Particle Physics: The Standard Model

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1. The Standard Model of Particle Physics: Overview
   - Kinematics
   - s channel and t channel
   - Cross section and total width
   - Description of an unstable particle

The Course Philosophy

- Emphasis of the course is on the phenomenology
- We will discuss experimental aspects but more important is the interpretation of measurements
- In an ideal world: construct theory and apply it
- Real (course) world: theory and application in parallel
- Build the theory knowledge to put the experiments into perspective
- Natural units: $\hbar = c = 1 \rightarrow \hbar c = 197.3 \mathrm{MeV} \cdot \mathrm{fm}$

Matter $=$ fermions (Spin-$\frac{1}{2}$ particles):

- Electrons with two spin orientations: L and R
- Neutrinos (L)
- Quarks L and R (proton$=$uud, neutron$=$udd)
- Three families $=$ heavier copies of the first family

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Interactions $=$ bosons (Spin–0 or –1 particles):

- Electromagnetism: Spin–1 massless
- Strong interaction (p=uud): Spin–1 massless
- Weak interaction: Spin–1 massive
- Masses: Spin–0 massive

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Properties: Electric Charge

- Fractional charges not observed in nature
- Strong interaction: uud, udd

$$\begin{array}{cccc}
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Properties: Color charge

- Sum of colors (RGB) white
- R$+$G$+$B$=$ (qqq $=$baryon)
- Color$+$anti-color$=$ White ($\mathrm{q\,\bar{q}}$$=$meson)
- Gluon carries color$+$anti-color
- 8 different gluons (not 9)

$$
\begin{array}{cccc}
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Rule of thumb for interactions

Interaction	Carrier	Relative strength
Gravitation	Graviton (G)	10^{-40}
Weak	Weak Bosons ($W^{\pm}$,Z°)	10^{-7}
Electromagnetic	Photon (γ)	10^{-2}
Strong	Gluon (g)	1

- Forget about Gravitation in particle physics problems
- The course and problem solving sessions will lead us to understand how the model describes the interactions and their strength.

$\mathbf{a} = (E_a, \vec{p}_a) = (p_0, p_1, p_2, p_3)$
$E_a \cdot E_a - \vec{p}_a \cdot \vec{p}_a = m_a^2$
$g^{\mu\nu} p_\mu p_\nu = m_a^2$

$g^{\mu\mu} = (1, -1, -1, -1)$
for $\mu \neq \nu : g^{\mu\nu} = 0$

Conservation of E and $\vec{p}$

$$\mathbf{a} + \mathbf{b} = \mathbf{c} + \mathbf{d}$$

therefore

$$\mathbf{a} - \mathbf{c} = \mathbf{d} - \mathbf{b}$$

Mandelstam Variables

$$a + b \rightarrow c + d$$

$$s = (\mathbf{a} + \mathbf{b})^2$$

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Theorem

$$s+t+u$$
$$= m_a^2+m_b^2+m_c^2+m_d^2$$
$$= 0$$

- High energy approx ($E \gg m \sim 0, E = |\vec{p}|$)
- CM-frame ($\vec{p}_a = -\vec{p}_b$)
- $\rightarrow E_a = E_b = E_c = E_d = \sqrt(s)/2$

Proof.

$$\begin{aligned}
s &= \mathbf{a}^2 + \mathbf{b}^2 + 2\cdot\mathbf{a}\cdot\mathbf{b} \\
&= m_a^2 + m_b^2 + 2(E_a\cdot E_b - \vec{p}_a\cdot\vec{p}_b) \\
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s &= \mathbf{a}^2 + \mathbf{b}^2 + 2 \cdot \mathbf{a} \cdot \mathbf{b} \\
&= m_a^2 + m_b^2 + 2(E_a \cdot E_b - \vec{p}_a \cdot \vec{p}_b) \\
&= 2(E_a \cdot E_b - \vec{p}_a \cdot \vec{p}_b) \\
&= 2(E_a \cdot E_a + \vec{p}_a \cdot \vec{p}_a) \\
&= 2(E_a^2 + E_a^2) \\
&= 4E_a^2 \\
t &= -2(E_a \cdot E_c - \vec{p}_a \cdot \vec{p}_c) \\
u &= -2(E_a \cdot E_d - \vec{p}_a \cdot \vec{p}_d) \\
&= -2(E_a \cdot E_c + \vec{p}_a \cdot \vec{p}_c)
\end{aligned}
$$

Theorem

$$
\begin{aligned}
& s+t+u \\
= \quad & m_a^2+m_b^2+m_c^2+m_d^2 \\
= \quad & 0
\end{aligned}
$$

- High energy approx ($E \gg m \sim 0$, $E = |\vec{p}|$)
- CM-frame ($\vec{p}_a = -\vec{p}_b$)
- $\rightarrow E_a = E_b = E_c = E_d = \sqrt{(}s)/2$

Proof.

$$
\begin{aligned}
s &= \mathbf{a}^2+\mathbf{b}^2+2\cdot\mathbf{a}\cdot\mathbf{b} \\
&= m_a^2+m_b^2+2(E_a\cdot E_b-\vec{p}_a\cdot\vec{p}_b) \\
&= 2(E_a\cdot E_b-\vec{p}_a\cdot\vec{p}_b) \\
&= 2(E_a\cdot E_a+\vec{p}_a\cdot\vec{p}_a) \\
&= 2(E_a^2+E_a^2) \\
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&= -2(E_a\cdot E_c+\vec{p}_a\cdot\vec{p}_c)
\end{aligned}
$$

$\square$

Proof.

$$
\begin{aligned}
t + u &= -2(2 \cdot E_a \cdot E_c) \\
&= -2(2 \cdot E_a \cdot E_a) \\
s + t + u &= 4 \cdot E_a \cdot E_a - 4 \cdot E_a \cdot E_a \\
&= 0
\end{aligned}
$$

□

2 particle reaction → 2 independent variables!

Proof.

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2 particle reaction → 2 independent variables!

Crossing relationship

$a + b \to c + d$	$a + \bar{c} \to \bar{b} + d$
$s = (\mathbf{a} + \mathbf{b})^2$	$s' = (\mathbf{a} + \bar{\mathbf{c}})^2 = (\mathbf{a} - \mathbf{c})^2 = t$
$t = (\mathbf{a} - \mathbf{c})^2$	$t' = (\mathbf{a} - \bar{\mathbf{b}})^2 = (\mathbf{a} + \mathbf{b})^2 = s$
$u = (\mathbf{a} - \mathbf{d})^2$	$u' = (\mathbf{a} - \mathbf{d})^2 = (\mathbf{a} - \mathbf{d})^2 = u$

- Calculate a process as function of s,t,u
- Derive crossed process by $s \to t$, $t \to s$, $u \to u$
- We can express one process in the kinematic variables of another process (Xcheck)
- Rigorous derivation $s \to -t$

Kinematics and Crossing and the - in Problem Solving

Crossing relationship

$a+b \to c+d$	$a+\bar{c} \to \bar{b}+d$		
$s=(\mathbf{a}+\mathbf{b})^2$	$s'=(\mathbf{a}+\bar{\mathbf{c}})^2$	$=(\mathbf{a}-\mathbf{c})^2$	$=t$
$t=(\mathbf{a}-\mathbf{c})^2$	$t'=(\mathbf{a}-\bar{\mathbf{b}})^2$	$=(\mathbf{a}+\mathbf{b})^2$	$=s$
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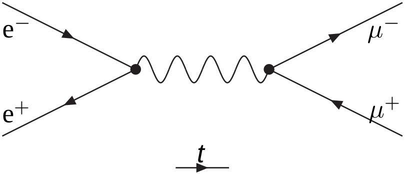
Kinematics and Crossing and the - in Problem Solving

s–channel: annihiliation

$$e^+ + e^- \to \gamma \to \mu^+\mu^-$$

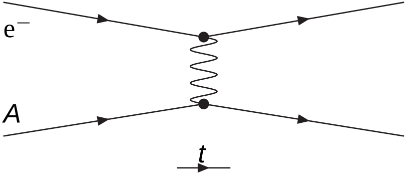


$$\begin{aligned} \mathbf{q}_\gamma &= \mathbf{p}_{e^-} + \mathbf{p}_{e^+} \\ s &= \mathbf{q}_\gamma^2 \\ (CM) &= (E_{e^-} + E_{e^+})^2 \\ &> 0 \end{aligned}$$

the photon is massive (virtual)
time-like

t–channel: scattering

$$e^- + A \to e^- + A$$



$$\begin{aligned} \mathbf{p}_{e_i^-} &= \mathbf{q}_\gamma + \mathbf{p}_{e_o^-} \\ t &= \mathbf{q}_\gamma^2 \\ &= m_e^2 + m_e^2 - 2 \cdot \mathbf{p}_{e_i^-} \cdot \mathbf{p}_{e_o^-} \\ &\approx -2(E_i E_o - |\vec{p}_i||\vec{p}_o| \cos\theta) \\ &\approx -2E_i E_o (1 - \cos\theta) \\ &\leq 0 \end{aligned}$$

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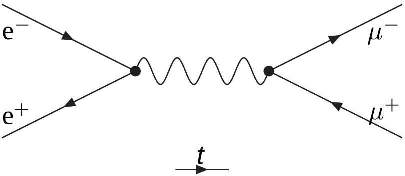

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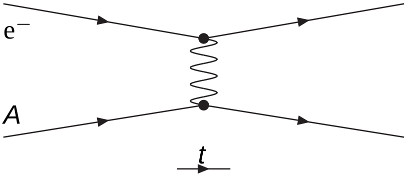

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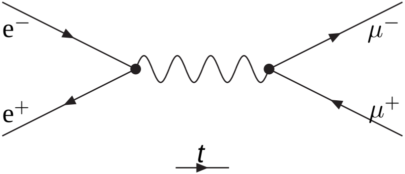

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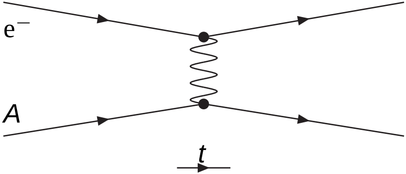

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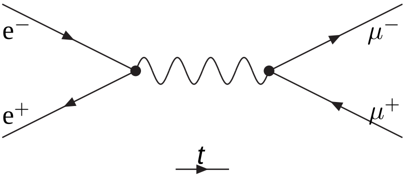

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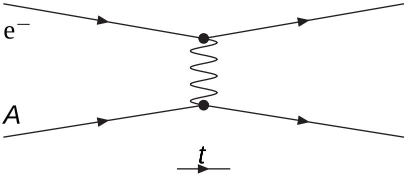

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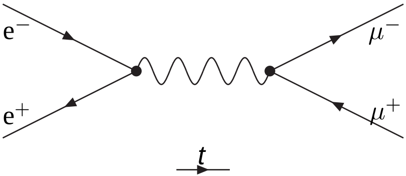

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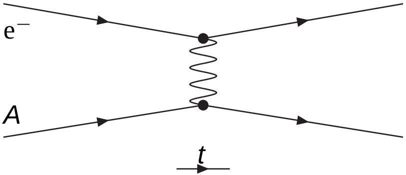

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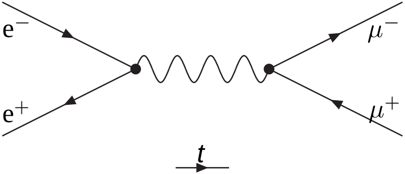

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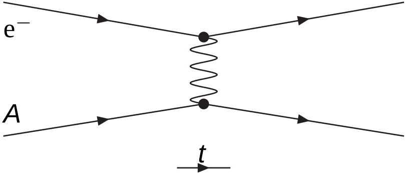

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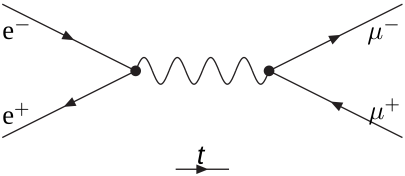

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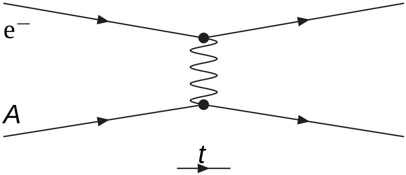

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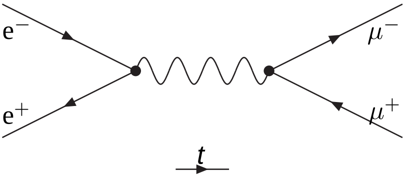

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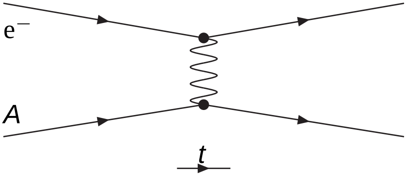

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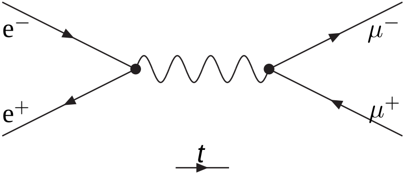

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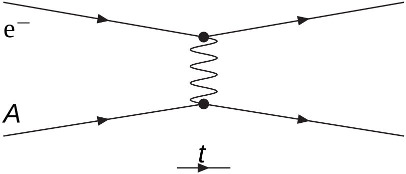

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\mathbf{p}_{\mathrm{e}_i^-} &= \mathbf{q}_\gamma + \mathbf{p}_{\mathrm{e}_o^-} \\
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&= m_e^2 + m_e^2 - 2 \cdot \mathbf{p}_{\mathrm{e}_i^-} \cdot \mathbf{p}_{\mathrm{e}_o^-} \\
&\approx -2(E_i E_o - |\vec{p}_i||\vec{p}_o| \cos\theta) \\
&\approx -2E_i E_o (1 - \cos\theta) \\
&\leq 0
\end{aligned}$$

the photon is massive space-like

Cross Section

- The cross section σ is the ratio of the transition rate and the flux of incoming particles.
- Its unit is cm^2
- $1\mathrm{b} = 10^{-24}\mathrm{cm}^2$ (puts barn in perspective, doesn't it?)

Two ingredients:

- the interaction tranforming initial state $|i\rangle$ to a final state $\langle f|$ of m particles with four-vectors $\mathbf{p}_i'$
- kinematics (including Lorentz-Invariant phase space element)

$$\mathrm{d}\sigma = \frac{1}{2S_{12}} \prod_{i=1}^{m} \frac{\mathrm{d}^3 p_i'}{(2\pi)^3 2E_i'^0} (2\pi)^4 \delta(\mathbf{p}_1' + \ldots + \mathbf{p}_m' - \mathbf{p}_1 - \mathbf{p}_2) |\mathcal{M}|^2$$

with $S_{12} = \sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}$

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Total Width or Decay Rate

- Total width is the inverse of the lifetime of the particle
- unit: energy, e.g., GeV.
- Closely related, but not identical to the cross section

$$\mathrm{d}\Gamma = \frac{1}{2E}\prod_{i=1}^{m}\frac{\mathrm{d}^3 p_i'}{(2\pi)^3 2E_i'^0}\delta(\mathbf{p_1'} + \ldots + \mathbf{p_m'} - \mathbf{p_1})|\mathcal{M}|^2$$

For the decay of an unpolarized particle of mass M into two particles (in the CM frame $\vec{p}_1' = -\vec{p}_2'$):

$$\mathrm{d}\Gamma = \frac{1}{32\pi^2}\frac{|\vec{p}_1'|}{M^2}|\mathcal{M}|^2\mathrm{d}\Omega$$

where Ω is the solid angle with $\mathrm{d}\Omega = \mathrm{d}\phi\mathrm{d}\cos\theta$

Cross section and total width for a final state with 2 particles

Cross section $2 \to 2$ reaction with four massless particles:

$$\mathrm{d}\sigma = \frac{1}{64\pi^2} \frac{|\mathcal{M}|^2}{s} \mathrm{d}\Omega$$

Width of a massive particle ($\sqrt{s} = M$) decaying to two massless particles in the final state $|\vec{p}_1'| = \sqrt{s}/2$:

$$\mathrm{d}\Gamma = \frac{1}{64\pi^2} \frac{|\mathcal{M}|^2}{\sqrt{s}} \mathrm{d}\Omega$$

Study of the phase space in Problem Solving with applications to 2-body and 3-body reactions.

- Particles: plane waves
  $\psi(\vec{x}, t) \sim \exp -im_0 t$
- $m_0 \rightarrow m_0 - i\Gamma/2$

$$\begin{aligned} N(t) &= N_0 \cdot \exp -t/\tau \\ \Gamma &= 1/\tau \end{aligned}$$

Fourrier transform to momentum space:

$$\begin{aligned} A &\sim \frac{1}{(m-m_0)+i\Gamma/2} \\ |A|^2 &\sim \frac{1}{(m-m_0)^2+\Gamma^2/4} \end{aligned}$$

Γ: full width half maximum
Similarity to classical mechanics: resonance

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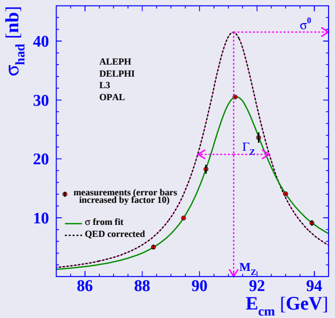
Example $e^- e^+ \to Z^\circ \to q\bar{q}$



lifetime too short to be measured directly: measure mass via decay products $q\bar{q}$
cross section measurement

- Particles: plane waves
  $\psi(\vec{x}, t) \sim \exp -im_0 t$
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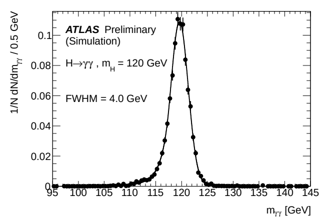
Example $pp \to H \to \gamma\gamma$



Beware: the width here has nothing to do with $\Gamma \sim 5\mathrm{MeV}$!
The experimental resolution is the origin (error propagation):

$$\begin{aligned} m_H &= \sqrt{(\mathbf{p_1}^\gamma + \mathbf{p_2}^\gamma)^2} \\ &= \sqrt{2E_1^\gamma E_2^\gamma (1-\cos\theta)} \end{aligned}$$

Suppose that we have two (and exactly two) possible decays for the particle a:

$$\begin{aligned} a &\rightarrow b + c \\ a &\rightarrow d + e \end{aligned}$$

then:

$$\Gamma = \Gamma_{bc} + \Gamma_{de}$$

If a particle of a given mass can decay to more final states than another one with the same mass, it will have a shorter lifetime

Branching ratio

$\mathcal{B}(\mathrm{a} \rightarrow \mathrm{b} + \mathrm{c}) = \Gamma_{bc}/\Gamma$
The branching ratio: Of N decays of particle a, a fraction $\mathcal{B}$ will be the final state with the particles b and c. Γ_{bc} is a partial width of particle a.

Remember: for the calculation Γ ALL final states (partial widths) have to be considered.

What do we know?

- Names of particles
- Kinematic description of interactions
- Defintion of cross section and decay width

What is next?

- Electromagnetic interactions (QED)
- Strong interaction (QCD)
- Electroweak interactions