Theory Progress in Top-Antitop Threshold Physics at the ILC

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Outline

- Top-antitop threshold: QCD effects

  [Hoang, MS: 1111.4486 [hep-ph], 1102.0269 [hep-ph]]

- Top-antitop threshold: EW effects

- Associated Higgs production: $e^+e^- \longrightarrow t\bar{t}H$

- Summary/Outlook

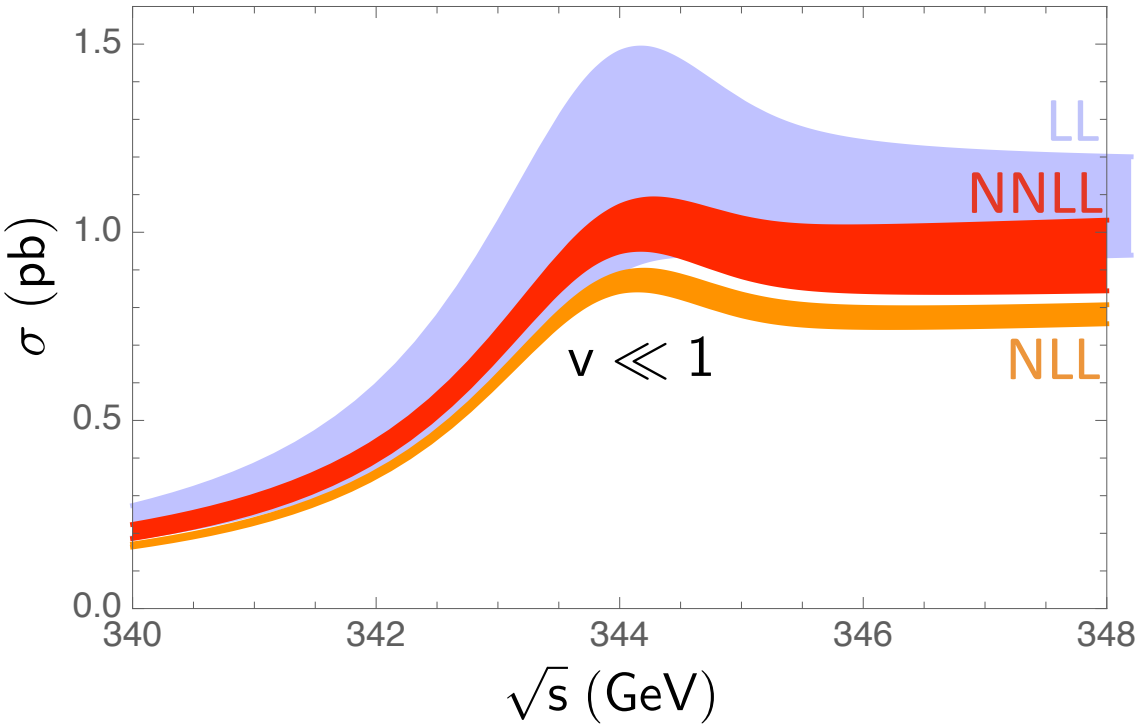
$$e^+e^- \longrightarrow t\bar{t}$$

Top-antitop threshold: QCD effects

[Hoang, MS: 1111.4486 [hep-ph], 1102.0269 [hep-ph]]

Top-antitop threshold: QCD effects

$t\bar{t}$ resonance @ Linear Colider:



$\Gamma_t \approx 1.5\text{GeV} \gg \Lambda_{QCD}$

- Nonpert. effects suppressed
  [Fadin, Khoze, '87]
- No sharp resonance peaks

Top-antitop threshold: QCD effects

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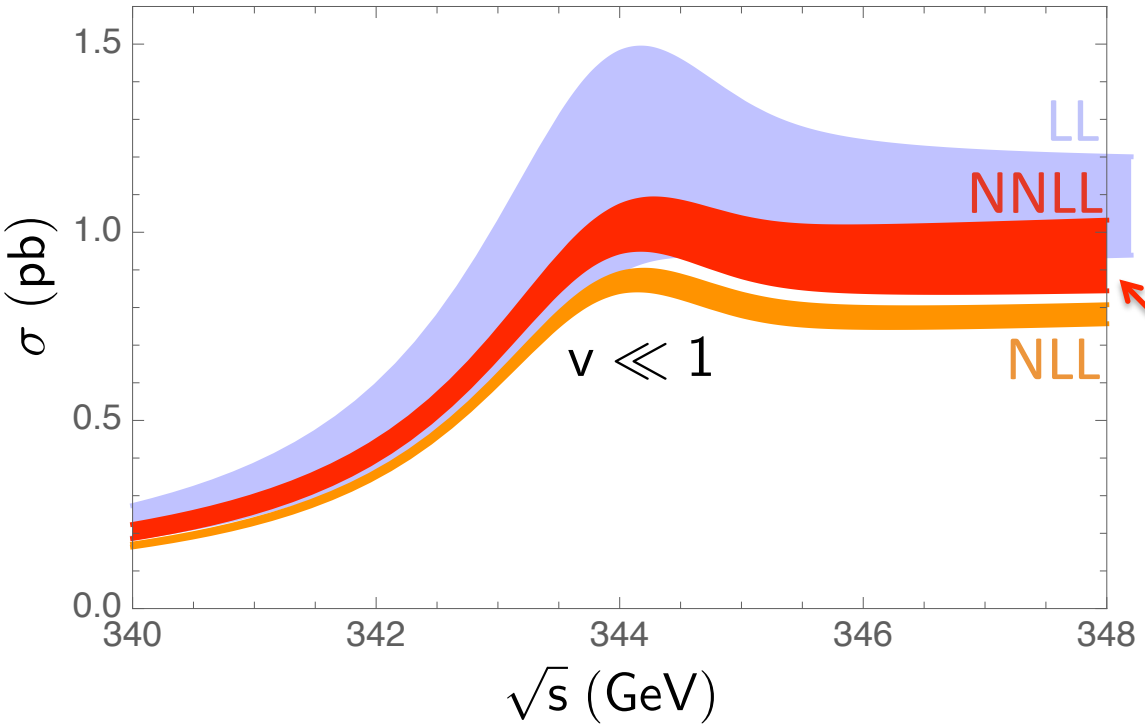


incomplete!!!

Theory status: 2010

$\delta m_t \sim 100\,\text{MeV}$ ✓

$\delta\sigma_{tot}/\sigma_{tot} \sim 6\,\%$

< 3% needed for precise Γ_t, y_t, α_s

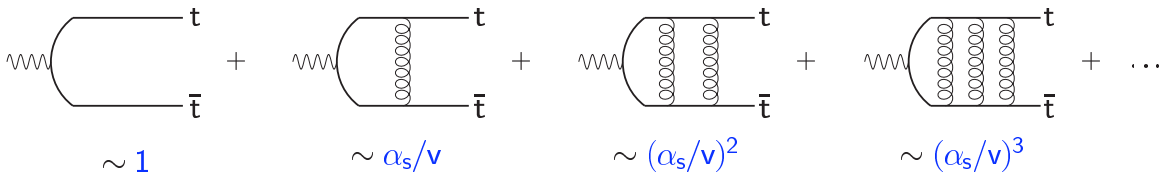
Top-antitop threshold: QCD effects

QCD near threshold: $v \sim \alpha_s \ll 1$

3 scales: $m_t \gg \vec{p} \sim m_t v \gg E_{kin} \sim m_t v^2$ $(\sim \Gamma_t \gg \Lambda_{QCD})$

"hard" "soft" "ultrasoft"

- "Coulomb singularities"



~ 1 $\sim \alpha_s/v$ $\sim (\alpha_s/v)^2$ $\sim (\alpha_s/v)^3$

→ vNRQCD

- Large logs: $\ln(E^2/m^2), \ln(\mathbf{p}^2/m^2), \ln(E^2/\mathbf{p}^2) \sim \ln v$

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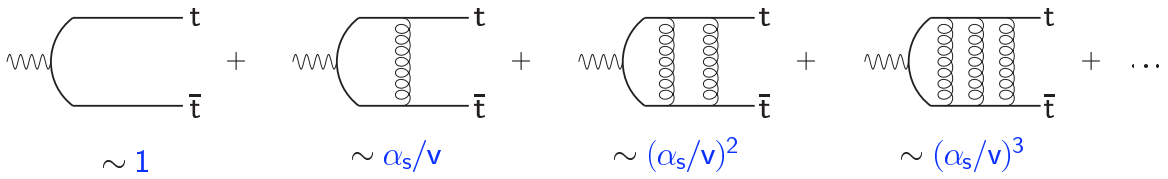

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↓

modified RG:

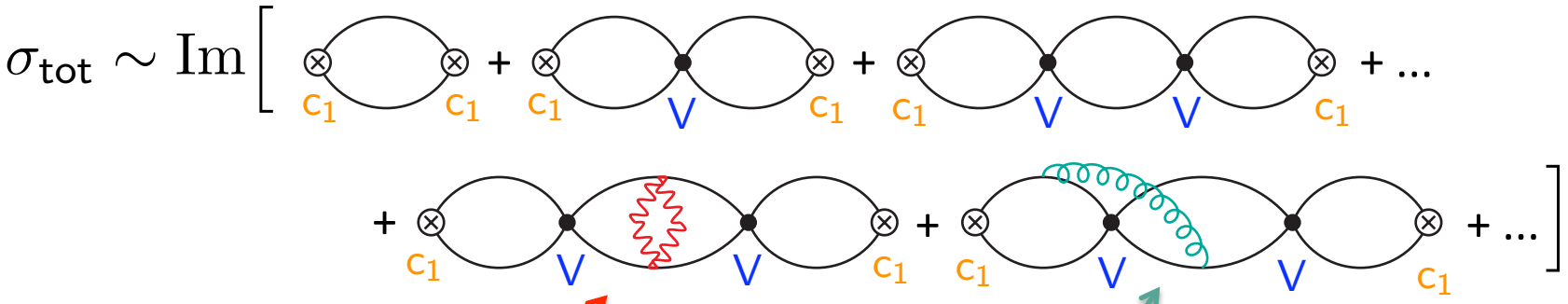
ν "subtraction velocity"

[Luke, Manohar, Rothstein '00]

→ RGE's resum logs: $[\alpha_s \ln v]^n$, $\alpha_s [\alpha_s \ln v]^n$, $\alpha_s^2 [\alpha_s \ln v]^n$ …

LL NLL NNLL

Top-antitop threshold: QCD effects

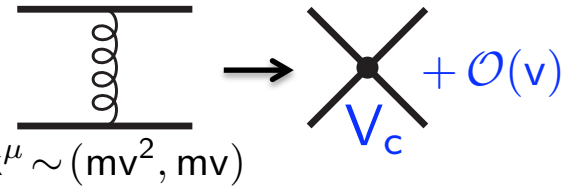


vNRQCD

soft gluon: $k^\mu \sim mv$

ultrasoft gluon: $k^\mu \sim mv^2$

potential:

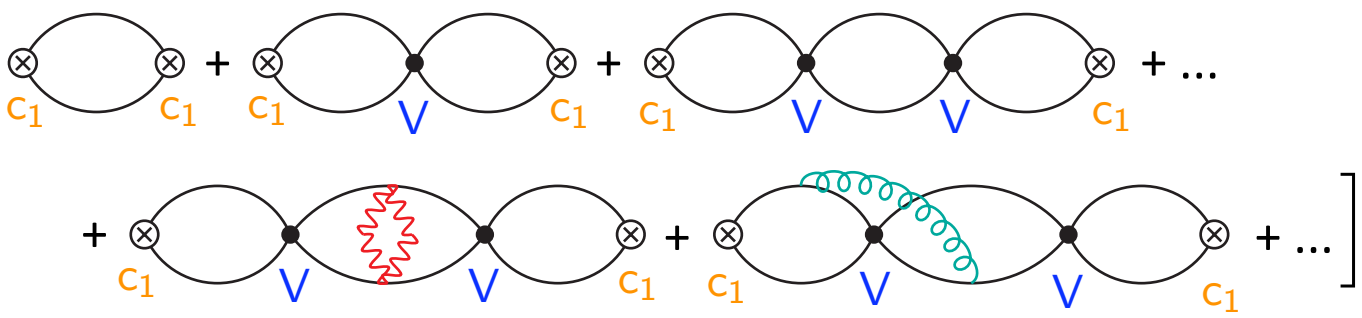


effective current:

→ power counting in $v \sim \alpha_s$

Top-antitop threshold: QCD effects

$$\sigma_{\text{tot}} \sim \text{Im}\Big[$$



$$\Big]$$

$$\sim \text{Im}\Big[\, c_1(\nu)^2 \cdot G(0,0,E,\nu) \,\Big]$$

$$\Big[-\frac{\nabla^2_{\vec{r}}}{m} + V_c(r) - E \Big] G(\vec{r},\vec{r}\,',E) = \delta^{(3)}(\vec{r}-\vec{r}\,') \quad \text{(LL)}$$

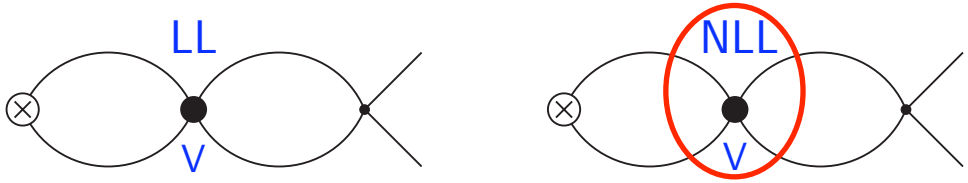
G^{NNLL} known ✓ [Hoang, Manohar, Stewart, Teubner, '02] [Pineda, Signer, '06]

G^{NNNLO} known ✓ [Beneke, Kiyo, Schuller, '07]

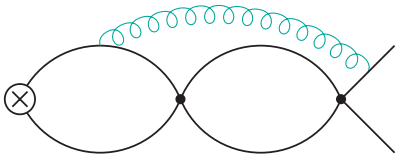
Top-antitop threshold: QCD effects

$$\sigma_{\text{tot}} \sim \text{Im}\left[c_1(\nu)^2 \cdot G(0,0,E,\nu) \right]$$

current renormalization



$$\ln\left[\frac{c_1(\nu)}{c_1(1)}\right] = \underbrace{\xi^{\text{LL}}}_{0} + \xi^{\text{NLL}} + \xi^{\text{NNLL}}_{\text{mix}} + \xi^{\text{NNLL}}_{\text{nonmix}}$$



[Hoang, '03]

missing

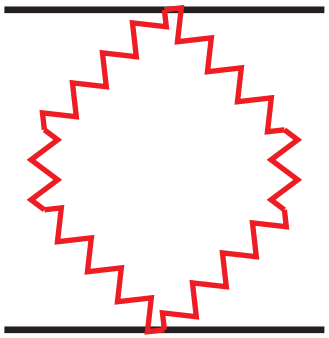
[Luke, Manohar, Rothstein, '00]
[Pineda, '02]
[Hoang, Stewart, '03]

Top-antitop threshold: QCD effects

NLL: $$\nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

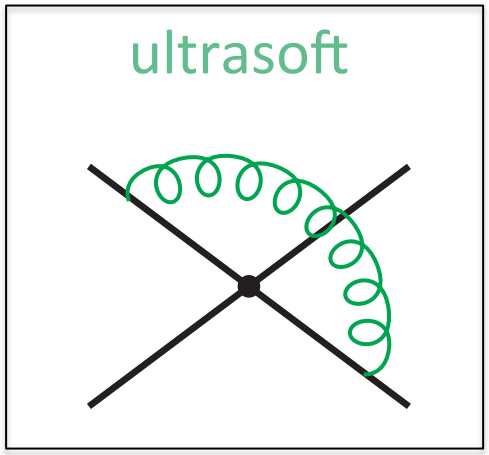
corrections

soft

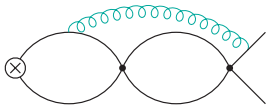


- known soft contributions small

ultrasoft



- dominant: $\alpha_s(mv^2) > \alpha_s(mv)$
- large contribution to $\xi^{\text{NNLL}}_{\text{nonmix}}$



[Hoang, '03]

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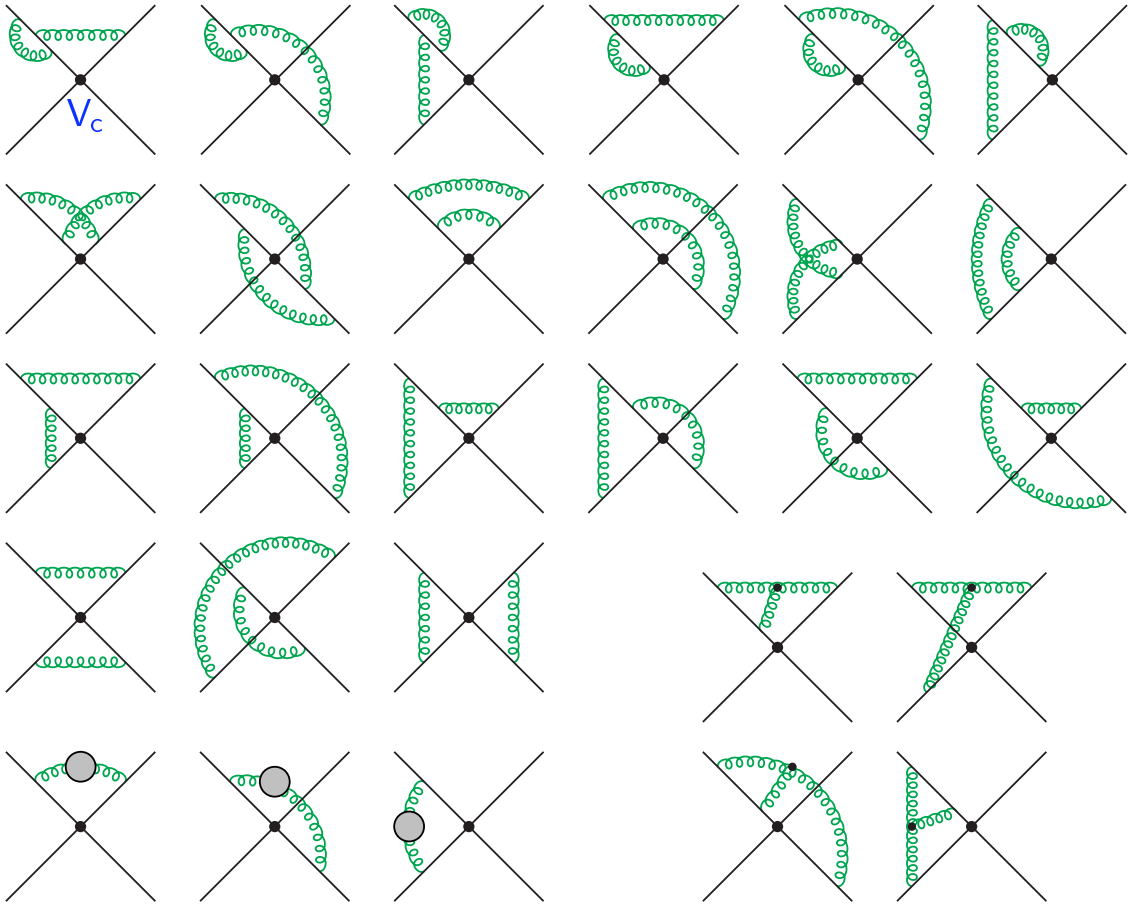
renormalize



- Feynman gauge
- $\overline{\rm MS}$, dim. reg.
- O(10^3) diagrams

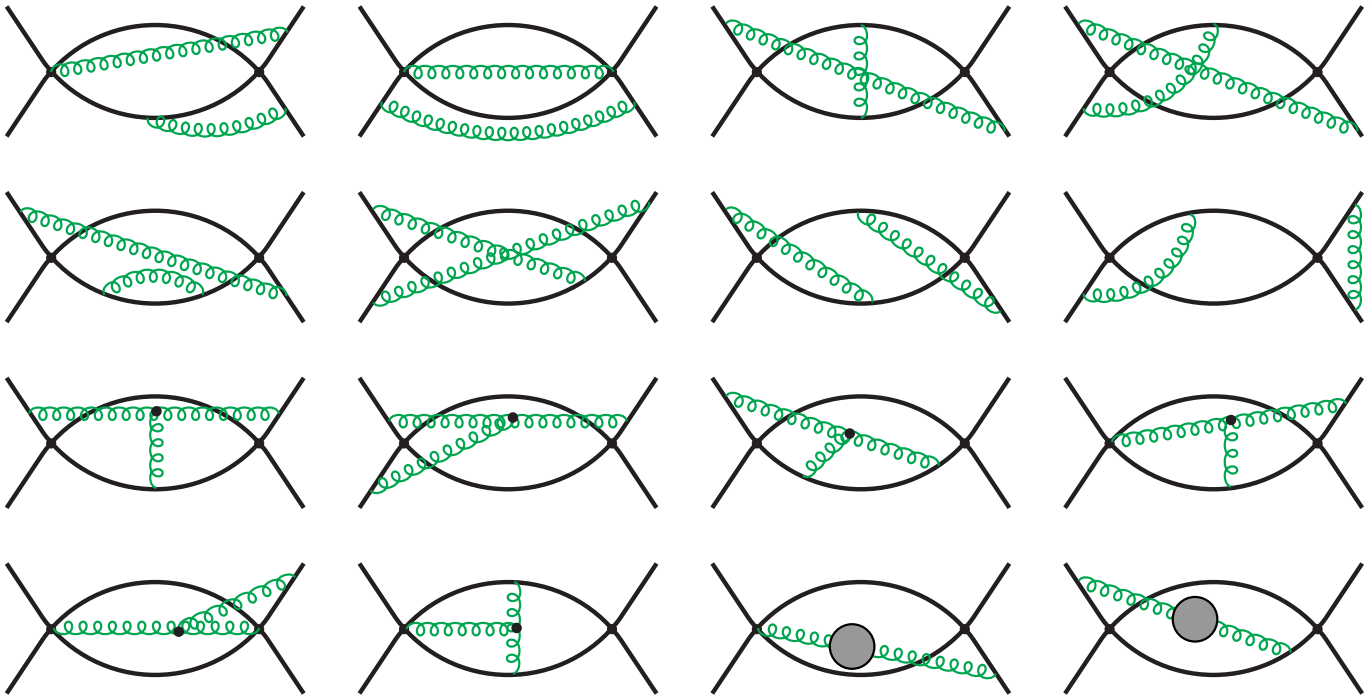
$$\Rightarrow \quad \boxed{\delta\mathcal{V}_{r,2}^{\text{2 loop}} \xrightarrow{\text{RGE}} \mathcal{V}_{r,2}^{\text{NLL}}(\nu)}$$

[Hoang, MS, '06]
[Pineda, '11]

Top-antitop threshold: QCD effects

NLL: $$\nu\frac{\partial}{\partial\nu}\ln[c_1(\nu)] = -\frac{\mathcal{V}_c(\nu)}{16\pi^2}\left[\frac{\mathcal{V}_c(\nu)}{4} + \mathcal{V}_2(\nu) + \mathcal{V}_r(\nu) + \mathbf{S}^2\,\mathcal{V}_s(\nu)\right] + \frac{1}{2}\mathcal{V}_k(\nu)$$

renormalize



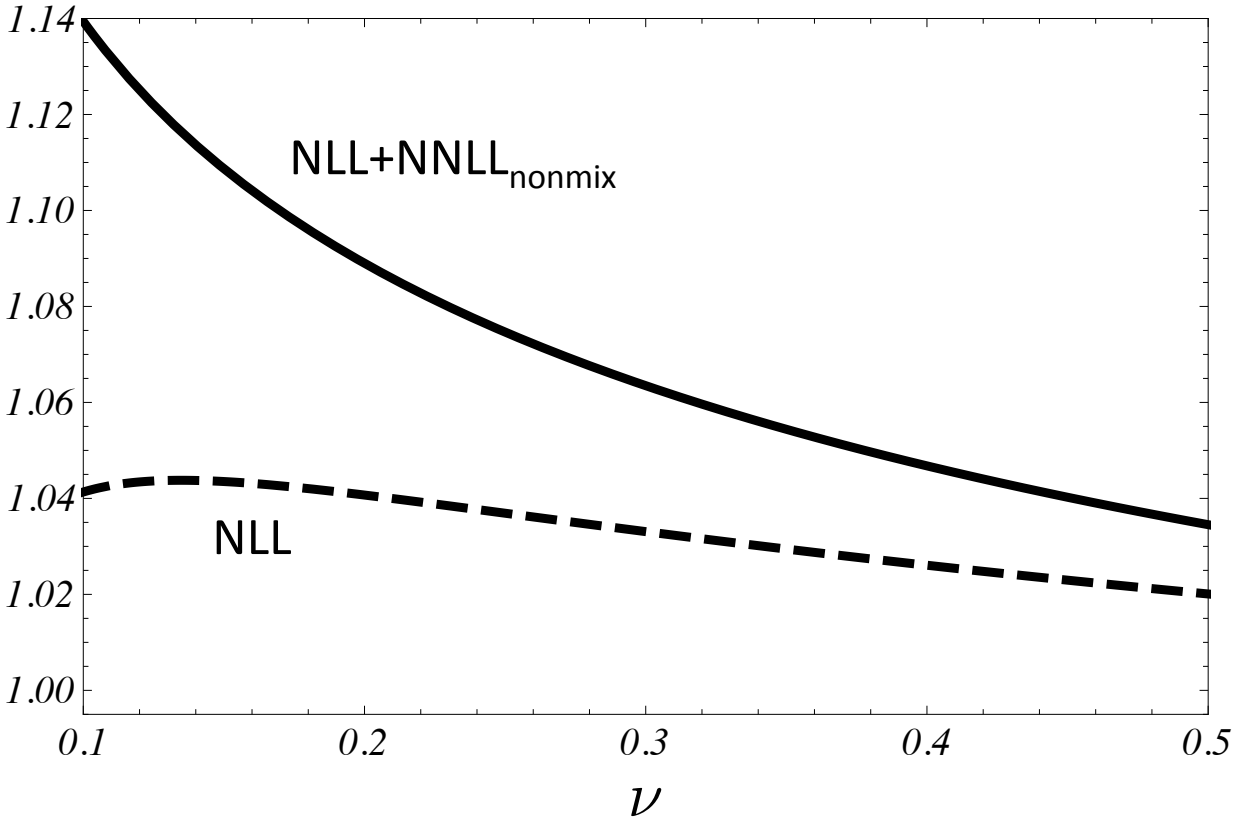
- <u>3 loops:</u>
  2 x usoft
  1 x potential (finite)
- <u>Feynman gauge</u>
- $\overline{\rm MS}$, dim. reg.
- O(10^4) diagrams
- Generation:
  own **Mathematica** code
- Integrals:
  IBP & partial frac.

$$\Rightarrow \boxed{\delta\mathcal{V}_k^{\text{2 loop}} \xrightarrow{\text{RGE}} \mathcal{V}_k^{\text{NLL}}(\nu)}$$

[Hoang, MS, '11]
[Pineda, '11]

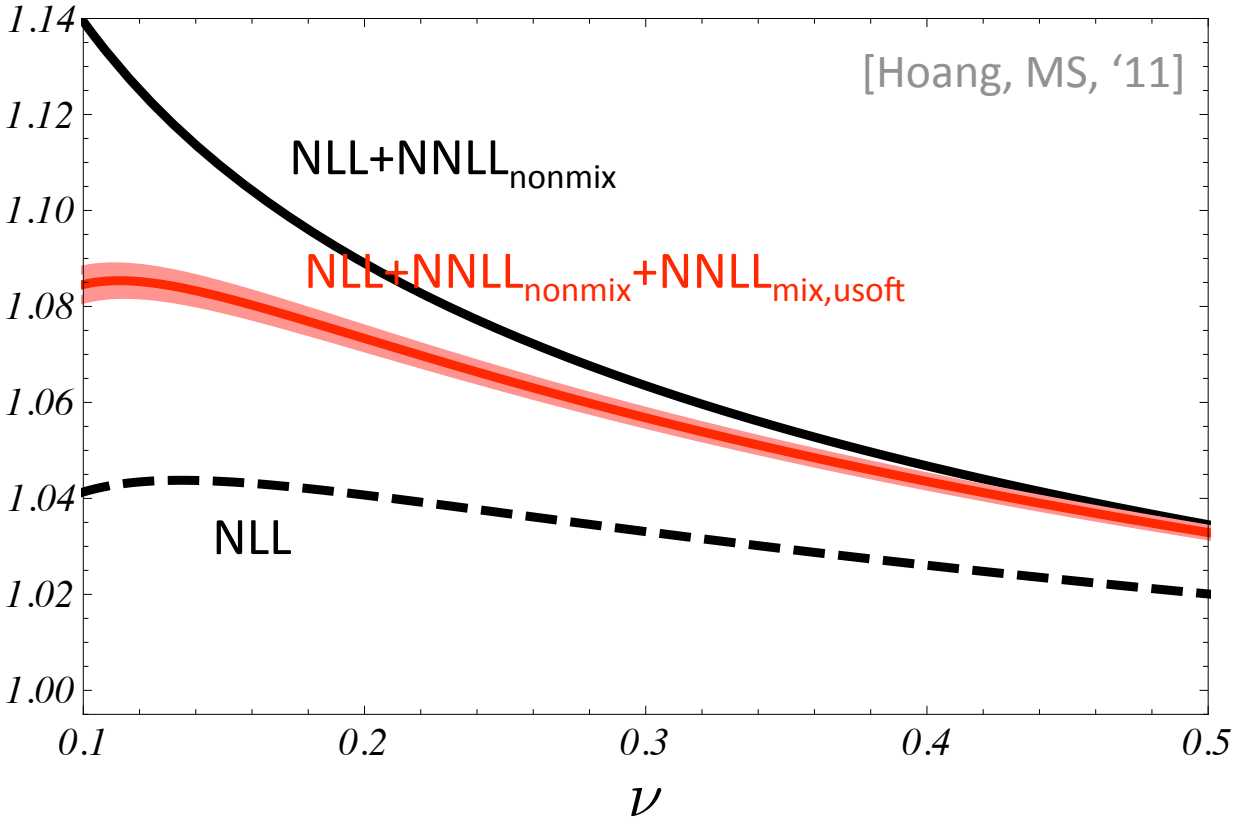
Top-antitop threshold: QCD effects

$$\frac{c_1(\nu)}{c_1(1)}$$



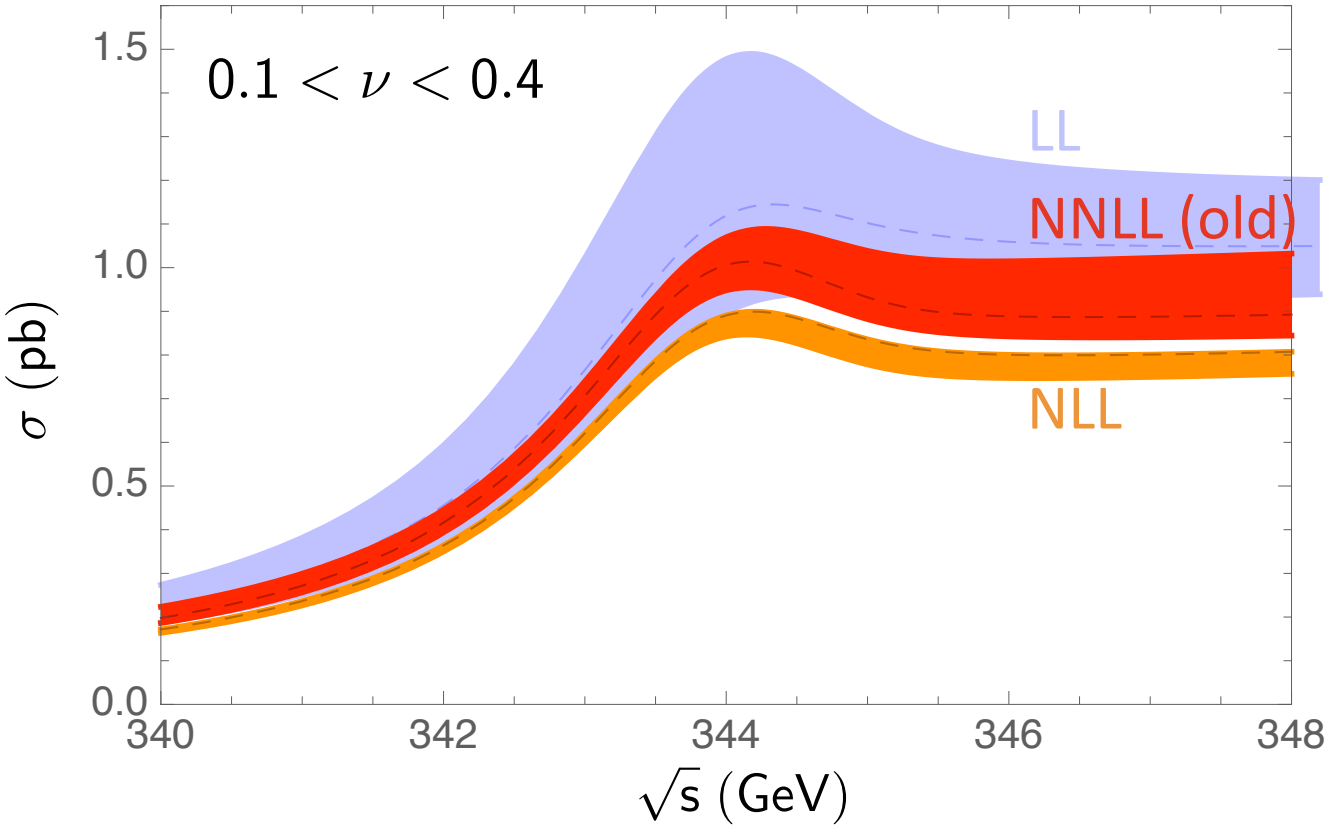
Top-antitop threshold: QCD effects

$\frac{c_1(\nu)}{c_1(1)}$



- large usoft NNLL contributions compensate each other!
- good convergence: $[c_1(\nu=0.15)]^2 = 1 - 0.096|_{NLL} - 0.029|_{NNLL}$
- known soft (nonmixing) contributions at NNLL are tiny!

Top-antitop threshold: QCD effects



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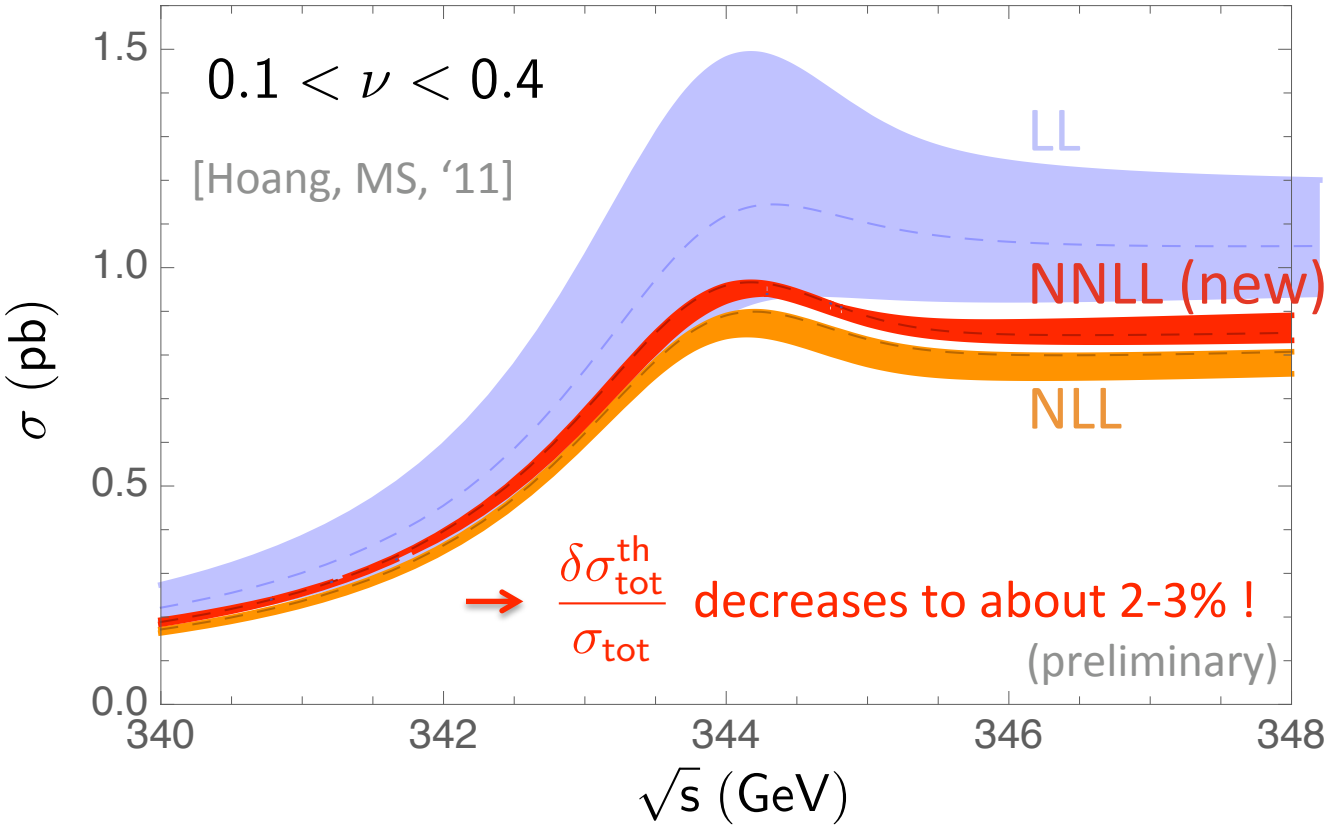


- LO EW effects included by $G^{LL}(0,0,E,\nu) \rightarrow G^{LL}(0,0,E+i\Gamma,\nu)$ [Fadin, Khoze, '87]
- Combination with NNLL EW effects and detailed error analysis → W.I.P

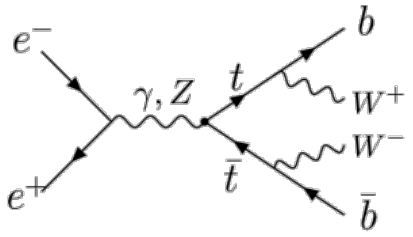
$$e^+e^- \longrightarrow t\bar{t}$$

Top-antitop threshold: EW effects

Top-antitop threshold: EW effects

- Power counting: $\boxed{\Gamma_t/m_t \sim \alpha_{EW} \sim \alpha_s^2 \sim v^2 \ll 1}$

- Physical final state: $e^+e^- \to W^+W^-b\bar{b}$

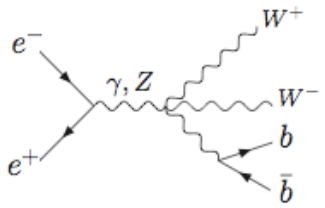


- Apply loose invariant mass cuts on reconstructed tops/antitops:

$$p_{t,\bar{t}}^2 = (m_t \pm \Delta M_t)^2 = m_t^2 + \Lambda^2 \qquad \boxed{m_t\Gamma_t \ll \Lambda^2 \lesssim m_t^2}$$

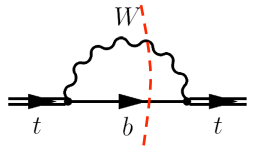
→ no effect on resonant contributions!

→ non-resonant background suppressed:



LO: $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ (replacement rule)

unstable top propagator: $\dfrac{i}{E/2 + p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2}$

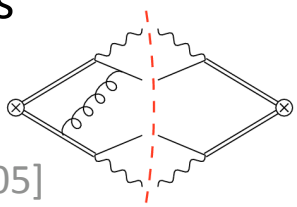


[Fadin, Khoze, '87]

Top-antitop threshold: EW effects

Beyond LO:

- QED: "Coulomb photon" → trivial extension of QCD corrections
- Gluon exchange with final state → negligible at NLO and NNLO



[Fadin, Khoze, Martin '94] [Hoang, Reisser '05]
[Melnikov, Yakovlev '94] [Beneke, Jantzen, Ruiz-Femenia '10]

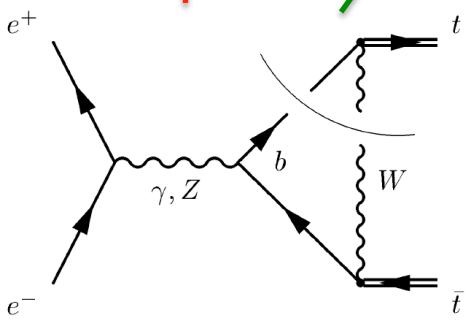
- Corrections to current matching:

$$c_1(1) = c_{1,\mathrm{LL}}^{\mathrm{born}} + c_{1,\mathrm{NLL}}^{\mathrm{QCD}} + c_{1,\mathrm{NNLL}}^{\mathrm{QCD}} + \mathrm{i}\, c_{1,\mathrm{NNLL}}^{\mathrm{bW,abs}} + c_{1,\mathrm{NNLL}}^{\mathrm{EW}} + \dots$$

[Kuhn, Guth '92]

complex matching

[Hoang, Reisser '06]



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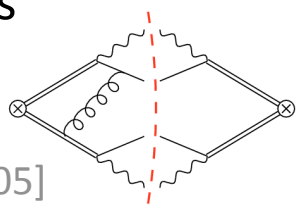

[Fadin, Khoze, Martin '94] [Hoang, Reisser '05]
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$$\Rightarrow \sigma_{\mathrm{tot}} \sim \mathrm{Im}\left[c_1(\nu)^2\, G(0,0,E+\mathrm{i}\Gamma_t,\nu)\right] \sim \frac{\alpha_s \Gamma_t}{\epsilon} + \text{finite}$$

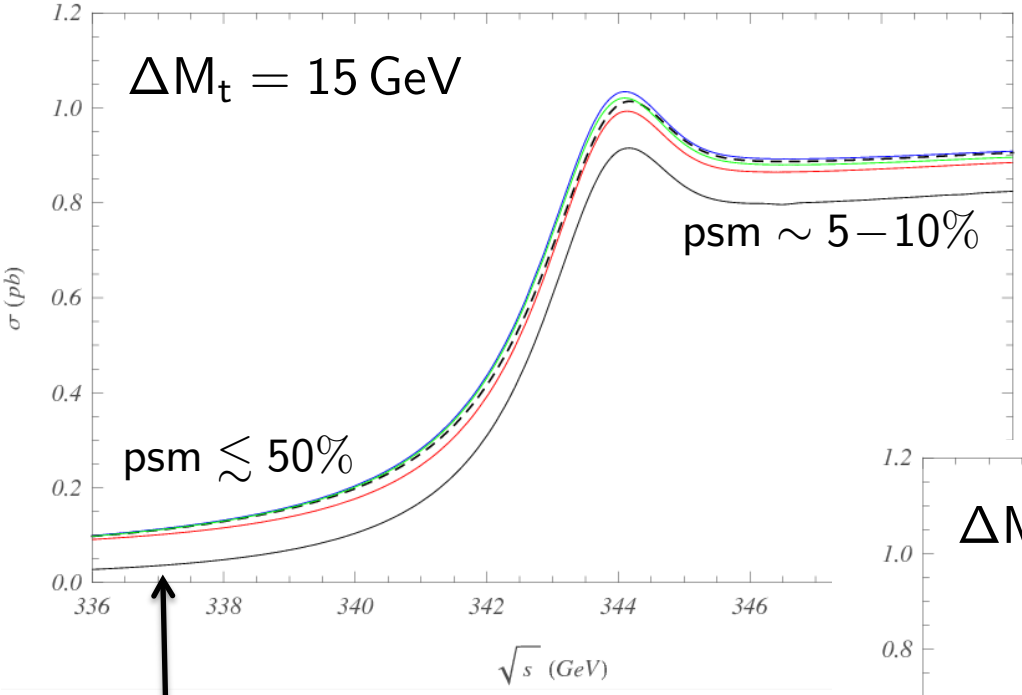
phase space divergence ↓ sums phase space logs in $C(\nu)$ at NLL ✔

$$\mathrm{i}\, C(\nu) \cdot$$

[Hoang, Reisser, Ruiz-Femenia '10]

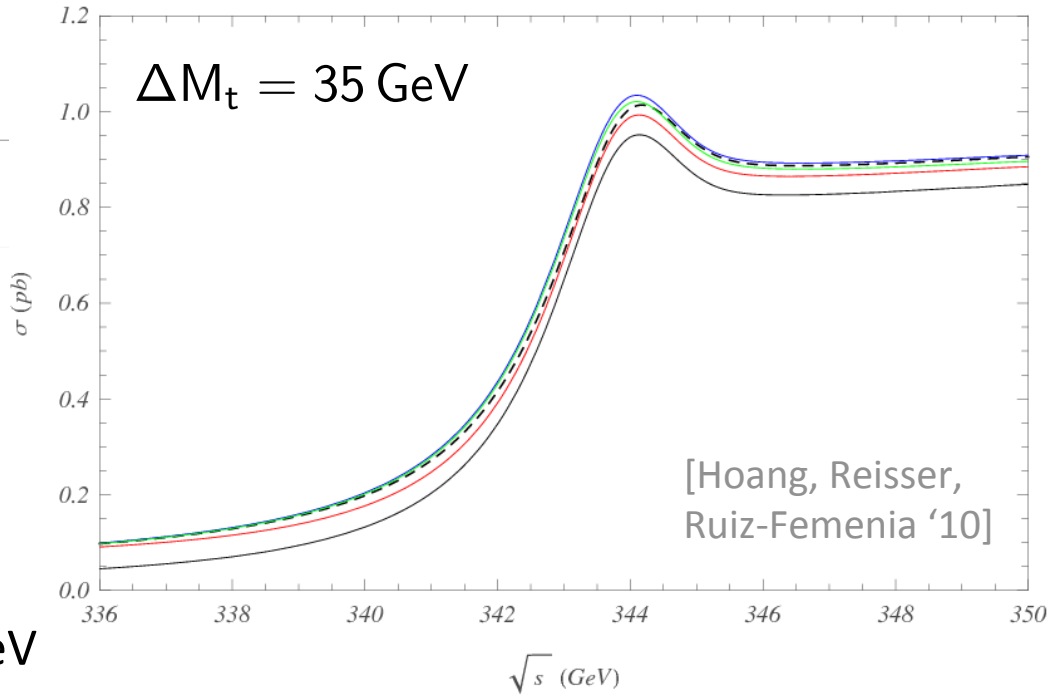
- "Phase space matching" for $C(\nu)$ to allow for Λ cuts: NLO, NNLO, N^{3}LO ✔

Top-antitop threshold: EW effects



dashed line: NNLL pure QCD prediction (add step by step)
+ NNLL QED effects
+ NNLL EW current matching (real)
+ NNLL EW current matching (absorbtive)
+ NLL+NNLL+N^{3}LL phase space matching contributions (psm)

large psm correction due to unphysical phase space in pure QCD prediction

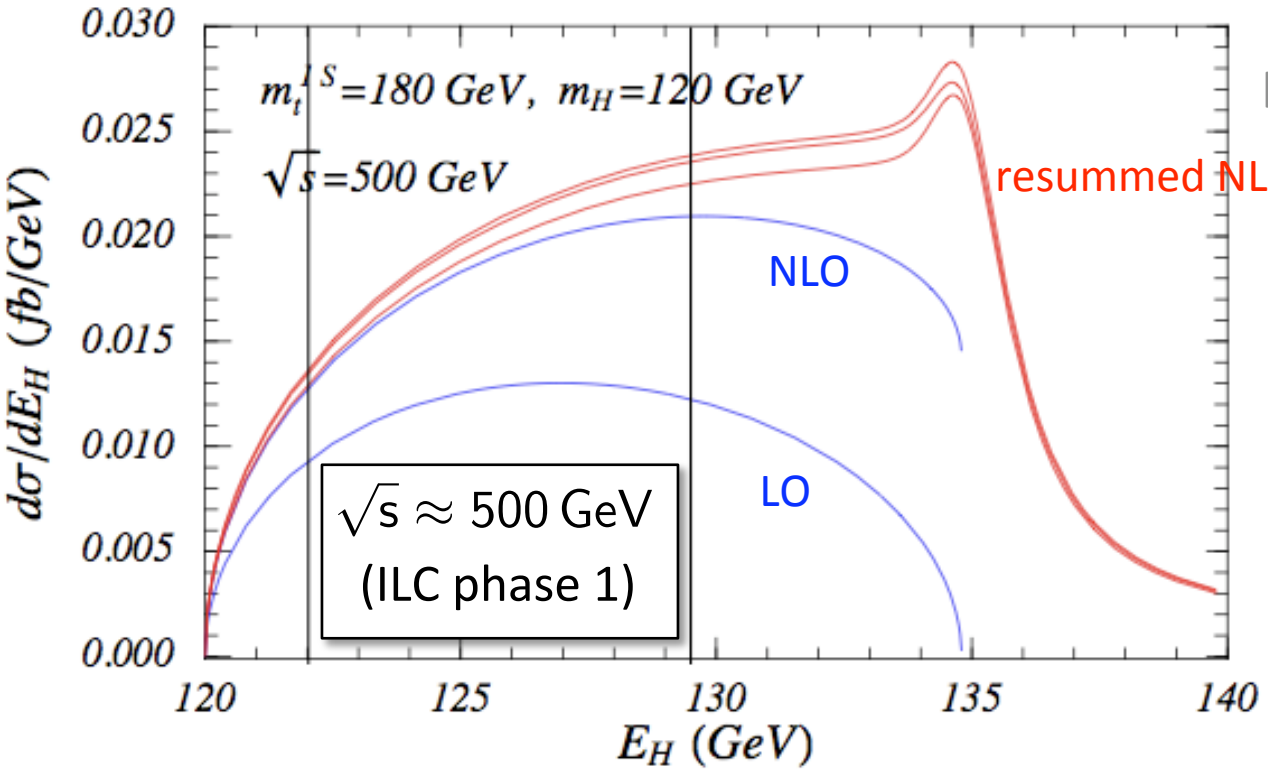


→ Shift in peak position: 30-50 MeV

$$e^+e^- \longrightarrow t\bar{t}H$$

Associated Higgs production

Associated Higgs production: $e^+e^- \to t\bar{t}H$



[Farrell, Hoang; 2006]

- For light Higgs ($m_H \approx 120$ GeV): full $t\bar{t}$ phase space nonrelativistic!
  → must sum $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$ terms → recycle $t\bar{t}$ results (vNRQCD)
  → factor 2 enhancement over tree level (+ factor 2 from polarized beams)
- realistic studies: $(\delta y_t/y_t)^{\text{ILC}}_{500\text{GeV}} \sim 30\% \to 10-15\%$? [Juste, '02,'06]

Summary/Outlook

- precise $m_t, y_t, \alpha_s, \Gamma_t$ from $t\bar{t}$ threshold physics
- $\sigma_{tot} \sim \mathrm{Im}\left[c_1(\nu)^2 \cdot G(0,0,E,\nu) \right] + \ldots$
- $G(0,0,E,\nu)$ known up to NNLL ✔
- New $c_1(\nu)$ at NNLL ✔

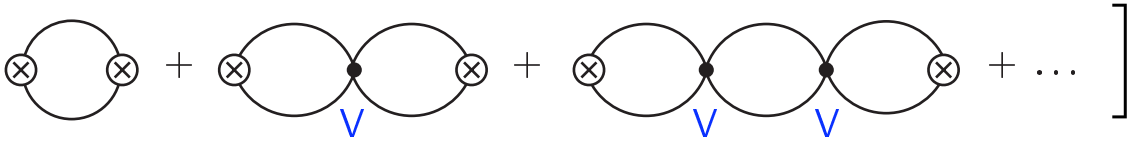
  → $\frac{\delta\sigma_{tot}^{th}}{\sigma_{tot}}$ decreases substantially! → RG improvement important!

- EW conributions up to NNLL ✔
- $e^+e^- \longrightarrow t\bar{t}H$ (ILC phase 1) at NLL (EW effects missing)
- <u>Outlook:</u>
  - Detailed error analysis for $\sigma_{tot}(e^+e^- \longrightarrow t\bar{t})$: EW+QCD
  - scalar (stop) pair production at threshold

W.I.P

Backup

Top-antitop threshold: QCD effects

$$\sigma_{\rm tot} \sim {\rm Im}\Big[$$ 

$$\Big]$$

$$\sim |c_1(\nu)|^2 \cdot {\rm Im}\left[-i \int d^4x\; e^{i\hat{q}x} \langle 0|\, T\, \vec{j}_1^{\,\rm eff\,*}(x)\, \vec{j}_1^{\,\rm eff}(0)\, |0\rangle \right] + \ldots$$

$$\sim |c_1(\nu)|^2 \cdot {\rm Im}\left[G(0,0,E,\nu) \right] + \ldots$$

LO SG: $\left[-\frac{\nabla^2_{\vec{r}}}{m} + V_c(r) - E \right] G(\vec{r},\vec{r}\,',E) = \delta^{(3)}(\vec{r}-\vec{r}\,')$

$$G^{\rm LL}(0,0,E,\nu) = \frac{m^2}{4\pi}\left\{ iv - C_F\alpha_s\left[\ln\left(\frac{-iv}{\nu}\right) - \frac{1}{2} + \ln 2 + \gamma_E + \Psi\left(1 - \frac{iC_F\alpha_s}{2v}\right)\right]\right\} + \frac{m^2 C_F \alpha_s}{16\pi\epsilon}$$

$G^{\rm NNLL}$ known ✓ [Hoang, Manohar, Stewart, Teubner; 2002] [Pineda, Signer; 2006]

$G^{\rm NNNLO}$ known✓ [Beneke, Kiyo, Schuller; 2007]

Results

RGE's + matching at hard scale $(\nu = 1)$ give:

LL NLL

$$\left[\mathcal{V}_2(\nu)\right]^{\text{NLL}}_{\text{usoft}} = 4\pi\alpha_s(m\nu)\left[-\frac{4\pi}{\beta_0}A_2 \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_2 - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_2\right)\right]$$

$$\left[\mathcal{V}_r(\nu)\right]^{\text{NLL}}_{\text{usoft}} = 8\pi\alpha_s(m\nu)\left[-\frac{4\pi}{\beta_0}A_r \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_r - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_r\right)\right]$$

$$\left[\mathcal{V}_k(\nu)\right]^{\text{NLL}}_{\text{usoft}} = 2\alpha_s^2(m\nu)\left[-\frac{4\pi}{\beta_0}A_k \ln\frac{\alpha_s(m\nu^2)}{\alpha_s(m\nu)} + \left(\frac{\beta_1}{\beta_0^2}A_k - \left[\alpha_s(m\nu^2)-\alpha_s(m\nu)\right]\frac{8\pi}{\beta_0}B_k\right)\right]$$

$$\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = C_F(C_A - 2C_F)\begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ B_r \end{bmatrix} = -C_A C_F \begin{bmatrix} A \\ B \end{bmatrix}$$

$$\begin{bmatrix} A_k \\ B_k \end{bmatrix} = -C_A C_F(C_A - 2C_F)\begin{bmatrix} A \\ B \end{bmatrix}$$

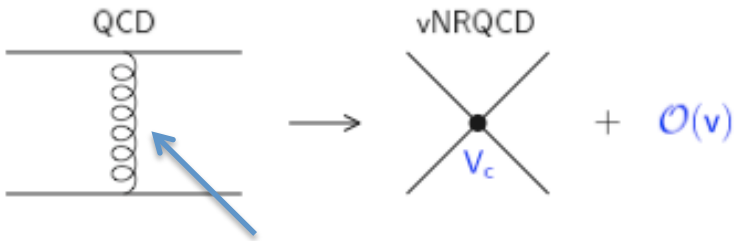
$$A = \frac{1}{3\pi}$$

$$B = \frac{C_A(47 + 6\pi^2) - 10 n_f T}{108\pi^2}$$

[Hoang, MS; 2011]
[Pineda; 2011]

vNRQCD

- Nonresonant dof's integrated out, e.g.:



- Resonant dof's $\rightarrow$ fields in the vNRQCD Lagrangian:

nonrel. quark:	$(E, \mathbf{p}) \sim (mv^2, mv)$	$\psi_{\mathbf{p}}(x)$	
soft gluon:	$(q_0, \mathbf{q}) \sim (mv, mv)$	$A_q(x)$	
ultrasoft gluon:	$(q_0, \mathbf{q}) \sim (mv^2, mv^2)$	$A(x)$	

- Systematic expansion in v $\Rightarrow$ consistent power counting in v $\sim \alpha_s$

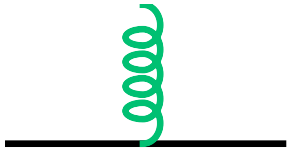
vNRQCD

[Luke, Manohar, Rothstein; 2000]

$$\mathcal{L}_{\text{vNRQCD}} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}$$

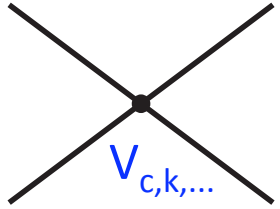
$$D^\mu = \partial^\mu + igA^\mu(x)$$

$$\mathcal{L}_{\text{usoft}} : \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \dots \right]\psi_{\mathbf{p}}(x) + \dots$$

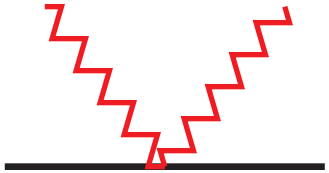


$$\mathcal{L}_{\text{pot}} : \quad - V \ \psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \dots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \dots$$



$$\mathcal{L}_{\text{soft}} :$$



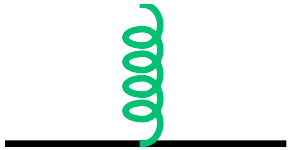
vNRQCD

[Luke, Manohar, Rothstein; 2000]

$$\mathcal{L}_{\text{vNRQCD}} = \mathcal{L}_{\text{usoft}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_{\text{soft}}$$

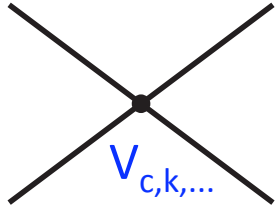
$$D^\mu = \partial^\mu + igA^\mu(x)$$

$$\mathcal{L}_{\text{usoft}} : \quad \psi^\dagger_{\mathbf{p}}(x)\left[iD^0 - \frac{(\mathbf{p}-i\mathbf{D})^2}{2m} + \dots \right]\psi_{\mathbf{p}}(x) + \dots$$

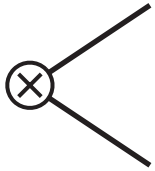


$$\mathcal{L}_{\text{pot}} : \quad - V \;\; \psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}}\chi^\dagger_{-\mathbf{p}'}\chi_{-\mathbf{p}} + \dots$$

$$V \sim \frac{\mathcal{V}_c}{\mathbf{k}^2} + \frac{\mathcal{V}_k \pi^2}{m\mathbf{k}} + \frac{\mathcal{V}_r(\mathbf{p}^2+\mathbf{p}'^2)}{2m^2\mathbf{k}^2} + \frac{\mathcal{V}_2}{m^2} + \frac{\mathcal{V}_s}{m^2}\mathbf{S}^2 + \dots$$



Production/annihilation current ($^3S_1$):

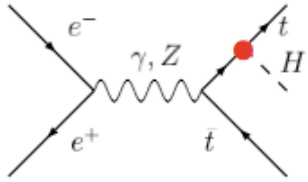


$$\sim \quad c_1(\nu) \cdot \underbrace{\vec{j}_1^{\,\text{eff}}(x)}_{\psi^\dagger_{\mathbf{p}}\vec{\sigma}(i\sigma_2)\chi^*_{-\mathbf{p}}} + \dots$$

(CMS)

Associated Higgs production: $e^+e^- \to t\bar{t}H$

- Dominant contributions from Higgs radiating of the top/antitop
- precise extraction of top Yukawa coupling possible
- At large E_H endpoint: $t\bar{t}$ dynamics nonrelativistic
- For $\sqrt{s} \lesssim 500\,\text{GeV}$ (ILC phase 1) and $m_H \approx 120\,\text{GeV}$:
  full $t\bar{t}$ phase space nonrelativistic !!! → must sum $(\alpha_s/v)^n$, $(\alpha_s \ln v)^n$!

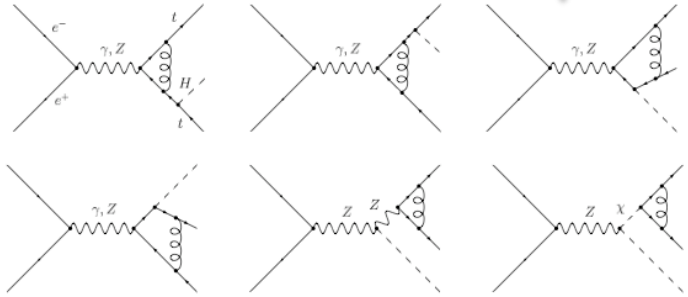


vNRQCD Recycle $t\bar{t}$ results: $c_1(\nu)/c_1(1)$, $G(0,0,v,\nu)$

$$\left(\frac{d\sigma}{dE_H}\right)_{E_H \approx E_H^{max}} \sim \left[c_{1,0}^2(\nu, \sqrt{s}, m_t, m_H) + c_{1,1}^2(\nu, \sqrt{s}, m_t, m_H)\right] \times \text{Im}[G(0,0,v,\nu)]$$

NLO matching

[Farrell, Hoang; 2005]



[Denner, Dittmaier, Roth, Weber; 2004]