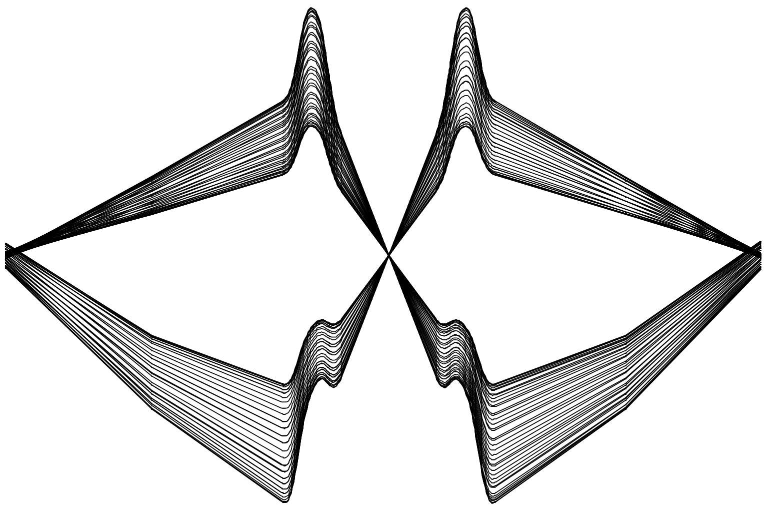
LAL, 14 Janvier 2010

THE RAY-TRACING CODE ZGOUBI



N. Monseu,

with a little help from my friend

Laboratoire de Physique Subatomique et de Cosmologie,

Université Joseph Fourier Grenoble 1, CNRS/IN2P3 , Institut Polytechnique de Grenoble, 53 avenue des martyrs, 38026 Grenoble France

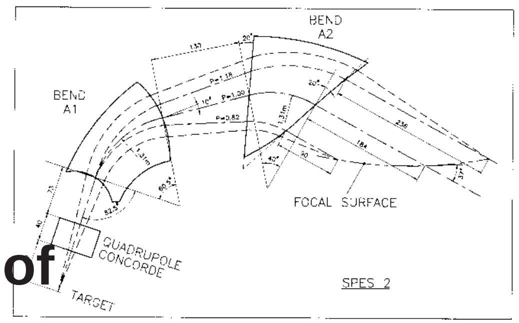
Contents

1 INTRODUCTION : What Zgoubi does

Calculate trajectories of charged particles in magnetic and electric fields.

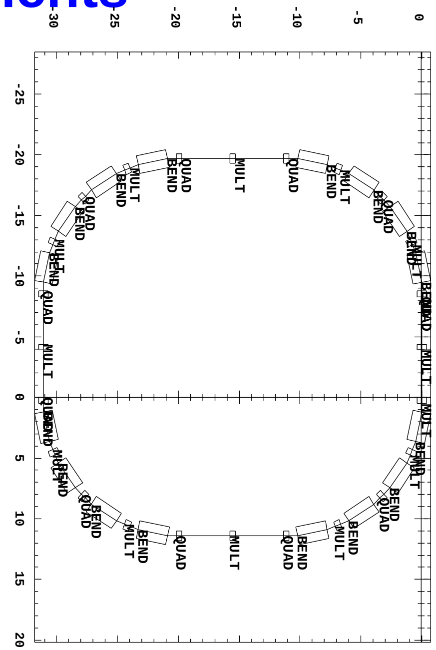
At the origin (early 1970's) developped for design and operation of

- **beam lines**
- **magnetic spectrometers**



Zgoubi has so evolved that it allows today the study of

- **systems including complex sequences of optical elements**
- **periodic structures**



and allows accounting for additional properties as

- **synchrotron radiation and its dynamical effects**
- **spin tracking**
- **in-flight decay**
- **etc...**
- **FAQ :** ***not space charge (not yet ?)***

Compared to other codes, Zgoubi presents several peculiarities (sometimes merits ?) :

- a numerical method for integrating the equation of motion,
  - based on Taylor series,
  - which optimizes computing time
  - provides high accuracy and strong symplecticity,

- spin tracking,
  - using the same Taylor-series based numerical method

- calculation of synchrotron radiation
  - effects on particle dynamics
  - electric field and spectra
    - in arbitrary magnetic fields

- **the possibility of using a mesh,**
  - **which allows ray-tracing from simulated or measured (1-D, 2-D, 3-D, 4-D (!)) field maps,**

- **numerous Monte Carlo procedures allowing**
  - **unlimited number of trajectories,**
  - **in-flight decay,**
  - **photon emission,**
  - **etc...**

- a built-in fitting procedure including
  - arbitrary variables
    - any data in the input file can be varied
  - a large variety of constraints,
    - easily extendable to even more

- multiturn tracking in circular accelerators including
  - features proper to machine parameter calculation and survey,
  - simulation of time-varying power supplies,
    - any element individually

The initial version of the Code

- was dedicated to ray-tracing in magnetic spectrometers and beam lines,

- it was developed by D. Garreta and J.C. Faivre at CEA-Saclay in the early 1970's.

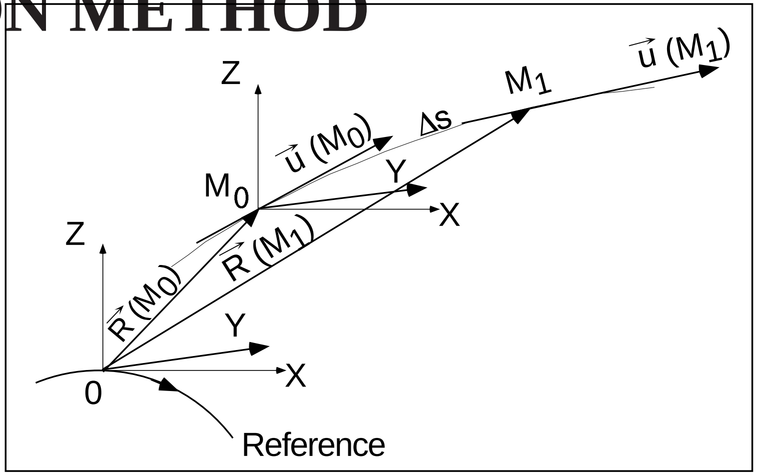
- **In these early times it was perfected for the purpose of studying**
  - **the four spectrometers SPES I, II, III, IV at SATURNE, Saclay**
  - **SPEG at Ganil, Caen**
  - **VENUS, PS170 at CERN-PS areas**
  - **and others...**

- **The 1980s' development period has strongly benefited of the environment of the *Groupe Théorie, SATURNE, CEA/DSM-Saclay*,**

  this is when he acquired its form, much develeopped since then, of a multi-prupose ray-tracing tool...

- **In the context of a pre-gnuplot era,**
  - **A graphic/data treatment interface, *zpop*, has also undergone extensive development,**
  - **a performant, well adapted tool for post-processing Zgoubi output**

Zgoubi's NUMERICAL INTEGRATION METHOD



MOTION, from M_0 to M_1

The equation of motion

$$d(m\vec{v}) \;=\; q\,(\vec{e} + \vec{v}\times\vec{b})\;dt$$

- **is solved using truncated Taylor expansions of $\vec{R}$ and $\vec{u} = \vec{v}/v$:**

$$\begin{aligned}\vec{R}(M_1) &\approx \vec{R}(M_0) + \vec{u}(M_0)\,\Delta s + \vec{u}'(M_0)\,\frac{\Delta s^2}{2!} + \ldots + \vec{u}'''''(M_0)\,\frac{\Delta s^6}{6!}\\ \vec{u}(M_1) &\approx \vec{u}(M_0) + \vec{u}'(M_0)\,\Delta s + \vec{u}''(M_0)\,\frac{\Delta s^2}{2!} + \ldots + \vec{u}'''''(M_0)\,\frac{\Delta s^5}{5!}\end{aligned} \quad (1)$$

- **In non-zero $\vec{E}$ environement, rigidity at M_1 is re-computed :**

$$(B\rho)(M_1) \approx (B\rho)(M_0) + (B\rho)'(M_0)\Delta s + \ldots + (B\rho)''''(M_0)\frac{\Delta s^4}{4!} \quad (2)$$

- **When necessary, time-of-flight is computed in a similar manner :**

$$T(M_1) \approx T(M_0) + \frac{dT}{ds}(M_0)\,\Delta s + \frac{d^2T}{ds^2}(M_0)\,\frac{\Delta s^2}{2} + \frac{d^3T}{ds^3}(M_0)\,\frac{\Delta s^3}{3!} + \frac{d^4T}{ds^4}(M_0)\,\frac{\Delta s^4}{4!} \quad (3)$$

• **In a general manner, the truncated Taylor series**

$$\begin{aligned} \vec{R}(M_1) &= \vec{R}(M_0) + \vec{u}(M_0)\,\Delta s + \ldots \\ \vec{u}(M_1) &= \vec{u}(M_0) + \vec{u}'(M_0)\,\Delta s + \ldots \\ (B\rho)(M_1) &= (B\rho)(M_0) + (B\rho)'(M_0)\Delta s + \ldots \\ T(M_1) &= T(M_0) + \tfrac{dT}{ds}(M_0)\,\Delta s + \ldots \end{aligned} \qquad (4)$$

require computation of the derivatives

$$\vec{u}^{(n)} = d^n\vec{u} \;/\; ds^n$$

$$(B\rho)^{(n)} = d^n(B\rho) \;/\; ds^n$$

$$d^n(T) \;/\; ds^n$$

• **This is the subject of the following slides.**

- **Let's introduce simplified notations :**

$$\vec{u} = \frac{\vec{v}}{v}, \quad ds = v\,dt, \quad \vec{u}' = \frac{d\vec{u}}{ds}, \quad m\vec{v} = mv\vec{u} = q\,B\rho\,\vec{u} \quad \vec{B} = \frac{\vec{b}}{B\rho} \tag{5}$$

$d(m\vec{v}) \;=\; q\,(\vec{e} + \vec{v} \times \vec{b})\;dt$ **(with $\vec{e} = 0$) then writes**

$$\boxed{\vec{u}' = \vec{u} \times \vec{B}}$$

This yields the $\vec{u}^{(n)} = d^n\vec{u}\;/\;ds^n$ needed in the Taylor expansions : $\vec{u}' = \vec{u} \times \vec{B}$

$$\begin{aligned}
\vec{u}'' &= \vec{u}' \times \vec{B} + \vec{u} \times \vec{B}' \\
\vec{u}''' &= \vec{u}'' \times \vec{B} + 2\vec{u}' \times \vec{B}' + \vec{u} \times \vec{B}'' \\
\vec{u}'''' &= \vec{u}''' \times \vec{B} + 3\vec{u}'' \times \vec{B}' + 3\vec{u}' \times \vec{B}'' + \vec{u} \times \vec{B}''' \\
\vec{u}''''' &= \vec{u}'''' \times \vec{B} + 4\vec{u}''' \times \vec{B}' + 6\vec{u}'' \times \vec{B}'' + 4\vec{u}' \times \vec{B}''' + \vec{u} \times \vec{B}''''
\end{aligned} \tag{6}$$

where $\vec{B}^{(n)} = d^n\vec{B}\;/\;ds^n$**.**

- **Field derivatives wrt. path length,**

$$\vec{B}^{(n)} = d^n\vec{B} \;/\; ds^n$$

- **are computed from the derivatives in magnet frame :**

$$d\vec{B} \;=\; \frac{\partial\vec{B}}{\partial X}dX + \frac{\partial\vec{B}}{\partial Y}dY + \frac{\partial\vec{B}}{\partial Z}dZ \;=\; \sum_{i=1,3}\frac{\partial\vec{B}}{\partial X_i}dX_i$$

- **following**

$$\begin{aligned}
\vec{B}' &= \textstyle\sum_i \frac{\partial\vec{B}}{\partial X_i}u_i \\
\vec{B}'' &= \textstyle\sum_{ij}\frac{\partial^2\vec{B}}{\partial X_i\partial X_j}u_iu_j + \sum_i\frac{\partial\vec{B}}{\partial X_i}u_i' \\
\vec{B}''' &= \textstyle\sum_{ijk}\frac{\partial^3\vec{B}}{\partial X_i\partial X_j\partial X_k}u_iu_ju_k + 3\sum_{ij}\frac{\partial^2\vec{B}}{\partial X_i\partial X_j}u_i'u_j + \sum_i\frac{\partial\vec{B}}{\partial X_i}u_i'' \\
&etc.
\end{aligned} \tag{7}$$

- **There are oodles of magnetic elements in Zgoubi,**
- **oodles of ways to simulate magnetic elements,**
- **allowing an answer to almost everyday's life magnetic situations**

What :	*Keyword :*
Decapole	DECAPOLE, MULTIPOL
Dipole	BEND, DIPOLE[S, -M], MULTIPOL, QUADISEX
Dodecapole	DODECAPO, MULTIPOL
FFAG magnets	FFAG, FFAG-SPI
Multipole	MULTIPOL, QUADISEX, SEXQUAD
Octupole	OCTUPOLE, MULTIPOL, QUADISEX, SEXQUAD
Quadrupole	QUADRUPO, MULTIPOL, SEXQUAD
Sextupole	SEXTUPOL, MULTIPOL, QUADISEX, SEXQUAD
Solenoid	SOLENOID
Helical dipole ?	yes...
Multiple-helix dipole ?	soon ?...

Field maps

1-D, cylindrical symmetry	BREVOL
2-D, mid-plane symmetry	CARTEMES, POISSON, TOSCA
2-D, no symmetry	MAP2D
2-D, polar mesh	POLARMES
3-D	TOSCA

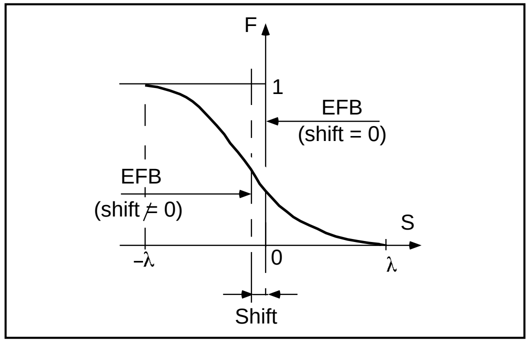
EXAMPLE 1 - Multipoles

- **Field and derivatives in magnet frame,** $\frac{\partial^{i+j+k}\vec{B}_n(X,Y,Z)}{\partial X^i \partial Y^i \partial Z^k}$ $\quad i+j+k=0$ **to** 4
- **are derived from scalar potential**

$$V_n(X,Y,Z) = (n!)^2 \left(\sum_{q=0}^{\infty} (-1)^q \frac{G^{(2q)}(X)(Y^2+Z^2)^q}{4^q q!(n+q)!} \right) \left(\sum_{m=0}^{n} \frac{sin\left(m\frac{\pi}{2}\right) Y^{n-m} Z^m}{m!(n-m)!} \right) \tag{8}$$

- **yielding** $\quad B_X = \frac{\partial V}{\partial X}, \quad B_Y = \frac{\partial V}{\partial Y}, \quad B_Z = \frac{\partial V}{\partial Z} \quad$ **etc.**
- $G(X)$ **is the longitudinal gradient, defined at the entrance or exit of the optical element by**

$$G(s) = \frac{G_0}{1+\exp(P(s))}, \quad G_0 = \frac{B_0}{R_0^n}$$



while

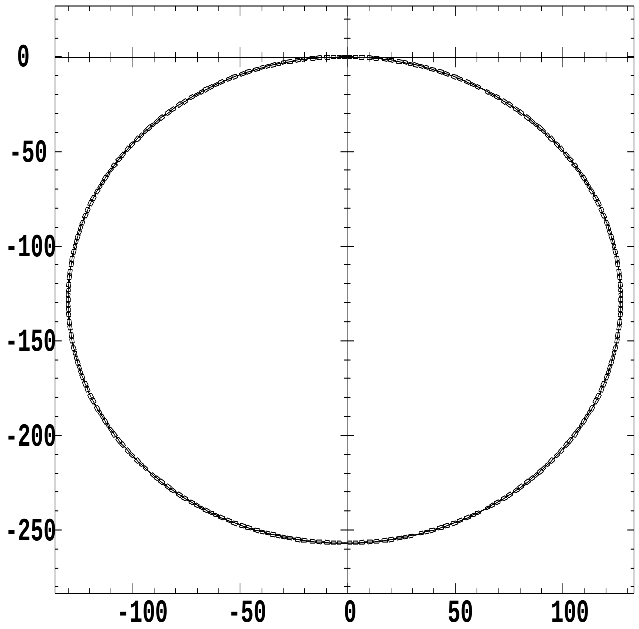
$$P(s) = C_0 + C_1\left(\frac{s}{\lambda}\right) + C_2\left(\frac{s}{\lambda}\right)^2 + C_3\left(\frac{s}{\lambda}\right)^3 + C_4\left(\frac{s}{\lambda}\right)^4 + C_5\left(\frac{s}{\lambda}\right)^5$$

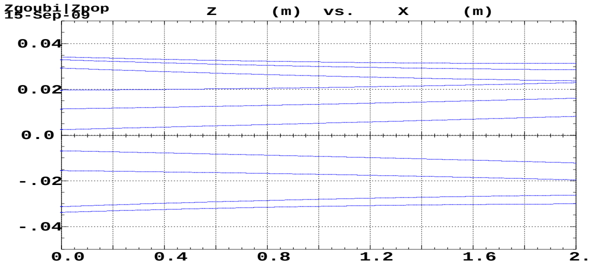
s **is the distance to the EFB.**

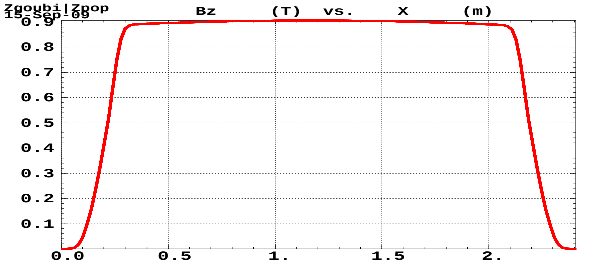
AGS ring based on 'MULTIPOL'

• zgoubi.dat input file, excerpt :

```
AGS ring. Xing 36+nu
'OBJET'
77.64321e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   .02e-7
  0  1  0.
 'SCALING'
 1  2
  MULTIPOL SBEN
  -1
  77.64321
  1
  MULTIPOL QUAD
  -1
  77.64321
  1
 'PARTICUL'
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
 'SPNTRK'
  3
   'PICKUPS'
   1
   #Start
   'FAISTORE'
   b_zgoubi.fai   #End
   1
  'SPNPRNL'
  b_zgoubi.spn
   'MARKER'  #Start
 'MARKER'   MARK      BSUPERA
 'MULTIPOL' SBEN      A1BF
 0  .Dip
  200.6554  10.00   0.11712499   0.04848519  -0.00050563 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|200|20    Dip  A1BF
              3   0.000000000000000        0.000000000000000      -1.175115045000000E-002
 'DRIFT'    DRIF      D2S
     60.9515
 'MULTIPOL' SBEN      A2BF
 0  .Dip
  200.6554  10.00   0.11712499   0.04848519  -0.00050563 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|200|20    Dip  A2BF
              3   0.000000000000000        0.000000000000000      -1.175115045000000E-002
 'DRIFT'    DRIF      DPUE
     28.7000
 'MARKER'   MARK      PUE_A02
 'MULTIPOL' HKIC      DHCA02
 0  .kicker
   0.1000E-03 10.00  0.000000E+00    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'MULTIPOL' VKIC      DVCA02
 0  .kicker
   0.1000E-03 10.00  0.000000E+00    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
```





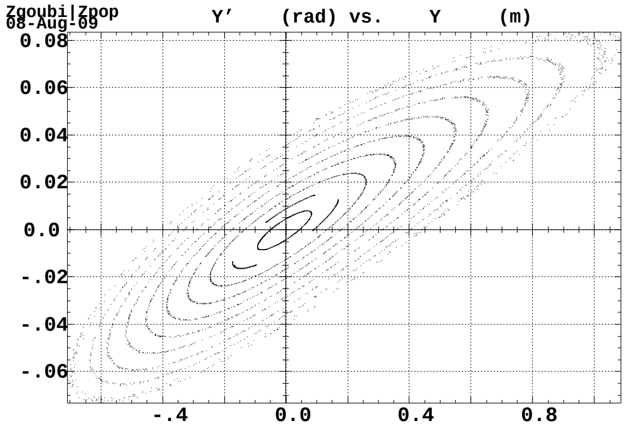


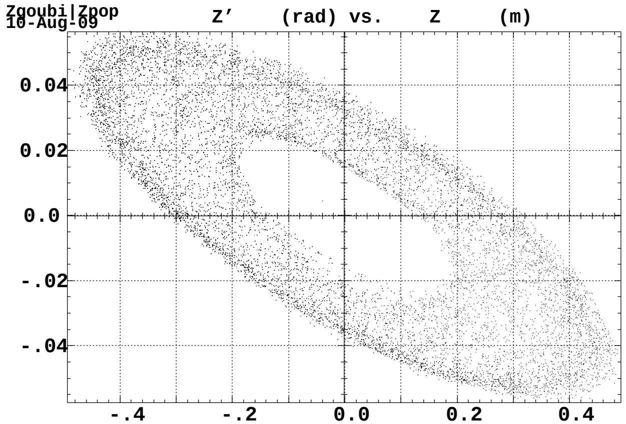

```
---------- A1BF, transfer matrix
MAD8
      0.903966      1.941764      0.000000      0.000000      0.025292
     -0.094164      0.903966      0.000000      0.000000      0.024799
      0.000000      0.000000      1.098925      2.072529      0.000000
      0.000000      0.000000      0.100184      1.098925      0.000000
     -0.024799     -0.025292      0.000000      0.000000      0.378389

Ray-tracing
      0.904036      1.94182       0.00000       0.00000       0.0235714
      0.094094      0.904042      0.00000       0.00000       0.0231068
      0.00000       0.00000       1.09884       2.07246       0.00000
      0.00000       0.00000       0.100096      1.09883       0.00000
      0.023107      0.023560      0.00000       0.00000       1.881443E-04?
```

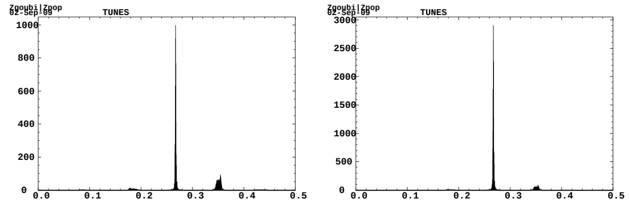
Multiturn tracking, DA finding

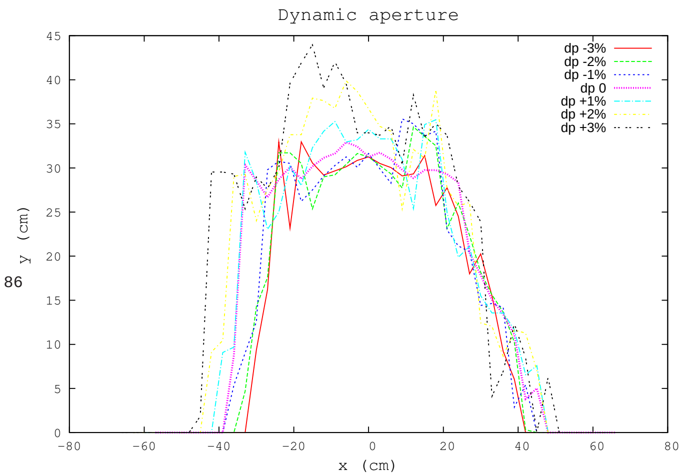
• zgoubi.dat input file, excerpt :

```
AGS ring. Xing 36+nu
'OBJET'
77.64321e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675   .02e-7
  0  1  0.
 'SCALING'
 1  2
  MULTIPOL SBEN
  -1
  77.64321
  1
  MULTIPOL QUAD
  -1
  77.64321
  1
 'PARTICUL'
 938.27203d0  1.602176487d-19  1.7928474d0 0. 0.
 'SPNTRK'
  3
   'PICKUPS'
   1
   #Start
   'FAISTORE'
   b_zgoubi.fai   #End
   1
  'SPNPRNL'
  b_zgoubi.spn
   'MARKER'  #Start
 'MARKER'   MARK      BSUPERA
 'MULTIPOL' SBEN      A1BF
 0  .Dip
  200.6554  10.00   0.11712499   0.04848519  -0.00050563 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 0. 0. 10.00  4.0  0.800 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
  #20|200|20    Dip  A1BF
            3   0.00000000000000        0.00000000000000      -1.175115045000000E-002
........................
........................
........................
 'MULTIPOL' HKIC      SML20
 0  .kicker
   0.2413E+03 10.00  0.000000E+00    0.000000 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 .0 .0  1.00 0.00 0.00 0.00 0.00 0. 0. 0. 0.
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
  0.000000000 0. 0. 0. 0. 0. 0. 0. 0. 0.
 #20|20|20  Kick
 1 0. 0. 0.
 'DRIFT'    DRIF      DSG10
      31.7339
 'MARKER'   MARK      ESUPERL
  'CAVITE'
  2.1         .1 is to fill zgoubi.CAVITE.Out for plot using zpop/7/20
  807.043778118095   12.
  290.d3  2.617993877991494365       9cavitiesx32kV, phi_s=30deg
   'MARKER'  #End
 'REBELOTE'
 9  0.2 99
 'END'
```

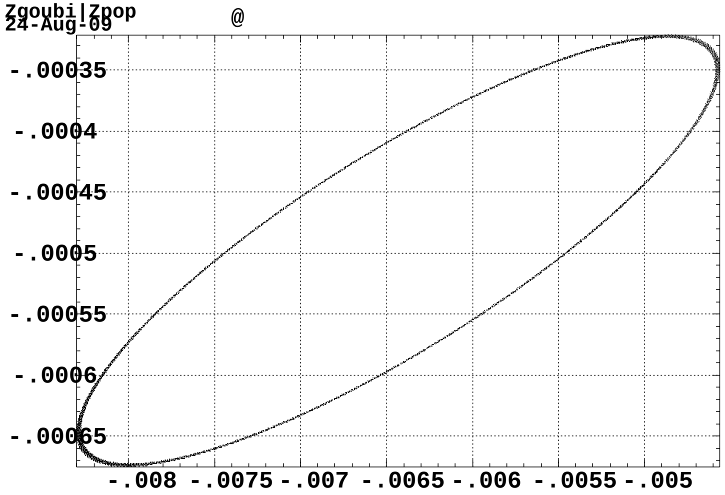




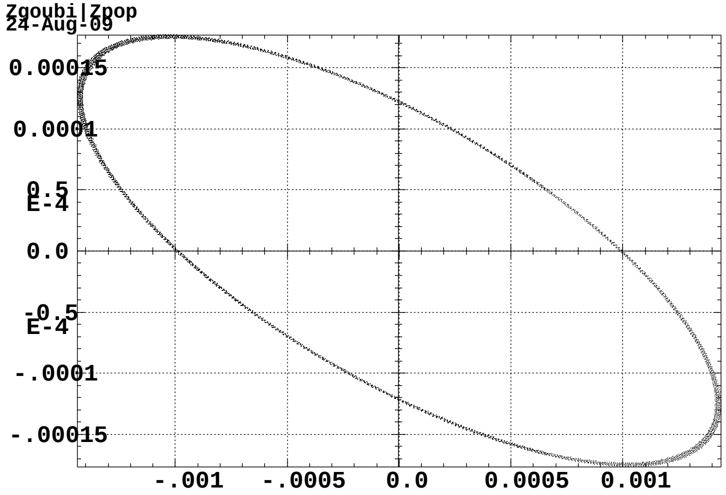


• 500000 turns in AGS, single particle...

x-x'



y-y'



• particle launched on $\epsilon_x/\pi = \epsilon_z/\pi = 0.35$ mm.mrad, at fixed rigidity,
• observed at "Begin AGS".

• Good symplecticity : no obvious broadening of the invariants, nor visible spiraling.

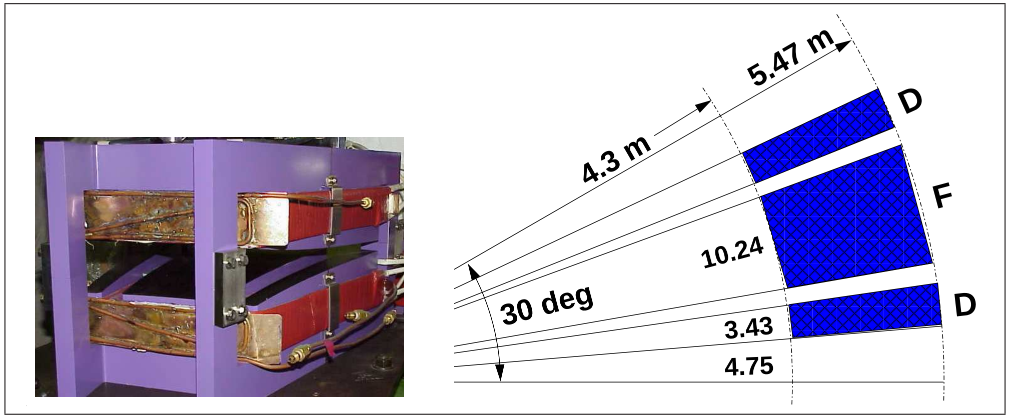
• Computiing speed, real time (*not* CPU), Dell Latitude D630 : 500 turns/minute

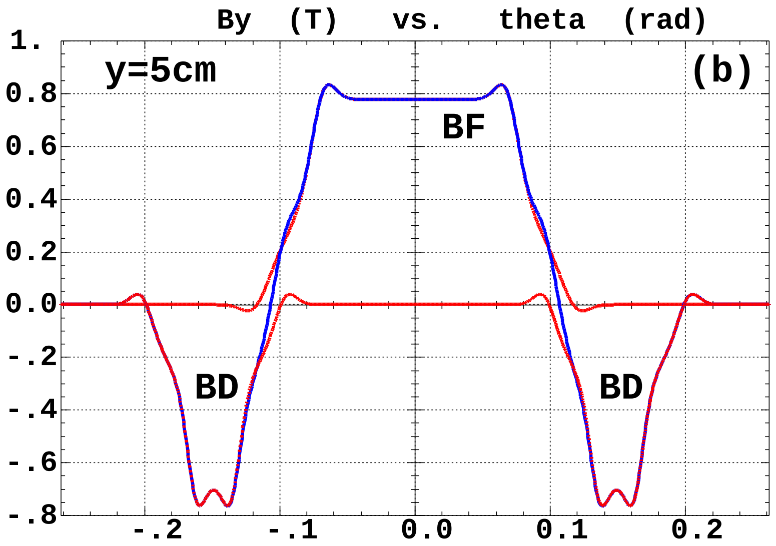
EXAMPLE 2 - “FFAG” and “DIPOLES” procedures

The bare AGS lattice is a rather simple case (sorry !). Zgoubi has been doing much more sophisticated.

- **A simulation of $B_{zi}(r,\theta) = B_{z0,i}\,\mathcal{F}_i(r,\theta)\,(r/R_0)^k$ including overlapping of fringe fields.**

- **Case of KEK 150 MeV scaling FFAG DFD triplet :**





- **The geometrical model is based on the superposition of the independent contributions of the N dipoles :**

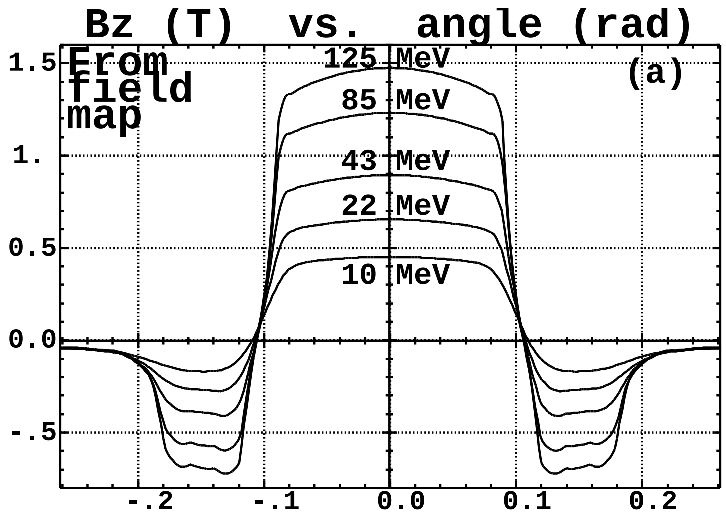
$$B_z(r,\theta) = \sum_{i=1,N} B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)$$

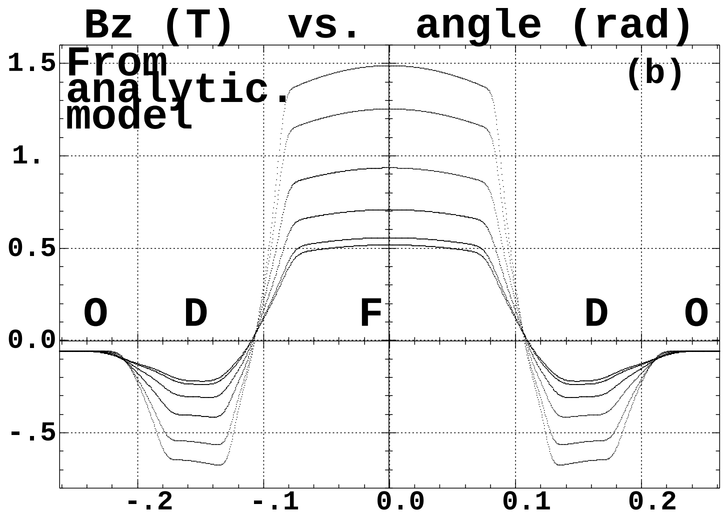
at all (r,θ) in the mid-plane.

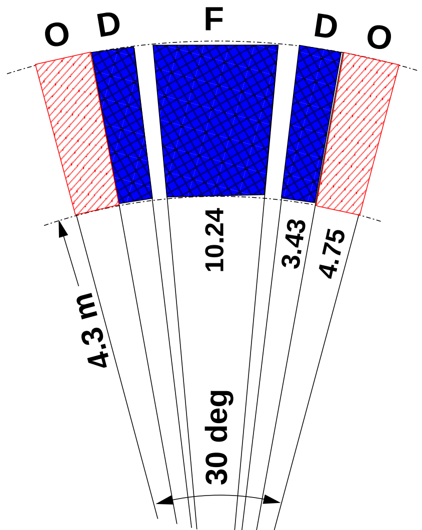
- **Field off mid-plane is obtained by Taylor expansion + Maxwell.**

Simulation of the field experienced by a praticle crossing along $r_0 = 4.87$ m, 5 cm off mid-plane. A superposition (blue line) of $N = 3$ independent sources (red lines), at all (r,θ,z).

EXAMPLE 3 - “ TOSCA” procedures







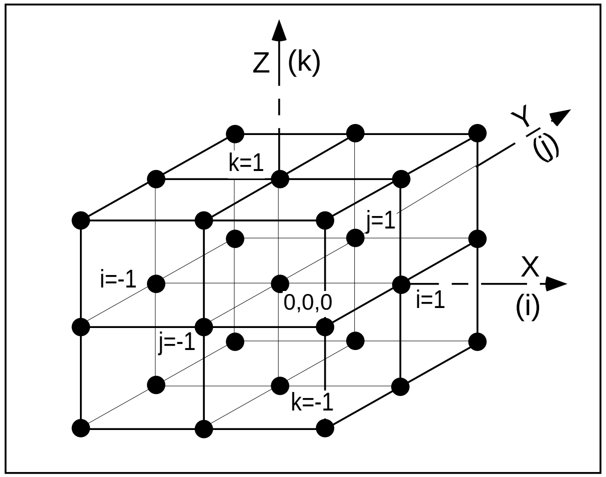
(c)

Comparison of magnetic field along closed orbits in the case of,

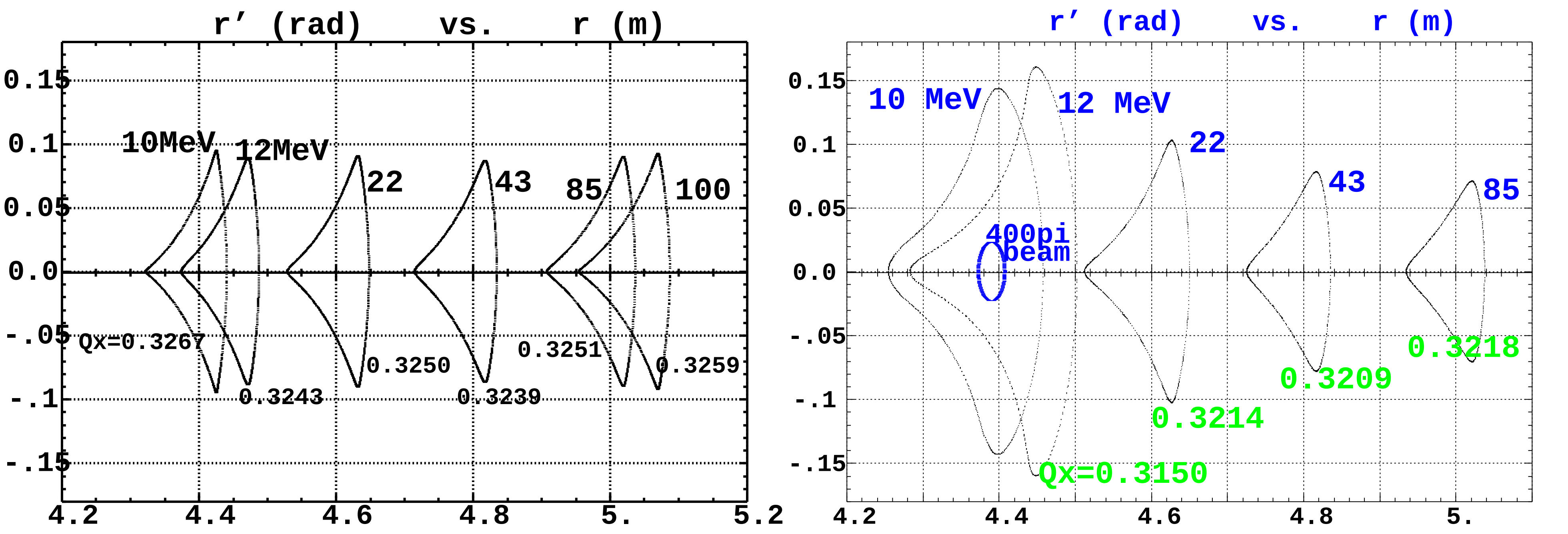
(a) : TOSCA 3-D map representative of the 150 MeV FFAG, and,

(b) : field from the “3+2”-dipole geometrical model.

(c) geometry of the “3+2”-dipole design, including two additional dipole regions (hatched) that simulate 700 G field extent over the two end drifts.



Tracking in scaling FFAG(cont'd) - Comparison between "FFAG" procedure and 3-D field maps



150MeV FFAG : horizontal phase space, the limits of stable motion, for 5 different values of energy/closed orbit radius.

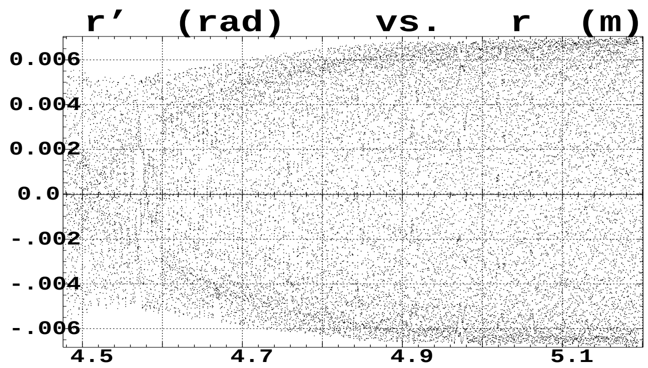
- **For comparison : tracking with geometrical model (left), or using TOSCA map (right).**

6-D tracking - difficult problem : 1/ using field map, 2/ scaling FFAG

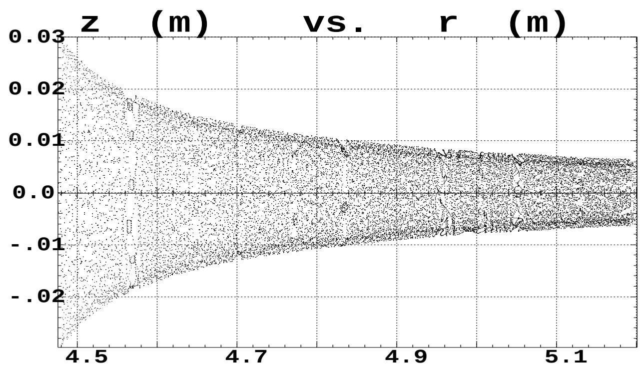
- Or, "where we observe that *high accuracy tracking in field maps* is doable"
- Acceleration from12 to 125 MeV, ≈50000 turns, 1 particle
- The orbit spirals out : 4.5 ⟶ 5.2 m

- **Analitycal modelling : "DIPOLES" or "FFAG"**

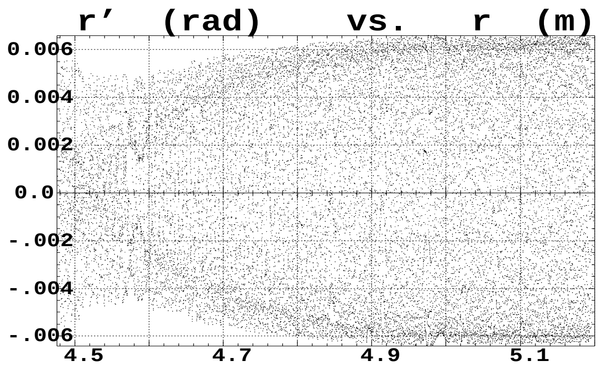
Some x-z coupling, yet $\epsilon_x \approx 0$

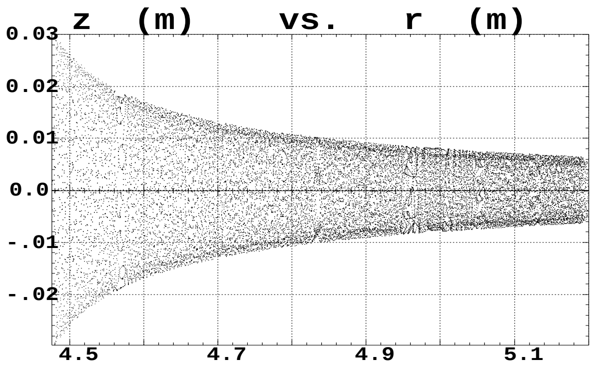


ϵ_z-damping $\alpha \sqrt{\beta\gamma}$



- **"TOSCA" procedure, 2-D field map**





- A (rather) large (considering the strong non-linearity of $B \sim (r/R_0)^k$, and fringe fields in addition) step size is used, in both cases : 0.5 cm, with success.

EXAMPLE 3 - HELICAL MAGNET : "HELIX" (analytical) or "MAP2D" (field map) modelling

• "HELIX", analytical

• Typical zgoubi.dat input :

```
'HELIX'
2
45.2  200.  19.855   0.
 3  10.
1.
1 0. 0. 0.
```

• "HELIX" is being extended to multiple-helix magnet,

• so to allow simulating warm-, cold-, any temperature-snake

• this will be based on the method used in "DIPOLES", "FFAG" as seen earlier :

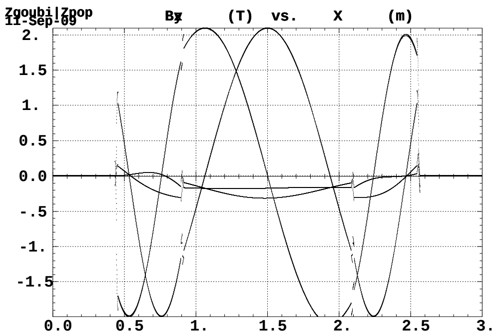
$$B_z(r,\theta) = \sum_{i=1,N} B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)$$

• Using a 2-D field map together with "MAP2D" procedure

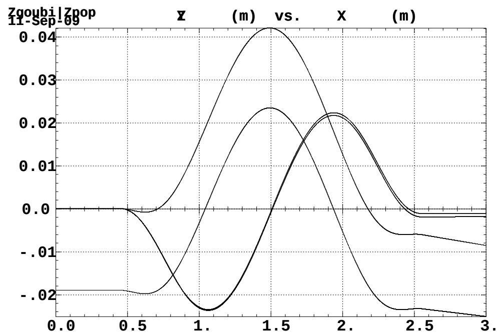
• Typical zgoubi.dat input :

```
'MAP2D'
 0  2
 10.  100.  100.
 HEADER_3 helical magnet field map
 301  41
csnk-se-1cm.map
 0 0 0 0
 2
  .1  stepSize
 2  0.  0.  0.  0.
```

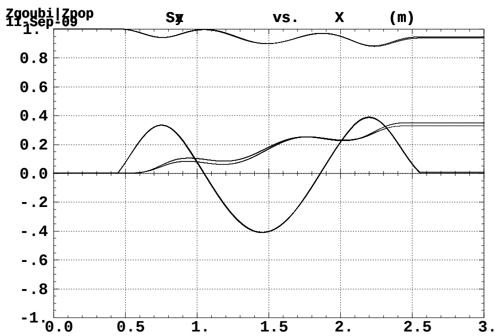
Bx, By, Bz versus z



x, y versus z



spin, starting vertical

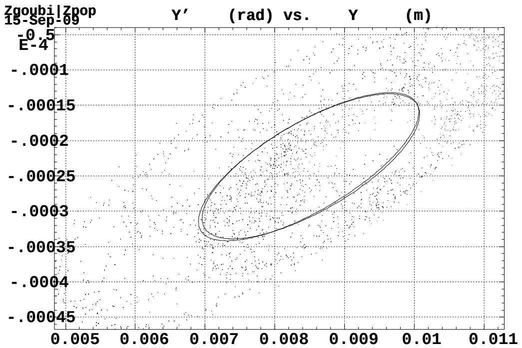


TRACKING AGS WITH HELICAL MAGNET

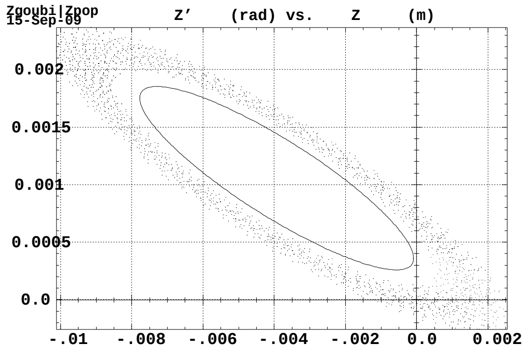
- **A 1 cm mesh size field map is used**
- **The symplecticity is bad. Two main defects, we are working on that :**
  - **- the mesh is loose, considering existence of strong local variations of $\vec{B}$**
  - **- the field has discontinuities (the derivatives in Zgoubi do no like that at all !)**

No acceleration

x-x'

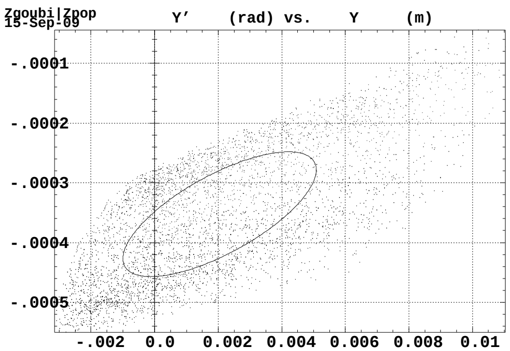


y-y'

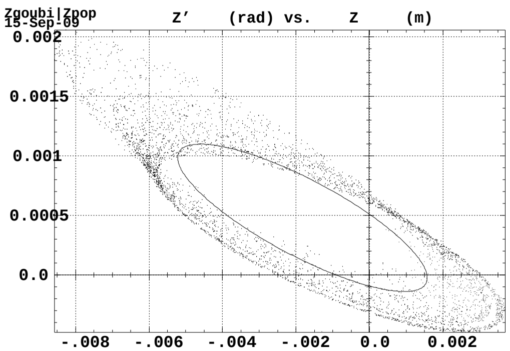


With acceleration, 2.104 Tesla/second
(I should not show that... well... I can't help - sorry)

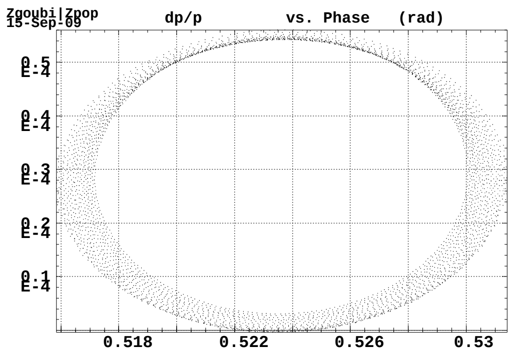
x-x'



y-y'



dp-phase



INTEGRATION IN ELECTRIC FIELDS

- **With the same ingredients as before,**

$d(m\vec{v}) \ = \ q\,(\vec{e}+\vec{v}\times\vec{b})\ dt$ **can be written under the form** $\boxed{(B\rho)'\vec{u} + B\rho\vec{u}' = \frac{\vec{e}}{v}}$

- **Then, based on**

$$(B\rho)'\vec{u} + B\rho\vec{u}' = \frac{\vec{e}}{v},\ (B\rho)''\vec{u} + 2(B\rho)'\vec{u}' + B\rho\vec{u}'' = \left(\frac{1}{v}\right)'\vec{e} + \frac{\vec{e}'}{v},\ etc.$$

$$\vec{u}' = \left(\frac{1}{v}\right)\ \vec{E} - \frac{(B\rho)'}{B\rho}\vec{u}\ ,\ \vec{u}'' = \left(\frac{1}{v}\right)'\vec{E} + \left(\frac{1}{v}\right)\ \vec{E}'\ |_{B\rho} - 2\frac{(B\rho)'}{B\rho}\vec{u}' - \frac{(B\rho)''}{B\rho}\vec{u}\ ,\ etc.$$

$$\left(\frac{1}{v}\right) = \frac{1}{c^2}\frac{W+m_0c^2}{qB\rho}\ ,\ \left(\frac{1}{v}\right)' = \frac{1}{c^2}\frac{(\vec{e}\cdot\vec{u})}{B\rho} - \frac{1}{v}\frac{(B\rho)'}{B\rho},\ etc.$$

one then calculate the Taylor series for the rigidity,

$$(B\rho)(M_1) \approx (B\rho)(M_0) + (B\rho)'(M_0)\Delta s + ... + (B\rho)''''(M_0)\frac{\Delta s^4}{4!} \tag{9}$$

• **The derivatives**

$$\vec{E}^{(n)}\mid_{B\rho}=\frac{1}{B\rho}\frac{d^n\vec{e}}{ds^n}$$

of the electric field so needed there are obtained from the total derivative in the frame of the electrostatic element,

$$d\vec{E}=\frac{\partial\vec{E}}{\partial X}\,dX+\frac{\partial\vec{E}}{\partial Y}\,dY+\frac{\partial\vec{E}}{\partial Z}\,dZ \tag{10}$$

which is to say

$$\begin{aligned}
\vec{E}' &= \sum_i \frac{\partial\vec{E}}{\partial X_i}\,u_i \\
\vec{E}'' &= \sum_{ij} \frac{\partial^2\vec{E}}{\partial X_i\partial X_j}\,u_iu_j+\sum_i \frac{\partial\vec{E}}{\partial X_i}\,u_i' \\
\vec{E}''' &= \sum_{ijk} \frac{\partial^3\vec{E}}{\partial X_i\partial X_j\partial X_k}\,u_iu_ju_k+3\sum_{ij} \frac{\partial^2\vec{E}}{\partial X_i\partial X_j}\,u_i'u_j+\sum_i \frac{\partial\vec{E}}{\partial X_i}\,u_i''\ , \\
\text{etc.}
\end{aligned} \tag{11}$$

- There are oodles of magnetic elements in Zgoubi, allowing simulation of almost everyday's life electric situations

What :	*Keyword :*
2-tube (bipotential) lens	EL2TUB
3-tube (unipotential) lens	UNIPOT
Decapole	ELMULT
Dipole	ELMULT
Dodecapole	ELMULT
Multipole	ELMULT
N-electrode mirror/lens, straight slits	ELMIR
N-electrode mirror/lens, circular slits	ELMIRC
Octupole	ELMULT
Quadrupole	ELMULT
R.F. (kick) cavity	CAVITE
Sextupole	ELMULT
Skewed multipoles	ELMULT

Electric field maps

1D, cylindrical symmetry	ELREVOL
2-D, no symmetry	MAP2D_E

INTEGRATION IN COMBINED E+B FIELDS

- **When both $\vec{e}$ and $\vec{b}$ are non-zero, the complete equation must be considered,**

$$(B\rho)'\vec{u} + B\rho\,\vec{u}' = \vec{e}\,/\,v \,+\, \vec{u}\times\vec{b}$$

- **Well, one can imagine...**

$$\begin{aligned}
&(B\rho)''\vec{u} + 2(B\rho)'\vec{u}' + B\rho\vec{u}'' = \left(\tfrac{1}{v}\right)'\vec{e} + \left(\tfrac{1}{v}\right)\vec{e}' + (\vec{u}\times\vec{b})' \\
&(B\rho)'''\vec{u} + 3(B\rho)''\vec{u}' + 3(B\rho)'\vec{u}'' + B\rho\vec{u}''' = \left(\tfrac{1}{v}\right)''\vec{e} + 2\left(\tfrac{1}{v}\right)'\vec{e}' + \left(\tfrac{1}{v}\right)\vec{e}'' + (\vec{u}\times\vec{b})'' \\
&(B\rho)''''\vec{u} + 4(B\rho)'''\vec{u}' + 6(B\rho)''\vec{u}'' + 4(B\rho)'\vec{u}''' + B\rho\vec{u}'''' = \\
&\qquad \left(\tfrac{1}{v}\right)'''\vec{e} + 3\left(\tfrac{1}{v}\right)''\vec{e}' + 3\left(\tfrac{1}{v}\right)'\vec{e}'' + \tfrac{1}{v}\vec{e}''' + (\vec{u}\times\vec{b})''' \\
&\vec{u}' = \left(\tfrac{1}{v}\right)\vec{E} + (\vec{u}\times\vec{B}) - \tfrac{(B\rho)'}{B\rho}\vec{u} \\
&\vec{u}'' = \left(\tfrac{1}{v}\right)'\vec{E} + \left(\tfrac{1}{v}\right)\vec{E}'\,|_{B\rho} + (\vec{u}\times\vec{B}')'\,|_{B\rho} - 2\tfrac{(B\rho)'}{B\rho}\vec{u}' - \tfrac{(B\rho)''}{B\rho}\vec{u} \qquad (12) \\
&\vec{u}''' = \left(\tfrac{1}{v}\right)''\vec{E} + 2\left(\tfrac{1}{v}\right)'\vec{E}'\,|_{B\rho} + \tfrac{1}{v}\vec{E}''\,|_{B\rho} + (\vec{u}\times\vec{B})''\,|_{B\rho} - 3\tfrac{(B\rho)'}{B\rho}\vec{u}'' - 3\tfrac{(B\rho)''}{B\rho}\vec{u}' - \tfrac{(B\rho)'''}{B\rho}\vec{u} \\
&\vec{u}'''' = \left(\tfrac{1}{v}\right)'''\vec{E} + 3\left(\tfrac{1}{v}\right)''\vec{E}'\,|_{B\rho} + 3(\tfrac{1}{v})'\vec{E}''\,|_{B\rho} + \left(\tfrac{1}{v}\right)\vec{E}'''\,|_{B\rho} \\
&\quad + (\vec{u}\times\vec{B})'''\,|_{B\rho} - 4\tfrac{(B\rho)'}{B\rho}\vec{u}''' - 6\tfrac{(B\rho)''}{B\rho}\vec{u}'' - 4\tfrac{(B\rho)'''}{B\rho}\vec{u}' - \tfrac{(B\rho)''''}{B\rho}\vec{u} \\
&\vec{E}^{(n)}\,|_{B\rho} = \tfrac{1}{B\rho}\tfrac{d^n\vec{e}}{ds^n}\ \textbf{and}\quad (\vec{u}\times\vec{B})^{(n)}\,|_{B\rho} = \tfrac{1}{B\rho}(\vec{u}\times\vec{b})^{(n)}\ldots\ldots\ldots
\end{aligned}$$

EXAMPLE - LESB3, BNL, TWO-STAGE SEPARATED 800-MeV/c KAON BEAMLINE USING 2 *WIENFILTERS*

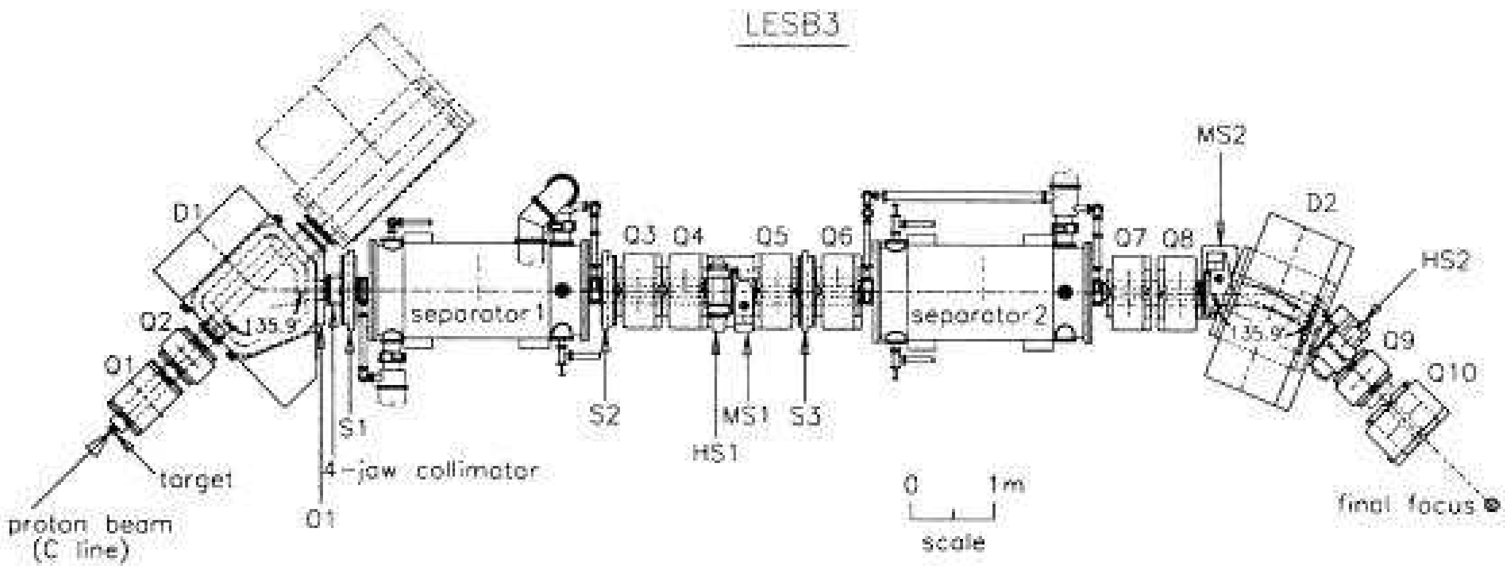


Fig. 1. Lay out of LESB3 beamline.

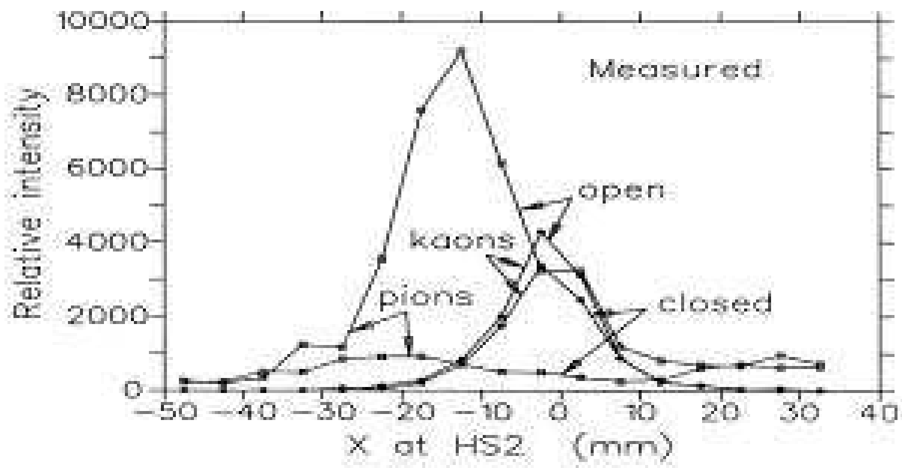


Fig. 16. Measured kaon and pion distributions at HS2 with the four-jaw collimator open and upper-right jaw closed. Compare with Fig. 10(a).

CALCULATION OF THE TIME OF FLIGHT

$$T(M_1) \approx T(M_0) + \frac{dT}{ds}(M_0)\,\Delta s + \frac{d^2T}{ds^2}(M_0)\,\frac{\Delta s^2}{2} + \frac{d^3T}{ds^3}(M_0)\,\frac{\Delta s^3}{3!} + \frac{d^4T}{ds^4}(M_0)\,\frac{\Delta s^4}{4!} \tag{13}$$

Calculation involves the derivatives $dT/ds = 1/v,\quad d^2T/ds^2 = d(1/v)/ds, etc.$**, these are obtained from :**

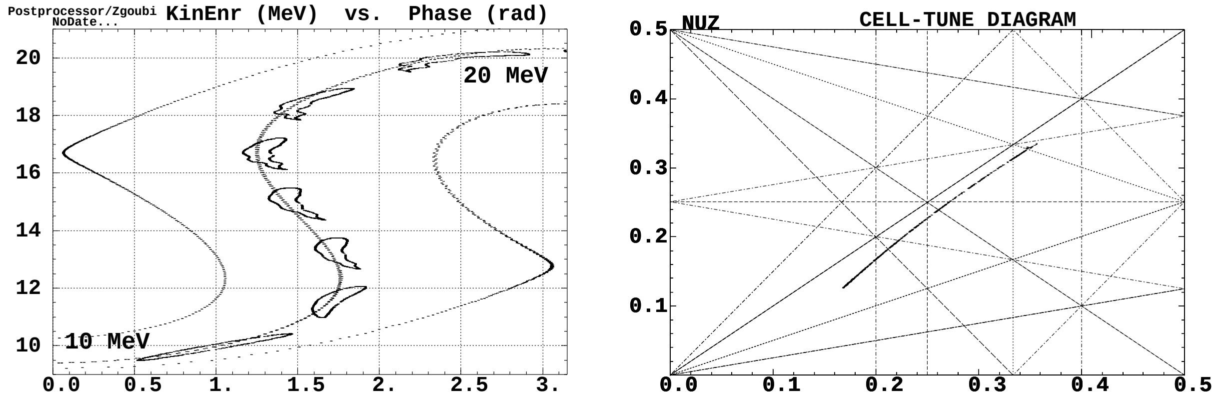
$$\begin{array}{rl}
\left(\frac{1}{v}\right) & = \frac{1}{c^2}\,\frac{W+m_0c^2}{qB\rho} \\
\left(\frac{1}{v}\right)' & = \frac{1}{c^2}\,\frac{(\vec{e}\cdot\vec{u})}{B\rho} - \frac{1}{v}\frac{(B\rho)'}{B\rho} \\
\left(\frac{1}{v}\right)'' & = \frac{1}{c^2}\,\frac{(\vec{e}\cdot\vec{u})'}{B\rho} - 2\left(\frac{1}{v}\right)'\frac{(B\rho)'}{B\rho} - \frac{1}{v}\,\frac{(B\rho)''}{B\rho} \\
\left(\frac{1}{v}\right)''' & = \frac{1}{c^2}\,\frac{(\vec{e}\cdot\vec{u})''}{B\rho} - 3\left(\frac{1}{v}\right)''\frac{(B\rho)'}{B\rho} - 3\left(\frac{1}{v}\right)'\frac{(B\rho)''}{B\rho} - \frac{1}{v}\,\frac{(B\rho)'''}{B\rho}
\end{array} \tag{14}$$

• Note that, in the absence of electric field, the Taylor expansion above reduces to (fortunately)

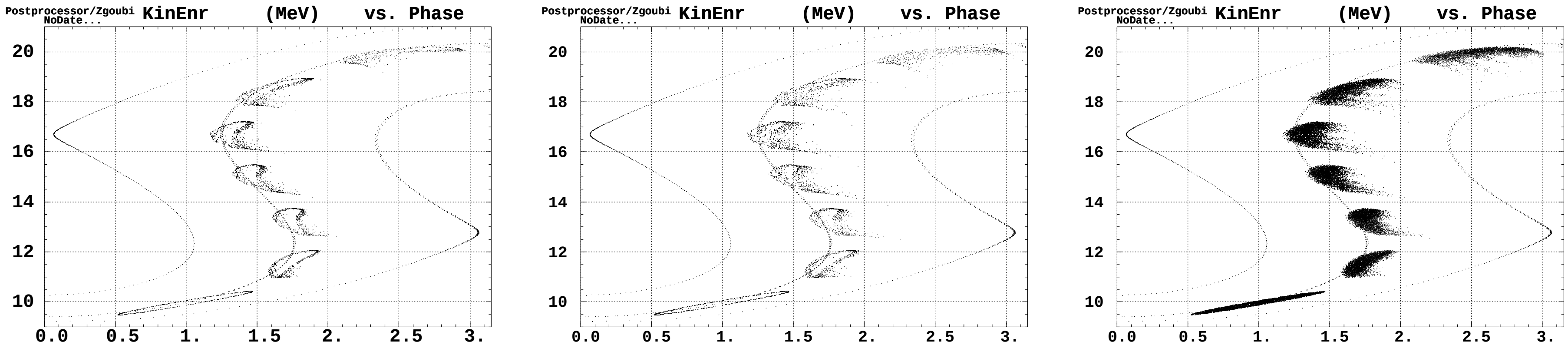
$$T(M_1) = T(M_0) + \Delta s/v \tag{15}$$

EXAMPLE - USE OF THE TOF BY 'CAVITE'

- **Longitudinal motion EMMA, electrons, $10 \rightarrow 20$ MeV - similar : muon aceleration in NuFact**

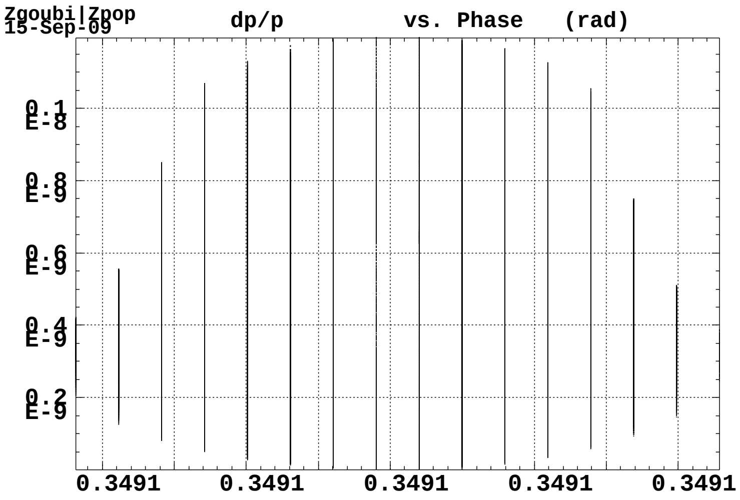


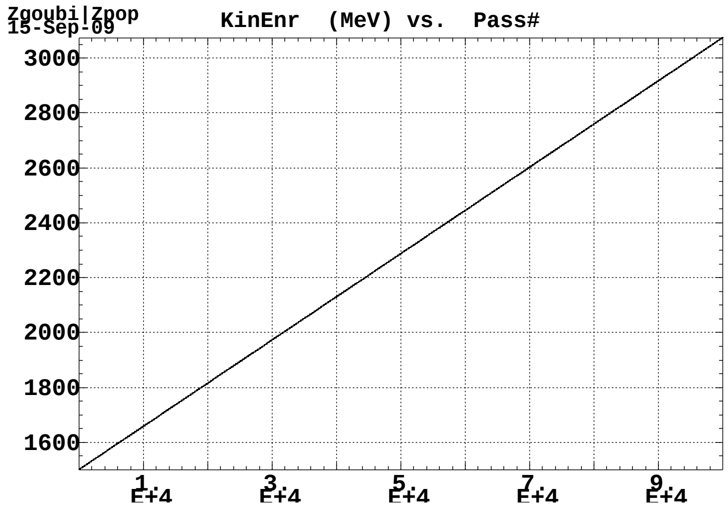
10 to 20 MeV, 125 cavity passes. Voltage : 70 kV peak, RF freq. : 1.3552 GHz.



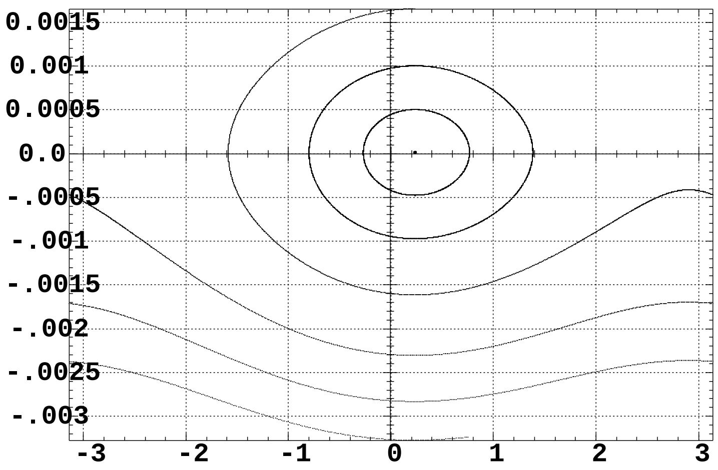
Case of non-zero transverse emittance. Left : $\epsilon_{x,z} = 90\,\pi$ **mm.mrad norm. Middle :** $\epsilon_{x,z} = 200\,\pi$ **mm.mrad norm. Right : full 6-D acceleration.**

Long term tracking including synchrotron motion





- **140000 turns in AGS,** $1.5 \rightarrow 3$ **GeV,** $\dot{B} = 2.104$ **T/s, synchrotronous particle.**



- **3 GeV proton synchrotron, 105 m circumference, accelerating bucket,** $\dot{B} = 4.2$ **T/s.**
  - **Momentum acceptance : theoretical** $1.65\,10^{-3}$**, numerical** $1.65\,10^{-3}$

3 SPIN TRACKING

• **The equation of spin precession**

$$\frac{d\vec{S}}{dt} = \frac{q}{m}\vec{S}\times\vec{\Omega}, \quad \textbf{with} \quad \vec{\Omega} = (1+\gamma G)\vec{b} + G(1-\gamma)\vec{b}_{//}$$

Normalizing as earlier (remember : "$u' = u \times \vec{B}$") using $ds = vdt$, $\gamma mv = qB\rho$, $\omega = \Omega / B\rho$, etc., yields the form handled in the Fortran :

$$\boxed{\vec{S}\,' = \vec{S} \times \vec{\omega}}$$

• $\vec{S}(M_1)$ **following a displacement Δs, is obtained from $\vec{S}(M_1)$ using truncated Taylor expansion**

$$\vec{S}(M_1) \approx \vec{S}(M_0) + \frac{d\vec{S}}{ds}(M_0)\,\Delta s + \frac{d^2\vec{S}}{ds^2}(M_0)\frac{\Delta s^2}{2} + \frac{d^3\vec{S}}{ds^3}(M_0)\frac{\Delta s^3}{3!} + \frac{d^4\vec{S}}{ds^4}(M_0)\frac{\Delta s^4}{4!}$$

• **Recurrent differenciation yields the $d^n\vec{S}\ /\ ds^n$ and $d^n\vec{B}_{//}\ /\ ds^n$ and at M_0 :**

$$\vec{S}\,' = \vec{S}\times\vec{\omega},\ \vec{S}\,'' = \vec{S}\,'\times\vec{\omega} + \vec{S}\times\vec{\omega}\,',\ \vec{S}\,''' = \vec{S}\,''\times\vec{\omega} + 2\vec{S}\,'\times\vec{\omega}\,' + \vec{S}\times\vec{\omega}\,'',\ ,etc.$$

$$\vec{B}_{//} = (\vec{B}\cdot\vec{u})\,\vec{u},\ \vec{B}'_{//} = (\vec{B}\,'\cdot\vec{u} + \vec{B}\cdot\vec{u}\,')\,\vec{u} + (\vec{B}\cdot\vec{u})\,\vec{u}\,',\ etc.$$

SPIN TRACKING AT AGS

$\gamma G = 48 - \nu_z$ **(19.59585 GeV)**

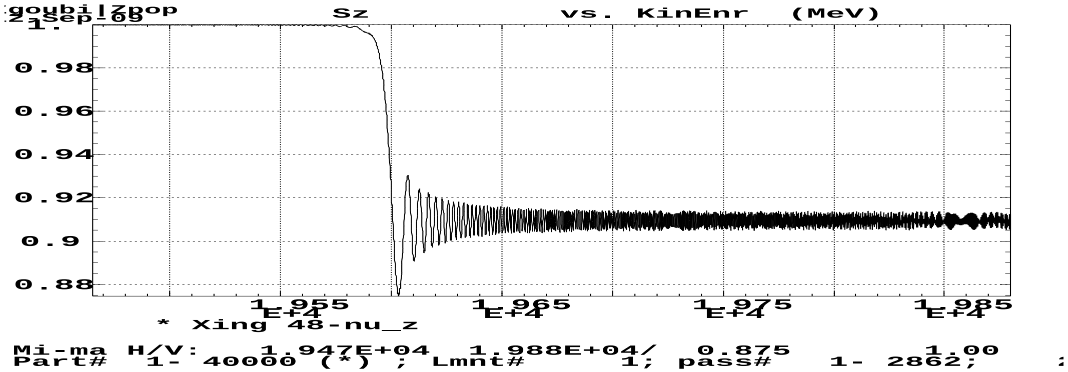
Typical particle and "power supply" data :

```
Xing 48-nu
'OBJET'
67.98605e3
8
  1  1  1
 -0.64319816e-2  -0.49829722e-3     0.0 0.0E+00    0.0E+00  1.    'p'
 -1.588  19.843  0.
 1.033  11.675     0.5 to 2e-6
  0  1  0.
 'SCALING'
 1  2
  MULTIPOL SBEN
  -1
  67.98605
  1
  MULTIPOL QUAD
  -1
  67.98605
  1
```

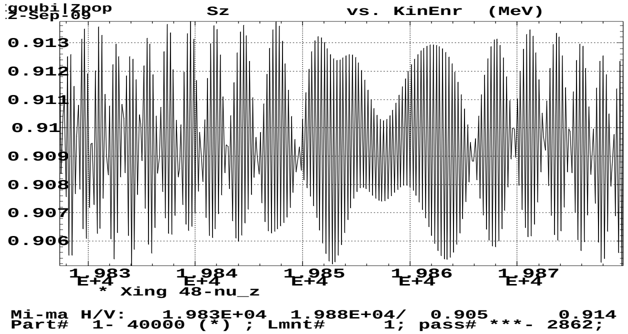

- **Resonance strength for various vertical invariant values.**

ϵ_z/π (×10^{-6})	A^2	$\|J_n\|^2$	$A^2/\epsilon_z/\pi$	$\|J_n\|^2/\epsilon_z/\pi$	p_{init}	p_{final}
.125	**0.04630575**	**1.2999233E-06**	**370446.0**	**10.39939**	**1**	**0.9095**
.25	**0.09862635**	**2.7686992E-06**	**394505.4**	**11.07480**	**0.9998**	**0.812**
.5	**0.1913054**	**5.3704412E-06**	**382610.8**	**10.74088**	**0.9996**	**0.6515**
1	**0.3895315**	**1.0935165E-05**	**389531.5**	**10.93517**	**0.9993**	**0.3545**
2	**0.8073937**	**2.2665648E-05**	**403696.8**	**11.33282**	**0.9985**	**-0.1078**

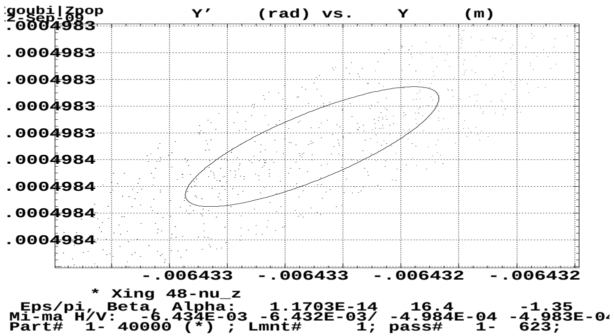
Crossing $\gamma G = 48 - \nu_z$ (19.59585 GeV), $\dot{B} = 2.104\ T/s$, $\epsilon_z/\pi = 0.125\,10^{-6}$

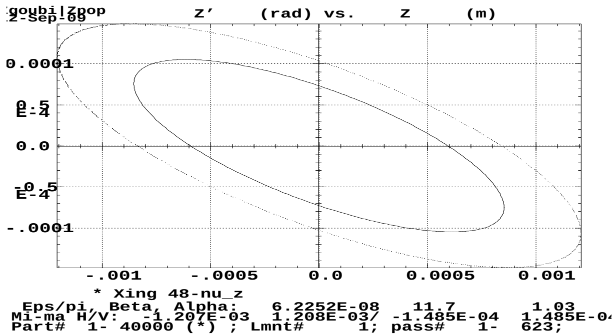


- S_z versus kinetic energy.

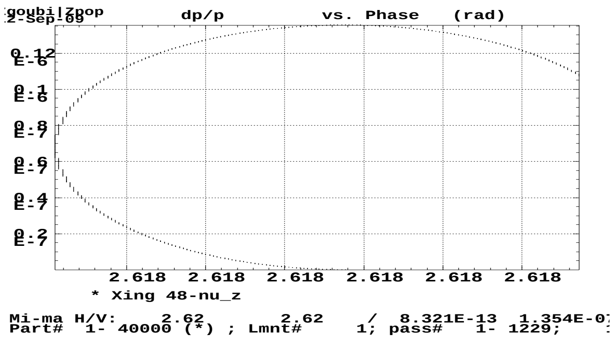


- Zoom on final S_z.



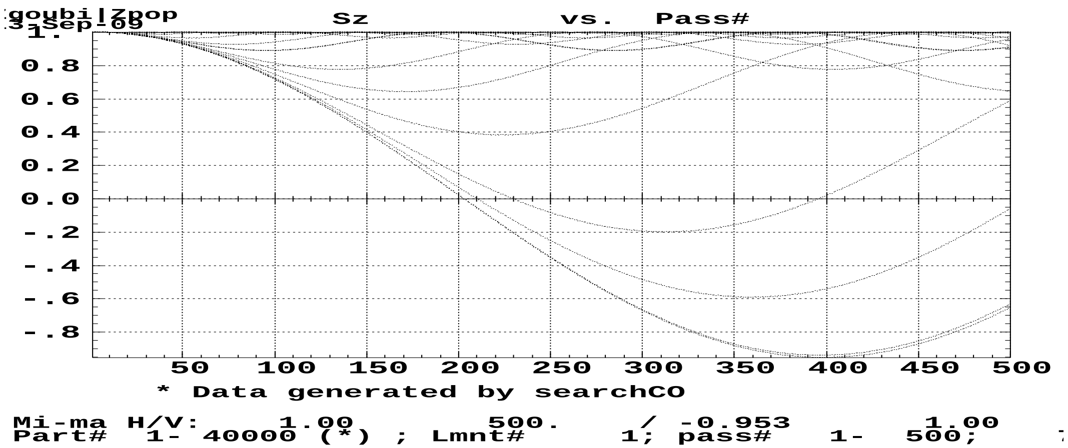


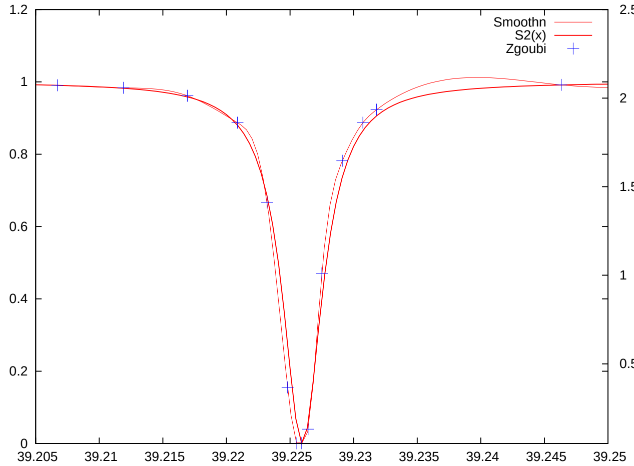
- x-x' and z-z'.



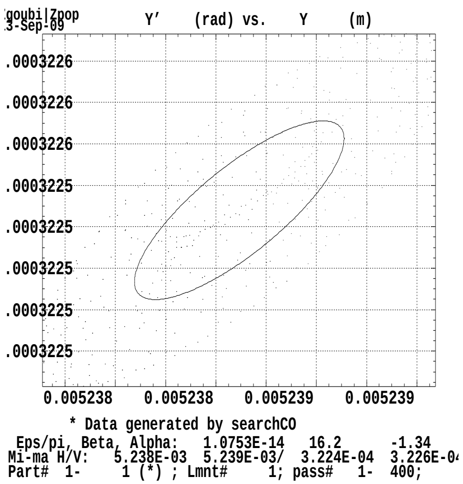
- dp-phase (1350 turns).

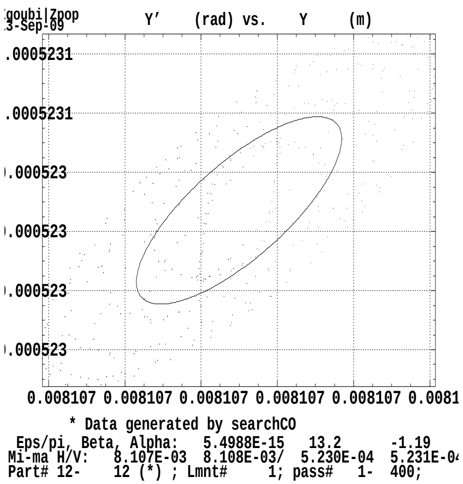
Static, $\gamma G = 48 - \nu_z$ (19.59585 GeV) - $\epsilon_z/\pi = 0.125\,10^{-6}$

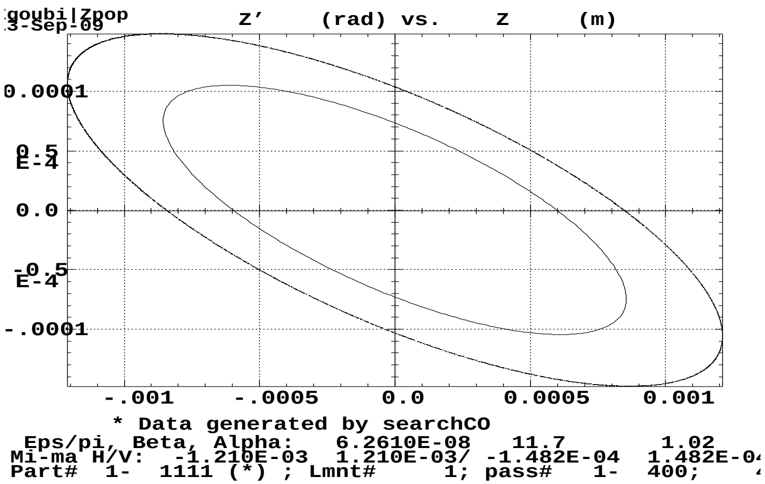




- **Top :** S_z **versus turn number. Bottom : matching with** $\bar{S}_z = \left(\frac{\Delta^2}{\Delta^2+\epsilon^2}\right)^{1/2}$







- **Left : x-x' of particles at min. and max.** p/p_0**. Right : z-z' (all particles superimposed).**

4 SYNCHROTRON RADIATION

• Zgoubi allows the simulation of two types of synchrotron radiation (SR) effects

- stochastic energy loss and ensuing perturbation on particle dynamics
- radiated spectral-angular energy densities observed in the lab.

Energy loss and related dynamical effects

• The energy loss is calculated after each integration step $\triangle s$, in a classical manner, accounting for two random processes :
  - probability of emission of a photon
  - energy of the photon.

• Effects on the dynamic of the emitting particle :
  - alteration of the energy, or extended to
  - angular kick effect

• **Main aspects of the method :**

- The probability of emission of a photon follows a Poisson distribution (very small number of photons emitted within a step Δs)

$$p(k) = \frac{\lambda^{-k}}{k!} exp(-k), \quad \textbf{with} \quad \lambda = \frac{20 e r_0}{8\bar{h}\sqrt{3}} \beta^2 B\rho\Delta s$$

- k is the number of photons emitted over Δs

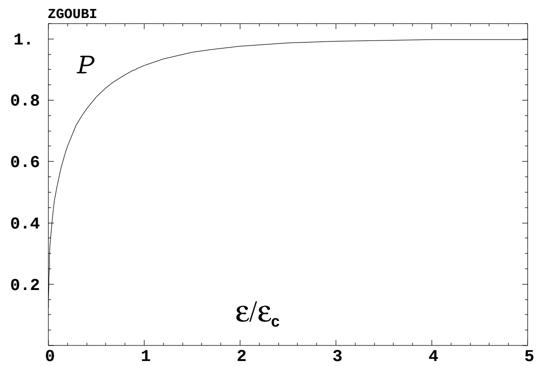
For instance :

a 1 GeV electron will emit about 20.6 photons per radian; an integration step size $\Delta s = 0.1$ m upon $\rho = 10$ m bending radius results in 0.2 photons per step

- λ is evaluated at each integration step from the current values β, $B\rho$ and Δs, then a value of k is drawn by a rejection method.

- Energy of the k photons is taken at random in :

$$\mathcal{P}(\epsilon/\epsilon_c) = \frac{3}{5\pi} \int_0^{\epsilon/\epsilon_c} \int_{\epsilon/\epsilon_c}^{\infty} K_{5/3}(x) dx$$



$\epsilon_c = \bar{h}\omega_c$, $\omega_c = 2\pi 3\gamma^3 c/2\rho$ **= critical frequency.**

Example - Emittance increase in the e^+e^- linear collider beam delivery system

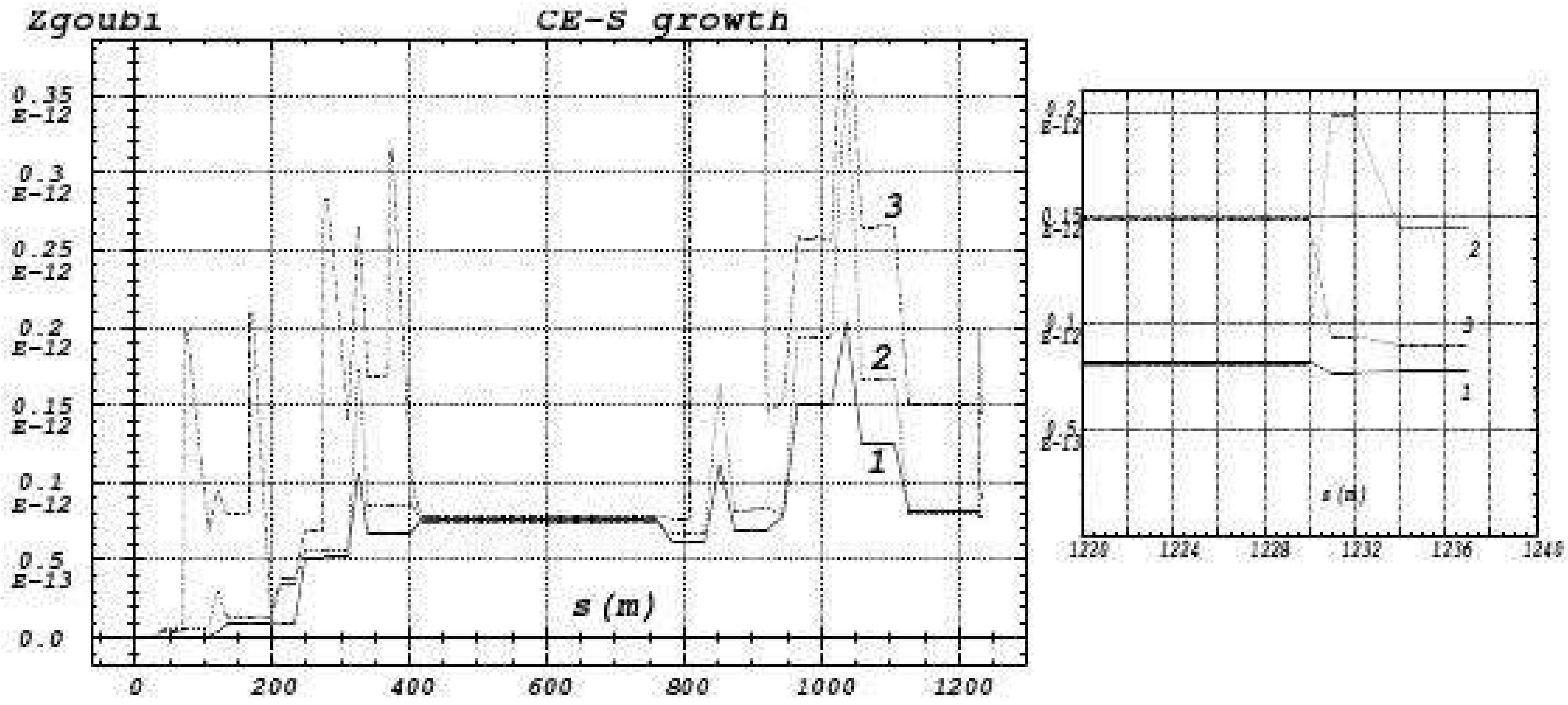
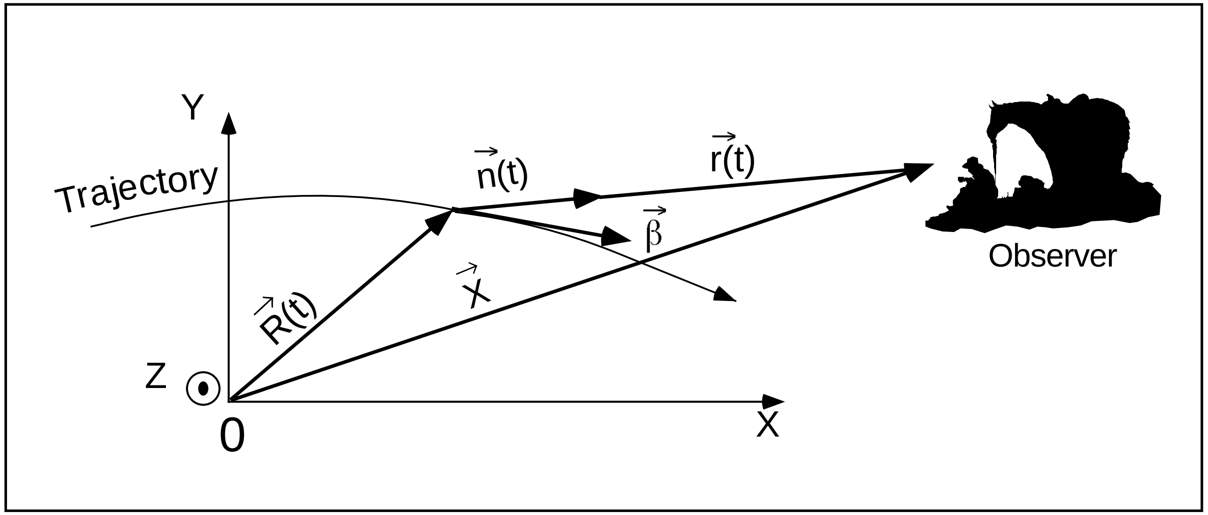


Figure 9: Horizontal CE-$\mathcal{S}$ variation $(\mathcal{S}_x/\pi(s) - \mathcal{S}_x/\pi(0))$ along TESLA-bds as obtained from the ray-tracing of $2\,10^4$ particles, in various cases of SR simulation (resp^{ly} 1, 2 and 3 in Table 1) :
- solid line : zero initial emittances, sextupoles off ;
- dashed line : initial emittances $\epsilon_x = 10^{-11}$, $\epsilon_z = 10^{-14}$ m.rad, sextupoles off ;
- dotted line : initial emittances $\epsilon_x = 10^{-11}$, $\epsilon_z = 10^{-14}$ m.rad, sextupoles are excited.

The last case shows a strong overshoot (cut out on the Figure) in the $s \approx 850$ m region due to chromatic distortions (see page 16) ; this effect appears also in the low-β quad and FF region zoomed on the right plot (broken lines are due to particle coordinates being saved only at optical element ends).

Spectral-angular radiated densities



The ray-tracing ingredients provide the toolkit to compute

$$\vec{\mathcal{E}}(\vec{n},\tau) = \frac{q}{4\pi\varepsilon_0 c} \frac{\vec{n}(t) \times \left[\left(\vec{n}(t) - \vec{\beta}(t)\right) \times d\vec{\beta}/dt\right]}{r(t)\left(1 - \vec{n}(t)\cdot\vec{\beta}(t)\right)^3}, \qquad \mathcal{B} = \vec{n} \times \vec{\mathcal{E}}/c$$

In the toolkit, amongst other tools :

$$d\tau/dt = 1 - \vec{n}(t)\cdot\vec{\beta}(t)$$

$$d\vec{\beta}/dt = (q/m)\ \vec{\beta}(t) \times \vec{b}(t)$$

$$\vec{r}(t) = \vec{X} - R(t) \textbf{ and } \vec{n}(t) = \vec{r}(t)/|\vec{r}(t)|$$

$$\vec{n} = (n_x, n_y, n_z) = (\cos\psi\ \cos\phi, \cos\psi\ \sin\phi, \sin\psi)$$
$$= \left[1 - 2(\sin^2\phi/2 + \sin^2\psi/2) + 4\sin^2\phi/2\sin^2\psi/2, \sin\phi(1 - 2\sin^2\psi/2), \sin\psi\right]$$

ϕ and ψ are the observation angles, given by

$$\vec{\beta} = (\beta_x, \beta_y, \beta_z) = \left[\sqrt{(\beta^2 - \beta_y^2 - \beta_z^2)}, \beta_y, \beta_z\right]$$
$$= \left[\sqrt{(1 - 1/\gamma^2 - \beta_y^2 - \beta_z^2)}, \beta_y, \beta_z\right] = (1 - a/2 + a^2/8 - a^3/16 + ..., \beta_y, \beta_z)$$

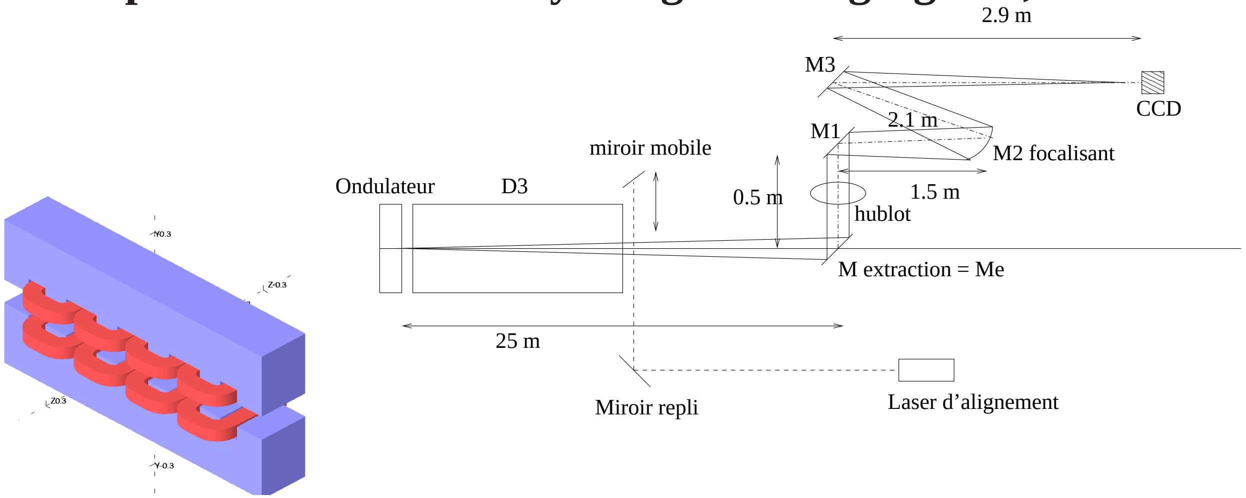
$$a = 1/\gamma^2 + \beta_y^2 + \beta_z^2$$

• The electric field of the radiation is then Fourier transformed, yielding the spectral angular energy density

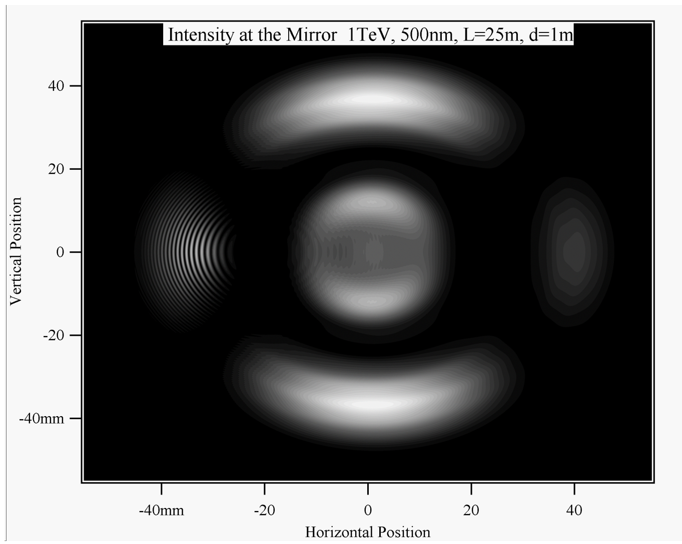
$$\partial^3 W/\partial\phi\,\partial\psi\,\partial\omega = 2r^2 \left| FT_\omega\left(\vec{\mathcal{E}}(\tau)\right)\right|^2 /\mu_0 c$$

EXAMPLE - Design of the SR beam diagnostics installations at LHC

This experience has been fully designed using Zgoubi, and then checked using SRW (ESRF).



LHC undulator upstream of a long dipole, and the optical system, drawn on that of LEP.



Intensity emitted (horizontal component) by 1 TeV protons, λ = 500 nm, with a distance d = 1 m between the two sources, simulated with Zgoubi (left) and with SRW (right).

5 THE FITTING PROCEDURE

An indispensable tool for
- preliminary adjustments (tunes, etc.) prior to 6-D simulations
- considered very useful for further assessment and optimisation of higher order behavior, DA, transmission, ...

FIT CONSTRAINTS :

Trajectory coordinates (e.g., final coordinates)

Several other types of quanttities that are deduced from trajectories, e.g. :
- first and higher order transport coefficients
- beam matrix coefficients (waist, divergence)
- particle fluxes through ellipses (→ transmission efficiency)
- *New ! Spin coordinates*

In the case of periodic structures :
- closed orbits
- tunes, chromaticites, anharmonicities
- *New ! Spin closed orbit*

FIT VARIABLES : all data !

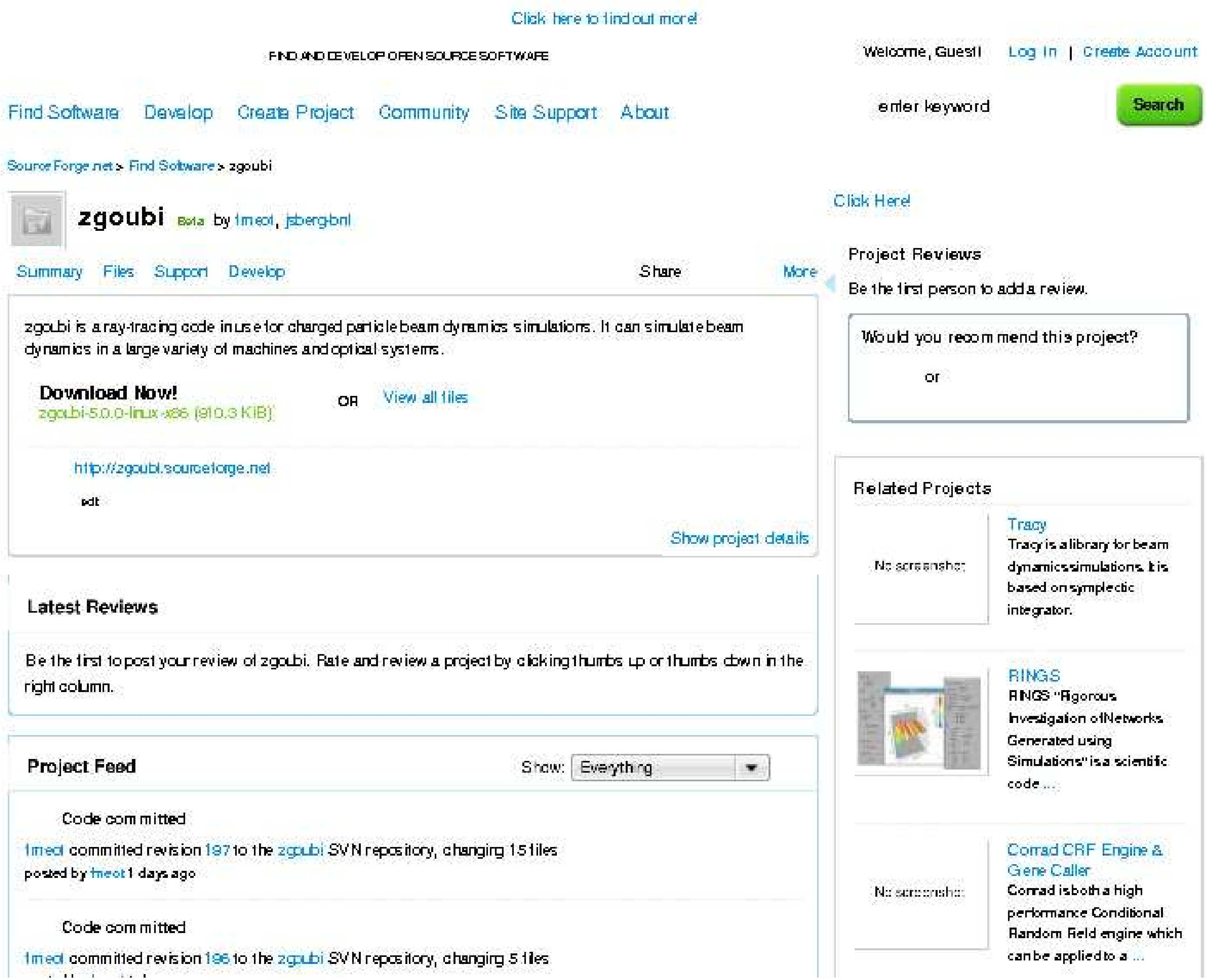
```
  'OBJET'    * c.o., constant Gap *
 226.8235847      68MeV/c muon
2
2  1
499.377    0.    0.  0. 0.  1.2     'b'
1 1 1 1 1 1 1 1 1
 'FFAG'
 0
 3   45.     500.                         NMAG, Sector angle, R0
 18.17   0.  -0.717  5.                     mag 1 : ACNT, dum, B0, K
 6.3  0.                                 EFB 1 : lambda, gap const/varbl
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 1.23 0.    1.E6  -1.E6  1.E6  1.E6
 6.3  0.                                 EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-1.23  0.    1.E6  -1.E6  1.E6  1.E6
 0. -1                                    EFB 3 : inhibited by iop=0
 0  0.      0.      0.      0.      0. 0.  0.
 0.  0.   0.    0.    0. 0.
 22.5    0.   3.2    5.                   mag 2 : ACNT, B0, K,dums
 6.3  0.                                 EFB 1
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 3.   0.    1.E6  -1.E6  1.E6  1.E6
 6.3  0.                                 EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-3    0.    1.E6  -1.E6  1.E6  1.E6
 0. -1                                    EFB 3
 0  0.      0.      0.      0.      0. 0.  0.
 0.  0.   0.    0.    0. 0.
26.83  0.  -0.717  5.                     mag 3 : ACNT, dum, B0, K
 6.3  0.                                 EFB 1
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 1.23 0.    1.E6  -1.E6  1.E6  1.E6
 6.3  0.                                 EFB 2
 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
-1.23  0.    1.E6  -1.E6  1.E6  1.E6
 0. -1                                    EFB 3
 0  0.      0.      0.      0.      0. 0.  0.
 0.  0.   0.    0.    0. 0.
   0     2  125.            KIRD anal/num, resol(mesh=step/resol)
.5                          integration step size
 2   0.  0. 0. 0.
```

6 Zgoubi's KEYWORDS/PROCEDURES

Glossary of keywords

CONCLUSIONS / 1

zgoubi | Get zgoubi at SourceForge.net http://sourceforge.net/projects/zgoubi/



CONCLUSIONS / 2

Possibility of a tutorial tomorow

2h , half a day, A full day !

Thank you for your attention