# Top Mass Precision Measurements at Linear Colliders

Frank Simon

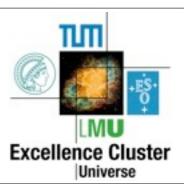
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Munich, Germany









#### Outline

- What are we measuring an how?
- Top mass at hadron colliders
- Top mass through invariant mass of decay products
- Top mass through threshold scan
- Summary / Outlook





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Disclaimer: Many different studies exist, and can not all be represented here.

I will show mostly studies performed in the CLIC context (since they are the most recent ones), inspired by ILC / Tesla studies, and in most cases following the same strategies.





## What are we measuring?

- Experimentally, masses of unstable particles are usually measured through the invariant mass of the decay products
- This is not what is used in theory!
- Several mass definitions exist for the top quark (1s, msbar, pole...) that are theoretically well defined, conversion possible (sometimes with uncertainties on the level of  $\Lambda_{QCD}$ )
  - Invariant mass probably closest to pole mass definition, with additional uncertainties
- Ideally: Measure mass in a theoretically well defined observable, or even better, in several ways





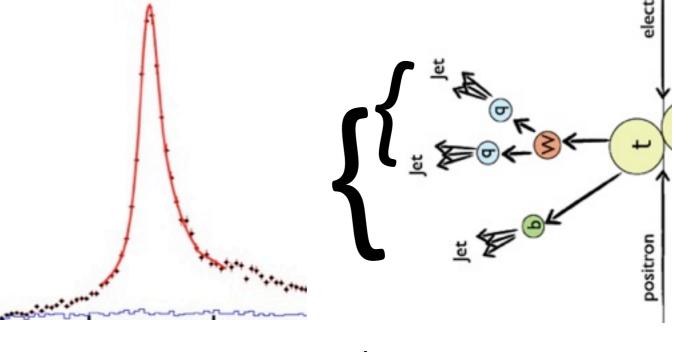
#### Top Mass at Linear Colliders

 Measurement in top pair production, two possibilities, each with advantages and dis-advantages:

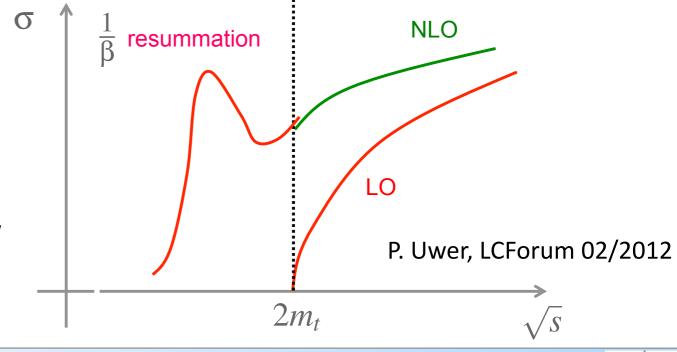
Invariant mass

experimentally well defined

 can be performed at arbitrary energy above threshold: high integrated luminosity



- Threshold scan
  - theoretically well understood
  - needs dedicated running of the accelerator (but still can also provide other measurements below top threshold - Higgs for example)



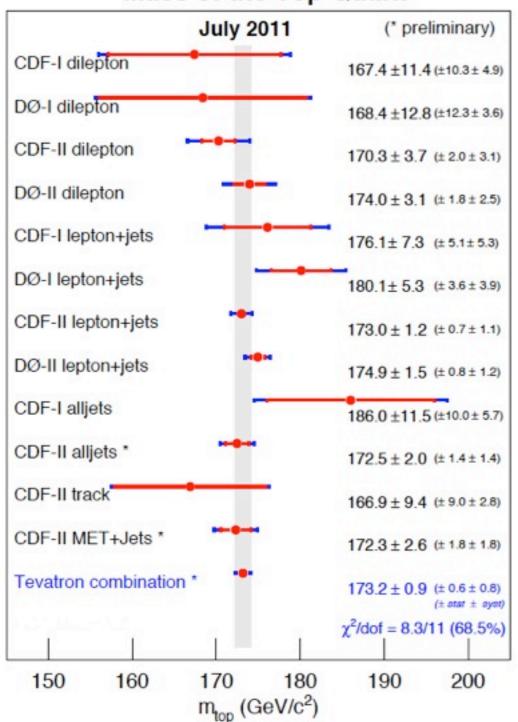




#### **Top Mass - Status & LHC Prospects**

At present, the best measurement still comes from the Tevatron

#### Mass of the Top Quark



Total error below 1 GeV
Systematics the biggest error source

arXiv:1203:5755

At hadron colliders: Systematics the limiting factor, in particular jet energy scale and MC modeling: Getting significantly below 1 GeV is tough.



#### Top Production and backgrounds at Linear Colliders

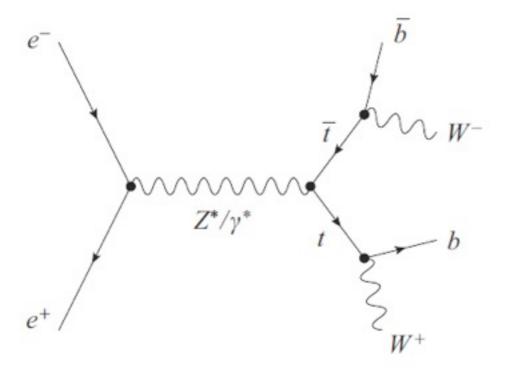
$$\sqrt{s} = 500\,\mathrm{GeV}$$
, CLIC beam energy spectrum

process type	$e^+e^- \rightarrow$	cross section $\sigma$	event generator
Signal $(m_{\rm t} = 174  {\rm GeV})$	$t ar{t}$	528 fb	PYTHIA
Background	WW	$7.1\mathrm{pb}$	PYTHIA
Background	ZZ	$410\mathrm{fb}$	PYTHIA
Background	$qar{q}$	$2.6\mathrm{pb}$	WHIZARD
Background	WWZ	$40\mathrm{fb}$	WHIZARD

- Always: All possible decay channels simulated, selection of final states of interest as part of the analysis
  - Use of PYTHIA to guarantee correct width of intermediate and final-state bosons

# Reconstructing Top Quarks at Lepton Colliders

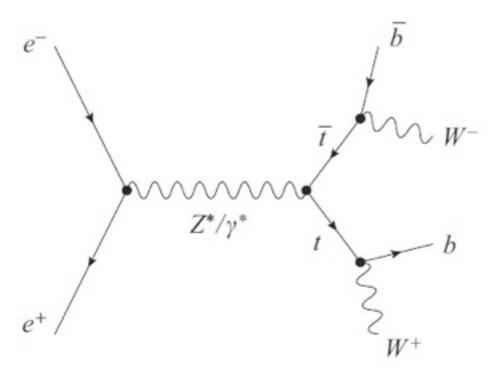
- Driven by production and decay:
  - Production in pairs, decay to W and b



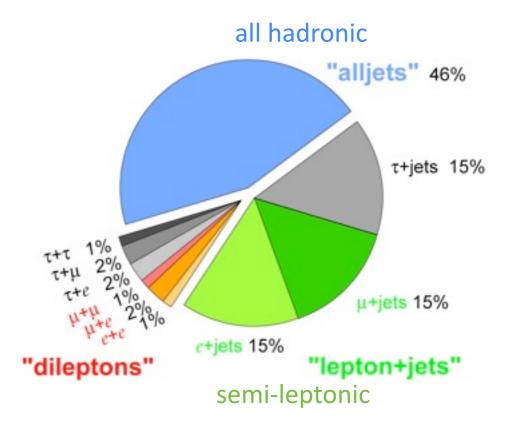


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Event signature entirely given by the decay of the W bosons:

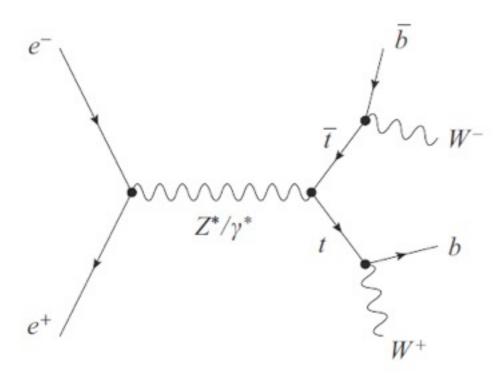




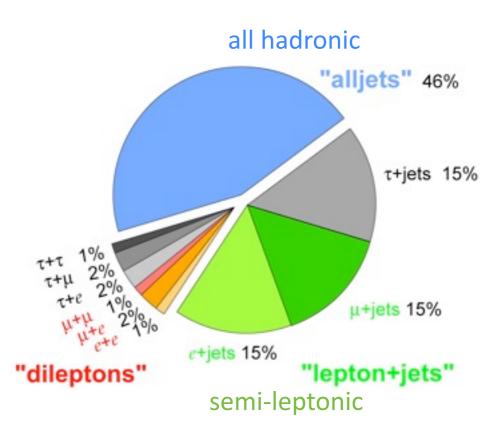


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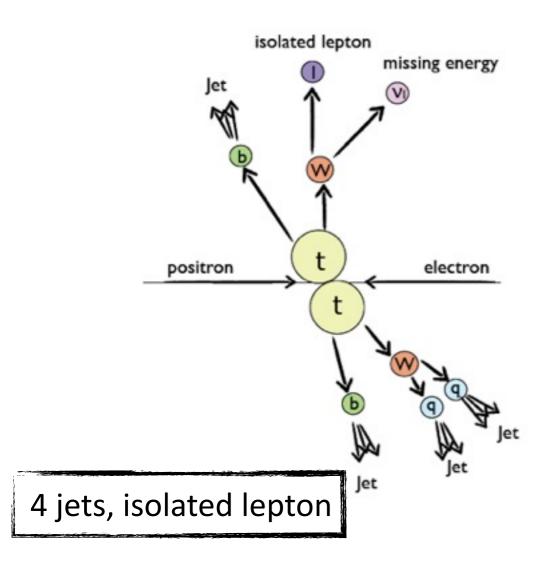


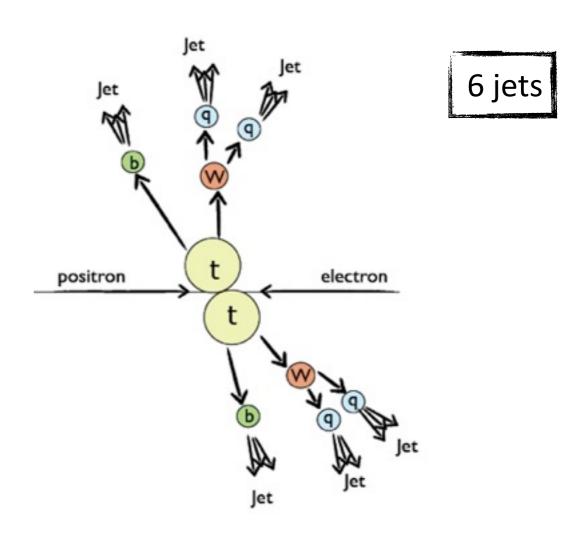
- At hadron colliders: Hard to pick out top pairs from QCD background Use one and two-lepton final states
- At lepton colliders: Top pairs easy to identify, concentrate on large branching fractions and controllable missing energy (not more than one neutrino!)





- By far dominating decays: All-hadronic (46%), semi-leptonic / lepton+jets (45%, 30% w/o τ)
  - $\bullet$  try to avoid decays into  $\tau$ , increased uncertainties from additional neutrino

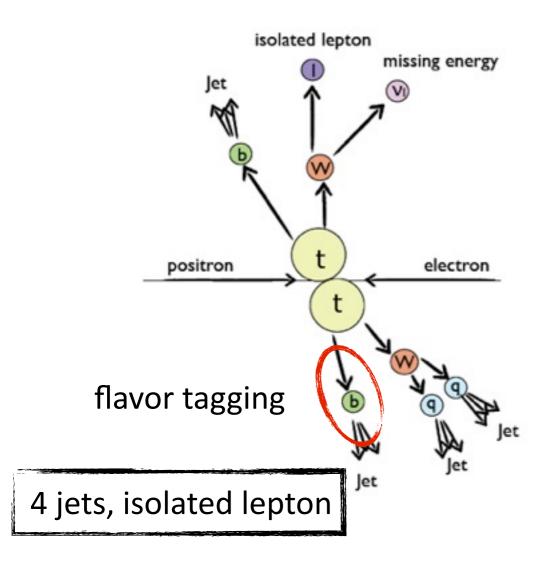


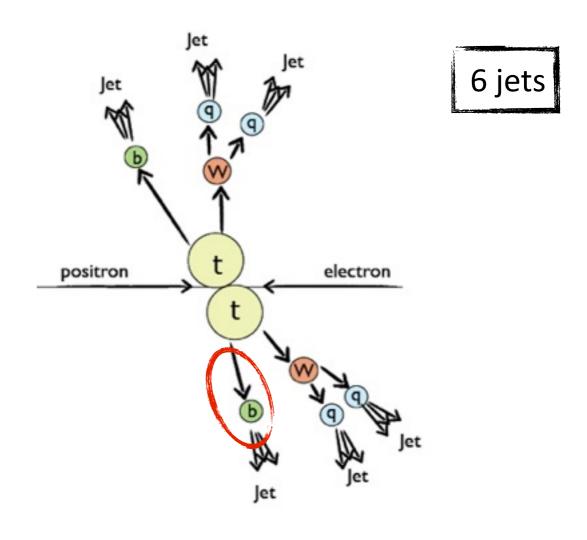




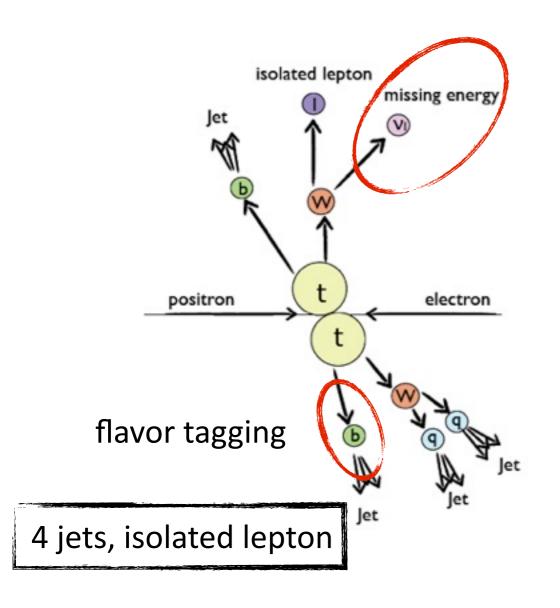


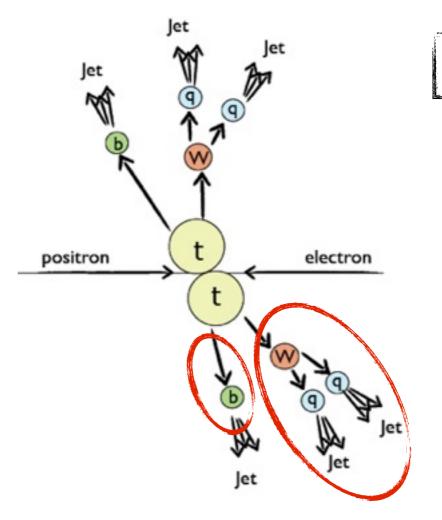
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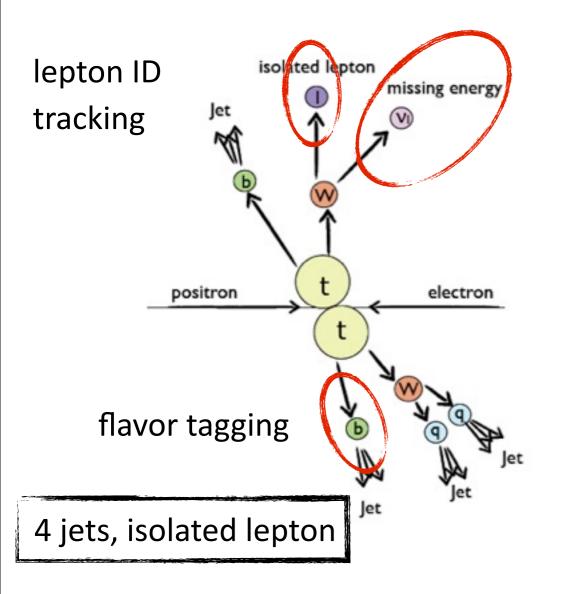
jet energy reconstruction, global event reconstruction

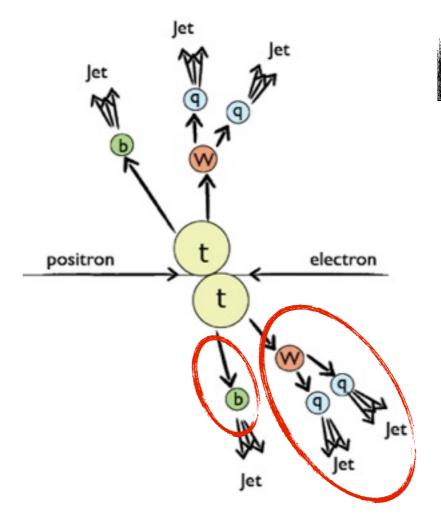




6 jets

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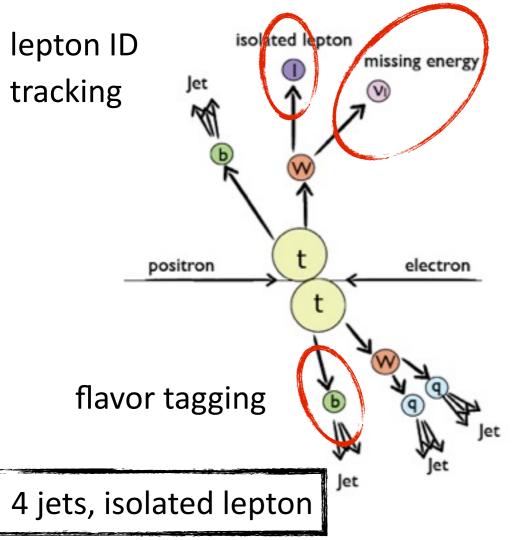
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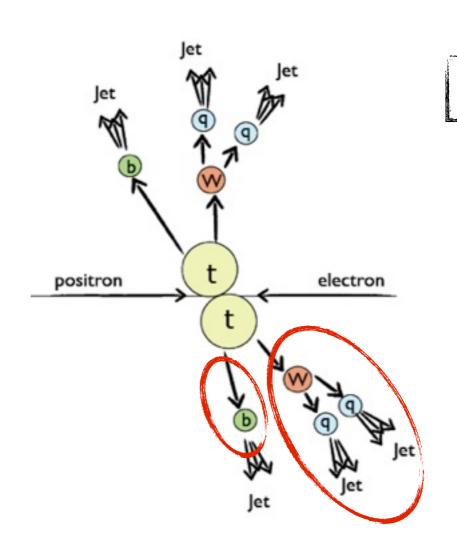




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jet energy reconstruction, global event reconstruction

Uses all aspects of LC detectors!





6 jets

## Invariant Mass Reconstruction - Exploiting ete

- Three key advantages at e<sup>+</sup>e<sup>-</sup> colliders:
  - Well-defined initial state: Can use full 3D energy constraints, not just transverse
  - Clean conditions: More powerful flavor tagging, reduction of background
  - Detectors optimized for precision: Improved jet energy resolution





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- The strategy:
  - Group all events (signal and background) in top candidates:
    - all-hadronic: No isolated lepton, event is clustered into six jets
    - semi-leptonic: One isolated lepton, neutrino from missing energy, event is clustered into four jets (excluding lepton)
    - fully leptonic: Two or more isolated leptons: These events are rejected large uncertainties in mass reconstruction due to two neutrinos, overall less than 10% of BR
  - Find two b jets: Flavor-tag all jets in the event, taking the two most probable bjets as b candidates





## **Building the Top: W Bosons**

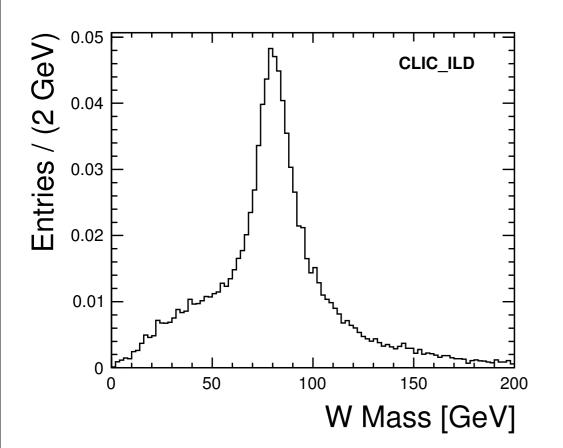
Reconstruct on-shell W bosons

#### Semi-leptonic events

- 2 b-jets
- 2 light-jets : first W
- 1 lepton

second W

missing energy / neutrino

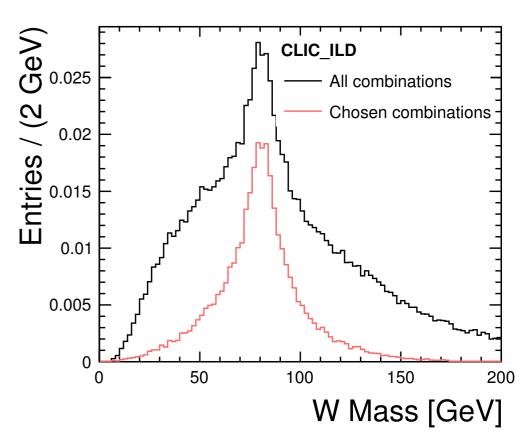


#### All-hadronic events

- 4 light-jets
- 2 b-jets
- Find two best W candidates:

$$|m_{ij} - m_W| + |m_{kl} - m_W|$$

Minimum value defines best permutation

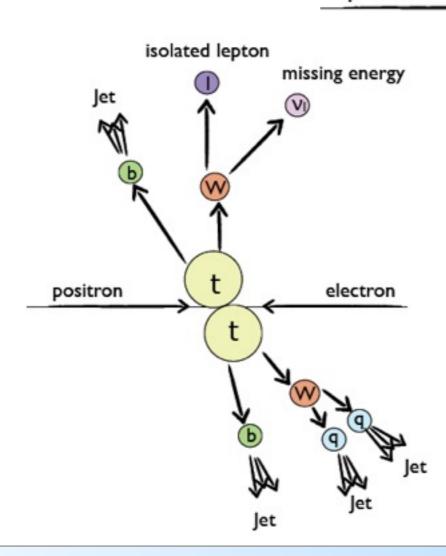




# Building the Top: Combining W and b

Kinematic fit uses constraints from signal event topology to correct measured properties of decay products

- Constraints for four and six jet events:
  - Energy conservation
  - Momentum conservation
  - W mass equals 80.4 GeV
  - Equal top masses







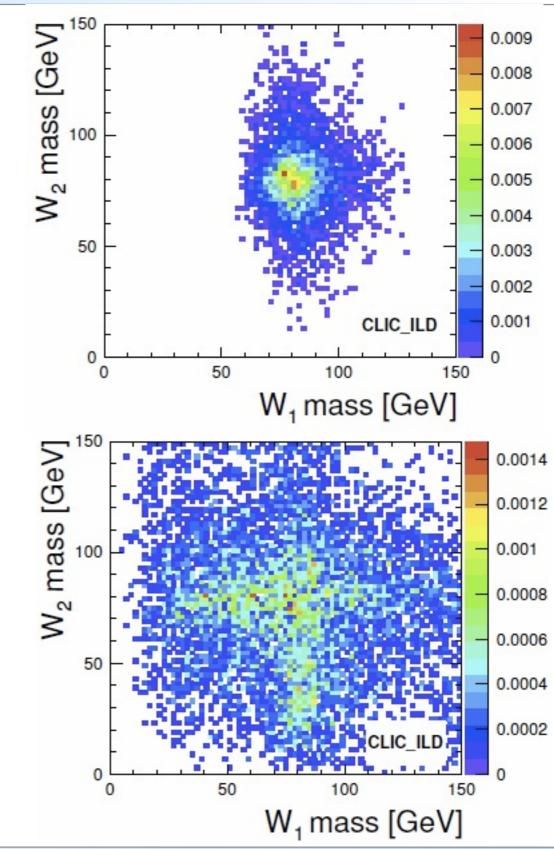
electron

positron

# Building the Top: Combining W and b

Kinematic fit uses constraints from signal event topology to correct measured properties of decay products

- Constraints for four and six jet events:
  - Energy conservation
  - Momentum conservation
  - W mass equals 80.4 GeV
  - Equal top masses
- Use kinematic fit for final Wb pairing
- Only very clean events pass kinematic fit
  - In case of fit failure: re-examine flavor assignment (recovers W decay into charm)
     10% increase in success rate

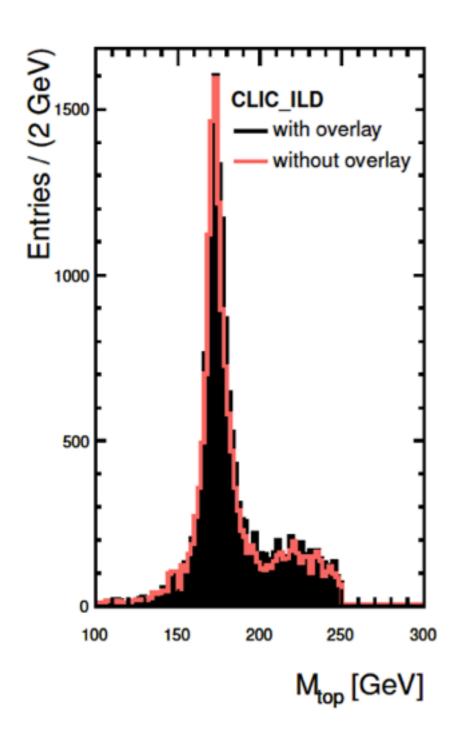


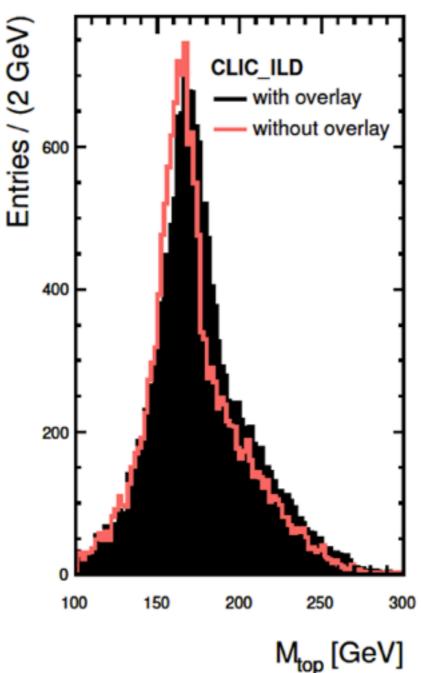




## The Power of Kinematic Fitting

• Improved resolution, increased stability towards pile-up of backgrounds at CLIC





all-hadronic top pairs at CLIC

Should also reduce systematics considerably!



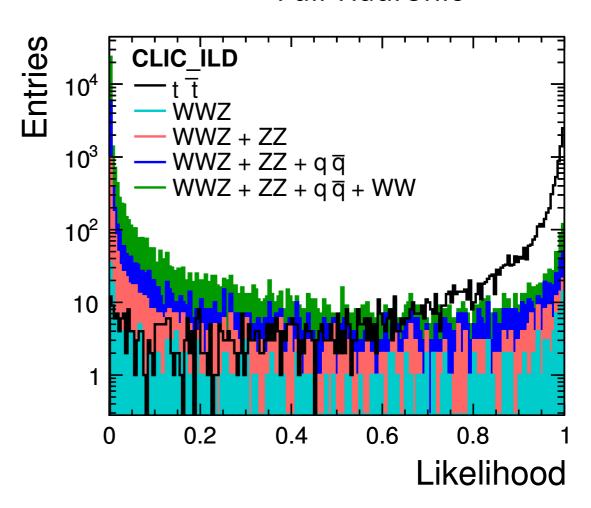
# Cleaning the Sample

#### **Kinematic Fit**

- Powerful Background Rejection for qq,
   WW, ZZ
- Rejection of unwanted signal events:
   full-leptonic events, tau- events

#### Binned likelihood rejection

- Seven input variables (Number of particles in event, value of b-tags, sphericity, ...)
- Likelihood cut of 0.6 chosen
- Training with independent sample Full-Hadronic







# Cleaning the Sample

#### **Kinematic Fit**

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Overall background rejection: > 99% Overall signal selection:

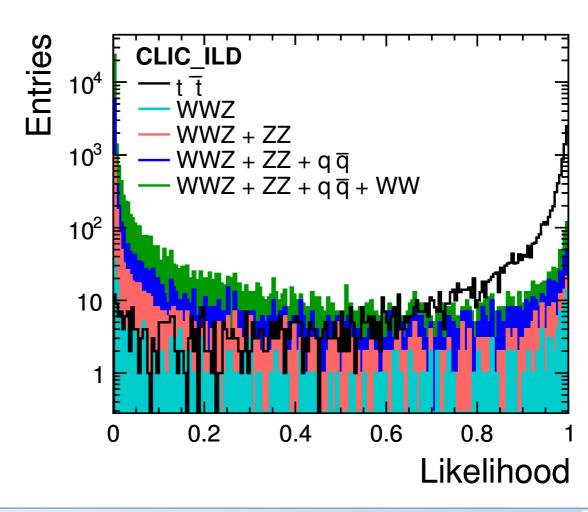
Full-Hadronic: 35%

Semi-Leptonic: 56%

- Signal efficiency could be improved
- Analysis goal: clean events, not maximized statistics

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# Cleaning the Sample

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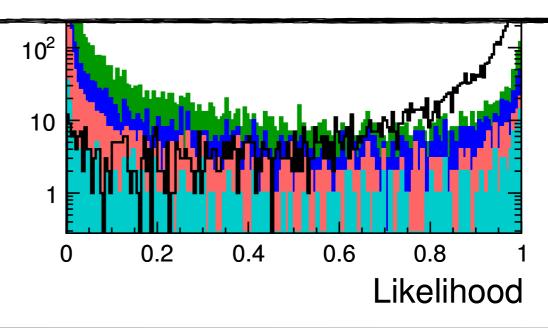
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Kinematic fit and background rejection using likelihood (or other multivariate techniques) can also be performed in reverse order (as was done for ILD LOI) Advantage of doing it this way: Correct assignment of Ws and bs to tops already found before likelihood

Semi-Leptonic: 56%

- Signal efficiency could be improved
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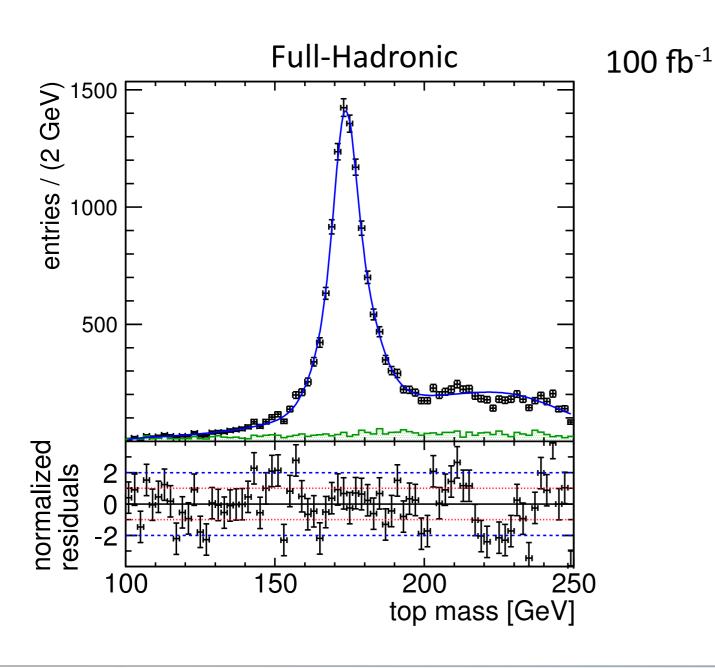


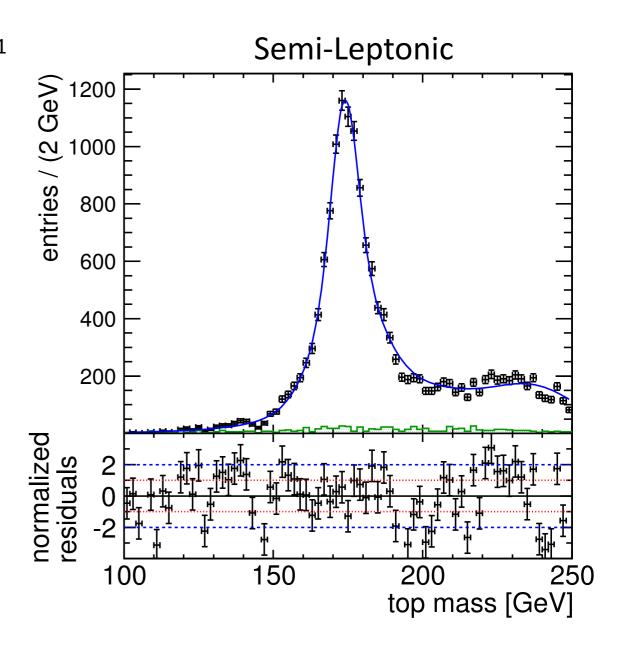


#### Measuring the Mass- CLIC CDR

Un-binned maximum likelihood fit over full range

- Combination of signal and background pdf
- Signal pdf is a convolution of a Breit-Wigner and a detector resolution function



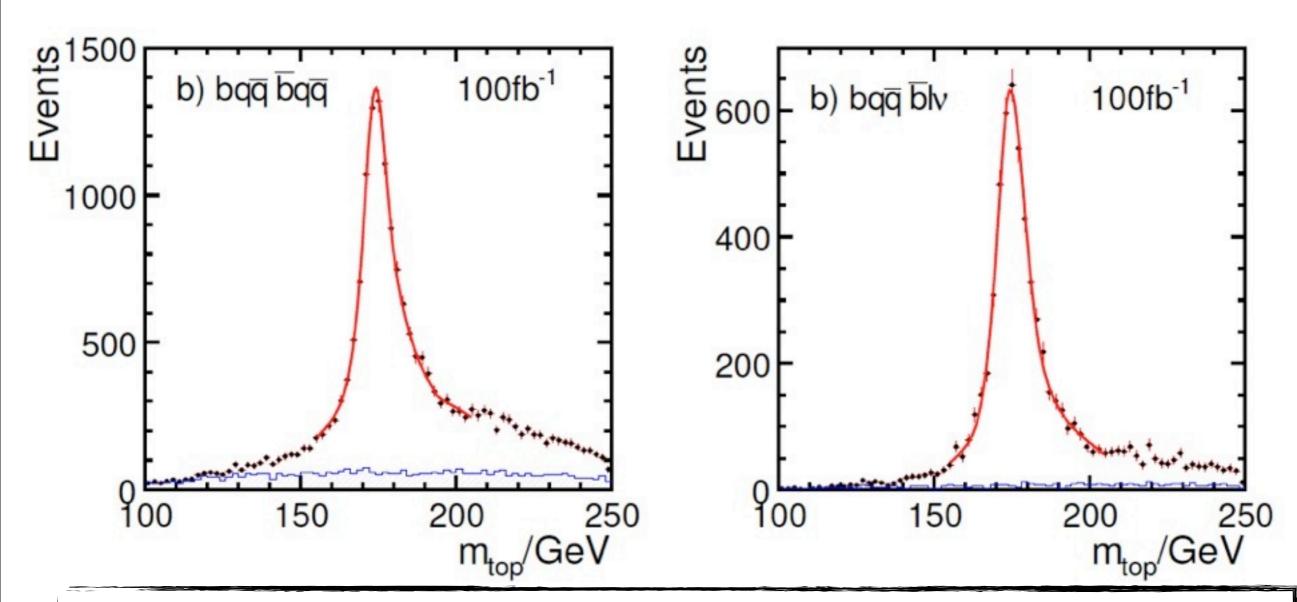






## Measuring the Mass-ILD LOI

Here: ChiSquared fit over limited mass range (some sensitivity to fit range!)



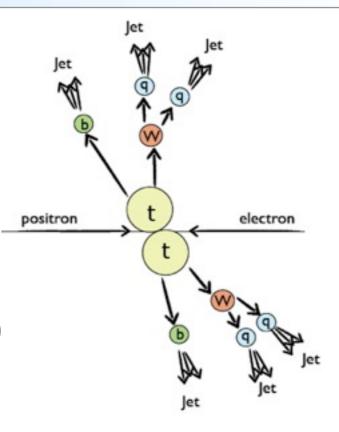
For both cases: Need "detector resolution function", determined by running the fit on an independent, high-statistics calibration sample, and then leaving only mass, width and normalization free in the final fit

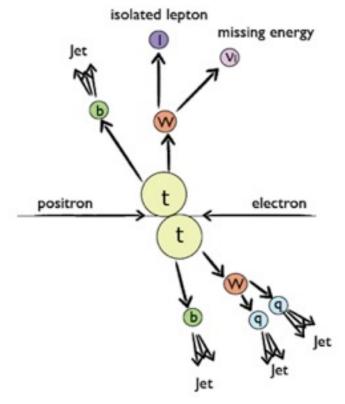




#### Results

- Correct mass is recovered within errors (no surprise, since full simulations are also used to determine detector resolution functions)
- Comparable results for CLIC and ILC:
   ~0.1 GeV to ~0.14 GeV statistical precision per channel
   (slightly larger errors for semi-leptonic due to reduced BR)
   (generator values: m<sub>top</sub> = 174 GeV, width: 1.37 GeV)
- Measurement of width is also possible (statistical uncertainty depends strongly on fit range and technique):
  - CLIC: stat errors of 220 MeV and 260 MeV
  - ILD LOI: stat errors of 60 MeV and 100 MeV



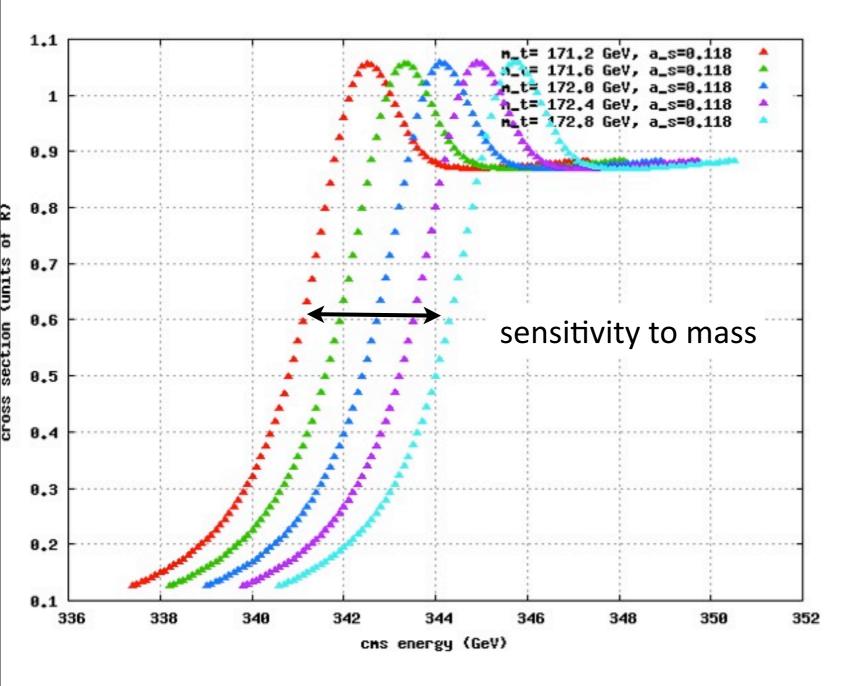






#### A Linear Collider Classic: Threshold Scan

• The primary variable: Top pair production cross section (other variables provide additional sensitivity - A<sub>FB</sub>, top quark momentum distribution)

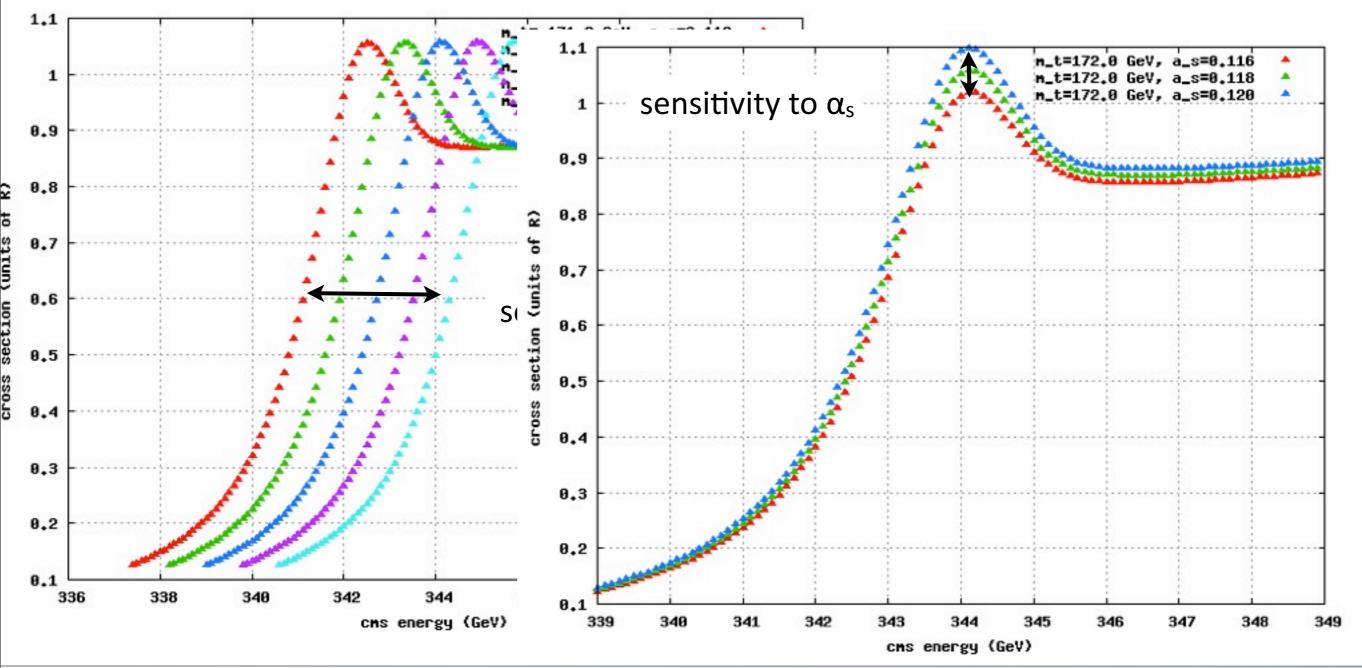






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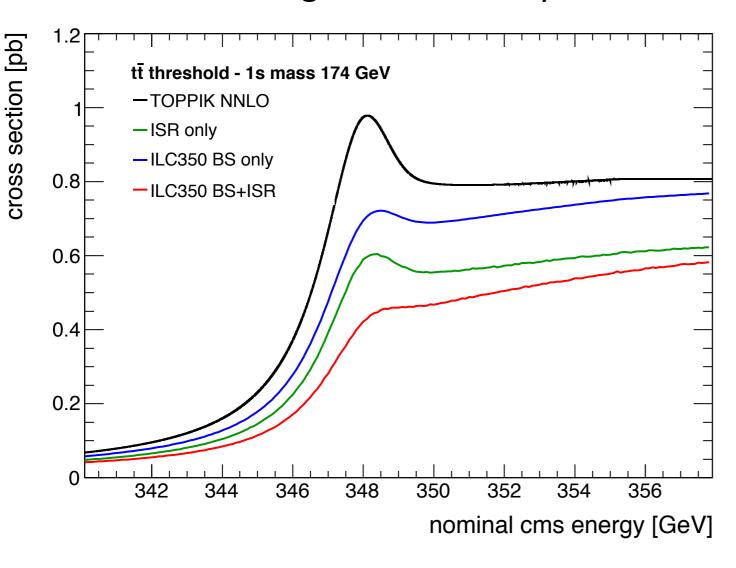






#### Beam Spectrum and ISR

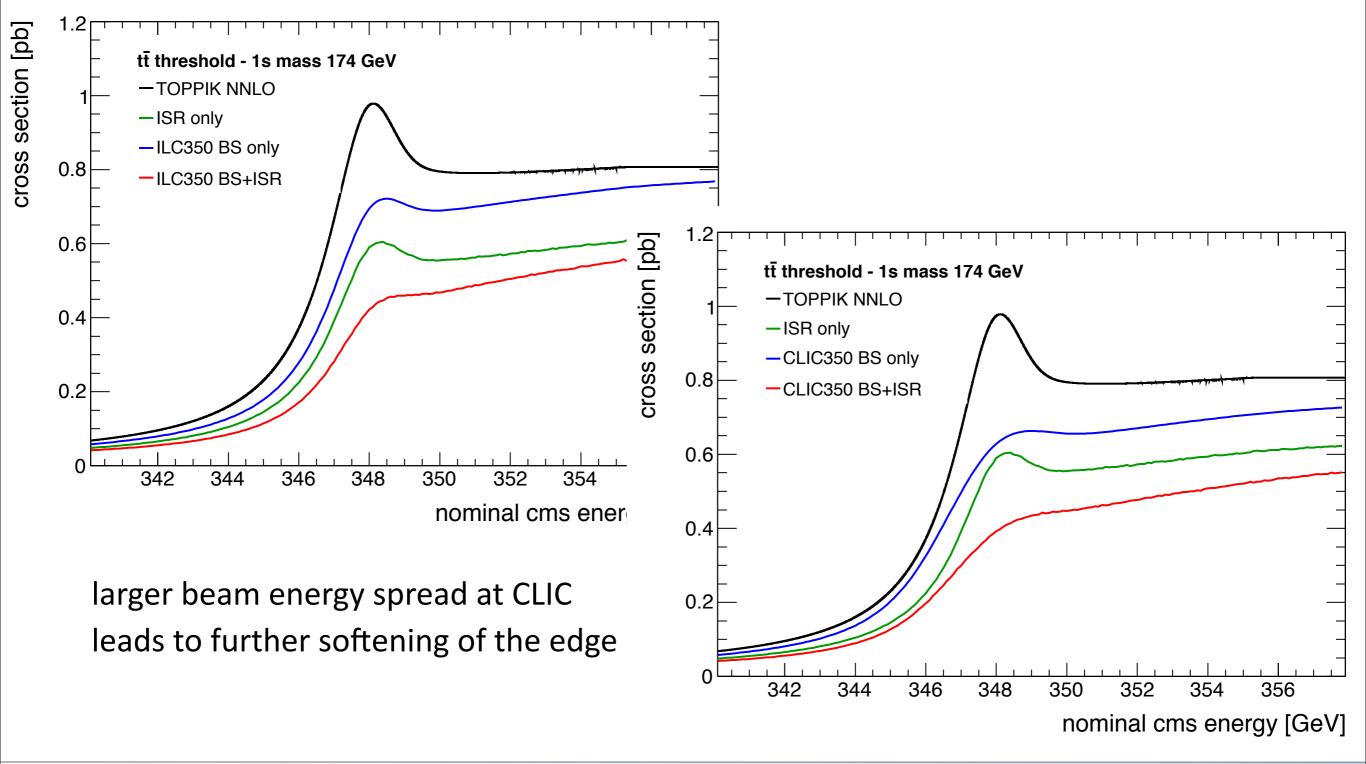
• The threshold gets distorted by the true collision energy distribution





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## The Measurement Strategy - And Simulations

- A simple cross section measurement:
  - Identify top pair events
    - Can follow the same strategy as for invariant mass measurement, potential optimizations to maximize significance (instead of mass resolution)
  - statistically subtract background





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- Simulation Studies: No public event generator for the top threshold exist PYTHIA for example is LO, with hadronization, does not get threshold right
- ▶ Use full NNLO theory calculations to determine cross section as a function of energy (for example TOPPIK, Hoang and Teubner, PRD 60, 114027 (1999))
- ▶ Determine signal efficiency and background contamination from full detector simulations above top threshold





#### Some Simple Games with Numbers

- Extract efficiency and background contamination from 500 GeV CLIC study (might be slightly optimistic due to effects on flavor tagging from lower boost)
- Scale background up by x2 to account for scaling with s (probably rather pessimistic)
- Assumption: 10 scan points with 10 fb<sup>-1</sup> each

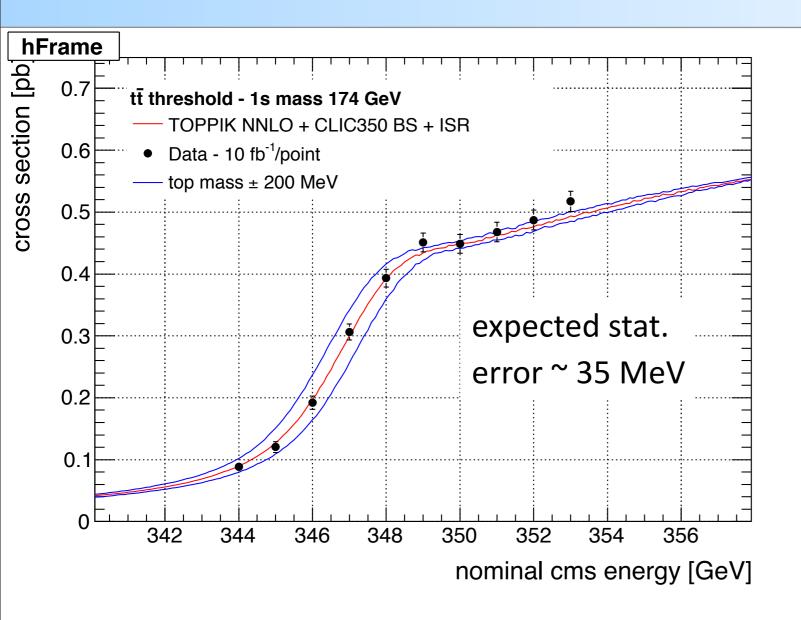
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Preliminary work in the CLIC Physics & Detector studies - No hard numbers yet!



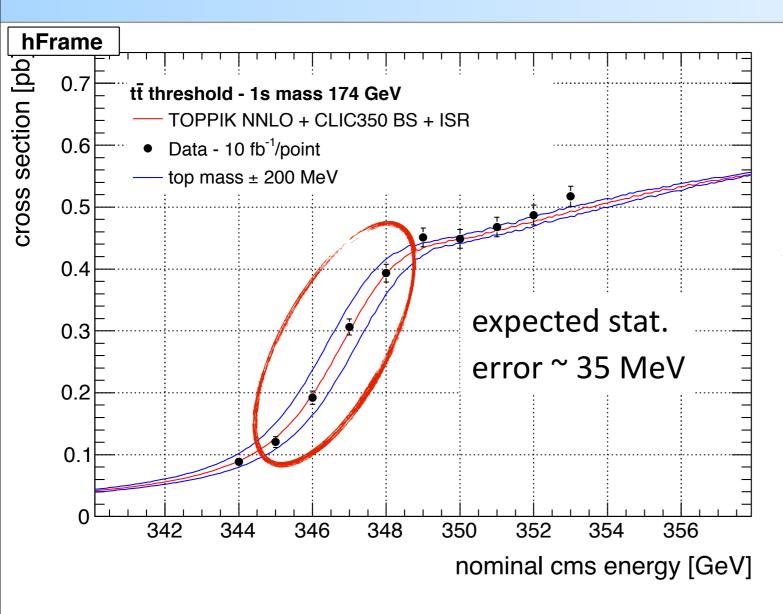
#### **Threshold Scan - Illustration**







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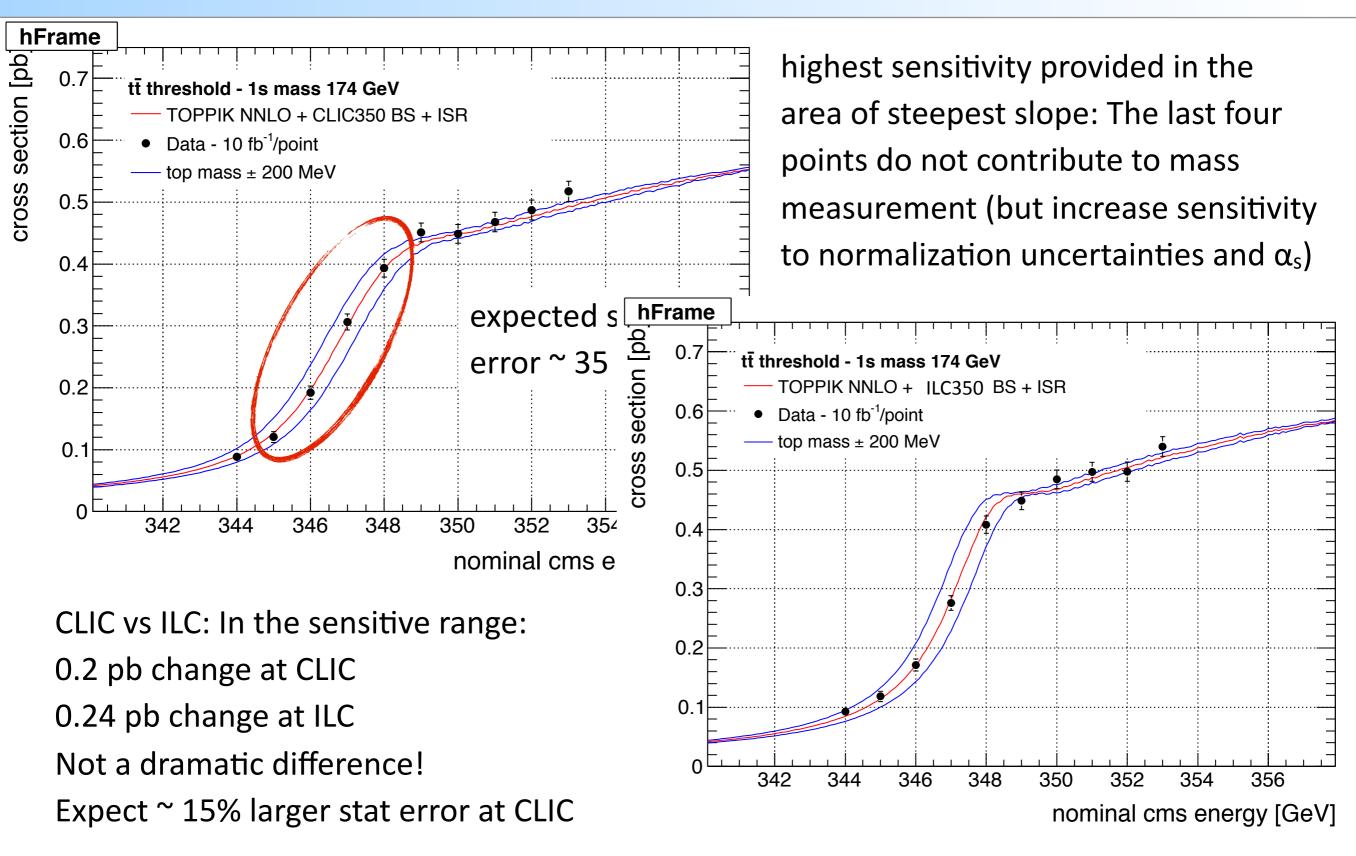


highest sensitivity provided in the area of steepest slope: The last four points do not contribute to mass measurement (but increase sensitivity to normalization uncertainties and  $\alpha_s$ )





#### Threshold Scan - Illustration

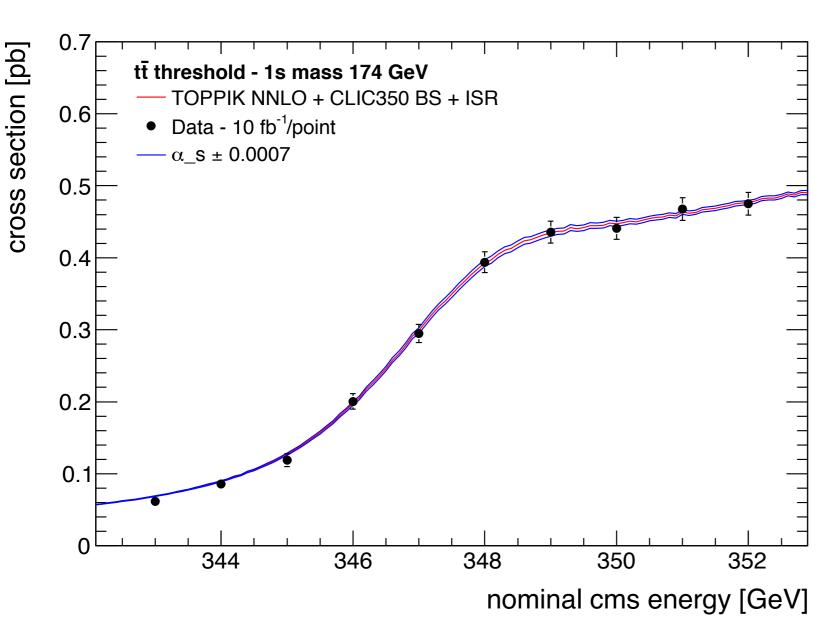






## Beyond Mass: Measuring α<sub>s</sub>

• The top mass and  $\alpha_s$  are correlated: Threshold scan also provides sensitivity to strong coupling

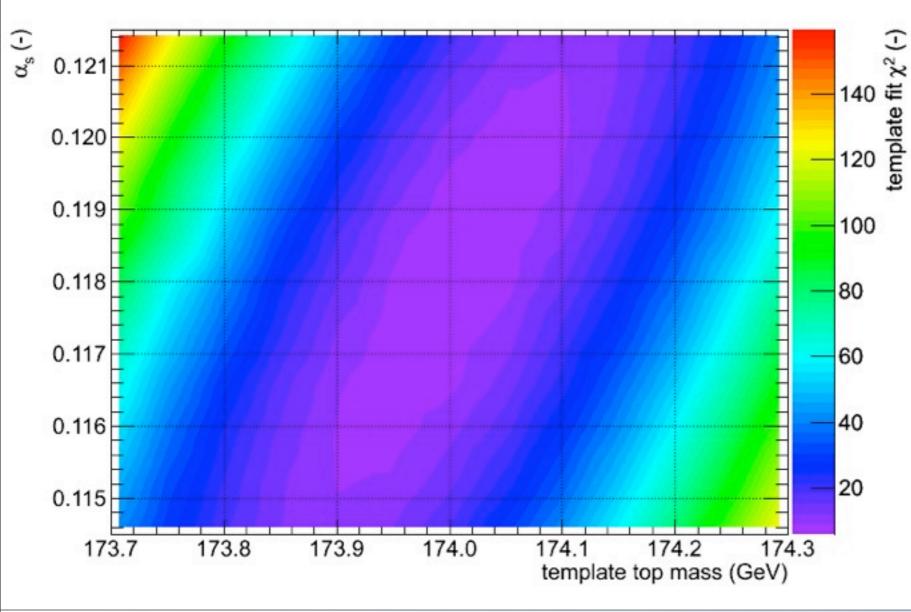


- Thorough previous studies have determined the top mass and  $\alpha_s$  simultaneously
- Current world average of α<sub>s</sub>:
   0.1184 +- 0.0007
   (EPJ C64, 689 (2009))
  - Error probably impossible to beat



#### Correlation between $m_t$ and $\alpha_s$

 High degree of correlation when using the cross section as observable: simultaneous fit of top mass and coupling constant increases uncertainties, requires significant integrated luminosity also above threshold



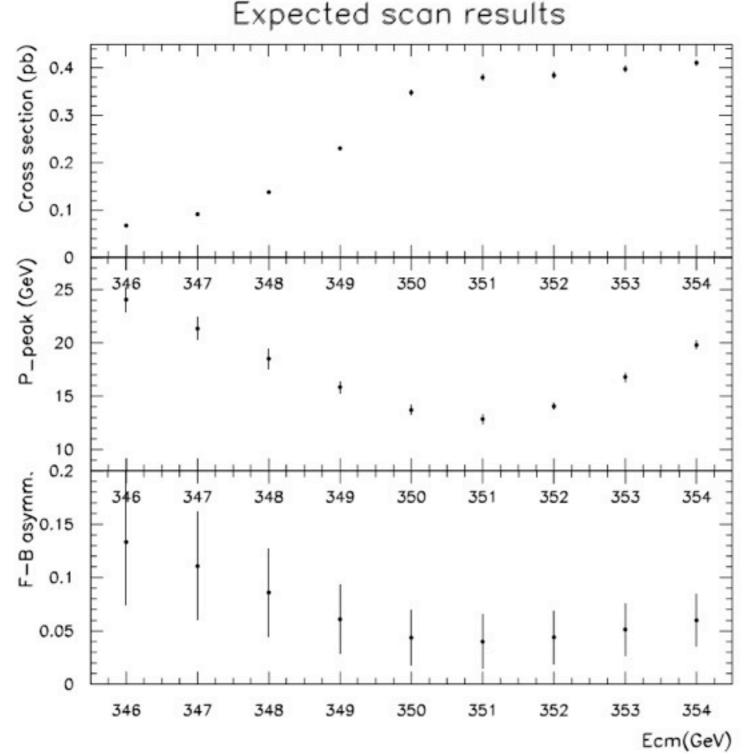
simultaneous extraction
will not provide the best
possible mass
measurement, but can be
an interesting cross-check





## A Previous Thorough Study

Using beam spectra and efficiencies for TESLA



With 30 fb<sup>-1</sup> per point (a total of 300 fb<sup>-1</sup>):

 $16 \text{ MeV on } m_t$   $0.0012 \text{ on } \alpha_s$ 

Width can also be determined, with  $^{\sim}32$  MeV precision (19 MeV on  $m_t$ ) in that case

Using multiple observables seems to help to control effect of systematics on cross section normalization (no other systematics considered in that study)



## Where are possible Limitations?

- Both threshold scans and invariant mass measurements allow to reach statistical precisions on m<sub>t</sub> quite a bit below 100 MeV with reasonable integrated luminosity
- ▶ Quite likely systematic limitations come into the game
  - Invariant mass:
    - Jet energy scale -> can be controlled by reconstruction of intermediate Ws,
       mitigated by kinematic fit (to be studied in more detail, first steps ongoing...)
    - Overall detector response: Our simulations assume "data" matches simulations perfectly
  - Threshold scan:
    - Event selection efficiencies: "data" simulations matching
    - Theory uncertainties: When using just the cross section, a 3% normalization uncertainty can lead to uncertainties of several 10 MeV (same for  $\alpha_s$ )
    - Knowledge of luminosity spectrum essential





# Summary / Conclusion

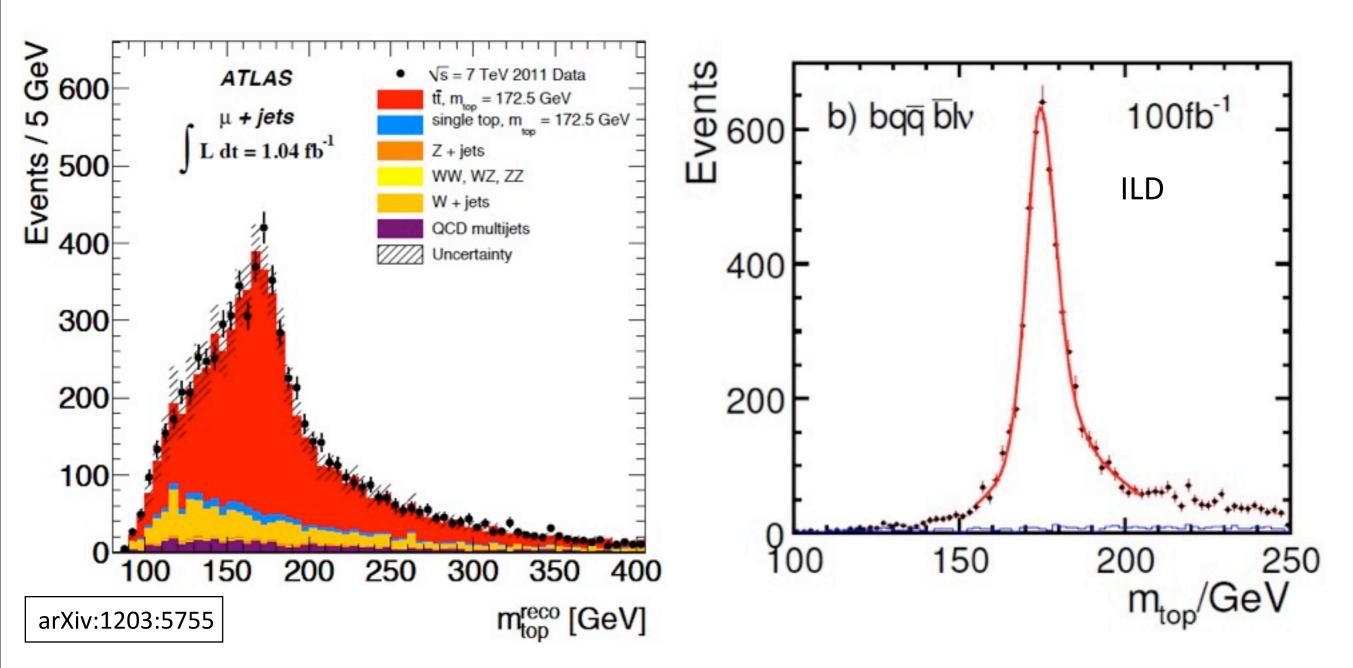
- Linear Colliders offer excellent possibilities for precision top mass measurements:
  - Invariant mass reconstruction above pair production threshold: Statistical errors below 100 MeV reachable with 100 fb<sup>-1</sup>, high integrated luminosity expected, with corresponding further reduction
  - Threshold scan, with extraction of theoretically well-defined mass: Statistical errors of a few 10 MeV reachable with high integrated luminosity (significantly below 100 MeV possible also with a few 10 fb<sup>-1</sup>), theory uncertainties seem manageable, keeping total uncertainty below 100 MeV
- Key questions: Systematic limitations
  - Jet energy scale, understanding of detector resolution for invariant mass (here also uncertainties when connecting to theory)
  - Theory uncertainties (scale, normalization), beam spectrum precision for threshold scan

possibilities for further interesting studies!





## Summary: The Powers of e<sup>+</sup>e<sup>-</sup> Colliders

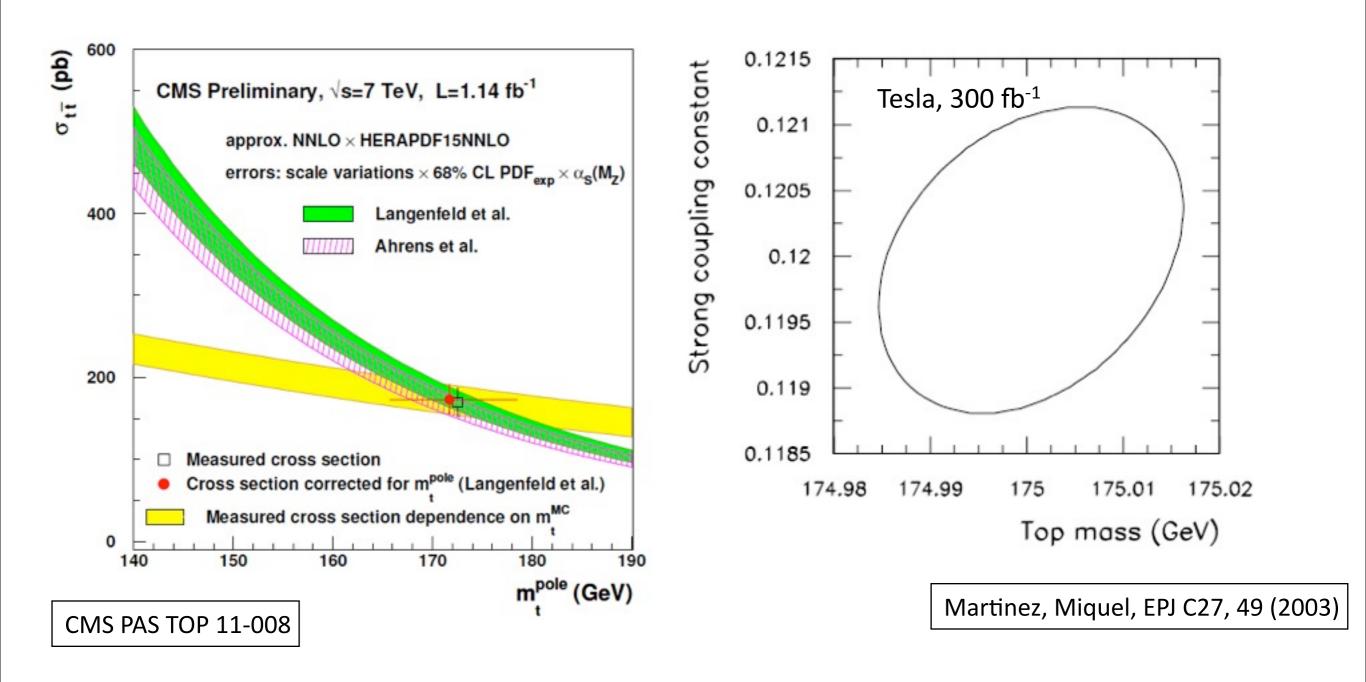


Very low background contributions, excellent mass resolution Clean environment, excellent detector performance





# Summary: The Powers of e<sup>+</sup>e<sup>-</sup> Colliders

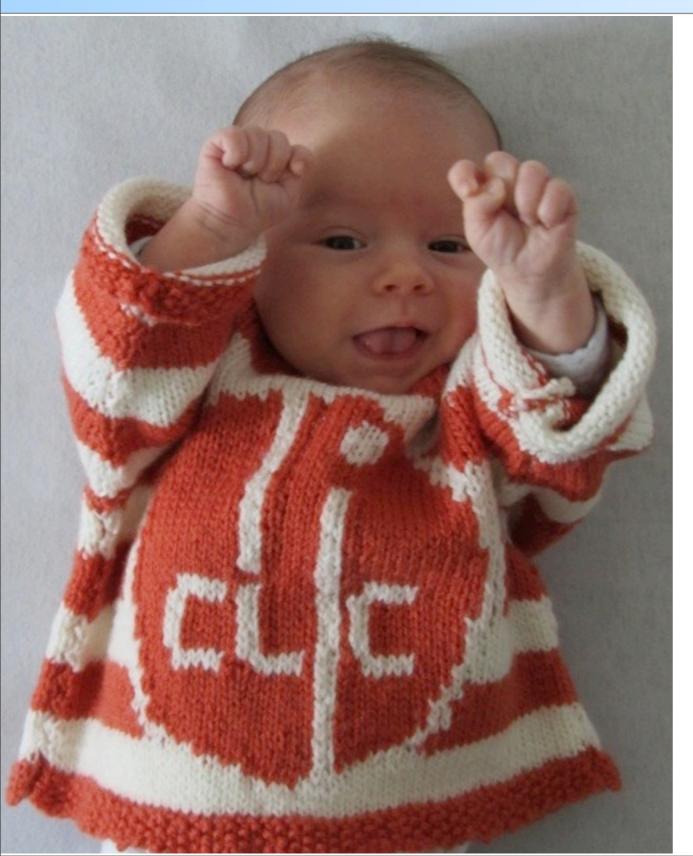


 Threshold scan to provide direct access to theoretically calculable mass definitions: Exploits low backgrounds, small theory uncertainties





# **Excellent Prospects...**

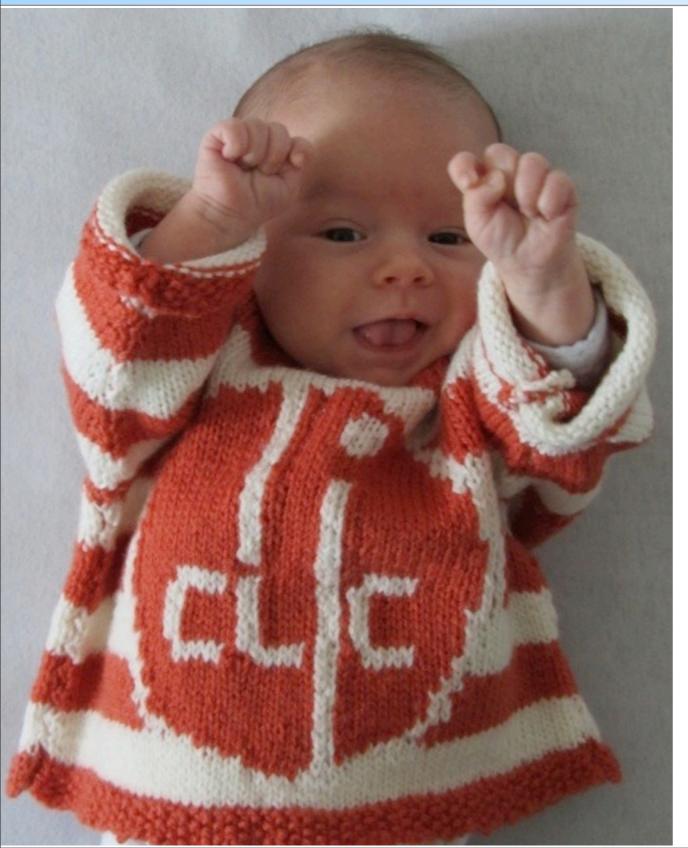


... for precision measurements of the Top mass at linear colliders...

... but it is not quite child's play!



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... but it is not quite child's play!

Any volunteers for an ILC sweater?



