Particle Physics: The Standard Model

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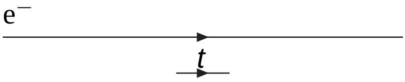
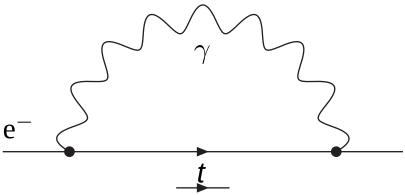
The History

- Introduction of particles ($\alpha\tau o\mu o\varsigma$)
- Particle-Wave dualism (deBroglie wave length)
- Particles are fields in a quantum field theory
- 1941: Stueckelberg proposes to interpret electron lines going back in time as positrons
- end of 1940s: Feynman, Tomonaga, Schwinger et al develop renormalization theory
- anomalous magnetic moment predicted (not today)

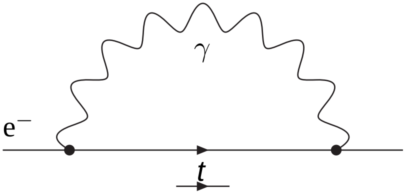
Quantum Field Theory in a nutshell



- Leading Order (LO) diagram is the simplest diagram
- The electron is on-shell ($\mathbf{p}^2 = m_e^2$), no interactions



- NLO (next-to-leading order) diagram
- Process not allowed in classical mechanics
- Heisenberg: $\Delta E \, \Delta t \geq 1 \rightarrow$ process allowed for reabsorption after $\Delta t \sim 1/\Delta E$



- Quantum mechanics: add all diagrams, but that would also include $N_\gamma = \infty$
- Each vertex is an interaction and each interaction has a strength ($|\mathcal{M}|^2 \sim \alpha = 1/137$)
- Perturbation theory with Sommerfeld convergence

- Construct the Lagrangian of Free Fields
- Introduce interactions via the minimal substitution scheme
- Derive Feynman rules ($\rightarrow$ courses by Adel Bilal, Pierre Binétruy, Pierre Fayet, Matteo Cacciari, Slava Ryshkov)
- Construct (ALL) Feynman diagrams of the process
- Apply Feynman rules

Some aspects are not part of these lectures, but will sketch the ideas

- Remember the particle zoo
- treat only the carrier of the interaction γ
- as well as the e

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ W^{\pm}, Z^{\circ} \\ H \end{matrix}$$

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$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

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$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\begin{matrix} \gamma \\ g \\ W^\pm, Z^\circ \\ H \end{matrix}$$

The photon

MAXWELL equations:

$$\begin{aligned} \partial_\mu F^{\mu\nu}(\mathbf{x}) &= j^\nu(\mathbf{x}) \\ \epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma}(\mathbf{x}) &= 0 \end{aligned}$$

with the photon field tensor:

$$F^{\mu\nu}(\mathbf{x}) = \partial^\mu A^\nu(\mathbf{x}) - \partial^\nu A^\mu(\mathbf{x})$$

Fermions

The DIRAC equation:

$$(i\gamma^\mu\partial_\mu - m)\psi(\mathbf{x}) = 0$$

leading to:

$$\bar\psi(\mathbf{x})(i\gamma^\mu\partial_\mu - m)\psi(\mathbf{x})$$

with $\bar\psi = \psi^\dagger\gamma^0 = \psi^{T^\star}\gamma^0$

The free Lagrangian ($\mathcal{L}_0$)

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}(\mathbf{x})F^{\mu\nu}(\mathbf{x}) + \bar\psi(\mathbf{x})(i\gamma^\mu\partial_\mu - m)\psi(\mathbf{x})$$

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Minimal Substitution

$$i\partial_\mu \to i\partial_\mu + eA_\mu(\mathbf{x})$$

$$\begin{aligned} & \bar{\psi}(\mathbf{x})\gamma^\mu i\partial_\mu\psi(\mathbf{x}) \\ \to\ & \bar{\psi}(\mathbf{x})\gamma^\mu(i\partial_\mu + eA_\mu(\mathbf{x}))\psi(\mathbf{x}) \\ =\ & \bar{\psi}(\mathbf{x})\gamma^\mu i\partial_\mu\psi(\mathbf{x}) + e\bar{\psi}(\mathbf{x})\gamma^\mu A_\mu(\mathbf{x})\psi(\mathbf{x}) \end{aligned}$$

leads to a coupling between photon and fermion fields:

Interaction Lagrangian $\mathcal{L}'$

$$\mathcal{L}' = -j^\mu A_\mu = e\bar{\psi}(\mathbf{x})\gamma^\mu A_\mu(\mathbf{x})\psi(\mathbf{x})$$

the negative sign for $j^\mu = -e\bar{\psi}(\mathbf{x})\gamma^\mu\psi(\mathbf{x})$

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Gauge Invariance

Principle

Invariance of the Lagrangian under local $U(1)$ transformations
or: why should physics at the Elysée be different at the ENS?

$$\begin{array}{lcl} A_\mu & \rightarrow & A_\mu + \partial_\mu \Lambda(\mathbf{x}) \\ \psi(\mathbf{x}) & \rightarrow & \exp\left(ie\Lambda(\mathbf{x})\right)\psi(\mathbf{x}) \end{array}$$

$$\mathcal{L}_0 + \mathcal{L}' = \mathcal{L} \rightarrow \mathcal{L}$$

Local gauge invariance under a $U(1)$ gauge symmetry (1929 Weyl)
if $\Lambda \neq f(\mathbf{x})$ it is a global $U(1)$ symmetry.

$U(1)$ Gauge invariance Photon field:

Proof.

$$
\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\
&= \partial_\mu (A_\nu + \partial_\nu \Lambda) - \partial_\nu (A_\mu + \partial_\mu \Lambda) \\
&= \partial_\mu A_\nu - \partial_\nu A_\mu + \partial_\mu \partial_\nu \Lambda - \partial_\nu \partial_\mu \Lambda \quad \partial_\mu \partial_\nu = \partial_\nu \partial_\mu \\
&= \partial_\mu A_\nu - \partial_\nu A_\mu \\
&= F_{\mu\nu}
\end{aligned}
$$

□

Photon field ok

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□

Photon field ok

Fermion field

Proof.

$$
\begin{aligned}
& \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \\
\rightarrow\quad & \psi^\dagger \exp(-ie\Lambda)\gamma^0(i\gamma^\mu\partial_\mu - m)(\psi\exp(ie\Lambda)) \\
=\quad & \exp(-ie\Lambda)\bar{\psi}(i\gamma^\mu\partial_\mu - m)(\psi\exp(ie\Lambda)) \\
=\quad & \exp(-ie\Lambda)\bar{\psi}i\gamma^\mu(\partial_\mu\psi)\exp(ie\Lambda) \\
+\quad & \exp(-ie\Lambda)\bar{\psi}i\gamma^\mu\psi\partial_\mu\exp(ie\Lambda)) \\
+\quad & \exp(-ie\Lambda)\bar{\psi}(-m)\psi\exp(ie\Lambda) \\
=\quad & \bar{\psi}i\gamma^\mu(\partial_\mu\psi) \\
+\quad & \bar{\psi}(-m)\psi \\
=\quad & \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - e\bar{\psi}\gamma^\mu(\partial_\mu\Lambda)\psi
\end{aligned}
$$

Fermion field

Proof.

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+ \; & \exp(-ie\Lambda)\bar{\psi}i\gamma^\mu\psi\partial_\mu\exp(ie\Lambda)) \\
+ \; & \exp(-ie\Lambda)\bar{\psi}(-m)\psi\exp(ie\Lambda) \\
= \; & \bar{\psi}i\gamma^\mu(\partial_\mu\psi) \\
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Fermion field

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= \quad & \exp(-ie\Lambda)\bar\psi(i\gamma^\mu\partial_\mu - m)(\psi\exp(ie\Lambda)) \\
= \quad & \exp(-ie\Lambda)\bar\psi i\gamma^\mu(\partial_\mu\psi)\exp(ie\Lambda) \\
+ \quad & \exp(-ie\Lambda)\bar\psi i\gamma^\mu\psi\partial_\mu\exp(ie\Lambda)) \\
+ \quad & \exp(-ie\Lambda)\bar\psi(-m)\psi\exp(ie\Lambda) \\
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=\ & \exp(-ie\Lambda)\bar{\psi}(i\gamma^\mu\partial_\mu - m)(\psi \exp(ie\Lambda)) \\
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+\ & \exp(-ie\Lambda)\bar{\psi} i\gamma^\mu \psi \partial_\mu \exp(ie\Lambda)) \\
+\ & \exp(-ie\Lambda)\bar{\psi}(-m)\psi \exp(ie\Lambda) \\
=\ & \bar{\psi} i\gamma^\mu(\partial_\mu\psi) \\
+\ & \bar{\psi}(-m)\psi \\
=\ & \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - e\bar{\psi}\gamma^\mu(\partial_\mu\Lambda)\psi
\end{aligned}
$$

Fermion field

Proof.

$$\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

$$\rightarrow \psi^\dagger \exp(-ie\Lambda)\gamma^0(i\gamma^\mu\partial_\mu - m)(\psi \exp(ie\Lambda))$$

$$= \exp(-ie\Lambda)\bar{\psi}(i\gamma^\mu\partial_\mu - m)(\psi \exp(ie\Lambda))$$

$$= \exp(-ie\Lambda)\bar{\psi}i\gamma^\mu(\partial_\mu\psi)\exp(ie\Lambda)$$

$$+ \exp(-ie\Lambda)\bar{\psi}i\gamma^\mu\psi\partial_\mu \exp(ie\Lambda))$$

$$+ \exp(-ie\Lambda)\bar{\psi}(-m)\psi \exp(ie\Lambda)$$

$$= \bar{\psi}i\gamma^\mu(\partial_\mu\psi)$$

$$+ \bar{\psi}(-m)\psi$$

$$= \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - e\bar{\psi}\gamma^\mu(\partial_\mu\Lambda)\psi$$

Fermion field

Proof.

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\begin{aligned}
& \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \\
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=\ & \bar{\psi}i\gamma^\mu(\partial_\mu\psi) + \exp(-ie\Lambda)\bar{\psi}i\gamma^\mu\psi ie\partial_\mu\Lambda\exp(ie\Lambda) \\
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Fermion field

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=\ & \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - e\bar{\psi}\gamma^\mu(\partial_\mu\Lambda)\psi
\end{aligned}
$$

Interaction

Proof.

$$
\begin{aligned}
& e\bar{\psi}\gamma^\mu A_\mu \psi(\mathbf{x}) \\
= \; & e \exp(-ie\Lambda)\bar{\psi}\gamma^\mu(A_\mu + \partial_\mu\Lambda)\psi \exp(ie\Lambda) \\
= \; & e\bar{\psi}\gamma^\mu(A_\mu + \partial_\mu\Lambda)\psi \\
= \; & e\bar{\psi}\gamma^\mu A_\mu\psi + e\bar{\psi}\gamma^\mu(\partial_\mu\Lambda)\psi
\end{aligned}
$$

□

- Interaction term combined with fermion field ($-ie\bar{\psi}\gamma^\mu\partial_\mu\Lambda\psi$) ok
- gauge invariance of the fermion field cries for the introduction of a gauge boson!

Interaction

Proof.

$$
\begin{aligned}
& e\bar{\psi}\gamma^{\mu}A_{\mu}\psi(\mathbf{x}) \\
= \;& e \exp(-ie\Lambda)\bar{\psi}\gamma^{\mu}(A_{\mu} + \partial_{\mu}\Lambda)\psi \exp(ie\Lambda) \\
= \;& e\bar{\psi}\gamma^{\mu}(A_{\mu} + \partial_{\mu}\Lambda)\psi \\
= \;& e\bar{\psi}\gamma^{\mu}A_{\mu}\psi + e\bar{\psi}\gamma^{\mu}(\partial_{\mu}\Lambda)\psi
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& e\bar{\psi}\gamma^{\mu}A_{\mu}\psi(\mathbf{x}) \\
= \;& e\exp(-ie\Lambda)\bar{\psi}\gamma^{\mu}(A_{\mu}+\partial_{\mu}\Lambda)\psi\exp(ie\Lambda) \\
= \;& e\bar{\psi}\gamma^{\mu}(A_{\mu}+\partial_{\mu}\Lambda)\psi \\
= \;& e\bar{\psi}\gamma^{\mu}A_{\mu}\psi + e\bar{\psi}\gamma^{\mu}(\partial_{\mu}\Lambda)\psi
\end{aligned}
$$

□

- Interaction term combined with fermion field $(-ie\bar{\psi}\gamma^{\mu}\partial_{\mu}\Lambda\psi)$ ok
- gauge invariance of the fermion field cries for the introduction of a gauge boson!

Interaction

Proof.

$$\begin{aligned} & e\bar{\psi}\gamma^\mu A_\mu \psi(\mathbf{x}) \\ = \; & e \exp(-ie\Lambda)\bar{\psi}\gamma^\mu(A_\mu + \partial_\mu \Lambda)\psi \exp(ie\Lambda) \\ = \; & e\bar{\psi}\gamma^\mu(A_\mu + \partial_\mu \Lambda)\psi \\ = \; & e\bar{\psi}\gamma^\mu A_\mu \psi + e\bar{\psi}\gamma^\mu(\partial_\mu \Lambda)\psi \end{aligned}$$

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Interaction

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□

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External lines

initial state electron	$u(p)$
initial state positron	$\bar{v}(p)$
initial state photon	ϵ^μ
final state electron	$\bar{u}(p)$
final state positron	$v(p)$
final state photon	$\epsilon^{\mu\star}$

Internal lines and vertex

virtual photon	$\frac{-ig_{\mu\nu}}{k^2+i\epsilon}$
virtual electron	$i\frac{\not{p}+m}{p^2-m^2+i\epsilon}$
interaction (vertex)	$ie\gamma^\mu$

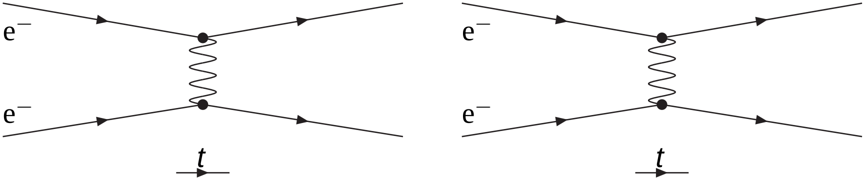
Matrix element

$$|\mathcal{M}|^2 = \sum_{fi}' T_{fi} T_{fi}^\dagger$$

Sum over final state, average over initial state

Moeller Scattering $e^-e^- \rightarrow e^-e^-$

- Simplest diagram with initial and final state of two electrons
- conserve electric charge and momentum at each vertex
- t channel only: $C(e^- + e^-) = -2e \neq C(\gamma) = 0$
- $\mathbf{p}$ conservation at each vertex $\rightarrow$ 2 diagrams
  $q_\gamma = \mathbf{p_2} - \mathbf{p_3} \neq \mathbf{p_2} - \mathbf{p_4}$



$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \rightarrow e^-(\mathbf{p_3})e^-(\mathbf{p_4})$ $e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \rightarrow e^-(\mathbf{p_4})e^-(\mathbf{p_3})$

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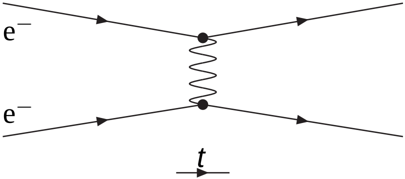
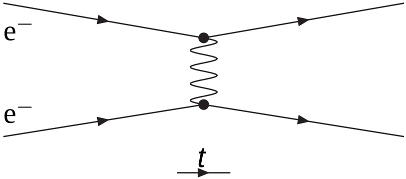


$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \rightarrow e^-(\mathbf{p_3})e^-(\mathbf{p_4})$



$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \rightarrow e^-(\mathbf{p_4})e^-(\mathbf{p_3})$

Moeller Scattering $e^- e^- \to e^- e^-$

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- conserve electric charge and momentum at each vertex
- t channel only: $C(e^- + e^-) = -2e \neq C(\gamma) = 0$
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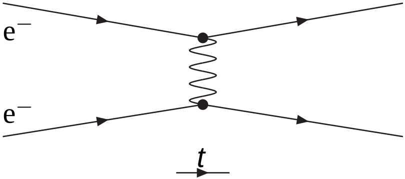
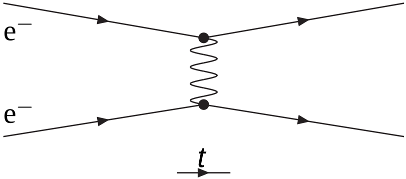


$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \to e^-(\mathbf{p_3})e^-(\mathbf{p_4})$



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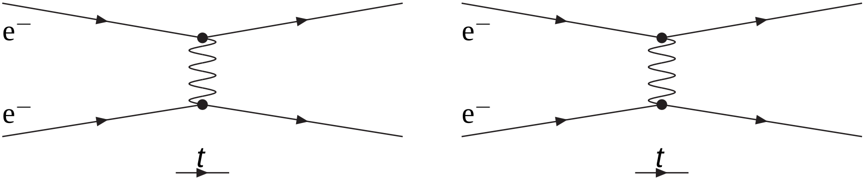
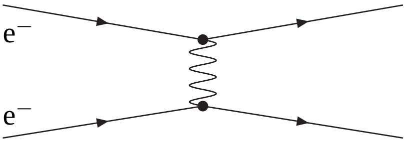
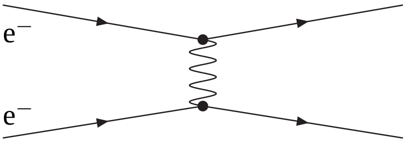


$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \to e^-(\mathbf{p_3})e^-(\mathbf{p_4})$ $\quad$ $e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \to e^-(\mathbf{p_4})e^-(\mathbf{p_3})$



- Fermion arrow tip to end
- Interaction
- propagator (internal line)
- second graph $\mathbf{p_3} \leftrightarrow \mathbf{p_4}$
- graphs fermion permutation: $-$
- $\mathbf{k} = f(\mathbf{p_i})$

$$T_{fi} = [\ \bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2})$$
$$-\ \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]$$



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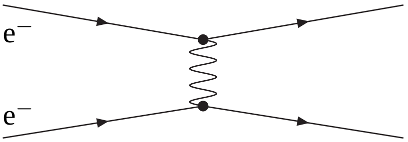

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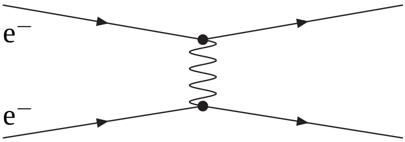

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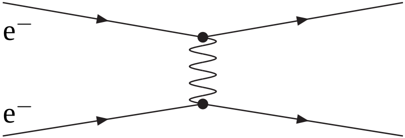

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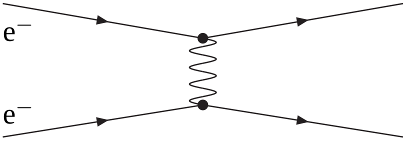

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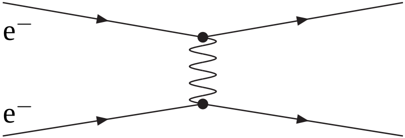
$$- \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})\left(\frac{-ig_{\rho\sigma}}{\;}\right)\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})\Big]$$



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\begin{aligned}
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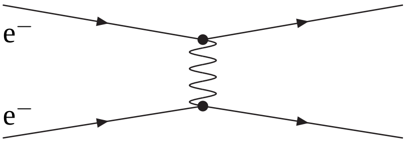
\end{aligned}
$$



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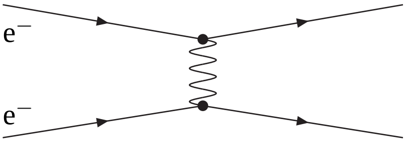

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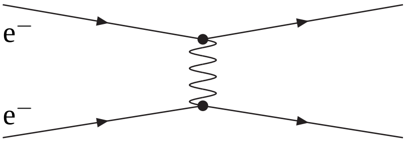
- \quad & \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{k^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]
\end{aligned}
$$



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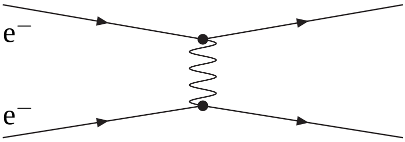
$$- \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{k^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]$$



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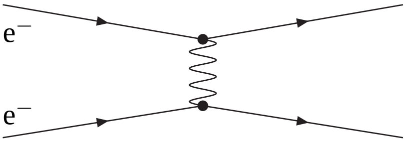

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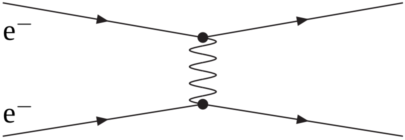
& - \quad \bar{u}(\mathbf{p_3})(-ie\gamma^{\rho})u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{k^2})\bar{u}(\mathbf{p_4})(-ie\gamma^{\sigma})u(\mathbf{p_2})]
\end{aligned}
$$



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T_{fi} \quad = \quad [\quad & \bar{u}(\mathbf{p_4})(-ie\gamma^{\mu})u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^{\nu})u(\mathbf{p_2}) \\
- \quad & \bar{u}(\mathbf{p_3})(-ie\gamma^{\rho})u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})(-ie\gamma^{\sigma})u(\mathbf{p_2})]
\end{aligned}
$$

$$\frac{1}{i}T_{fi} = \frac{1}{i}[\ \bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2})$$

$$- \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]$$

$$= e^2[\ \bar{u}(\mathbf{p_4})\gamma^\mu u(\mathbf{p_1})(\frac{g_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})\gamma^\nu u(\mathbf{p_2})$$

$$- \bar{u}(\mathbf{p_3})\gamma^\rho u(\mathbf{p_1})(\frac{g_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})\gamma^\sigma u(\mathbf{p_2})]$$

$$|\mathcal{M}|^2 = \sum\nolimits'_{fi} T_{fi}T_{fi}^\dagger$$

$$= \frac{1}{4}\sum\nolimits_{fi} T_{fi}T_{fi}^\dagger$$

Calculation to be continued in Problem Solving

$$|\mathcal{M}|^2 = \frac{64\pi^2\alpha^2}{t^2u^2}[(s-2m^2)^2(t^2+u^2)+ut(-4m^2s+12m^4+ut)]$$

$$\frac{1}{i}T_{fi} = \frac{1}{i}[\quad \bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2})$$

$$- \quad \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]$$

$$= e^2[\quad \bar{u}(\mathbf{p_4})\gamma^\mu u(\mathbf{p_1})(\frac{g_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})\gamma^\nu u(\mathbf{p_2})$$

$$- \quad \bar{u}(\mathbf{p_3})\gamma^\rho u(\mathbf{p_1})(\frac{g_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})\gamma^\sigma u(\mathbf{p_2})]$$

$$|\mathcal{M}|^2 = \sum'_{fi} T_{fi}T^\dagger_{fi}$$

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Calculation to be continued in Problem Solving

$$|\mathcal{M}|^2 = \frac{64\pi^2\alpha^2}{t^2u^2}[(s-2m^2)^2(t^2+u^2)+ut(-4m^2s+12m^4+ut)]$$

$$\frac{1}{i}T_{fi} = \frac{1}{i}\left[\bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})\left(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2}\right)\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2})\right.$$
$$\left. - \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})\left(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2}\right)\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})\right]$$
$$= e^2\left[\bar{u}(\mathbf{p_4})\gamma^\mu u(\mathbf{p_1})\left(\frac{g_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2}\right)\bar{u}(\mathbf{p_3})\gamma^\nu u(\mathbf{p_2})\right.$$
$$\left. - \bar{u}(\mathbf{p_3})\gamma^\rho u(\mathbf{p_1})\left(\frac{g_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2}\right)\bar{u}(\mathbf{p_4})\gamma^\sigma u(\mathbf{p_2})\right]$$
$$|\mathcal{M}|^2 = \sum\nolimits'_{fi} T_{fi}T_{fi}^\dagger$$
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Calculation to be continued in Problem Solving

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$$
\begin{aligned}
\frac{1}{i}T_{fi} &= \frac{1}{i}[&& \bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2}) \\
& - && \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})] \\
&= e^2[&& \bar{u}(\mathbf{p_4})\gamma^\mu u(\mathbf{p_1})(\frac{g_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})\gamma^\nu u(\mathbf{p_2}) \\
& - && \bar{u}(\mathbf{p_3})\gamma^\rho u(\mathbf{p_1})(\frac{g_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})\gamma^\sigma u(\mathbf{p_2})] \\
|\mathcal{M}|^2 &= \sum\nolimits'_{fi} && T_{fi}T^\dagger_{fi} \\
&= \frac{1}{4}\sum\nolimits_{fi} && T_{fi}T^\dagger_{fi}
\end{aligned}
$$

Calculation to be continued in Problem Solving

$$|\mathcal{M}|^2 = \frac{64\pi^2\alpha^2}{t^2u^2}[(s-2m^2)^2(t^2+u^2)+ut(-4m^2s+12m^4+ut)]$$

$$\begin{aligned}
\frac{1}{i}T_{fi} &= \frac{1}{i}[\quad \bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2}) \\
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Calculation to be continued in Problem Solving

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$$\frac{d\sigma}{d\Omega} = |\mathcal{M}|^2 \frac{1}{64\pi^2 s}$$

$0 \leq \theta \leq \pi/2$ (electrons) $m_e \approx 0$

$$\begin{aligned}
t &= -2\mathbf{p_1 p_3} = -2(\sqrt{s}/2\sqrt{s}/2 - s/4\cos\theta) = -s/2(1-\cos\theta) \\
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\frac{d\sigma}{d\Omega} &= \frac{\alpha^2}{st^2u^2}[s^2(t^2+u^2)+u^2t^2] \\
&= \frac{\alpha^2}{s}[\frac{s^2}{u^2}+\frac{s^2}{t^2}+1] \\
&= \frac{\alpha^2}{s}\frac{(3+\cos^2\theta)^2}{\sin^4\theta}
\end{aligned}$$

$s\frac{d\sigma}{d\Omega}$ is scale invariant: measure of the pointlikeness of a particle

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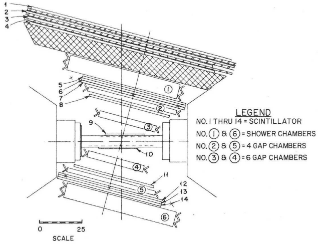


FIG. 1. Storage-ring interaction region and detector system for 556-MeV/electron scattering experiment.

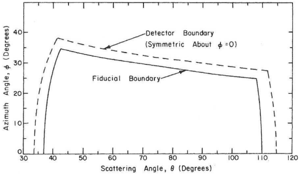


FIG. 2. Detector boundary and fiducial boundaries.

- limited detector acceptance
- differential cross section measurement and prediction

- Stanford-Princeton Storage ring
- $2e^-$ beams $\sqrt{s} = 556 MeV$

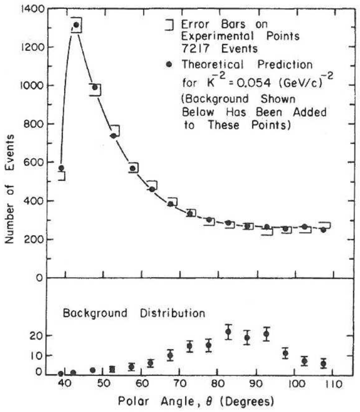
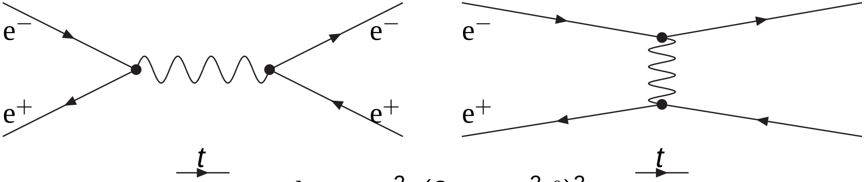


FIG. 3. Comparison of experimental result with Møller scattering modified by radiative corrections. Because the detector geometry is included, the theoretical curve is not symmetric about 90°.

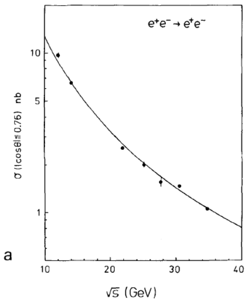
- Typical t channel $\theta = 0 \rightarrow d\sigma/d\Omega \rightarrow \infty$
- Extremely good agreement between the measurement and the theory prediction
- e^-e^- colliders discontinued (1971)

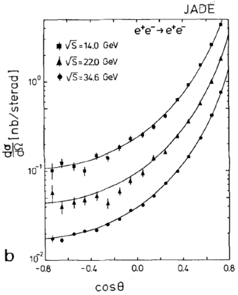
The Bhabha Process Homi Bhabha studied in the 1930s in Great Britain, worked in India afterwards



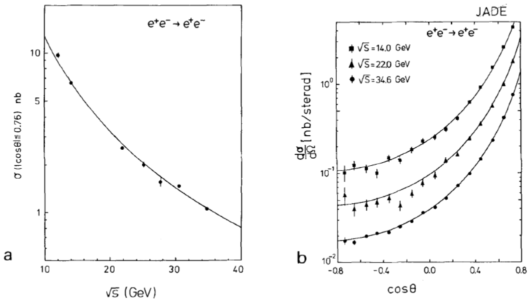
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16s} \frac{(3+\cos^2\theta)^2}{\sin^4\frac{\theta}{2}}$$

- $0 \leq \theta \leq \pi$
- t channel: $\sim \sin^{-4}(\theta/2)$
- s channel: $\sim 1 + \cos^2\theta$





- PETRA $\mathrm{e^+e^-}$ collider $\sqrt{s} \leq 35\mathrm{GeV}$
- JADE, TASSO, CELLO
- total cross section
- differential cross section



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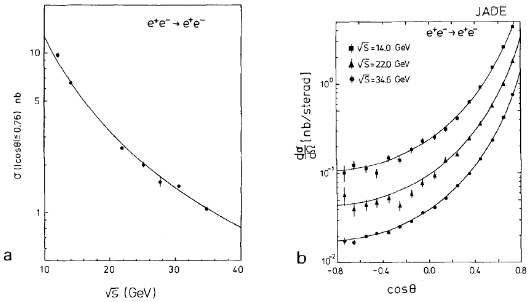

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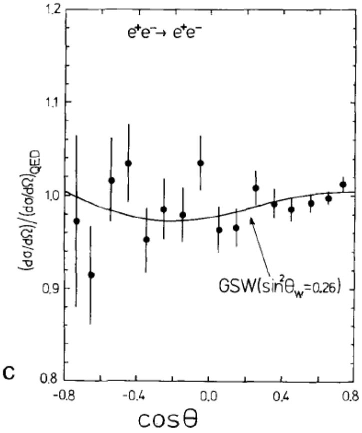

- Excellent agreement with QED
- Errors reflect statistics
- QED deviation : $s/\Lambda^2 < 5\%$ with $s = 35^2\mathrm{GeV}^2$
- $\to (\hbar c)/\Lambda = (0.197\mathrm{GeV}\cdot\mathrm{fm})/\Lambda \approx 0.13 \cdot 10^{-3}\mathrm{fm}$
- $N = \int Ldt \cdot \sigma$
- Today Bhabha is a luminosity measurement