MC tools and NLO Monte Carlos

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Higgs Hunting 2012
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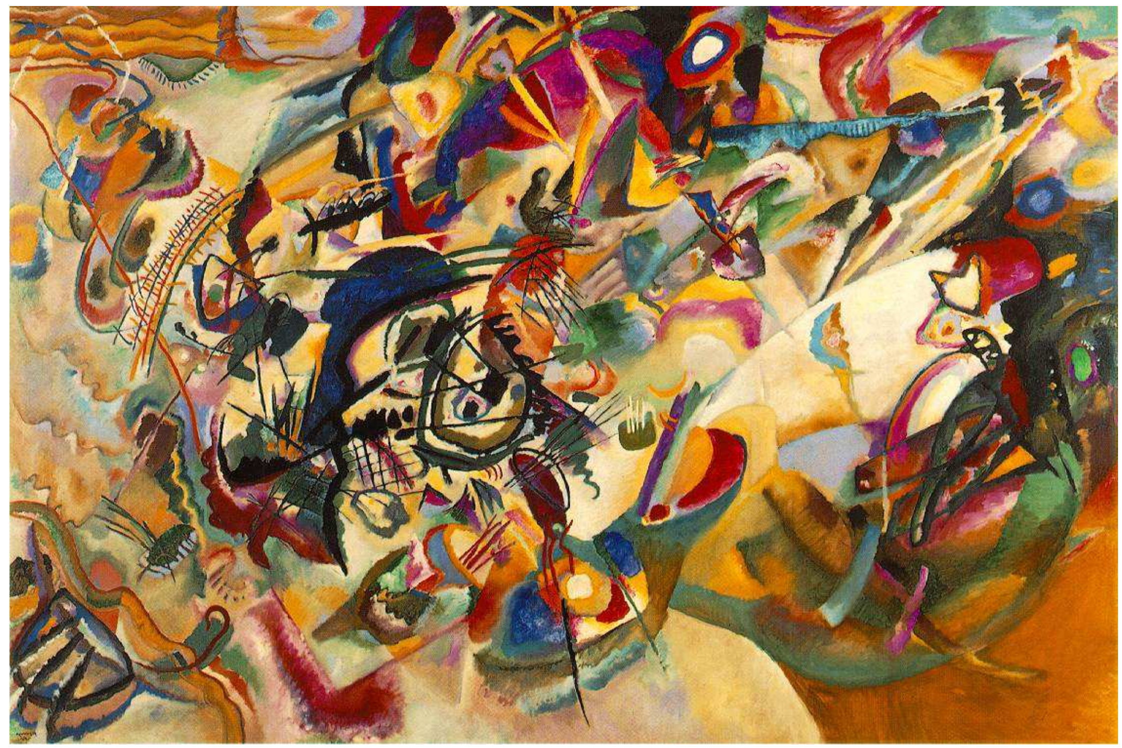
- Introduction to NLO+Parton Shower Monte Carlo programs
- Higgs boson production in gluon fusion: H, Hj and Hjj
- H in VBF
- $t\bar{t}H$
- WH/ZH
- $tH^{\pm}$



NLO vs Shower Monte Carlo

NLO

- ✓ accurate shapes at high p_T
- ✓ normalization accurate at NLO order
- ✓ reduced dependence on renormalization and factorization scales
- ✗ wrong shapes at small p_T
- ✗ description only at the parton level

SMC (LO + shower)

- ✗ bad description at high p_T
- ✗ normalization accurate only at LO
- ✓ correct Sudakov suppression at small p_T
- ✓ simulate events at the hadron level

It is natural to try to merge the two approaches, keeping the good features of both

MC@NLO [Frixione and Webber, 2001] and POWHEG [Nason, 2004] do this in a consistent way

Higgs boson production

- H in gluon fusion: MC@NLO, POWHEG BOX, POWHEG+SHERPA, POWHEG+HERWIG++, MC@NLO+SHERPA
- H+1jet: POWHEG+SHERPA, MC@NLO+SHERPA, POWHEG BOX
- H+2jet: POWHEG BOX
- H in VBF: POWHEG BOX, POWHEG+HERWIG++
- $t\bar{t}H$: POWHEG BOX + HELAC, aMC@NLO
- VH: POWHEG+HERWIG++, MC@NLO
- $tH^{\pm}$: MC@NLO, POWHEG BOX
- $H \to Q\overline{Q}$: POWHEG+HERWIG++

The POWHEG differential cross section

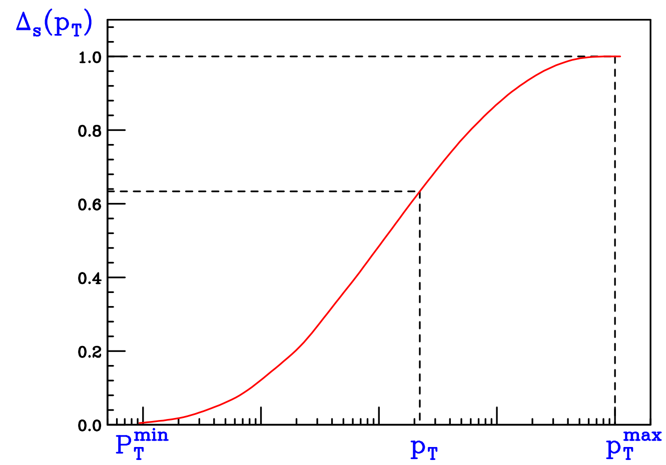
$R = R_s + R_f$ with $R_s > 0$, $R_f > 0$, R_s singular in the infrared regions, R_f finite in collinear and soft limit [Nason 2004]. Define

$$d\sigma = \overline{B}_s(\mathbf{\Phi}_n) \underbrace{\left\{ \Delta_s(p_T^{min}) + \Delta_s(p_T) \frac{R_s(\mathbf{\Phi}_{n+1})}{B(\mathbf{\Phi}_n)} \, d\Phi_r \right\}}_{\text{1 by unitarity}} d\mathbf{\Phi}_n + R_f(\mathbf{\Phi}_{n+1}) \, d\mathbf{\Phi}_{n+1}$$

$$\overline{B}_s(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \left[R_s(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r) \right]$$

$$\Delta_s(p_T) = \exp \left[- \int d\Phi'_r \frac{R_s(\mathbf{\Phi}_n, \Phi'_r)}{B(\mathbf{\Phi}_n)} \theta(p'_T - p_T) \right]$$

The expansion of $d\sigma$ up to the NLO level is exactly equal to $d\sigma_{\text{NLO}}$.
The part of the real cross section that is treated with the shower technique can be varied.



MC@NLO in the POWHEG language

The MC@NLO hardest emission cross section can be written in the POWHEG language

$$d\sigma = \underbrace{\overline{B}_{\rm HW}\, d\mathbf{\Phi}_n}_{\mathbb{S}\text{ event}} \underbrace{\left[\Delta_{\rm HW}(p_T^{min}) + \Delta_{\rm HW}(p_T)\frac{R_{\rm HW}(\mathbf{\Phi}_{n+1})}{B(\mathbf{\Phi}_n)}\, d\Phi_r\right]}_{\text{HERWIG event}} + \underbrace{\left[R(\mathbf{\Phi}_{n+1}) - R_{\rm HW}(\mathbf{\Phi}_{n+1})\right] d\mathbf{\Phi}_{n+1}}_{\mathbb{H}\text{ event}}$$

$$\overline{B}_{\rm HW}(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int \left[R_{\rm HW}(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r)\right] d\Phi_r$$

$$\Delta_{\rm HW}(p_T) = \exp\left[-\int d\Phi_r' \frac{R_{\rm HW}(\mathbf{\Phi}_n, \Phi_r')}{B(\mathbf{\Phi}_n)}\, \theta(p_T' - p_T)\right]$$

Like POWHEG with $\begin{cases} R_s = R_{\rm HW} & \\ R_f = R - R_{\rm HW} & \Longleftarrow \text{ can be negative} \end{cases}$

This formula illustrates why MC@NLO and POWHEG are equivalent at NLO.
But differences can arise at NNLO.

The radiation cross section

$$\overline{B}_s(\mathbf{\Phi}_n) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n) + \int d\Phi_r \left[R_s(\mathbf{\Phi}_n, \Phi_r) - C(\mathbf{\Phi}_n, \Phi_r) \right]$$

$$d\sigma = \overline{B}_s(\mathbf{\Phi}_n) \left\{ \Delta_s\left(p_T^{min}\right) + \Delta_s(p_T) \frac{R_s(\mathbf{\Phi}_{n+1})}{B(\mathbf{\Phi}_n)} \, d\Phi_r \right\} d\mathbf{\Phi}_n + R_f(\mathbf{\Phi}_{n+1}) \, d\mathbf{\Phi}_{n+1}$$

The differential cross section describing the hard radiation is given by

$$\begin{aligned} d\sigma_{\rm rad} &\approx \frac{\overline{B}_s(\mathbf{\Phi}_n)}{B(\mathbf{\Phi}_n)} R_s(\mathbf{\Phi}_{n+1}) \, d\mathbf{\Phi}_{n+1} + R_f(\mathbf{\Phi}_{n+1}) \, d\mathbf{\Phi}_{n+1} \\ &= \left\{ \underbrace{R_s(\mathbf{\Phi}_{n+1}) + R_f(\mathbf{\Phi}_{n+1})}_{R(\mathbf{\Phi}_{n+1})} + \left[\frac{\overline{B}_s(\mathbf{\Phi}_n)}{B(\mathbf{\Phi}_n)} - 1 \right] R_s(\mathbf{\Phi}_{n+1}) \right\} d\mathbf{\Phi}_{n+1} \\ &= R(\mathbf{\Phi}_{n+1}) \, d\mathbf{\Phi}_{n+1} + \mathcal{O}(\alpha_s) R_s(\mathbf{\Phi}_{n+1}) \end{aligned}$$

- We expect differences at the NNLO level. While formally at NNLO, they may be large for particular processes (see i.e. Higgs boson production in gluon fusion).
- Notice that the $\overline{B}_s(\mathbf{\Phi}_n)/B(\mathbf{\Phi}_n)$ also depends on how the real contribution R has been split into R_s and R_f.

Sources of possible differences

$$d\sigma_{\rm rad} \approx \frac{\overline{B}_s(\mathbf{\Phi}_n)}{B(\mathbf{\Phi}_n)} R_s(\mathbf{\Phi}_{n+1})\, d\mathbf{\Phi}_{n+1} + R_f(\mathbf{\Phi}_{n+1})\, d\mathbf{\Phi}_{n+1}$$

In an NLO+Parton Shower implementation, visible differences of the radiation cross section with respect to the fixed-order result will be present, due to

1. The $\Delta_s(p_T)$ factor, dropped in the NLO-accuracy derivation.
   The Sudakov factor yields resummation-improved results at NLO.
   It is less than 1: it always reduces the transverse-momentum spectrum of radiation with respect to the pure NLO result

2. The $\overline{B}_s(\mathbf{\Phi}_n)/B(\mathbf{\Phi}_n)$ factor, also dropped.
   This factor spreads the K factor over the finite p_T region. The spreading of the K factor depends upon the R_s and R_f separation.

3. The choice of scales used in the process.

In summary

Experience in comparing MC@NLO and POWHEG results (various papers from the POWHEG BOX and from the HERWIG++ collaborations) has shown that

- all important differences between MC@NLO and POWHEG can be tracked back to the rôle of the $\overline{B}_s / B$ factor and to scale-choice issues.
- Exponentiation in $\Delta_s(p_T)$ does not seem to yield important differences. This is understood as due to the fact that the integral in

$$\Delta_s(p_T) = \exp\left[- \int d\Phi'_r \, \frac{R_s(\mathbf{\Phi}_n, \Phi'_r)}{B(\mathbf{\Phi}_n)} \, \theta(p'_T - p_T) \right]$$

  is dominated by the region of soft p_T, where all R_s agree.

Scale dependence

$$d\sigma = \overline{B}_s(\boldsymbol{\Phi}_n, \mu_R)\, d\boldsymbol{\Phi}_n \left\{ \Delta_s(\boldsymbol{\Phi}_n, p_T^{min}) + \Delta_s(\boldsymbol{\Phi}_n, p_T)\, \frac{R_s(\boldsymbol{\Phi}_n, \Phi_r, \alpha_s(k_T))}{B(\boldsymbol{\Phi}_n)}\, d\Phi_r \right\}$$

$$+ R_f(\boldsymbol{\Phi}_{n+1}, \alpha_s(\mu_R))\, d\boldsymbol{\Phi}_{n+1}$$

$$\overline{B}_s(\boldsymbol{\Phi}_n, \mu_R) = B(\boldsymbol{\Phi}_n) + V(\boldsymbol{\Phi}_n, \alpha_s(\mu_R)) + \int d\Phi_r \left[R_s(\boldsymbol{\Phi}_n, \Phi_r, \alpha_s(\mu_R)) - C(\boldsymbol{\Phi}_n, \Phi_r, \alpha_s(\mu_R)) \right]$$

$$\Delta_s(\boldsymbol{\Phi}_n, p_T) = \exp\left[-\int d\Phi'_r\, \frac{R_s(\boldsymbol{\Phi}_n, \Phi'_r, \alpha_s(k_T))}{B(\boldsymbol{\Phi}_n)}\, \theta\left(k_T(\boldsymbol{\Phi}_n, \Phi'_r) - p_T\right) \right]$$

- A scale variation in the curly braces $\{\}$ is in practice never performed (in order not to spoil the NLL accuracy of the Sudakov form factor). The scale in the Sudakov has to go exactly to the transverse momentum of the radiation in the collinear and soft region.

- Scale dependence affects $\overline{B}_s$ and R_f differently: $\overline{B}_s$ is a quantity integrated over the radiation kinematics $\Longrightarrow$ milder scale dependence

Similar conclusions for the factorization scale μ_F

Scale dependence in gg→H

$$d\sigma = \overline{B}_s(\mathbf{\Phi}_n, \mu_R)\, d\mathbf{\Phi}_n \left\{ \Delta_s(\mathbf{\Phi}_n, p_T^{min}) + \Delta_s(\mathbf{\Phi}_n, p_T)\, \frac{R_s(\mathbf{\Phi}_n, \Phi_r, \alpha_s(k_T))}{B(\mathbf{\Phi}_n)}\, d\Phi_r \right\}$$
$$+ R_f(\mathbf{\Phi}_{n+1}, \alpha_s(\mu_R))\, d\mathbf{\Phi}_{n+1}$$

$$\overline{B}_s(\mathbf{\Phi}_n, \mu_R) = B(\mathbf{\Phi}_n) + V(\mathbf{\Phi}_n, \alpha_s(\mu_R)) + \int d\Phi_r \left[R_s(\mathbf{\Phi}_n, \Phi_r, \alpha_s(\mu_R)) - C(\mathbf{\Phi}_n, \Phi_r, \alpha_s(\mu_R)) \right]$$

- The $\overline{B}$ prefactor is of order α_s^2 at the Born level, and it includes NLO corrections of order α_s^3. Its scale variation must therefore be of order α_s^4.
  Therefore the relative scale variation $\delta\overline{B}/\overline{B}$ is of order α_s^2.
- On the other hand, the R_f term ($\mathbb{H}$ in MC@NLO) is of order α_s^3, and its scale variation is of order $\alpha_s^4 \Longrightarrow$ its relative scale variation is of order α_s.

Thus, the larger the contribution to the transverse momentum distribution coming from R_s (or $\mathbb{S}$ in MC@NLO) events, the smaller its relative scale dependence will be.

Scale dependence in gg→H

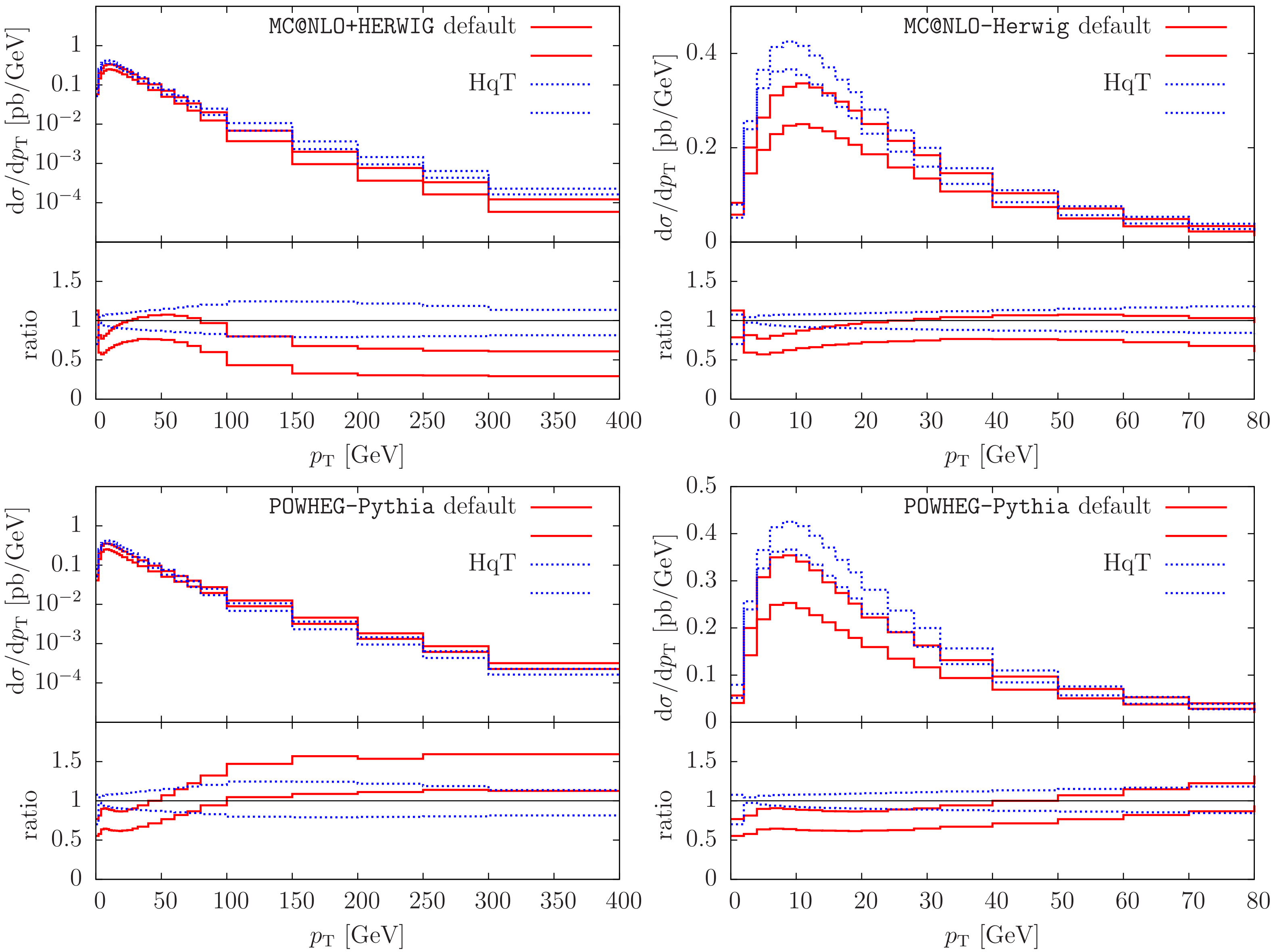
- $gg \to H$ at NLO+PS
- $m_{\mathrm{H}} = 120$ GeV
- $0.5 < \mu_{\mathrm{R}}/\mu_{\mathrm{F}} < 2$ around central reference scale μ
- Comparison with HqT [Catani, Grazzini et al.]: NNLL + NNLO. "Switched" result, with resummation scale $Q = m_{\mathrm{H}}/2$ and reference factorization and renormalization scale $\mu = m_{\mathrm{H}}$, as recommended by the authors
- MSTW2008NNLO central pdf for all the curves. This pdf set is needed by HqT. Used for all the other programs, since we want to focus on the differences that have to do with the calculation, rather than the pdf

The R_s and R_f terms are chosen such that

$$R_s = \frac{h^2}{p_T^2 + h^2} R, \qquad R_f = \frac{p_T^2}{p_T^2 + h^2} R, \qquad R = R_s + R_f$$

If $h \to 0$, the NLO prediction is recovered, but the Sudakov region is dangerously squeezed and distorted.

If $h \to \infty$, $R_s = R$ and $R_f \to 0$ and the whole real contribution enters the Sudakov form factor. This is the default POWHEG BOX setting.
The NLO K factor, $\overline{B}/B$ multiplies uniformly the whole transverse-momentum distribution



DEFAULT VALUES

POWHEG

$\mu = m_{\mathrm{H}}$

$h = \infty$

MC@NLO

$\mu = m_{\mathrm{T}} = \sqrt{m_{\mathrm{H}}^2 + p_T^2}$

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high p_T

$\frac{\text{POWHEG}}{\text{MC@NLO}} \approx 3 = 2 \times 1.6$

K fac ≈ 2

$(\alpha_s(m_{\mathrm{T}})/\alpha_s(m_{\mathrm{H}}))^3 \approx 1.6$

in the last bin

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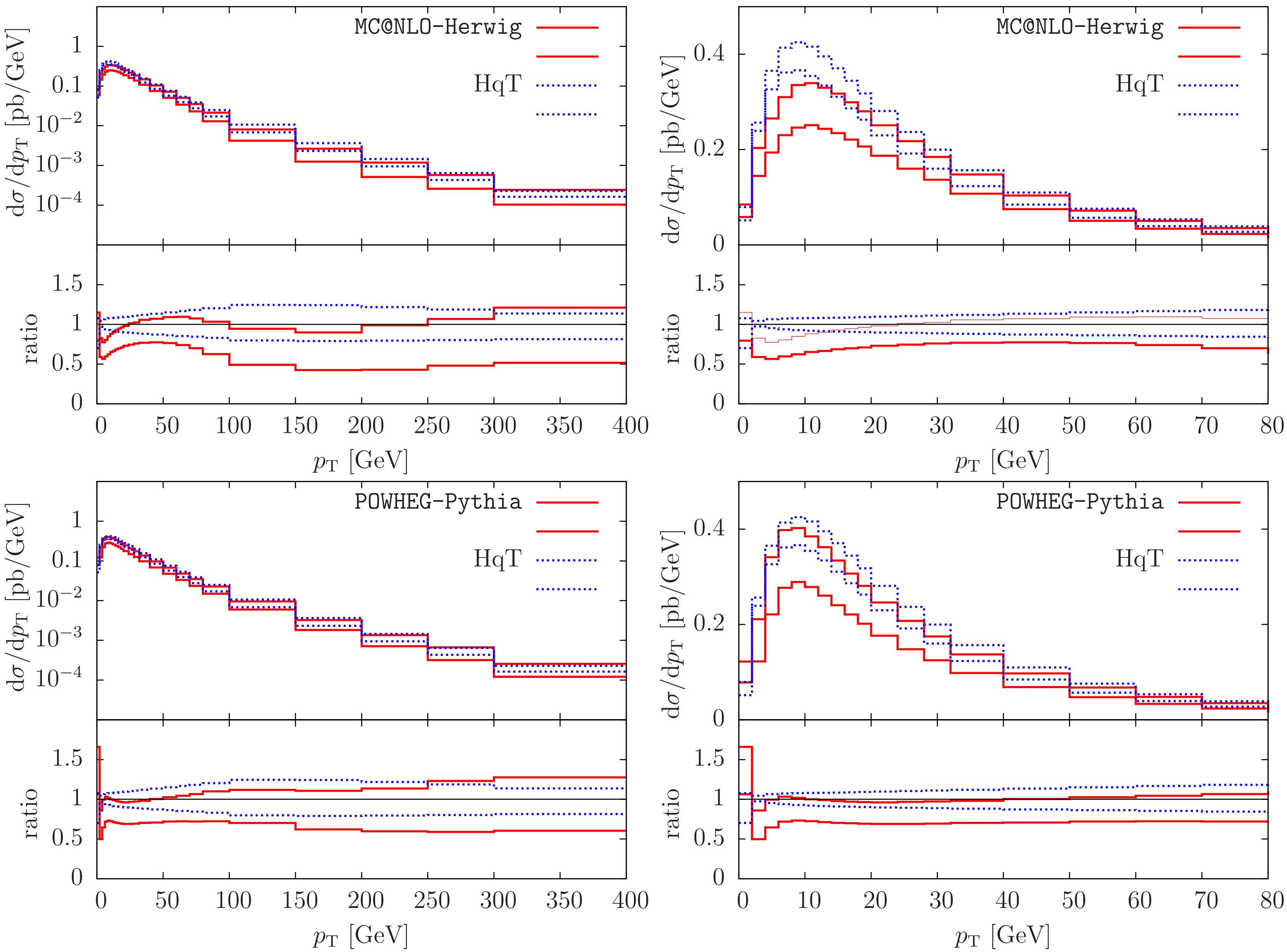
narrow band at small p_T

larger band at large p_T

[YRHXS2; Nason and Webber, 2012]



BEST VALUES

POWHEG

$\mu = m_{\mathrm{H}}$

$h = \mu/1.2$

MC@NLO

$\mu = m_{\mathrm{H}}$

$h = 100$ GeV, the large-p_T tail in POWHEG and MC@NLO are now very similar

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narrow band at small p_T

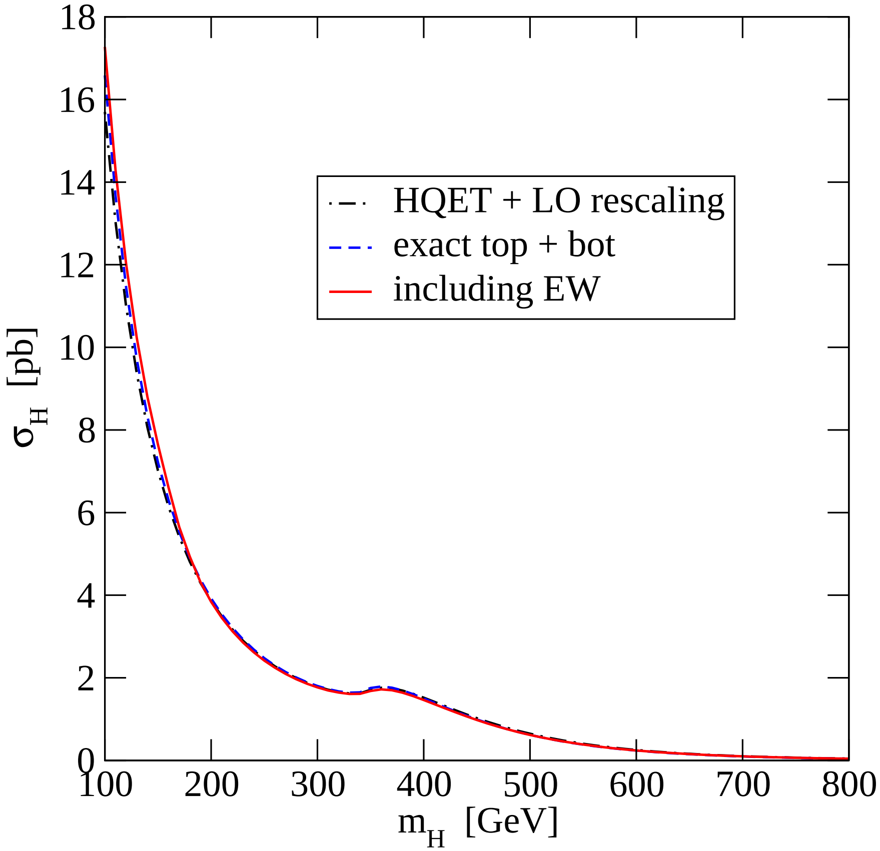
larger band at large p_T

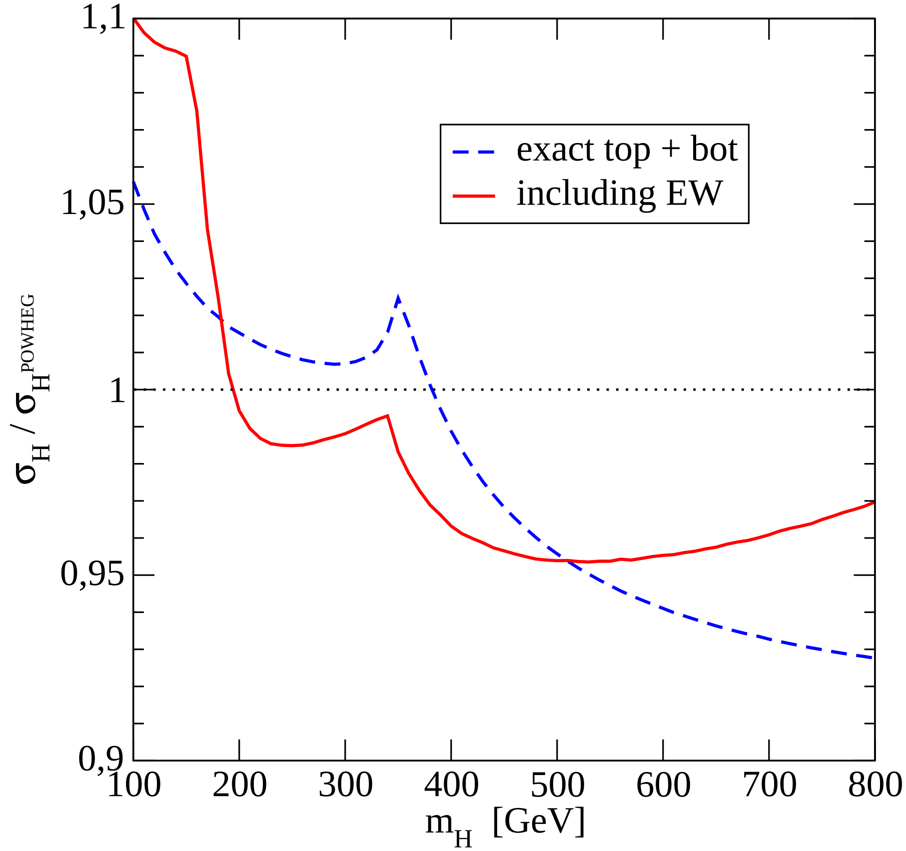
Questions: 1) What if HqT didn't exist?

2) What is the most "appropriate" scale in the high-p_T region?

Higgs boson with heavy quarks in the loop

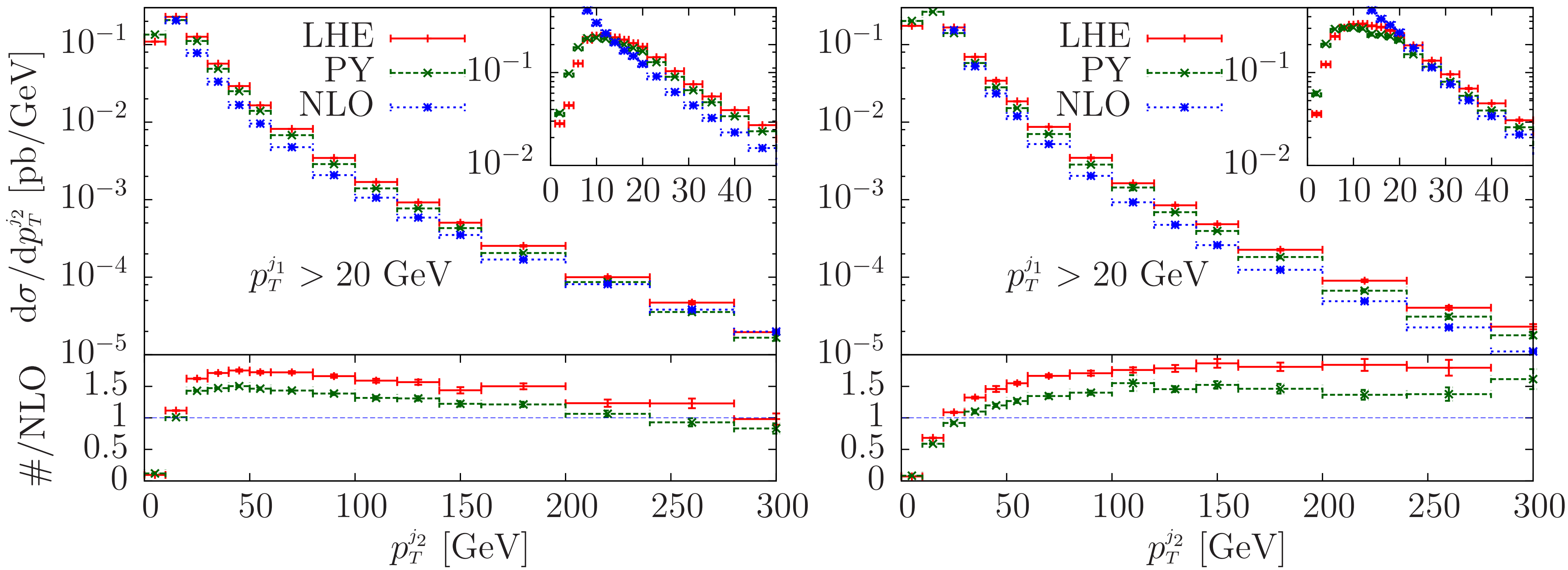
Done for SM and MSSM [Bagnaschi, Degrassi, Slavich, Vicini, 2011]. For the SM case, include exact m_t and m_b dependence, and two-loop EW corrections included as an overall (mass-dependent) global factor.





See Bagnaschi's talk

Higgs boson plus 1 jet production



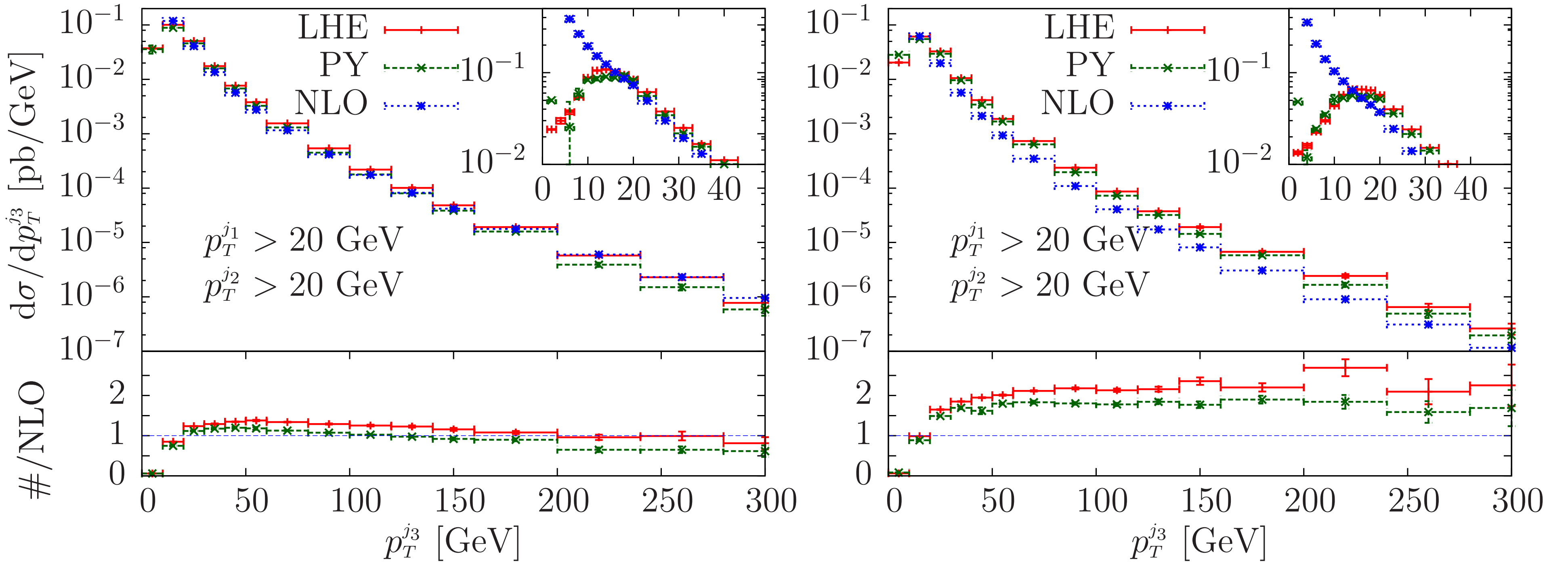
Left: $\mu_F = \mu_R = m_H$. Right: $\mu_F = \mu_R = p_T^{UB} = p_T$ of the underlying Born configuration

- Diverging NLO, Sudakov suppression in LHE
- The trend of the $\overline{B}_s/B$ factor very evident
- In LHE results, one power of α_s is evaluated at the p_T of the radiation

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

See also [Hoeche, Krauss, Schonherr and Siegert, 2011]

Higgs boson plus 2 jet production

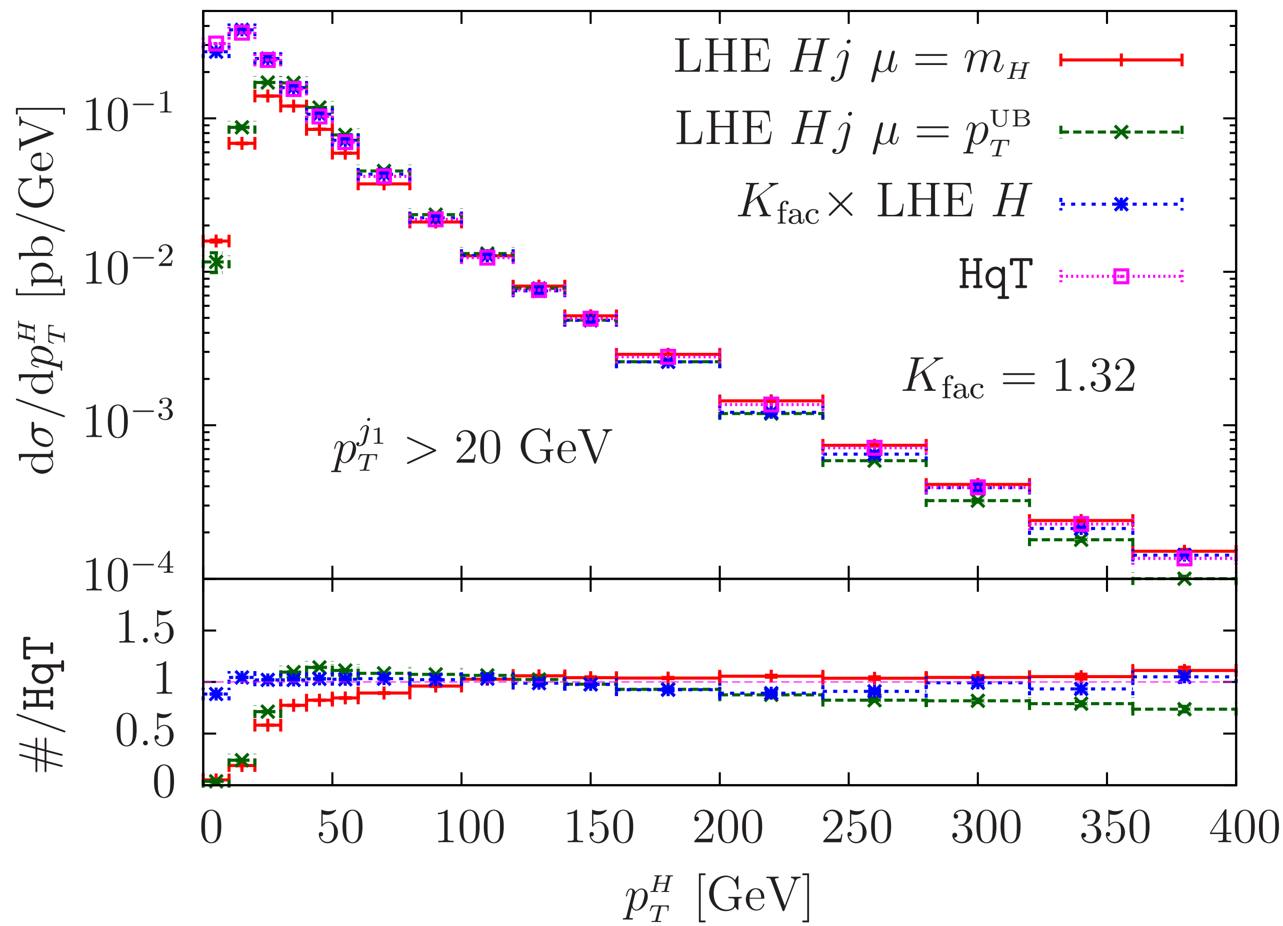


Left: $\mu_F = \mu_R = m_H$. Right: $\mu_F = \mu_R = \hat{H}_T = m_T^H + \sum_i p_{T\,i}$ where $m_T^H = \sqrt{m_H^2 + (p_T^H)^2}$ and $p_{T\,i}$ are the final-state parton transverse momenta in the underlying-Born kinematics.

- The trend of the $\overline{B}_s/B$ factor very evident
- K factor close to 1 for fixed scales
- These are two "extreme" scales. $\hat{H}_T$ too big at large p_T

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

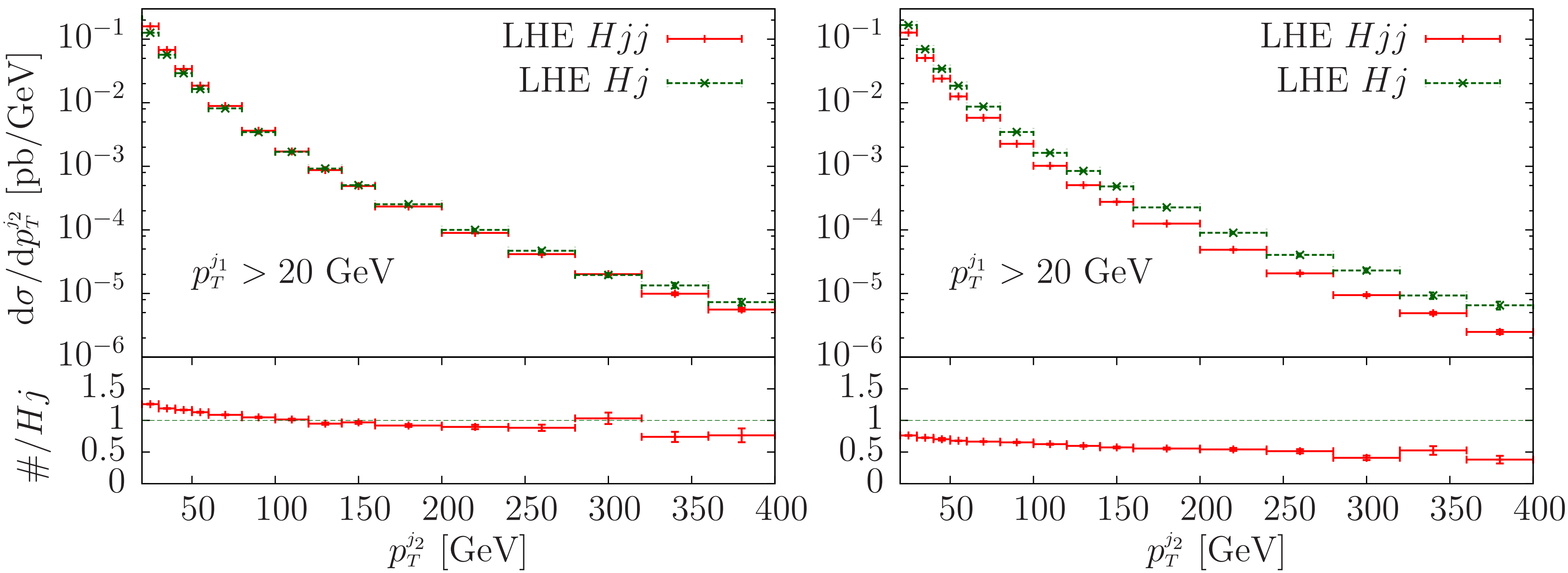
H and Hj comparison



- $\sigma_H^{\rm NLO} = 10.85$ pb, $\sigma_H^{\rm NNLO} = 14.35$ pb $\Longrightarrow K = 1.32$
- H generator: NLL accuracy in the low p_T region but only LO at high p_T
- Hj generator: NLO accuracy only in the high p_T region. No Sudakov resummation.

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

Hj and Hjj comparison



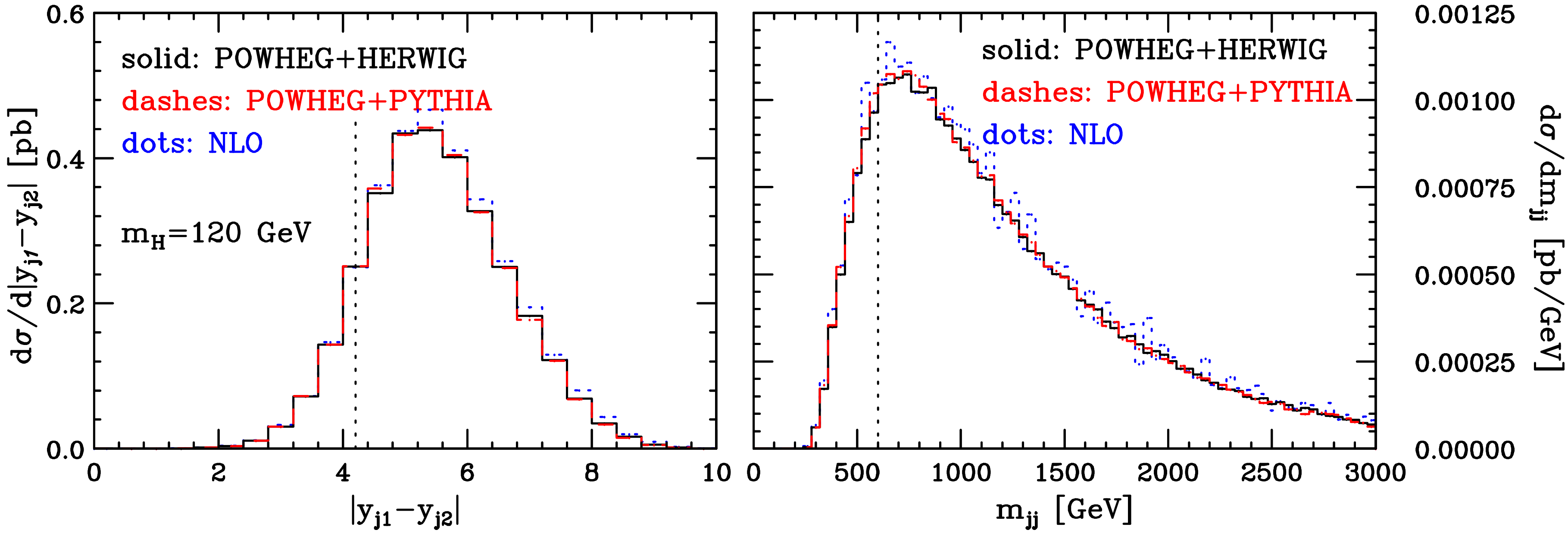
Left: $\mu_F = \mu_R = m_{\mathrm{H}}$. Right: $\mu_F = \mu_R = p_T^{\mathrm{UB}}$ for Hj and $\mu_F = \mu_R = \hat{H}_T$ for Hjj

- Hj generator: NLL accuracy in the low p_T region but only LO at high p_T
- Hjj generator: NLO accuracy only in the high p_T region. No Sudakov resummation.

Work in progress to merge the H, Hj and Hjj samples

[Campbell, Ellis, Frederix, Nason, Williams, C.O., 2012]

Higgs in vector-boson fusion

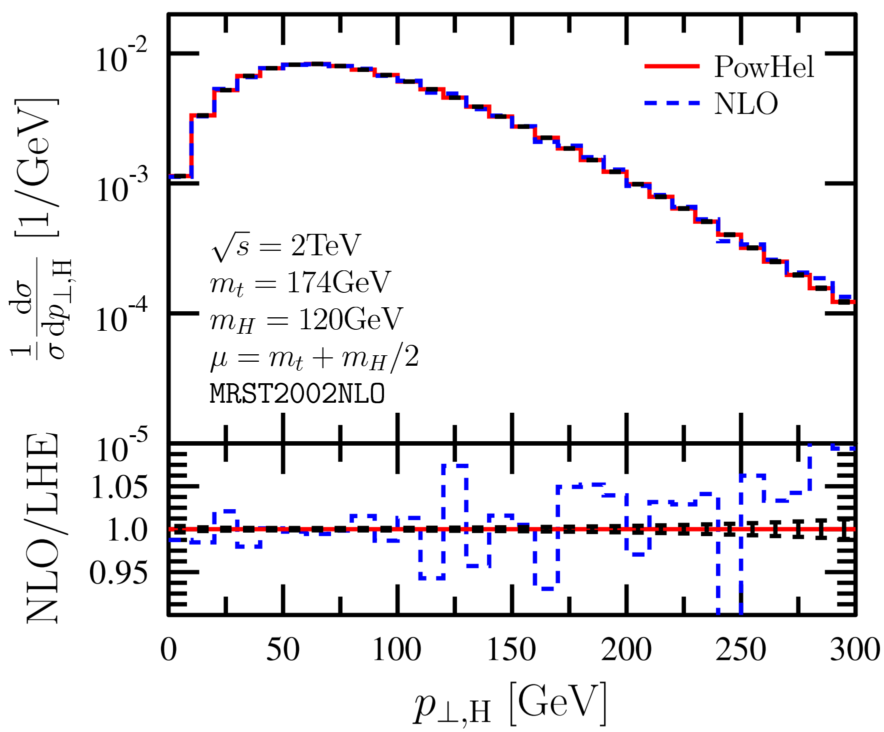


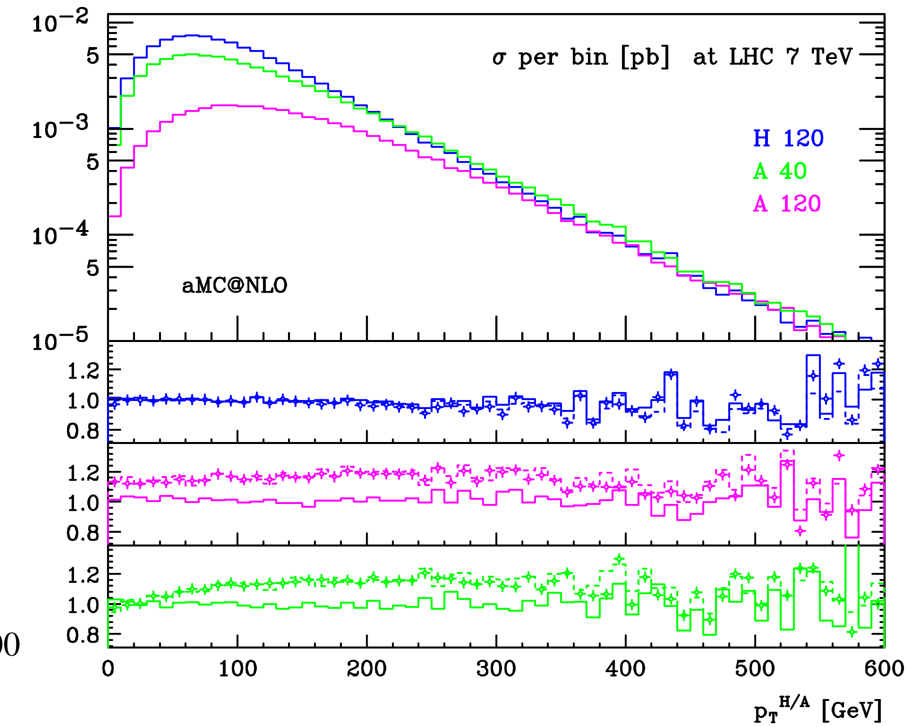
- Only t-channel vector-boson exchange diagrams: built having in mind VBF cuts [Nason and C.O, 2010].
- Nevertheless, the cross section is finite even with no cuts.

This is what has been used in the Yellow Report Higgs Cross Section 1 and 2.

See also POWHEG+HERWIG++ [D'Errico, Richardson, 2011].

$t\bar{t}H$

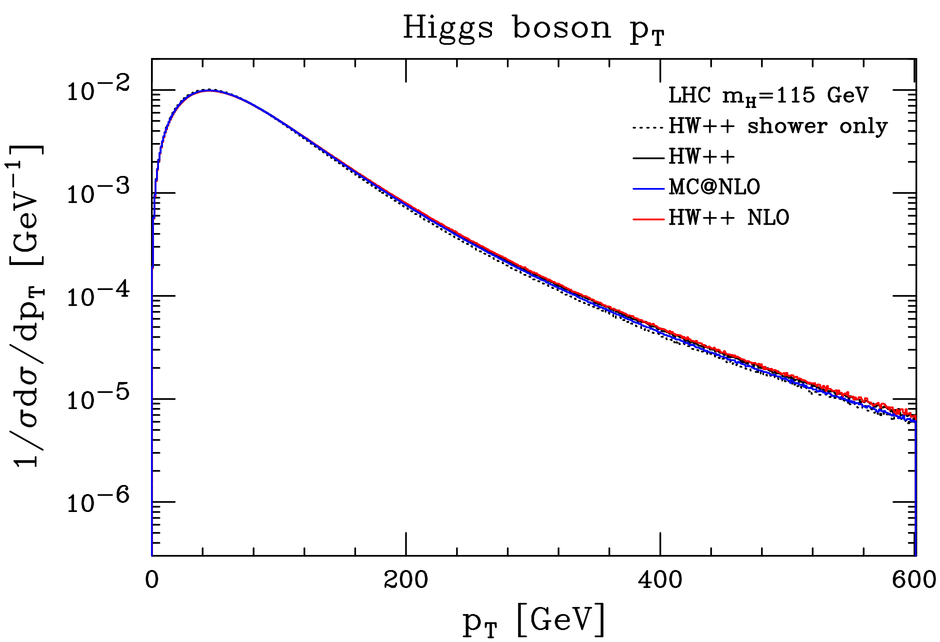


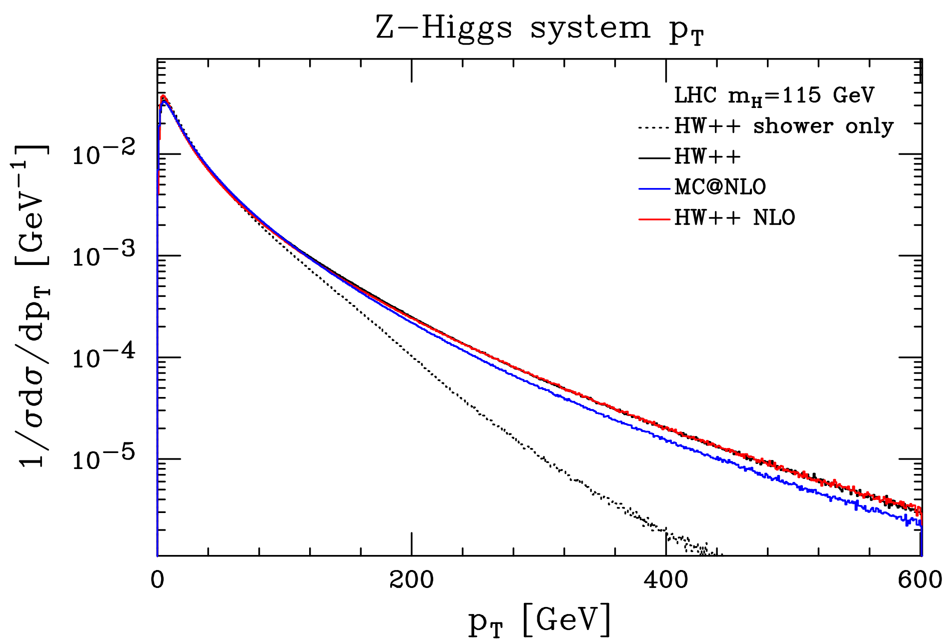


- POWHEG BOX+HELAC [Garzelli, Kardos, Papadopoulos and Trocsanyi, 2011]
- aMC@NLO: scalar and pseudoscalar Higgs boson [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, 2011]

WH/ZH

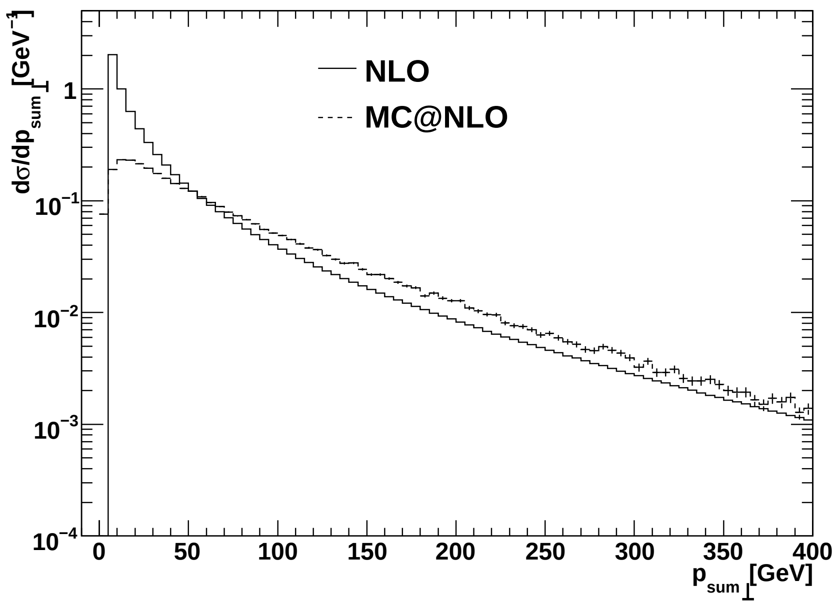
- POWHEG+HERWIG++ [Hamilton, Richardson, Tully, 2009]
- MC@NLO [Frixione, Webber]

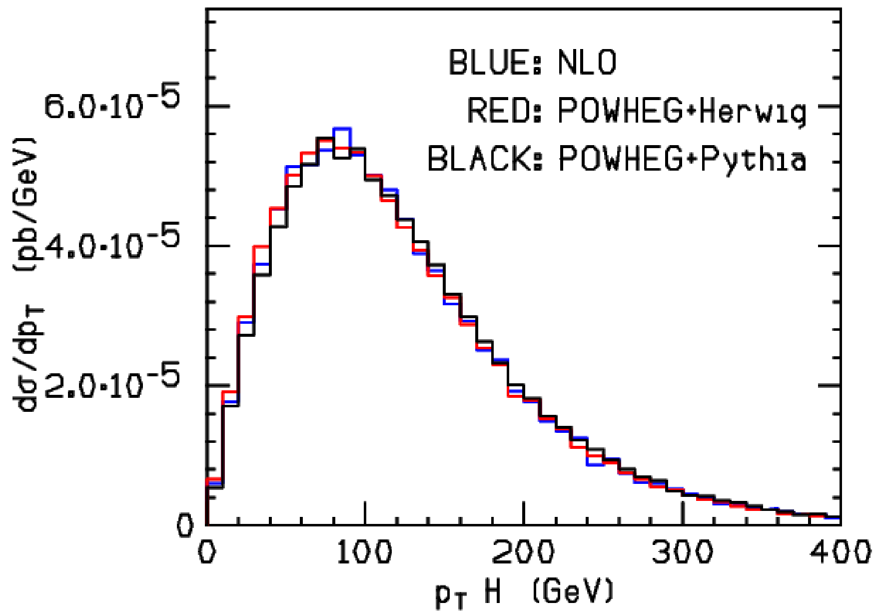




$tH^{\pm}$

MC@NLO [Weydert, Frixione, Herquet, Klasen, Laenen, Plehn, Stavenga, White, 2009]
POWHEG BOX [Klasen, Kovarik, Nason, Weydert, 2012]

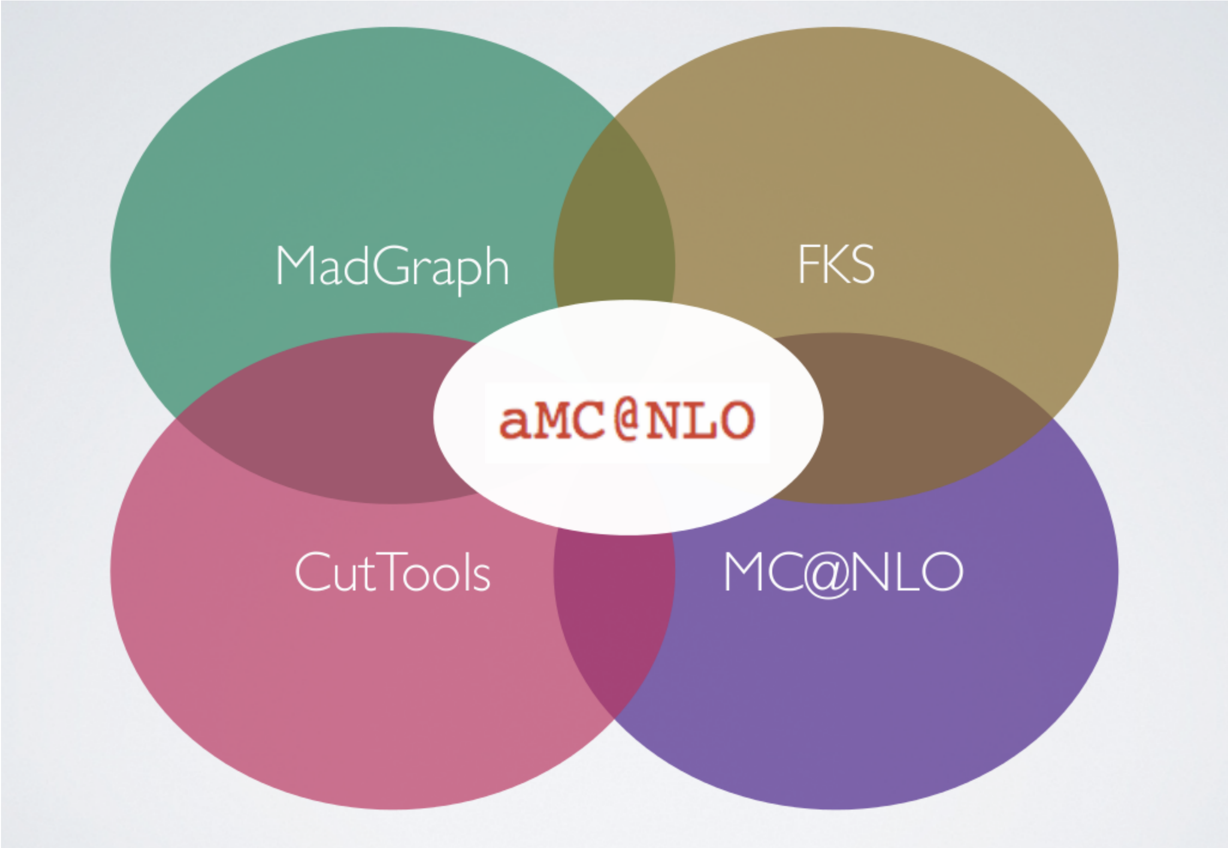




Left: transverse momentum of $H^- t$ at LHC@14, HDM-II, $m_{H^-} = 300$ GeV, $\tan\beta = 30$
Right: transverse momentum of H^- at LHC@7, HDM-II, $m_H = 300$ GeV, $\tan\beta = 10$

Towards total automation

aMC@NLO = MadGraph+CutTools+FKS+MC@NLO
[Hirshi, Frederix, Frixione, Maltoni, Garzelli, Pittau, Torrielli]



http://amcatnlo.cern.ch

Towards total automation

Two useful interfaces exists now in the POWHEG BOX:

- ✓ an interface to MadGraph 4, built in collaboration with Rikkert Frederix, that automatically builds the codes to compute the Born, Born color- and spin-correlated amplitudes, the real amplitude and the Born color structure in the limit of large number of colors. This is done just once and for all, when a new process is implemented in the POWHEG BOX.

- ✓ an interface to GoSam [Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano], built in collaboration with Gionata Luisoni, that writes automatically the code for the computation of the finite part of the virtual contributions.

`http://powhegbox.mib.infn.it`

Conclusions

- There are several NLO+PS programs that describe the production of a Higgs boson in different channels.
- Although they formally all agree at NLO, NNLO terms can be large for processes with large K factors.
- Differences among each other are well understood and have been studied in the past few years.
- A great effort is being done towards the fully automation of the generation of the NLO+PS codes.