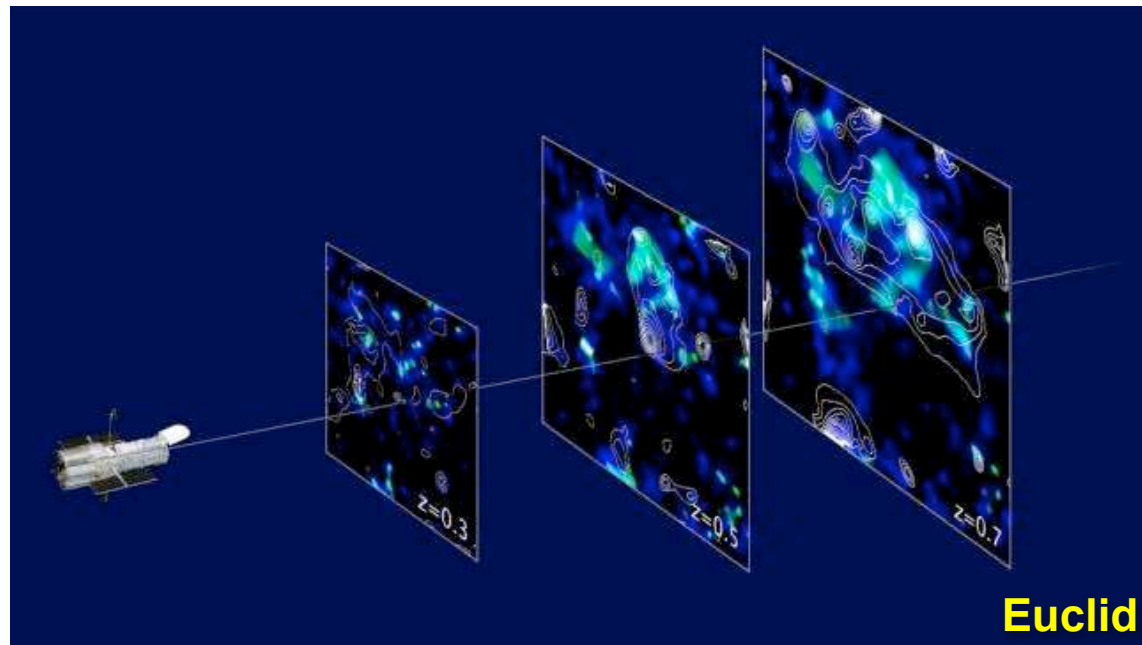
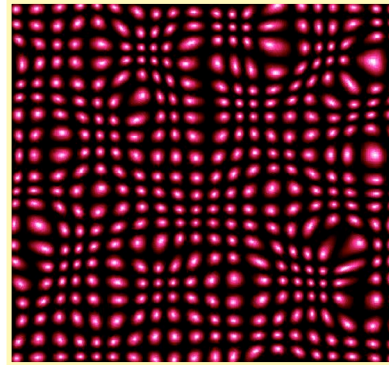
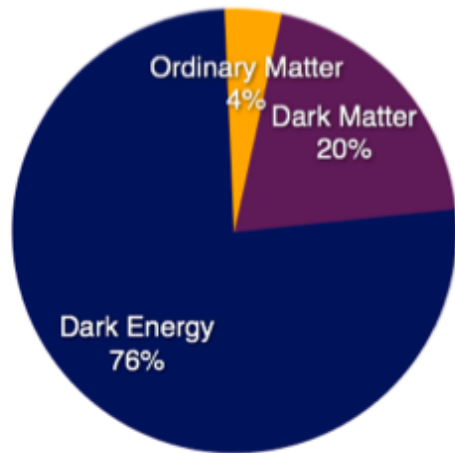


# Weak Lensing

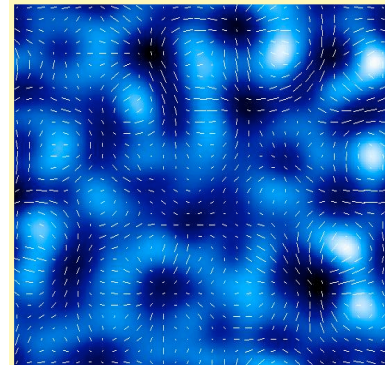
*Saclay Weak Lensing Team:* J.-L. Starck, S. Paulin-Henriksson, S. Pires, A. Leonard, M. Kilbinger, S. Beckouche



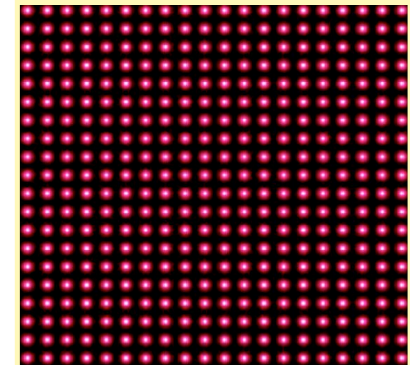
# Weak Lensing



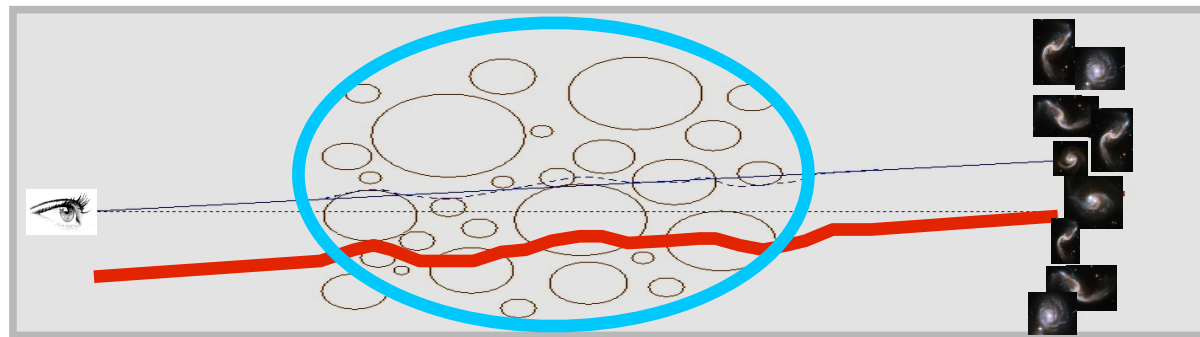
OBSERVER



GRAVITATIONAL LENS



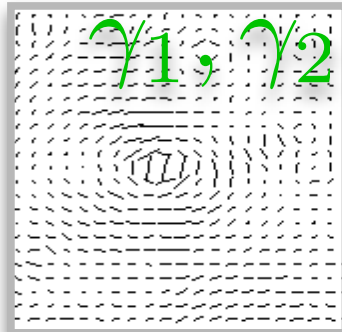
BACKGROUND GALAXIES



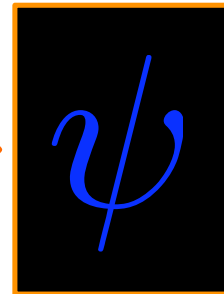
gravitational lens

# Weak Lensing

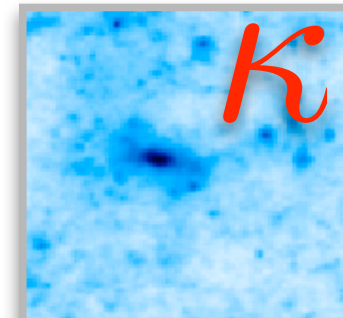
simulated Shear map



lensing potential



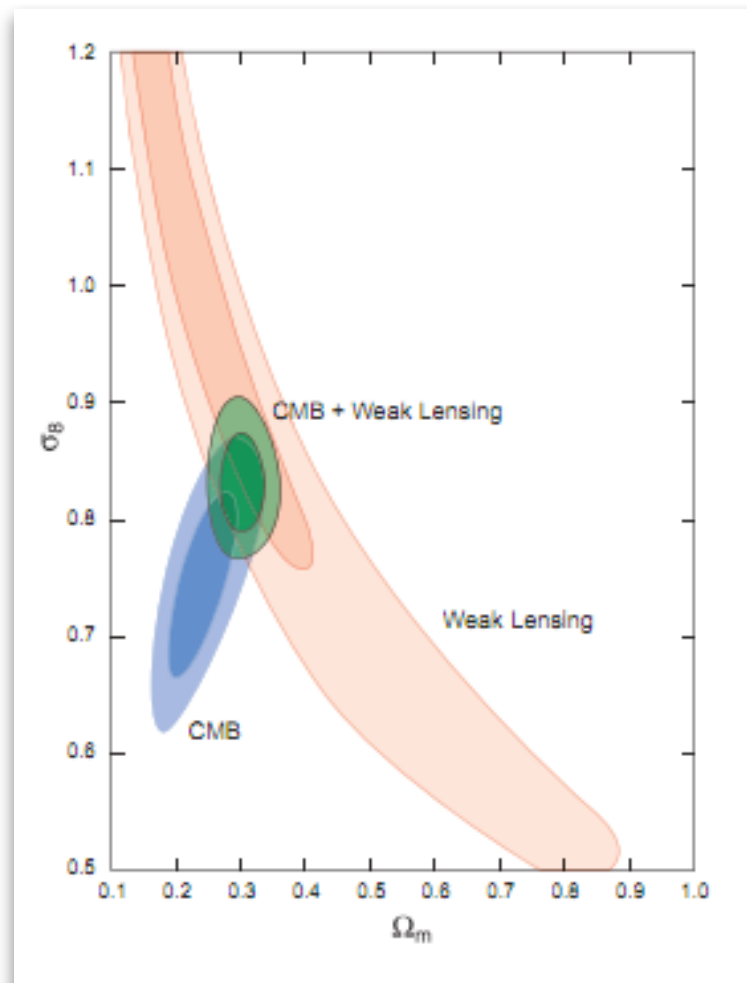
simulated mass map



$$\begin{aligned} \gamma_1 &= \frac{1}{2} (\partial_1^2 - \partial_2^2) \psi \\ \gamma_2 &= \partial_1 \partial_2 \psi \end{aligned} \quad \longleftrightarrow \quad \psi \quad \longleftrightarrow \quad \frac{1}{2} (\partial_1^2 + \partial_2^2) \psi = \kappa$$

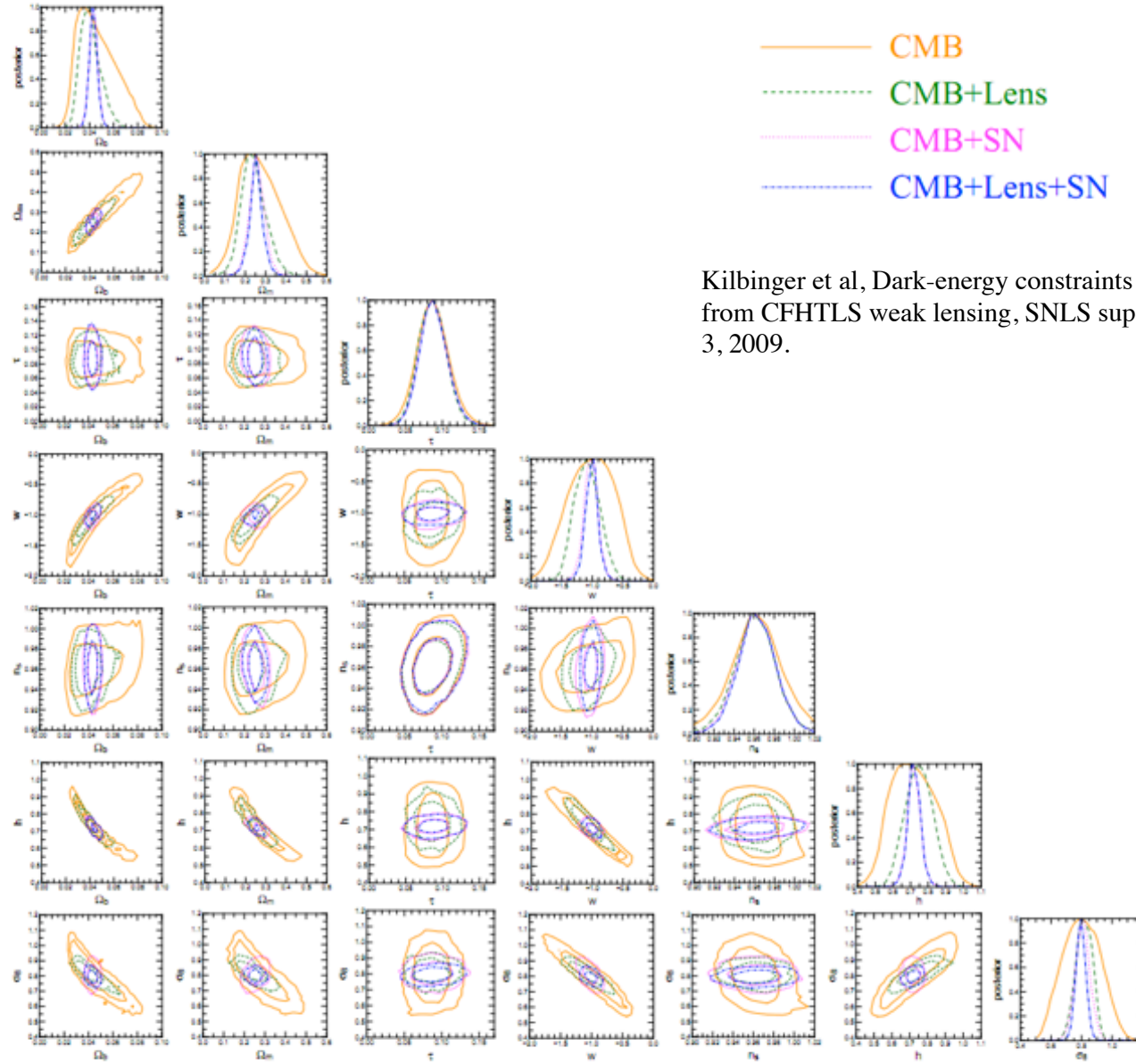
$$\begin{pmatrix} \hat{E}(\mathbf{k}) = \hat{\kappa}(\mathbf{k}) \\ \hat{B}(\mathbf{k}) \end{pmatrix} = \underbrace{\frac{1}{|\mathbf{k}|^2} \begin{pmatrix} k_1^2 - k_2^2 & 2k_1 k_2 \\ 2k_1 k_2 & -k_1^2 + k_2^2 \end{pmatrix}}_{A_{\kappa}} \begin{pmatrix} \hat{\gamma}_1(\mathbf{k}) \\ \hat{\gamma}_2(\mathbf{k}) \end{pmatrix}$$

# Weak Lensing



Massey et al, "The dark matter of gravitational lensing", Reports on Progress in Physics, 73, 8, 2010.





Kilbinger et al, Dark-energy constraints and correlations with systematics from CFHTLS weak lensing, SNLS supernovae Ia and WMAP5, AA, 497, 3, 2009.

**Fig. 9.** 68% and 95% confidence levels for CMB (orange, solid lines), CMB+lensing (green dashed), CMB+SNIa (magenta dotted) and CMB+lensing+SNIa (blue dash-dotted). Systematics are ignored in this plot.

# Tomographic Weak Lensing

THE ASTROPHYSICAL JOURNAL, 522:L21–L24, 1999 September 1

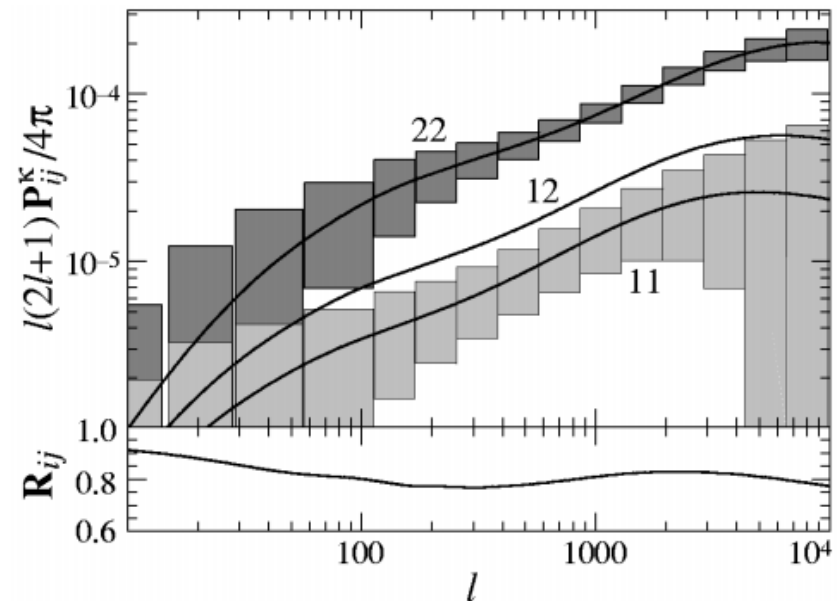
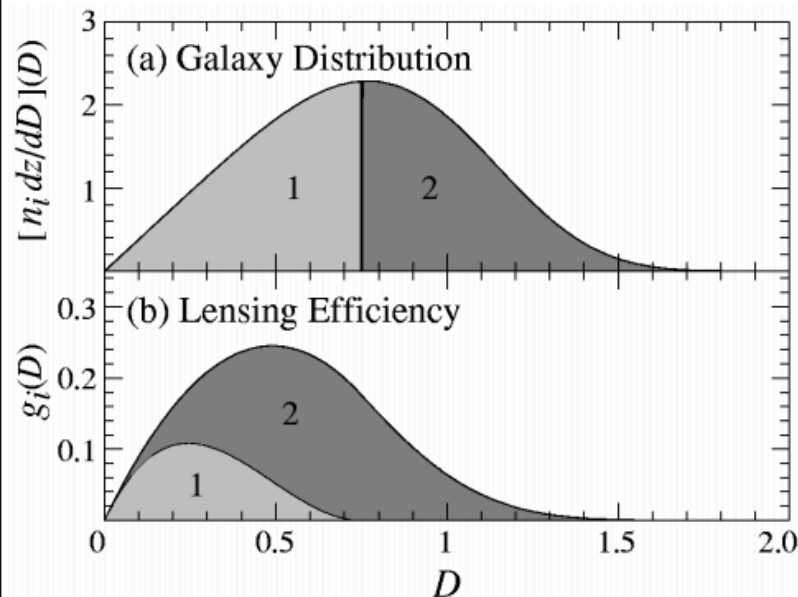
© 1999. The American Astronomical Society. All rights reserved. Printed in U.S.A.

## POWER SPECTRUM TOMOGRAPHY WITH WEAK LENSING

WAYNE HU

Institute for Advanced Study, Princeton, NJ 08540

*Received 1999 April 13; accepted 1999 June 30; published 1999 July 21*



# Euclid Red Book

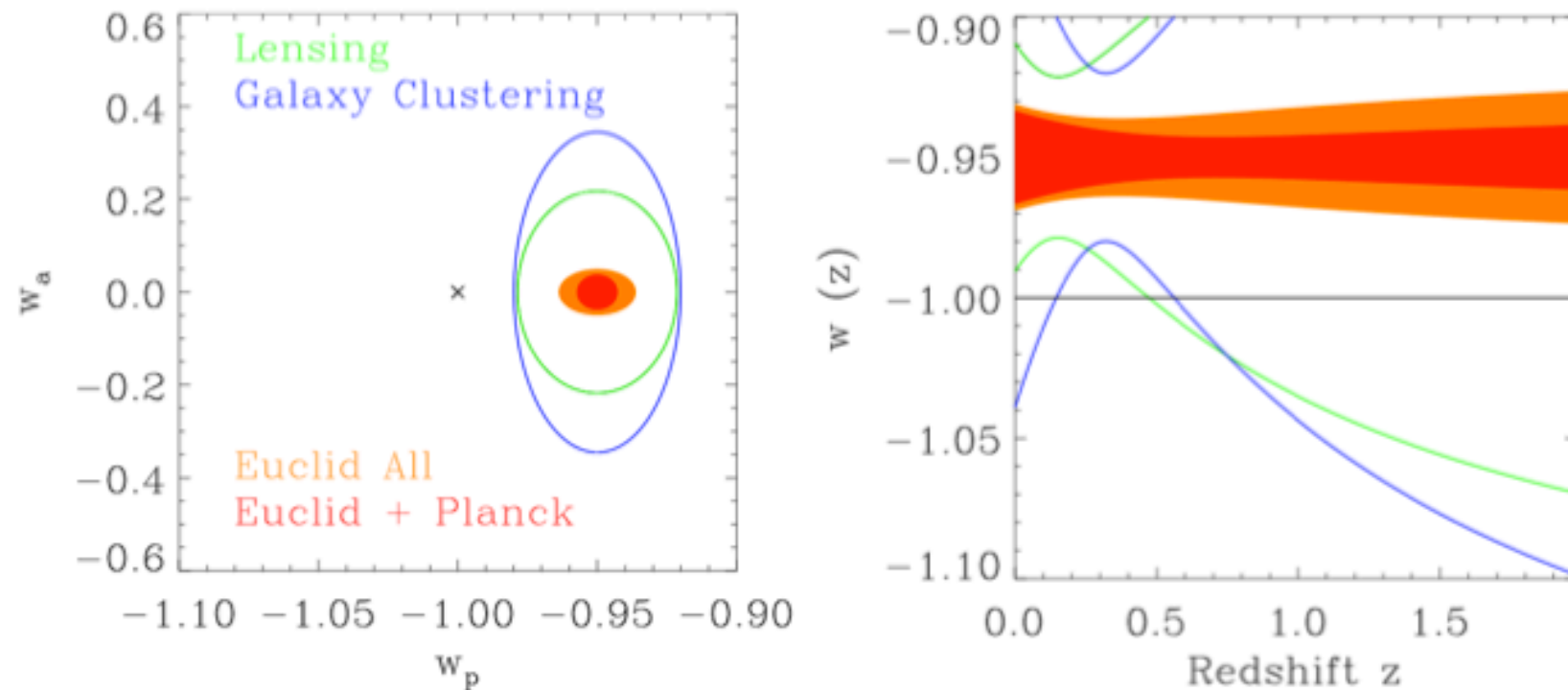


Figure 2.4: The expected constraints from Euclid in the dynamical dark energy parameter space. We show lensing only (green), galaxy clustering only (blue), all the Euclid probes (lensing+galaxy clustering+clusters+ISW; orange) and all Euclid with Planck CMB constraints (red). The cross shows a cosmological constant model. Left panel: the expected 68% confidence contours in the  $(w_p, w_a)$ . Right panel: the  $1\sigma$  constraints on the function  $w(z)$  parameterised by  $(w_p, w_a)$  as a function of redshift (green-lensing alone, blue-galaxy clustering alone, orange-all of the Euclid probes, red-Euclid combined with Planck).

# Detection + Classification stars/galaxies



# Shape Parameters

## The shear map $(\gamma_1, \gamma_2)$

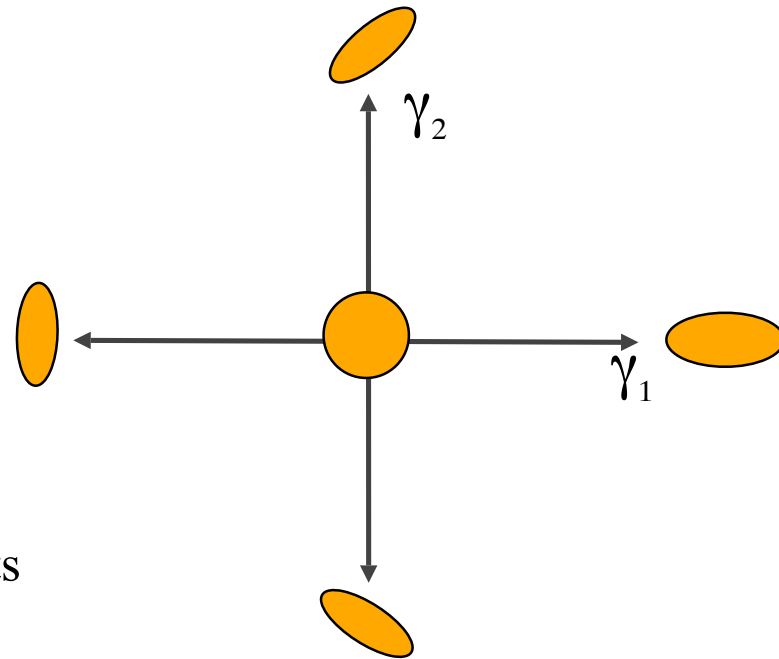
$\gamma_1$  = deformation along the x-axis,  
and  $\gamma_2$  at 45 degrees from it.

$$\gamma = \gamma_1 + i\gamma_2 = |\gamma|e^{2i\theta}$$

Where the modulus represents the  
amount of shear and the phase represents  
its direction.

$$\gamma_1 = \frac{M_{1,1} - M_{2,2}}{M_{1,1} + M_{2,2}}, \gamma_2 = \frac{2M_{1,2}}{M_{1,1} + M_{2,2}}, \quad M_{i,j} = \int \theta_i \theta_j S(\theta) w(\theta) d\theta^2$$

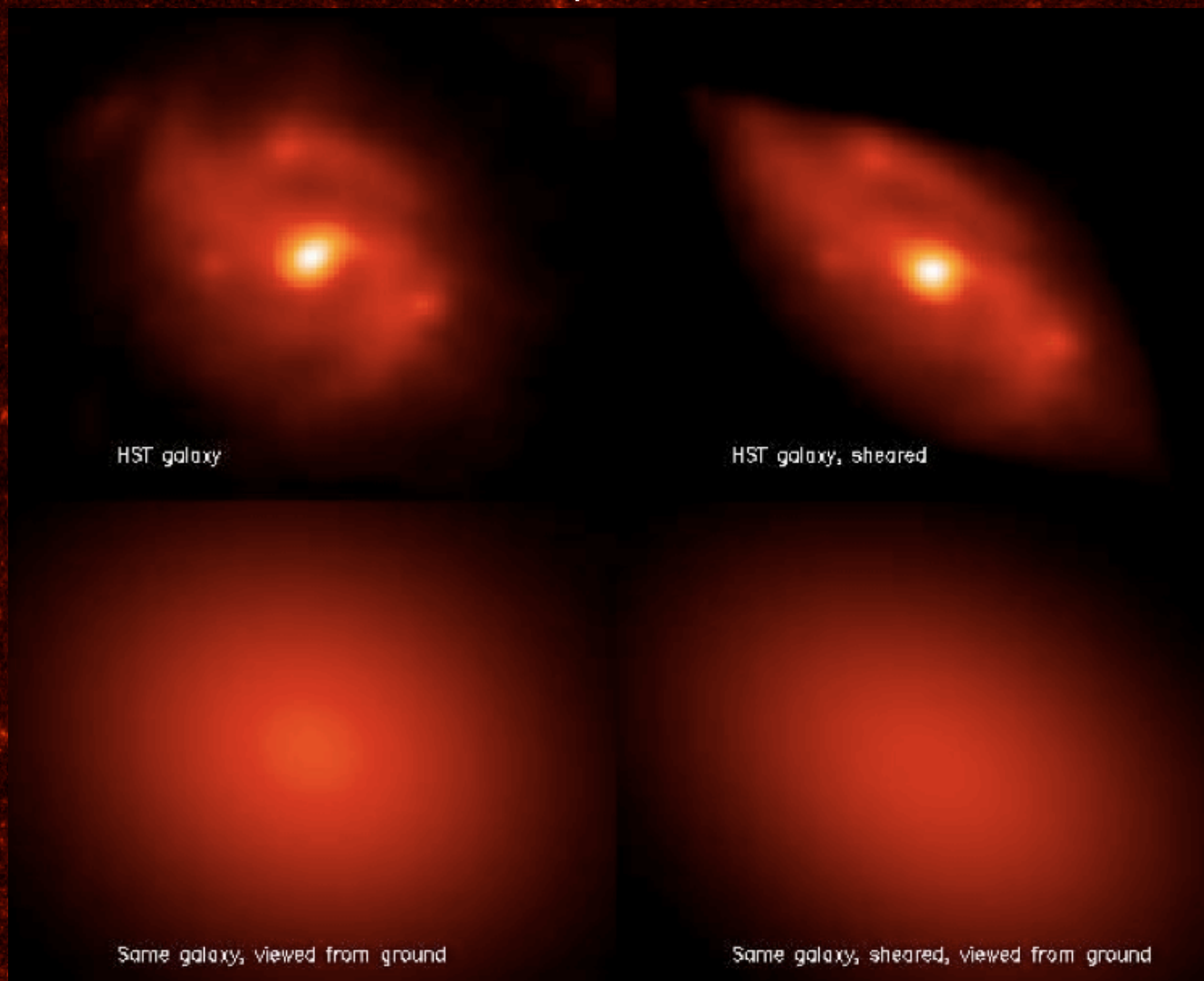
$M_{1,1} - M_{2,2}$  and  $2M_{1,2}$  correspond respectively to the flattening along the  $x$  axis and the  $45^\circ$  axis.  $M_{1,1} + M_{2,2}$  is related to the size.



**PB 1: We need accurate measurements from noisy data**



## Motivation for spatial observations



Convolution with an isotropic PSF circularises galaxies.

Convolution with an anisotropic PSF also changes their shapes... coherently!

Worst from ground (large PSF, with unpredictable spatial / temporal variation).

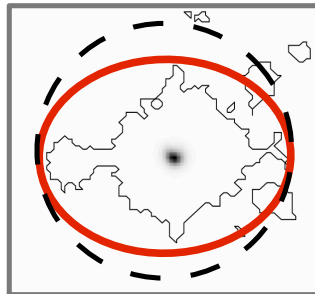
Massey, MGA

10

# Point Spread Function

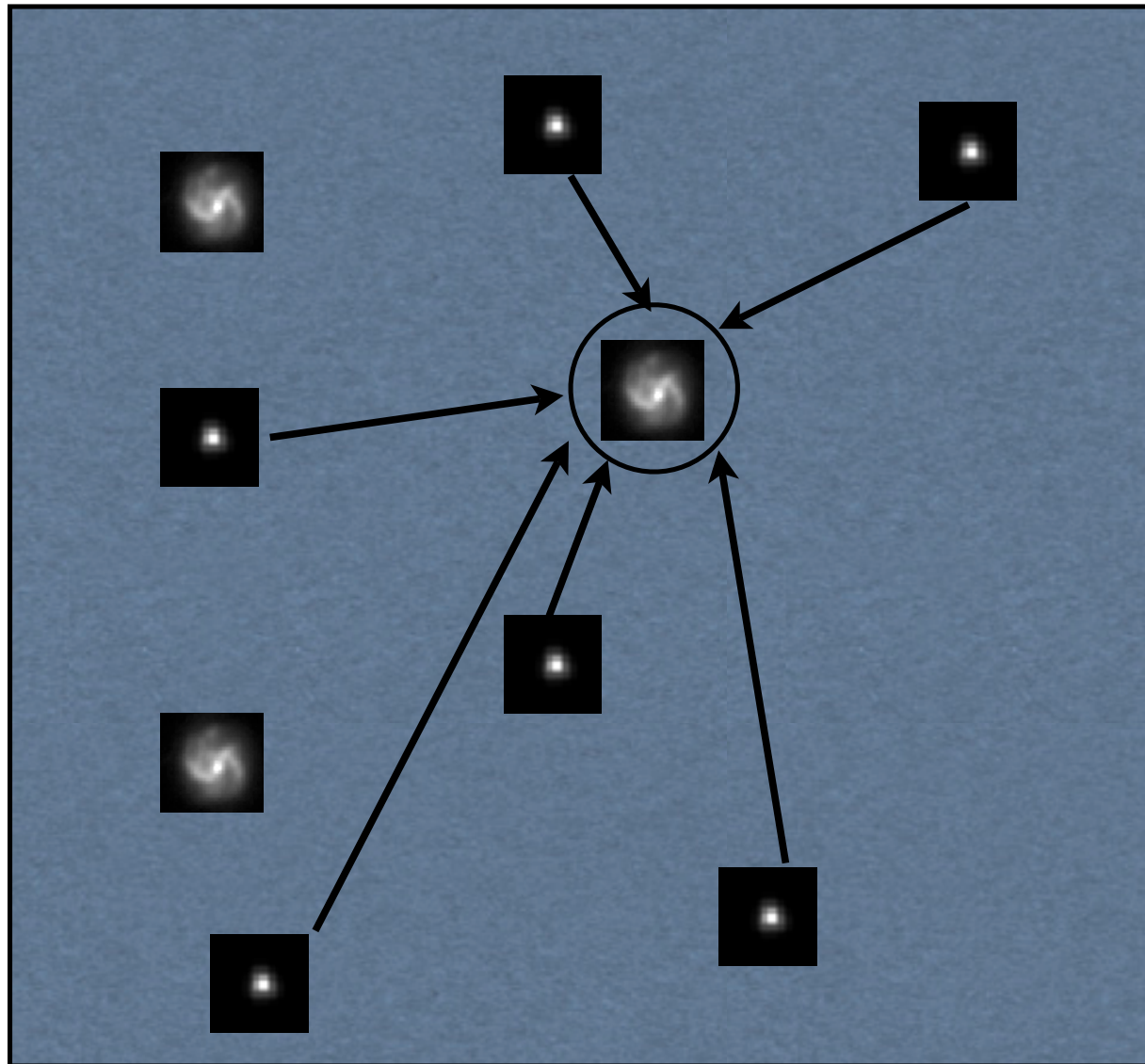
---

Galaxies are convolved by an asymmetric PSF



**PB 2: Shape measurements must be deconvolved**

## Space Variant PSF



**PB 3: We need to interpolate the PSF shape !**



# Intrinsic Ellipticities

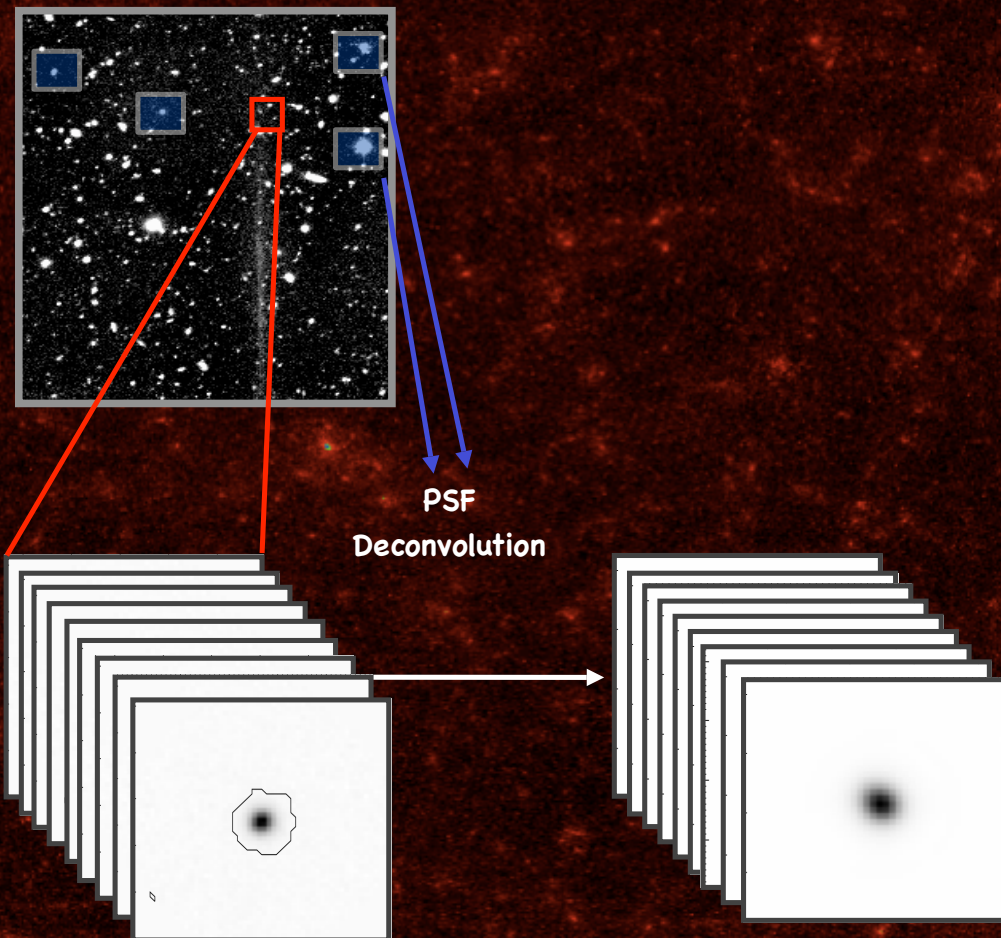
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✓ Galaxies have an intrinsic ellipticity



**PB 4: We need to correct the measurements from the intrinsic ellipticity**

# From Shear Measurements to Shear Map



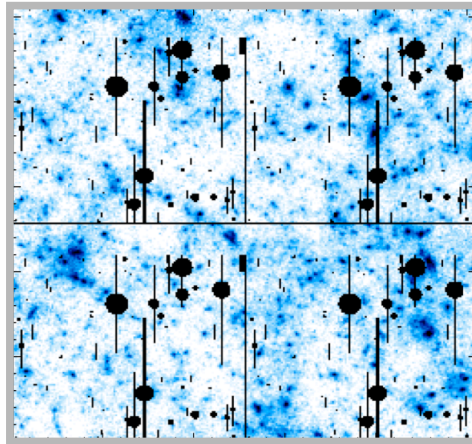
# **We need to solve a triple inverse problem !!!**

Determine the PSF at any position from the measured PSF.

Correct the galaxy shear from the PSF shear.

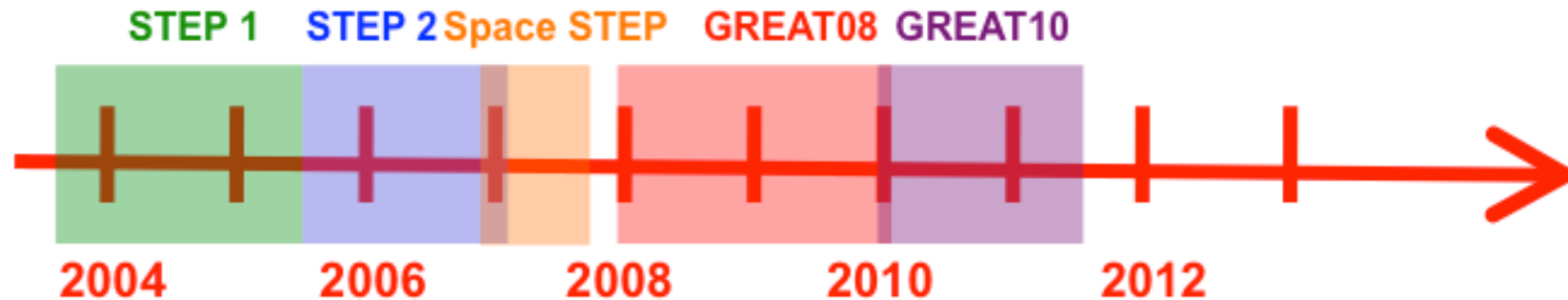
Correct the shear from intrinsic ellipticities

+ noise and missing data!!!



**Missing data**

## Shape measurement techniques: chronology of challenges

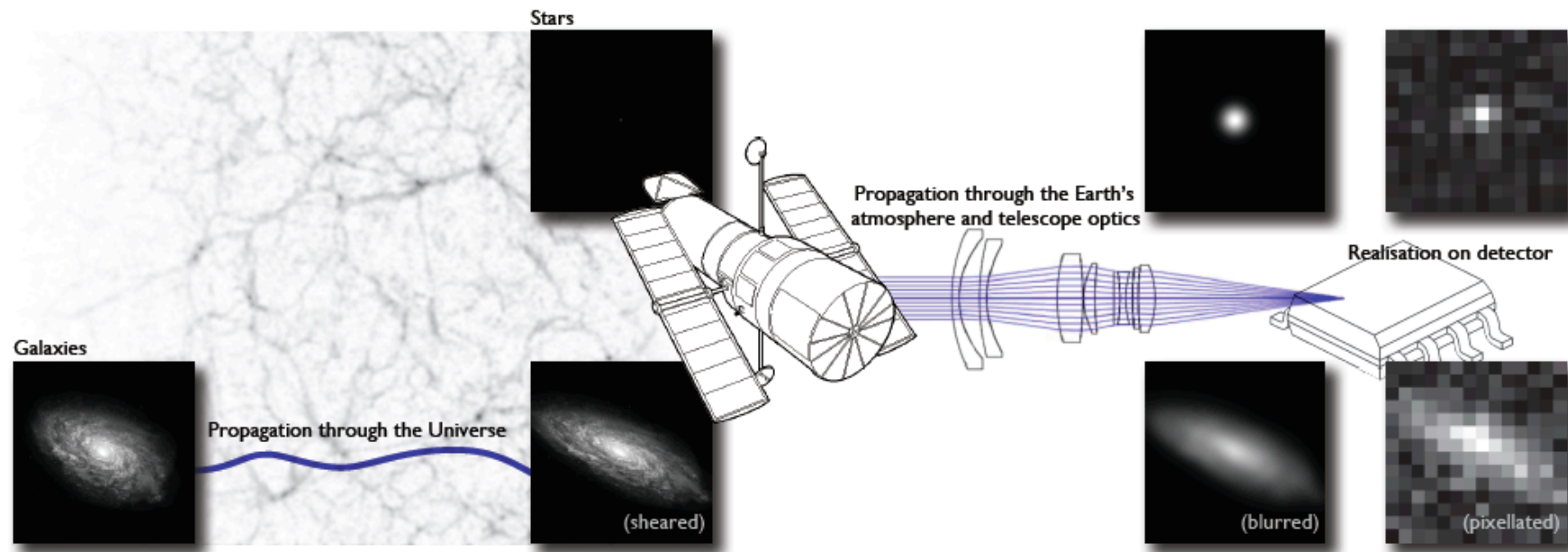


**~3 more challenges should be  
organised before 2020**

**In GREAT10, there were 3 sub-challenges:**

- 1.the main (galaxy) challenge: to measure the ellipticity of galaxies, assuming the PSF is known**
- 2.a star challenge: to estimate the PSF and interpolate it at the position of galaxies**
- 3.a 'light' challenge (named 'kaggle') to attract more people**





Need to measure the shear (which is of  $\sim 10^{-2}$ ) with an accuracy of  $10^{-2}$   
 → Need an absolute accuracy of  $\sim 10^{-4}$  in the measure of the shear  
 → Need an absolute accuracy of  $\sim \sqrt{N_{\text{gal}}} \cdot 10^{-4}$  in the measure of the ellipticity for individual galaxies

# Star Challenge

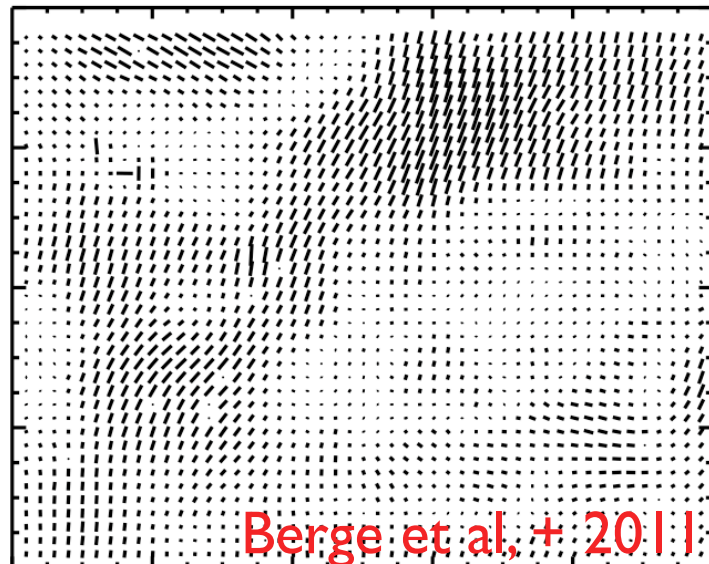
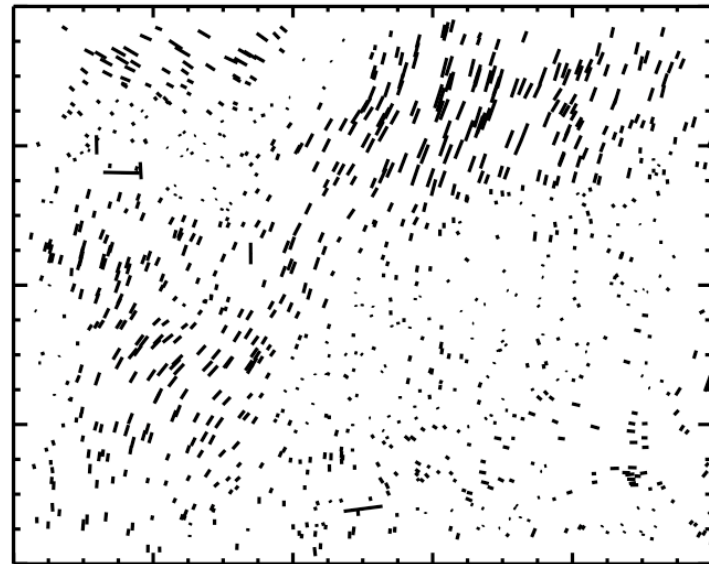
Estimate the PSF and interpolate it at the position of galaxies



around 40000 stars

## Drawbacks:

- 1.unrealistically simple PSF (could be easily modeled with a profile).
- 2.the SNR of stars was high and the noise unrealistically simple.

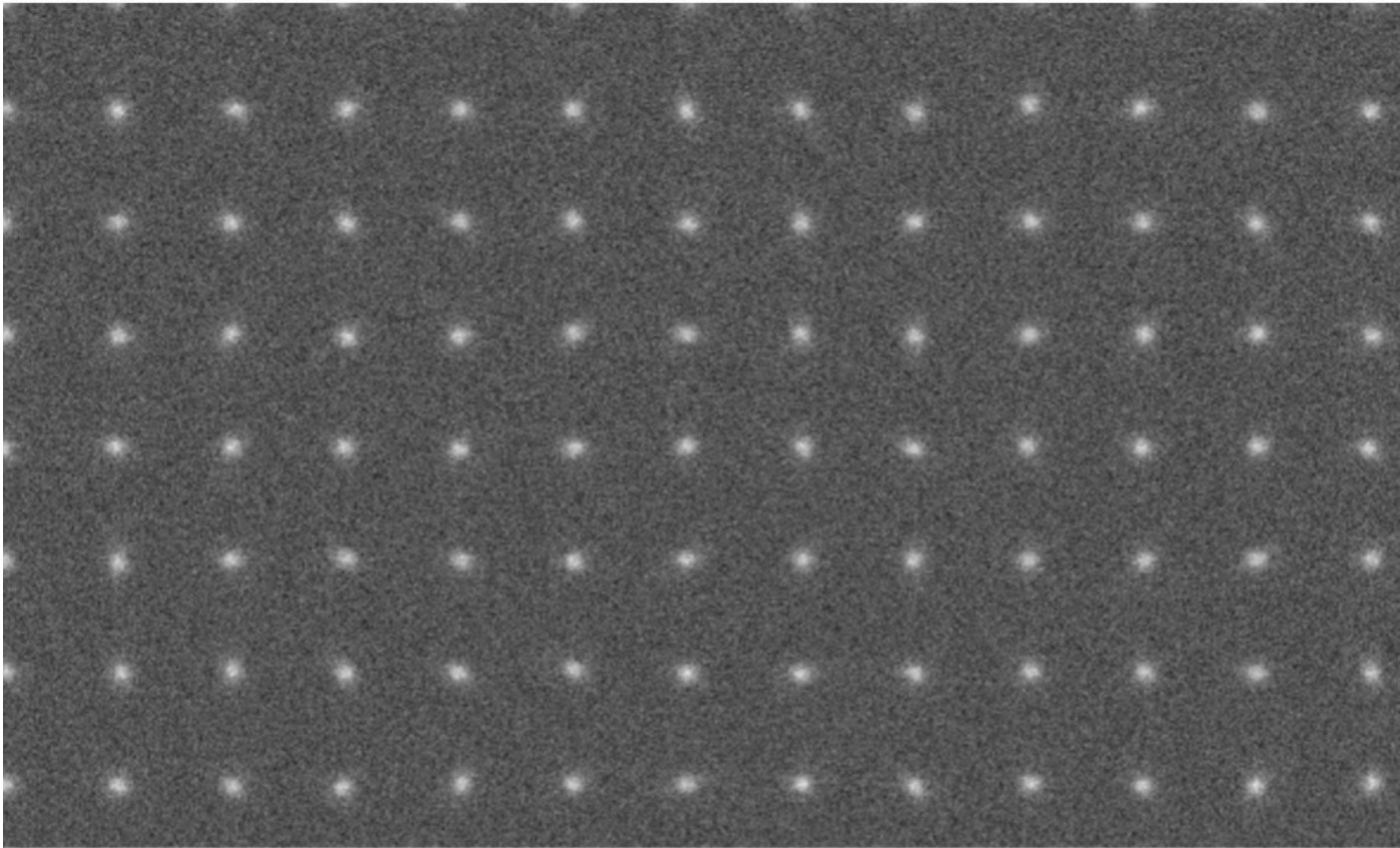


Berge et al, + 2011

**Winner: EPFL(Lausanne) ==> PSF model (moffat, 8 parameters: position, amplitude and 5 shape parameters) + interpolation (spline, kriging, etc).**

Rank	Group Name	Method Name	Submission Date	Q	Sigma Sys
1	EPFL LASTRO	B-Splines	Aug. 25, 2011, 3:17 p.m.	13.3	7.52602E-05
2	EPFL LASTRO	IDW	July 18, 2011, 12:38 p.m.	13.2	7.59258E-05
3	EPFL LASTRO	RBF	Aug. 15, 2011, 4:42 p.m.	12.7	7.86296E-05
4	EPFL LASTRO	RBF-thin	Aug. 16, 2011, 4:33 p.m.	12.6	7.92756E-05
5	EPFL LASTRO	IDW	June 23, 2011, noon	8.7	0.000114747
6	EPFL LASTRO	IDW	June 2, 2011, 7:07 p.m.	7.2	0.000138215
7	EPFL LASTRO	Kriging	June 3, 2011, 4:13 p.m.	7.2	0.000138215
8	Purdue	Gaussianlets	July 19, 2011, 11:32 p.m.	4.1	0.000245255
9	EPFL LASTRO	IDW Stk	Aug. 25, 2011, 3:17 p.m.	4.0	0.000249888
10	AstrOmatic	PSFEx 3.10 baseline	Sept. 1, 2011, 4:40 p.m.	4.0	0.000252947
11	UPenn/USM	PSFEx	April 28, 2011, 4:45 p.m.	3.7	0.000268701
12	U. Penn	Shapelets	Sept. 1, 2011, 7:27 p.m.	3.5	0.000287593
13	Purdue	PCA+kriging	July 6, 2011, 9:34 p.m.	2.9	0.000342539
14	Purdue	PAC+kriging	July 20, 2011, 2:46 p.m.	2.1	0.000476479
15	AstrOmatic	PSFEx 3.10 tuned lo-mid	Sept. 2, 2011, 3:25 p.m.	1.9	0.00052613
16	AstrOmatic	PSFEx 3.10 baseline scaled	Sept. 2, 2011, 4:34 p.m.	1.9	0.00052613
17	AstrOmatic	PSFEx 3.10 tuned lo-hi	Sept. 2, 2011, 10:20 a.m.	1.9	0.000533332
18	AstrOmatic	PSFEx 3.10 tuned mid-hi	Sept. 2, 2011, 2:02 p.m.	1.8	0.000545185
19	InfEd	MoffatGP	Sept. 1, 2011, 4:39 p.m.	1.2	0.000800598
20	GREAT10		Dec. 7, 2010, 11:15 a.m.	1.0	0.00104465
21	BayesPSF	Basic SEx retry	May 2, 2011, 8:42 a.m.	1.0	0.00104465
22	InfEd	PcaGP	Aug. 31, 2011, 2:28 a.m.	0.8	0.00120245
23	I2PRG-ITB	Thin Plate	June 29, 2011, 12:27 p.m.	0.8	0.00125409
24	I2PRG-ITB	Correction of Thin-Plate	Aug. 23, 2011, 6:12 a.m.	0.7	0.00136537
25	I2PRG-ITB	SimpleQM+Thin Plate	Aug. 5, 2011, 10:39 a.m.	0.6	0.00155256
26	I2PRG-ITB	Thin-Plate (revised)	Aug. 25, 2011, 6:33 a.m.	0.5	0.00199096
27	I2PRG-ITB	Sparse Random Field	Aug. 25, 2011, 10:43 a.m.	0.5	0.00199158
28	I2PRG-ITB	Thin Plate (V.2.0)	Aug. 28, 2011, 10:18 a.m.	0.5	0.00207088
29	Purdue	Moffatlets	July 20, 2011, 7:05 a.m.	0.2	0.00496487

## Galaxy challenge: to measure the ellipticity of galaxies, assuming the PSF is known



volume :  $10^4$  galaxies  $\times$   $10^3$  images  $\sim$  1 TB



# GREAT10 Challenge

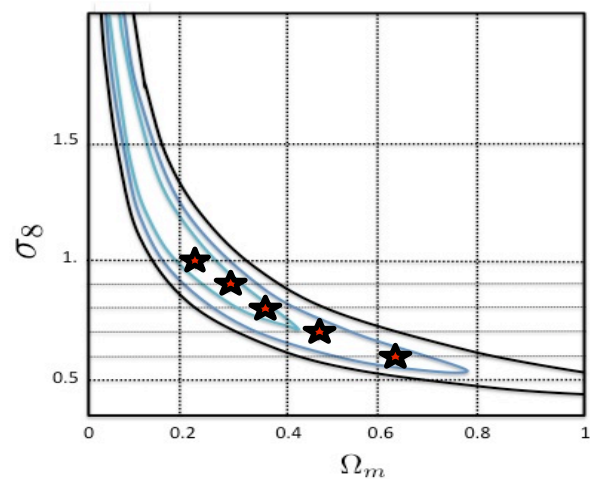
Winner: DeepZot (KSB + galaxy model fitting + bias correction from MC using Neural Network)

Rank	Group Name	Method Name	Submission Date	Q	Sigma Sys
1	DeepZot	fit2-unfold (ps)	Sept. 1, 2011, 4:53 p.m.	319.5	3.12987E-06
2	DeepZot	fit1-unfold (ps)	Sept. 1, 2011, 4:51 p.m.	291.5	3.4304E-06
3	Ohio State University	KSB	Aug. 29, 2011, 10:58 p.m.	119.6	8.36359E-06
4	EPFL LASTRO	gfit_den_cs	Sept. 2, 2011, 10:05 a.m.	118.8	8.4204E-06
5	Ohio State University	ARES	Sept. 2, 2011, 6:22 a.m.	115.5	8.6578E-06
6	Ohio State University	ARES2	Sept. 2, 2011, 4:36 p.m.	114.0	8.76837E-06
7	mpi-is	method04 (set21)	Sept. 2, 2011, 11:16 a.m.	109.7	9.11972E-06
8	mpi-is	method04	Sept. 1, 2011, 2:25 p.m.	109.3	9.15092E-06
9	mpi-is	method04 (set_21 corrected)	Sept. 1, 2011, 5:53 p.m.	109.3	9.15092E-06
10	mpi-is	method05 (set21)	Sept. 2, 2011, 1:33 p.m.	96.4	1.03681E-05
11	mpi-is	method05	Sept. 2, 2011, 10:18 a.m.	95.0	1.05228E-05
12	Ohio State University	KSB_BSA (ps)	Sept. 1, 2011, 11:38 a.m.	92.0	1.08703E-05
13	UCL CoGS	Im3shape NBC0	Aug. 31, 2011, 12:54 p.m.	89.1	1.12279E-05
14	UCL CoGS	Im3shape NBC1	Sept. 2, 2011, 1:37 a.m.	88.9	1.12478E-05
15	UCL CoGS	Im3shape NBC0XS	Sept. 2, 2011, 3:12 p.m.	88.6	1.12827E-05
16	UCL CoGS	Im3shape Uncalibrated	Aug. 30, 2011, 11:57 p.m.	87.6	1.14111E-05
17	UCL CoGS	Im3shape Uncalibrated XS	Sept. 2, 2011, 4:11 p.m.	87.2	1.14683E-05
18	Ohio State University	KSB analytic	Sept. 2, 2011, 4:07 p.m.	81.0	1.23428E-05
19	Ohio State University	DEIMOS C6	Aug. 24, 2011, 12:14 a.m.	79.9	1.251E-05
20	Ohio State University	DEIMOS C4	Aug. 23, 2011, 12:16 a.m.	77.9	1.28418E-05
21	UCL CoGS	Im3shape NBC0	Aug. 29, 2011, 6:13 p.m.	75.7	1.32041E-05
22	mpi-is	method01	Aug. 30, 2011, 9:29 p.m.	74.1	1.34928E-05
23	EPFL LASTRO	gfit	Sept. 2, 2011, 9:52 a.m.	73.8	1.35452E-05
24	EPFL LASTRO	gfit_cs	Sept. 2, 2011, 10:12 a.m.	73.8	1.35452E-05
25	UCL CoGS	Im3shape Uncalibrated	Aug. 27, 2011, 11:52 a.m.	73.5	1.36001E-05
26	UCL CoGS	Im3shape Uncalibrated	Aug. 23, 2011, 1:21 a.m.	73.2	1.36676E-05
27	UCL CoGS	Im3shape NBC1XS	Sept. 2, 2011, 2:27 p.m.	72.2	1.38468E-05
28	mpi-is	method02	Aug. 31, 2011, 6:05 p.m.	71.8	1.39286E-05
29	UCL CoGS	Im3shape Uncalibrated	Sept. 2, 2011, 10:55 a.m.	71.7	1.39533E-05
30	GREAT10	None	Dec. 3, 2010, 5:43 p.m.	67.6	1.47931E-05
31	Ohio State University	DEIMOS C6 analytic	Sept. 1, 2011, 9:51 a.m.	64.6	1.54684E-05
32	Ohio State University	DEIMOS C8 analytic	Sept. 1, 2011, 8:16 p.m.	63.1	1.58405E-05
33	Oxford/Malta	LensfitG10 -uses incorrect PSF due to FUNCPSF form	Aug. 31, 2011, 1:15 p.m.	63.1	1.58555E-05
34	Ohio State University	DEIMOS C4 analytic	Sept. 1, 2011, 9:15 p.m.	62.8	1.59281E-05
35	Oxford/Malta	LensfitG10 -uses incorrect PSF due to FUNCPSF form	Aug. 31, 2011, 5 p.m.	61.7	1.6215E-05

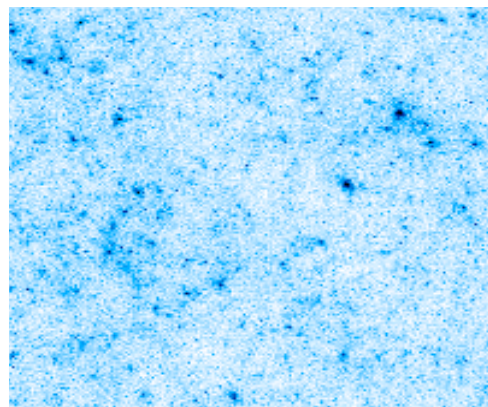
EPFL  
(Lausanne)

GREAT08 winner

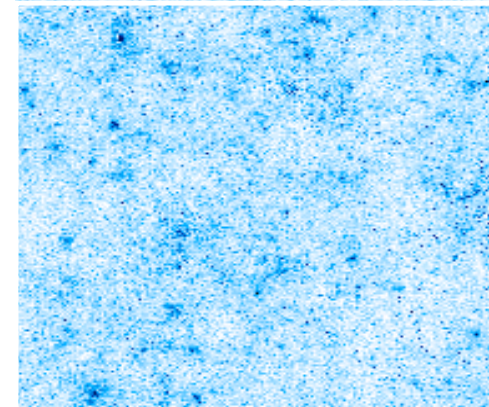
# Cosmological Parameters Constraints and High Order Statistics



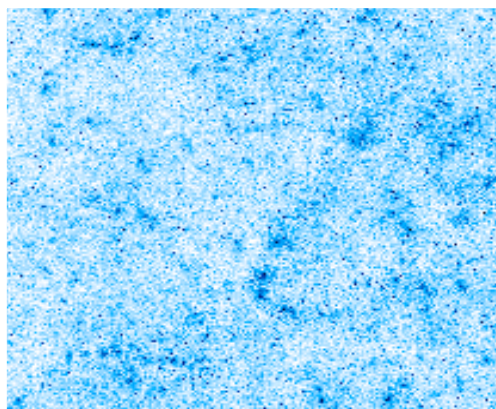
Model1 ( $\sigma_8=1$ ,  $\Omega_m=0.23$ )



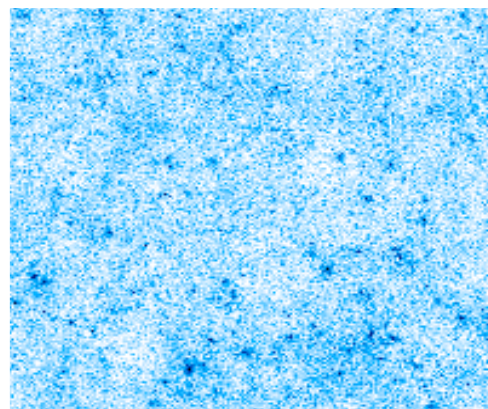
Model2 ( $\sigma_8=0.9$ ,  $\Omega_m=0.3$ )



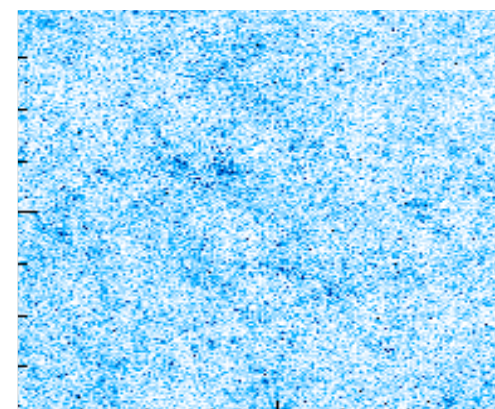
Model3 ( $\sigma_8=0.8$ ,  $\Omega_m=0.36$ )



Model4 ( $\sigma_8=0.7$ ,  $\Omega_m=0.47$ )

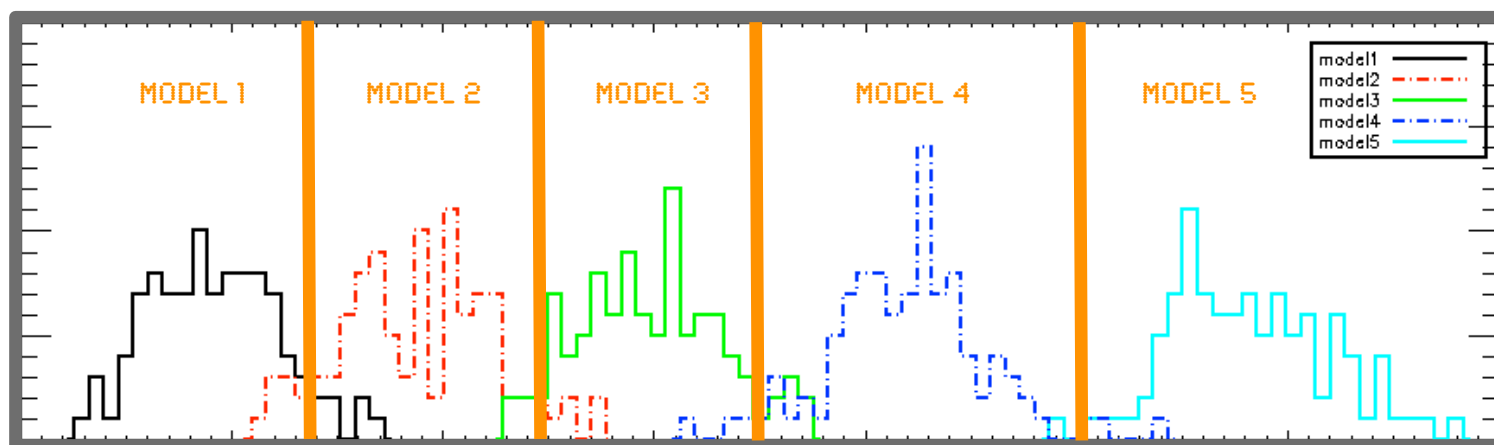


Model5 ( $\sigma_8=0.6$ ,  $\Omega_m=0.64$ )



# Cosmological Parameters Constraints and High Order Statistics

- S. Pires, J.-L. Starck, A. Amara, A. Refregier, R. Teyssier, "Cosmological models discrimination with Weak Lensing", 505, A&A, pp 969-979, 2009.
- S. Pires, J.-L. Starck and A. Refregier, "[Light on Dark Matter with Weak Gravitational Lensing](#)", IEEE Signal Processing Magazine, 27, 1, pp 76--85, 2010.
- S. Pires, J.-L. Starck, A. Amara, A., R. Teyssier, A. Refregier and J. Fadili, "[FASTLens \(FAST Statistics for weak Lensing\) : Fast method for Weak Lensing Statistics and map making](#)", MNRAS , 395, 3, pp. 1265-1279, 2009.
- S. Pires, A. Leonard, J.-L. Starck, "Cosmological Parameters Constraint from Weak Lensing Data", MNRAS, submitted, 2011.
- A. Leonard, S. Pires, J.-L. Starck, "Fast Calculation of the Weak Lensing Aperture Mass Statistic", MNRAS, submitted, 2011.



WAVELET PEAK COUNTING ON MRLENS FILTRED MAPS (AT SCALE OF ABOUT 1 ARCMIN)



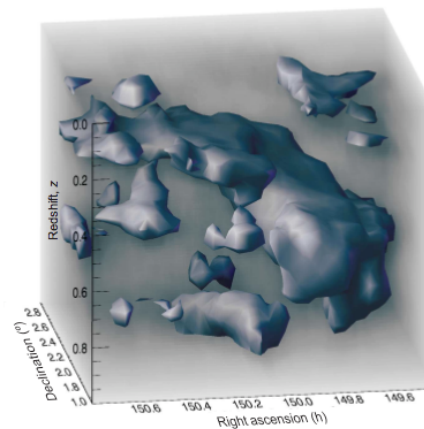
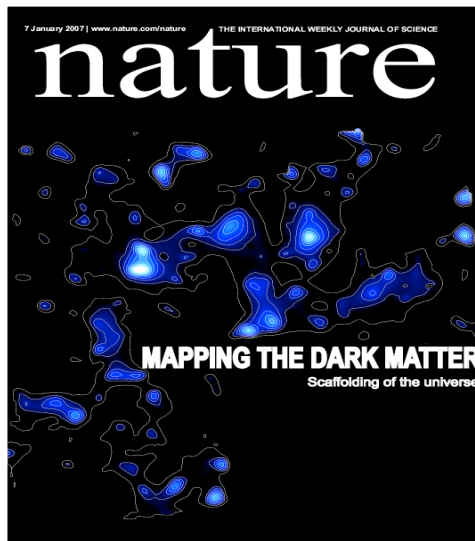
# Mass Map Reconstruction and High Order Statistics

J.-L. Starck, S. Pires and A. Réfrégier, Astronomy and Astrophysics, 451, 3, 2006, pp.1139-1150, 2006

S. Pires, J.-L. Starck, A. Amara, R. Teyssier, A. Refregier and J. Fadili, MNRAS, Volume 395, Issue 3, pp. 1265-1279, 2009.

R. Massey et al, Maps of the Universe's Dark matter scaffolding,, Nature, Vol. 445, pp. 286-290, 2007

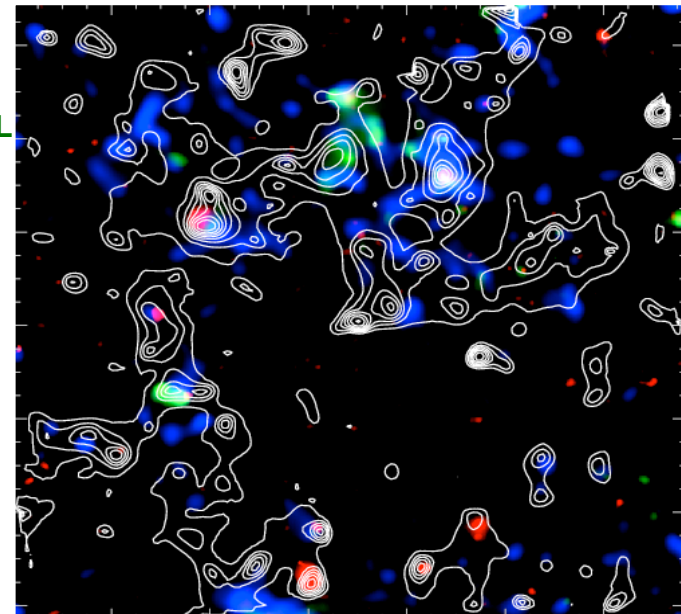
$$\gamma_i \rightarrow \min_{\kappa} \|\Phi^t \kappa\|_{l_0} \text{ subject to } \sum_i \|\gamma_i - M(P_i * \kappa)\|_{l_2}^2 \leq \varepsilon \rightarrow \kappa$$



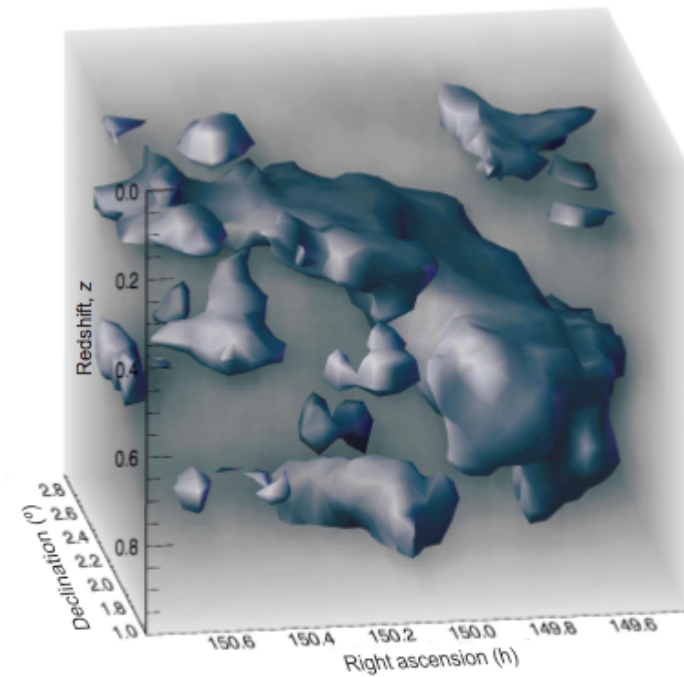
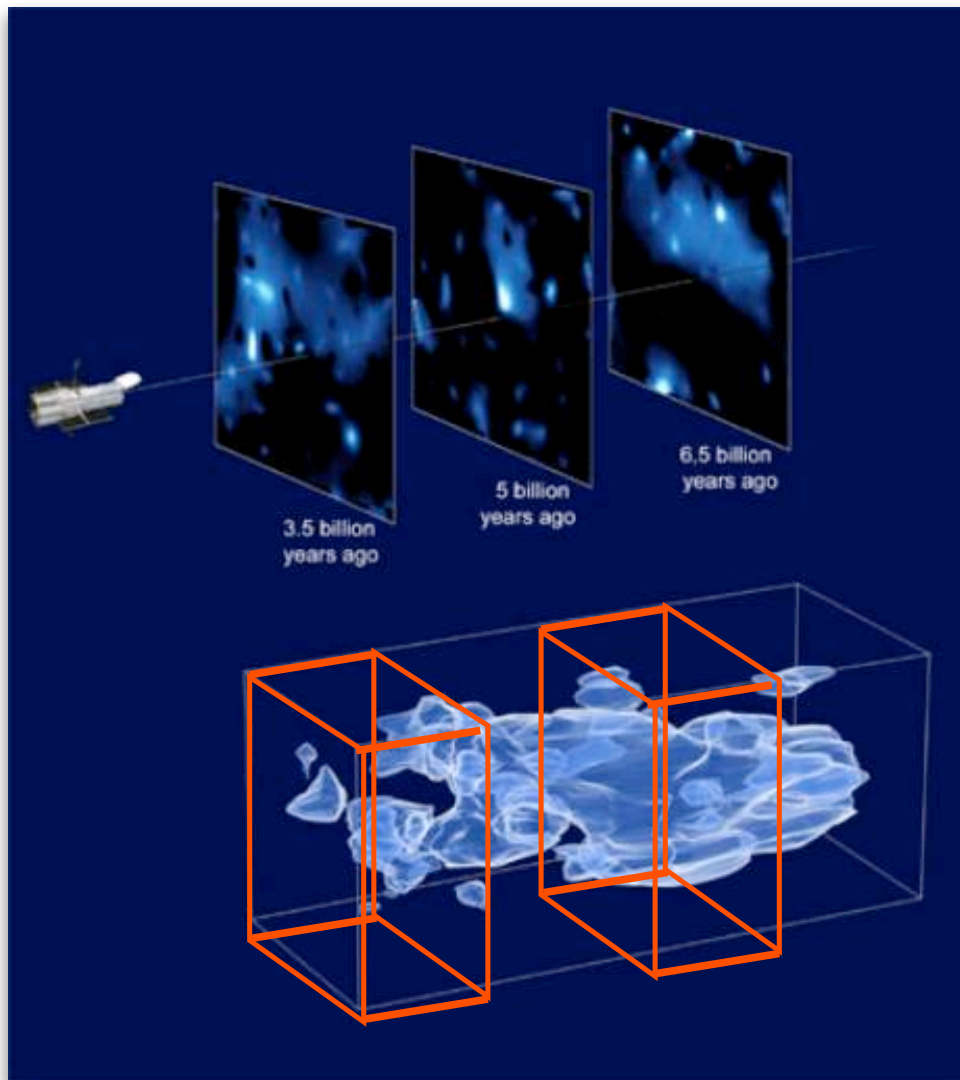
Stellar mass

Galaxy density and WL

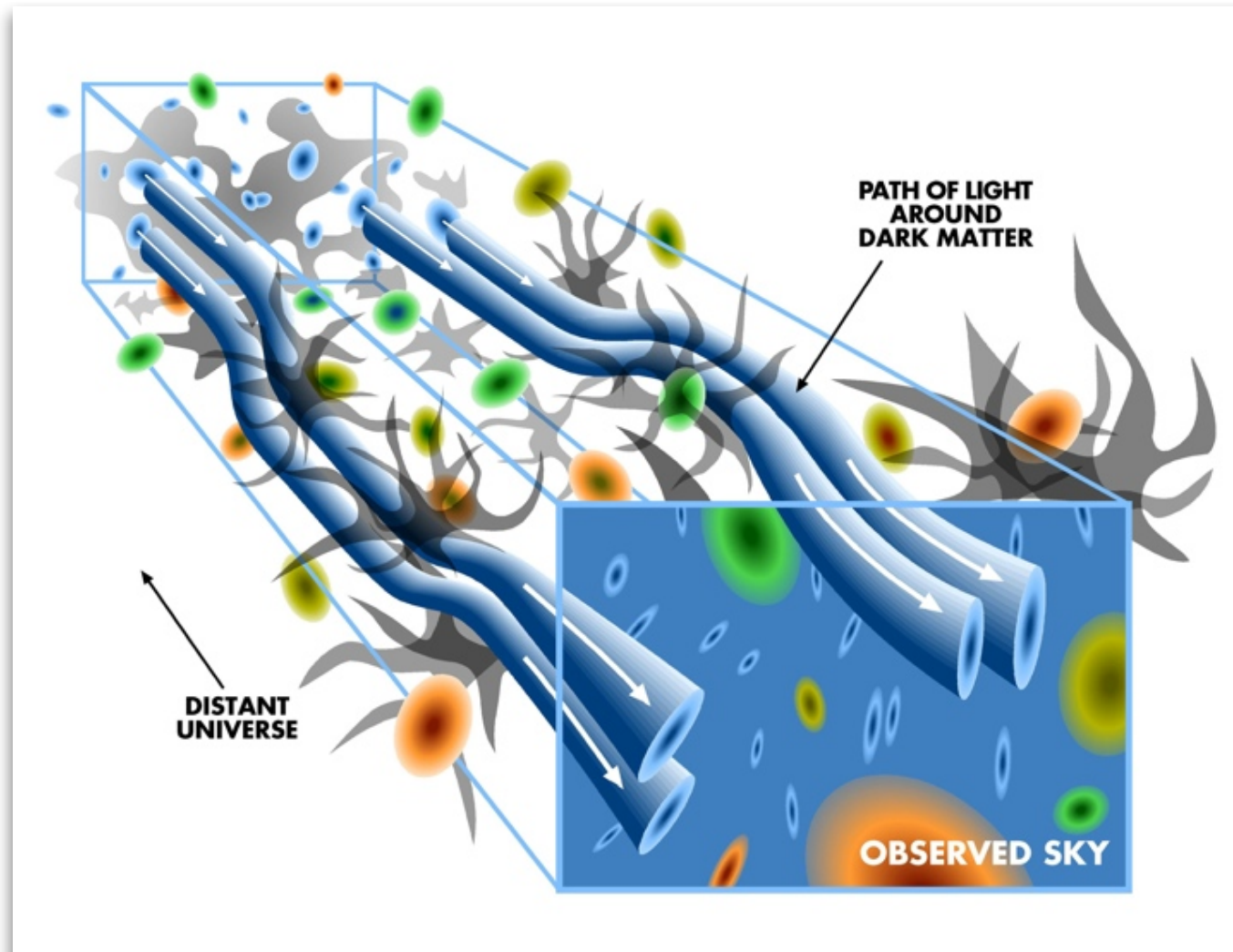
Hot gas



# Pseudo-3D Weak Lensing



# 3D Weak Lensing



# 3D Weak Lensing

The convergence  $\kappa$ , as seen in sources of a given redshift bin, is the linear transformation of the matter density contrast,  $\delta$ , along the line-of-sight (Simon et al 2009):

$$\kappa = Q\delta + N \quad \text{with} \quad \delta(r) \equiv \rho(r)/\bar{\rho} - 1$$

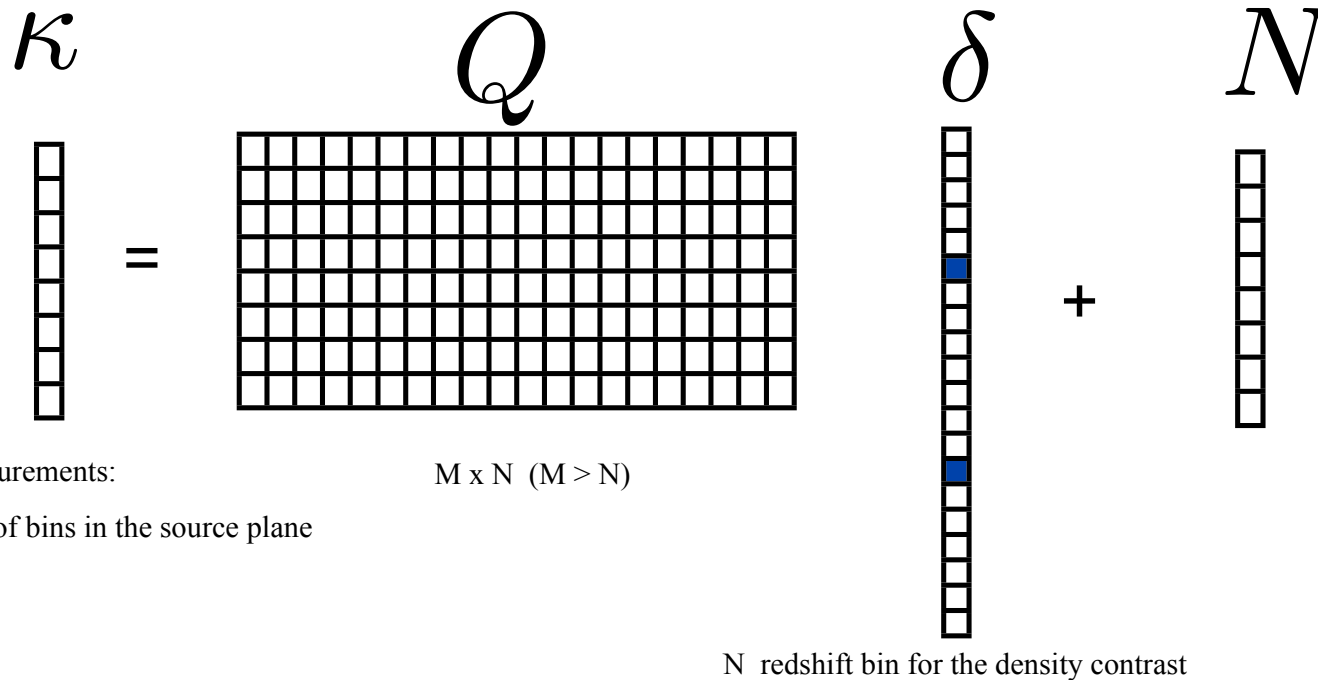
$$Q_{i\ell} = \frac{3H_0^2\Omega_M}{2c^2} \int_{w_\ell}^{w_{\ell+1}} dw \frac{\overline{W}^{(i)}(w) f_K(w)}{a(w)}, \quad \overline{W}^{(i)}(w) = \int_0^{w^{(i)}} dw' \frac{f_K(w-w')}{f_K(w')} \left( p(z) \frac{dz}{dw} \right)_{z=z(w')}$$

where  $H_0$  is the hubble parameter,  $\Omega_M$  is the matter density parameter,  $c$  is the speed of light,  $a(w)$  is the scale parameter evaluated at comoving distance  $w$ , and

$$f_K(w) = \begin{cases} K^{-1/2} \sin(K^{1/2}w), & K > 0 \\ w, & K = 0 \\ (-K)^{-1/2} \sinh([-K]^{1/2}w) & K < 0 \end{cases},$$

gives the comoving angular diameter distance as a function of the comoving distance and the curvature,  $K$ , of the Universe.

# 3D Weak Lensing



$\delta$  is sparse.

$Q$  spreads out the information in  $\delta$  along  $\kappa$  bins.

More unknown than measurements





# Compressed Sensing



- \* E. Candès and T. Tao, "Near Optimal Signal Recovery From Random Projections: Universal Encoding Strategies? ", IEEE Trans. on Information Theory, 52, pp 5406–5425, 2006.
- \* D. Donoho, "Compressed Sensing", IEEE Trans. on Information Theory, 52(4), pp. 1289–1306, April 2006.
- \* E. Candès, J. Romberg and T. Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information", IEEE Trans. on Information Theory, 52(2) pp. 489 – 509, Feb. 2006.

## A non linear sampling theorem

**“Signals with exactly  $K$  components different from zero can be recovered perfectly from  $\sim K \log N$  incoherent measurements”**

Replace samples with *few linear projections*  $y = \Theta x$

$$\begin{array}{c}
 \begin{array}{c} y \\ M \times 1 \\ \text{measurements} \end{array} = \begin{array}{c} \Theta \\ M \times N \end{array} \begin{array}{c} x \\ N \times 1 \\ \text{sparse signal} \\ K \\ \text{nonzero entries} \end{array} \\
 y = \Theta x \\
 K < M \ll N
 \end{array}$$

Reconstruction via non linear processing:

$$\min_x \|x\|_1 \quad \text{s.t.} \quad y = \Theta x$$

⇒ Application: Compression, tomography, ill posed inverse problem.

# 3D Weak Lensing

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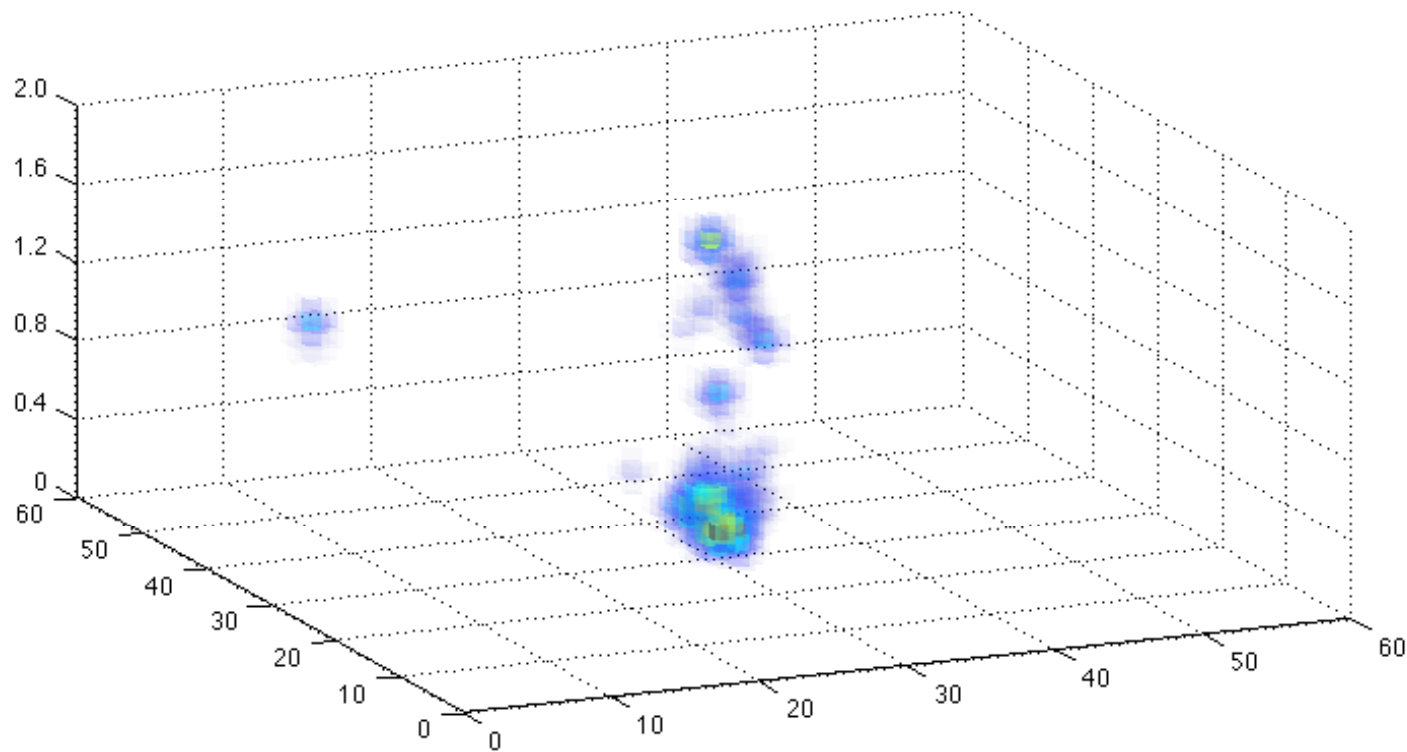
## Matter in the Universe as a Natural Compressed Sensing Operator

$$\min_{\delta} \|\delta\|_1 \quad s.t. \quad \frac{1}{2} \|\kappa - Q\delta\|_{\Sigma^{-1}}^2 \leq \epsilon$$

Recent optimization method, based on proximal theory, such as Chambolle & Pock (2010) can be used to find the solution.

A. Leonard, F.-X. Dupe, J.-L. Starck, "A compressed sensing approach to 3D weak lensing", Astronomy and Astrophysics, [arXiv:1111.6478](https://arxiv.org/abs/1111.6478), A&A, in press.

# 3D Weak Lensing



Reconstructions of two clusters along the line of sight, located at a redshift 0.2 and 1.0 (data binned into  $N_{sp} = 20$  redshift bins, but aim to reconstruct onto  $N_{lp} = 25$  redshift bins).

# Conclusions/Perspectives

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- Weak Lensing is a powerful technique to measure large-scale structure
- It directly measures the mass (as opposed to light)
- But require tight control of systematic
- Algorithms need clearly to be improved in order to meet EUCLID scientific requirements.
  - \* Psf measurements
  - \* Shear on individual galaxies
  - \* Lensing statistics.
- High order statistics should be used to better constraint the cosmological parameters
- Compressed sensing theory useful to recover the 3D density map.
  - \*  $l_1$  norm minimization is required to find the best solution with such operator.
  - \* Compressed Sensing approach may allow us to map the cosmic web in far greater detail than what has previously been achieved.