Indirect constraints on the Higgs boson

Gino Isidori
[*INFN, Frascati & CERN*]

Theoretical ~~Indirect~~ constraints on the Higgs boson mass (*and some implications of its recent measurement*)

Gino Isidori
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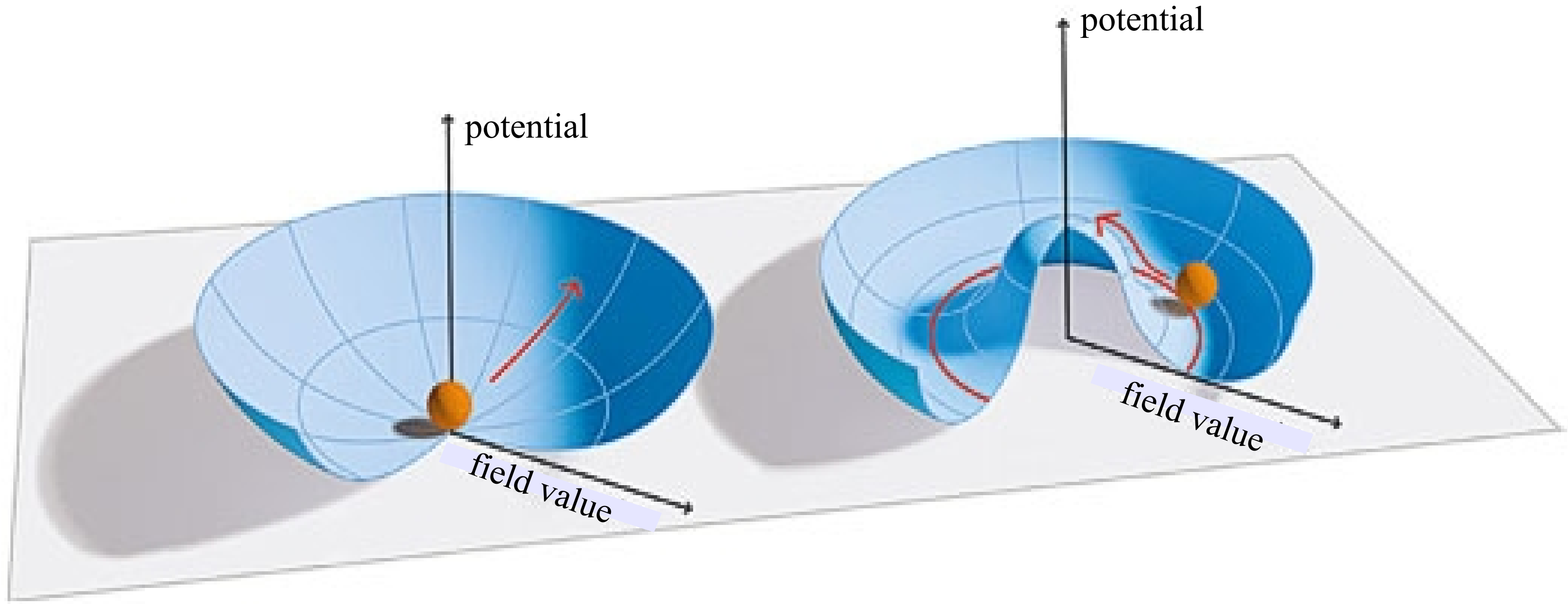
- Introduction
- The Higgs potential at high energies
- Stability and metastability bounds
- Vacuum stability at NNLO
- Speculations on Planck-scale dynamics
- Conclusions

Introduction

The Higgs mechanism, namely the introduction of an elementary $SU(2)_L$ scalar doublet, with ϕ^4 potential, is the most economical & simple choice to achieve the spontaneous symmetry breaking [$SU(2)_L \times U(1)_Y \to U(1)_Q$]. that we observe in nature.



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$$\mathscr{L}_{higgs}(\phi, A_a, \psi_i) = D\phi^+ D\phi - V(\phi)$$

$$V(\phi) = -\mu^2 \phi^+\phi + \lambda(\phi^+\phi)^2 + Y^{ij}\, \psi_L{}^i\, \psi_R{}^j\, \phi$$

Till very recently only the ground state determined by this potential (*and the corresponding Goldstone boson structure*) was tested with good accuracy:

$$v = \langle \phi^+\phi \rangle^{1/2} \sim 246 \text{ GeV} \quad [\; m_W = \tfrac{1}{2}\, g\, v \;]$$

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The situation has substantially changed a few weeks ago, with the observation of the 4th degree of freedom of the Higgs field (or its *massive excitation*):

$$\lambda_{\rm (tree)} = \tfrac{1}{2} m_h^{\,2} / v^2 \sim 0.13$$

Introduction

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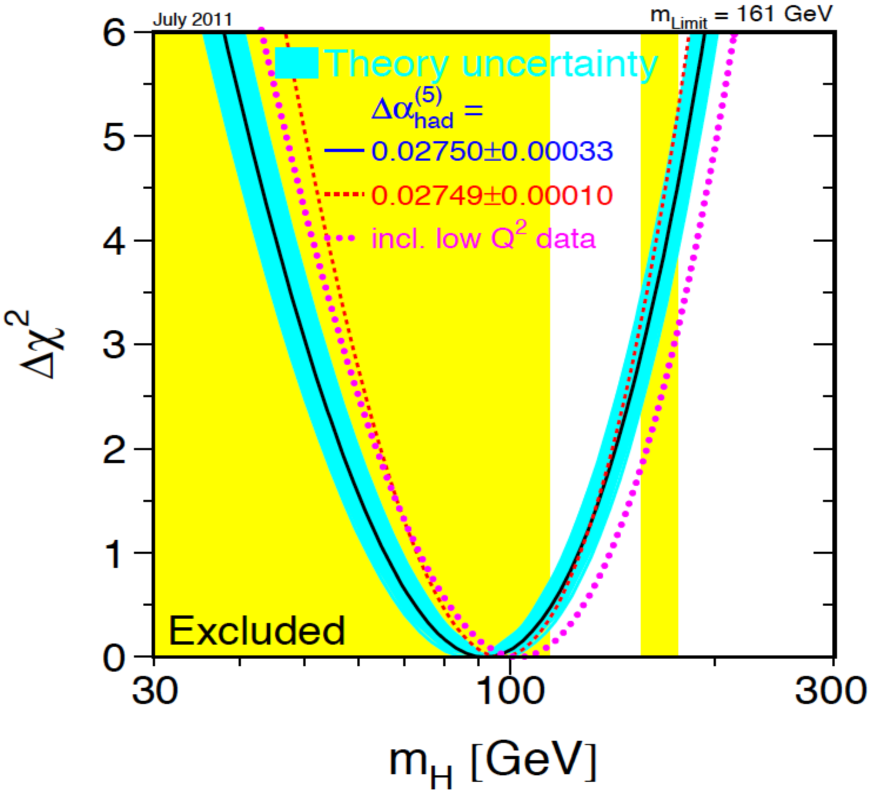

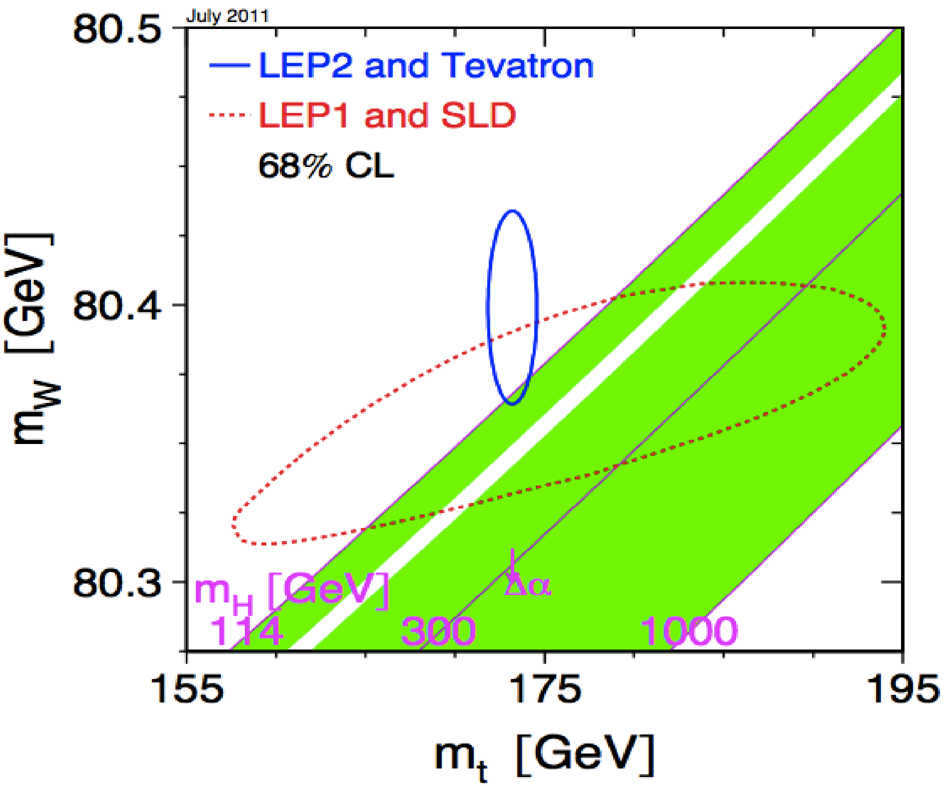

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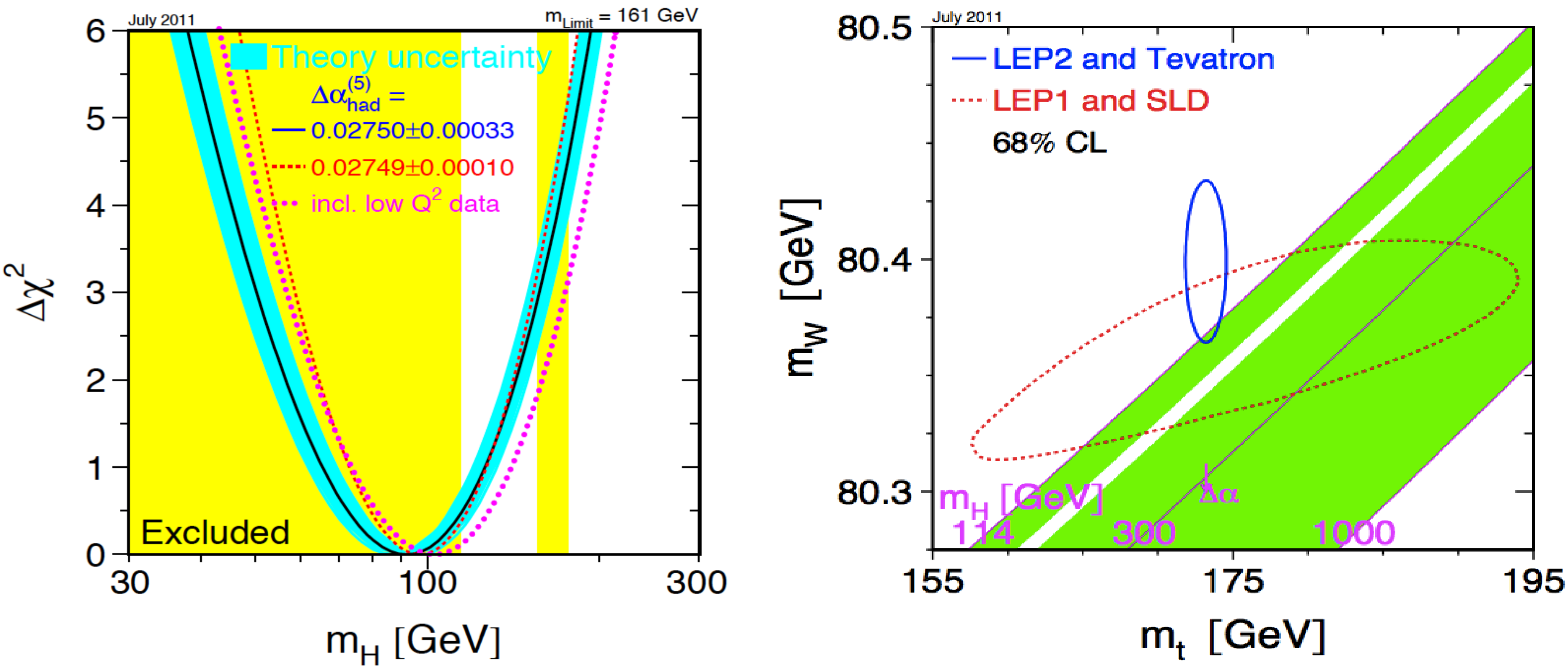

Message n.1: The observation of the physical Higgs boson with m_h well consistent with the (indirect) prediction of the e.w. precision tests is a *great success of the SM !*

Introduction

Still, the SM Higgs potential is "ugly" and hides the most serious *theoretical problems* of this highly successful theory:

$$V(\phi) = -\mu^2 \phi^+\phi + \lambda (\phi^+\phi)^2 + Y^{ij} \psi_L^{\,i} \psi_R^{\,j} \phi$$

vacuum instability

possible <u>internal inconsistency</u> of the model ($\lambda < 0$) at large energies

[*key dependence on* m_h]

Quadratic sensitivity to the cut-off

$$\Delta\mu^2 \sim \Delta m_h^2 \sim \Lambda^2$$

(indication of *new physics* close to the electroweak scale ?)

SM flavour problem

(unexplained span over 5 orders of magnitude and strongly hierarchical structure of the Yukawa coupl.)

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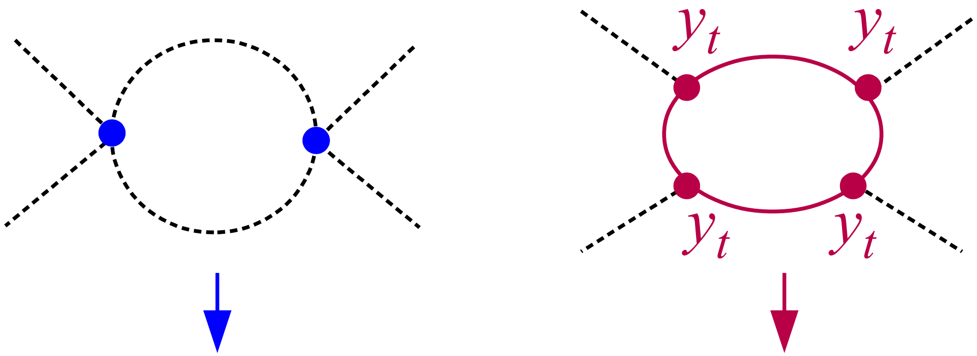
Stability and metastability bounds

At large field values the shape of the Higgs potential is determined by the RGE evolution of the Higgs self coupling:

$$V_{eff}(\ |\phi| \gg v\ ) \approx \lambda(|\phi|) \times |\phi|^4 + O(v^2|\phi|^2)$$

The evolution of λ is determined by two main effects:



growing λ at large energies

decreasing λ

$$\lambda(v) \propto \frac{m_h^2}{v^2}$$

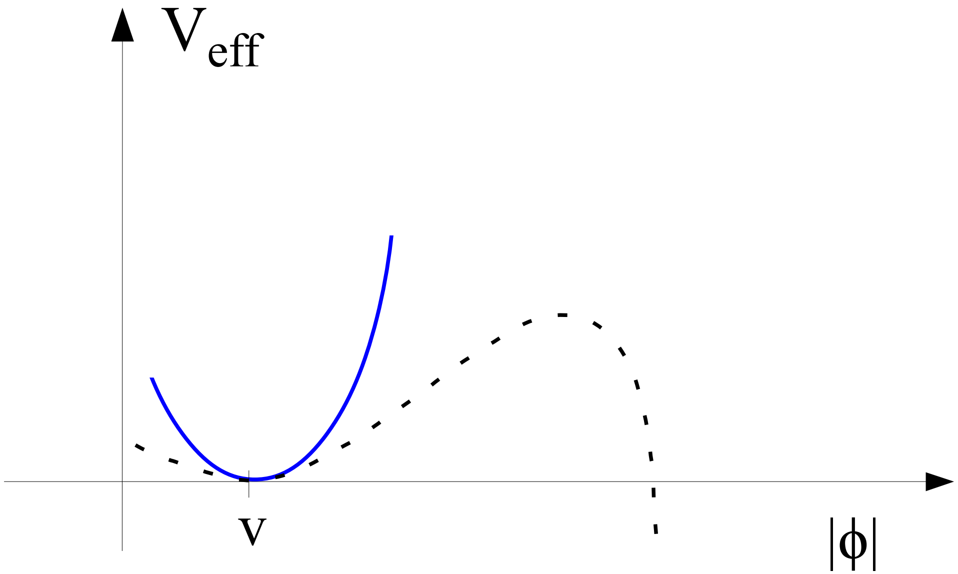
$$y_t(v) \propto \frac{m_t}{v}$$

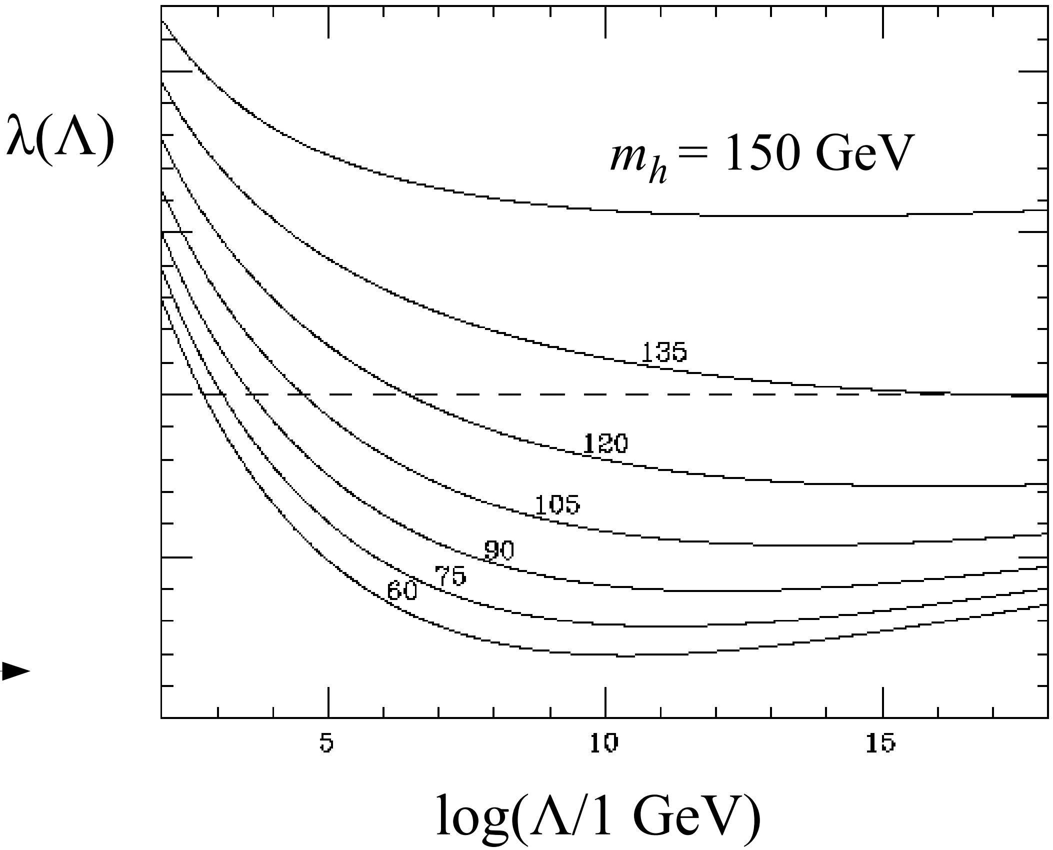
Given the large value of y_t, the destabilization due to top-quark loops is quite relevant

▶ *Stability and metastability bounds*

At large field values:

$$V_{eff}(|\phi|) \approx \lambda(|\phi|) \times |\phi|^4$$

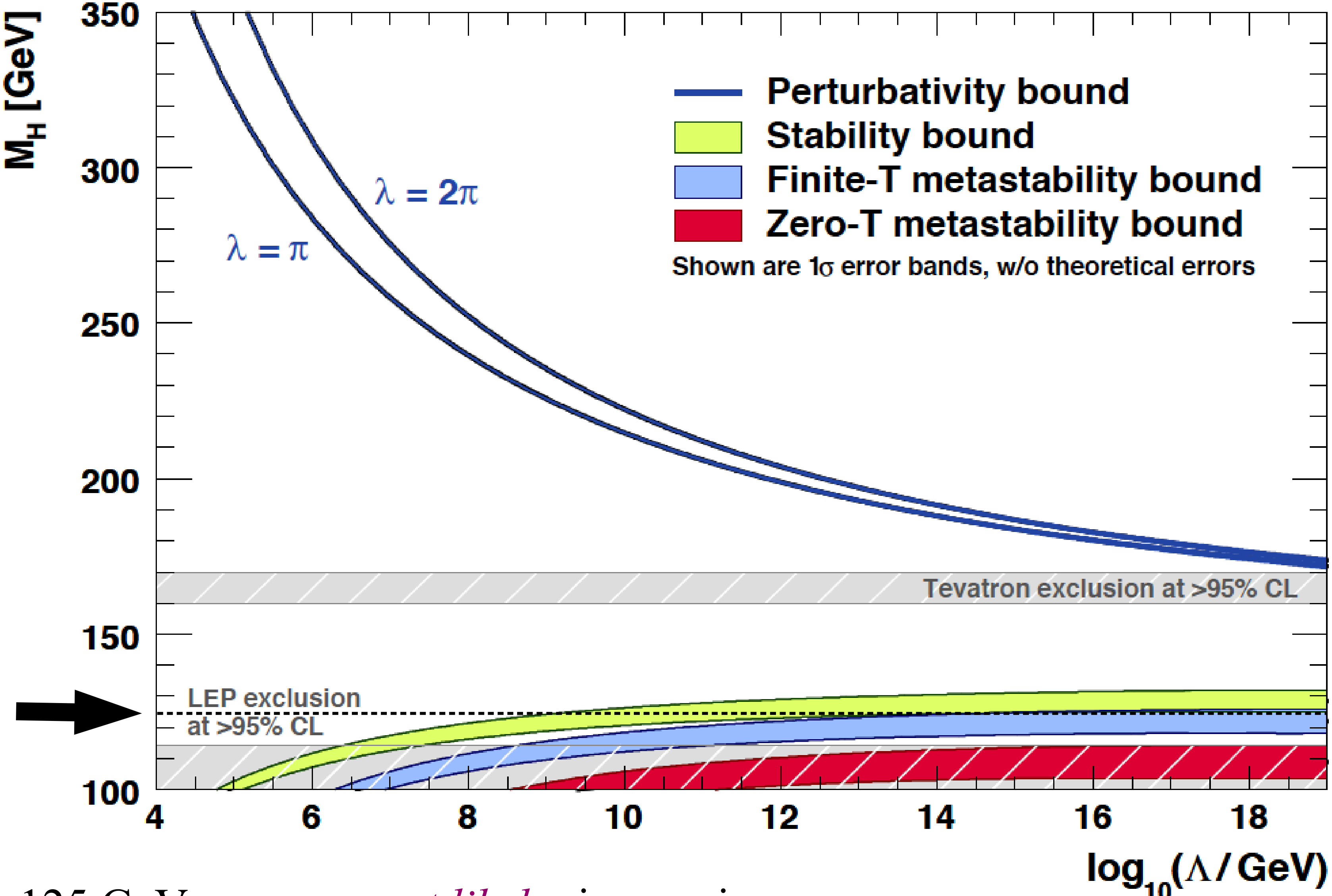




Cabibbo *et al.* '79; Hung '79;
Lindner 86; Sher '89; ....

Stability and metastability bounds

Ellis et al. '09



For $m_h \sim 125$ GeV we are -*most likely*- in a region where the Higgs potential is not absolutely stable

The metastability condition:

Can we rule out the model (and determine an upper bound on the new-physics scale Λ) if there is a second (deeper) minimum at large field values ?

Not really: The model could still be consistent if the lifetime of the (unstable) e.w. minimum is sufficiently long (i.e. longer than the age of the Universe)

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↓

We need to estimate the transition probability between false and true vacua.

Model-independent transition via quantum tunneling (occurring also a T=0).

Bubbles of true vacuum can from in the homogeneous background of the false vacuum. These bubbles are nothing but solutions of the e.o.m. (instantons) that interpolate between the two vacua (bounces).

Coleman '79

The bounces of the SM potential are characterised by a size R:

$$h(r) = \left(\frac{2}{|\lambda|}\right)^{1/2} \frac{2R}{r^2+R^2} \qquad (r = x_\mu x_\mu)$$

At the semiclassical level, this leads to: $p \sim e^{-\frac{8\pi^2}{3|\lambda|}}$

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A precise evaluation of the tunneling probability (integrated over the full volume of the Universe) can only be obtained going beyond the semiclassical approximation.

Highly non-trivial problem, which has been solved in the SM case:

G.I., Ridolfi, Strumia '01

- The tunneling is dominated by bounces of size R, such that $\lambda(1/R)$ reaches its minimum value
- The critical R determine the reference scale of the volume pre-factor:

$$p \sim \max_R \frac{V_U}{R^4} e^{-\frac{8\pi^2}{3|\lambda(1/R)|}}$$

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The leading gravitational effects are also calculable when 1/R is not far from (but below) M_{pl}

G.I., Rychkov, Strumia, Tetradis '08

The metastability condition:

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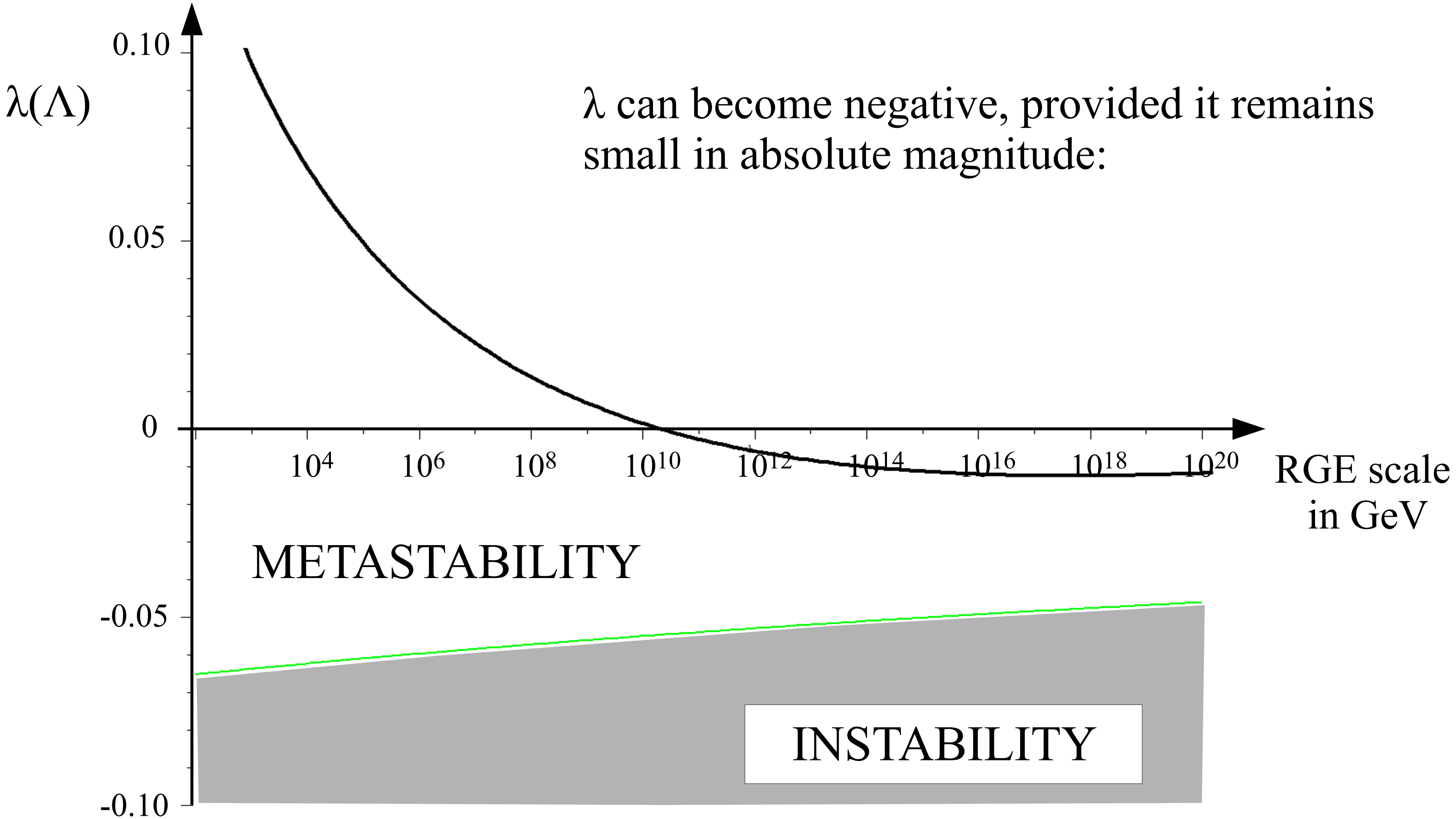
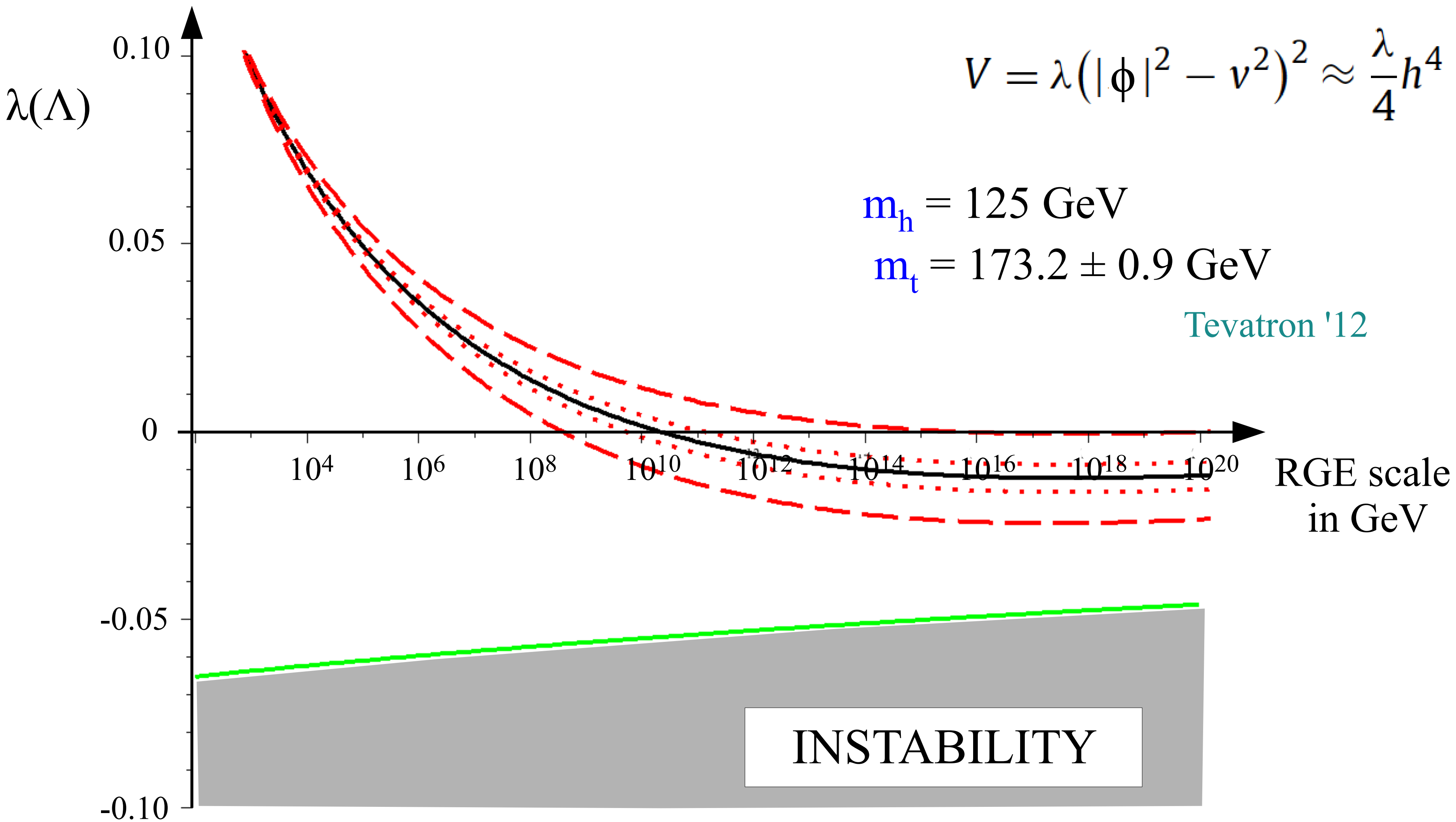
λ can become negative, provided it remains small in absolute magnitude:



Message n.2: For m_h =125 GeV and the present central value of m_{top} , the SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe



▶ *Vacuum stability at NNLO* (for m_h ~125 GeV)

For m_h =125 GeV and the present central value of m_{top} , the SM vacuum is unstable but sufficiently long-lived, compared to the age of the Universe

How "precise" is this statement?

A full NNLO analysis has recently become possible:

- Two-loop potential — Ford, Jack, Jones '92, '01
- Three-loop beta functions — Mihaila, Salomon, Steinhauser 1201.5868; Chetyrkin, Zoller, 1205.2892
- Two-loop threshold corrections in relating $\lambda(v)$ to the Higgs mass:

$$\lambda(\mu) = \frac{G_F\, m_h^2}{\sqrt{2}} + \Delta\lambda(\mu)$$

(*dominant uncertainty*)

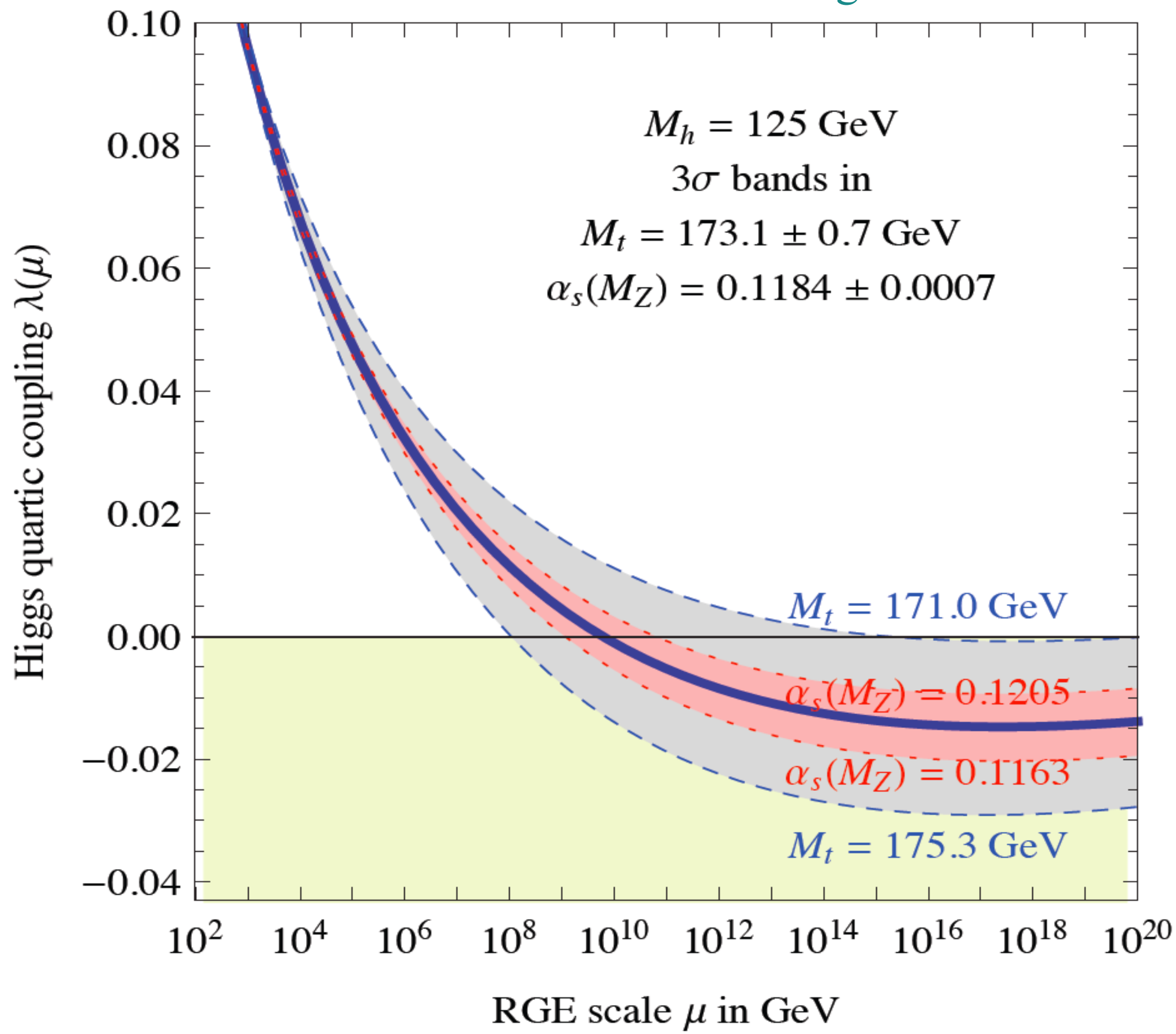
Yukawa×QCD — Bezrukov, Kalmykov, Kniehl, Shaposhnikov, 1205. 2893

Yukawa×QCD, Yuk.×Yuk. — Degrassi, Di Vita, Elias-Miro', Espinosa, Giudice, G.I., Strumia 1205.6497

Given the fast running of λ close to the e.w. scale, the dominant uncertainty comes from threshold (non-log enhanced) corrections at the electroweak scale (or in the precise evaluation of the initial condition).

While the smallness of λ (and the other couplings) at high energies imply that the 3-loop terms in the beta functions play a very minor role (useful to control the error).

Degrassi *et al.* '12



With the NNLO calculation we are able to derive a <u>very precise</u> relation between Higgs and top masses from vacuum stability:

Absolute stability:

$$M_h\ [\mathrm{GeV}] > 129.4 + 2.0\left(\frac{M_t\ [\mathrm{GeV}] - 173.1}{1.0}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\mathrm{th}}$$

Conservative th. error given the size of the shifts from NLO to NNLO:

$+\,0.6\,\mathrm{GeV}$ due to the QCD threshold corrections to λ

$+\,0.2\,\mathrm{GeV}$ due to the Yukawa threshold corrections to λ

$-\,0.2\,\mathrm{GeV}$ from RG equation at 3 loops

$-\,0.1\,\mathrm{GeV}$ from the effective potential at 2 loops.

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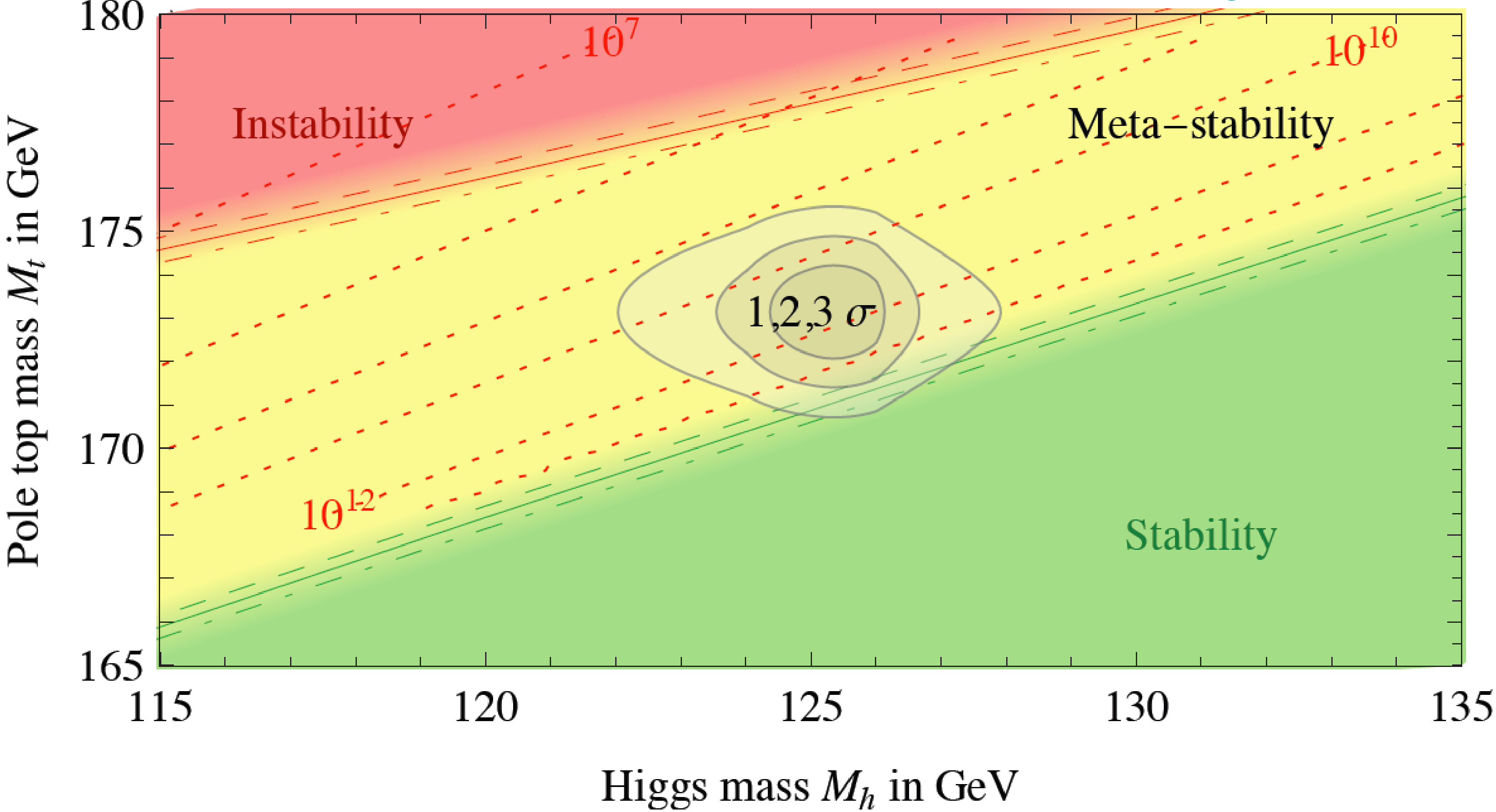

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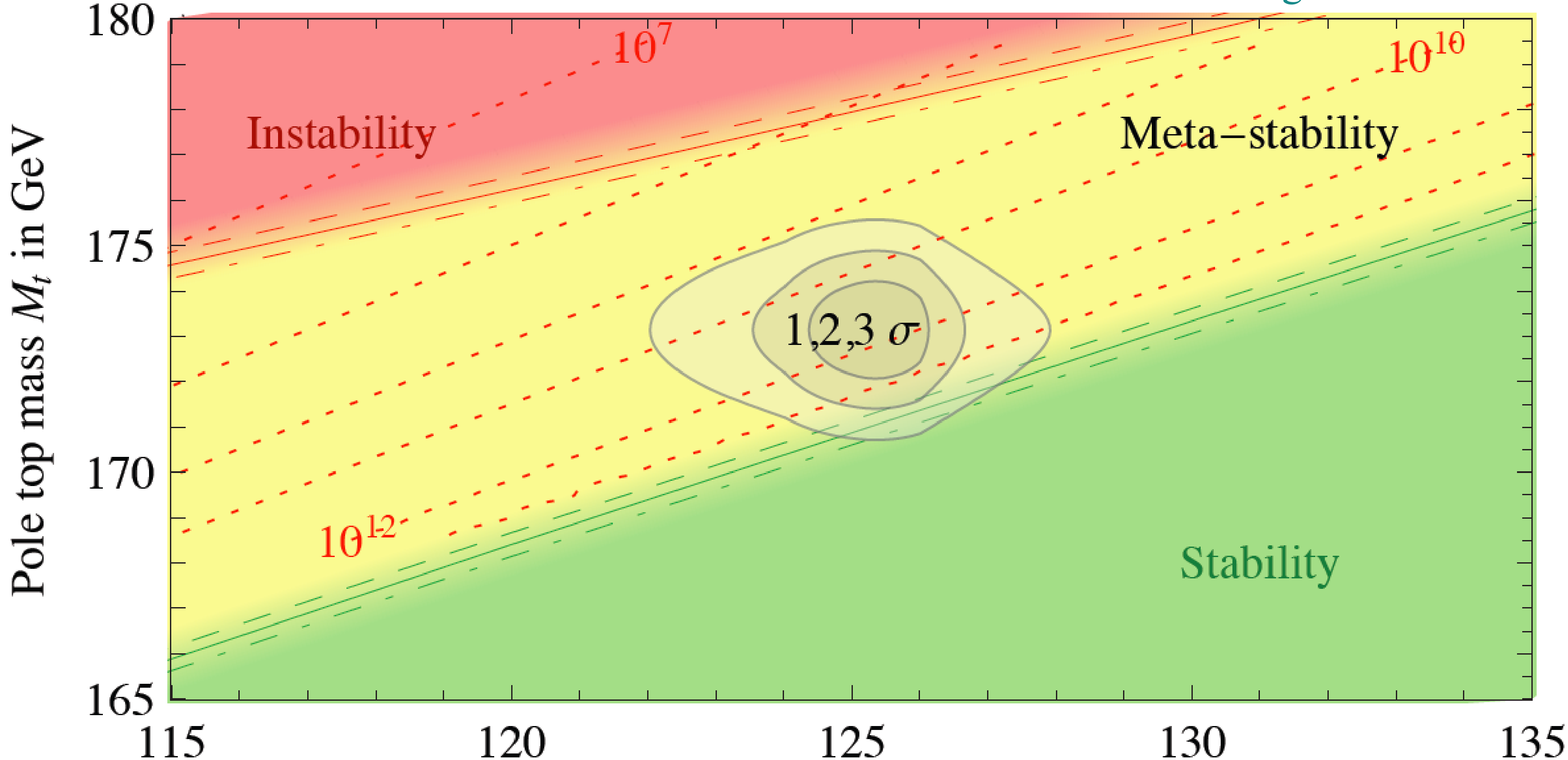

Assuming a precise determination of m_h by ATLAS & CMS in a short time, the main uncertainty will remain the top mass.

Note also that the m_t measured by Tevatron is not really the pole mass (*possible larger error...* Alekhin, Djouadi, Moch '12, Hoang & Stewart, '07-'08)

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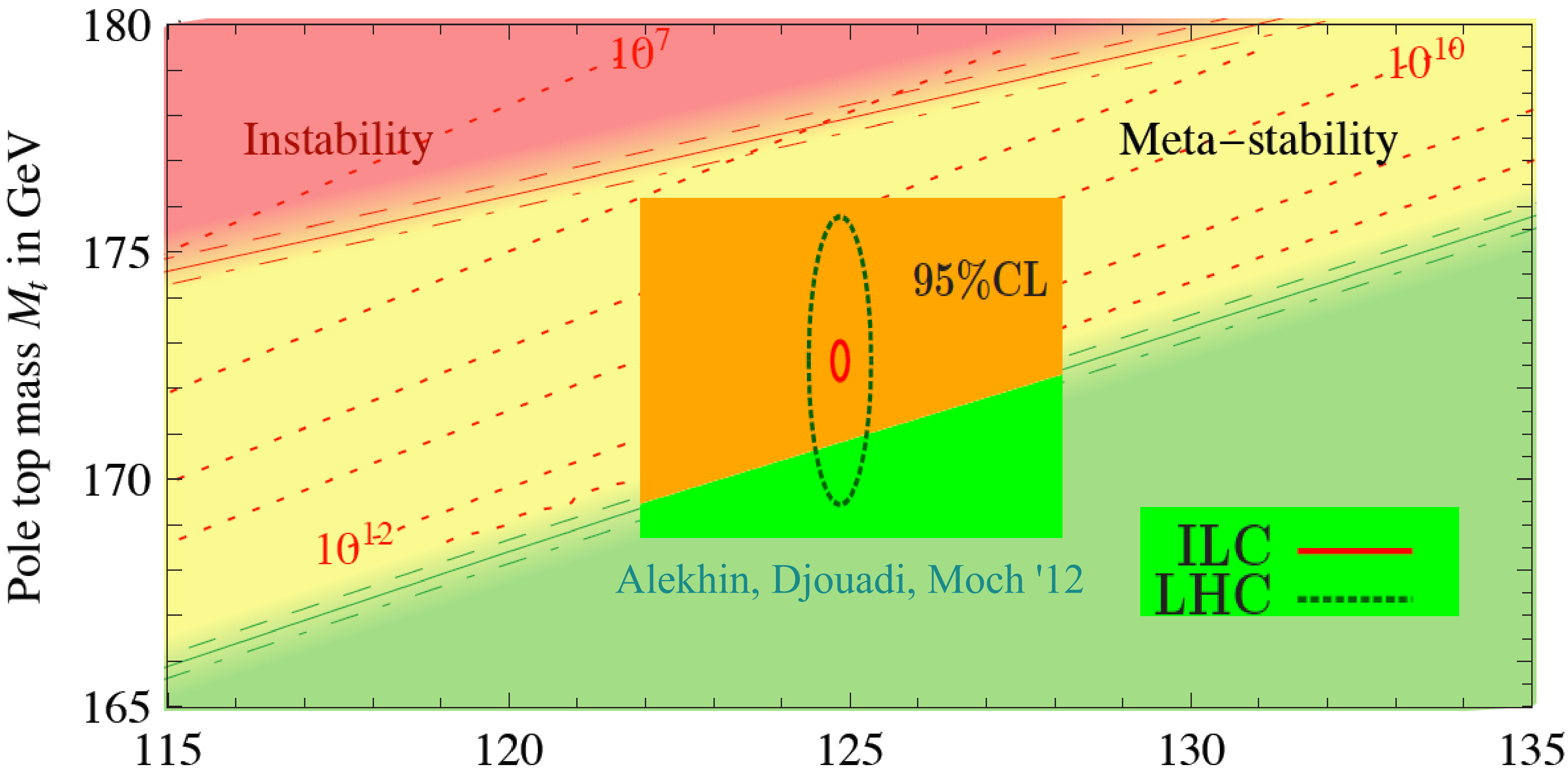
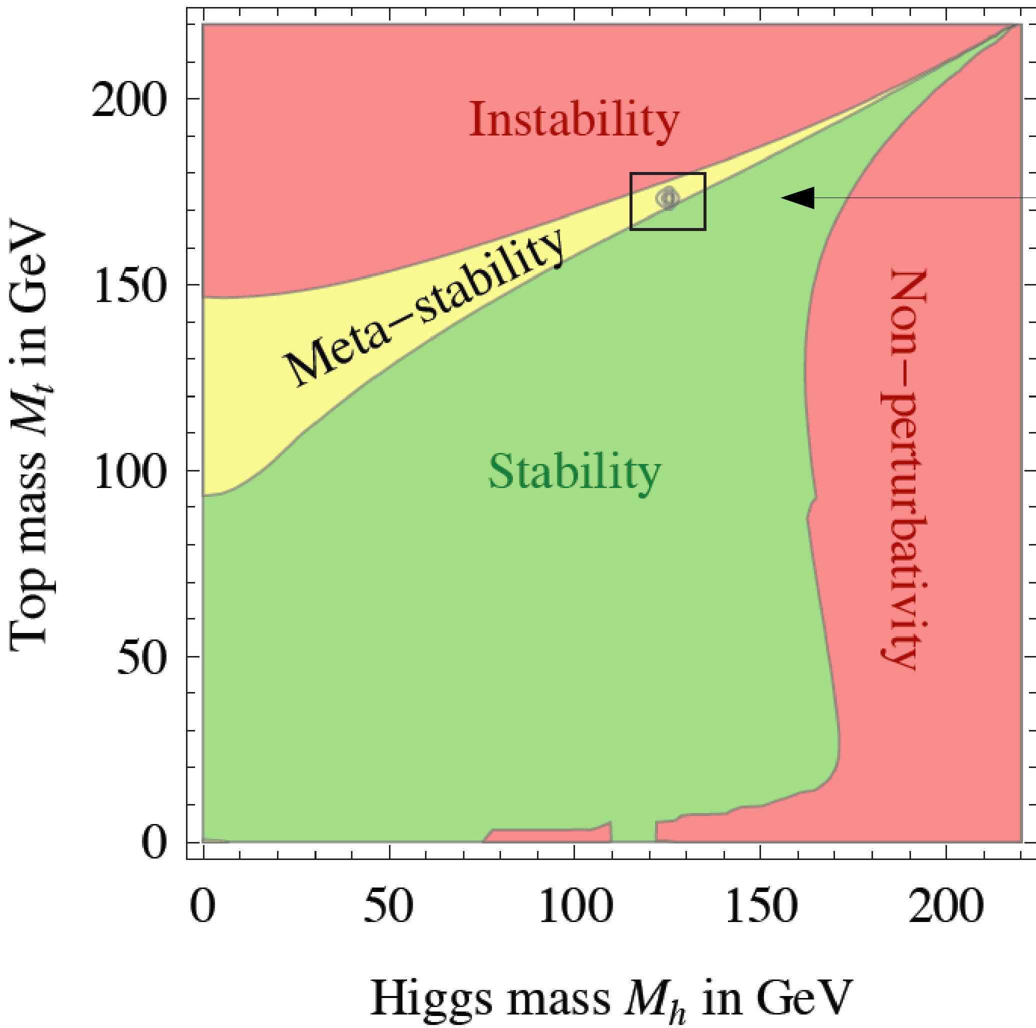
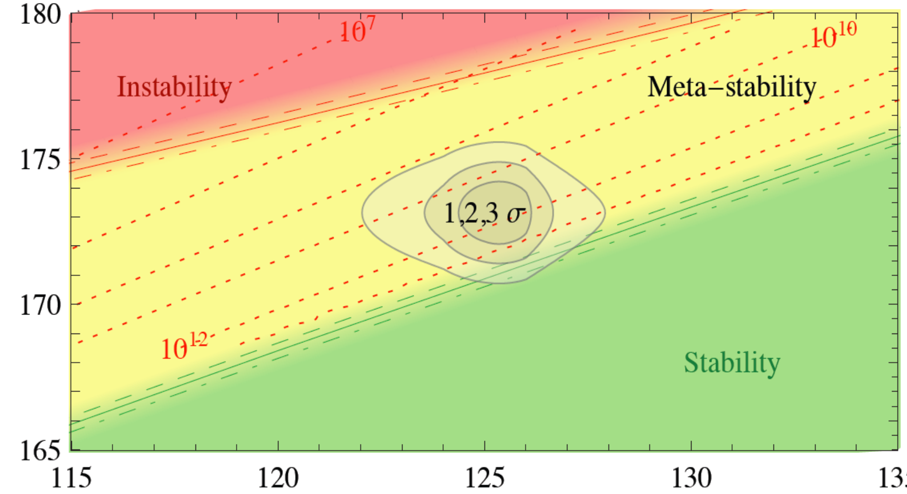


A linear collider would be the ideal machine to bring down this uncertainty, determining more precisely the fate of the SM vacuum (*if in the meanwhile we have not found anything else...!*)

▶ Speculations on Planck-scale dynamics





Looking at the plane from a more distant perspective, it appears more clearly that "we live" in a quite "peculiar" region...

And moving m_t down by ~ 2 GeV, we reach the even more peculiar configuration where $\lambda(M_{pl})=0$

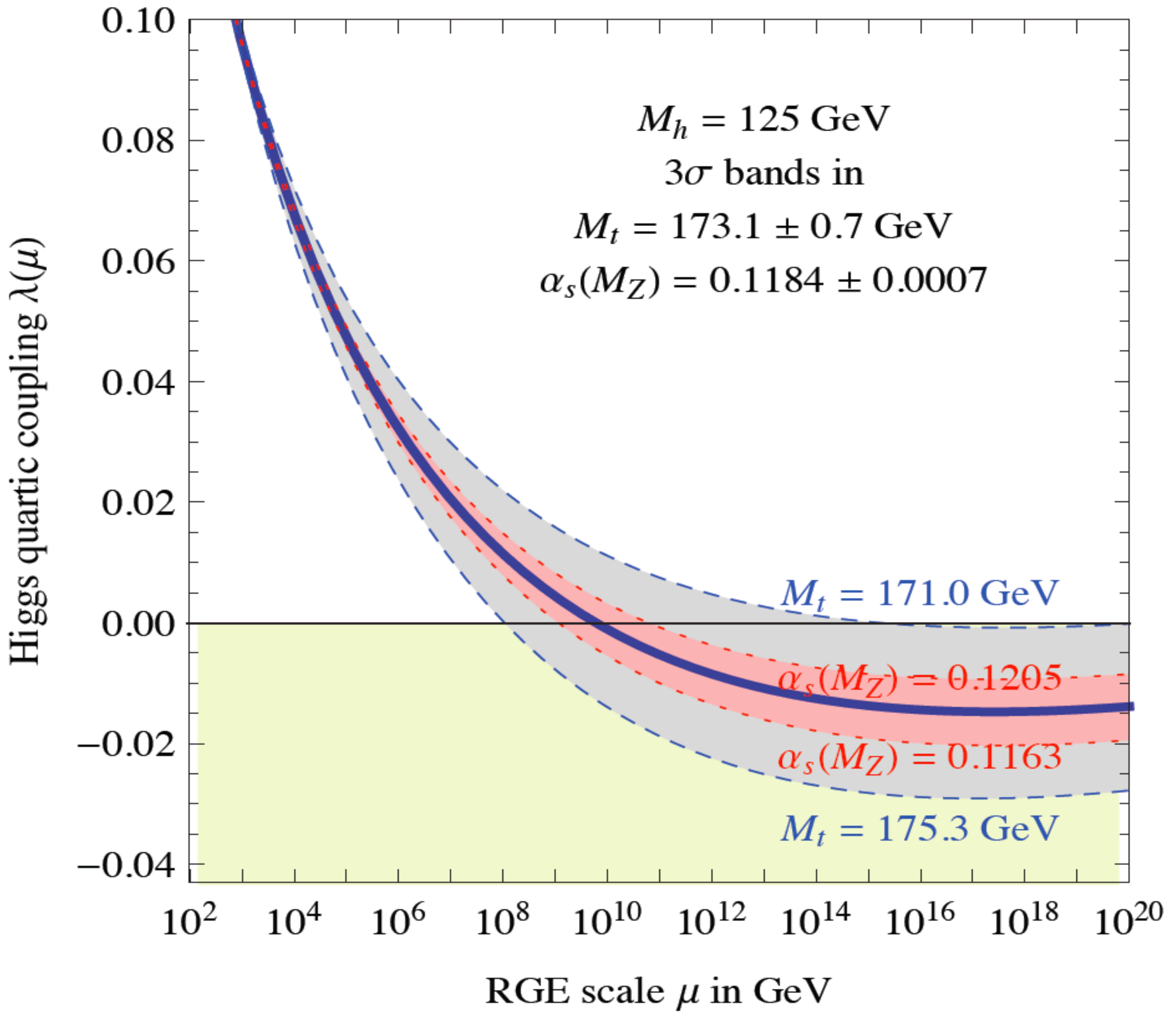
Froggatt, Nielsen, Takanishi, '01
Arkani-Hamed *et al.*, '08
Shaposhnikov, Wetterich, '10
...

Speculations on Planck-scale dynamics

What's special about $\lambda(M_{pl})=0$?

Despite also the beta function vanishes, is not a true fixed point (other coupl. $\neq 0$).

Maybe more interesting the overall smallness of λ compared to the other couplings.



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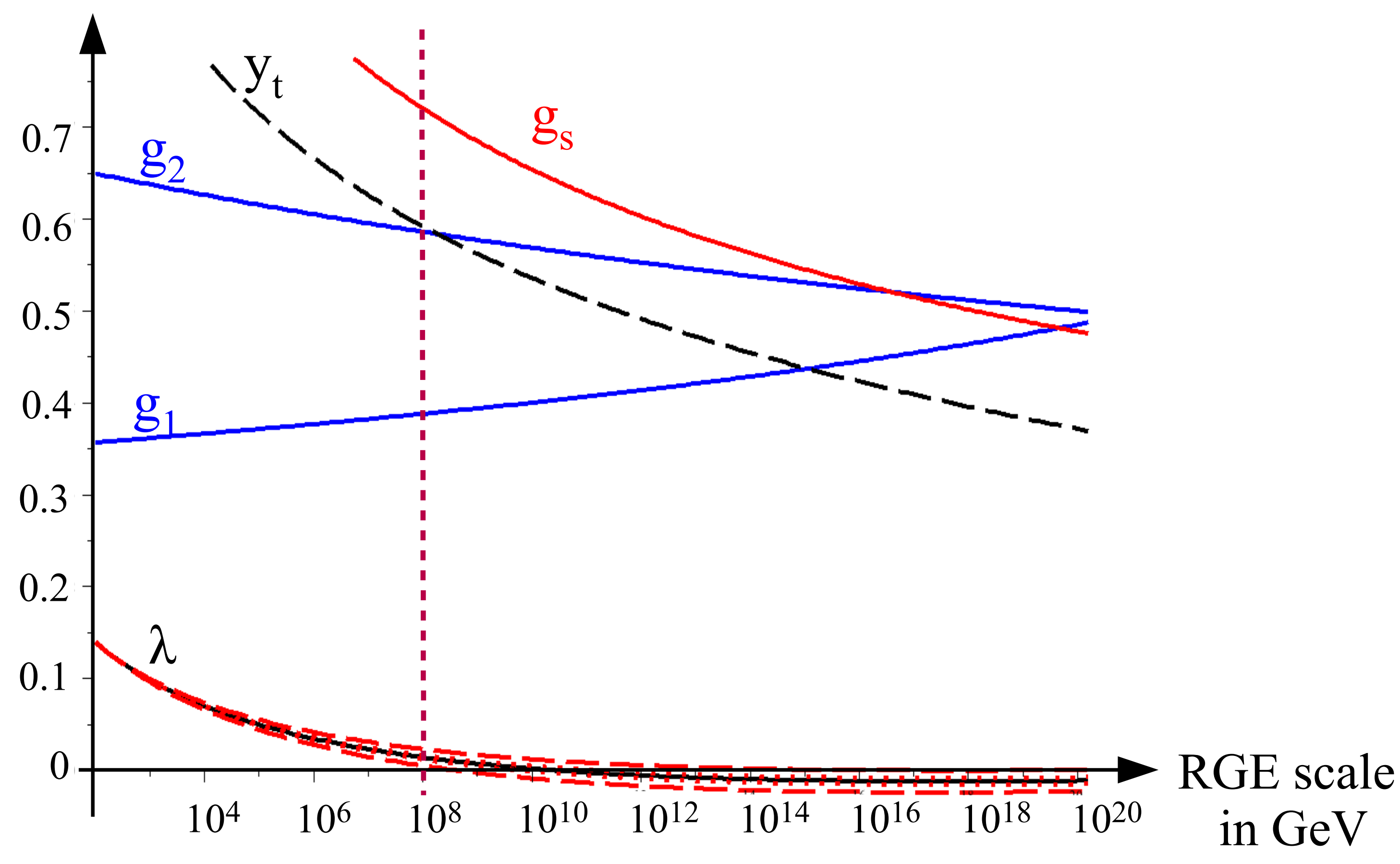
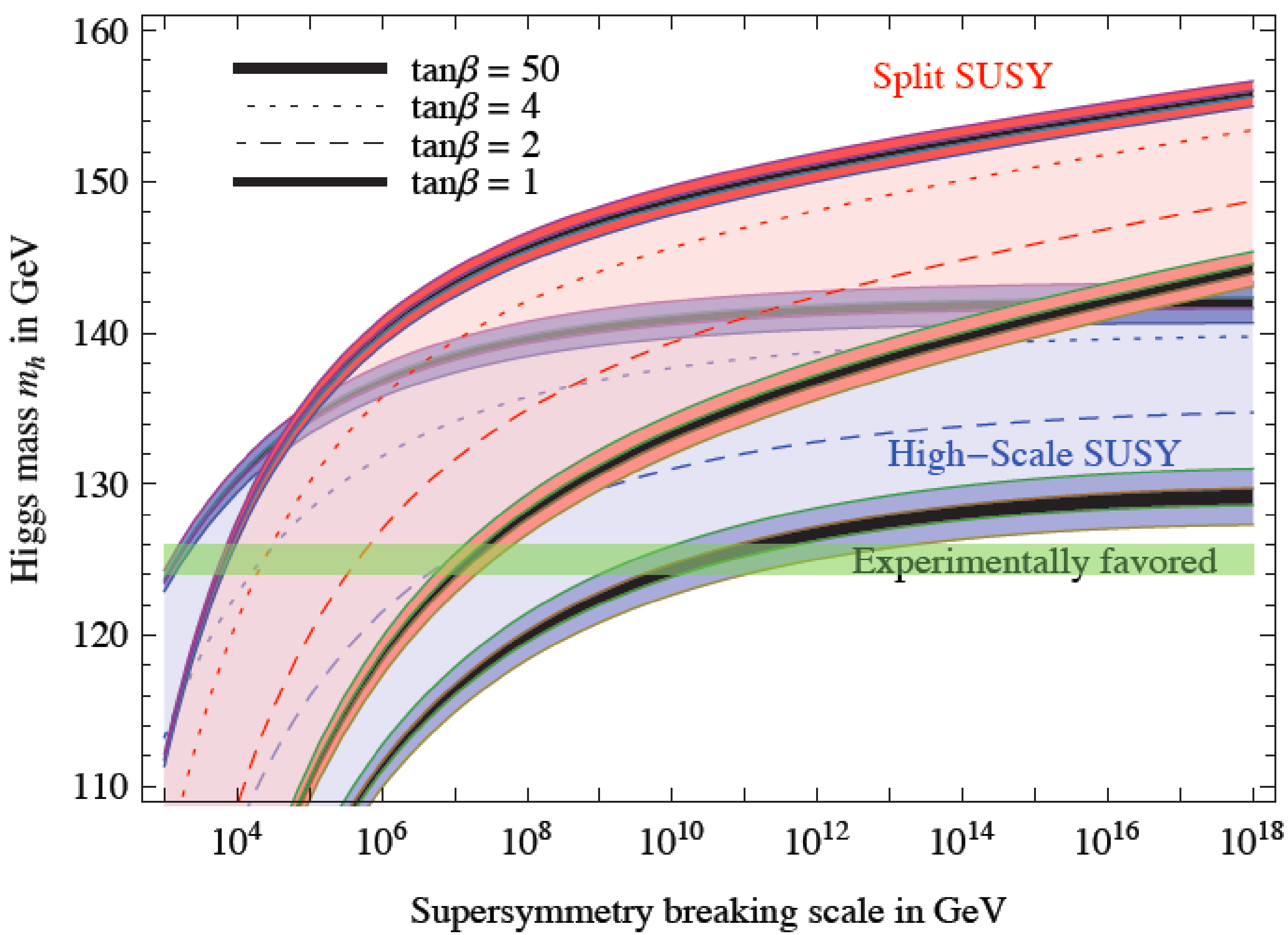
At a scale $\Lambda \gtrsim 10^8$ GeV λ becomes of the same order of its typical e.w. quantum corrections: *hints of a radiatively generated coupling*?



Speculations on Planck-scale dynamics

The smallness of λ certainly fits well with the possibility of a high-scale matching with a weakly coupled theory

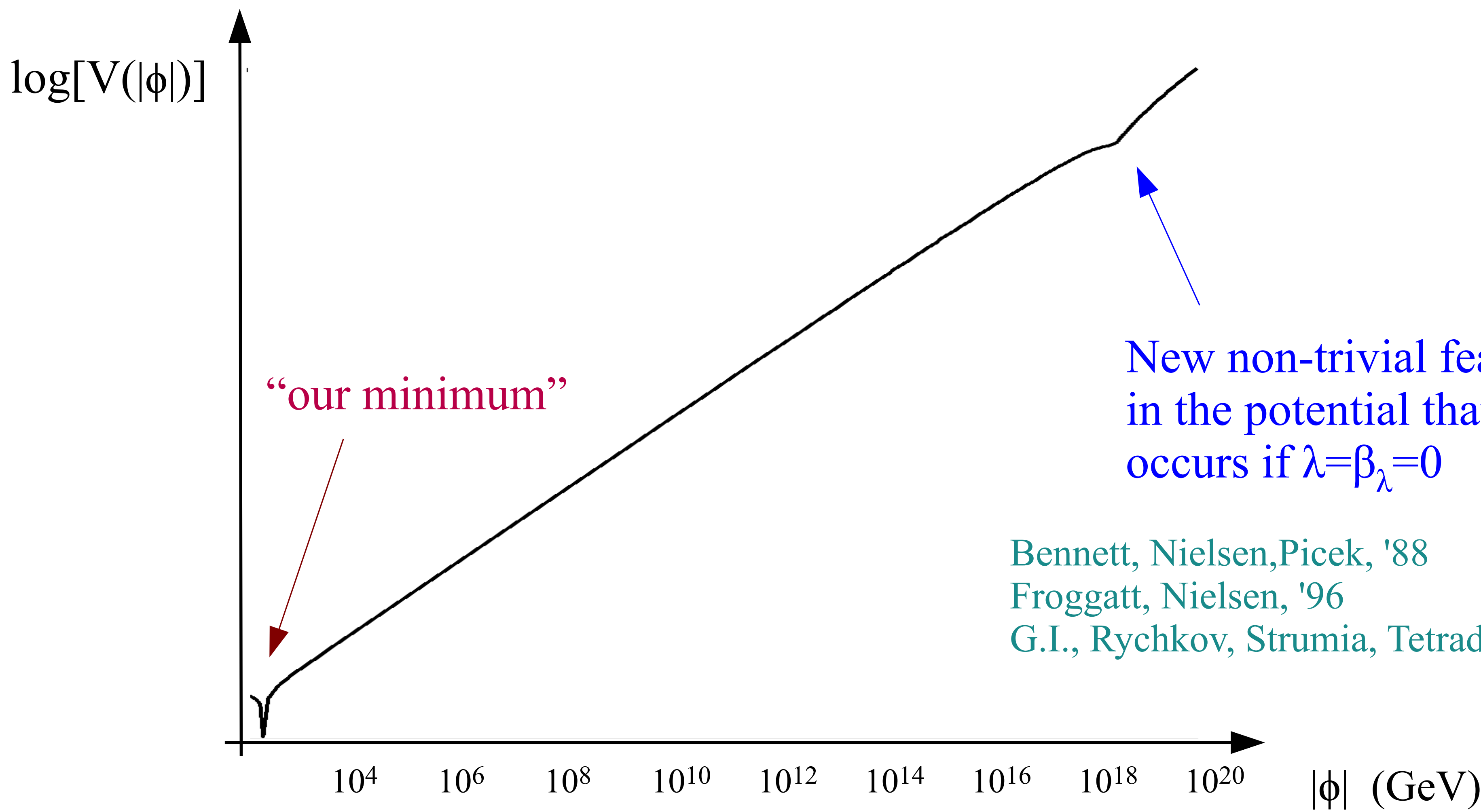
Giudice & Strumia '11-'12



Speculations on Planck-scale dynamics

Probably the most attractive feature of having λ=0 close to M_{pl} (*assuming no new physics below such scale*) is the possibility that the Higgs field has played some role in the early Universe, during inflation.

Bezrukov & Shaposhnikov, '08
Notari & Masina '11-'12



New non-trivial feature in the potential that occurs if $\lambda=\beta_\lambda=0$

Bennett, Nielsen,Picek, '88
Froggatt, Nielsen, '96
G.I., Rychkov, Strumia, Tetradis '08

<u>*Speculations on Planck-scale dynamics*</u>

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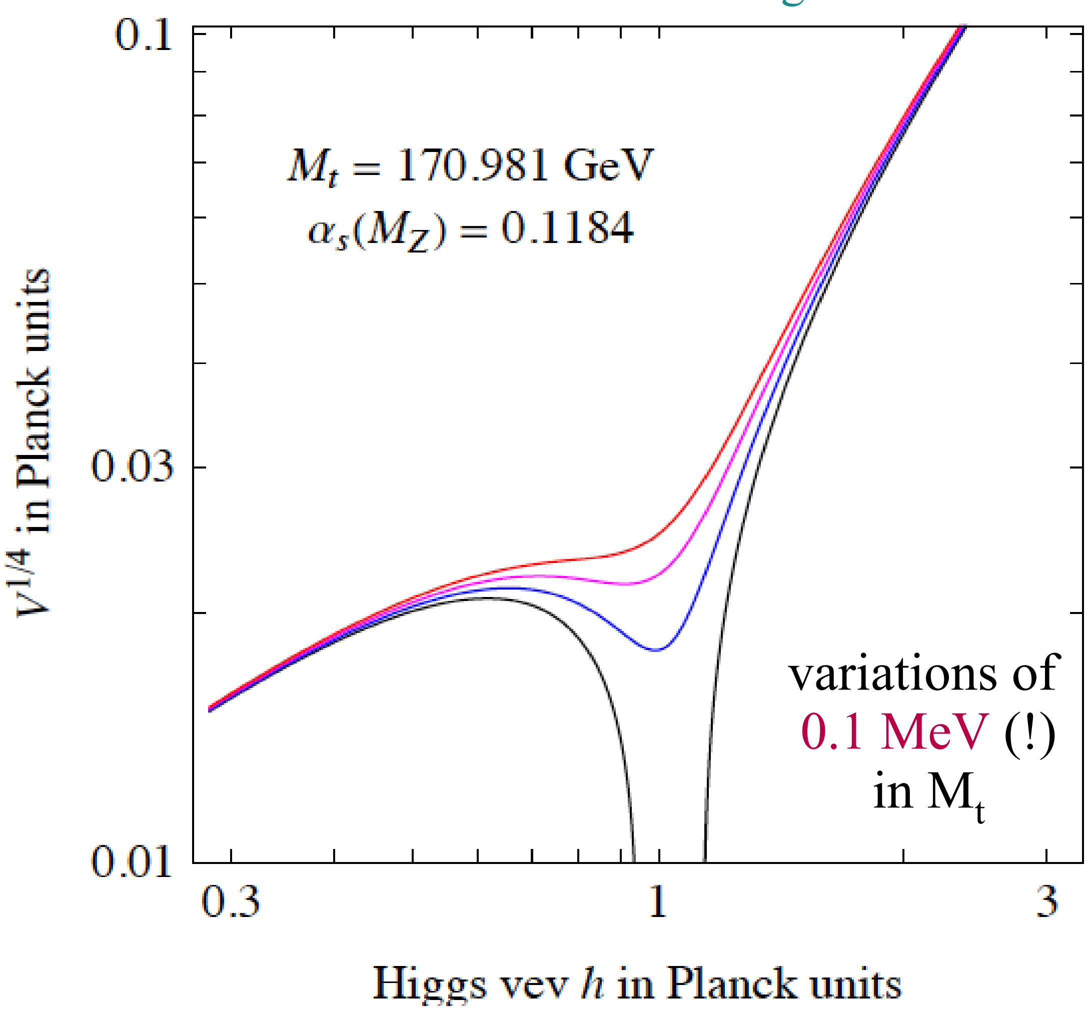
Degrassi *et al.* '12

The minimal set-up (SM only) does not work (*field trapped into the new minimum or too large fluctuations*)

But the problem can be solved with non-minimal couplings of the Higgs field to gravity and/or to other fields

Bezrukov & Shaposhnikov, '08
Notari & Masina '11-'12

The minimality of the scheme is lost, but it remains an intriguing possibility.



Two final remarks about the instability of the SM potential:

I. What about the instability because of thermal fluctuations?

II. What about adding to the model heavy right-handed neutrinos?

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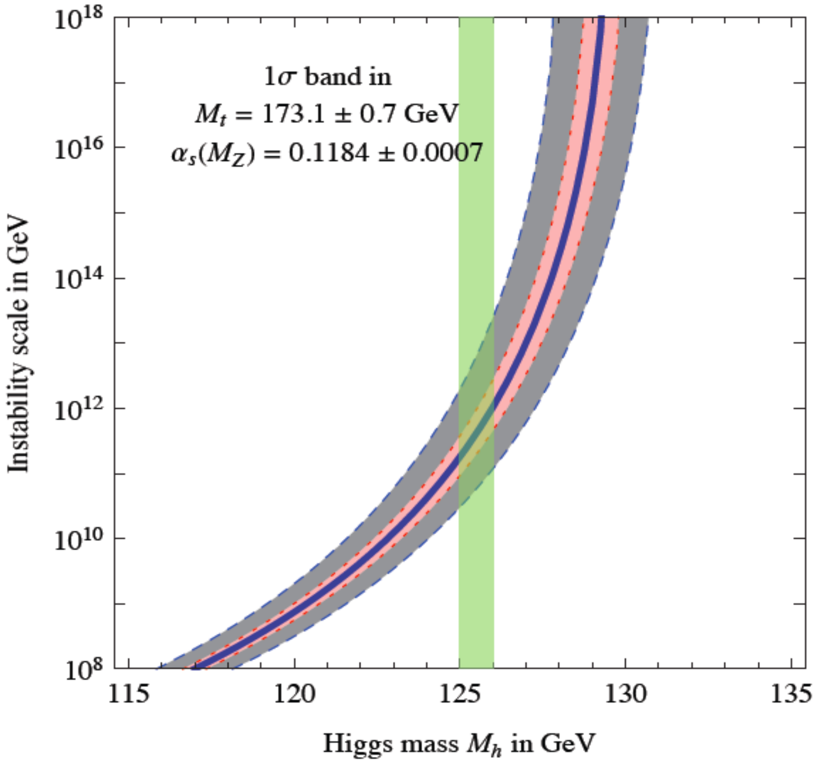
Since the instability occurs at very high energies, thermal corrections do not play a significant role in destabilizing the potential.



II. What about adding to the model heavy right-handed neutrinos?

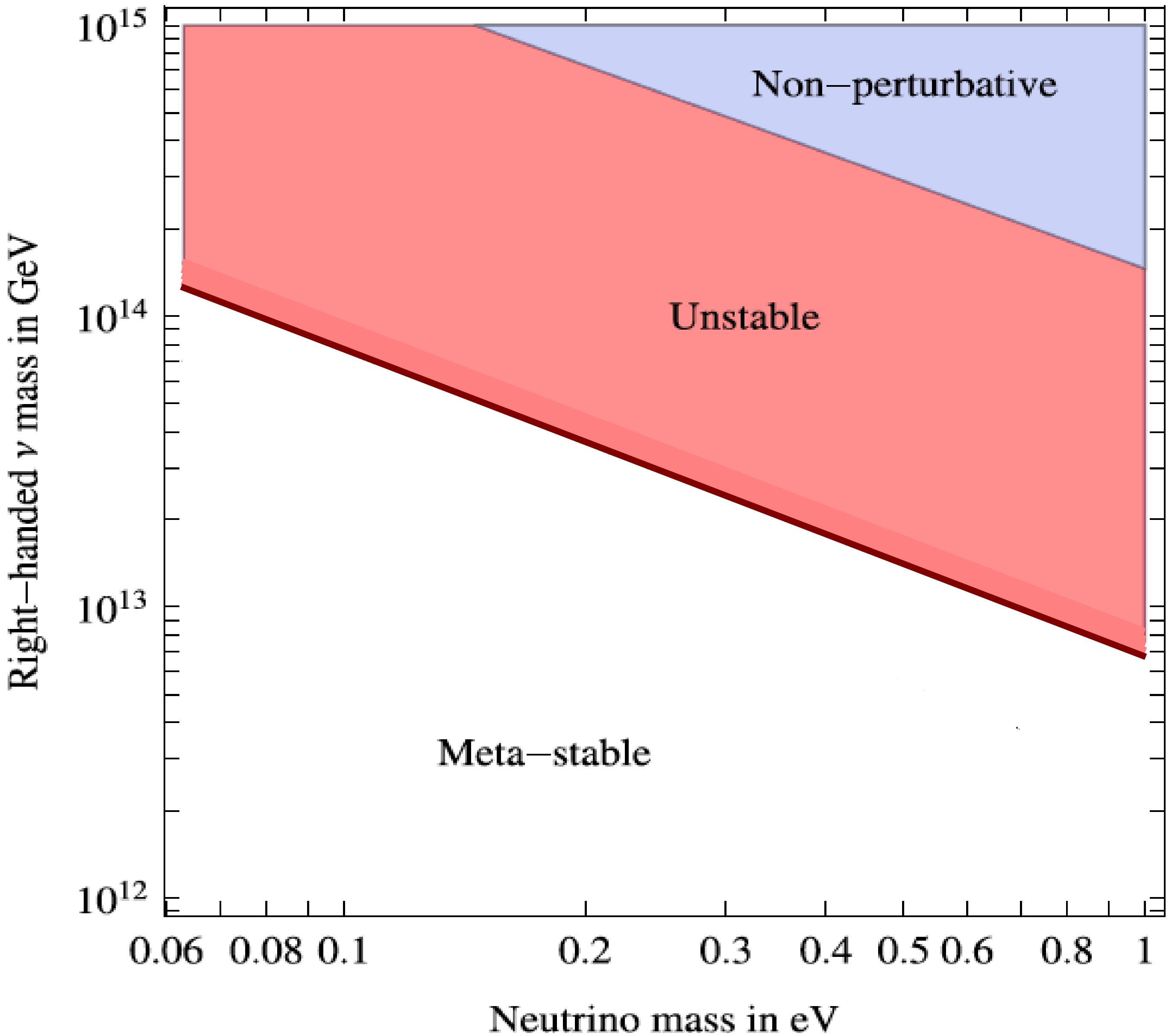
On general ground, adding new fermions may induce a furhter destabilization of the potential. However, the effect depend on the size of the new Yukawa couplings:

$$m_\nu \sim Y_n^T \frac{v^2}{M_R} Y_n \longrightarrow$$

Requiring a sufficiently stable Higgs potential allow us to derive an upper bound on M_R

$$m_\nu \sim Y_n^T \frac{v^2}{M_R} Y_n$$

Elias-Miro *et al.* '11



Still enough room for leptogenesis to take place.

Conclusions

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- A SM-like Higgs with $m_h \sim 125$ GeV does not allow us to derive model-independent conclusions about the scale of New Physics: the Higgs potential is most likely unstable, but the e.w. vacuum is certainly sufficiently long-lived.

- Clear indication about a small, or even vanishing, Higgs self-coupling at high energies: if the SM is only an effective theory, we have to match it into a model where the Higgs

  - is a weakly interacting particle, if the matching occurs close to the e.w. scale [*as indicated by naturalness*]

  - may have no intrinsic self-coupling (trivial $\lambda\phi^4$), if the matching occurs above $\sim 10^8$ GeV

- More precise determinations of both m_h & m_t would be very useful, especially in absence of other NP signals, to better investigate the structure of the Higgs potential at high energies