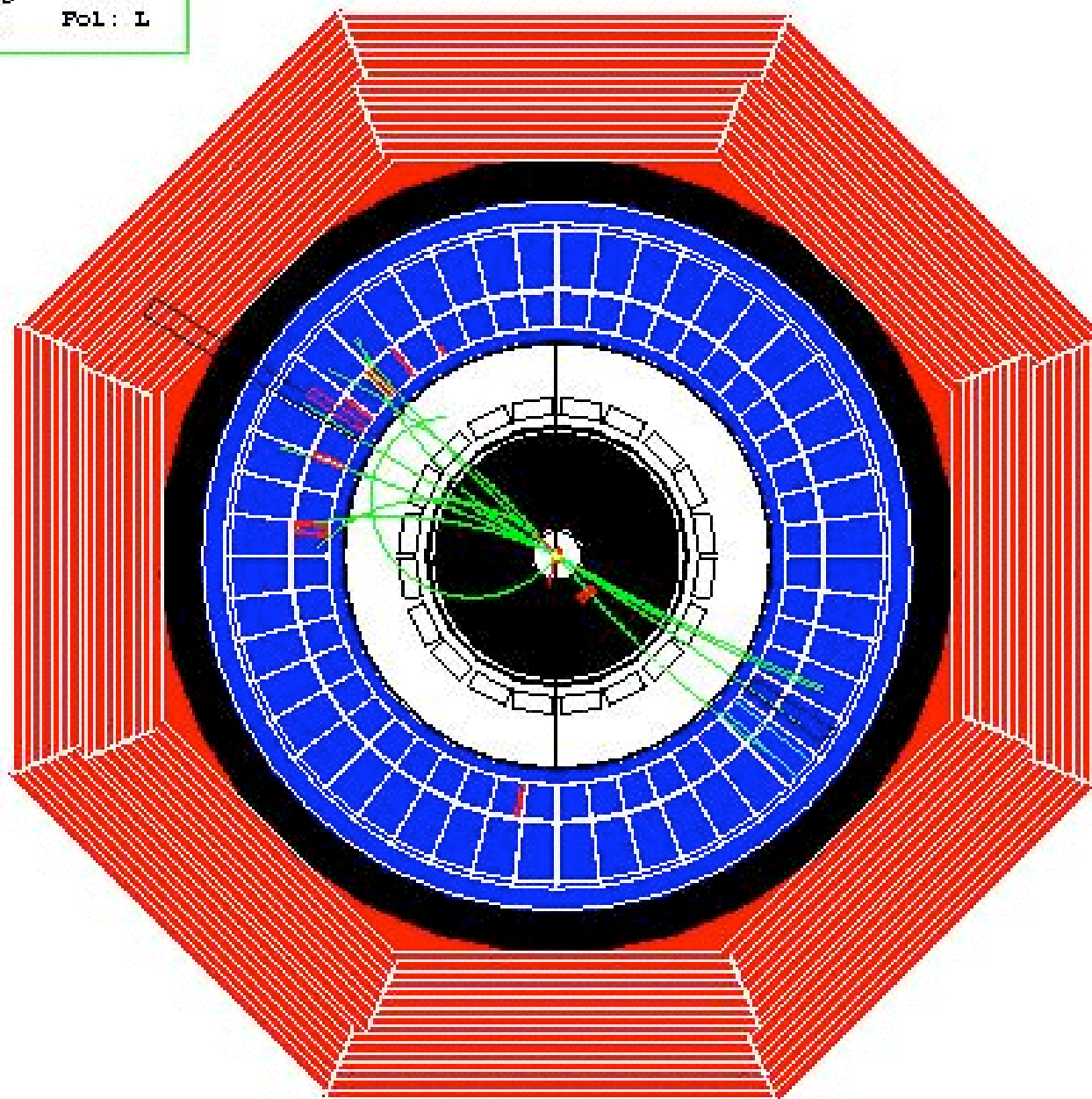
The first question is:

What do these events look like ?



SLD

Run 12576, EVENT 2540  
4-JUL-1992 19:18  
Source: Run Data Pol: L



SLD



The collimated bundles of strongly interacting particles are called “jets”. As a first approximation, we can identify jets with quarks and antiquarks produced at high momentum.

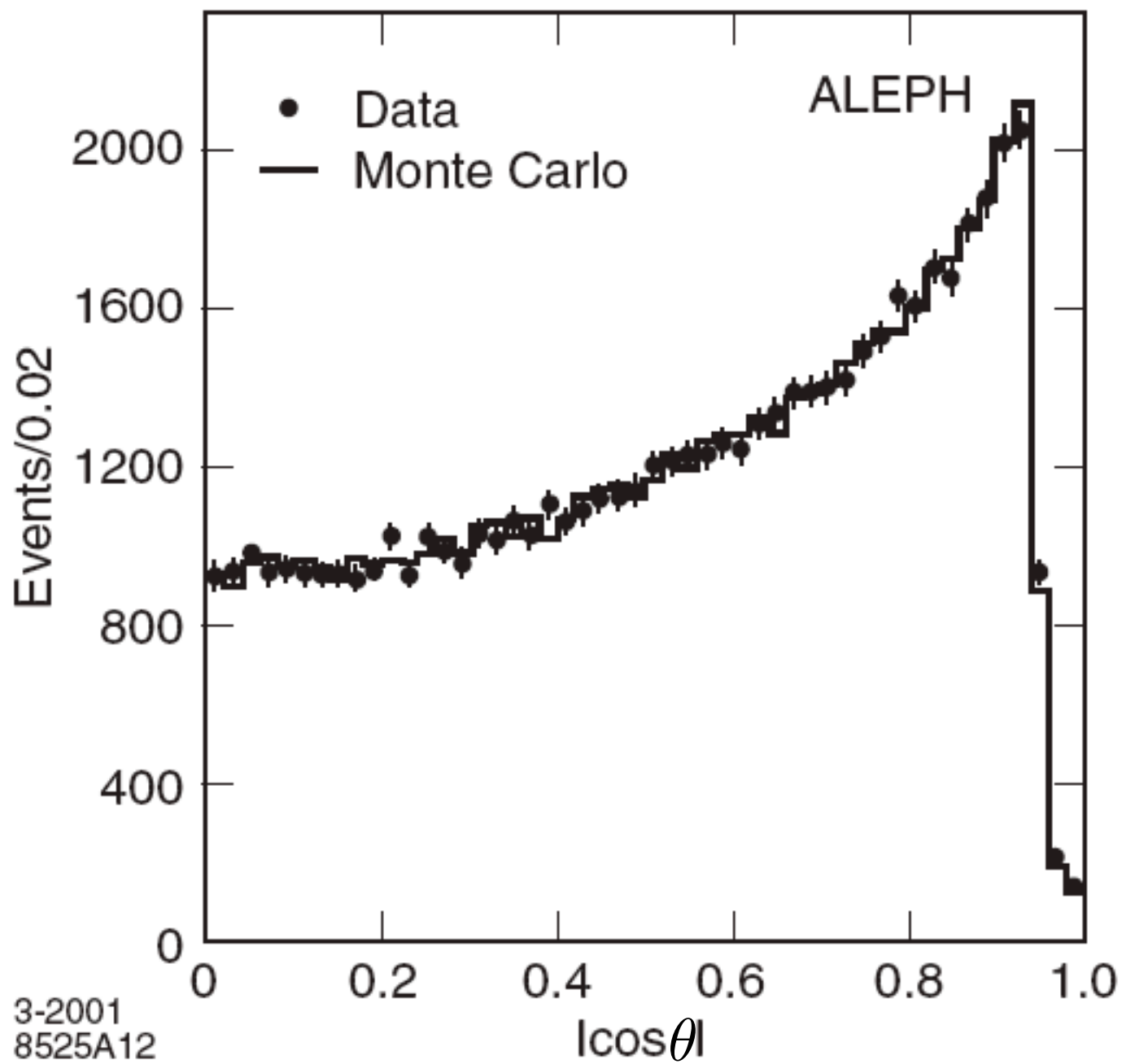
QED predicts

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) \sim (1 + \cos^2\theta)$$

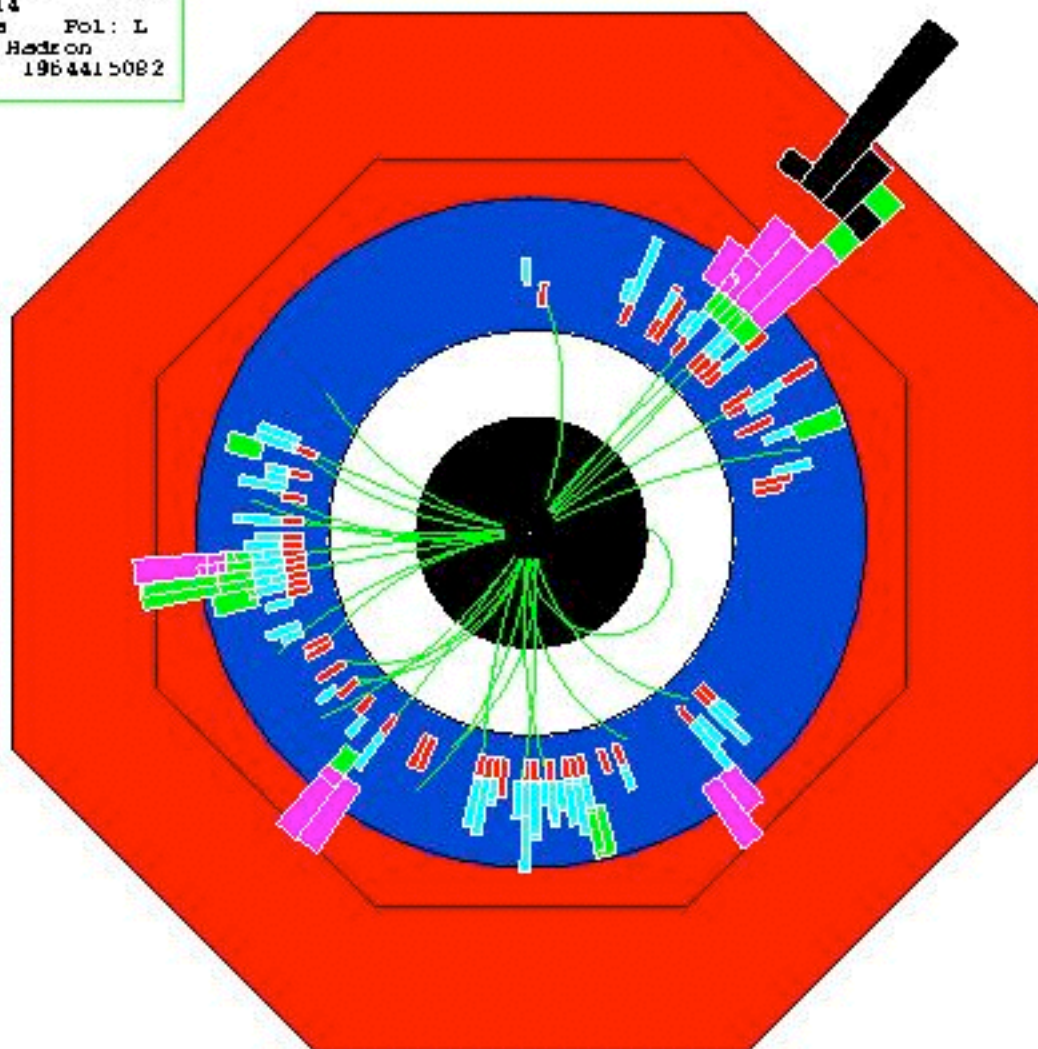
or, for later reference

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow q\bar{q}) \sim (1 + \cos\theta)^2 + (1 - \cos\theta)^2$$

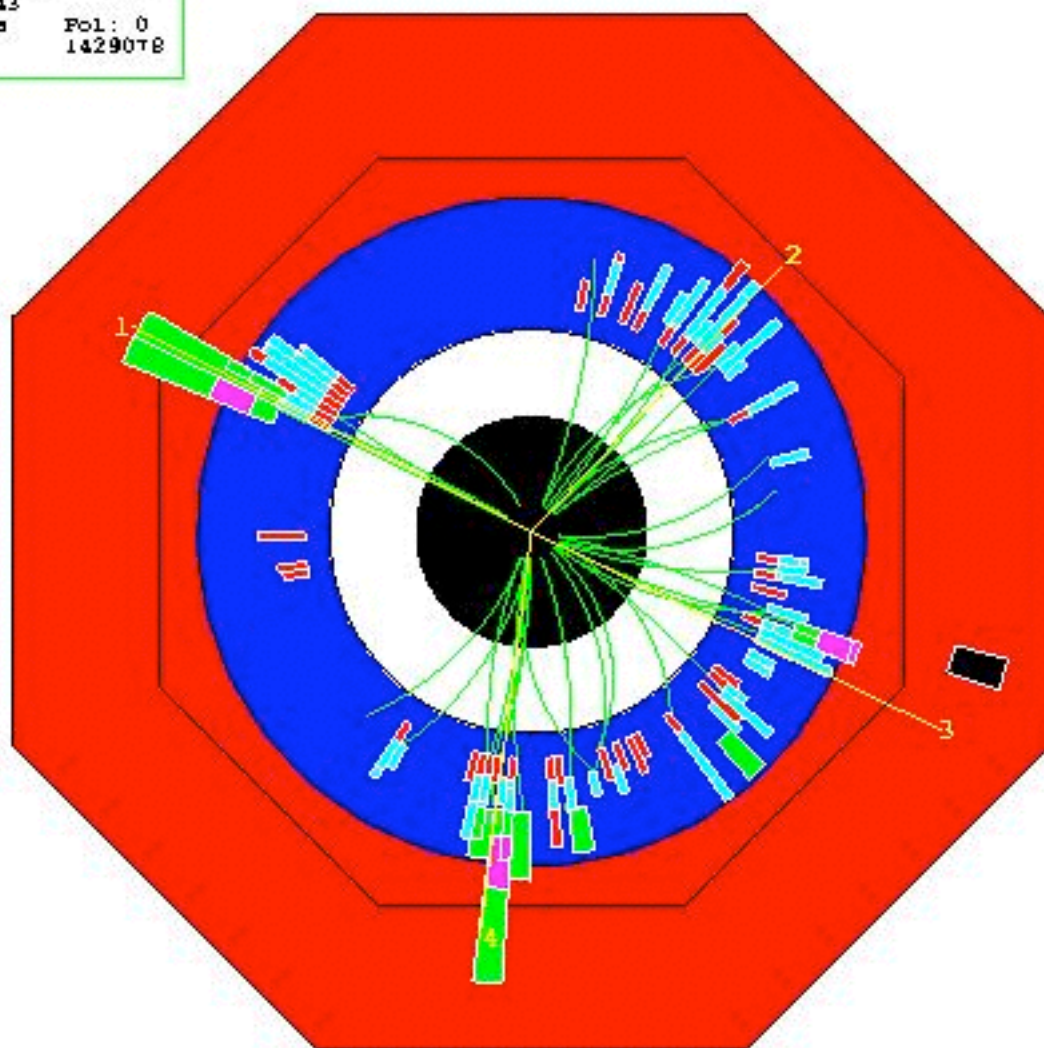
How do the jets compare ?



Run 12637, EVENT 6353  
8-JUL-1992 10:14  
Source: Run Data Fol: L  
Trigger: Energy Hadron  
Beam Crossing 1964415082



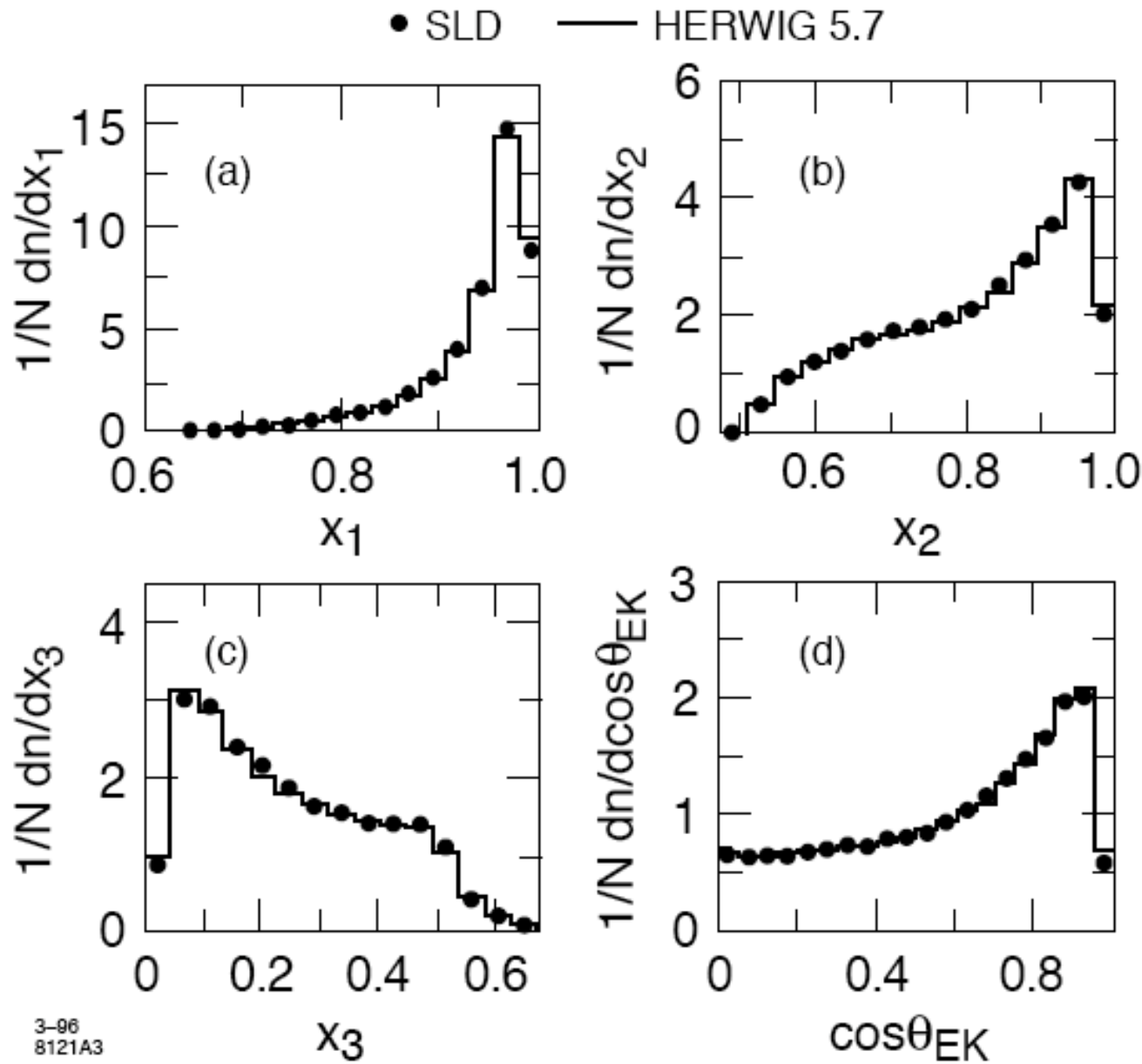
Run 1536, EVENT 2126  
6-JUL-1991 14:43  
Source: Run Data Pol: 0  
Beam Crossing 1429078



The sharing of energy with the third jet follows exactly the pattern expected for **photon** radiation.

$$P(z) \sim \frac{1 + (1 - z)^2}{z}$$

This indicates radiation of a spin-1 particle, the **gluon**.



The charge to which gluons couple in QCD is called **color**. Quarks come in three colors (**r,g,b**). Gluons carry a color and an anticolor. Through this, gluons couple to one another.

This nonlinear interaction, in quantum theory of QCD, leads to an amazing effect, **asymptotic freedom** (Gross, Politzer, Wilczek)

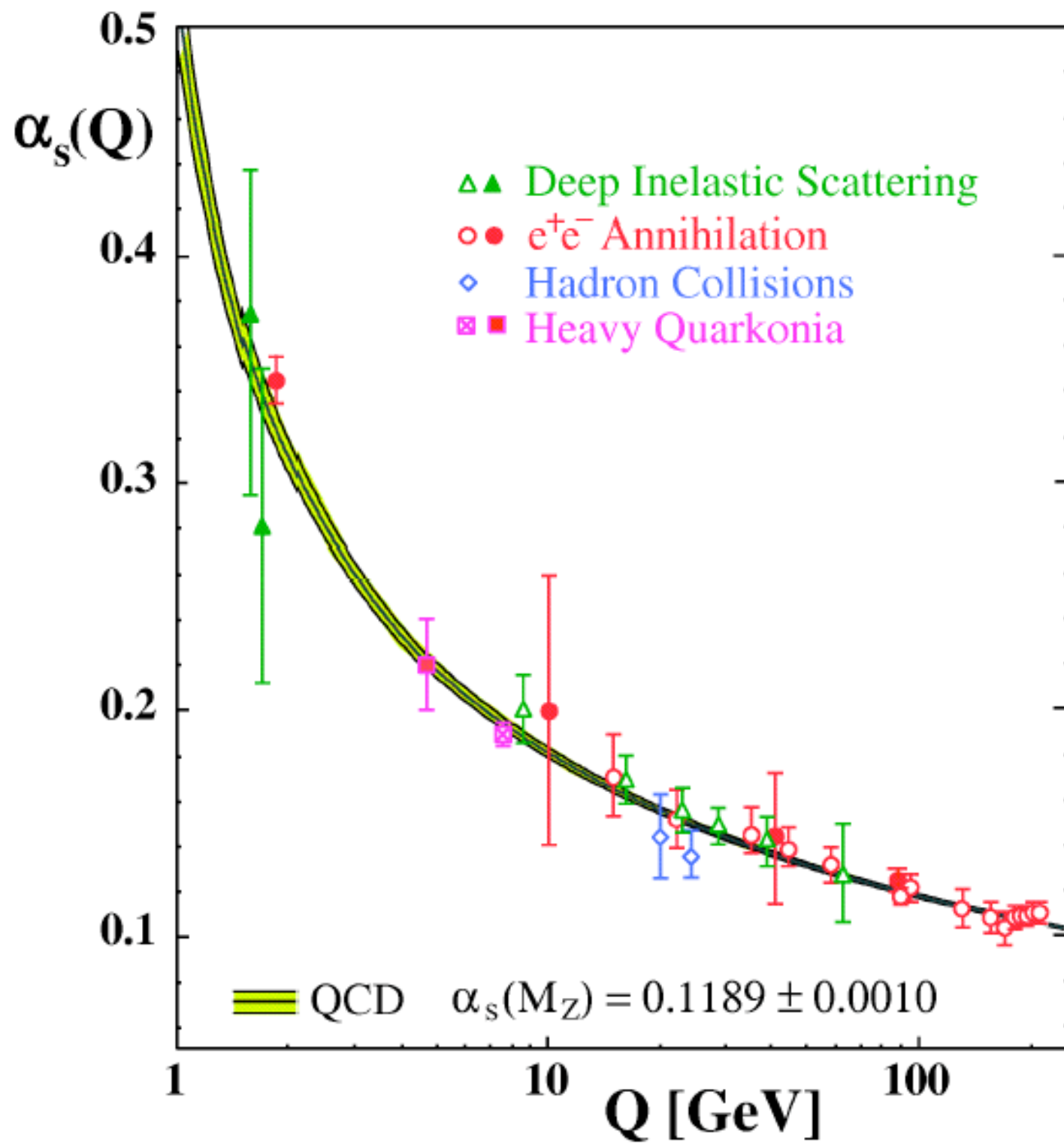
Due to quantum corrections, the QCD coupling constant is

strong at large distances (small momentum transfer)

weak at short distances (large momentum transfer)

$$\alpha_s(Q) = \frac{\alpha_s(m_Z)}{1 + (b_0 \alpha_s(m_Z)/2\pi) \log(Q/m_Z)}$$

It is this that justifies the use of weak-coupling perturbation theory in the theory of strong interactions.



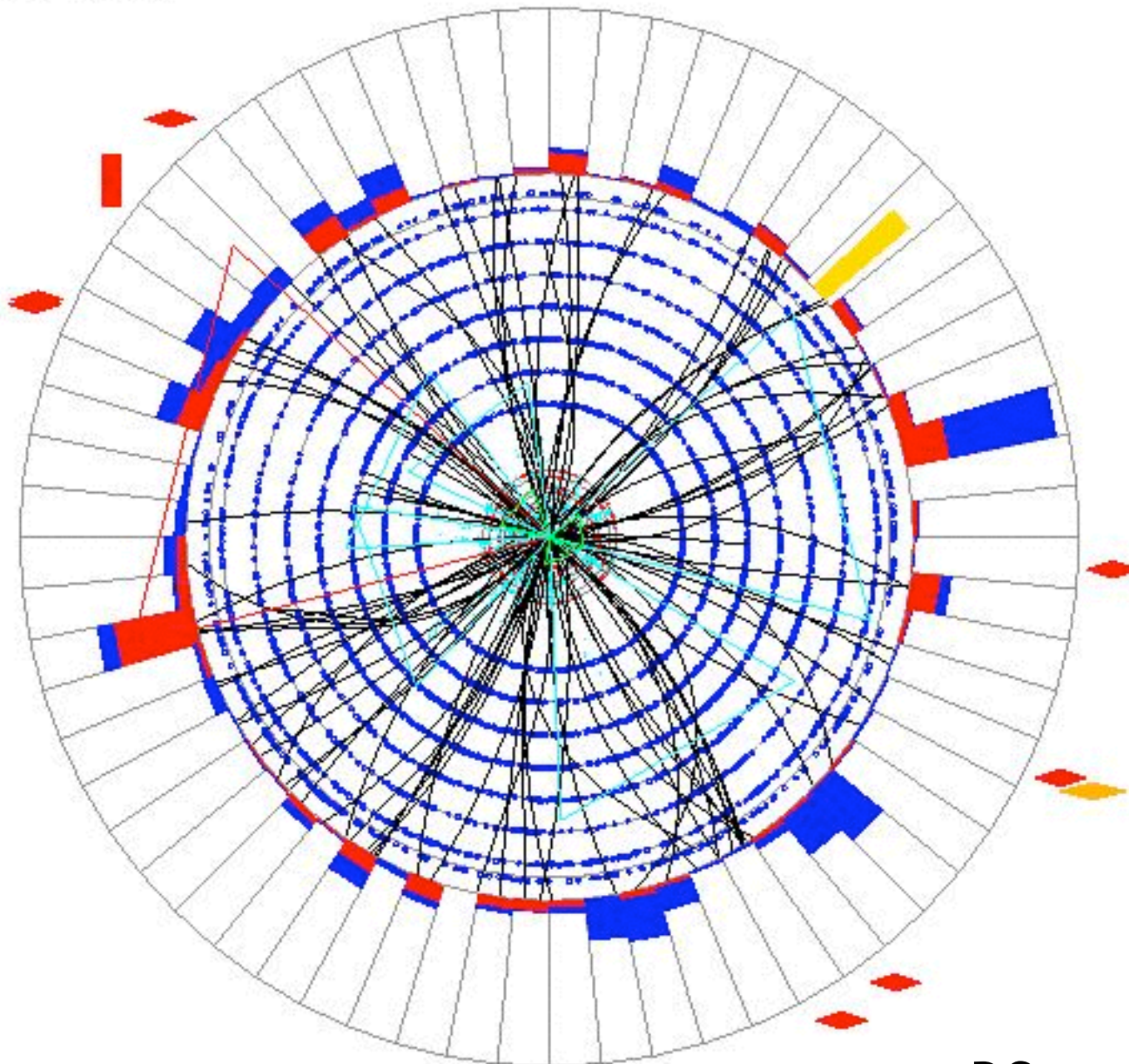
Bethke

At the LHC, we collide protons. These are basically bags of quarks and gluons. Typically events should look like quark or gluon scatterings, giving rise to two jets...

... as long as the scattering is sufficient hard that we are in the regime of asymptotic freedom.

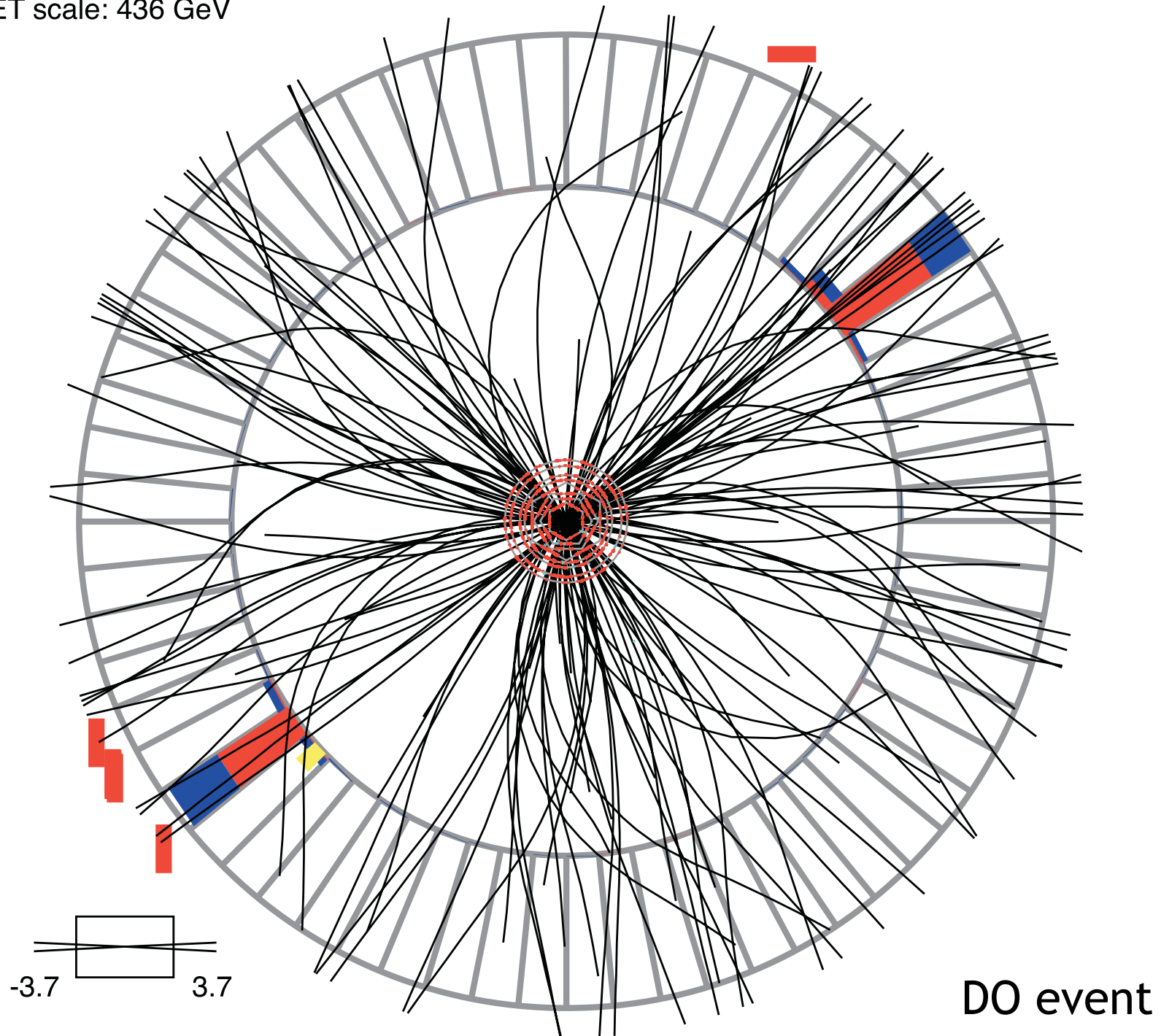
Run 223385 Evt 9802792 Thu Jul 20 17:14:11 2006

ET scale: 10 GeV



D0 event

ET scale: 436 GeV



A useful discriminator is the **Lego Plot**.

Unroll the plane of  $(\theta, \phi)$  . Draw

$p_T$  = component of  $p$  perpendicular to the beam axis

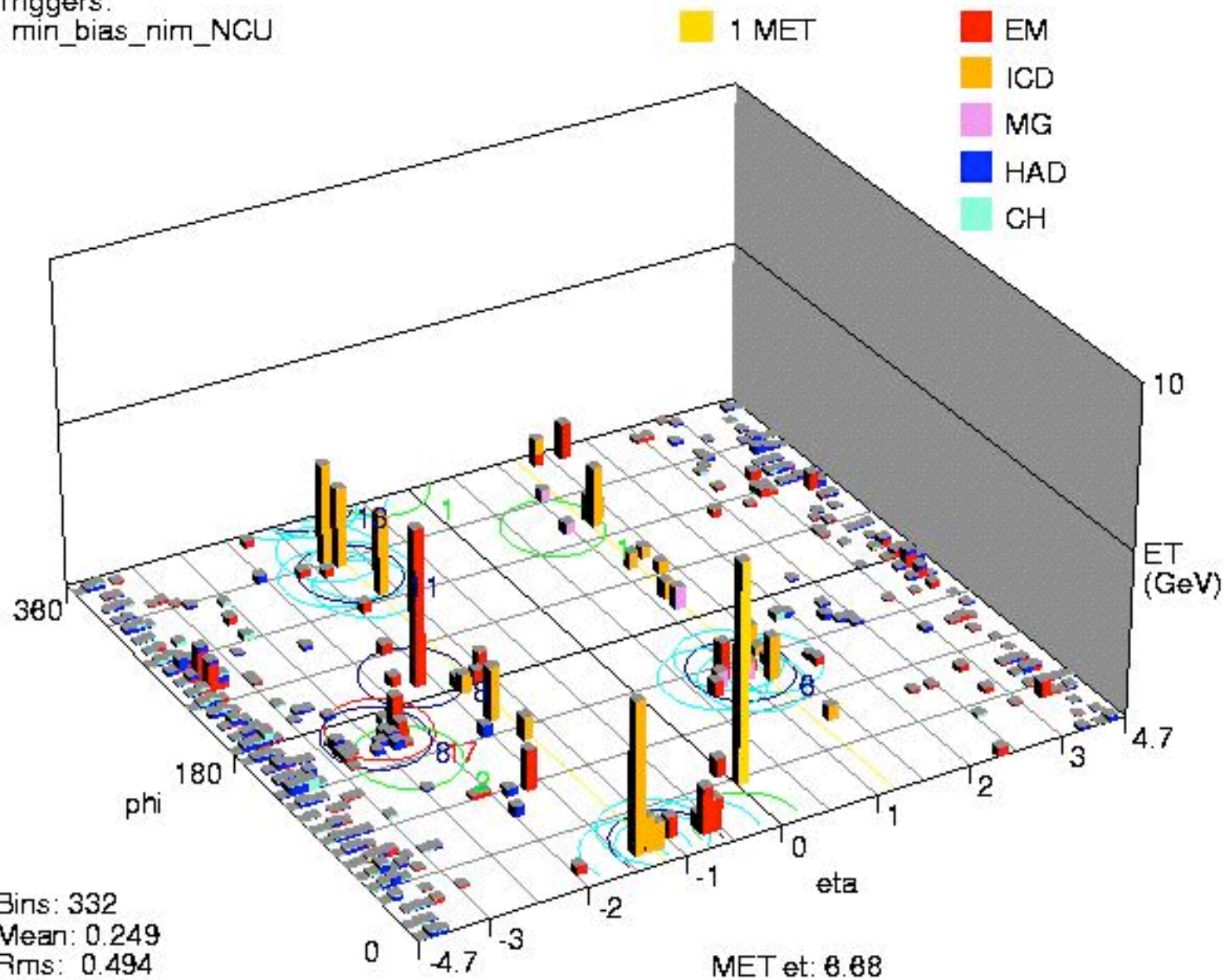
as a tower on each unit square of the  $(\theta, \phi)$  plane. This plot gives a more direct picture of quark and gluon hard scattering.

In the advanced class, we use **pseudo-rapidity** instead of  $\theta$  ,

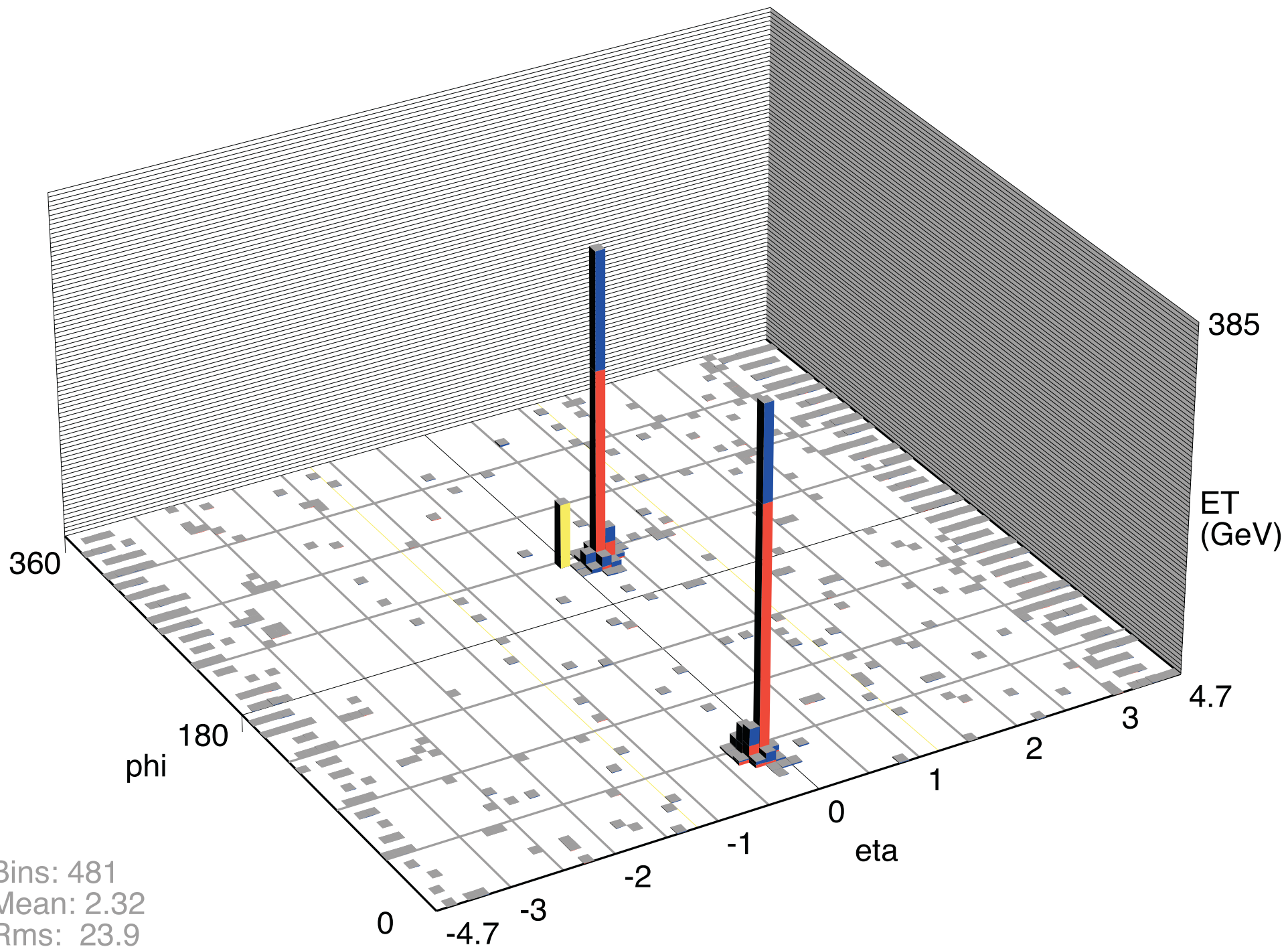
$$\eta = \frac{1}{2} \log \frac{1 + \cos \theta}{1 - \cos \theta}$$

Triggers:

min\_bias\_nim\_NCU

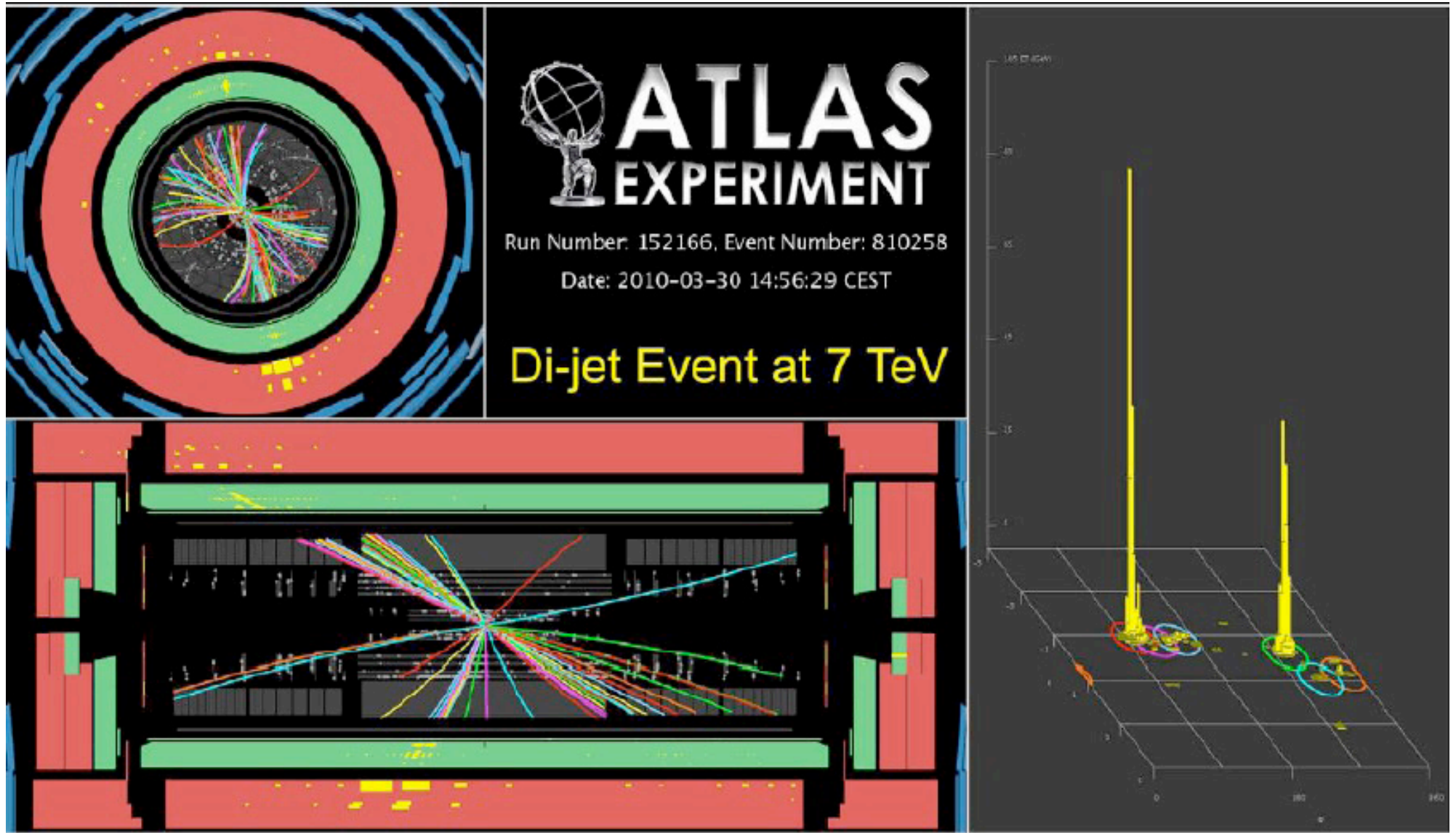


lego plot of D0 event



Bins: 481  
Mean: 2.32  
Rms: 23.9  
Min: 0.00933  
Max: 384

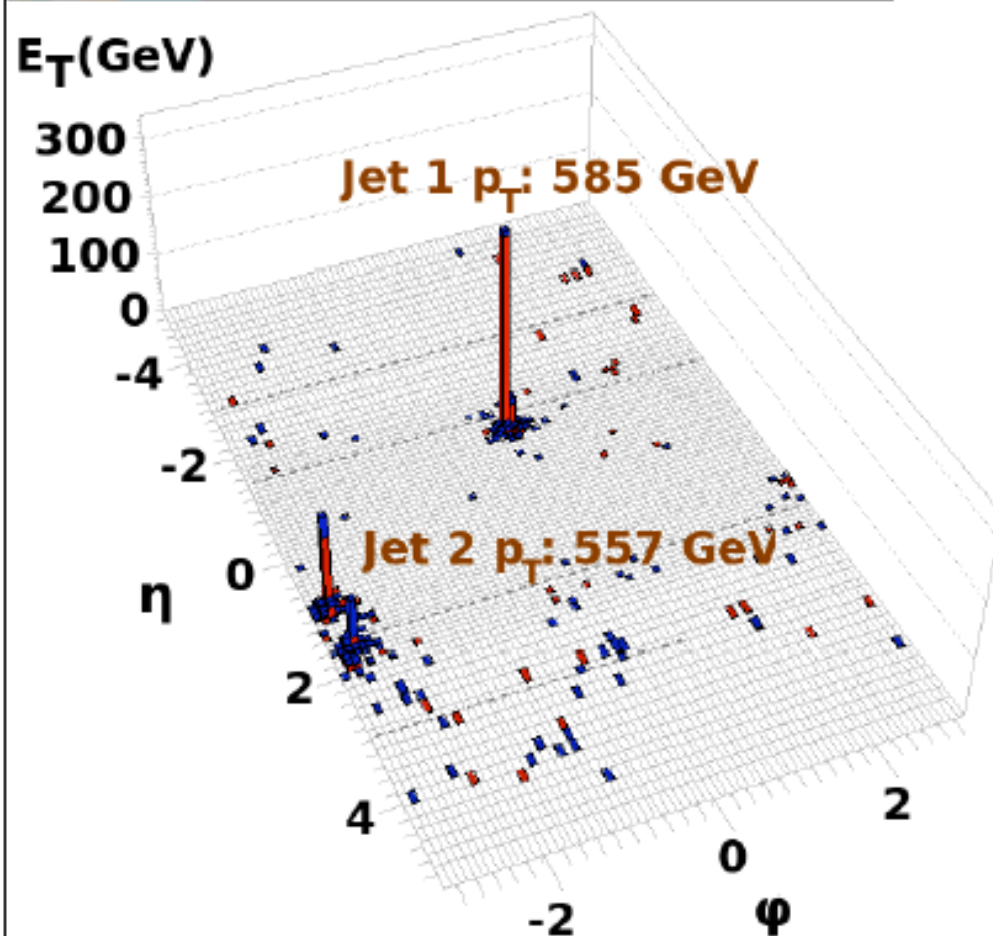
lego plot of D0 event



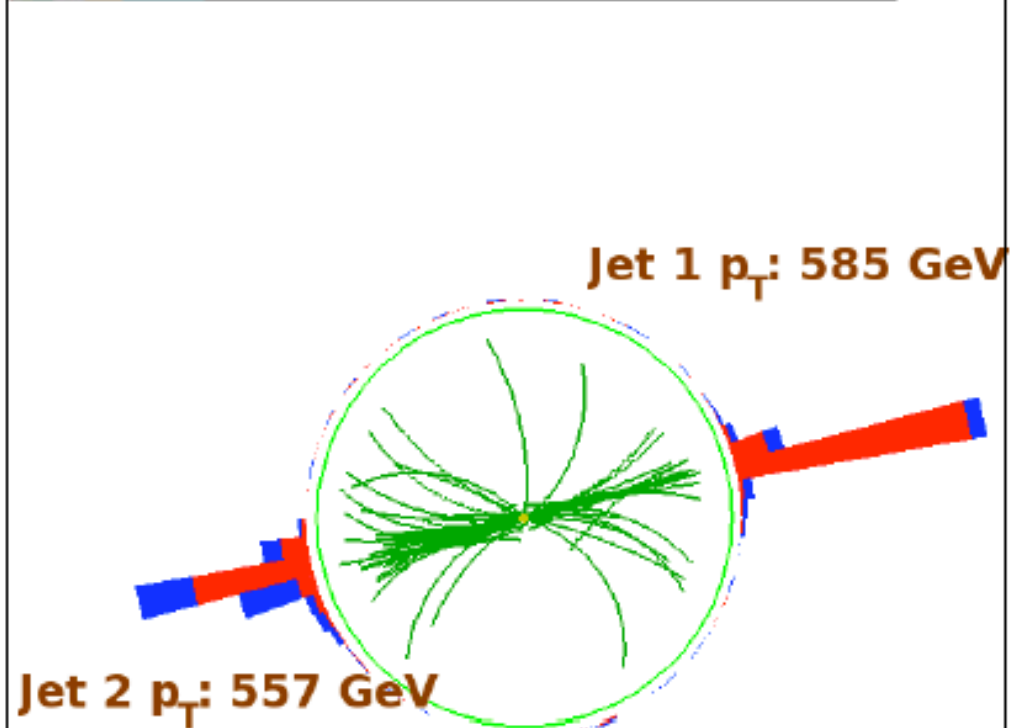
dijet event w. jets of  $ET = 310$  and  $350$  GeV



Run : 138919  
Event : 32253996  
Dijet Mass : 2.130 TeV



Run : 138919  
Event : 32253996  
Dijet Mass : 2.130 TeV



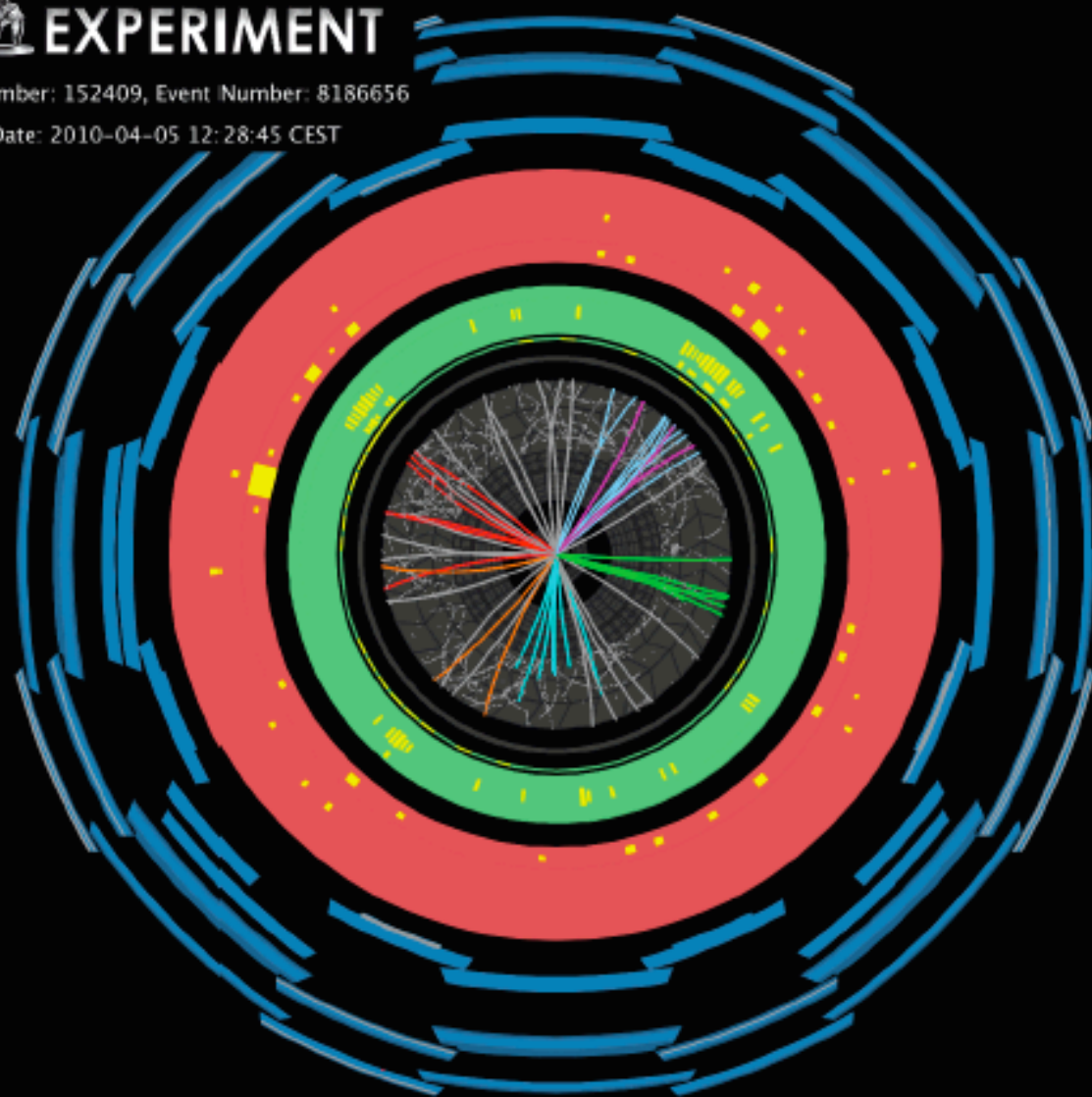


# 6 jets event in ATLAS

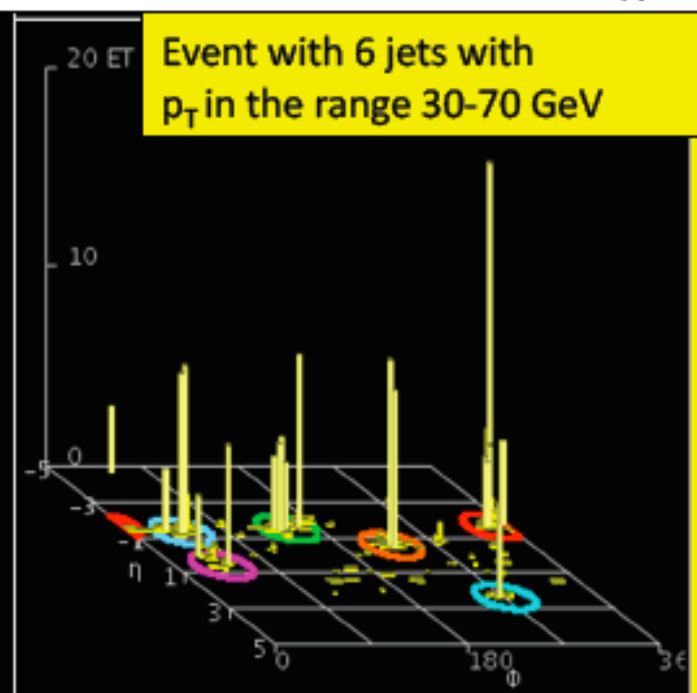
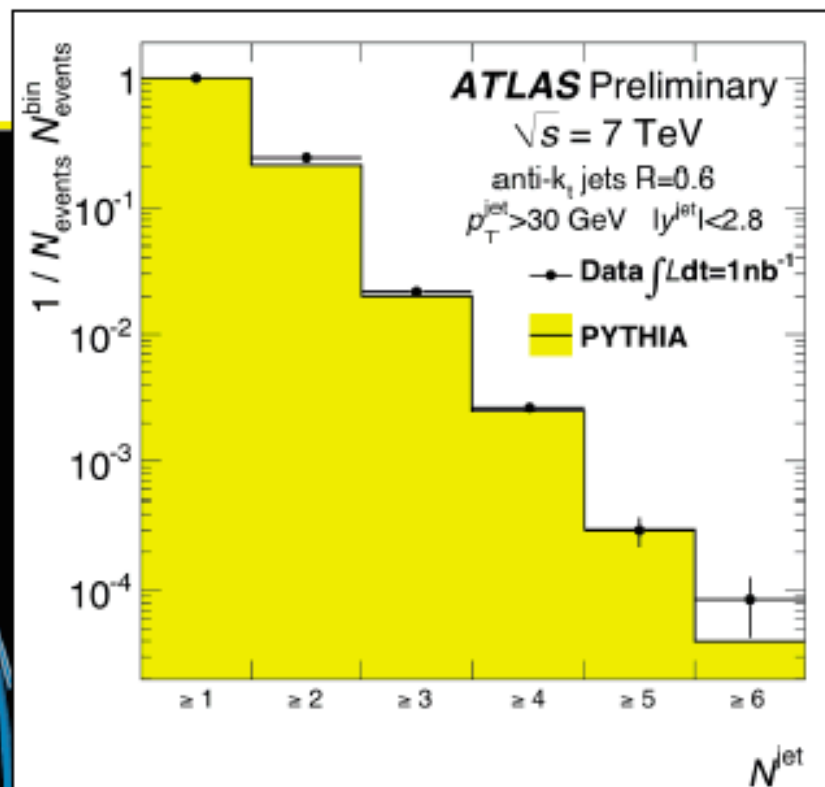


Run Number: 152409, Event Number: 8186656

Date: 2010-04-05 12:28:45 CEST



6 Jet Event in 7 TeV Collisions

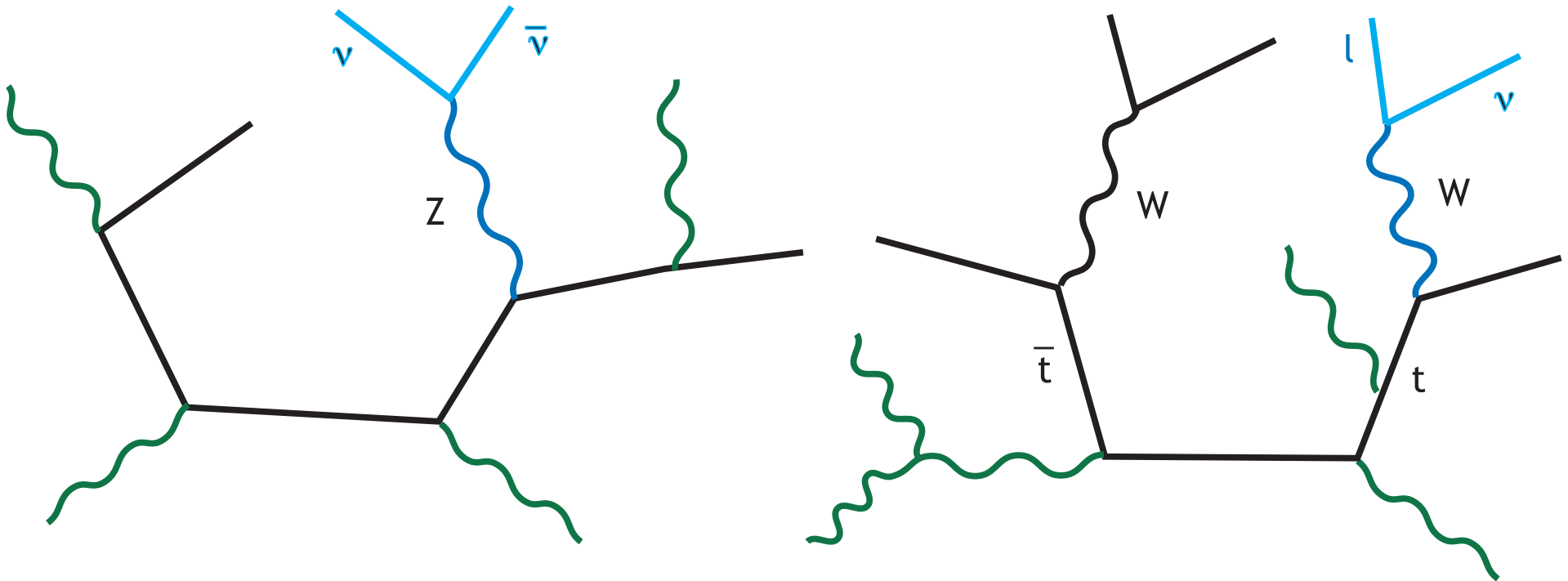


Pure QCD processes are dominantly 2-jet-like and have little unbalanced transverse momentum.

Controlling backgrounds to new physics from these events is mainly a problem of building a detector with no large gaps in solid angle and uniform calibration.

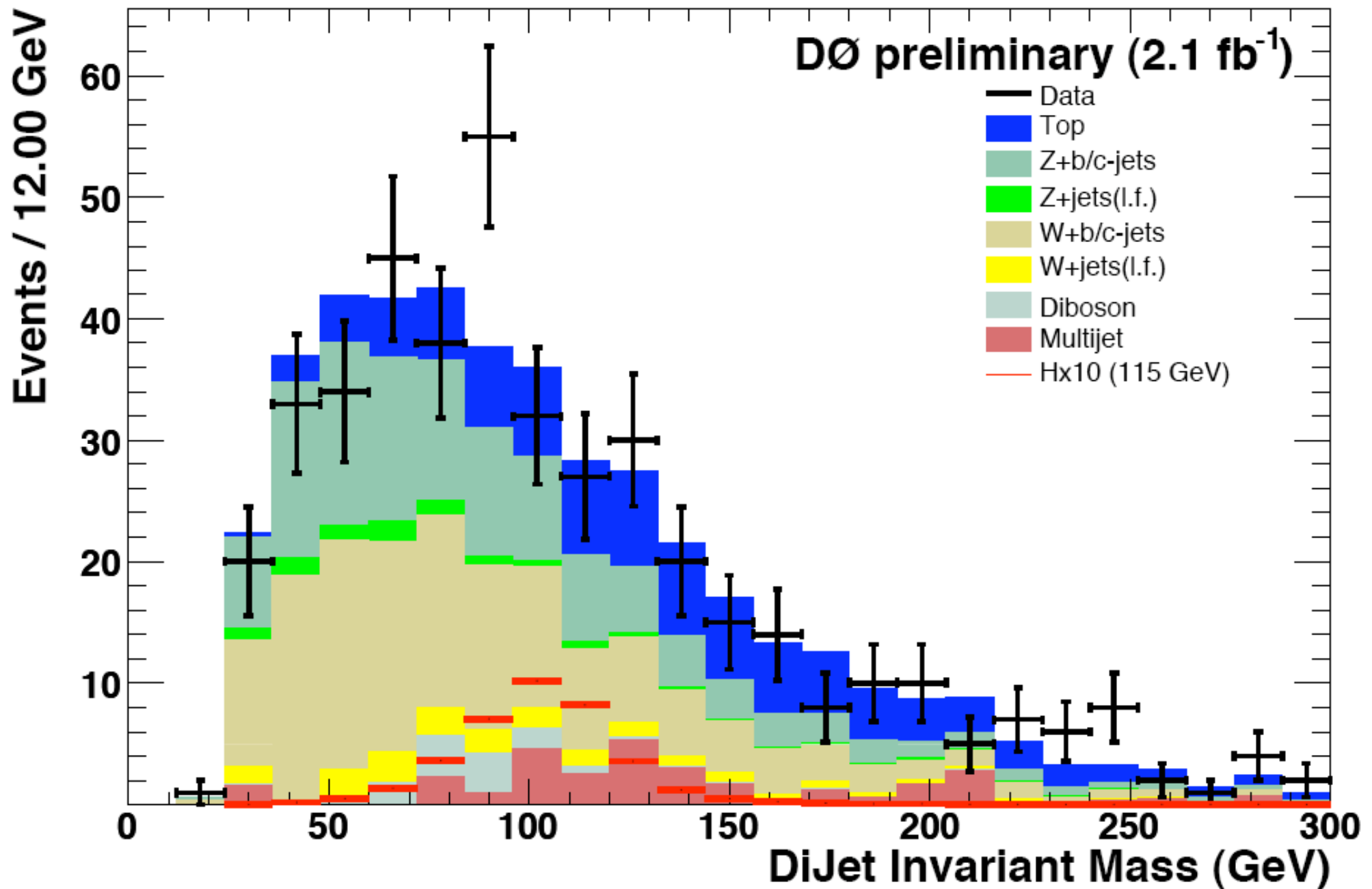
However, we can add QCD processes of the same complexity to reactions with heavy particles -  $W$ ,  $Z$ ,  $t$  - that decay to electrons, muons, and neutrinos.

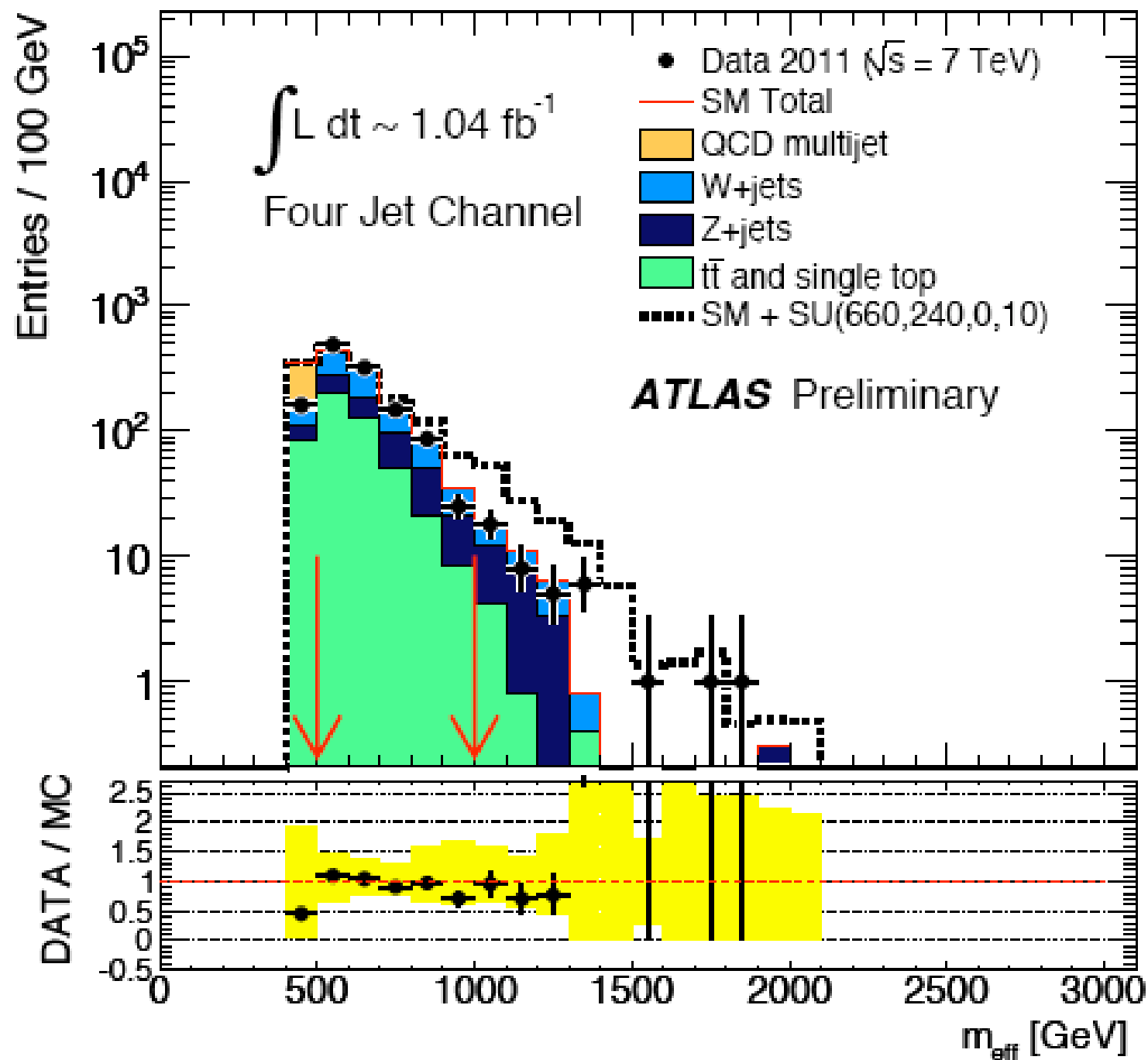
This is genuinely scary. Processes such as



have cross sections comparable to new physics signals and might compete with it.

DØ Higgs search in  $p\bar{p} \rightarrow b\bar{b} + \text{invisible}$  (2008)





For the rest of the lecture, I will discuss:

How can we obtain theoretical control  
over these difficulties ?

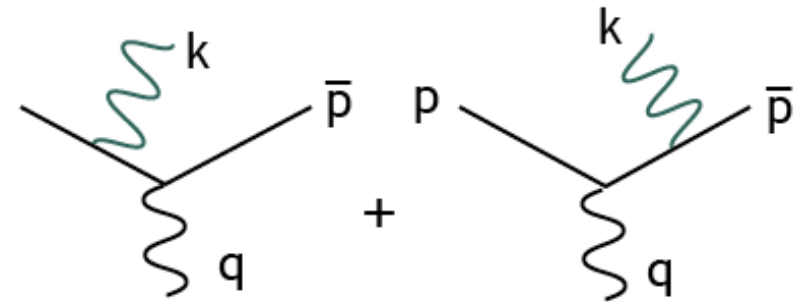
The first question to address is,

What makes up the structure of a jet ?

In the pictures I showed you, much of the development of structure occurs in the regime of asymptotic freedom.  
Can we use QCD to understand how the jet develops ?

Start with the original quark antiquark system emitting one gluon.

Use the fact that we are at high energy, so we can ignore the quark masses.



In this circumstance, these diagrams have an odd property: They diverge if  $k$  soft. But, also, if  $k$  and  $p$  are parallel, then

$$k \sim p \sim (k + p)$$

and all are lightlike, so the diagrams diverge even if  $k$  is not soft.

The basic move is a collinear splitting:

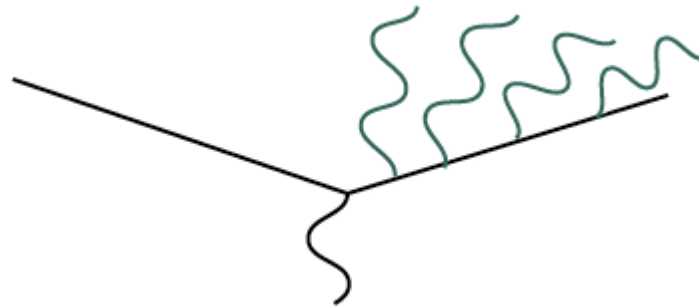
$$(P, \vec{0}, P) \rightarrow (zP, \vec{k}_T, zP) + ((1 - z)P, -\vec{k}_T, (1 - z)P)$$

$$P \rightarrow k + p$$

For one gluon emission,

$$P \sim \frac{2\alpha_s}{3\pi} \int \frac{dk_T}{k_T} \int \frac{dz}{z} (1 + (1-z)^2)$$

but, the logs from the integrals compensate  $\alpha_s$ , so repeat:



The series of emissions is summed by a differential equation, called the Altarelli-Parisi equation.

$$\frac{d}{d \log P_T} \mathcal{P}_g(z, P_T) = \frac{\alpha_s}{\pi} \int \frac{dw}{w} \mathcal{P}_q\left(\frac{z}{w}, P_T\right) P_{q \rightarrow gq}(w)$$

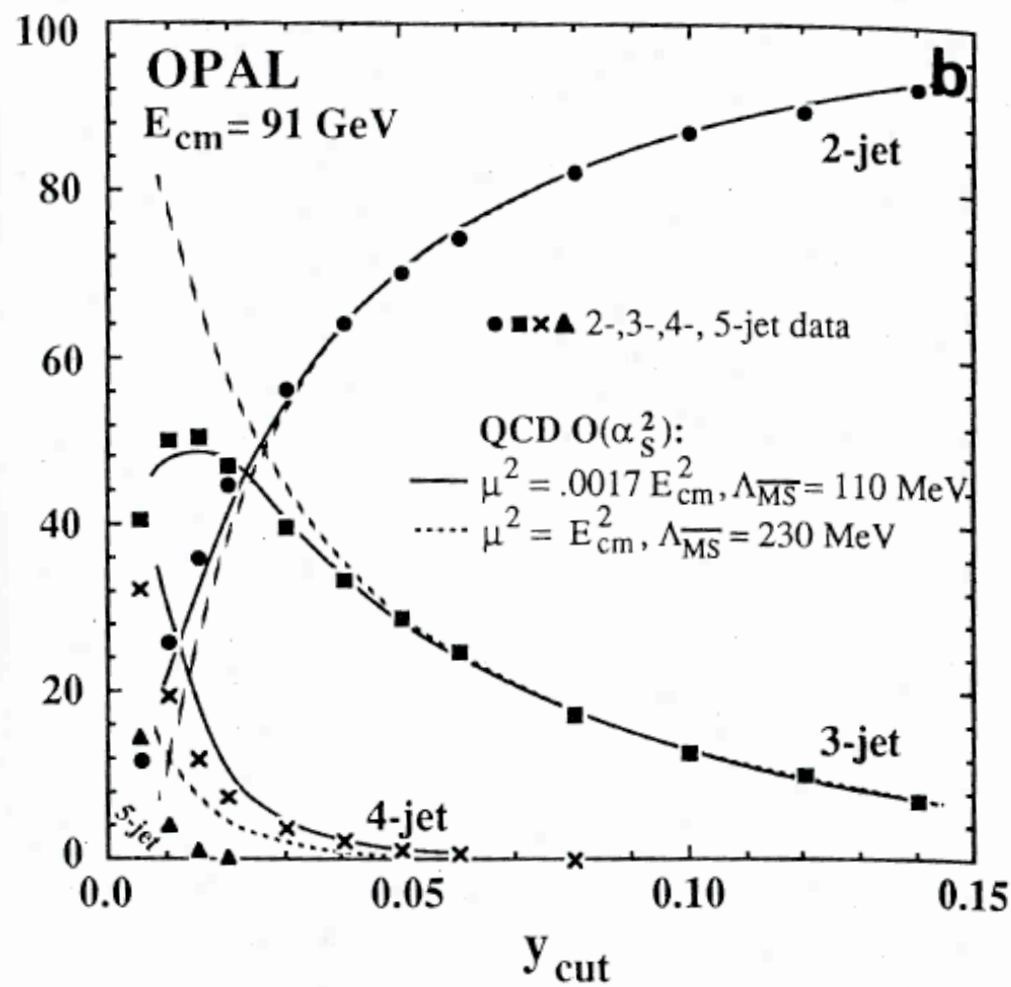
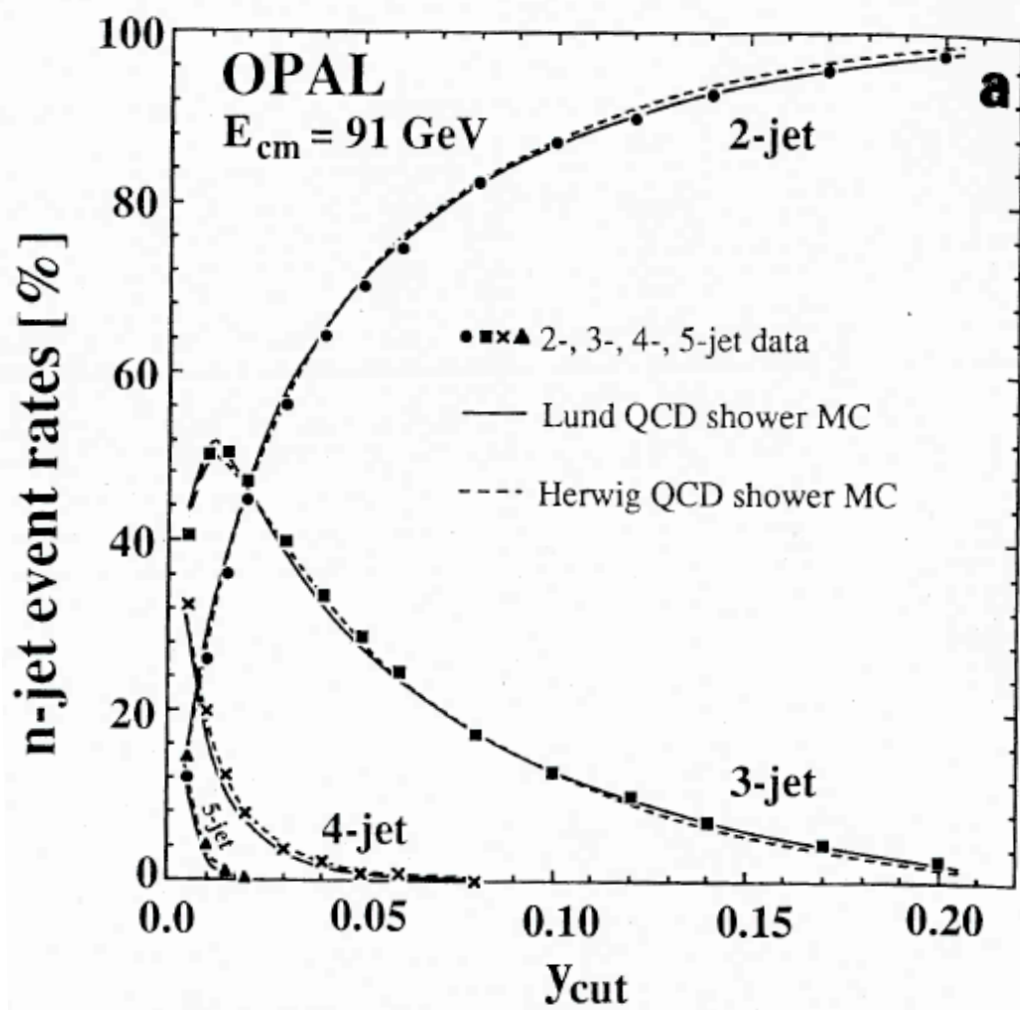
If  $\alpha_s$  were constant, the process would be **scale invariant**, and jets would be **fractals**.

It is a matter of convention whether we cluster an event into 2, 3, 4, .. jets. For example cluster tracks into jets and stop when

$$y = \frac{(M(\text{cluster}))^2}{E_{CM}^2}$$

takes a fixed maximum value.

As we move the boundary, the Altarelli-Parisi equation tells us how the number of jets changes.

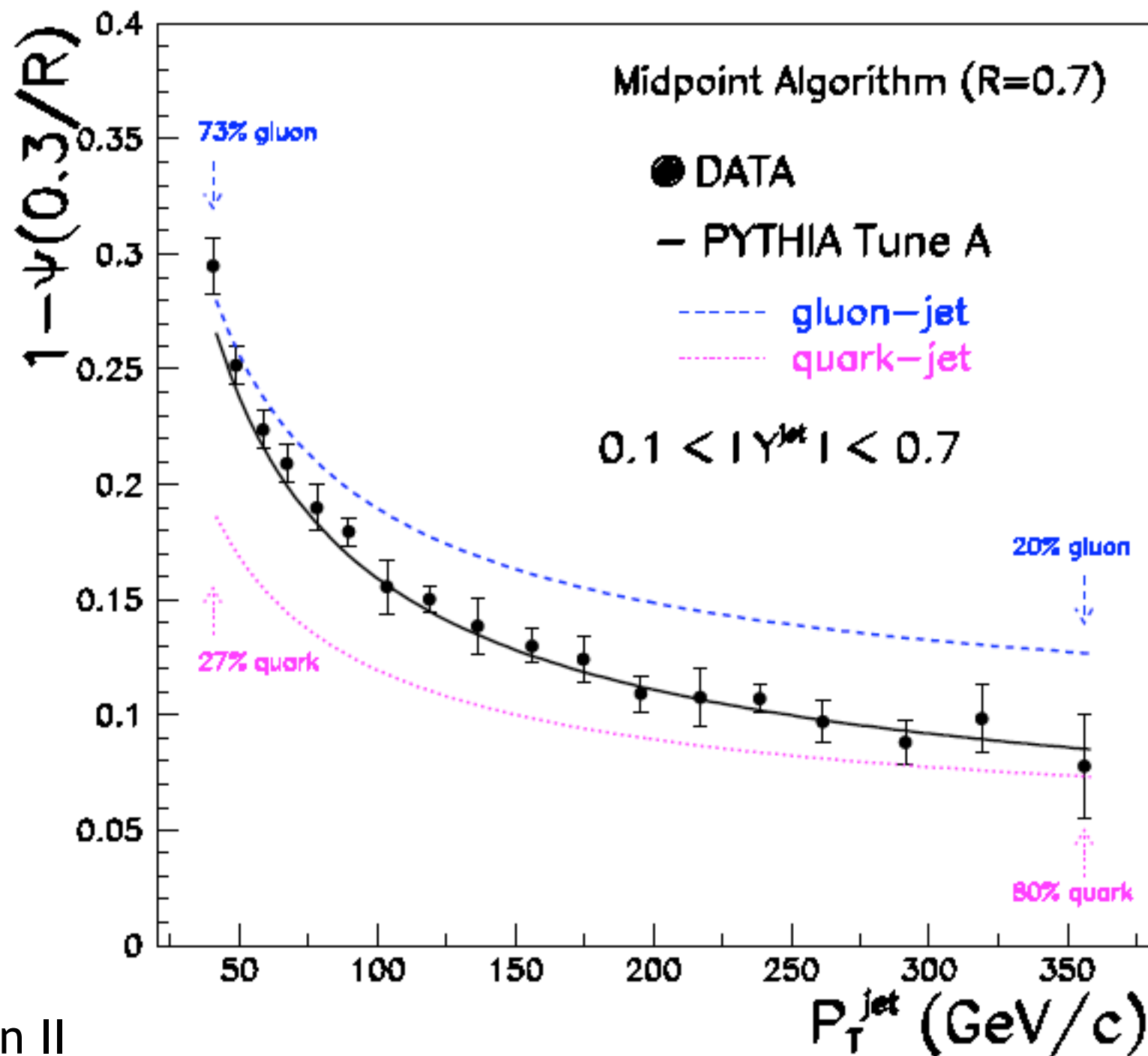


The solution of the Altarelli-Parisi equation can be represented as a Markov process, with quark or gluon emissions at random values but appropriately distributed values of  $(k_t, z)$ .

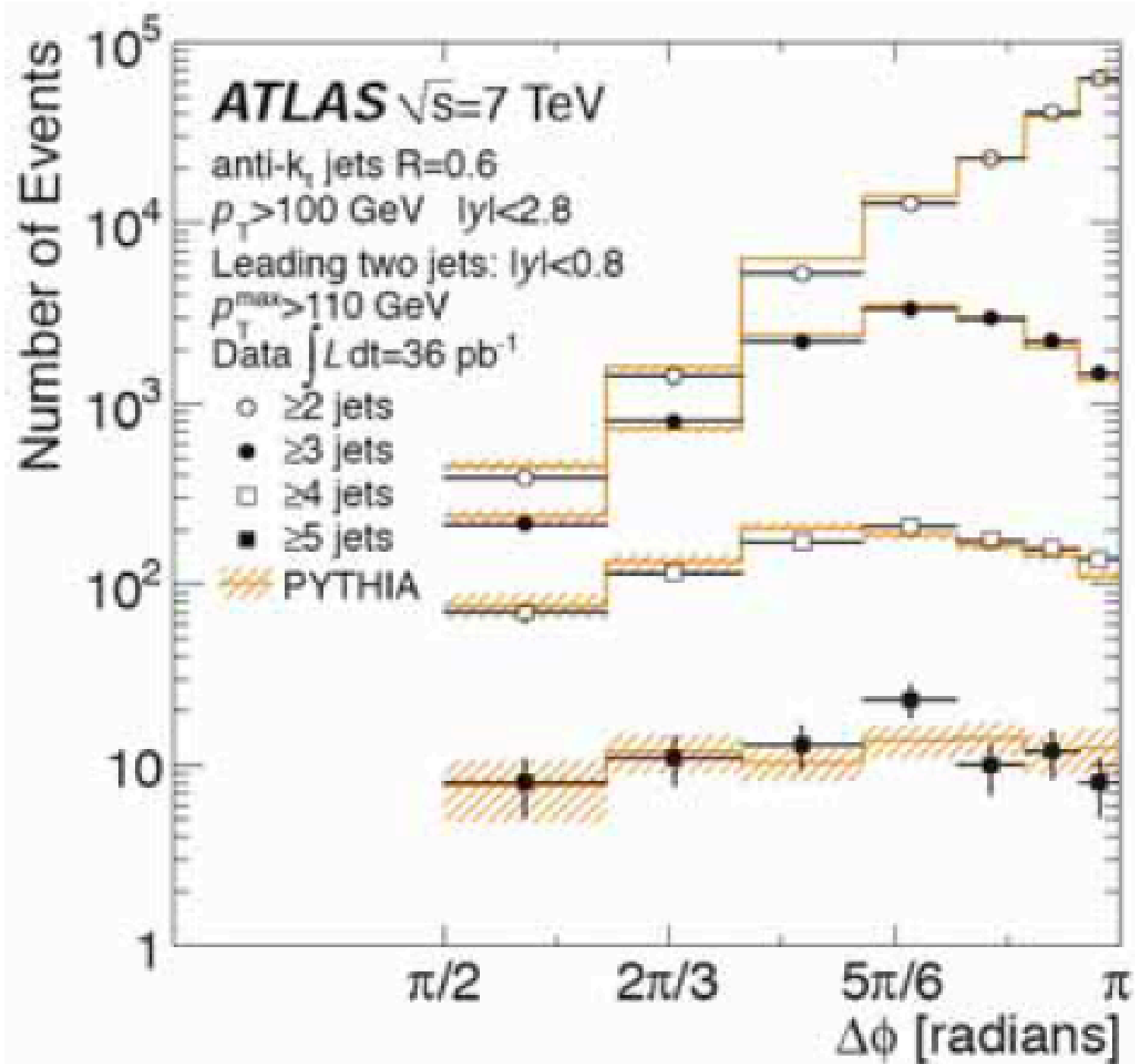
A computer program that implements this process gives us a model of jets:

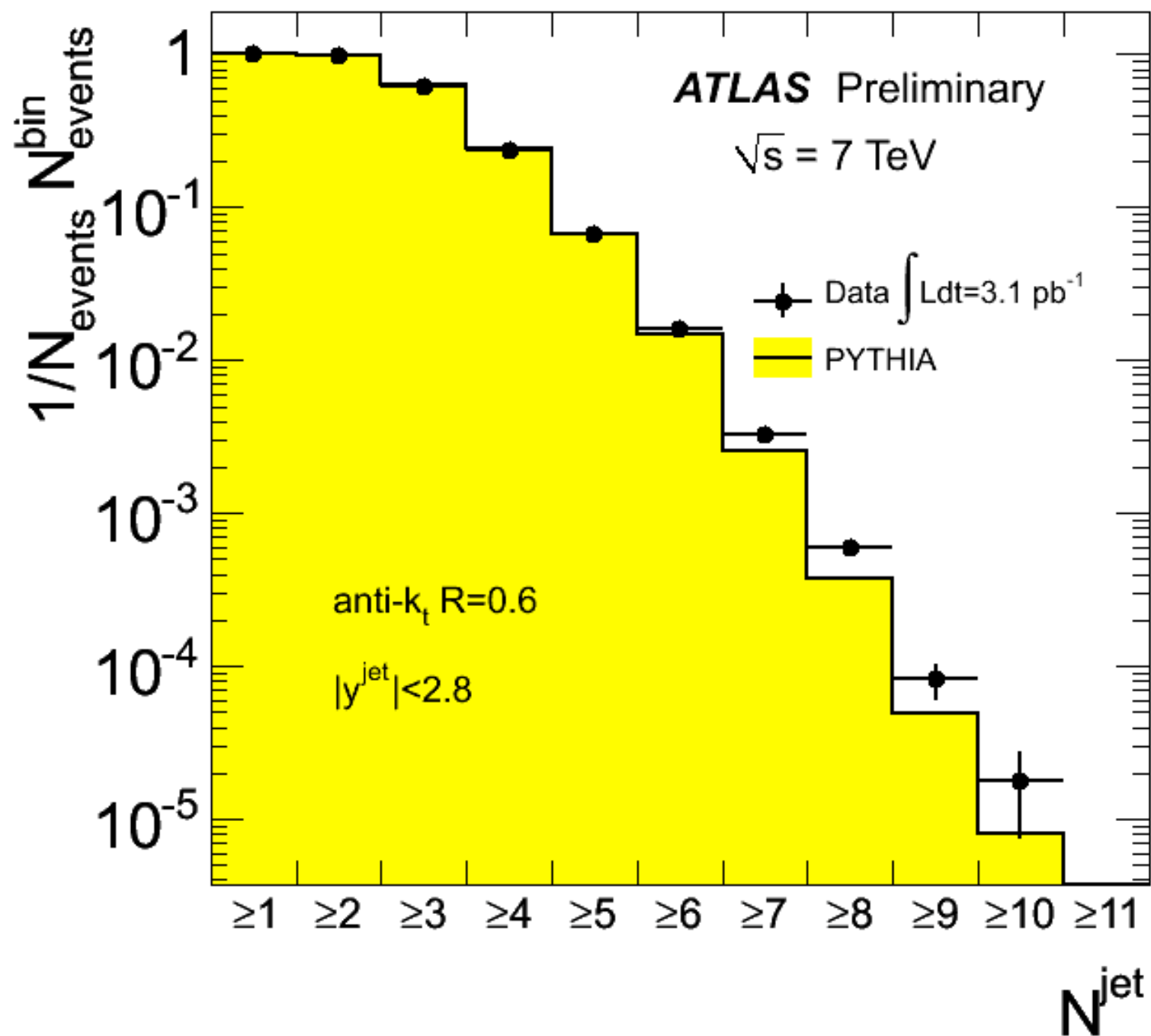
**HERWIG** -- Webber

**PYTHIA** -- Sjostrand



CDF, run II





So far, so good.

But, not good enough. The parton shower is accurate for small-angle emissions that make up the general structure of jets, but is inaccurate for wide-angle emissions. It is these emissions, though, that give complex events that are backgrounds to new physics.

To treat these emissions, we need to compute complex Feynman diagrams completely.

Fortunately, this task is simplified if we deal only with massless particles. Berends and Wu introduced a wonderful technique to exploit this.

They noted that computations with **massless** particles can be dramatically simplified by the use of **spinors of lightlike momenta**

$$|1\rangle = u_R(1) \quad [1] = u_L(1) \quad \langle 1| = \bar{u}_L(1) \quad [1| = \bar{u}_R(1)$$

These objects are related to more familiar objects by

$$|1\rangle [1| = \frac{1}{2} (1 + \gamma^5) \not{1}$$

The spinor products are square roots of Lorentz vector products:

$$\langle 12 \rangle = \bar{u}_L(1) u_R(2) \quad [12] = \bar{u}_R(1) u_L(2)$$

$$|\langle 12 \rangle|^2 = |[12]|^2 = 2k_1 \cdot k_2$$

The spinor products are antisymmetric. They obey the following useful identities:

$$\langle 1 \gamma^\mu 2 \rangle [3 \gamma_\mu 4] = 2 \langle 13 \rangle [42] \quad \text{Fierz}$$

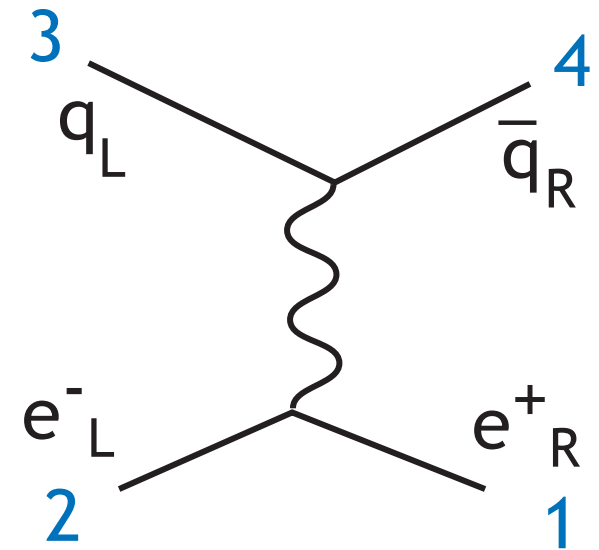
$$\langle 12 \rangle \langle 34 \rangle + \langle 13 \rangle \langle 42 \rangle + \langle 14 \rangle \langle 23 \rangle = 0 \quad \text{Schouten}$$

It is simplest to label the helicities as if all particles were outgoing.  
An incoming L corresponds to an outgoing R.

Try this on our first diagram, in the polarization state

$$e_L^- e_R^+ \rightarrow q_L \bar{q}_R$$

$$\begin{aligned} i\mathcal{M} &= (-ie)^2 \langle 1\gamma^\mu 2 \rangle \frac{-i}{(1+2)^2} \langle 3\gamma_\mu 4 \rangle \\ &= 2ie^2 \frac{\langle 12 \rangle [42]}{|\langle 12 \rangle|^2} \end{aligned}$$



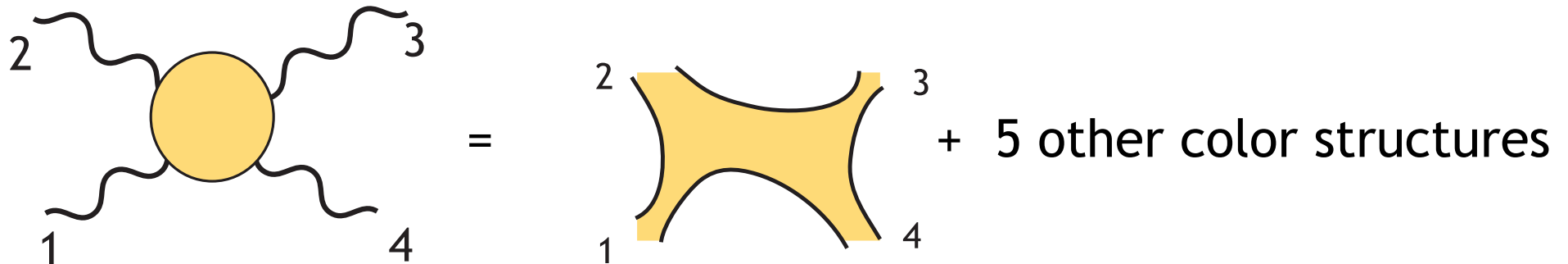
Then

$$|\mathcal{M}|^2 = 4e^4 \frac{(4 \cdot 2)^2}{(1 \cdot 2)^2} = e^4 (1 + \cos \theta)^2$$

which is correct !

Similarly tricky methods make similar simplifications for external photon and gluons.

Apply this technology to QCD. It is convenient to decompose QCD amplitudes into color structures.



The diagram shows a yellow circle with four wavy external lines labeled 1, 2, 3, and 4. This is equal to a yellow four-pointed star-like shape with the same external lines, plus five other color structures. The equation is:

$$\text{Diagram 1} = \text{Diagram 2} + 5 \text{ other color structures}$$

$$= \text{tr}[T_1 T_2 T_3 T_4] i\mathcal{M}_1 + 5 \text{ more}$$

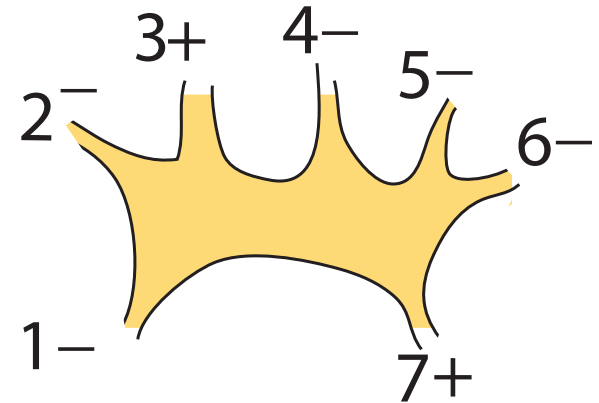
The color-ordered amplitudes can be computed using color-ordered Feynman rules. Using the technology above, we can compute color-ordered amplitudes with definite helicity external particles (all outgoing).

Some remarkable results are obtained.

Parke and Taylor showed that there is a general property that applies to tree amplitudes with arbitrarily many gluons:

Notate:

$$i\mathcal{M}(1^-, 2^-, 3^+, 4^-, 5^-, 6^-, 7^+) =$$



Then:

All amplitudes with all + or only one - vanish. Similarly, all amplitudes with all - or only one + vanish.

Amplitudes with two - and all the rest + have the following simple form:

$$i\mathcal{M}(1^+ \dots i^- \dots j^- \dots n^+) = ig^{n-2} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$

These are called **Maximum Helicity Violating (MHV) amplitudes**.

There are connections here to some deep ideas.

In supersymmetric Yang-Mills theory, amplitudes with all-+ or one - helicity vanish by the supersymmetric Ward Identities.

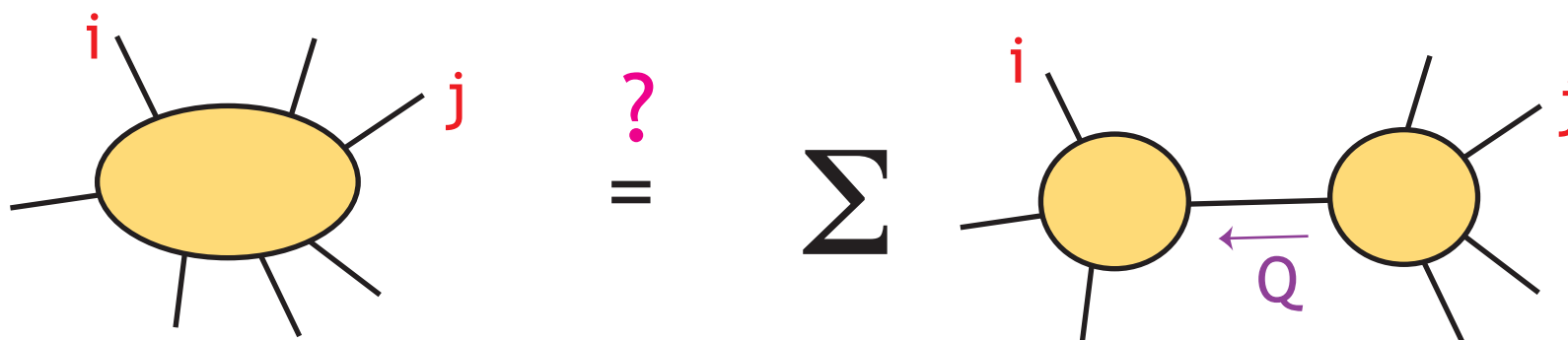
Ordinary YM is not supersymmetric, but it is an orbifold of a supersymmetric theory. That is good enough for tree amplitudes.

The representation of lightlike momenta by spinors invites the use of twistors as variables.

Witten suggested that MHV amplitudes are holomorphic functions of twistors and that they can be computed by a string theory in twistor space.

Starting from this insight, **Britto, Cachazo, and Feng** showed how to use MHV amplitudes as building blocks of more general QCD amplitudes.

It would be wonderful to build up non-MHV amplitudes from the simple MHV expressions. But it is not so obvious how to do this. MHV amplitudes are physical expressions, but if we cut the more complex amplitudes, we will have to evaluate them off the mass shell, in an unphysical configuration.



Or do we ? BCF suggested that we pick legs **i** and **j** and shift

$$i] \rightarrow i] + z j] \quad j\rangle \rightarrow j\rangle - z i\rangle$$

for  $z$  a complex variable. Then the shifted  $Q$  can be on-shell.

Now consider (BCF+Witten)

$$\oint \frac{dz}{2\pi i} \frac{i\mathcal{M}(z)}{z} = i\mathcal{M}(z=0) + (\text{other poles}) = (\text{contour at } \infty)$$

The first term is the amplitude that we wish to evaluate. The contour at  $\infty$  vanishes if we choose **i** and **j** correctly (e.g **i** a - gluon and **j** a + gluon). The additional poles result when a momentum on an intermediate line satisfies

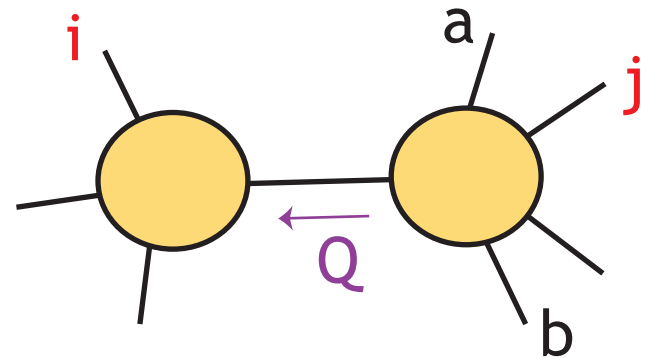
$$Q(z)^2 = 0$$

Looking again at the diagram,

$$Q^\mu(z)\gamma_\mu = \sum_{k=a}^b k^\mu [k - z i] [j$$

and so

$$z_* = \frac{s_{a\dots b}}{\langle i(\sum k) [k] j \rangle}$$



Tidying up the formula, one finds the following relation:

$$i\mathcal{M}(1 \cdots n) = \sum_{splits} i\mathcal{M}(b+1 \cdots \hat{i} \cdots a-1 - \hat{Q}) \cdot \frac{1}{s_{a \cdots b}} \cdot i\mathcal{M}(a \cdots \hat{j} \cdots b \hat{Q})$$

called the **Britto-Cachazo-Feng-Witten (BCFW) recursion formula**.

Momenta with hats have the shift with  $z_*$ . The hatted momenta are complex but satisfy  $\hat{Q}^2 = 0$ , so the amplitudes on the right-hand side need to be evaluated only at “physical”, but complex, values.

This allows the n-point amplitudes to be recursively evaluated in terms of amplitudes with fewer legs. We can stop when we reach MHV. At 5 points all amplitudes are MHV or anti-MHV.

BCF recursion is a general method for computing QCD tree amplitudes of arbitrarily high order. Even more heavy duty recursion formulae were developed by Berends and Giele, Mangano and Moretti, and others.

BCF recursion can also be used in certain non-renormalizable theories, including gravity and  $N=8$  supergravity.

Unfortunately, the tree level is not good enough.

In QCD, the coupling runs according to

$$\alpha_s(Q) = \frac{\alpha_s(m_Z)}{1 + (b_0 \alpha_s(m_Z)/2\pi) \log(Q/m_Z)}$$

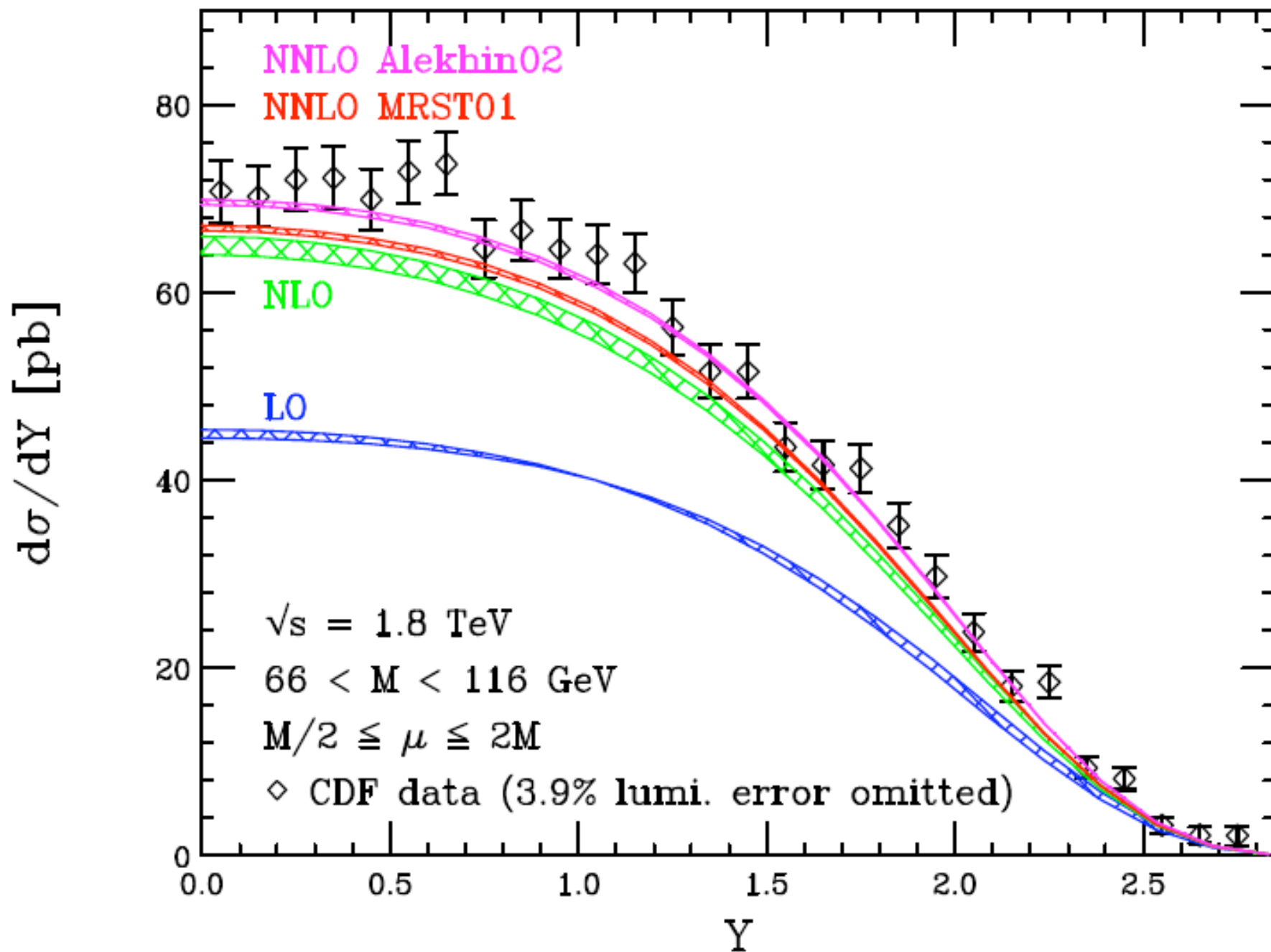
What  $Q$  should we use to compute a given cross section.

Guesses might differ by more than a factor of 2. In a process with  $n$  gluon radiation,  $(\alpha_s(Q))^n$  appear. This can lead in practice to 30-50% uncertainty.

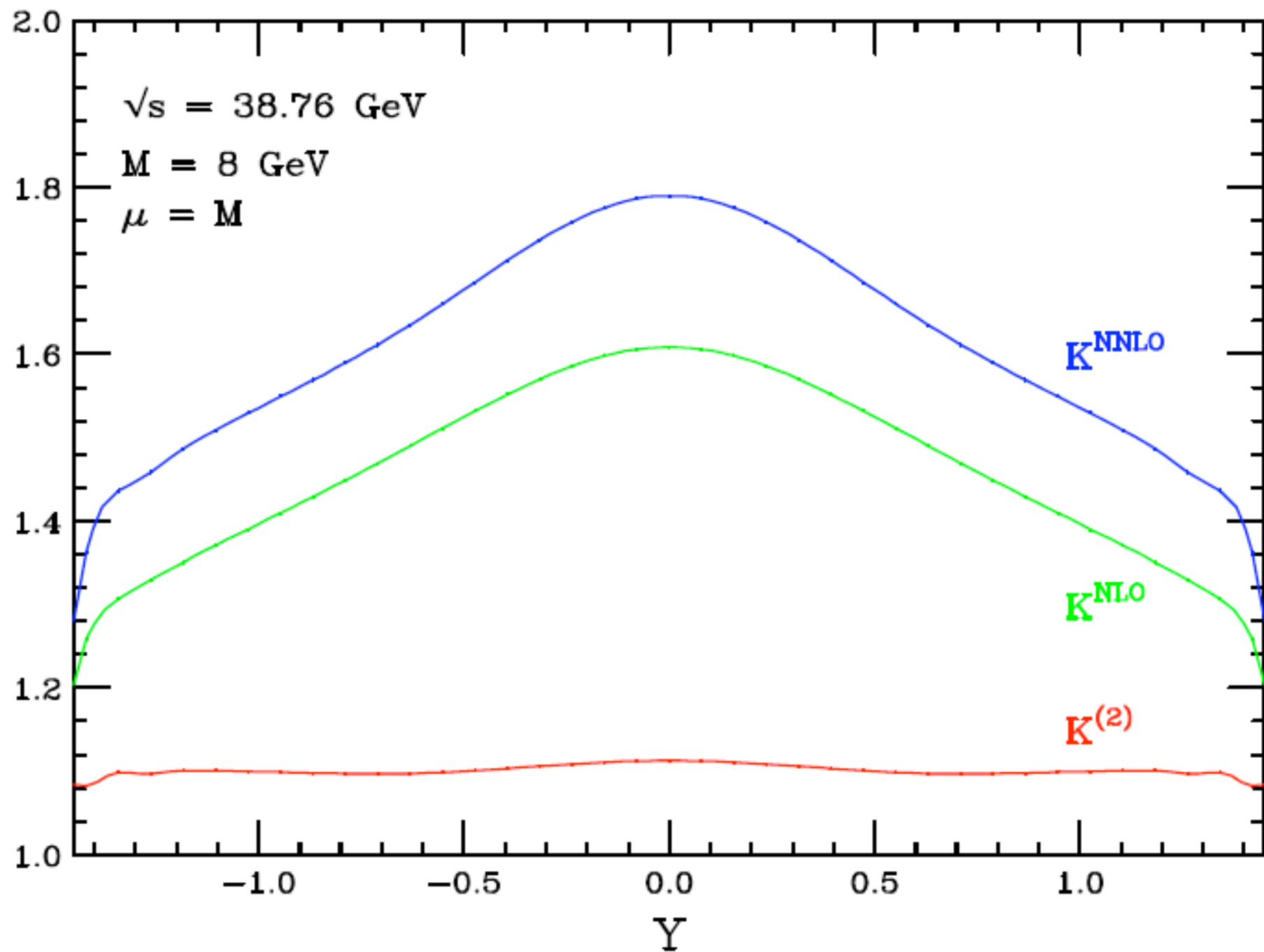
This is called scale ambiguity. It is conventional to estimate the accuracy of a perturbation expansion by varying the renormalization scale for  $\alpha_s(Q)$  by a factor of 2 in either direction.

In many cases, this underestimates the error!

$$p\bar{p} \rightarrow (Z, \gamma^*) + X$$



Anastasiou, Dixon, Melnikov, Petriello



Anastasiou, Dixon, Melnikov, Petriello

Explicit perturbative corrections gives a large coefficient of  $\alpha_s(Q)$ :

so 
$$\frac{8\pi^2}{9} - \frac{7}{3} = 6.44$$

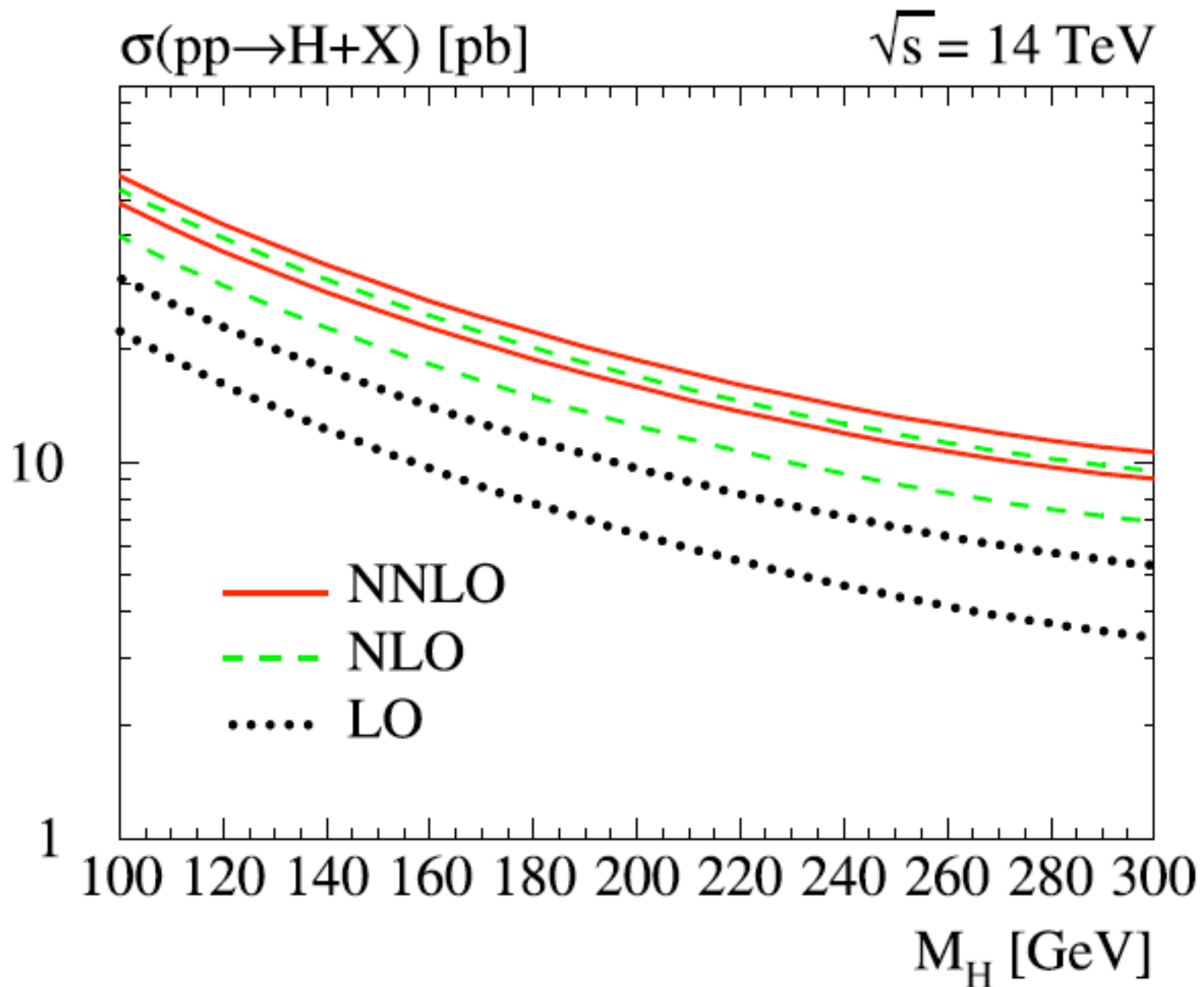
$$1 + \left(\frac{8\pi^2}{9} - \frac{7}{3}\right) \frac{\alpha_s(Q)}{\pi} = \begin{cases} 1.25 & Q = m_W \\ 1.6 & Q = 2 \text{ GeV} \end{cases}$$

The actual correction to the Drell-Yan rate is even larger, because in higher orders there is a new contribution proportional to the proton's gluon distribution.

It is typical the the NLO corrections to hadron collider cross sections give large positive corrections to the rate.

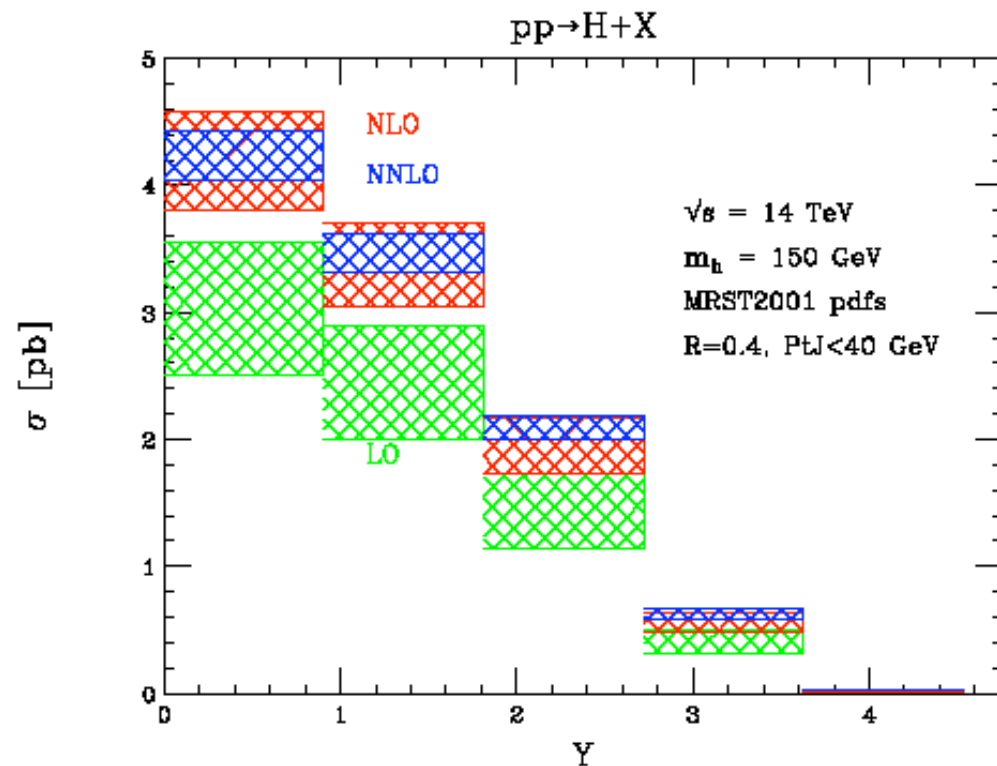
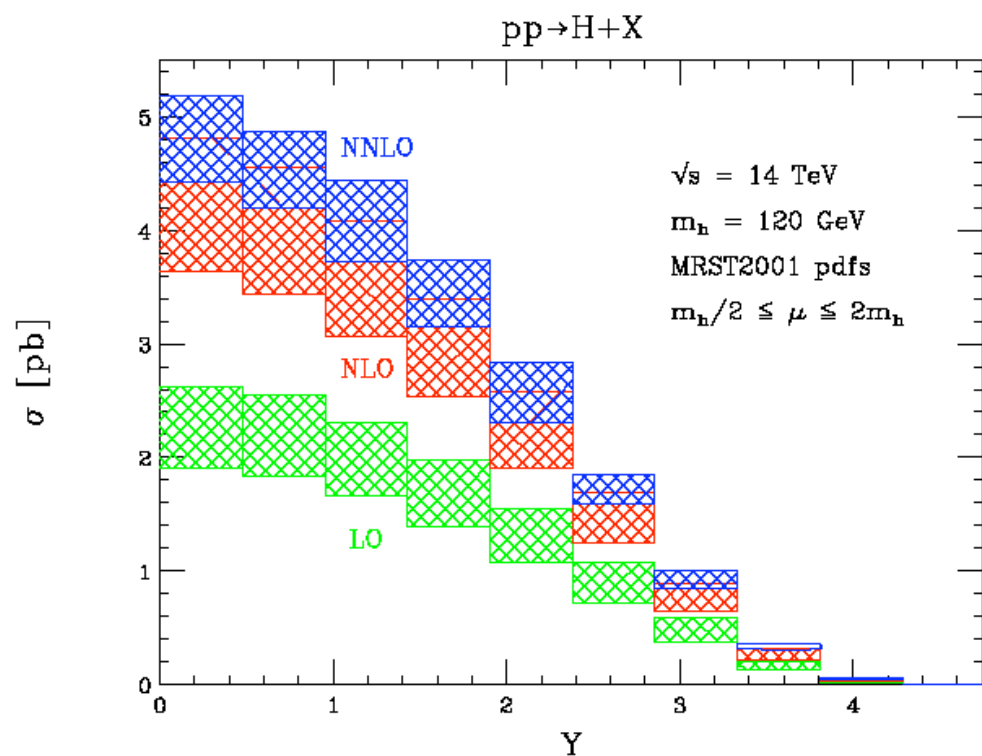
This correction is often expressed as a multiplicative factor on top of the leading order cross section, called a K-factor.

gluon fusion cross section (pb)



Harlander and Kilgore

# rapidity distribution of $gg \rightarrow \text{Higgs}$ , without and with a jet veto



Anastasiou, Melnikov, Petriello

For  $2 > 2$  processes, these NLO calculations can be done by hand. For  $2 > 3$  processes, they already involve thousands of diagrams, with thousands of terms each.

At NNLO, or at NLO for  $2 > 4$  and beyond, brute force Feynman diagram evaluation fails even when done by computers. New methods are needed.

Passarino and Veltman:

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{k_1^2 k_2^2 \cdots k_n^2} = \sum A_\pi \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k_{\pi(1)}^2 k_{\pi(2)}^2 k_{\pi(3)}^2 k_{\pi(4)}^2}$$

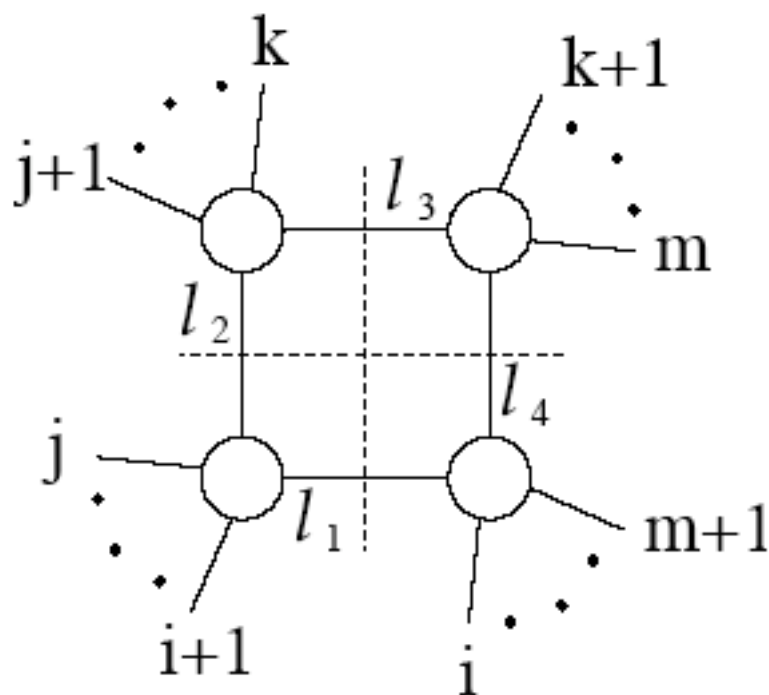
There are a finite number of integrals; do them once and for all.

Bern, Dixon, Dunbar, Kosower:

In N=4 super-Yang-Mills, a much smaller set of integrals appear. These include only boxes, no triangles or bubbles. Identity these by unitary cuts.

Britto, Cachazo, and Feng:

The coefficients can be identified easily by putting 4 intermediate momenta on shell:



Ossola, Papadopoulos, Pittau:

Even in non-supersymmetric QCD, the set of integrals is more limited: boxes, plus a small number of triangles, bubbles, tadpoles.

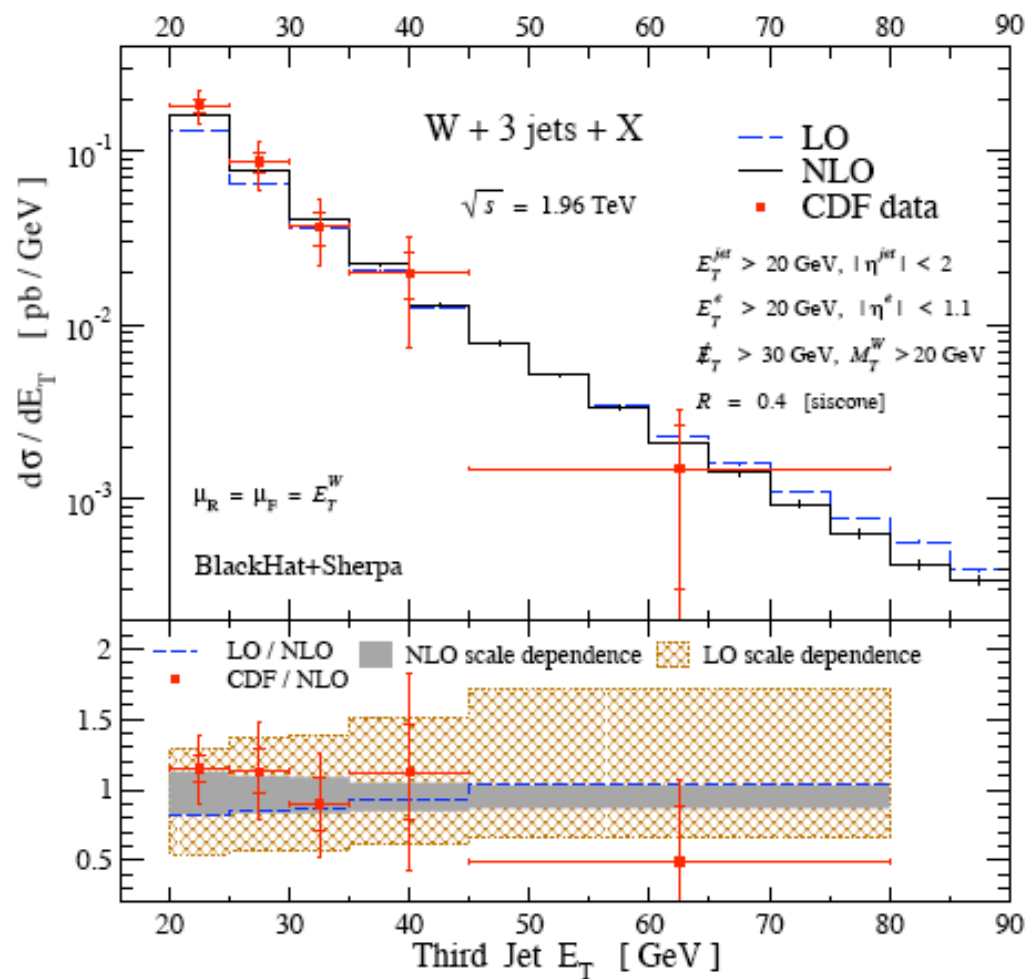
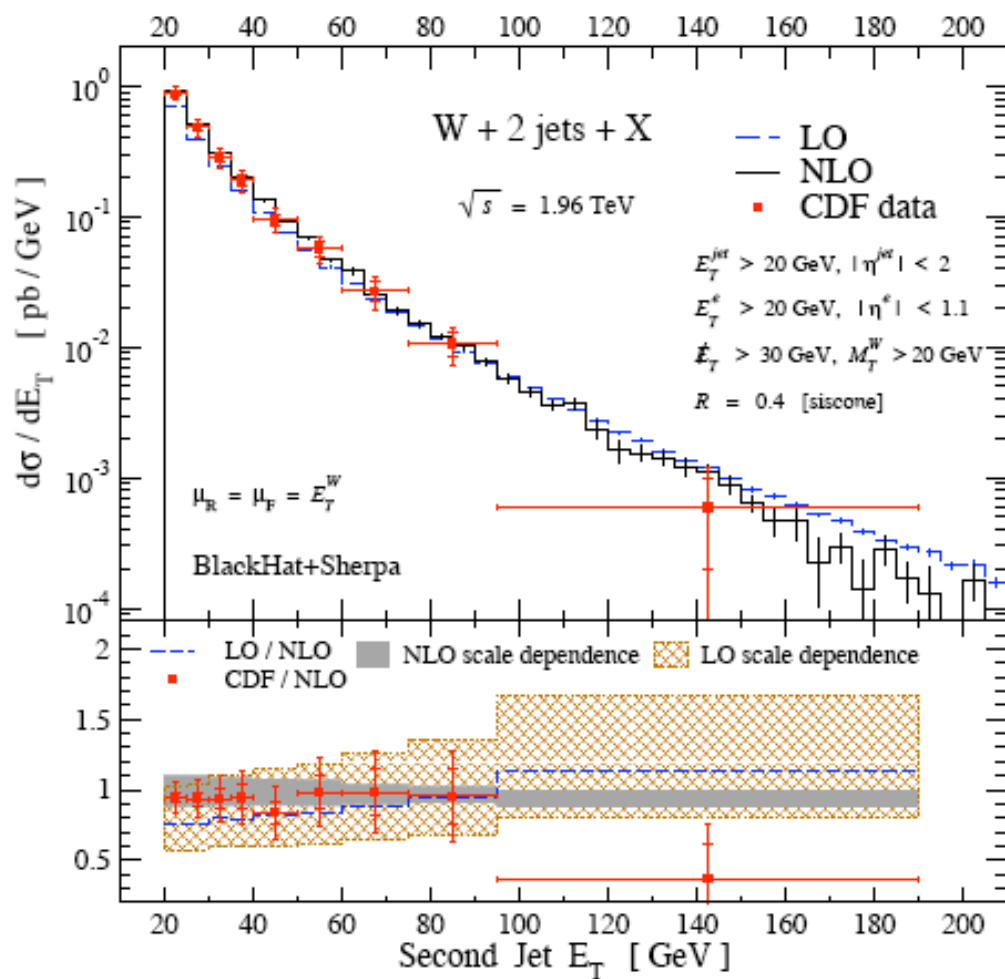
Given a minimal basis of integrals, systematically cutting in all channels identifies the coefficients.

Berger, Forde, ...

Ellis, Kunszt, Zanderighi

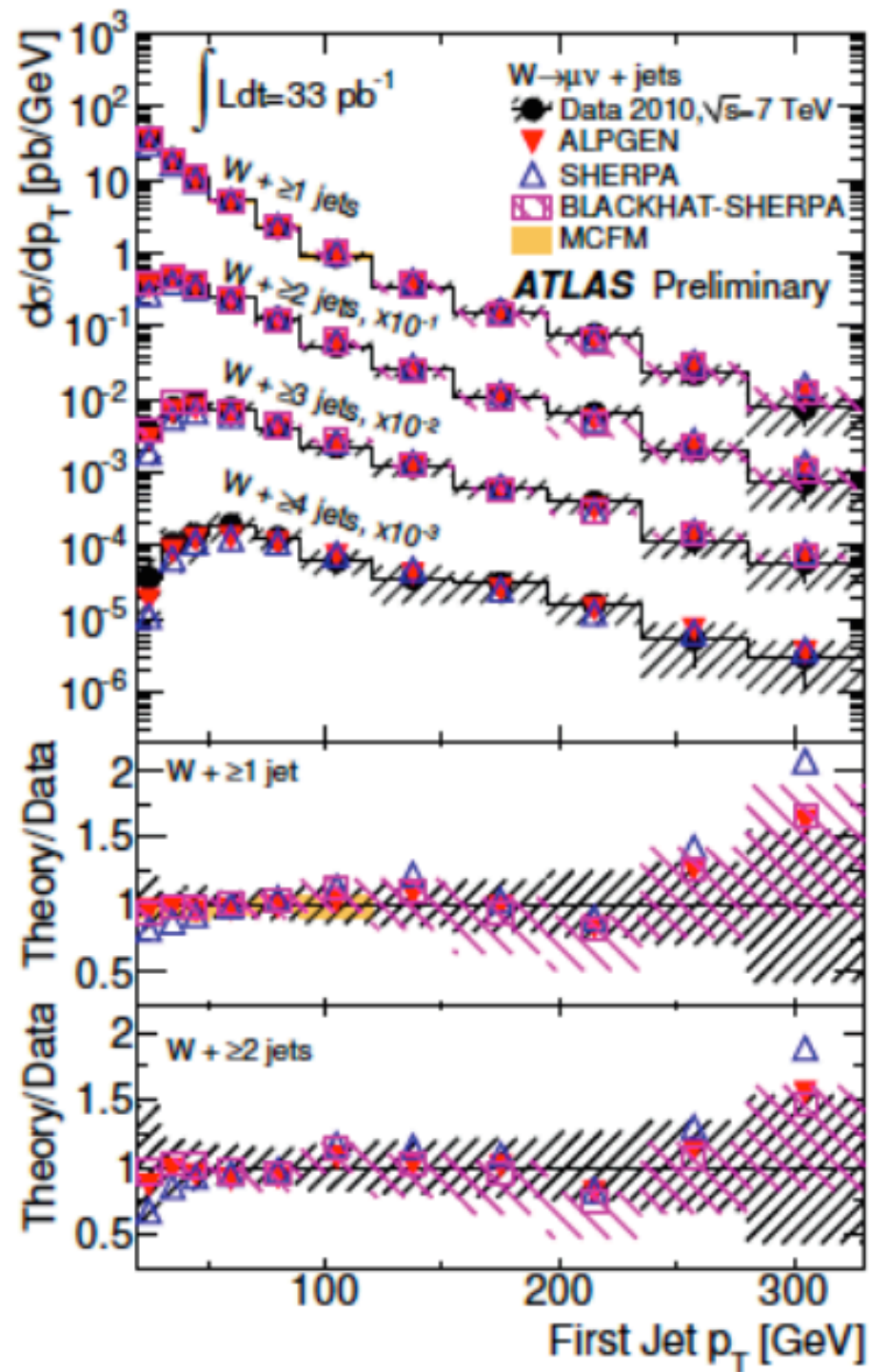
The integrals may also contain terms with no unitary cuts that are rational functions of the kinematic variables. These can also be obtained by systematic algorithms (e.g., extension to  $4 + \epsilon$  dimensions).

Using these tools, it is possible to automate the calculation of NLO contributions to multiparticle processes.

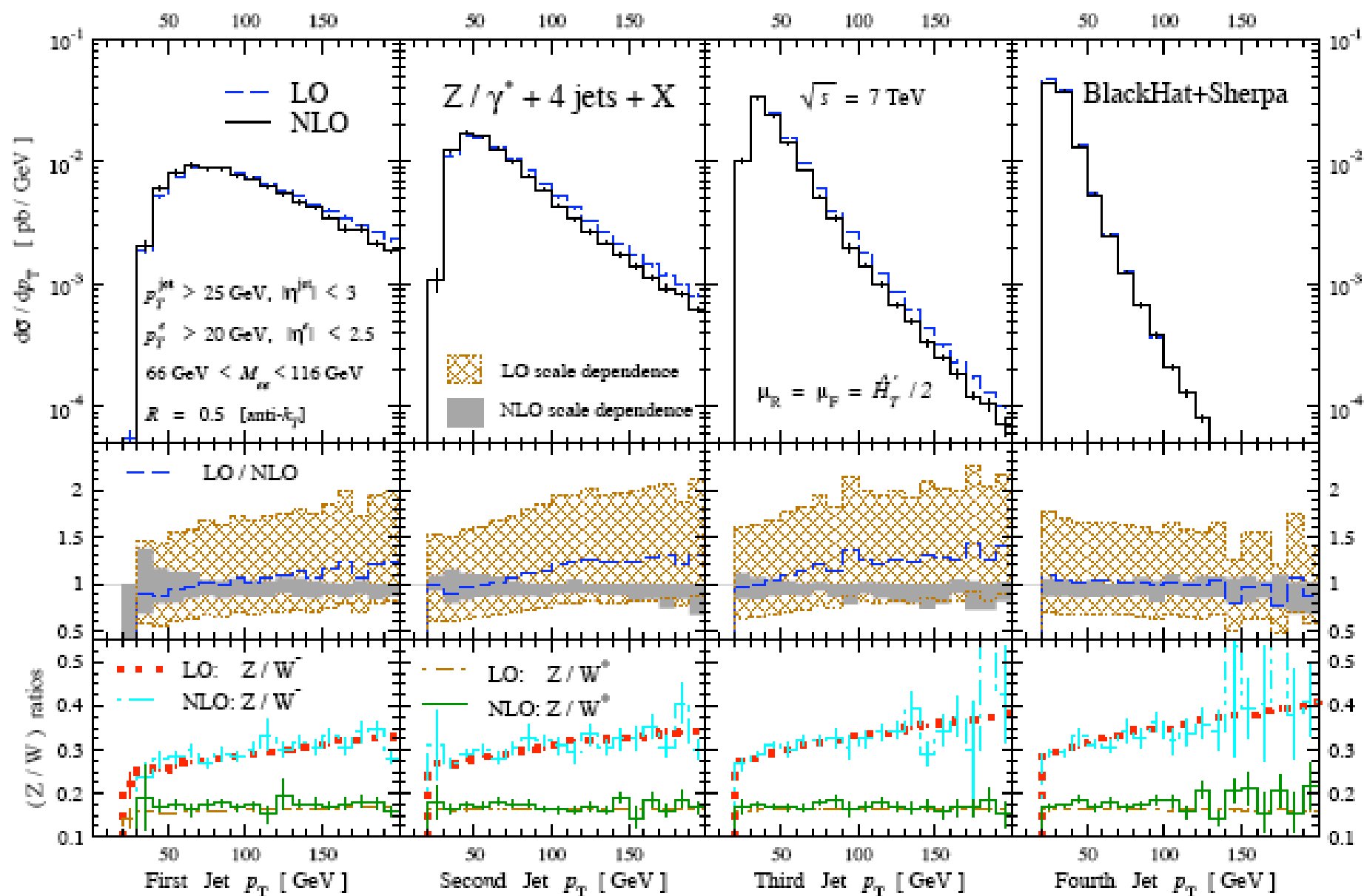


C.Berger et al (BlackHat)

Here is a sample  
theory/data  
comparison in W + jets  
for 1,2,3,4 jets

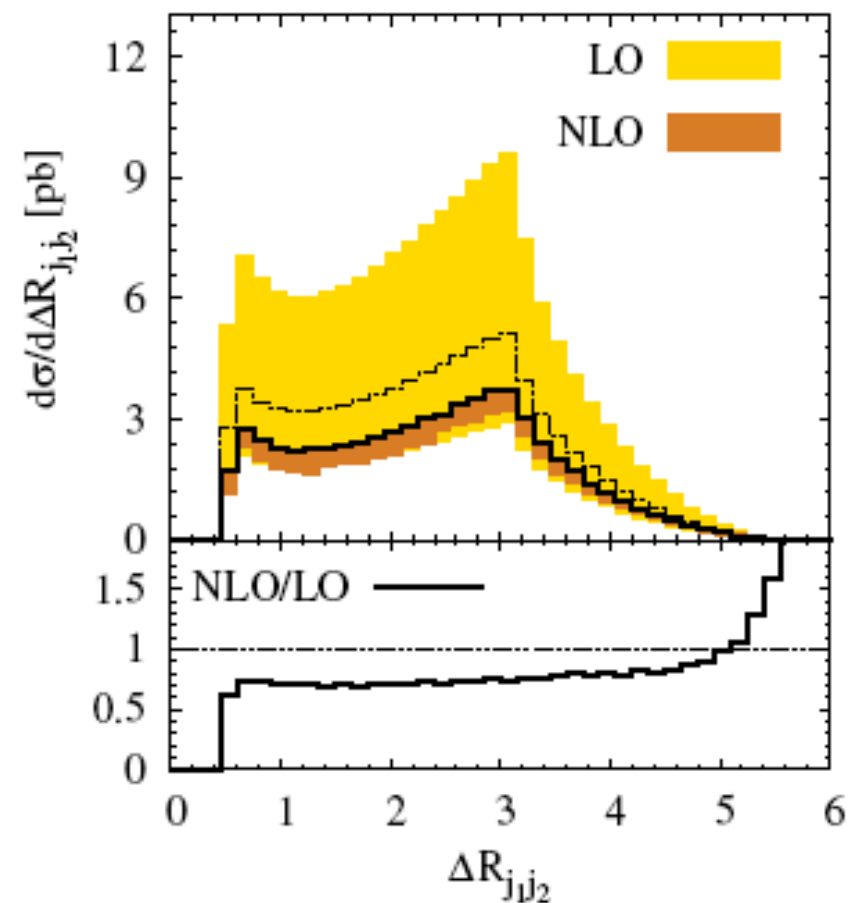
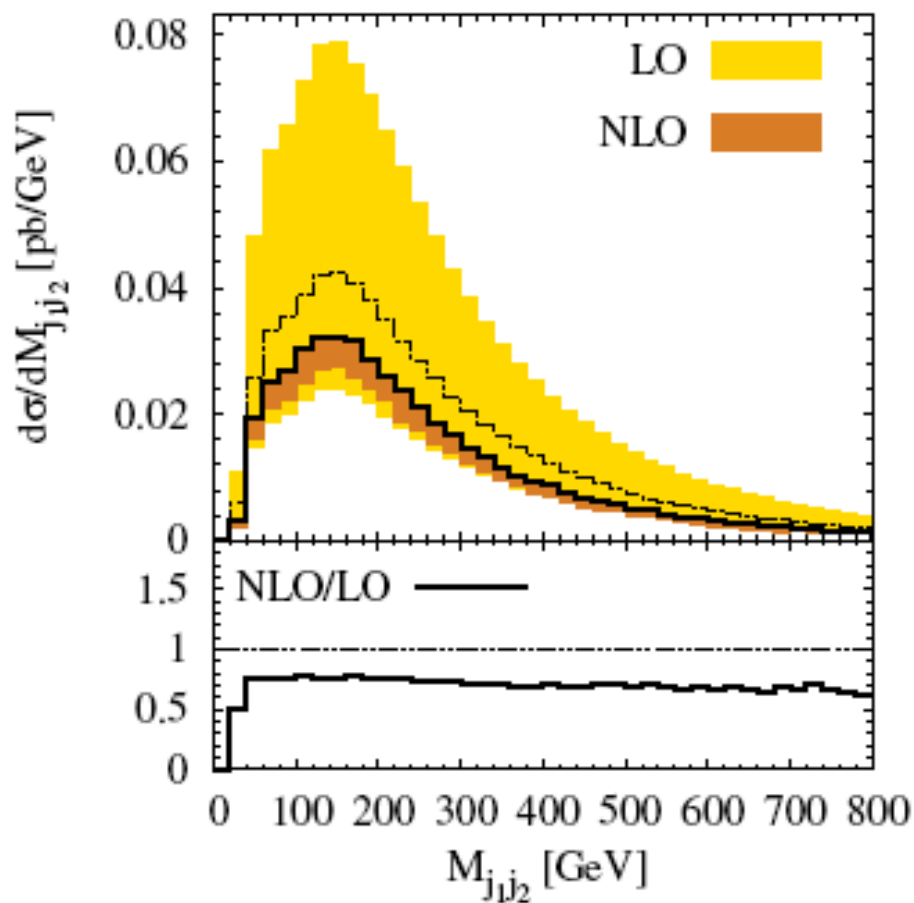


# pT distribution of the leading 4 jets in Z + jets



Ita et al. (BlackHat)

distribution of 2j observables in  $pp \rightarrow t\bar{t}jj$ ,  
comparison of LO and NLO



Bevilacqua, Czakon, Papadopoulos, Worek

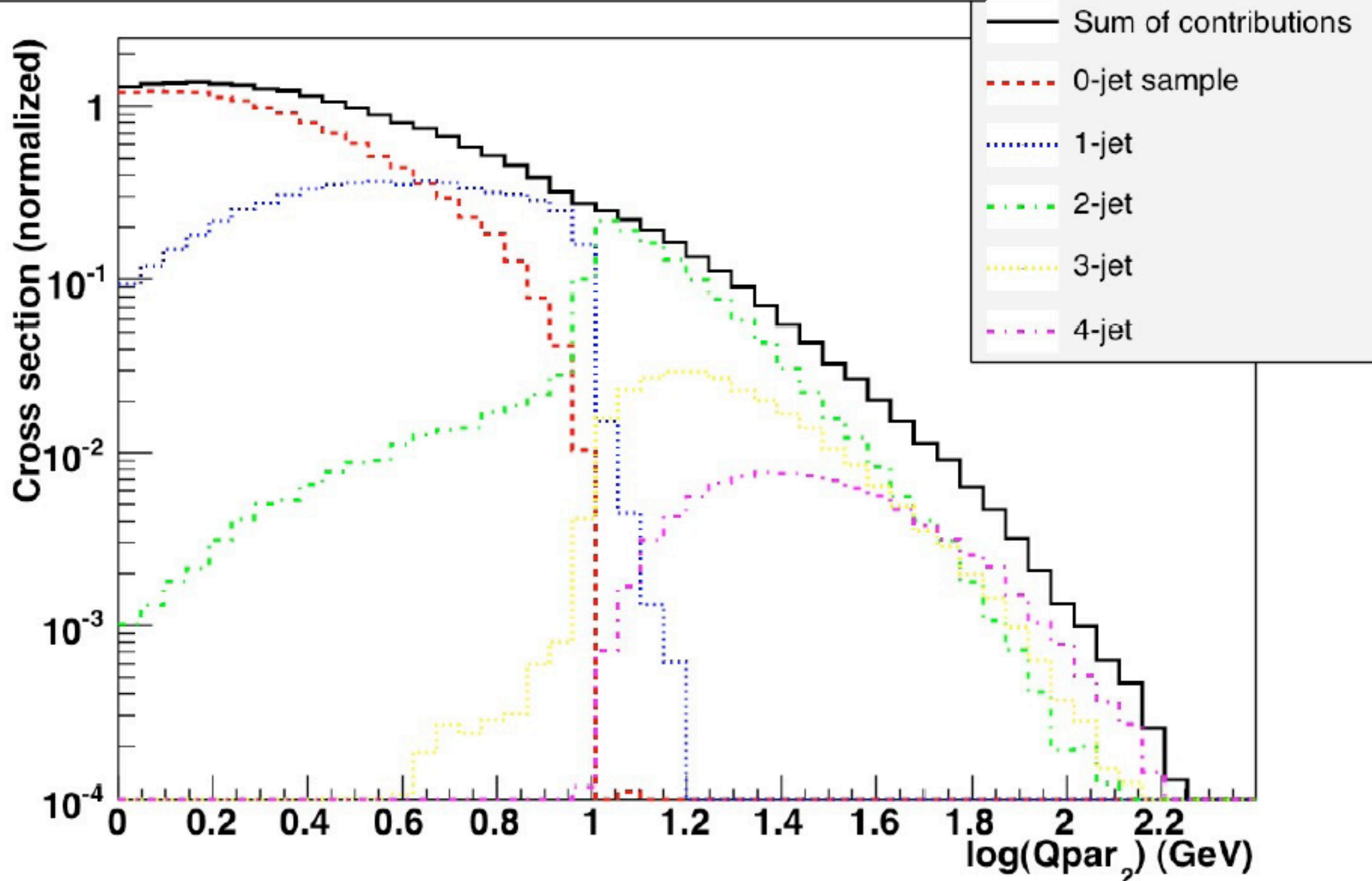
The final step is to merge these approaches:

Use direct Feynman diagram calculation for the wide-angle emissions. This gives the correct global pattern for complex events.

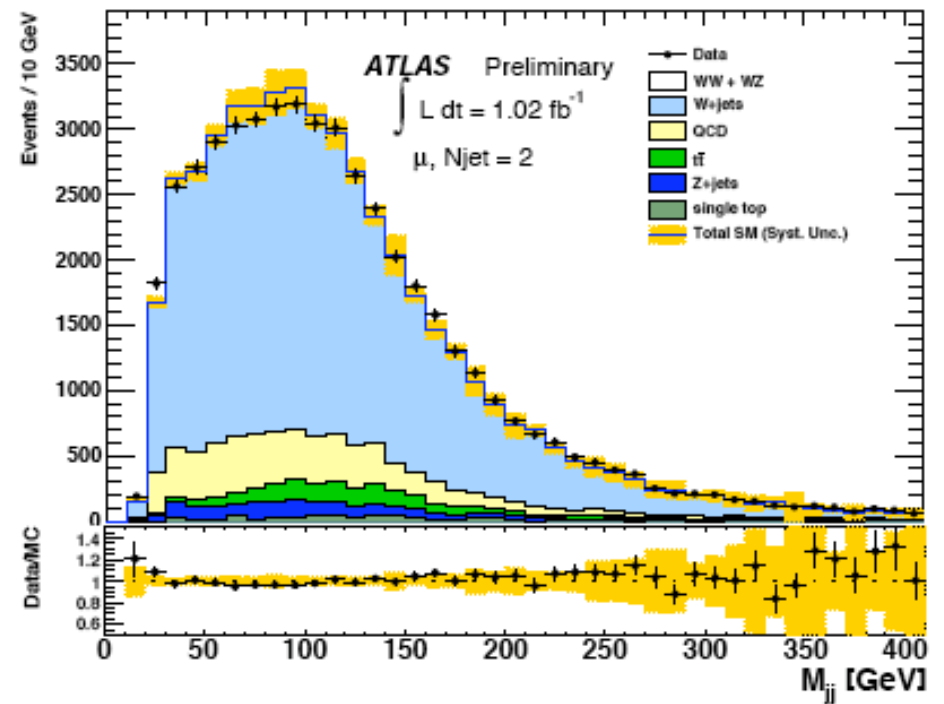
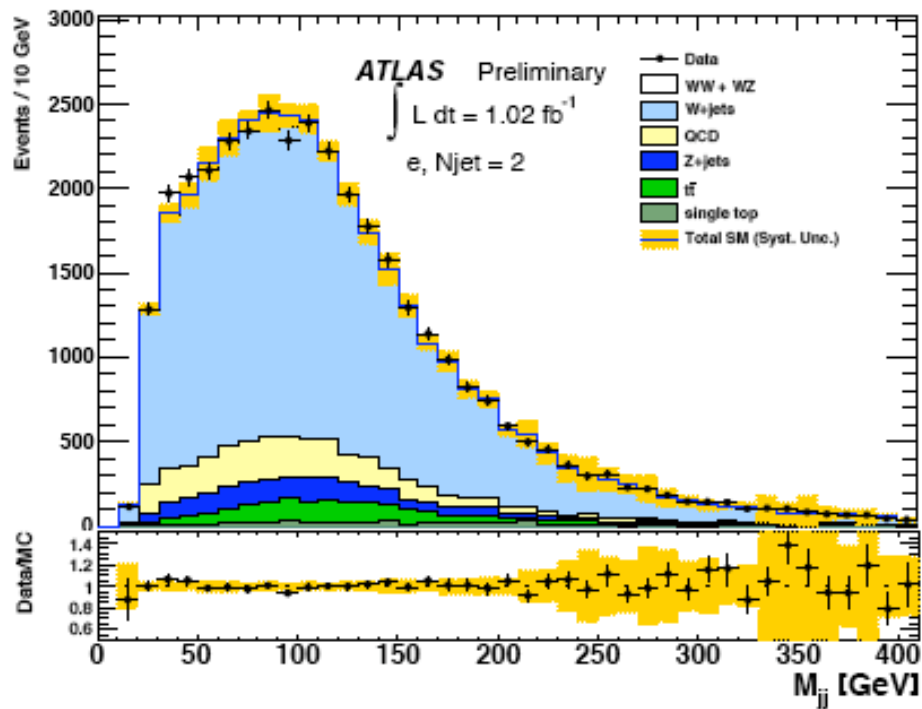
Use the parton shower process for small-angle emissions. This gives the correct internal structure of jets.

It is understood how to do this merging only at leading order in QCD. Merging at higher order, for arbitrarily many emissions, is an open research problem.

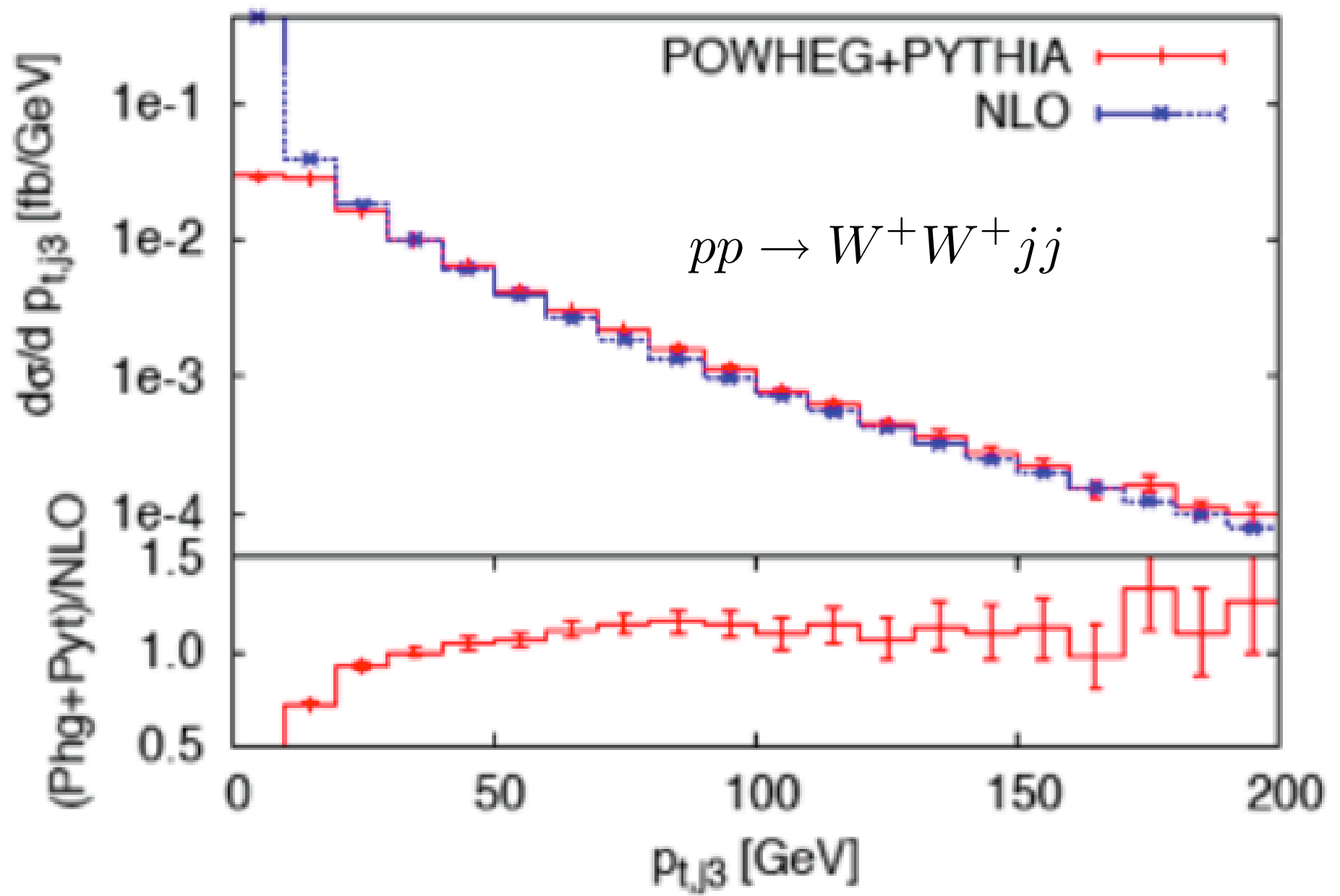
Diff. 1->2 jet rate (parton level) in W + jets at Tevatron by MadEvent/Pythia



J. Alwall: W + 2 jet rates from MadGraph



$m(jj)$  distribution in  $Wjj$   
 SM = ALPGEN matched to HERWIG



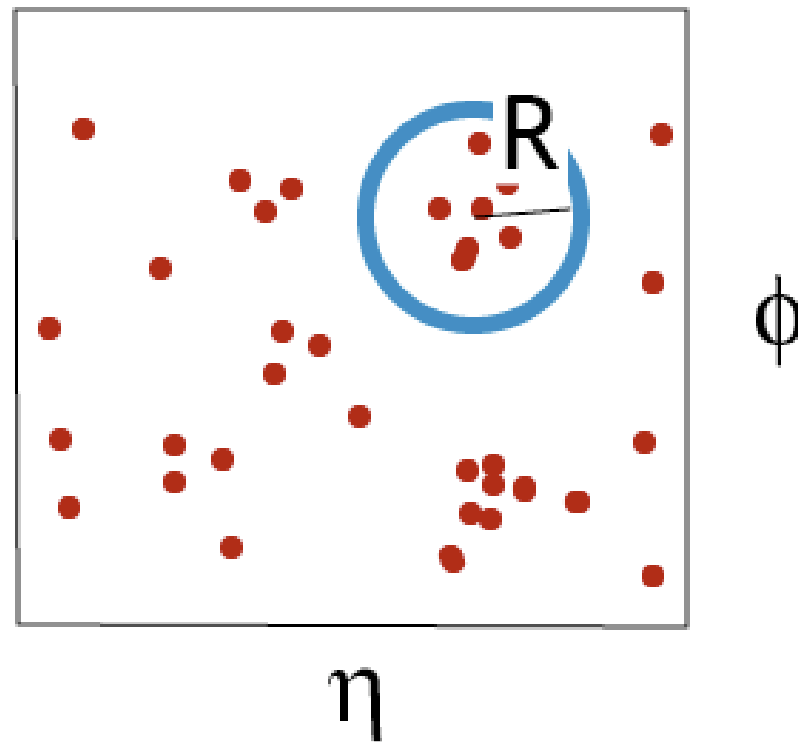
Melia, Nason, Rontsch, Zanderighi

Finally, I will review some ideas on how to use our knowledge of the structure of jets to search more effectively for new physics processes.

To begin, I remind you that jets are defined conventionally. We start from the hadrons in the event and cluster them together to form jets.

At hadron colliders, the pieces of the disrupted proton go forward, with large momenta but small  $p_T$ . To get these out of the way, use the  $p_T$  as the criterion for clustering.

Jets then look like towers on the Lego Plot. You might think of them as a collection of particles in a cone localized in  $(\theta, \phi)$ .



Define a distance measure, including distance from the beam direction. Combine objects  $i, j$  at the smallest distance.

If the distance of  $i$  to the beam is smaller than the distance to any other particle, consider  $i$  as a jet, and drop it from the list of particles.

Continue until no particles remain.

**kt algorithm:**

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2) \cdot \Delta R_{ij}^2 / R^2, \quad d_{iB} = p_{Ti}^2$$

**Cambridge-Aachen:**

$$d_{ij} = \Delta R_{ij}^2 / R^2, \quad d_{iB} = 1$$

**anti-kt:** (Cacciari, Salam, Soyez)

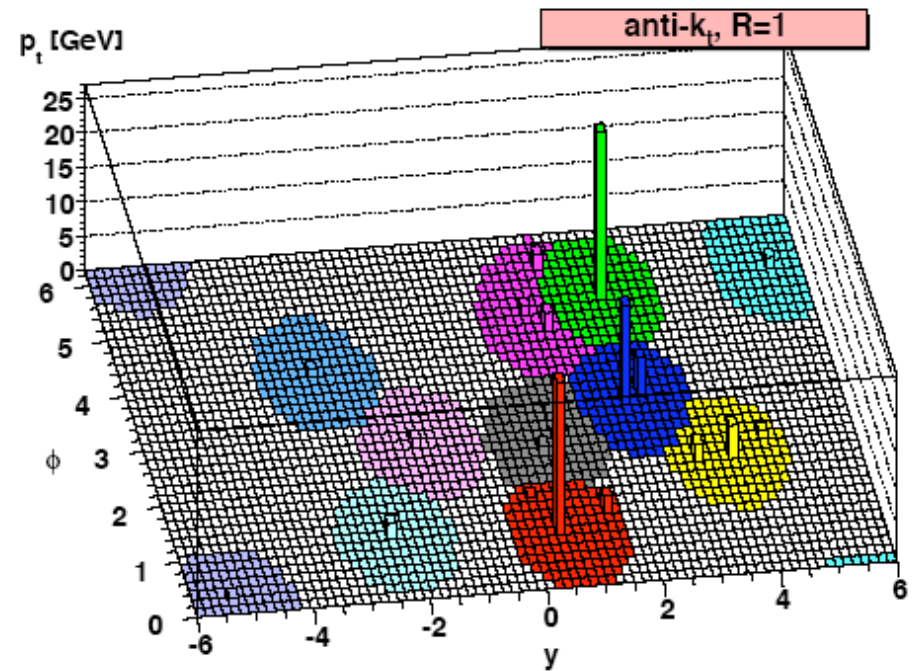
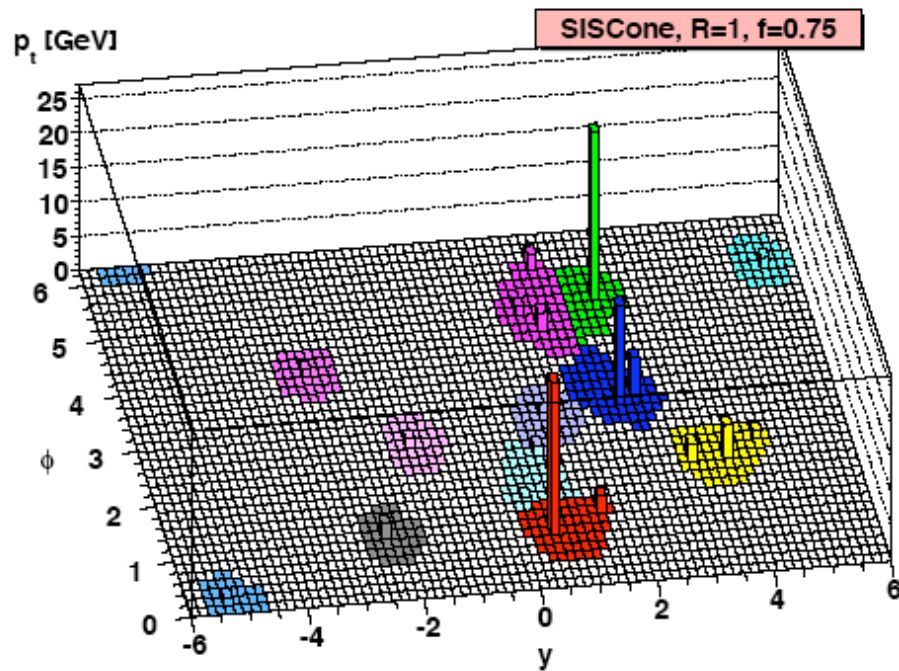
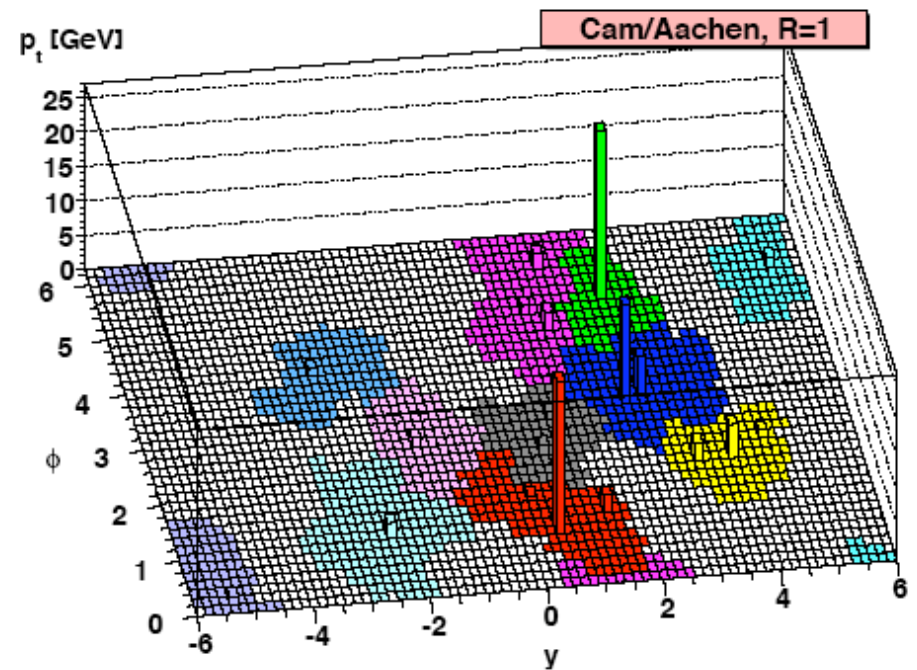
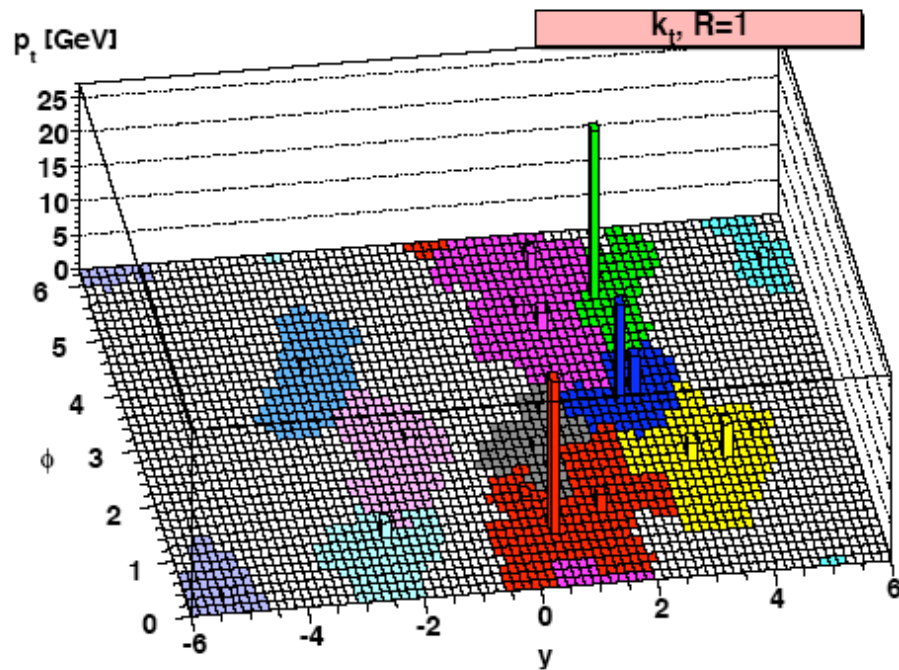
$$d_{ij} = \min(p_{Ti}^{-2}, p_{Tj}^{-2}) \cdot \Delta R_{ij}^2 / R^2, \quad d_{iB} = p_{Ti}^{-2}$$

The three algorithms collect particles into jets in different ways. The variation might suit different purposes in analysis.

Anti-kt has the nice property of producing round, cone-like jets.

ATLAS and CMS now use anti-kt jets with  $R = 0.4 - 0.7$  for most jet-analysis purposes.

With a definite jet definition, we can compute n-jet differential cross sections and compare to experiment.



recommended reference: G. Salam, arXiv 0906.1833

Ordinary QCD jets, initiated by quarks and gluons, have a structure described earlier in the lecture.

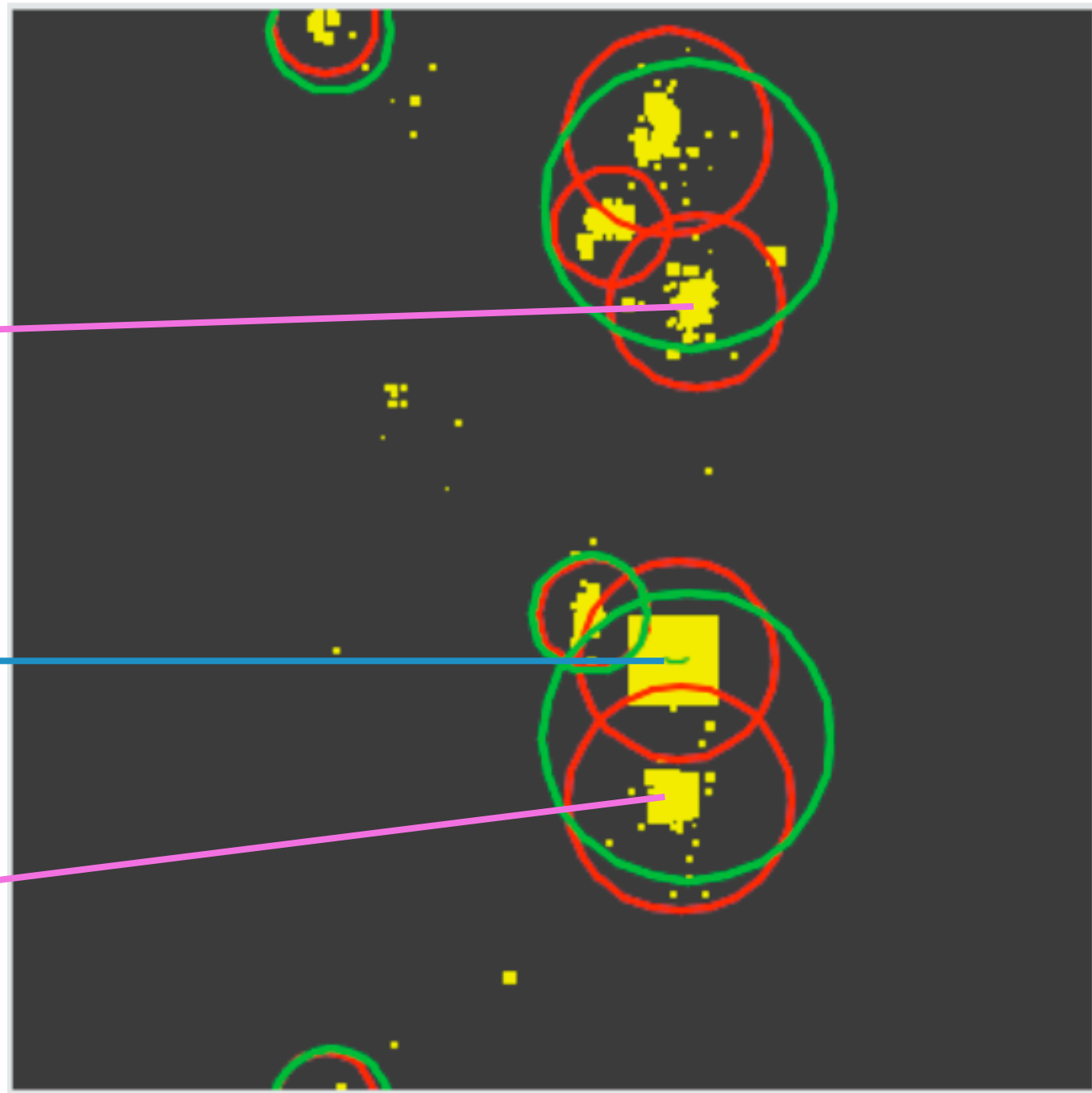
Heavy particles - W, Z, top, Higgs - can also look like jets if they are boosted so that their decay products lie in a single cone.

If we can look inside jets, we can tell the difference and then search effectively for heavy particles.

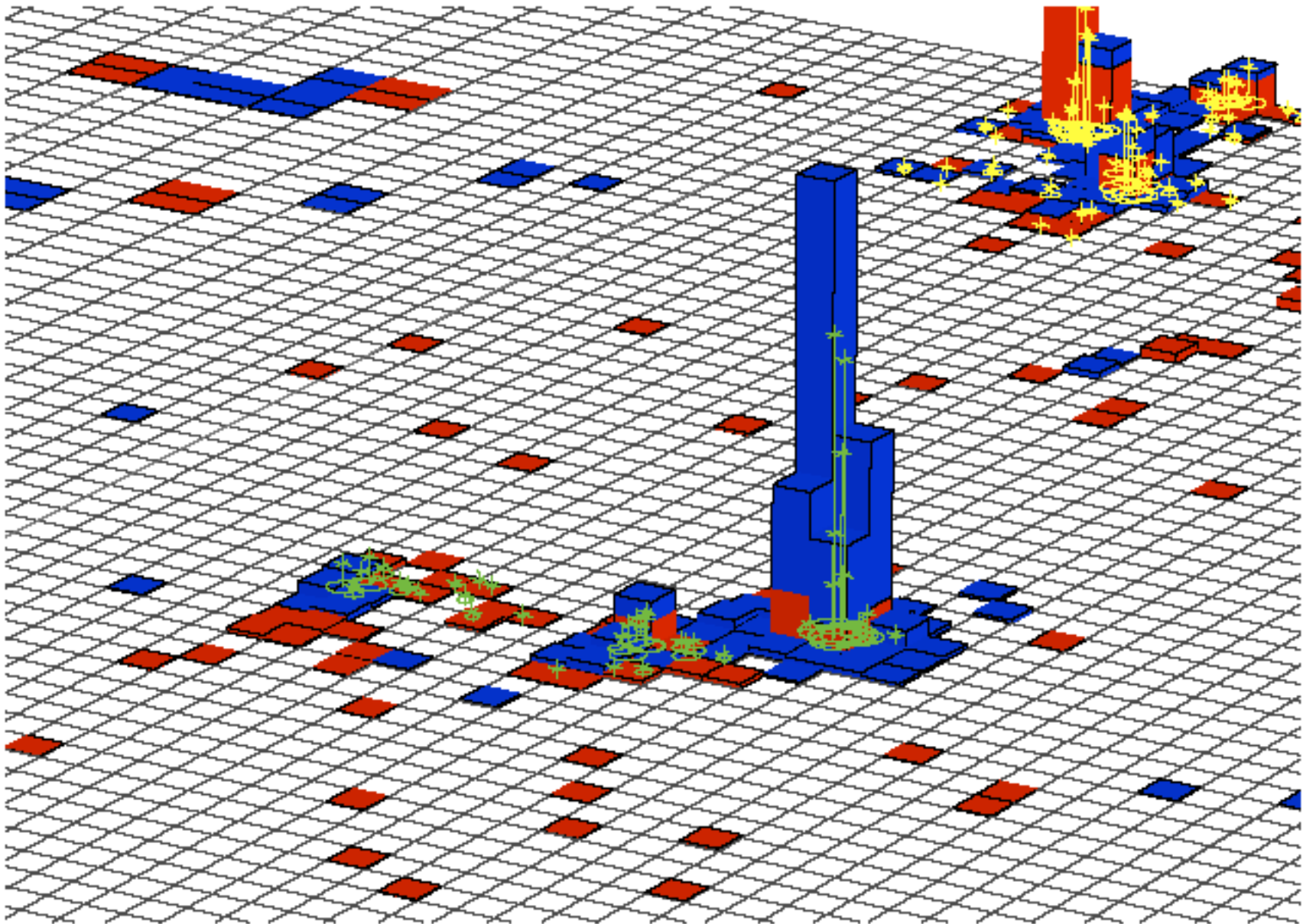
b-tagged jet

electron

b-tagged jet



ATLAS tagged  $t\bar{t}b\bar{b}$  event



CMS tagged  $t\bar{t}$  event

There are two important ideas we can use:

A jet has a **tree structure**, defined by clustering. We can systematically undo the clustering. This reveals the components or **subjets**.

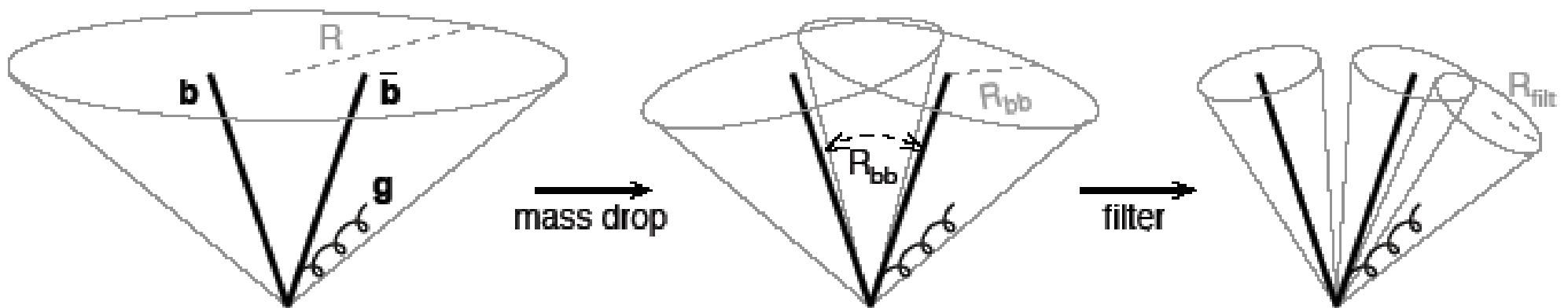
In QCD, splittings typically give one soft and one hard component. If a subjet is soft, it is probably a gluon. If we are looking for heavy particles, we can ignore this piece.

Ellis, Vermillion, and Walsh call this **“pruning”**.

Butterworth, Davison, Rubin, and Salam propose to look for boosted Higgs bosons using these tools.

Higgs bosons can be produced in  $q\bar{q} \rightarrow W h$  with high boosts. They decay by  $h^0 \rightarrow b\bar{b}$  into a single cone.

Decompose the cone by looking at the low branches of the tree.



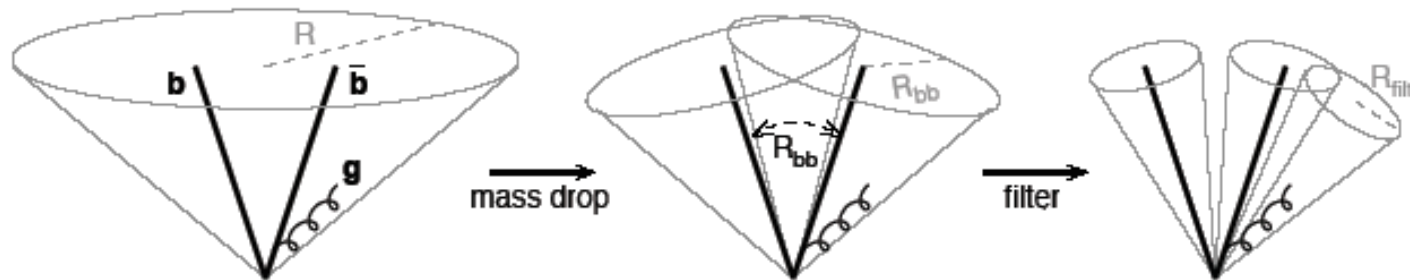
Prune soft components. This removes extra gluons, and also debris from unrelated pieces of the initial protons.

Try to be more sophisticated about this.

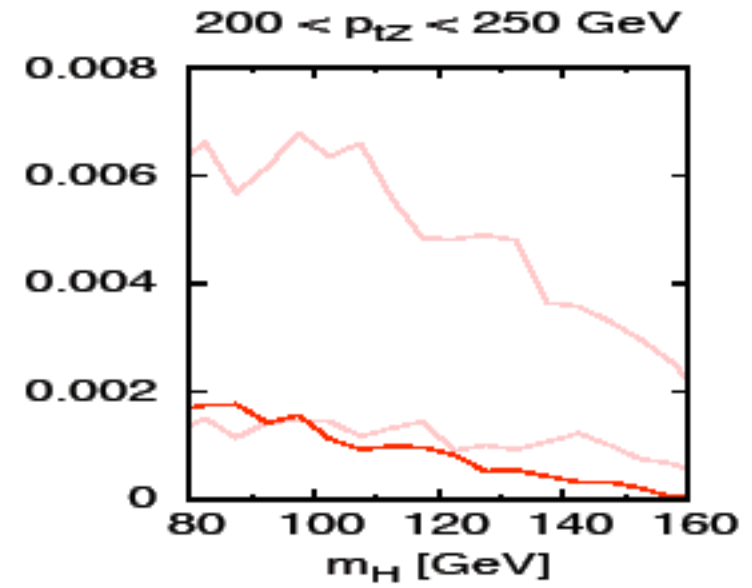
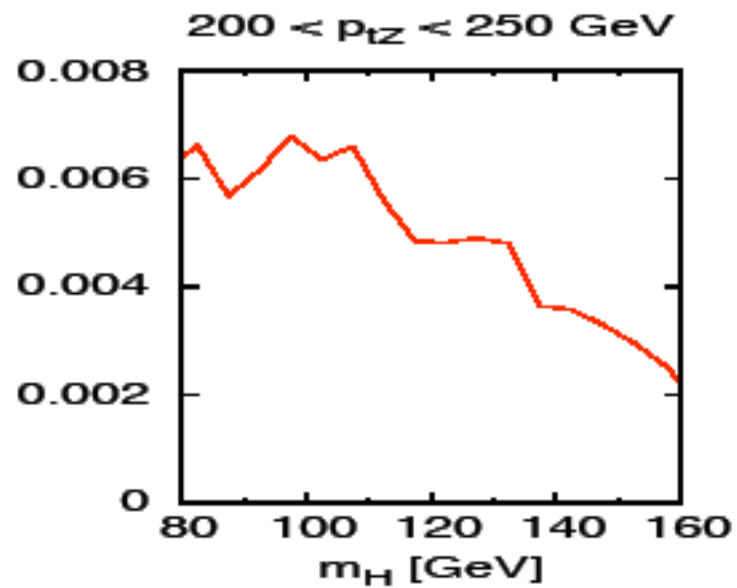
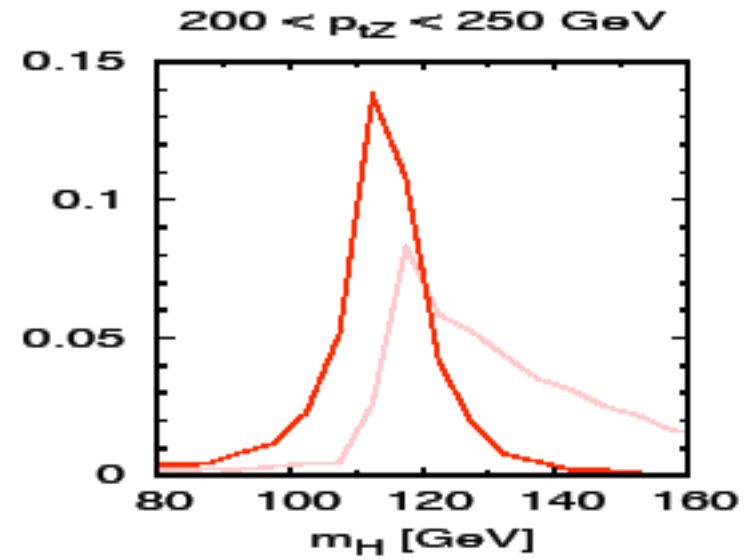
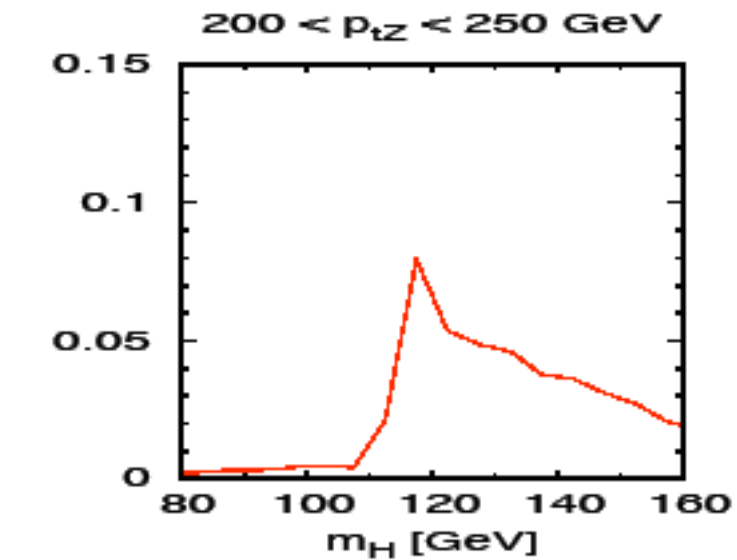
Jets are assembled from particles by jet algorithms. Some jet algorithms (kT, Cambridge-Aachen, but NOT anti-kT) systematically create clusters of increasing mass or internal kT. These clusters can be considered as subjects.

Heavy-particle decay leads to two subjects of relatively equal  $z$ . Gluon radiation leads to a subject that typically has a small  $z$ . So, to isolate heavy particles, look at the tree of subjects and remove ('prune') subjects of small  $z$ .

This destroys jets built from successive gluon radiations, but it has little effect on 'exotic jets' formed from a boosted heavy particle.



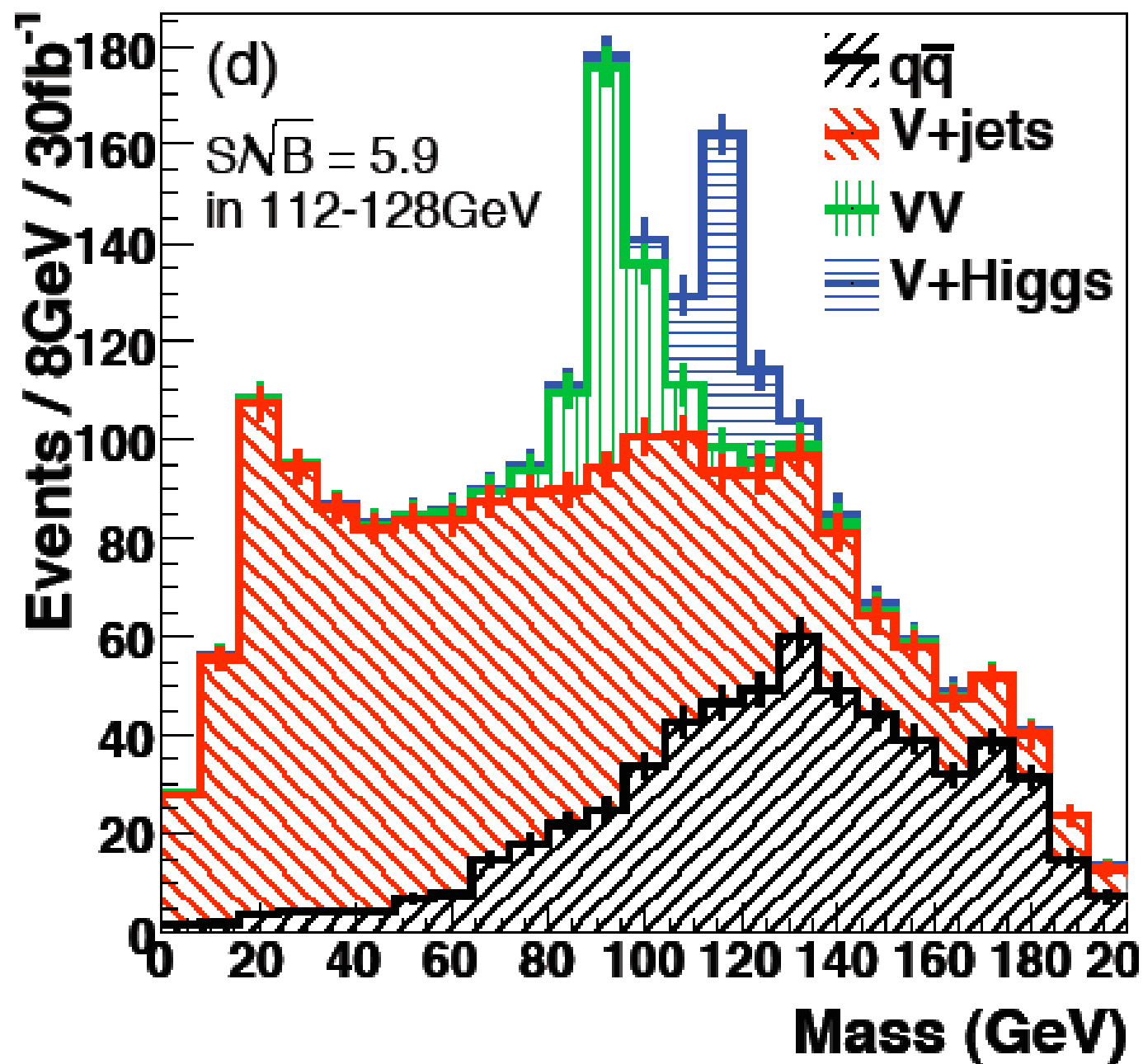
signal



background from W+jets

The main effect on the Higgs mass search turns out to come from the sharpening of the Higgs mass distribution and softening of the background.

Pruning has the effect of measuring the **QCD color** of the boosted system.



By understanding QCD precisely, we can turn the strong interactions from an impediment into a tool for discovering new physics processes at the LHC.

We will see, over the next few years, what wonders these tools can bring to light.