

# The Top Mass in the SM and MSSM (and BMSSM)

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1. What to expect from  $m_t$  @ the LC
2. Implications for the SM
3. Implications for SUSY
4. Implications for other models
5. Conclusions

# 1. What to expect from GigaZ and $m_t$ @ the LC

(Sad) Reality: LC will start in 2020 earliest

## World of High Energy Physics in the year 2020:

Both LHC detectors will have accumulated  $\sim 300 \text{ fb}^{-1}$

Initial LHC physics goals are accomplished:

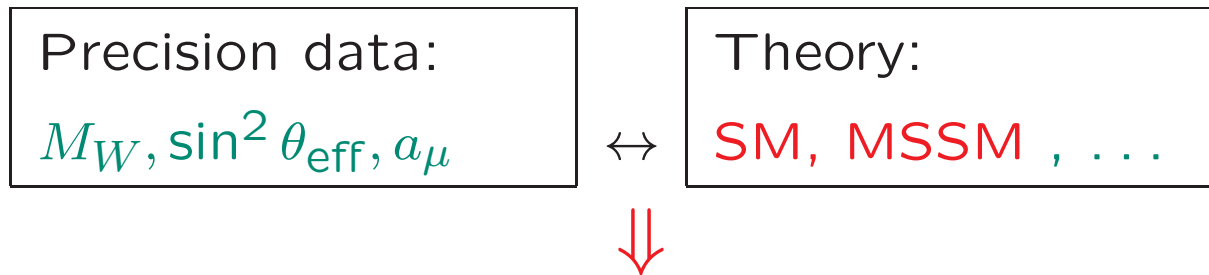
- state compatible with a Higgs found  
corresponding couplings measured to 10–30%
- SUSY-like signatures observed (if realized at the EW scale)  
(or not ... ???)
- Extra dimensions or ...-like signatures observed  
(or not ... ???)

LHC may await luminosity upgrade

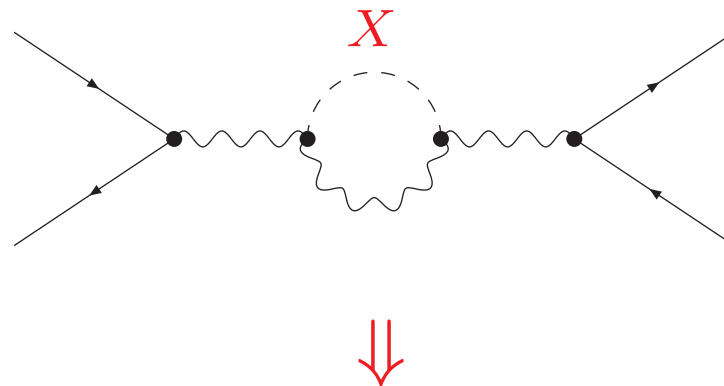
## **What can LC/GigaZ add?**

Important test for **any** model:

Comparison of observables with theory:



Test of theory at quantum level: Sensitivity to loop corrections, e.g.  $X$



**SM: limits on  $M_X$**

Very high accuracy of measurements and theoretical predictions needed

## Important: three different types of errors:

### Experimental error ( $\Rightarrow$ included in the figure):

- current error
  - future expectations
- $\Rightarrow$  sets the scale, has to be matched by other errors

### Theory error:

- $\Rightarrow$  error due to missing higher order corrections
- only estimates possible
  - even more complicated for the future

### Parametric error:

- current uncertainty in the prediction due to error in the input parameters
  - future uncertainty
- $\Rightarrow$  focus on SM parameters
- $\Rightarrow$  derive information about (unknown) SUSY(?) parameters  
(BSM parametric uncertainties highly model dependent)

Precision observables:  $M_W$ ,  $\sin^2 \theta_{\text{eff}}$ ,  $M_h$ ,  $(g-2)_\mu$ ,  $b$  physics, ...

A) Theoretical prediction for  $M_W$  in terms

of  $M_Z$ ,  $\alpha$ ,  $G_\mu$ ,  $\Delta r$ :

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r)$$

$\Updownarrow$

loop corrections

Evaluate  $\Delta r$  from  $\mu$  decay  $\Rightarrow M_W$

One-loop result for  $M_W$  in the SM:

[A. Sirlin '80] , [W. Marciano, A. Sirlin '80]

$$\begin{aligned} \Delta r_{1\text{-loop}} &= \Delta\alpha & - & \frac{c_W^2}{s_W^2} \Delta\rho & + & \Delta r_{\text{rem}}(M_H) \\ &\sim \log \frac{M_Z}{m_f} & & \sim m_t^2 & & \log(M_H/M_W) \\ &\sim 6\% & & \sim 3.3\% & & \sim 1\% \end{aligned}$$

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loop corrections

B) Effective mixing angle:

$$\sin^2 \theta_{\text{eff}} = \frac{1}{4 |Q_f|} \left( 1 - \frac{\text{Re } g_V^f}{\text{Re } g_A^f} \right)$$

Higher order contributions:

$$g_V^f \rightarrow g_V^f + \Delta g_V^f, \quad g_A^f \rightarrow g_A^f + \Delta g_A^f$$

Experimental errors of the precision observables:

	today	Tev./LHC	LC	GigaZ
$\delta \sin^2 \theta_{\text{eff}} (\times 10^5)$	16	16	–	1.3
$\delta M_W$ [MeV]	15	$\leq 15$	10	7
$\delta m_t$ [GeV]	0.9	$\leq 1$	0.2	0.1

Relevant SM parametric errors:  $\delta(\Delta\alpha_{\text{had}}) = 5 \times 10^{-5}$ ,  $\delta M_Z = 2.1$  MeV

	$\delta m_t = 2$	$\delta m_t = 1$	$\delta m_t = 0.1$	$\delta(\Delta\alpha_{\text{had}})$	$\delta M_Z$
$\delta \sin^2 \theta_{\text{eff}} [10^{-5}]$	6	3	0.3	1.8	1.4
$\Delta M_W$ [MeV]	12	6	1	1	2.5

## Current and future errors:

### Current:

$$\delta m_t^{\text{exp}} = 0.9 \text{ GeV},$$

$$\delta(\Delta\alpha_{\text{had}}) = 3.5 \times 10^{-4}$$

$$\delta M_W^{\text{theory,SM}} \approx \pm 4 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{theory}} \approx \pm 10 \times 10^{-5}$$

$$\delta m_t : \quad \delta M_W^{\text{para}} \approx \pm 5.5 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 7 \times 10^{-5}$$

$$\delta(\Delta\alpha_{\text{had}}) : \quad \delta M_W^{\text{para}} \approx \pm 6.5 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 13 \times 10^{-5}$$

$$\delta M_W^{\text{exp}} \approx \pm 15 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{exp}} \approx \pm 16 \times 10^{-5}$$

### Future:

$$\delta M_W^{\text{theory}} \gtrsim \pm 2 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{theory}} \gtrsim \pm 2 \times 10^{-5}$$

$$\delta m_t : \quad \delta M_W^{\text{para}} \approx \pm 1 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 0.4 \times 10^{-5}$$

$$\delta(\Delta\alpha_{\text{had}}) : \quad \delta M_W^{\text{para}} \approx \pm 1 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{para}} \approx \pm 1.8 \times 10^{-5}$$

$$[\text{GigaZ}] : \quad \delta M_W^{\text{exp}} \approx \pm 7 \text{ MeV},$$

$$\delta \sin^2 \theta_{\text{eff}}^{\text{exp}} \approx \pm 1.3 \times 10^{-5}$$



The top is guaranteed at the LC  $\Rightarrow$  sure physics case

Top-quark mass is a fundamental parameter of the electroweak theory

By far the largest quark mass,  
largest mass of all known fundamental particles

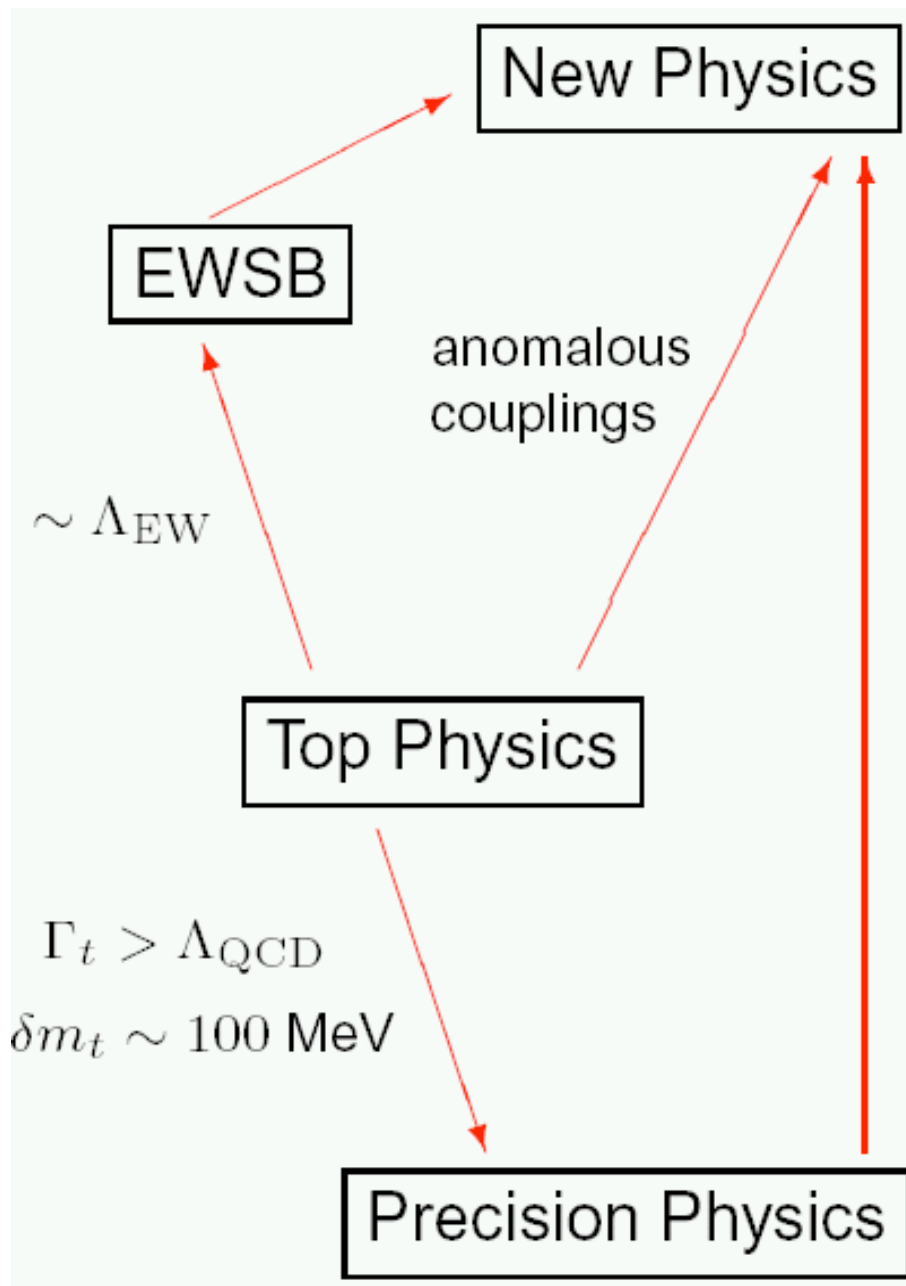
Window to new physics?

Large coupling to the Higgs boson; physics of flavor;  
prediction of  $m_t$  from underlying theory?

Radiative corrections

$\Rightarrow$  non-decoupling effects proportional to powers of  $m_t$

$\Rightarrow$  Need to know  $m_t$  very precisely in order to have  
sensitivity to effects of new physics



EWSB: just a heavy quark?  
 special role for  $t$  in EWSB?  
 strong constraint on any model

Precision physics:

$\delta m_t^{\text{exp}}$  leading parametric uncertainty  
 → could obscure new physics

SUSY:  $m_t$  crucial input parameter  
 drives SSB/unification

Little Higgs: heavier top

Tevatron: "rough" measurements  
 of mass, couplings, BRs

LHC: the same (but better!) & more

LC: high precision of everything

## What is the top mass?

Particle masses are **not** observables  
one can only measure cross sections, decay rates, ...

Additional problem for the top mass:

**what is the mass of a colored object?**

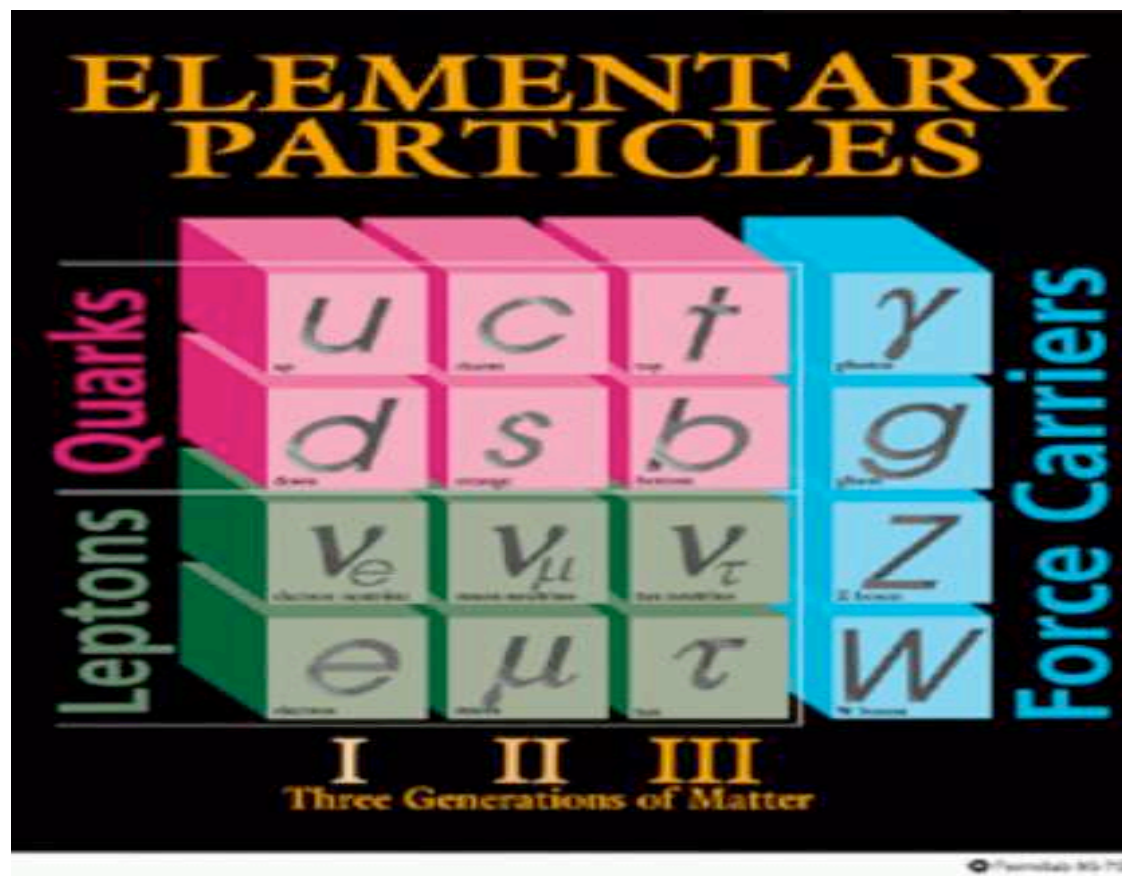
Top pole mass is not IR safe (affected by large long-distance contributions), cannot be determined to better than  $\mathcal{O}(\Lambda_{\text{QCD}})$

## Measurement of $m_t$ :

- At Tevatron, LHC:  
kinematic reconstruction, fit to invariant mass distribution  
 $\Rightarrow$  “pole” mass
- At the LC:  
mainly from threshold behavior  $\Rightarrow$  threshold mass  $\Rightarrow$  **SAFE!**

## 2. Implications for the SM

Current status of knowledge: the Standard Model (SM)



⇒ Last remaining free parameter:  $M_H$

# Comparison of SM prediction of $M_W$ with direct measurements:

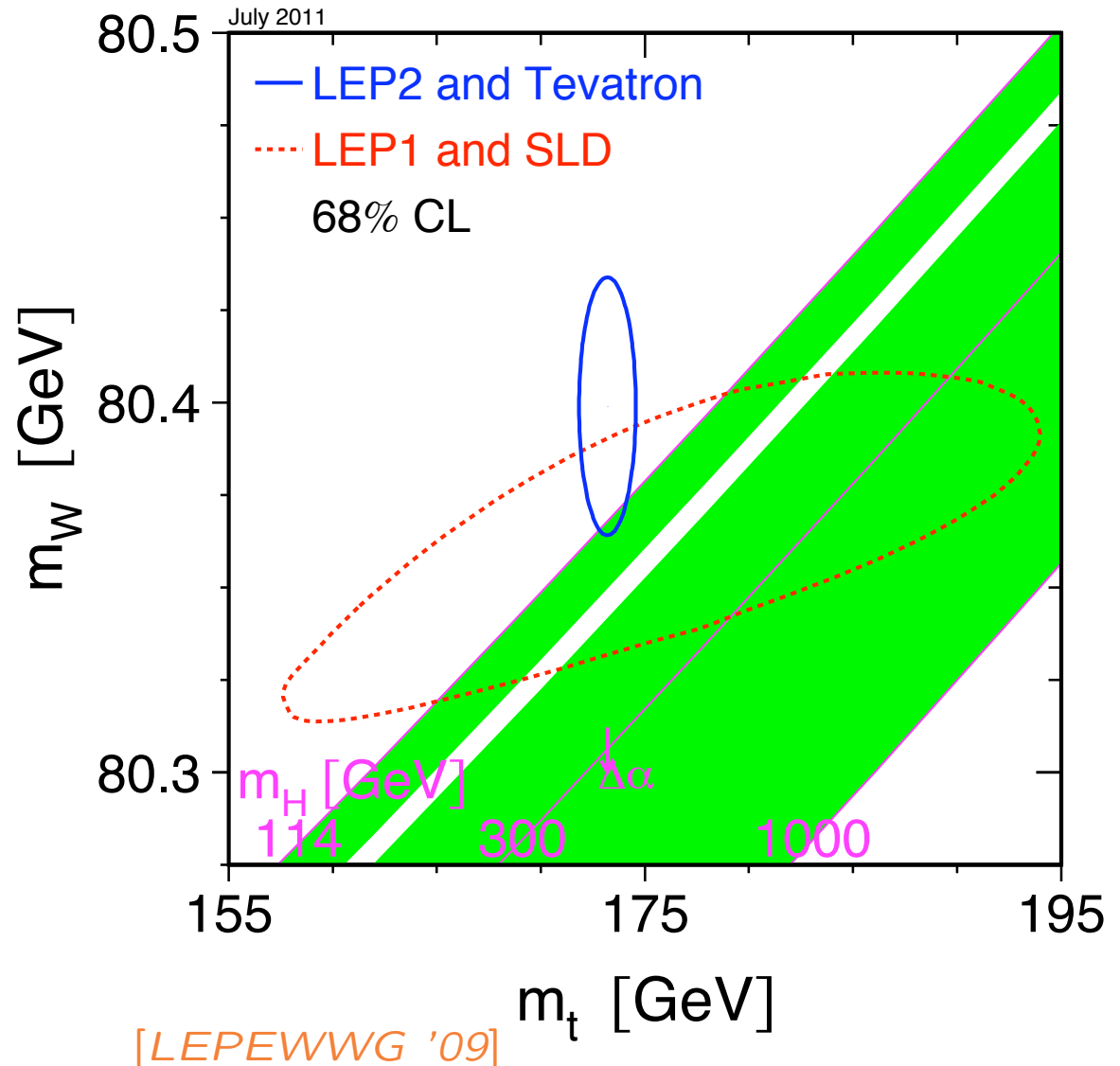
$$\Delta r = -\frac{11g_2^2 s_W^2}{96 \pi^2 c_W^2} \log\left(\frac{M_H}{M_W}\right)$$

general for EWPO:

$$\Delta \sim g_2^2 \left[ \log\left(\frac{M_H}{M_W}\right) + g_2^2 \frac{M_H^2}{M_W^2} \right]$$

leading term:  $\log(M_H)$

first term  $\sim M_H^2$  with  $g_2^4$



⇒ light Higgs boson preferred

Global fit to all SM data:

[LEPEWWG '09]

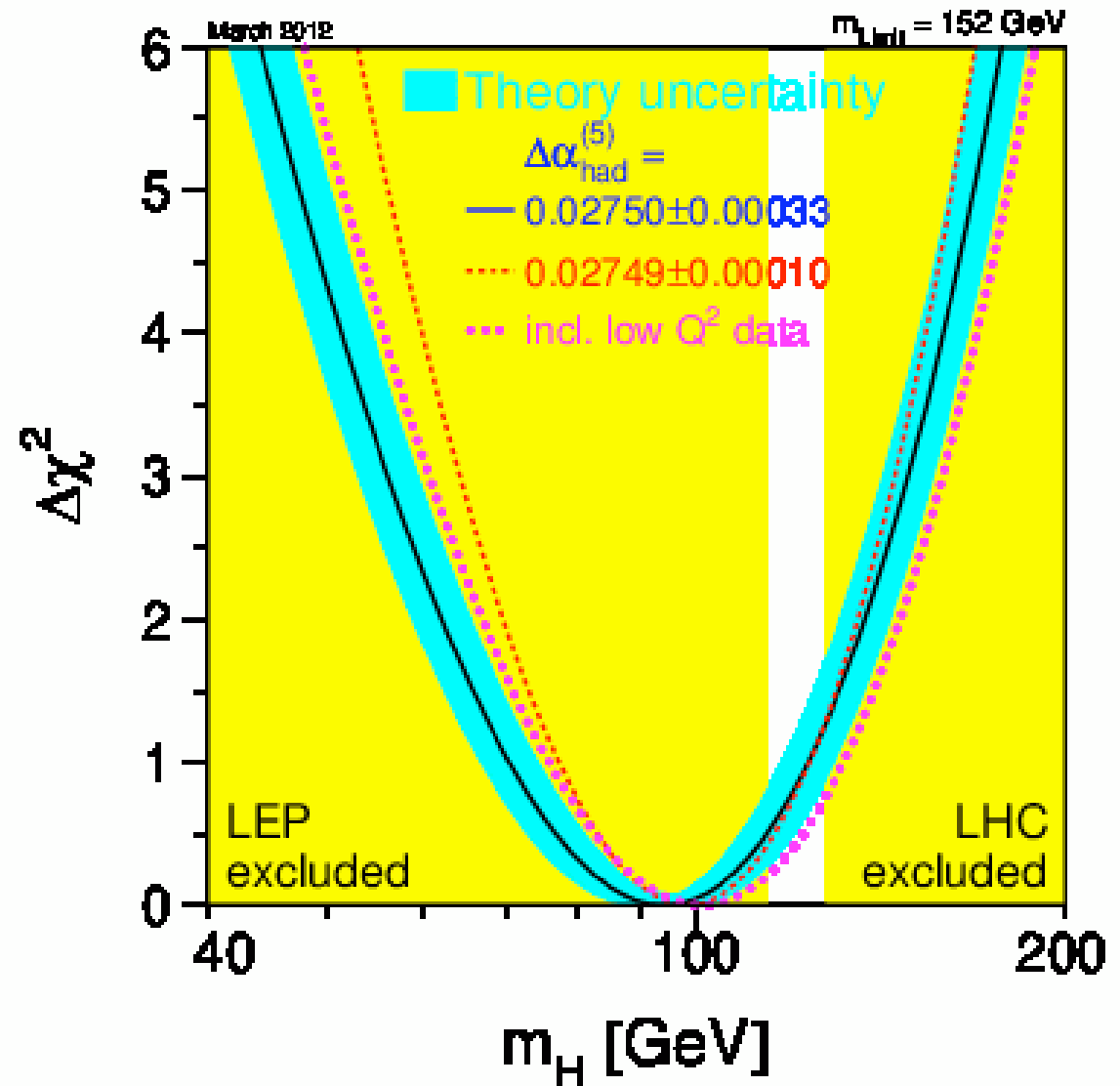
$$\Rightarrow M_H = 94^{+29}_{-24} \text{ GeV}$$

$$M_H < 152 \text{ GeV, 95\% C.L.}$$

Assumption for the fit:

SM incl. Higgs boson

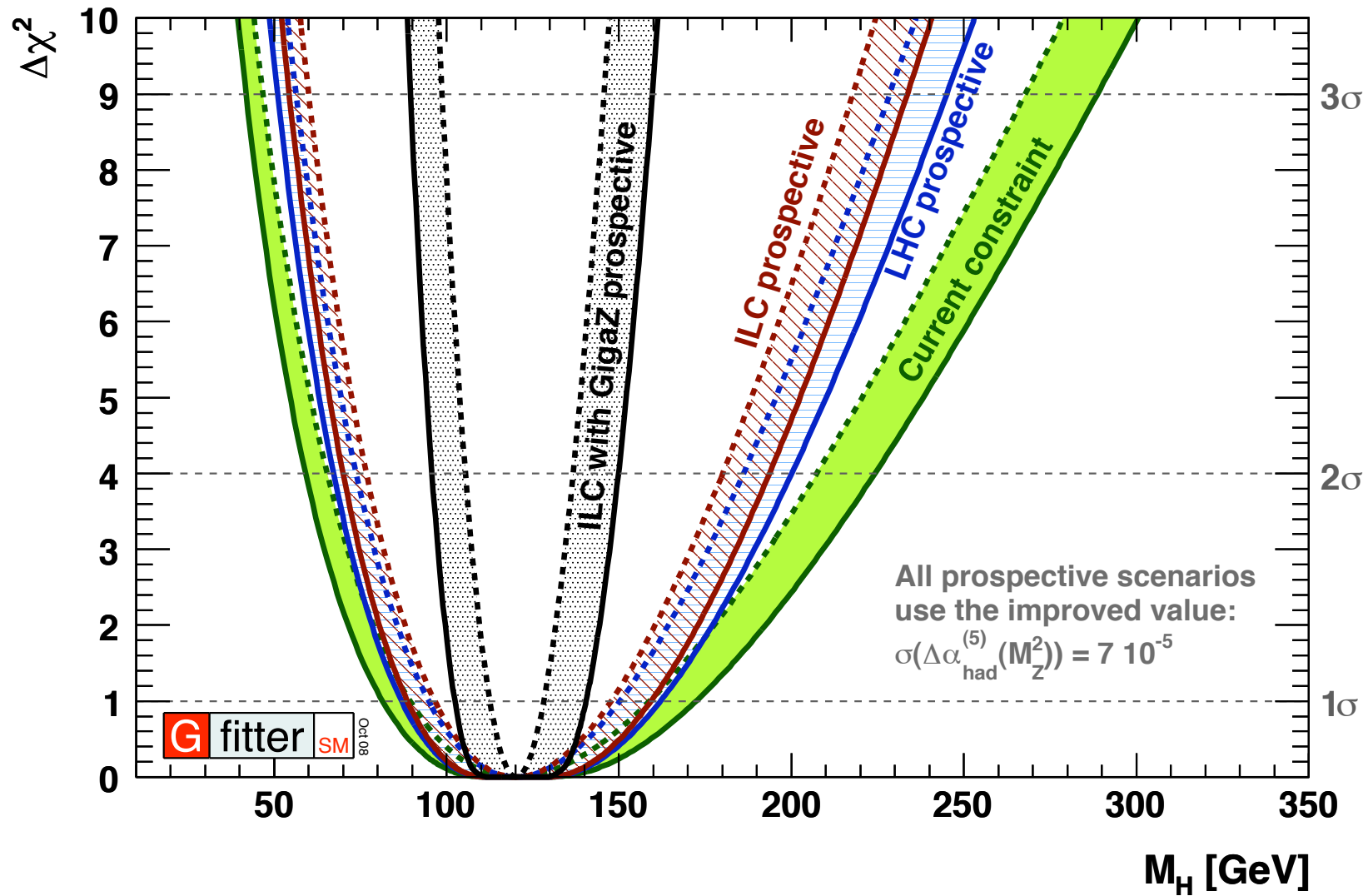
$\Rightarrow$  no confirmation of Higgs mechanism



$\Rightarrow$  Higgs boson seems to be light,  $M_H \lesssim 160 \text{ GeV}$

# GigaZ: Improvement in the Blue Band plot:

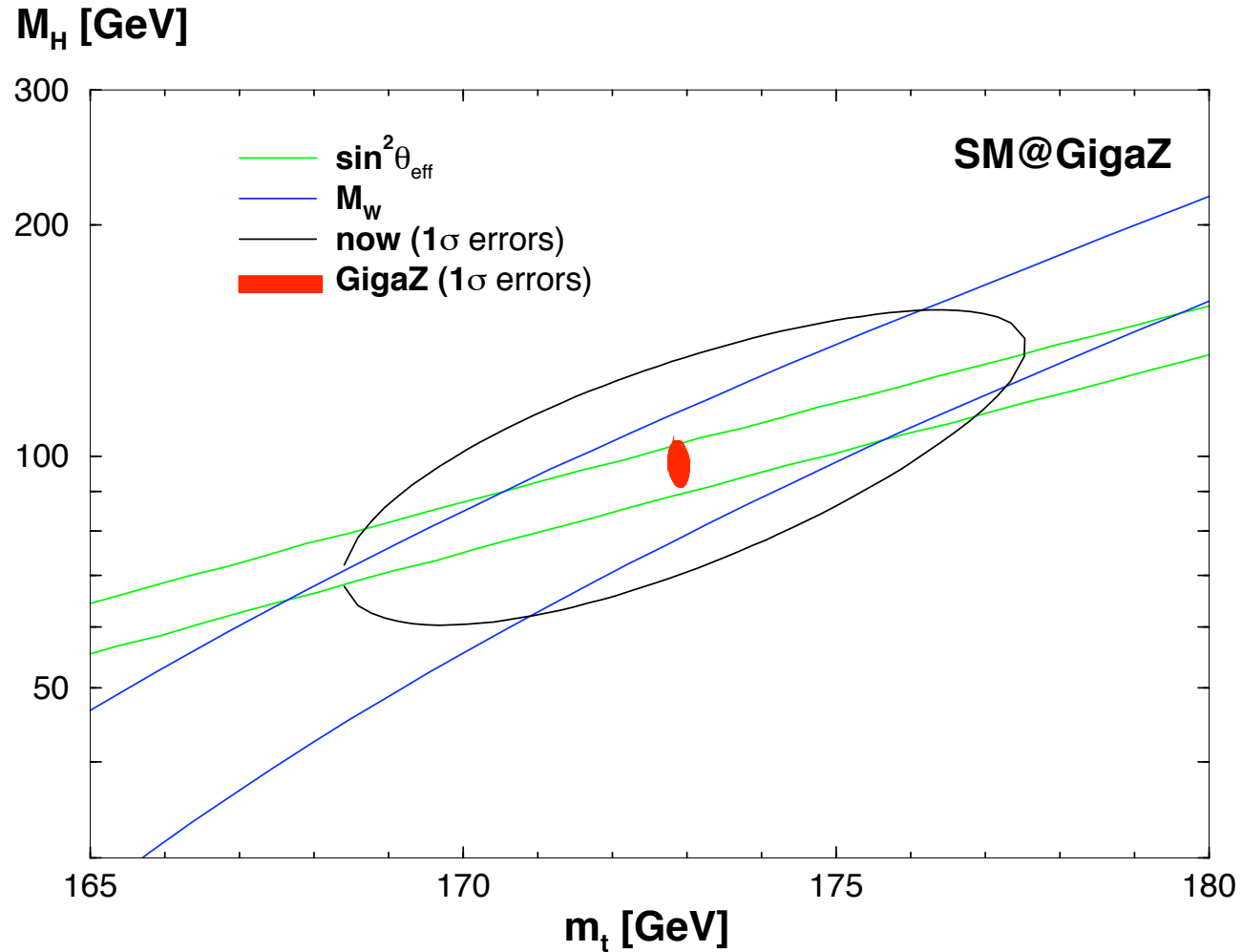
[GFitter '09]



(note: artificially  $M_H^{\text{SM}} = 120$  GeV)

**GigaZ:**  $\Rightarrow$  Improvement in  $M_H$  determination:

[J. Erler, S.H., W. Hollik, G. Weiglein, P. Zerwas '00]



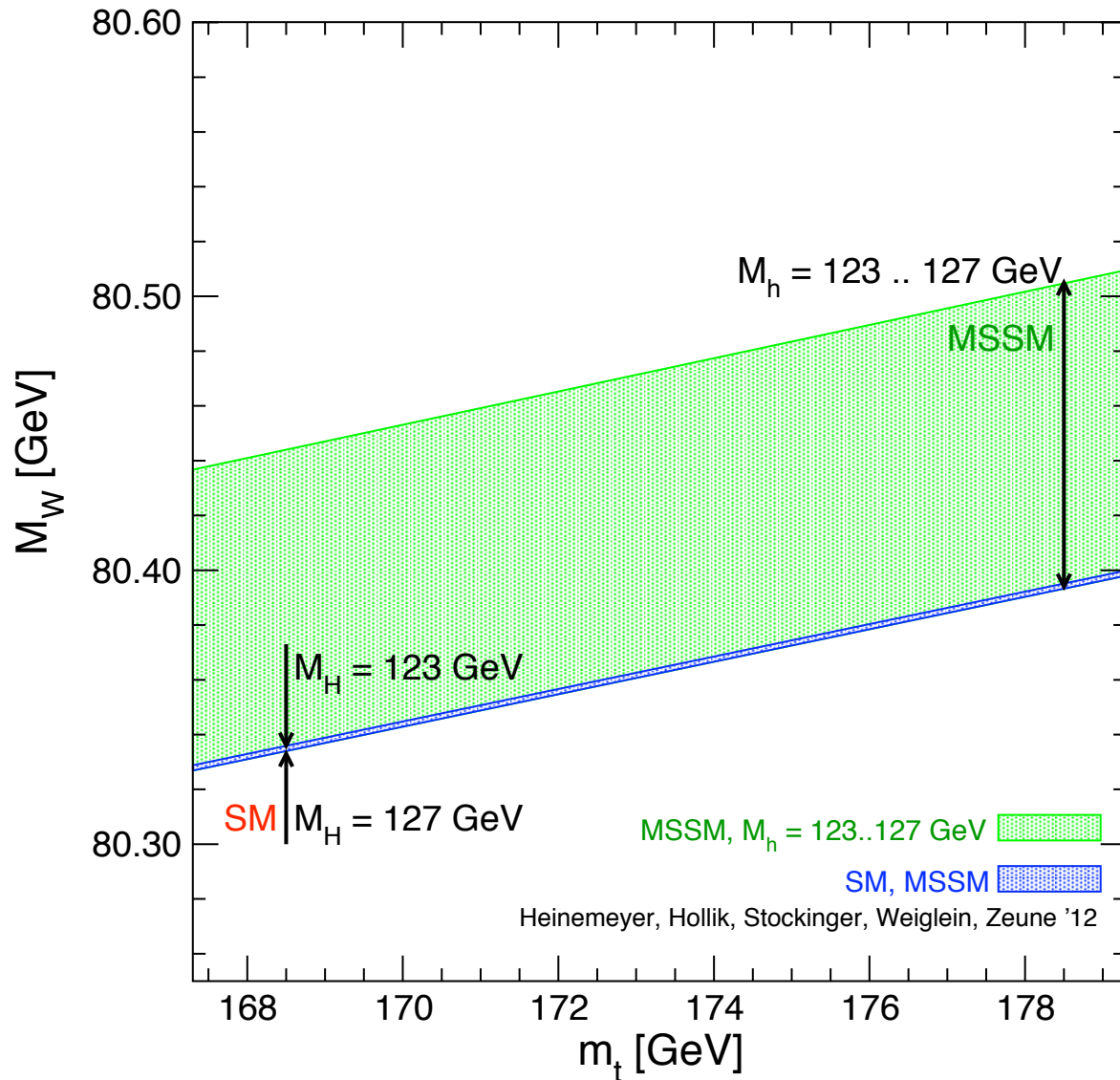


### 3. Implications for SUSY

- Precision observables to test the MSSM
- top mass measurement for the MSSM Higgs sector
- discriminate between SM and MSSM
- limits on MSSM extensions

Example: Prediction for  $M_W$  in the **SM** and the **MSSM** :

[S.H., W. Hollik, D. Stockinger, G. Weiglein, L. Zeune '12]



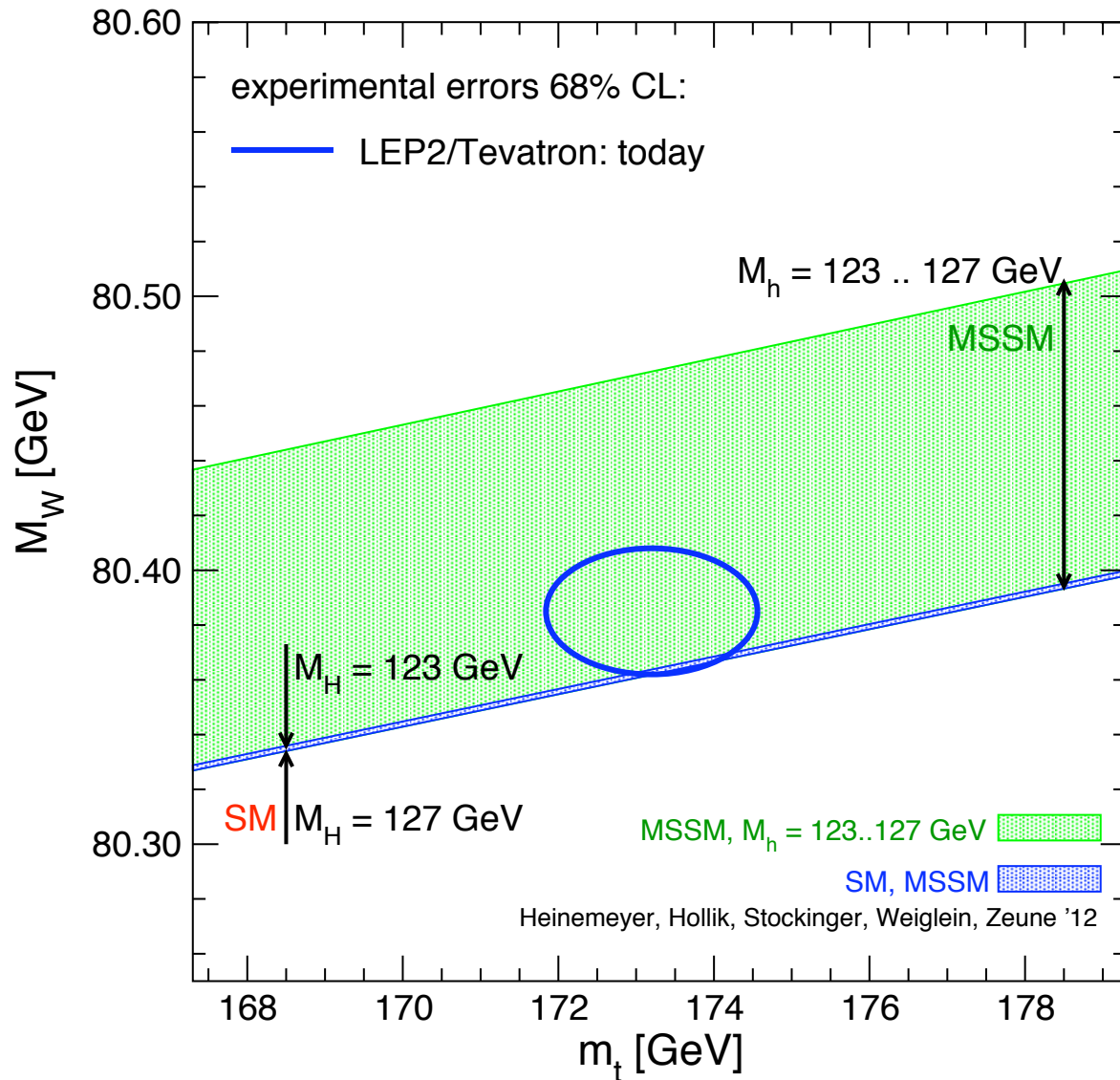
**MSSM band:**  
scan over  
SUSY masses

**overlap:**  
SM is MSSM-like  
MSSM is SM-like

**SM band:**  
variation of  $M_H^{\text{SM}}$

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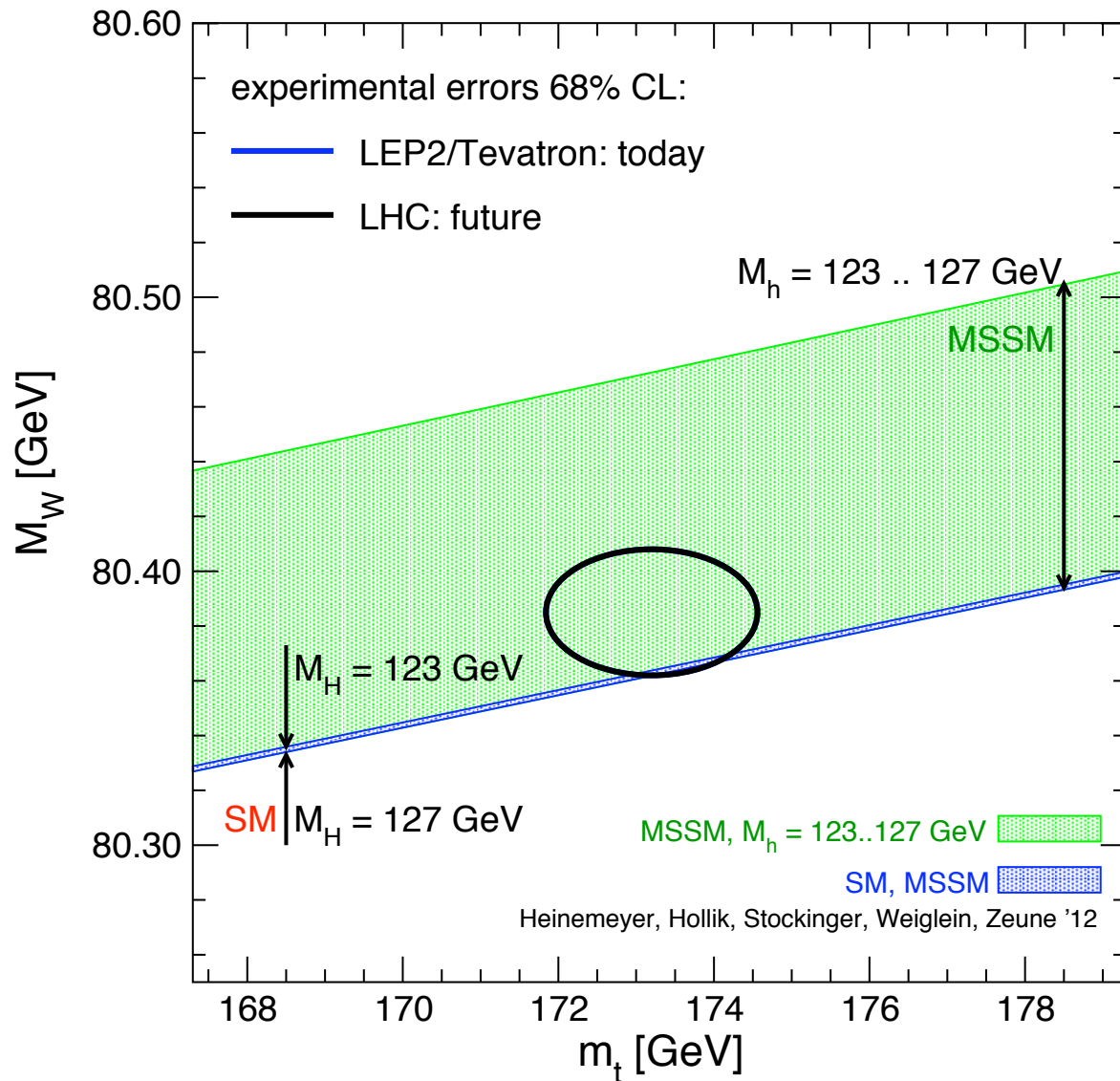
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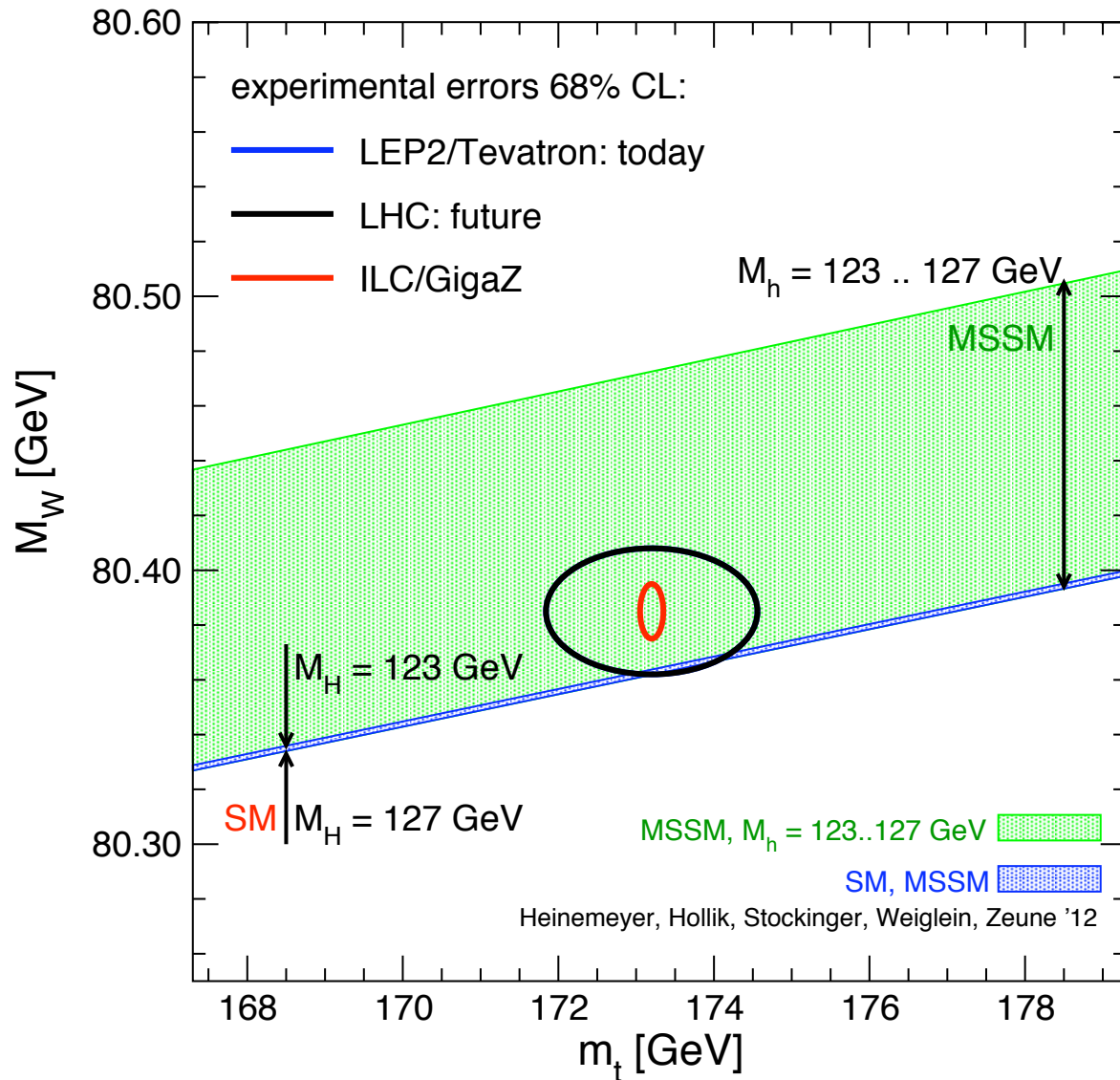
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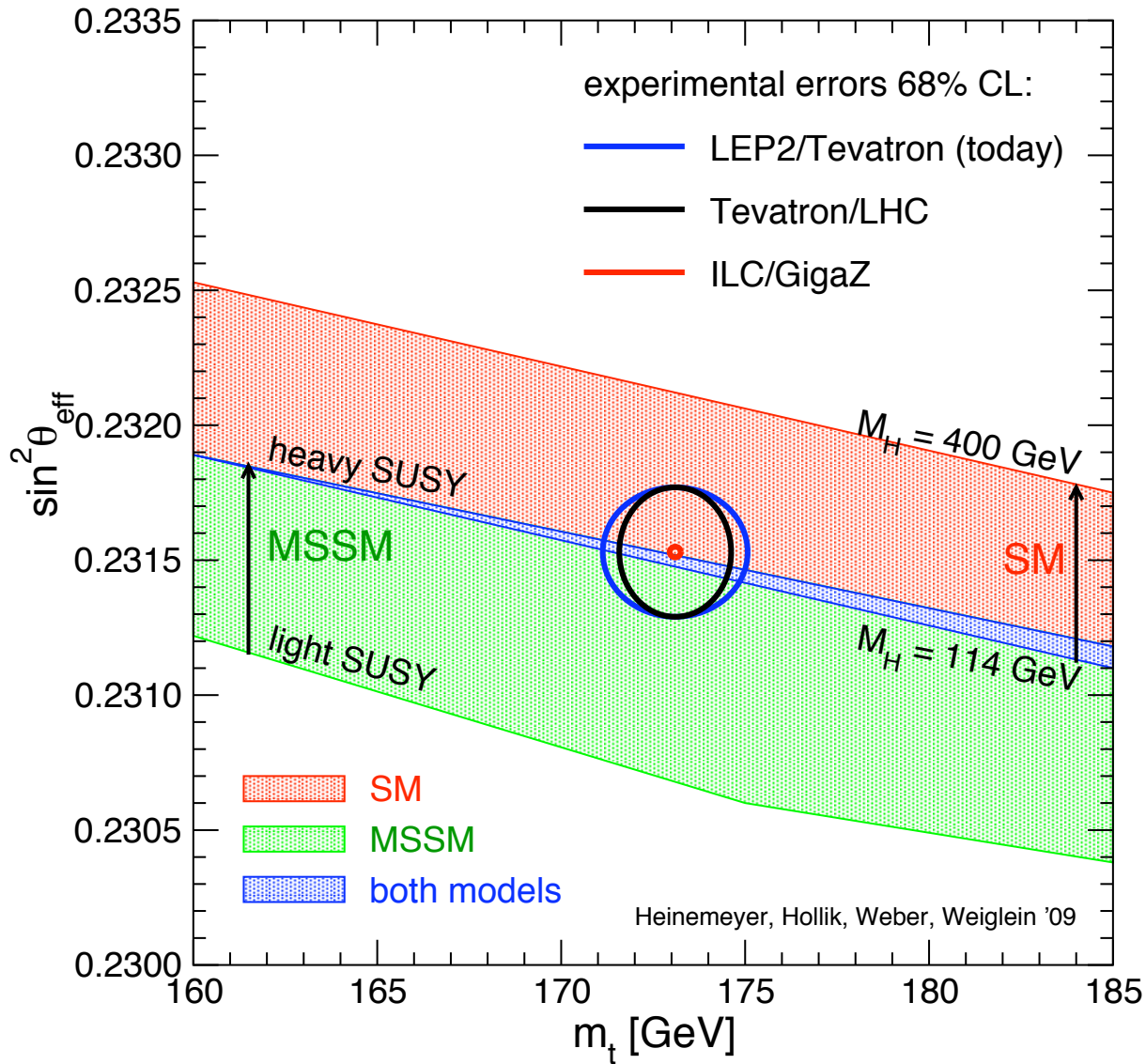
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**SM band:**

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# Prediction for $\sin^2 \theta_{\text{eff}}$ in the SM and the MSSM :

[S.H., W. Hollik, A. Weber, G. Weiglein '07]



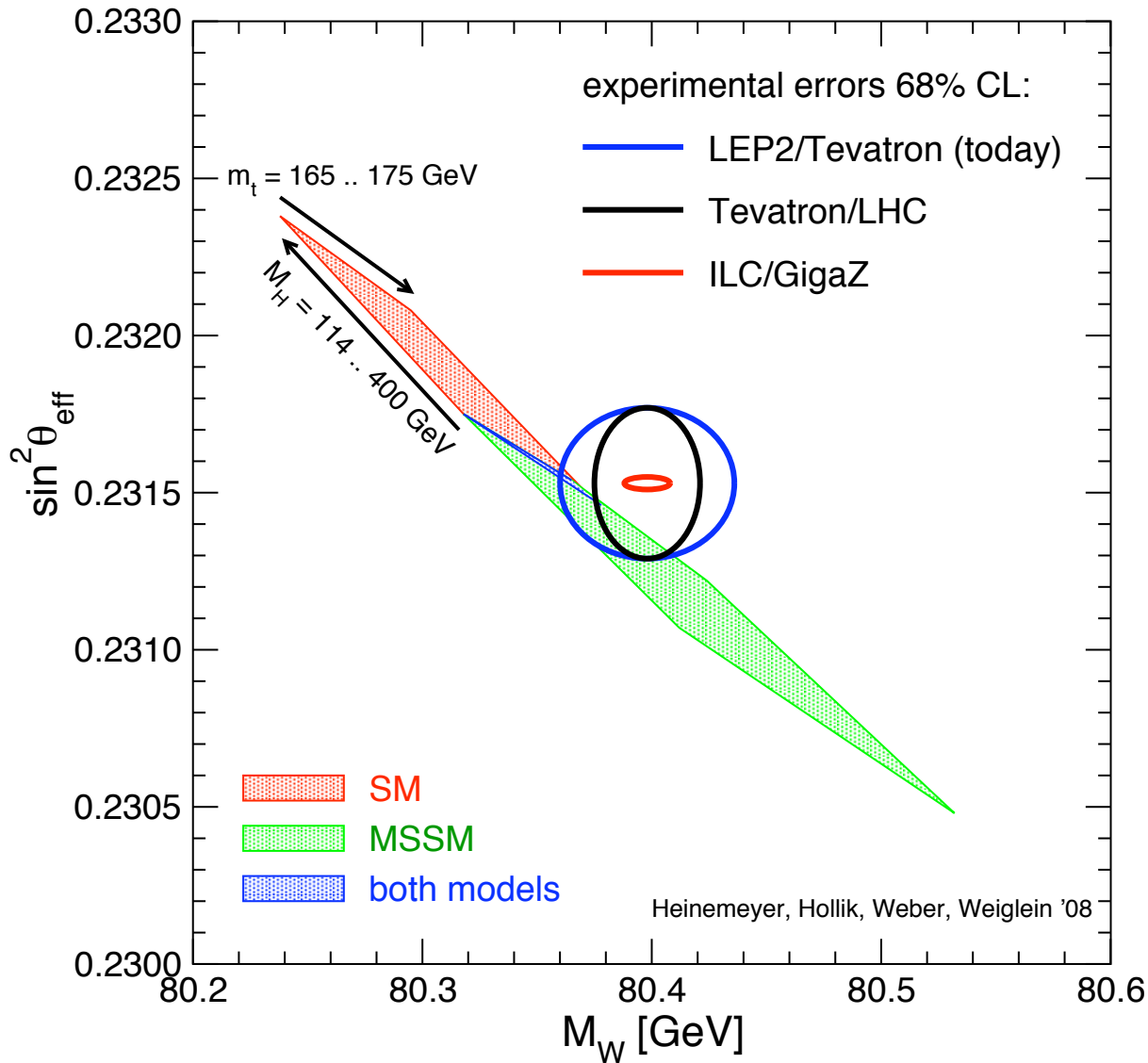
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MSSM is SM-like

**SM band:**  
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Prediction for  $M_W$  and  $\sin^2 \theta_{\text{eff}}$  in the SM and the MSSM :

[S.H., W. Hollik, A. Weber, G. Weiglein '07]



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SUSY masses

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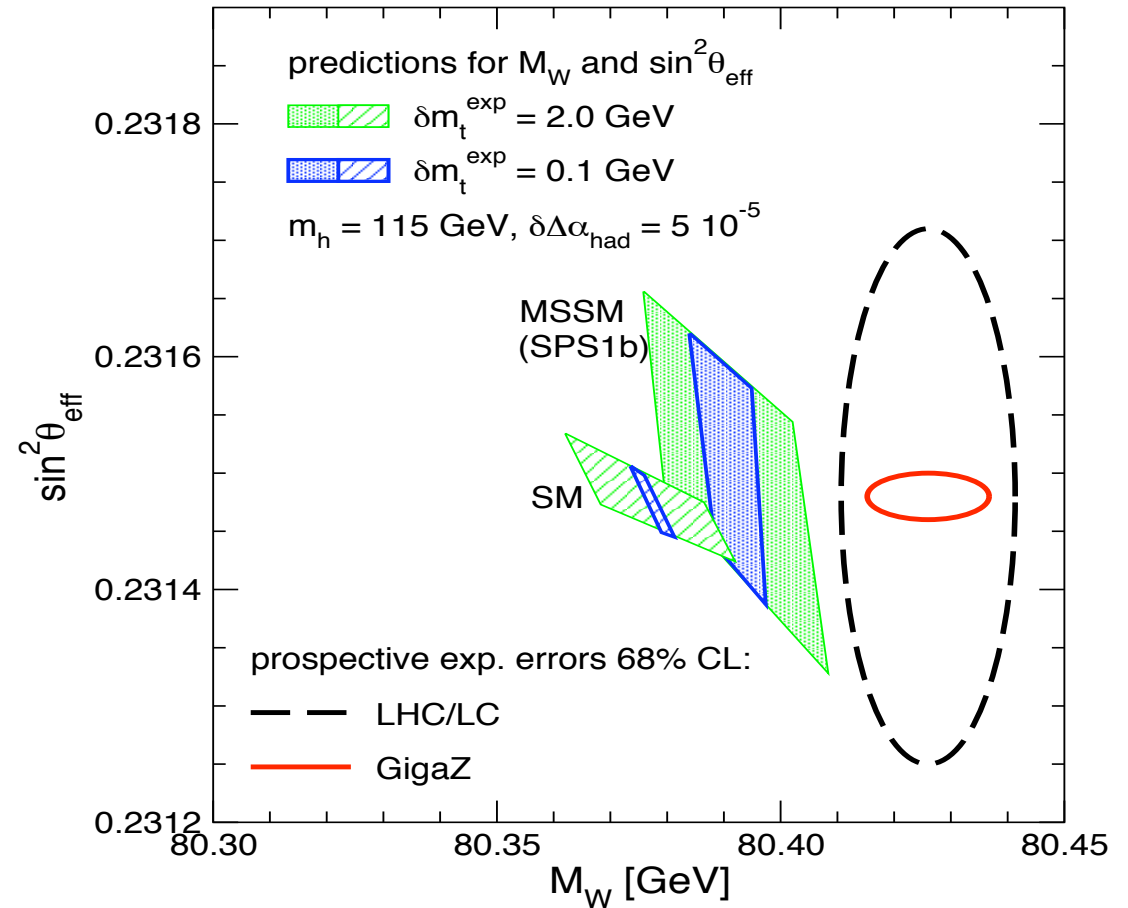
## Possible future scenario:

[S.H., S. Kraml, W. Porod, G. Weiglein '03]

SM:  $M_H = 115$  GeV

MSSM: SPS 1b

all SUSY parameters varied  
within realistic errors



$\delta m_t = 0.1$  GeV vs.  $\delta m_t = 2$  GeV

$\Rightarrow$  SM: improvement by a factor  $\sim 10$

$\Rightarrow$  MSSM: improvement by a factor  $\sim 2 - 3$



# Scenario with no SUSY particles at the LHC:

→  $\sin^2 \theta_{\text{eff}}$  investigation

→ SPS 1a with heavy scalars

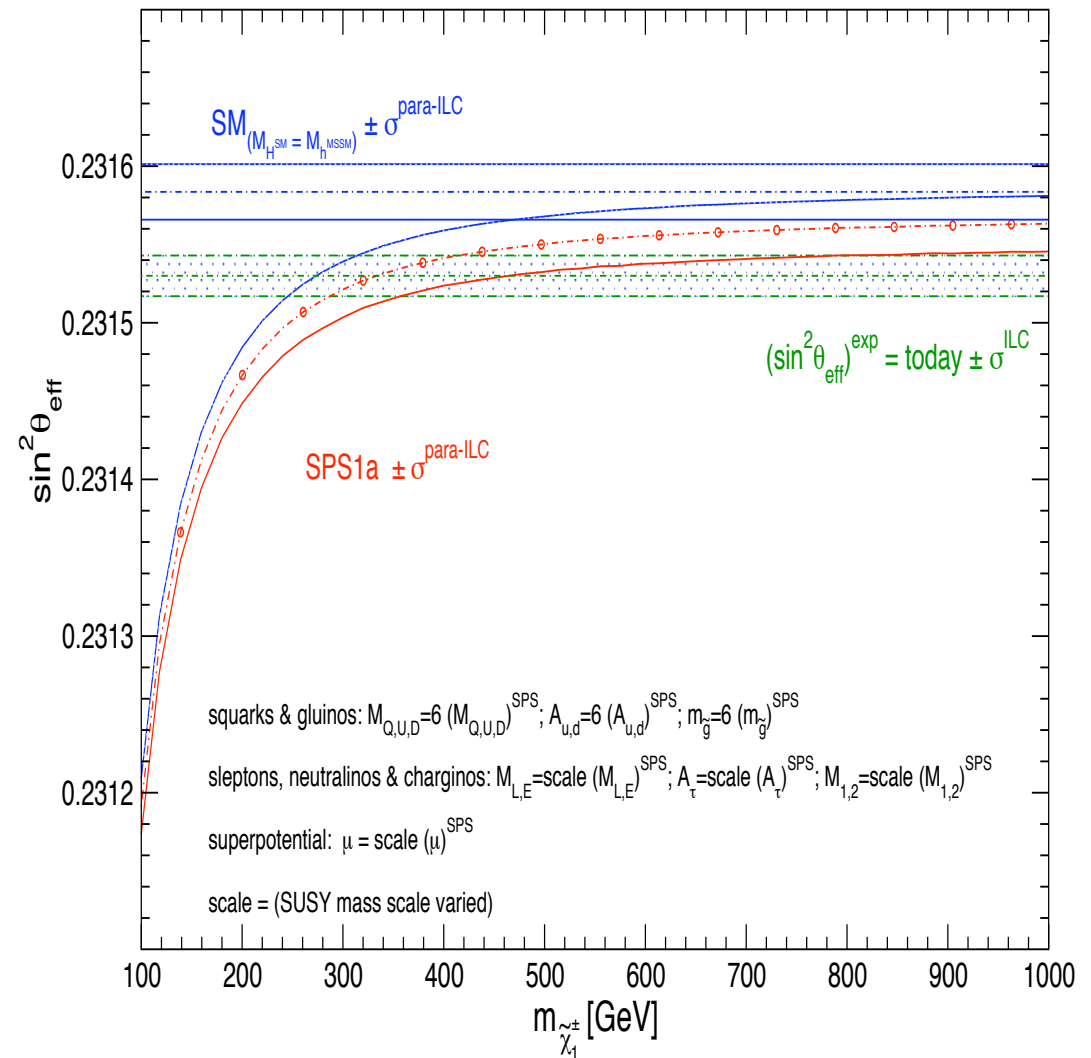
SM prediction

vs.

MSSM (SPS 1a) prediction

vs.

LC resolution



⇒ the LC(1000)/GigaZ could detect SUSY directly/indirectly

Theoretical prediction of the lightest MSSM Higgs boson mass:  $M_h$

Contrary to the SM:  $M_h$  is not a free parameter

MSSM tree-level bound:  $m_h < M_Z$ , excluded by LEP Higgs searches

Large radiative corrections:

Dominant one-loop corrections:  $\Delta M_h^2 \sim G_\mu m_t^4 \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right)$

The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

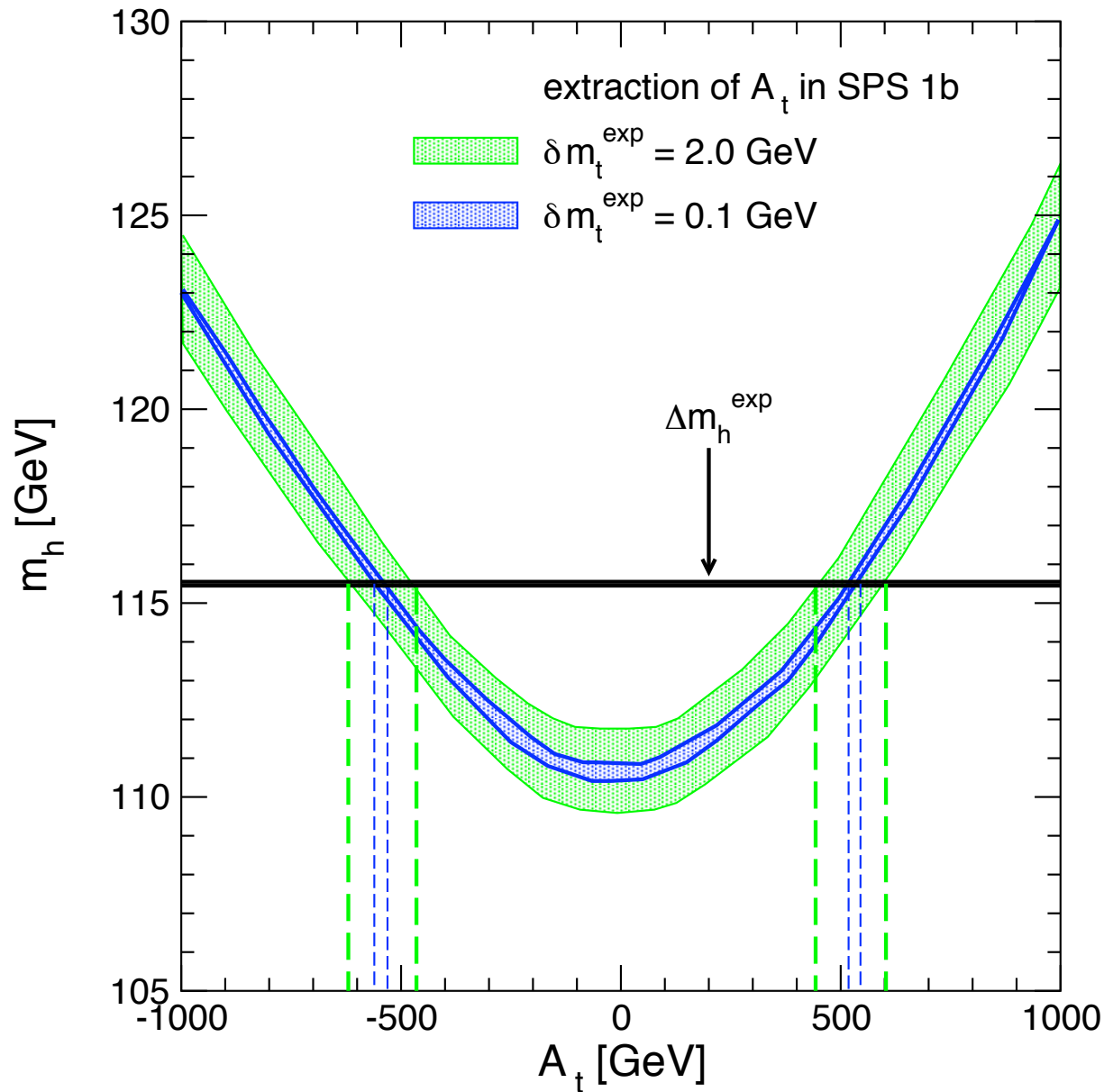
Measurement of  $M_h$ , Higgs couplings  $\Rightarrow$  test of the theory

LHC:  $\Delta M_h \approx 0.2$  GeV

LC:  $\Delta M_h \approx 0.05$  GeV

$\Rightarrow M_h$  will be (the best?) electroweak precision observable

Example of application:  $M_h$  prediction as a function of  $A_t$



SPS1b:

$m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_{\tilde{b}_1}, m_{\tilde{b}_2}$  known,

$A_t$  unknown

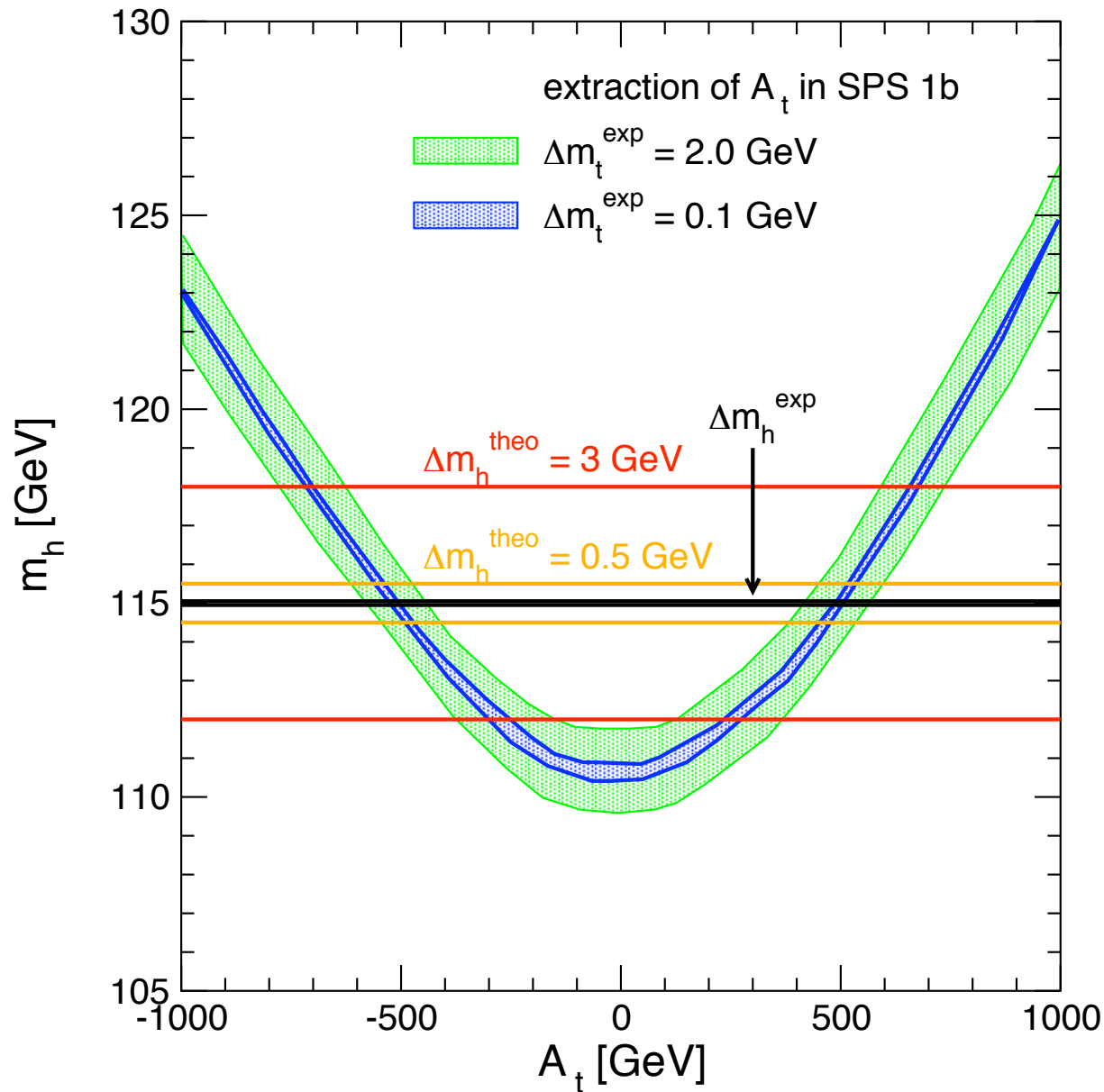
$\tan \beta, M_A$  known,

realistic experimental errors assumed

$\Rightarrow$  extraction of  $A_t$  possible

but theory errors neglected!

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$A_t$  unknown

$\tan \beta, M_A$  known,

realistic experimental errors assumed

$\Rightarrow$  extraction of  $A_t$  possible

$\Rightarrow \Delta m_h^{\text{theo}}$  has to be under control

$\Rightarrow$  crucial for SUSY fits

## 4. Implications for other models

### Precision observables in BMSSM:

Problem: Theorists are lazy ...

No “real” precision observables are calculated

At most:  $S, T, U$  (with  $U = 0$  ...)

[*M. Peskin, T. Takeuchi '92*]

$$\Delta\rho \sim -\alpha T$$

$$\rho = \frac{1}{1 - \Delta\rho} \quad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2} \quad \Delta\rho^{\text{SM}} \sim m_t^2$$

(leading, process independent terms)

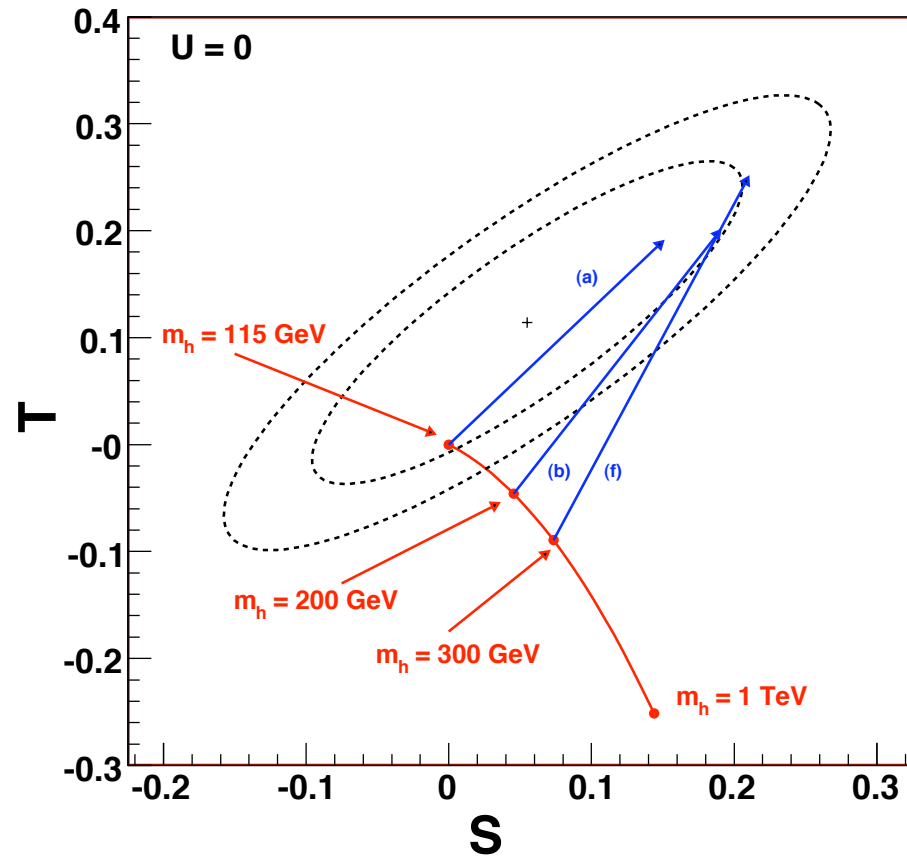
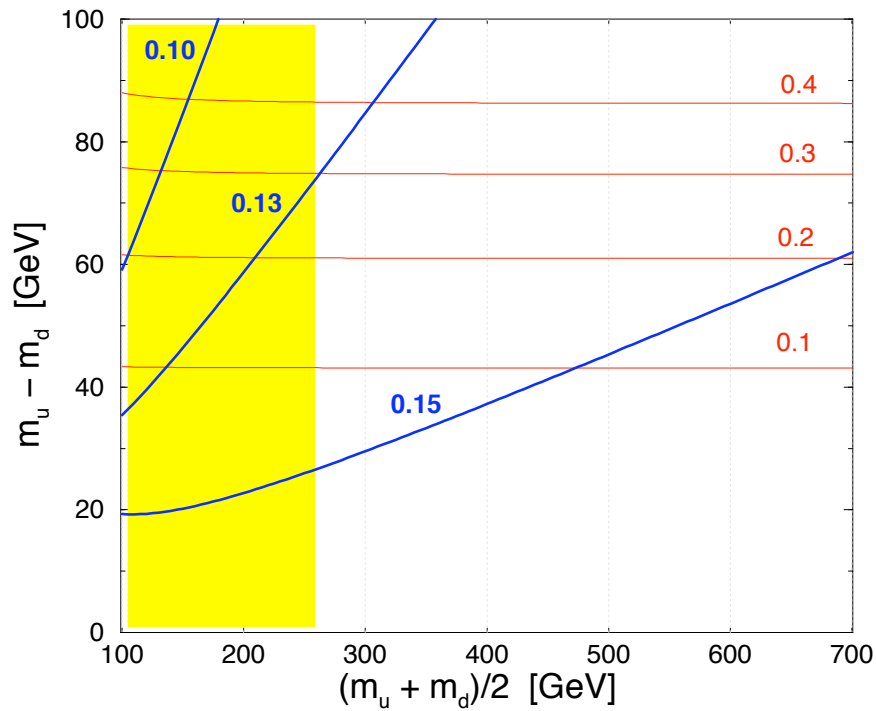
$\Delta\rho$  gives the main contribution to EW observables:

$$\Delta M_W \approx \frac{M_W}{2} \frac{c_W^2}{c_W^2 - s_W^2} \Delta\rho, \quad \Delta \sin^2 \theta_W^{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2} \Delta\rho$$

## Example for SM4:

SM4 = SM + 4th generation of quarks and leptons

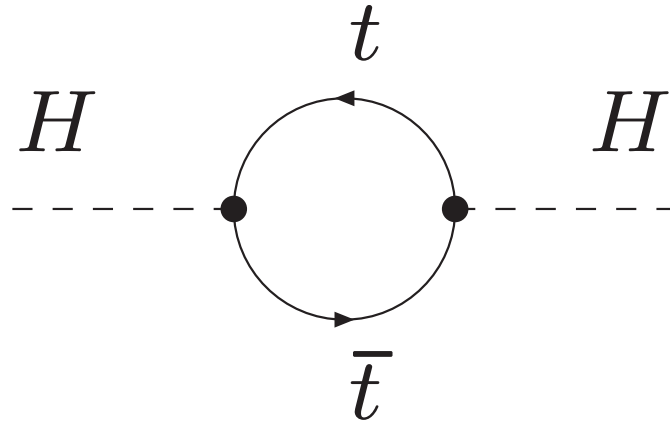
[G. Kribs, T. Plehn, M. Spannowsky, T. Tait '07]



⇒ to play this game requires LC precision for  $m_t$ !

## Higgs physics in BSM:

Nearly any model: large coupling of the Higgs to the top quark:



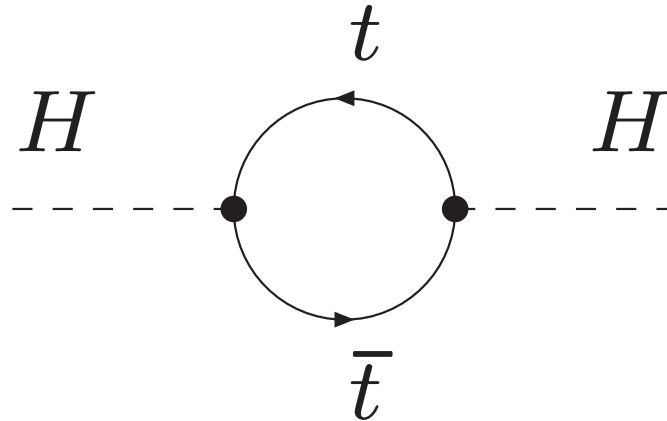
⇒ one-loop corrections  $\Delta M_H^2 \sim G_\mu m_t^4$

⇒  $M_H$  depends sensitively on  $m_t$  in all models where  $M_H$  can be predicted (SM:  $M_H$  is free parameter)

SUSY as an example:  $\Delta m_t \approx \pm 1 \text{ GeV} \Rightarrow \Delta M_h \approx \pm 1 \text{ GeV}$

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⇒ Precision Higgs physics needs precision (LC!) top physics



## Tricky scenario:

The LHC finds only a **SM-like Higgs** and nothing else

**Q:** Do we still need the **LC** with **GigaZ**?

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**A:** Of course!

## Tricky scenario:

The LHC finds only a **SM-like Higgs** and nothing else

**Q:** Do we still need the **LC** with **GigaZ**?

**A:** Of course! Or better: **even more!**

The **LC+GigaZ** provides:

- precise **Higgs coupling** measurements (**LC**)
- precision **observable** measurements (**GigaZ**)
- precise **top mass** measurement (**LC/GigaZ**)

⇒ Only the **LC+GigaZ** can find deviations from the SM predictions via the various precision measurements

⇒ **Only the LC+GigaZ** can point towards extensions of the SM

## 5. Conclusions

- What does LC/GigaZ add to the LHC measurements?
  - The LC will add a precise  $m_t$  measurement (+ much more)
  - GigaZ will add precise measurements of  $M_W$ ,  $\sin^2 \theta_{\text{eff}}$  and  $m_t$

⇒ crucial for indirect model testing
- SM: precise indirect determination of  $M_H$
- MSSM: strong constraints on the parameter space:
  - possibly: discriminate between SM and MSSM
  - precise  $m_t$  crucial for precision Higgs physics  
→ extraction of  $A_t$  (crucial for SUSY fits)
- Other models: much less advanced, mostly  $S, T, U$ 
  - $M_H$  is not a free parameter (as in nearly any BSM):  
precise  $m_t$  crucial for precision Higgs physics
  - “only” a SM-like Higgs at the LHC: LC+GigaZ are the only option!