École de Gif
LAL Orsay, September 2012

QCD at the LHC

Matteo Cacciari
LPTHE Paris

Many thanks to Guenther Dissertori, Rikkert Frederix, Fabio Maltoni, Paolo Nason, Gavin Salam, Maria Ubiali, and many others, from whose talks/lectures I have drawn inspiration, as well as extracted many slides

Outline

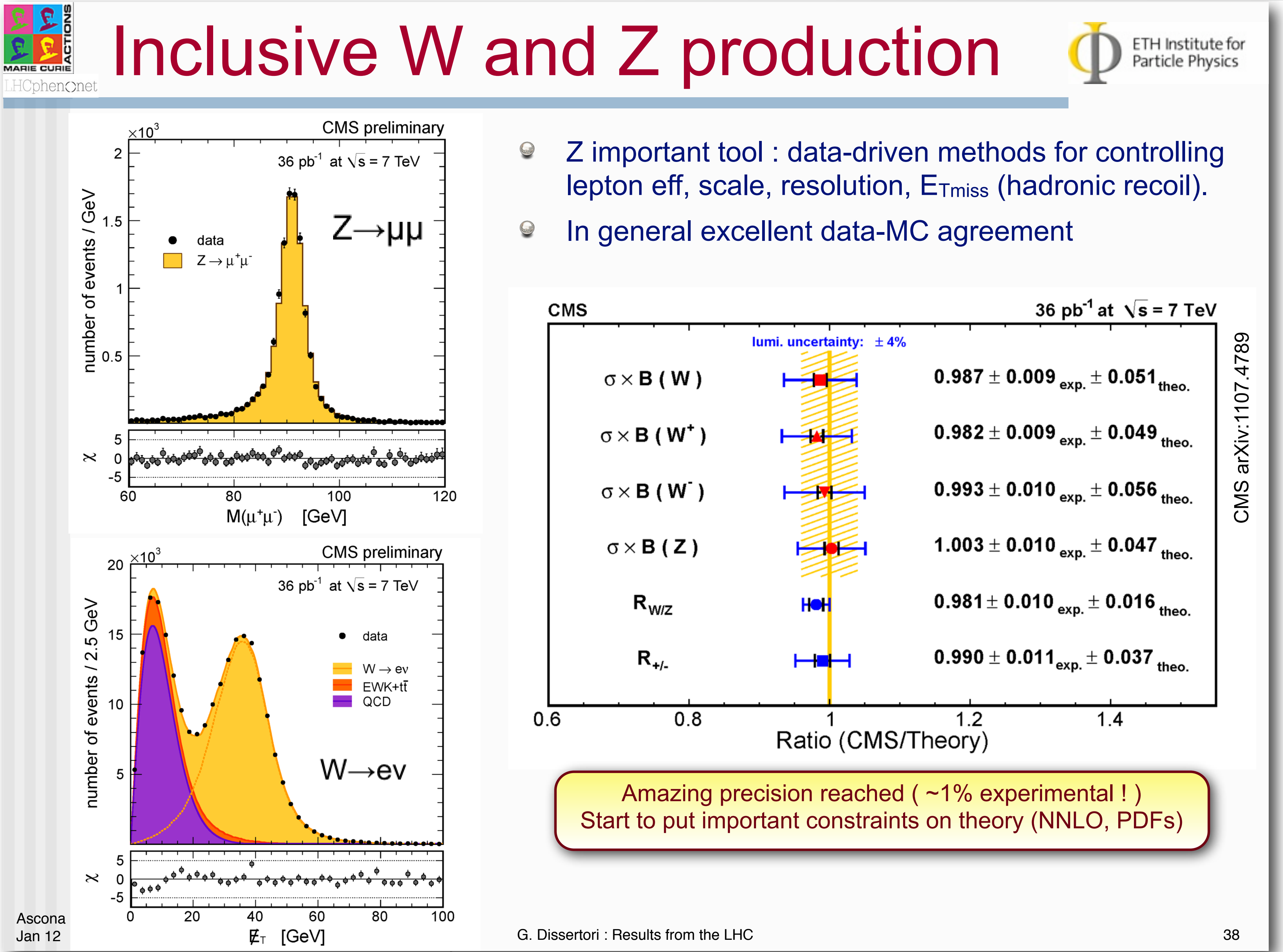
- Show some LHC experimental results (many of them possibly already outdated by now) and their (mostly successful) comparison with theory

- Briefly discuss the theoretical results, advances, tools that have allowed such good comparisons

By no means a set of systematic lectures.
You'll mainly see ***what*** *exists, rather than how and why*

Slide from G. Dissertori

LHC physics results



Inclusive W and Z production

- Z important tool : data-driven methods for controlling lepton eff, scale, resolution, E_{Tmiss} (hadronic recoil).
- In general excellent data-MC agreement

$\sigma\times B(W)$	$0.987 \pm 0.009_{exp.} \pm 0.051_{theo.}$
$\sigma\times B(W^+)$	$0.982 \pm 0.009_{exp.} \pm 0.049_{theo.}$
$\sigma\times B(W^-)$	$0.993 \pm 0.010_{exp.} \pm 0.056_{theo.}$
$\sigma\times B(Z)$	$1.003 \pm 0.010_{exp.} \pm 0.047_{theo.}$
$R_{W/Z}$	$0.981 \pm 0.010_{exp.} \pm 0.016_{theo.}$
$R_{+/-}$	$0.990 \pm 0.011_{exp.} \pm 0.037_{theo.}$

Amazing precision reached (~1% experimental !)
Start to put important constraints on theory (NNLO, PDFs)

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G. Dissertori : Results from the LHC

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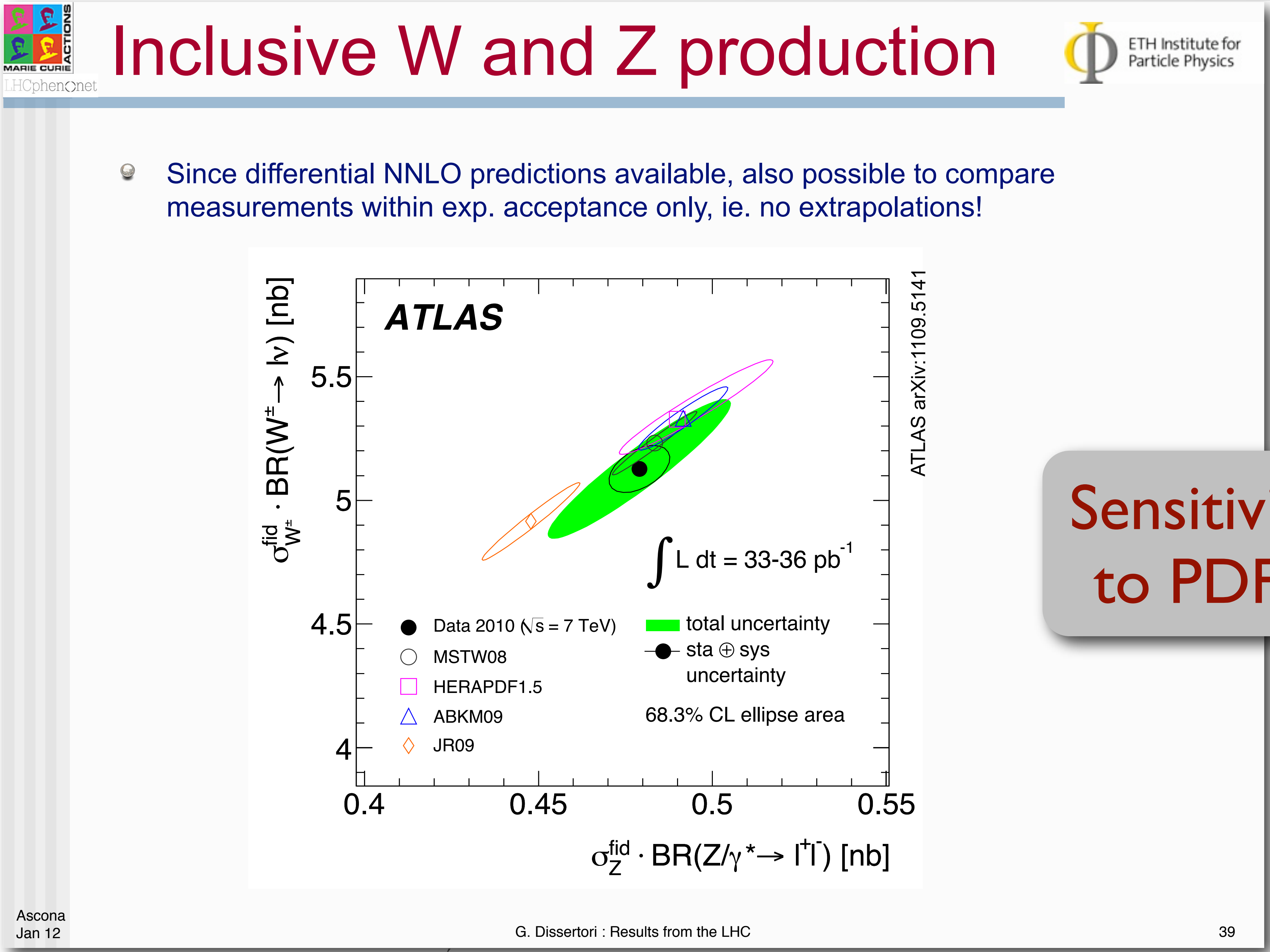
Slide from G. Dissertori

LHC physics results



Inclusive W and Z production

- Since differential NNLO predictions available, also possible to compare measurements within exp. acceptance only, ie. no extrapolations!

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Sensitivity to PDFs

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LHC physics results

Top cross section

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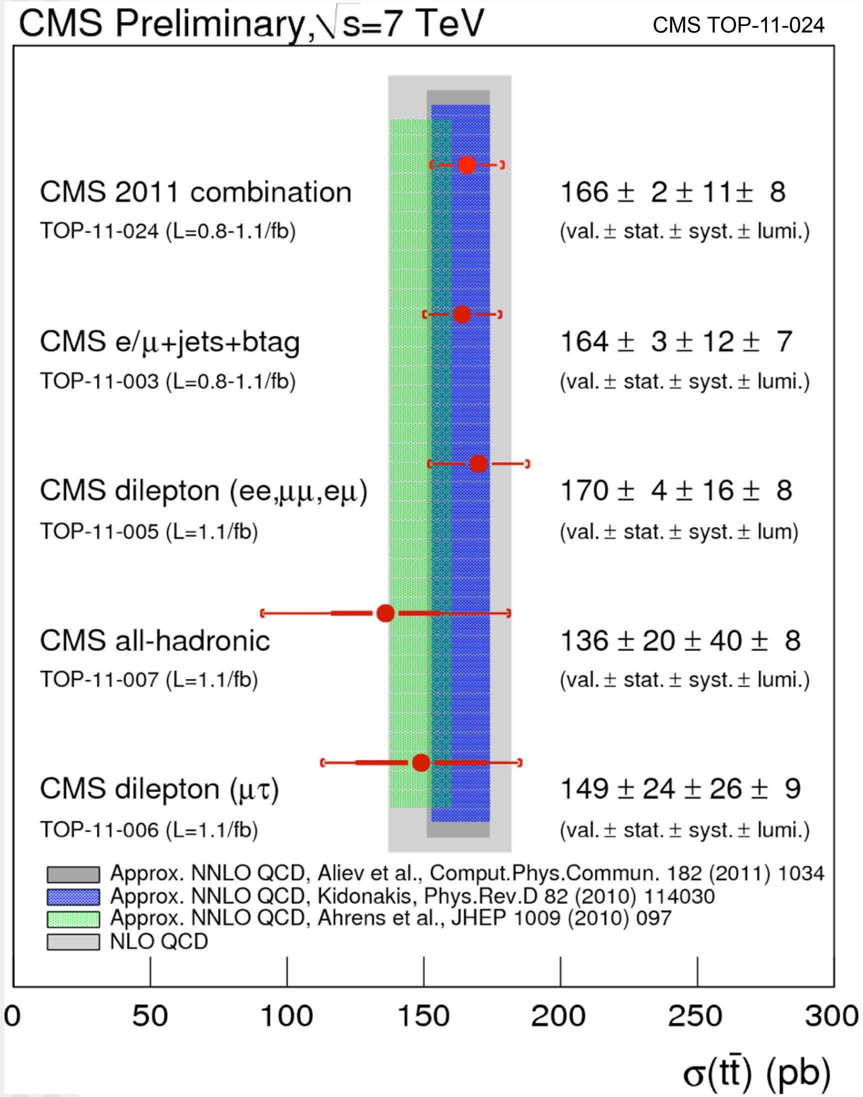
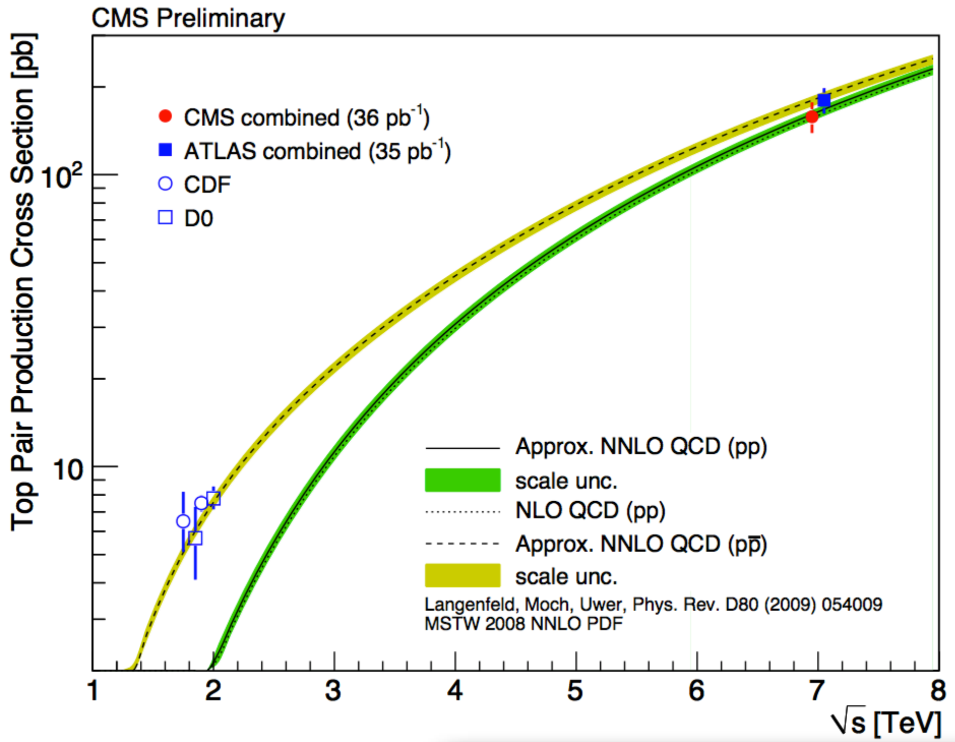


also tau channels included by now!

Similar results by ATLAS.

Excellent agreement with theoretical calculations so far...



- **Other measur**
  - **Top Charge a**
ppbar collider,
  - Top-AntiTop **m**
  - single top pro

LHC did in 1 year what the Tevatron did in 10. Theory keeping up...

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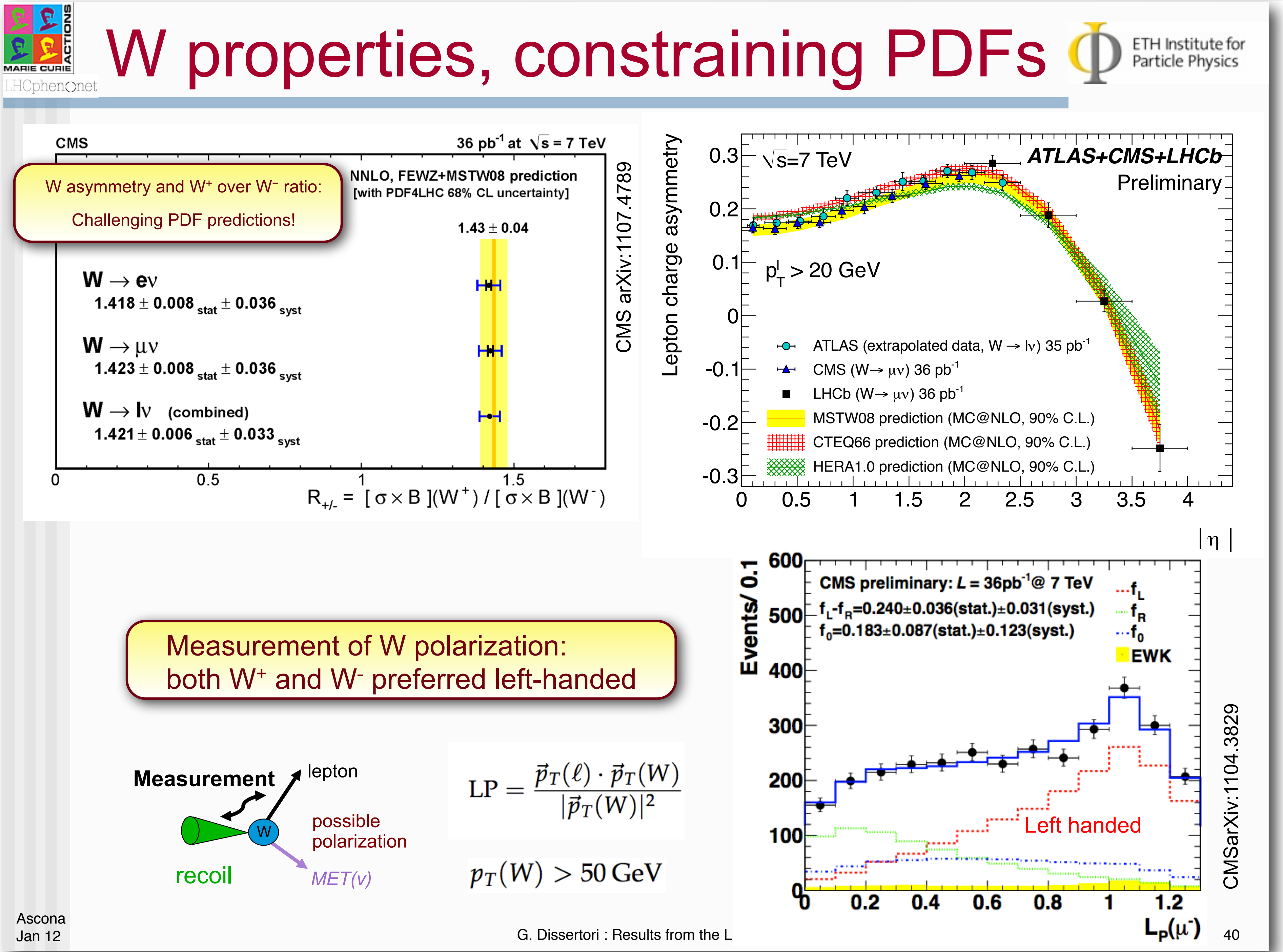
LHC physics results



W properties, constraining PDFs

W asymmetry and W⁺ over W⁻ ratio:

Challenging PDF predictions!

Measurement of W polarization:
both W⁺ and W⁻ preferred left-handed

$$\mathrm{LP} = \frac{\vec{p}_T(\ell)\cdot\vec{p}_T(W)}{|\vec{p}_T(W)|^2}$$

$$p_T(W) > 50\ \mathrm{GeV}$$

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LHC physics results

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W+jets

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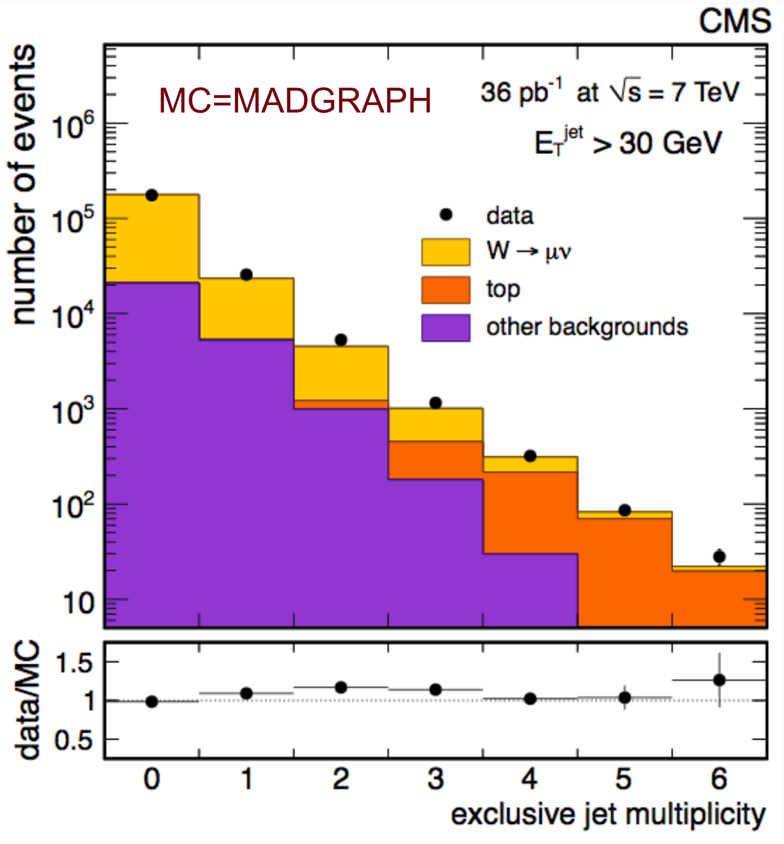
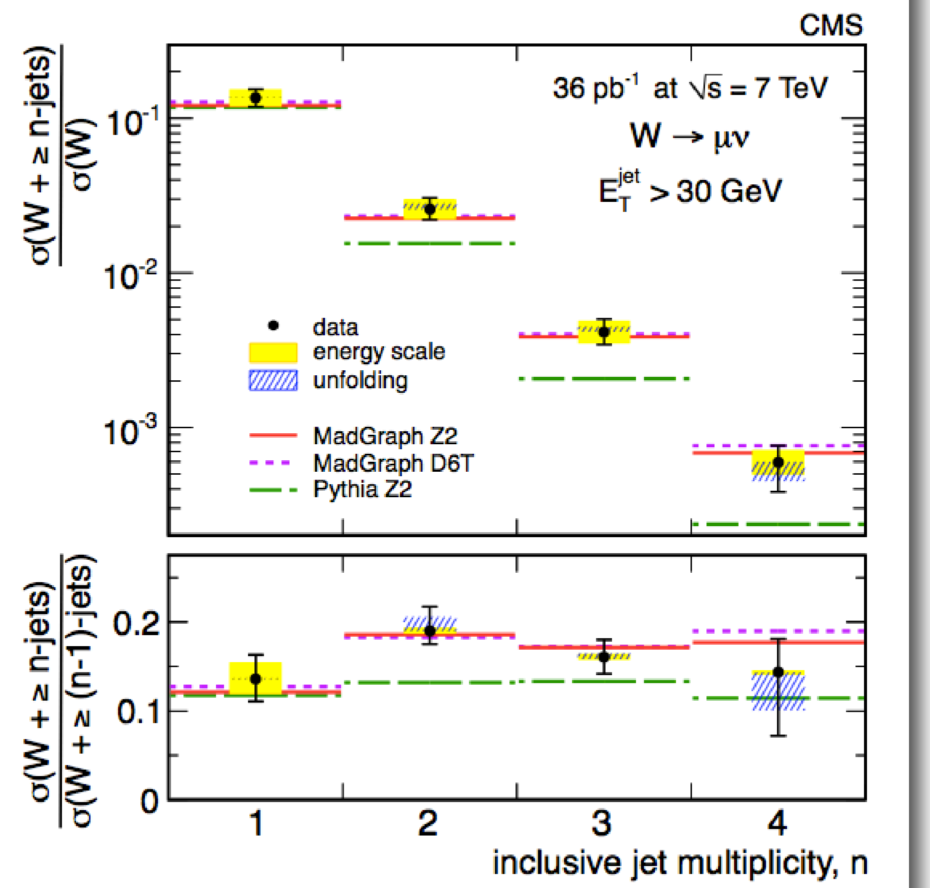




- **simultaneous** extraction of W signal and top background
- final distributions: **unfolded to particle level**
- presented for experimental lepton and jet acceptance, eg. p_{Tjet} > 30 GeV

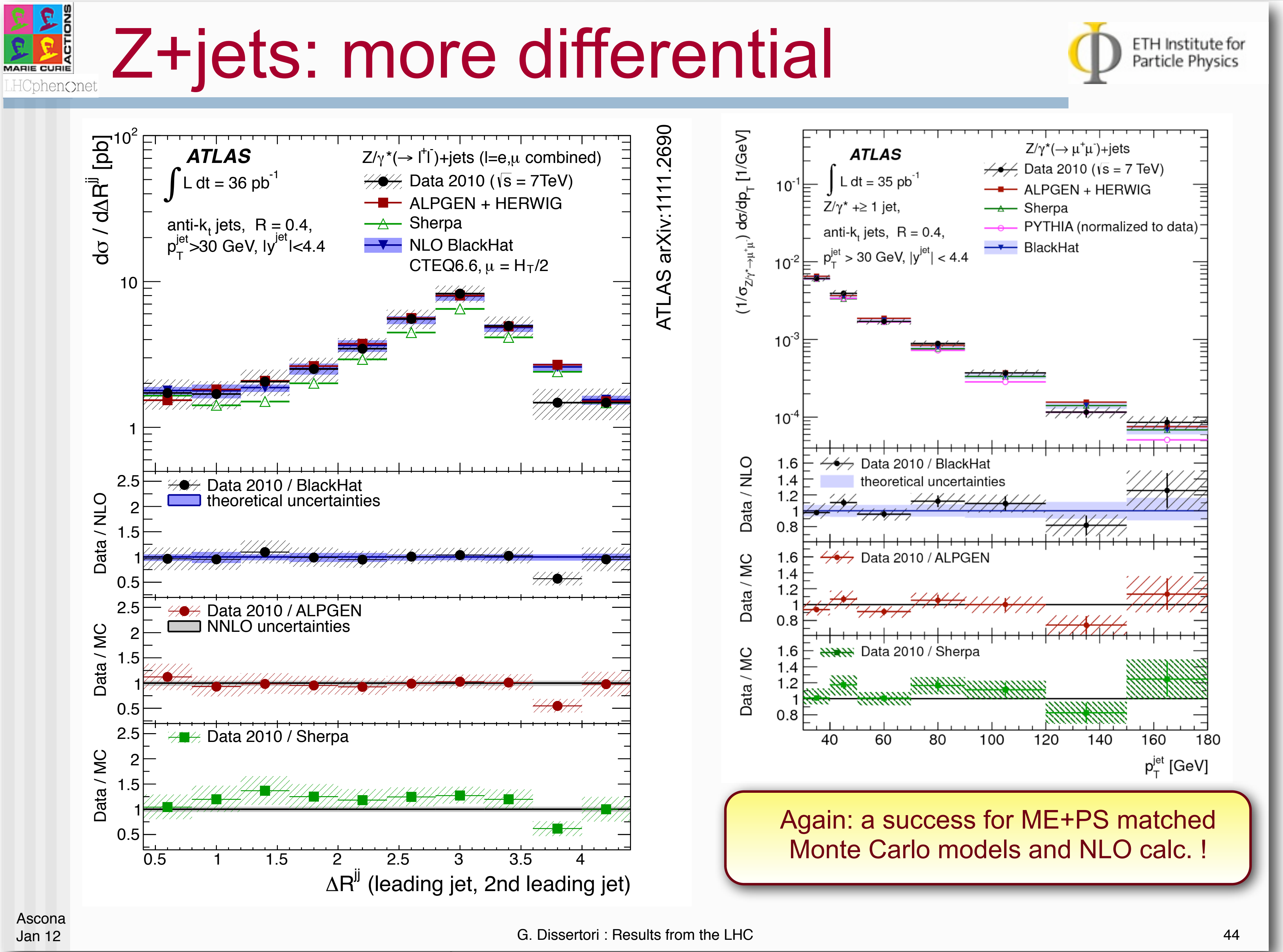
An additional jets “costs” ~1 alpha$_s$

Excellent agreement with ME+PS matched Monte Carlo model.

Great predictivity up to large multiplicities

Slide from G. Dissertori

LHC physics results



Z+jets: more differential

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Again: a success for ME+PS matched Monte Carlo models and NLO calc. !

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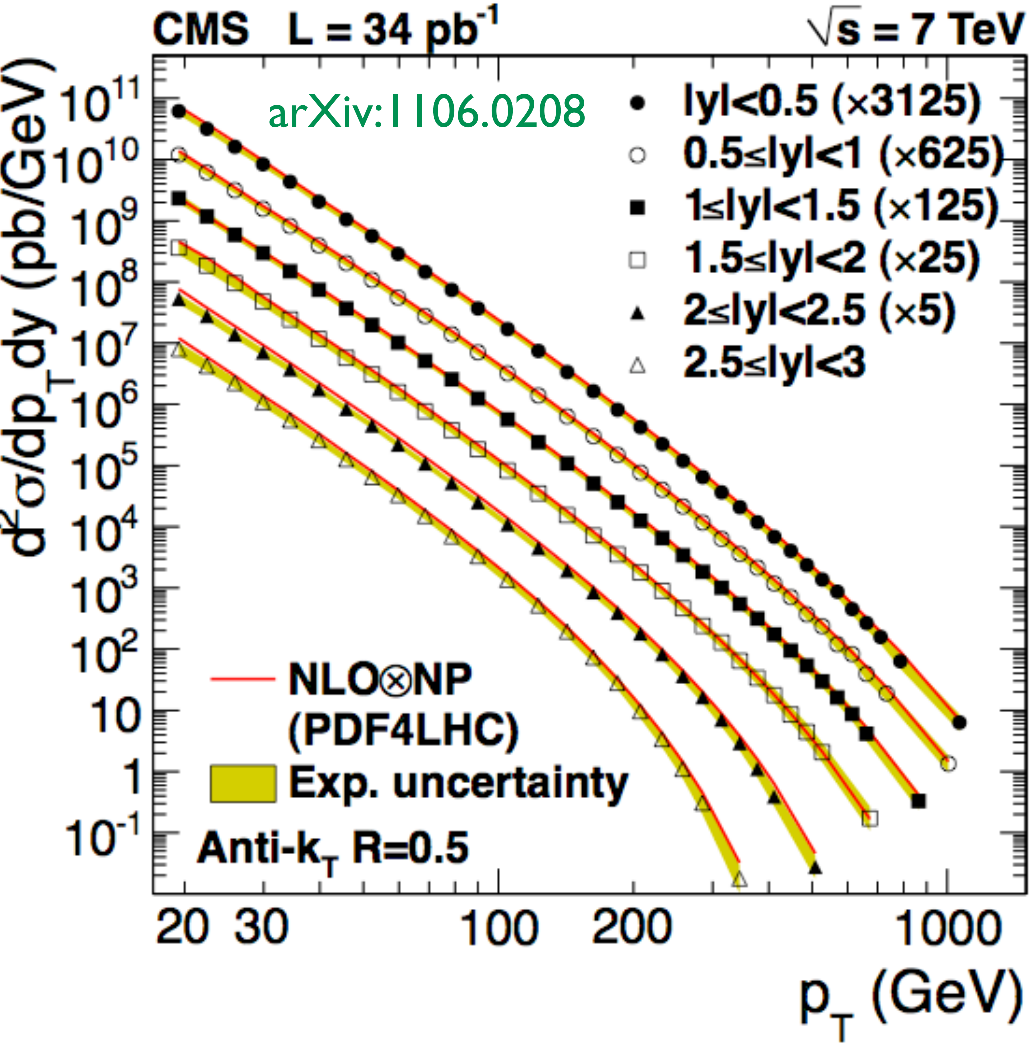
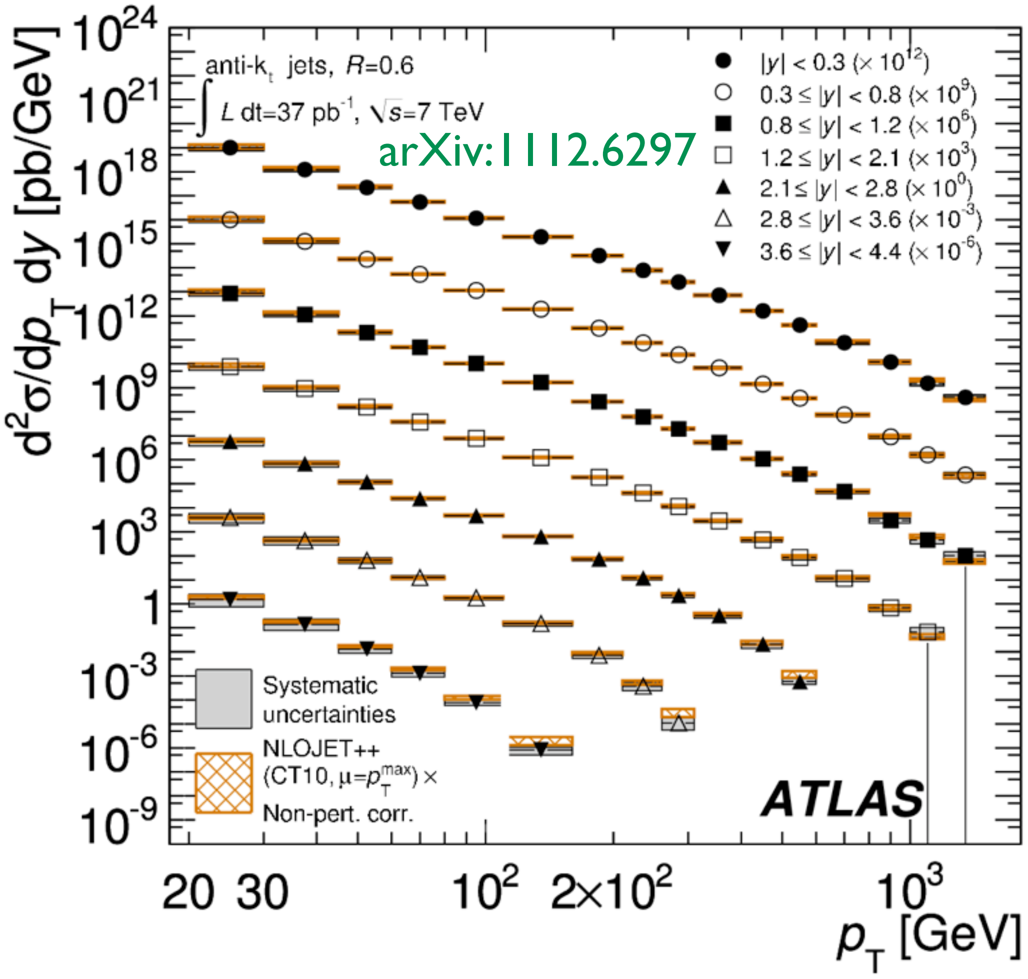
LHC physics results

Jet inclusive cross section

Jets extensively measured in hadronic collisions.

One of the most basic observables.



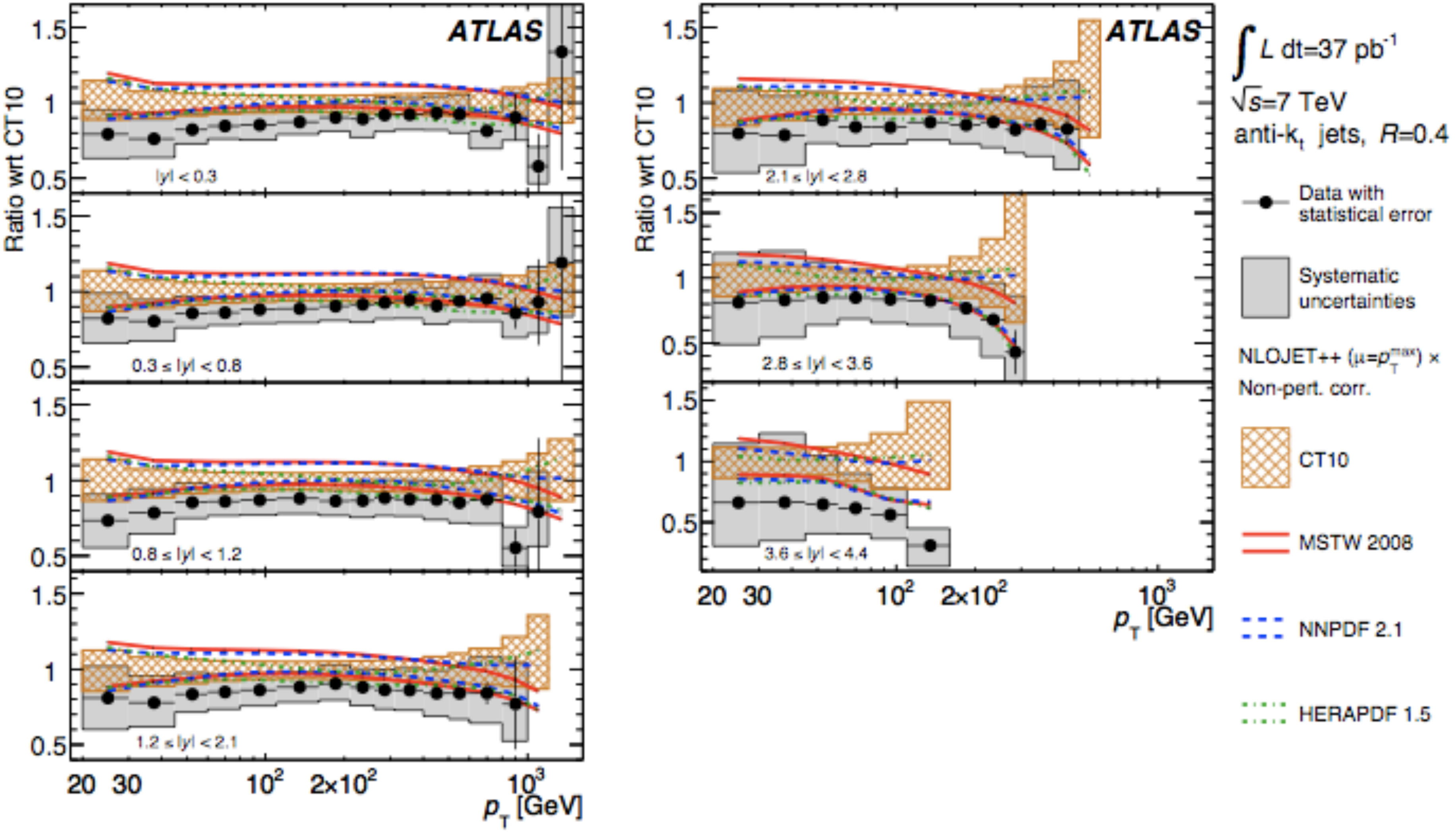


Very good agreement with pQCD predictions over 10 orders of magnitude

LHC physics results

Jet inclusive cross section

Ratio to theory, **sensitivity to PDFs**

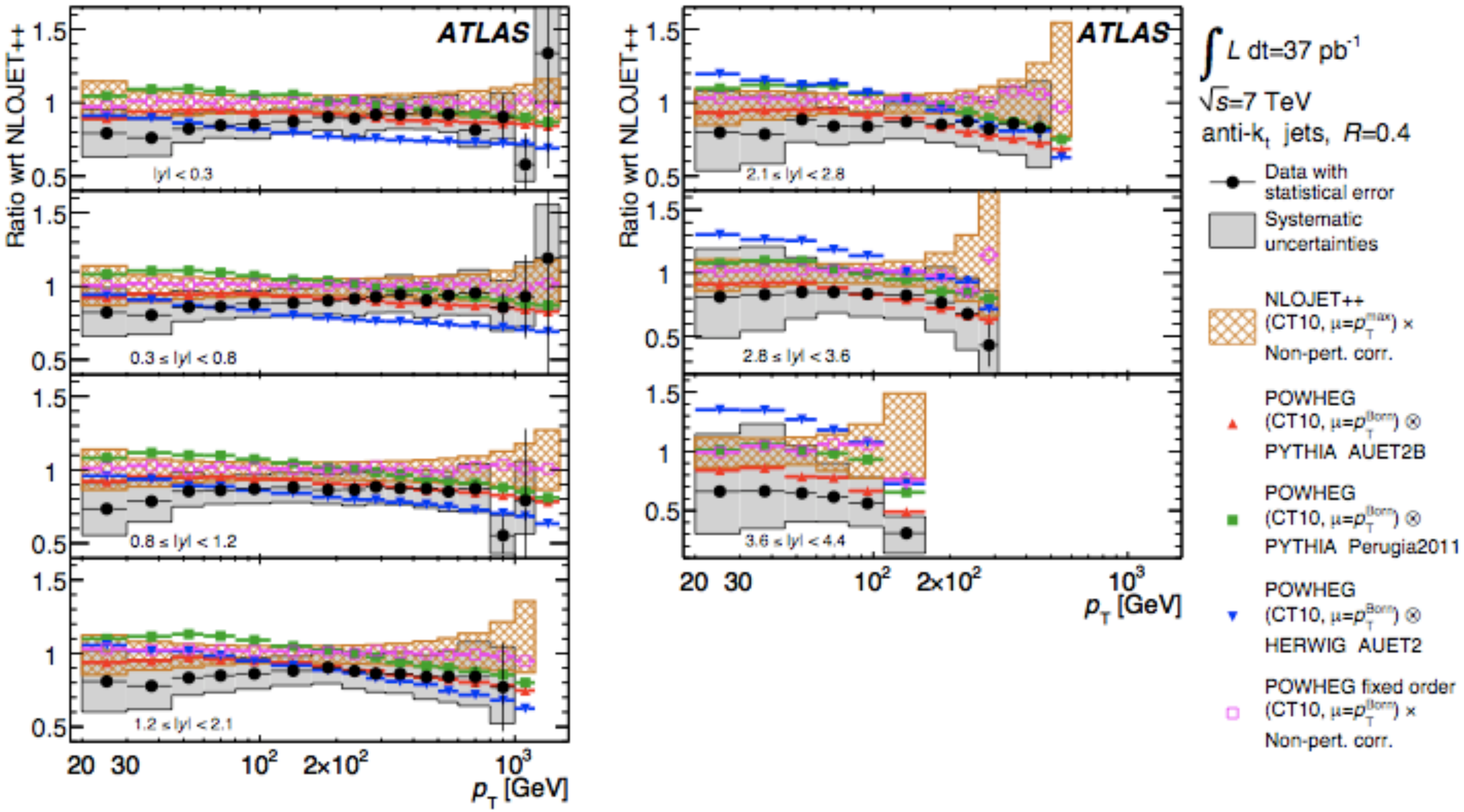


LHC physics results

Jet inclusive cross section

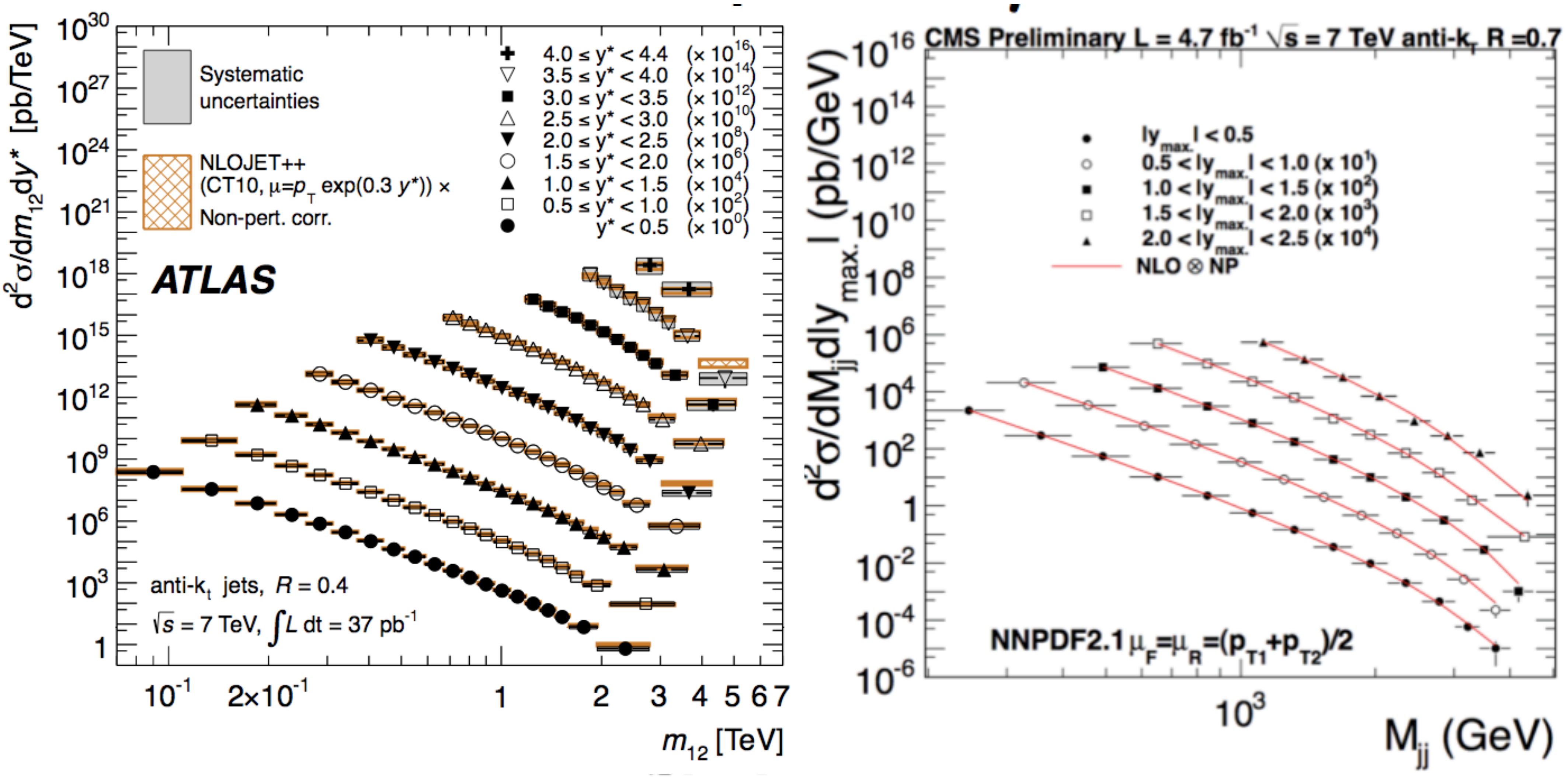
Ratio to theory,

sensitivity to parton shower and non-perturbative physics



LHC physics results

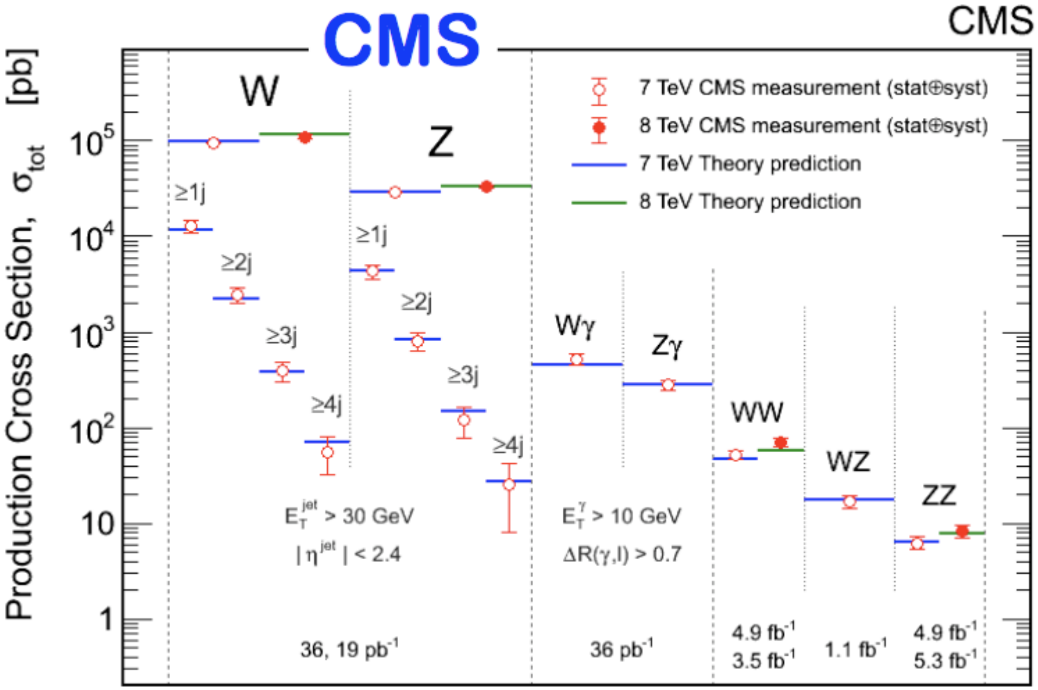
Dijet mass

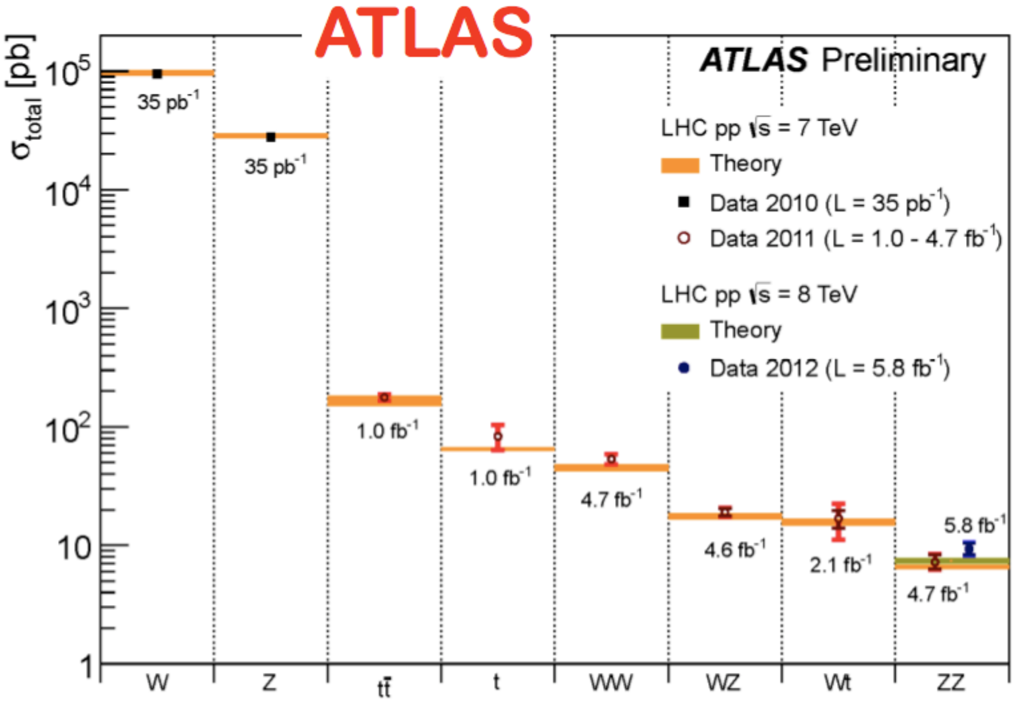


Good agreement with theory up to 4-5 TeV

LHC physics results

The mother of all data/theory comparisons





Mostly excellent agreement

LHC physics results

It is worth noting that the data/theory comparison does not (yet) **always** work perfectly.

On the other hand, theoretical progress continues to be made, and often wrongs are righted

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Di-Photon Production: Results

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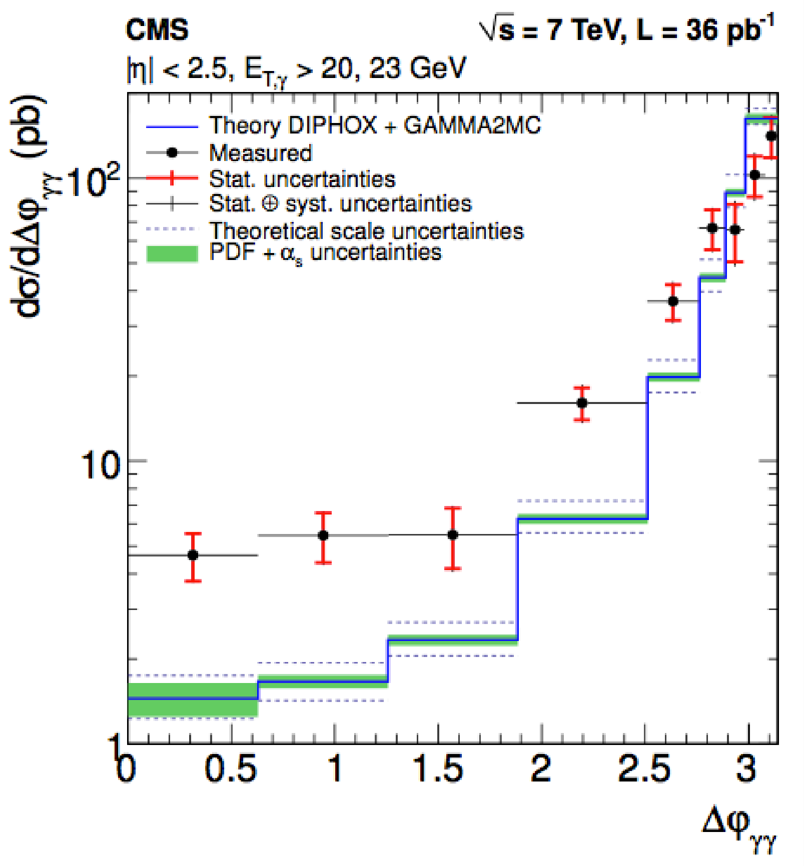
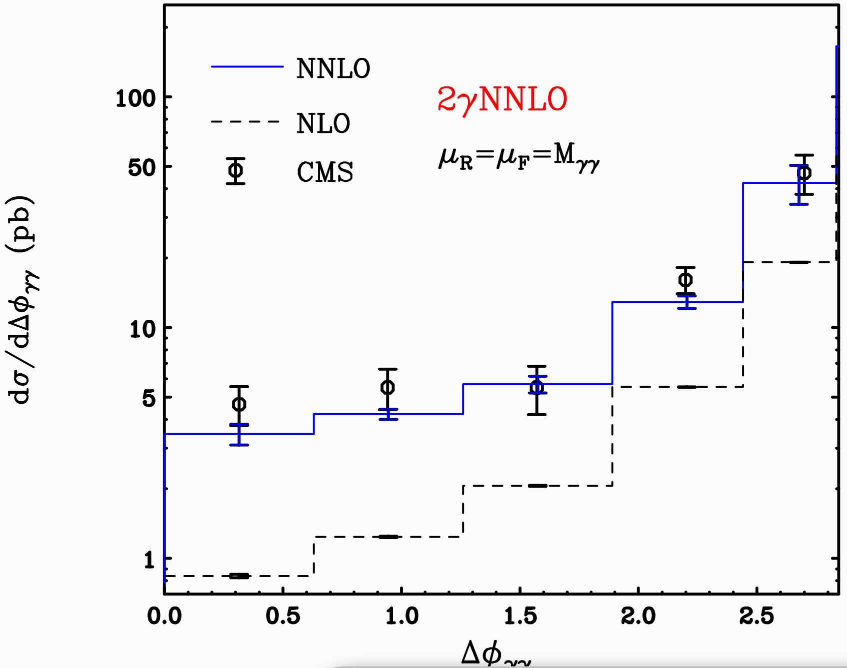


from D. de Florian, M. Grazzini, et al

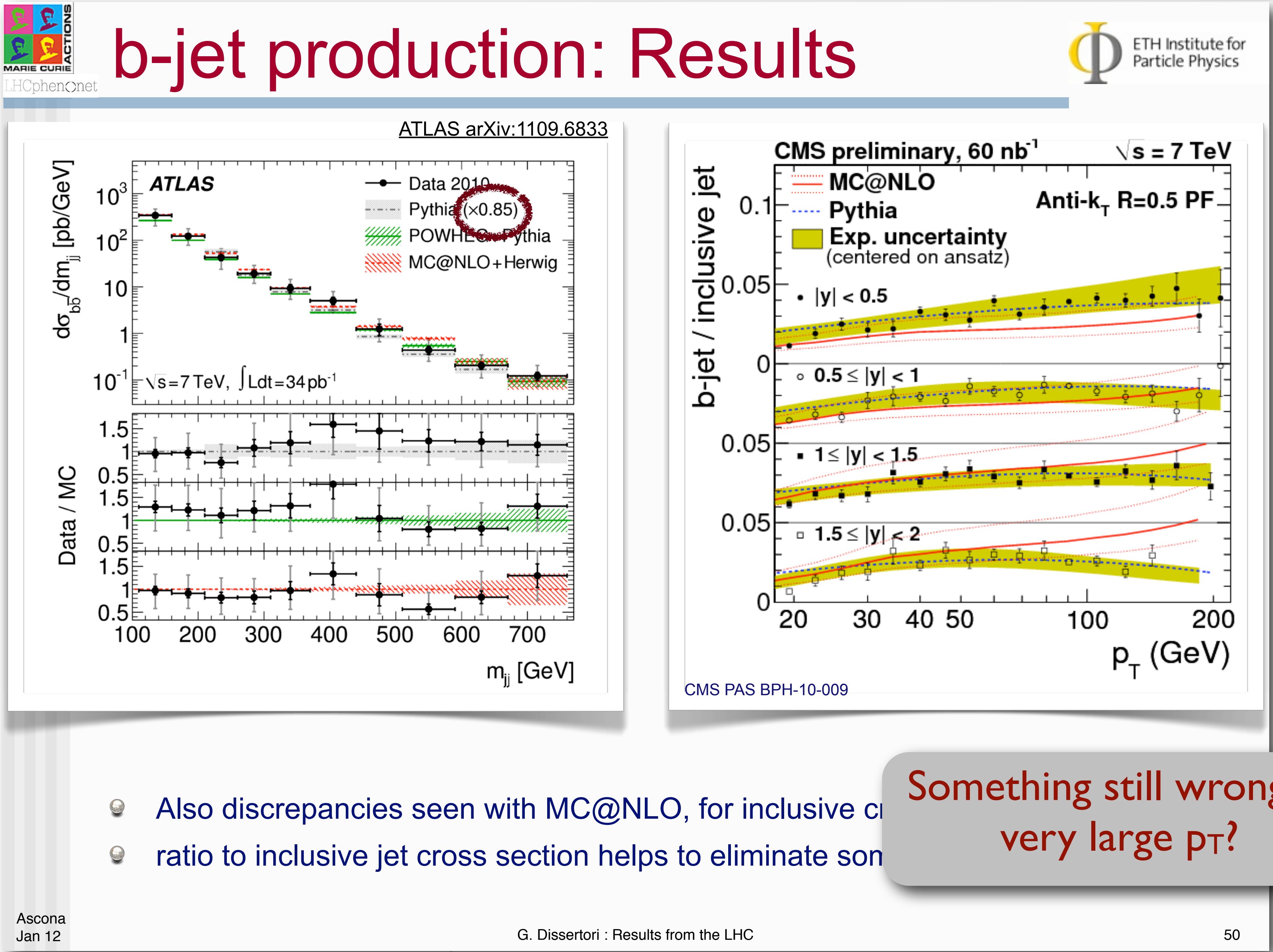


Very recent NNLO calculation seems to eliminate the discrepancy

- **Big discrepancy at small angles???**
  - But note: at very small angles, the NLO calculation is actually a “LO” calculation
  - confirmed by very recent calculation (see plot on the right)

Slide from G. Dissertori

LHC physics results



b-jet production: Results

ATLAS arXiv:1109.6833

CMS PAS BPH-10-009

- Also discrepancies seen with MC@NLO, for inclusive c
- ratio to inclusive jet cross section helps to eliminate son

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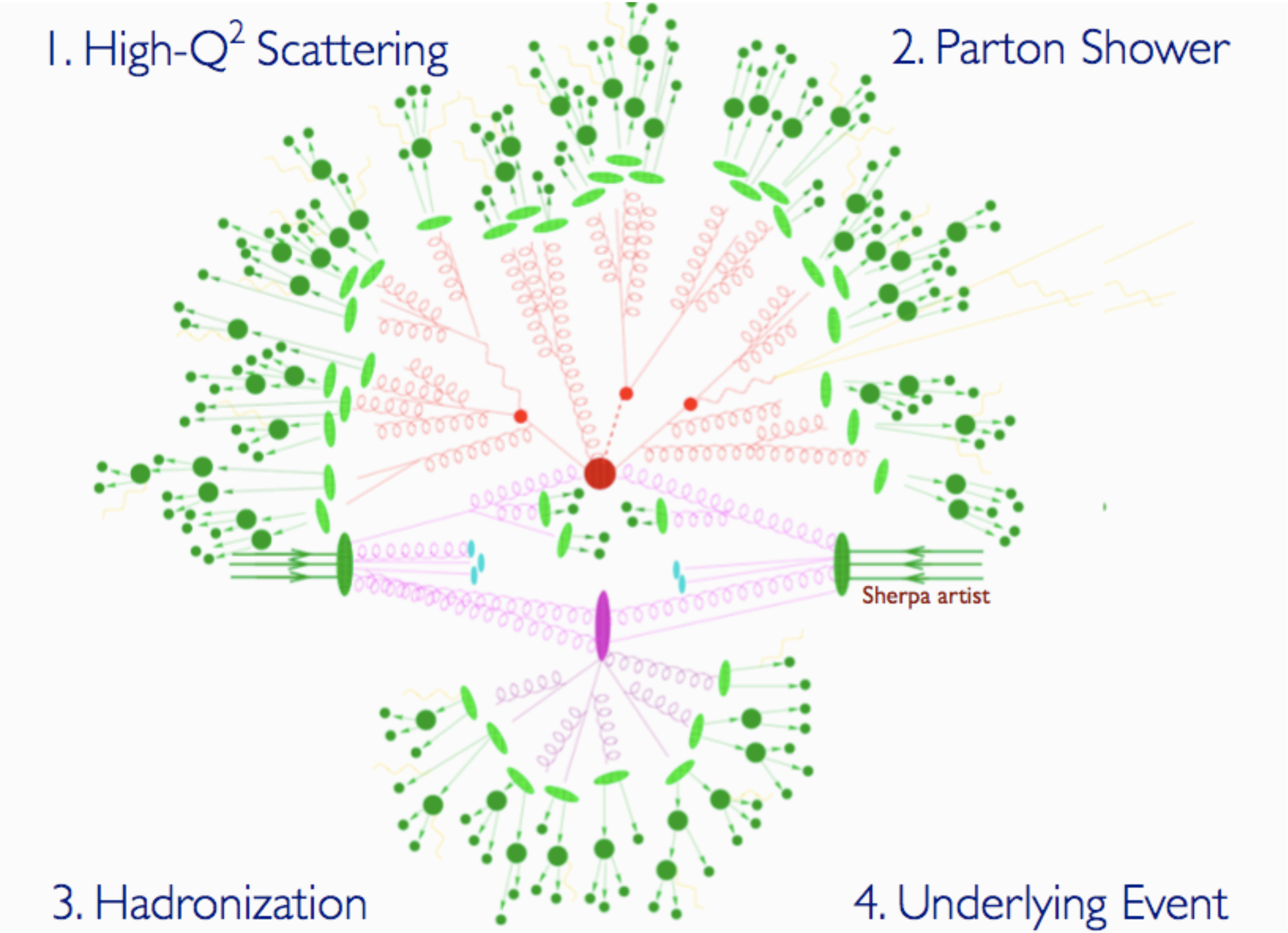
Something still wrong at very large p_T?

The scene

With just a few months of operation, the LHC is largely a ***sub-10% accuracy*** machine
(possibly on its way to a 1% level)

In what kind of environment have these measurements and calculations taken place?

A hadronic process



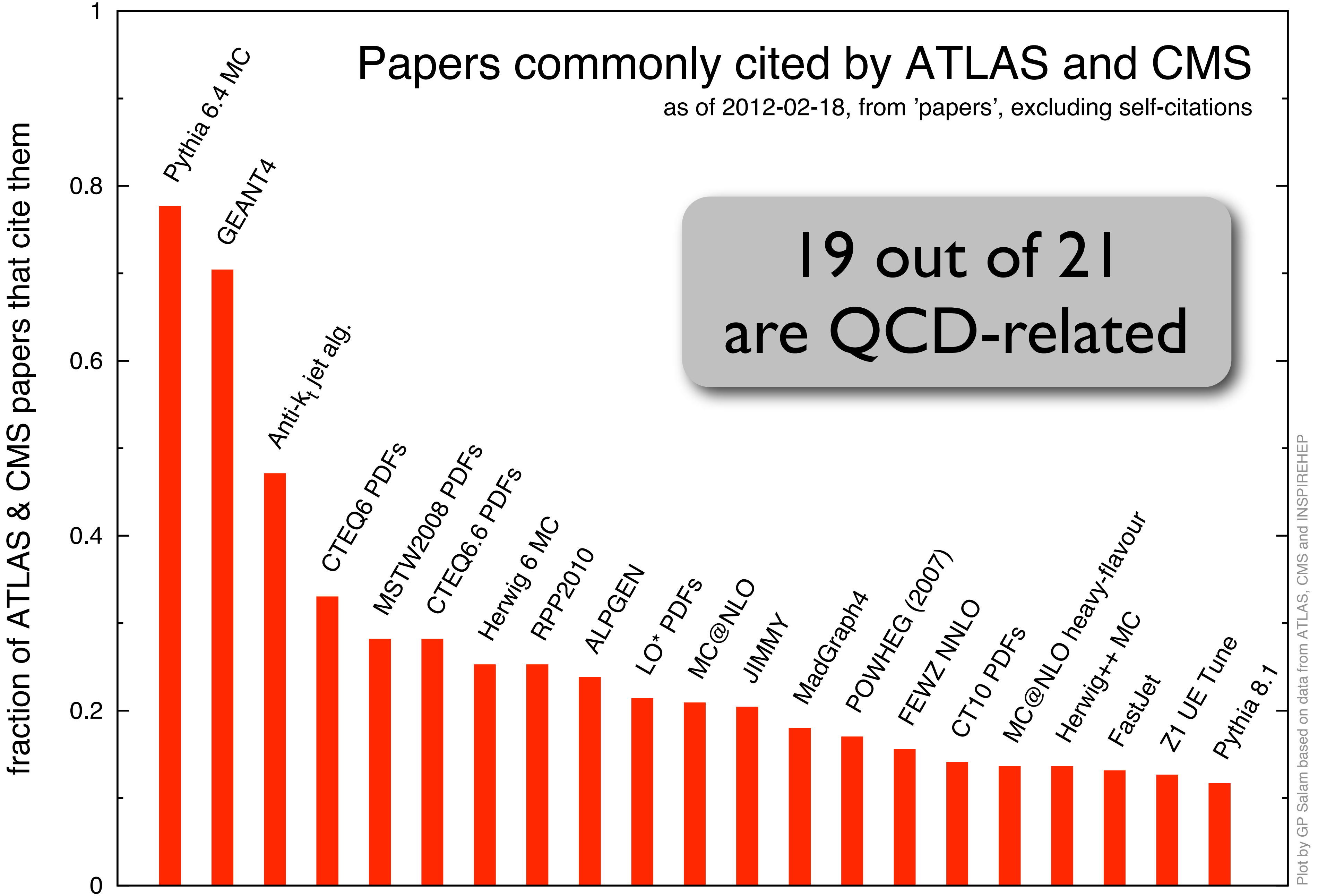
Describing complexity

A large part of the success of LHC physics (and the speed with which it has come) must be due to the excellent quality of the simulation tools for detectors and physics employed there.

Tevatron did not have such good tools, especially at the beginning of its 25 years run. It took a lot longer to understand the detector and to extract physics.

[I think it was at LEP that the need/usefulness of high-precision simulations/predictions became evident]

Role of tools in ATLAS and CMS



Plot by GP Salam based on data from ATLAS, CMS and INSPIREHEP

19 out of 21
are QCD-related

Evolution of (physics) tools

- 10 years ago we had
  - PYTHIA, HERWIG (parton shower MCs)
  - GRV, CTEQ, MRST (NLL PDFs)
  - first automated tools for tree level (CompHEP,...)
  - dedicated NLO codes, for fairly simple processes
  - Infrared and collinear unsafe (and/or slow) jet algorithms

Evolution of (physics) tools

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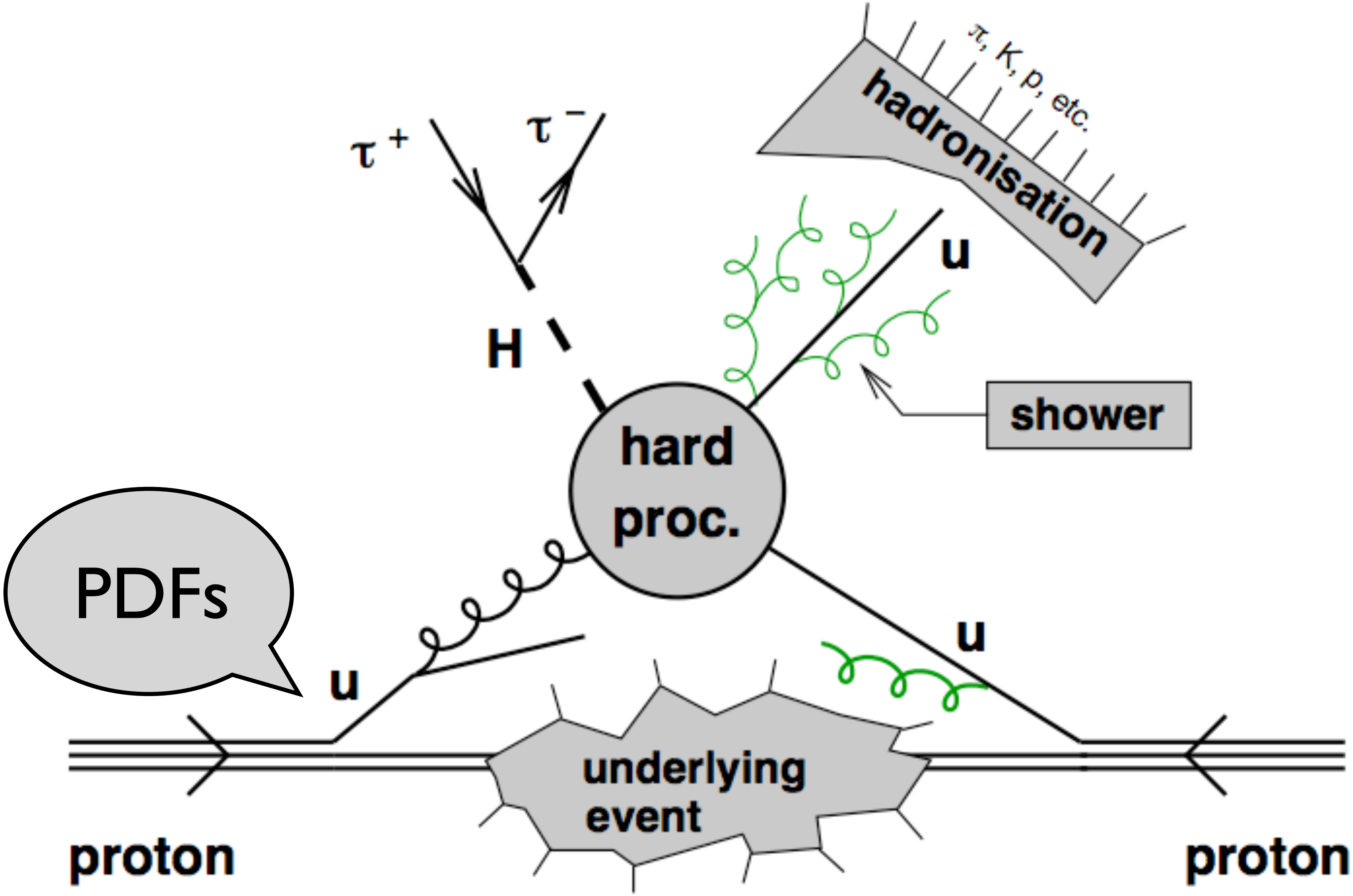
  - first automated tools for tree level (CompHEP,...)
  - dedicated NLO codes, for fairly simple processes
  - Infrared and collinear unsafe (and/or slow) jet algorithms
- now we also have
  - PYTHIA8, HERWIG++, SHERPA
  - MC@NLO, POWHEG (matching of NLO with PS)
  - matching of PS with matrix elements (CKKW, MLM)
  - more PDFs sets, some at NNLL (NNPDF, HERAPDF, ABKM, JR,...)
  - many more NLO calculations, including for complex processes
  - automated tools for LO and NLO (MadGraph, aMC@NLO,...)
  - dedicated NNLO codes, for fairly simple processes
  - Infrared and collinear safe and fast jet algorithms

A hadronic process



Final state

Hard process

Initial state

The template for an hadronic process

$$H_1\,H_2 \rightarrow H_3 + X$$

$$\frac{d\sigma}{d^3p}(Q) \sim F(\mu_F) \times F(\mu_F) \times \frac{d\hat{\sigma}}{d^3\hat{p}}(\mu_f, \mu_R, \alpha_s(\mu_R)) \times D(\mu_F) + O\left(\frac{\Lambda}{Q}\right)^p$$

The template for an hadronic process

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$$\frac{d\sigma}{d^3p}(Q) \sim F(\mu_F) \times F(\mu_F) \times \frac{d\hat{\sigma}}{d^3\hat{p}}(\mu_f, \mu_R, \alpha_s(\mu_R)) \times D(\mu_F) + O\left(\frac{\Lambda}{Q}\right)^p$$

short-distance,
calculable
in pQCD

The template for an hadronic process

$$H_1\, H_2 \longrightarrow H_3 + X$$

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fit from data,
use in other predictions

short-distance,
calculable
in pQCD

fit from data,
use in other predictions

‘*leading twist*’ long-distance non-perturbative contributions

The template for an hadronic process

$$H_1\, H_2 \to H_3 + X$$

$$\frac{d\sigma}{d^3p}(Q) \sim F(\mu_F) \times F(\mu_F) \times \frac{d\hat{\sigma}}{d^3\hat{p}}(\mu_f, \mu_R, \alpha_s(\mu_R)) \times D(\mu_F) + O\left(\frac{\Lambda}{Q}\right)^p$$

fit from data,
use in other predictions

short-distance,
calculable
in pQCD

fit from data,
use in other predictions

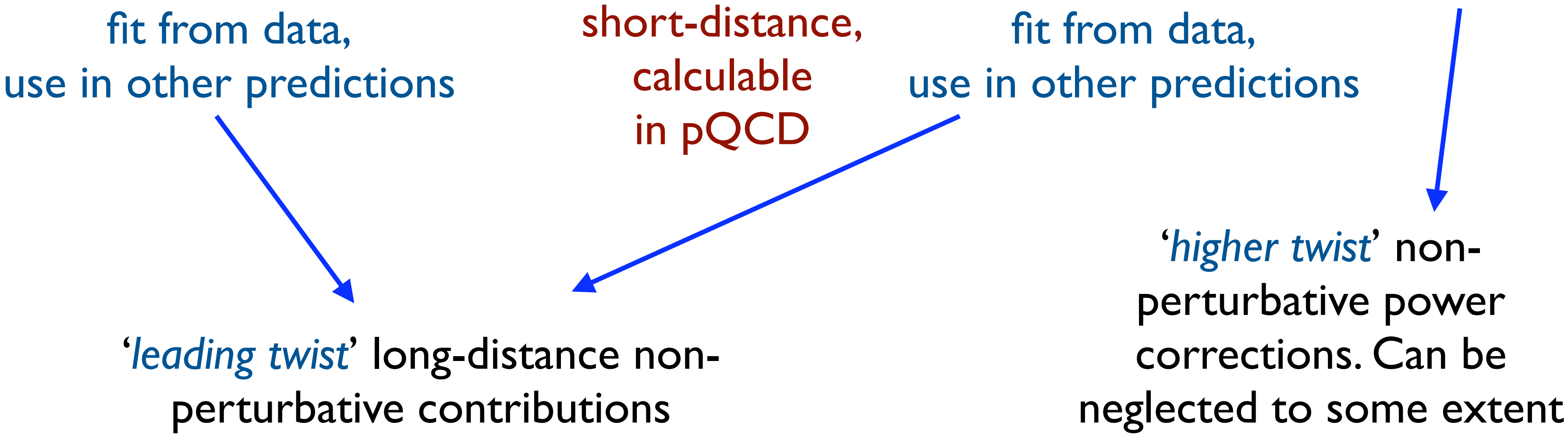
‘*leading twist*’ long-distance non-perturbative contributions

‘*higher twist*’ non-perturbative power corrections. Can be neglected to some extent

The template for an hadronic process

$$H_1\, H_2 \longrightarrow H_3 + X$$

$$\frac{d\sigma}{d^3p}(Q) \sim F(\mu_F) \times F(\mu_F) \times \frac{d\hat{\sigma}}{d^3\hat{p}}(\mu_f, \mu_R, \alpha_s(\mu_R)) \times D(\mu_F) + O\left(\frac{\Lambda}{Q}\right)^p$$



fit from data,
use in other predictions

short-distance,
calculable
in pQCD

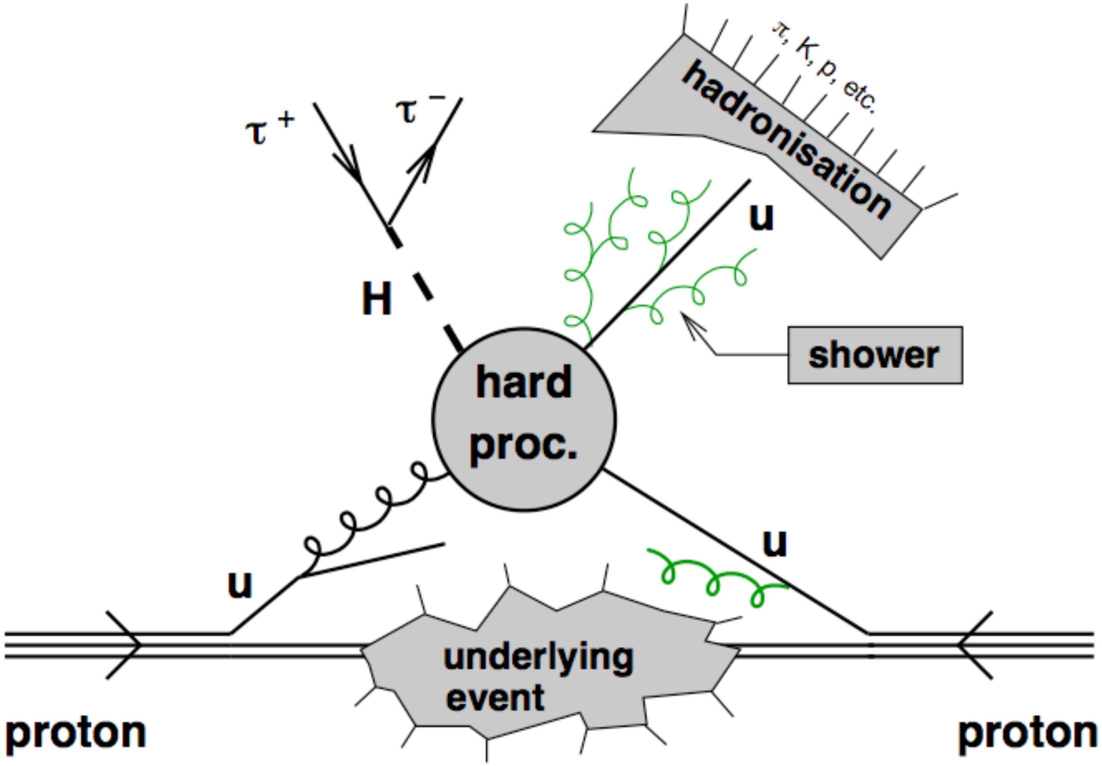
fit from data,
use in other predictions

‘*leading twist*’ long-distance non-perturbative contributions

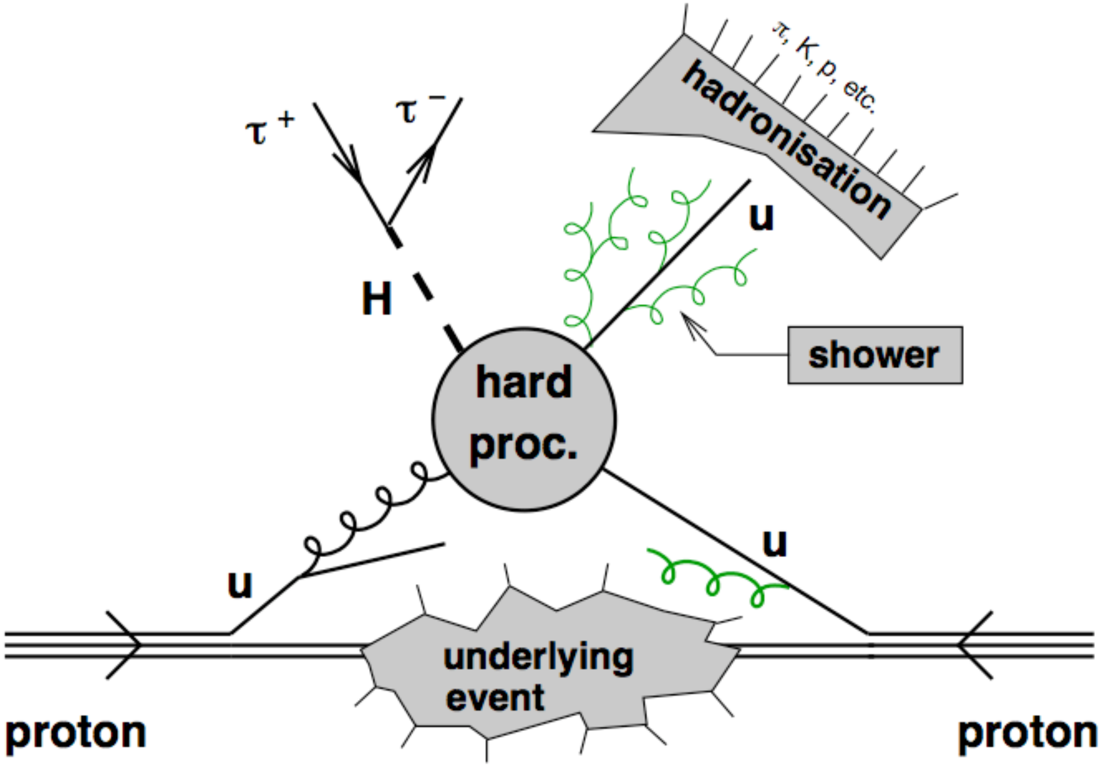
‘*higher twist*’ non-perturbative power corrections. Can be neglected to some extent

Testing (and using) QCD is essentially an iterative procedure which amounts to running an equation like this one through many sets of data, extracting ingredients and using them for predictions, always checking for consistency

Ingredients and tools



Ingredients and tools

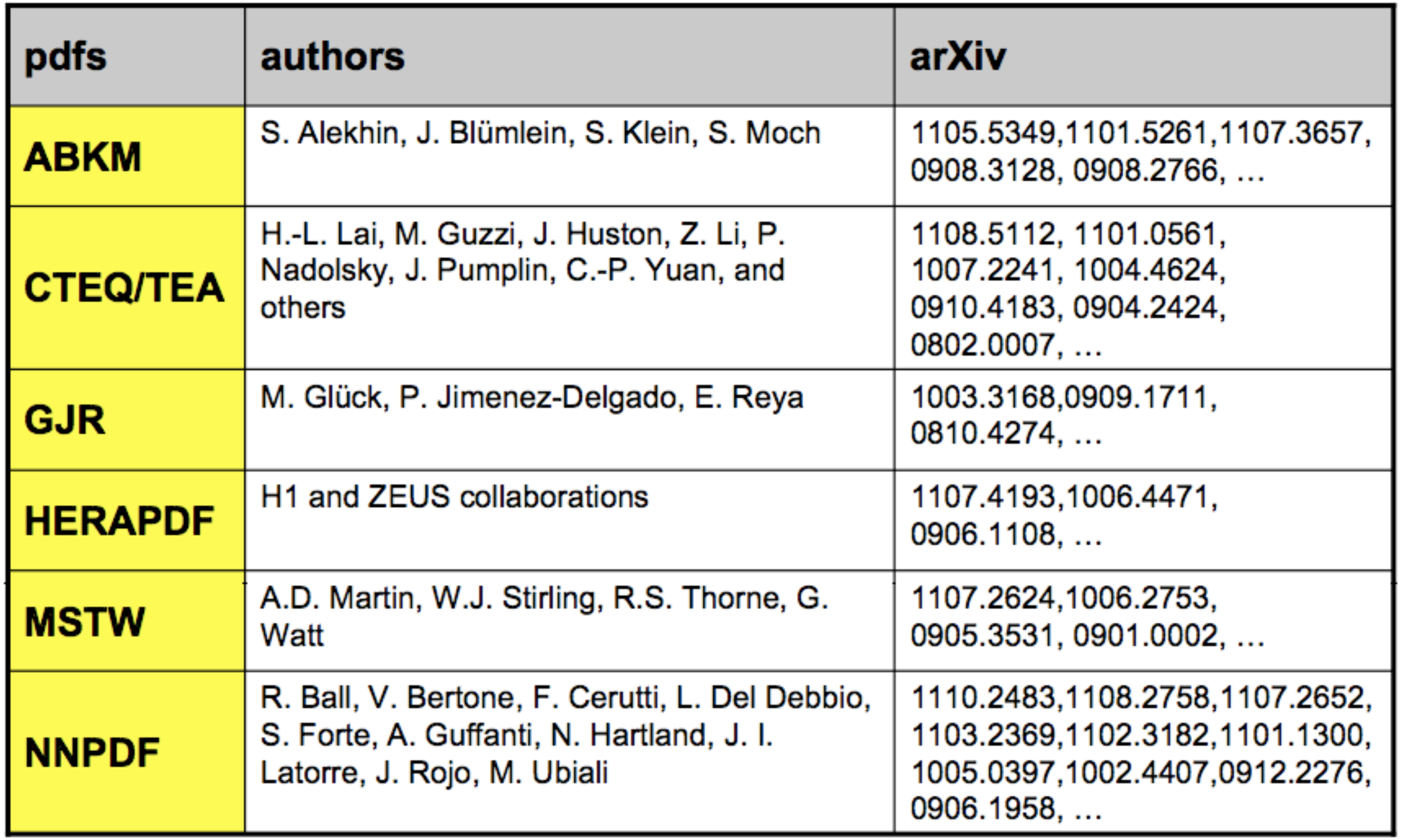


- PDFs
- Hard scattering
- Final state tools

PDFs: choices

Extracting PDFs from data has become a favourite pastime

- Then: CTEQ, MRST, GRV, ...
- Today: CTEQ, MSTW, NNPDF, HERAPDF, ABKM,GJR, ...



pdfs	authors	arXiv
ABKM	S. Alekhin, J. Blümlein, S. Klein, S. Moch	1105.5349,1101.5261,1107.3657, 0908.3128, 0908.2766, …
CTEQ/TEA	H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others	1108.5112, 1101.0561, 1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, …
GJR	M. Glück, P. Jimenez-Delgado, E. Reya	1003.3168,0909.1711, 0810.4274, …
HERAPDF	H1 and ZEUS collaborations	1107.4193,1006.4471, 0906.1108, …
MSTW	A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt	1107.2624,1006.2753, 0905.3531, 0901.0002, …
NNPDF	R. Ball, V. Bertone, F. Cerutti, L. Del Debbio, S. Forte, A. Guffanti, N. Hartland, J. I. Latorre, J. Rojo, M. Ubiali	1110.2483,1108.2758,1107.2652, 1103.2369,1102.3182,1101.1300, 1005.0397,1002.4407,0912.2276, 0906.1958, …

M. Ubiali

PDFs: choices

Is the abundance of PDF sets redundant?

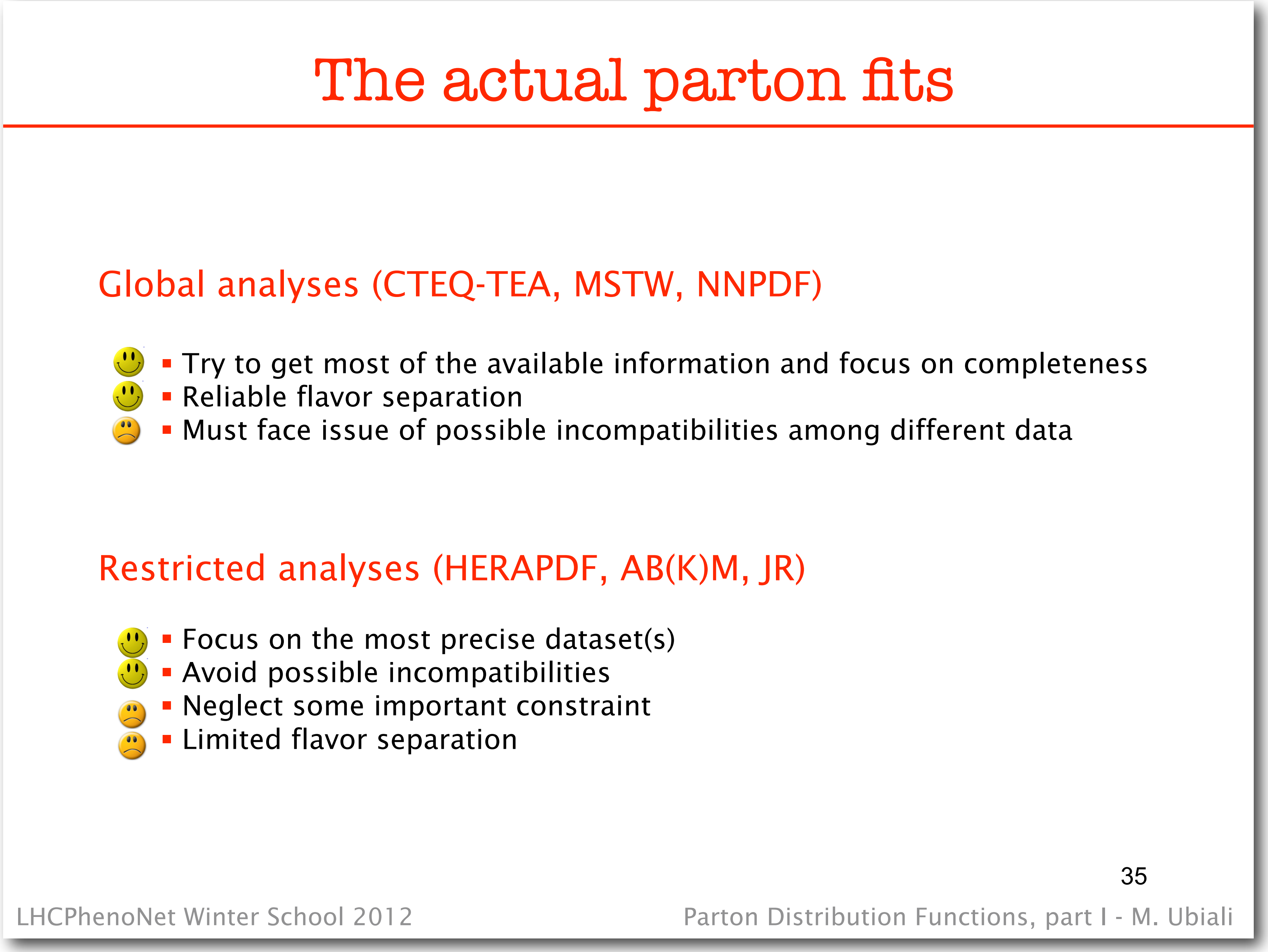
Only up to a point, since many different choices can be made

- What data to fit? Everything? A more limited and more consistent set?
- What technique to use to describe the PDFs? Parametric form? Neural network?
- Fit α_s with PDFs, or use external value?
- What treatment for heavy quark masses?
- How to exploit higher order calculations? K-factors or exact results?
- .....

There is value in having (a reasonable number of) independently obtained PDF sets

Slide from M. Ubiali

PDFs: choices



The actual parton fits

Global analyses (CTEQ-TEA, MSTW, NNPDF)

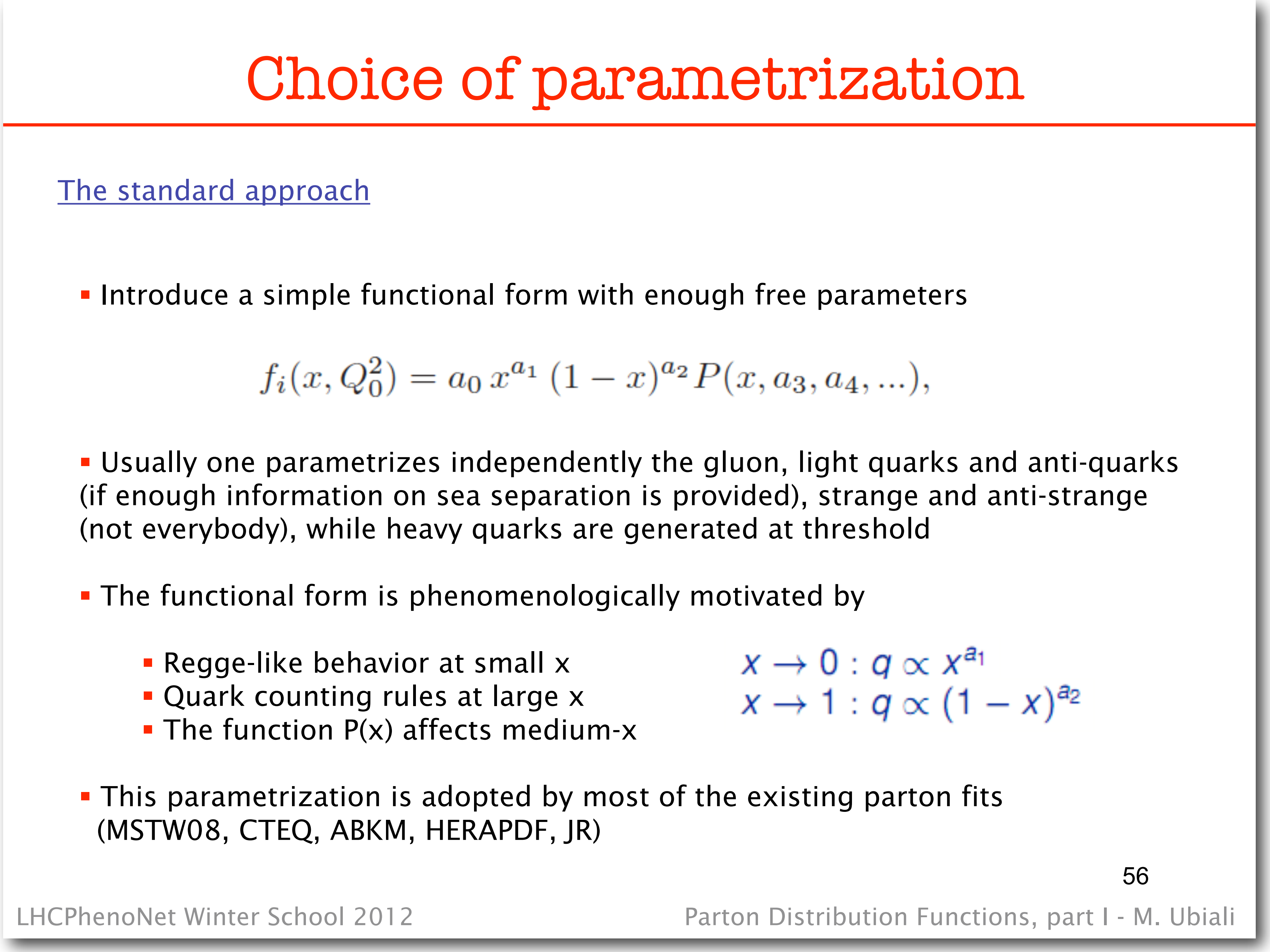
- ☺ Try to get most of the available information and focus on completeness
- ☺ Reliable flavor separation
- ☹ Must face issue of possible incompatibilities among different data

Restricted analyses (HERAPDF, AB(K)M, JR)

- ☺ Focus on the most precise dataset(s)
- ☺ Avoid possible incompatibilities
- ☹ Neglect some important constraint
- ☹ Limited flavor separation

Slide from M. Ubiali

PDFs: choices



Choice of parametrization

The standard approach

- Introduce a simple functional form with enough free parameters

$$f_i(x, Q_0^2) = a_0\, x^{a_1}\, (1-x)^{a_2}\, P(x, a_3, a_4, \ldots),$$

- Usually one parametrizes independently the gluon, light quarks and anti-quarks (if enough information on sea separation is provided), strange and anti-strange (not everybody), while heavy quarks are generated at threshold

- The functional form is phenomenologically motivated by
  - Regge-like behavior at small x
  - Quark counting rules at large x
  - The function P(x) affects medium-x

$x \to 0 : q \propto x^{a_1}$

$x \to 1 : q \propto (1-x)^{a_2}$

- This parametrization is adopted by most of the existing parton fits (MSTW08, CTEQ, ABKM, HERAPDF, JR)

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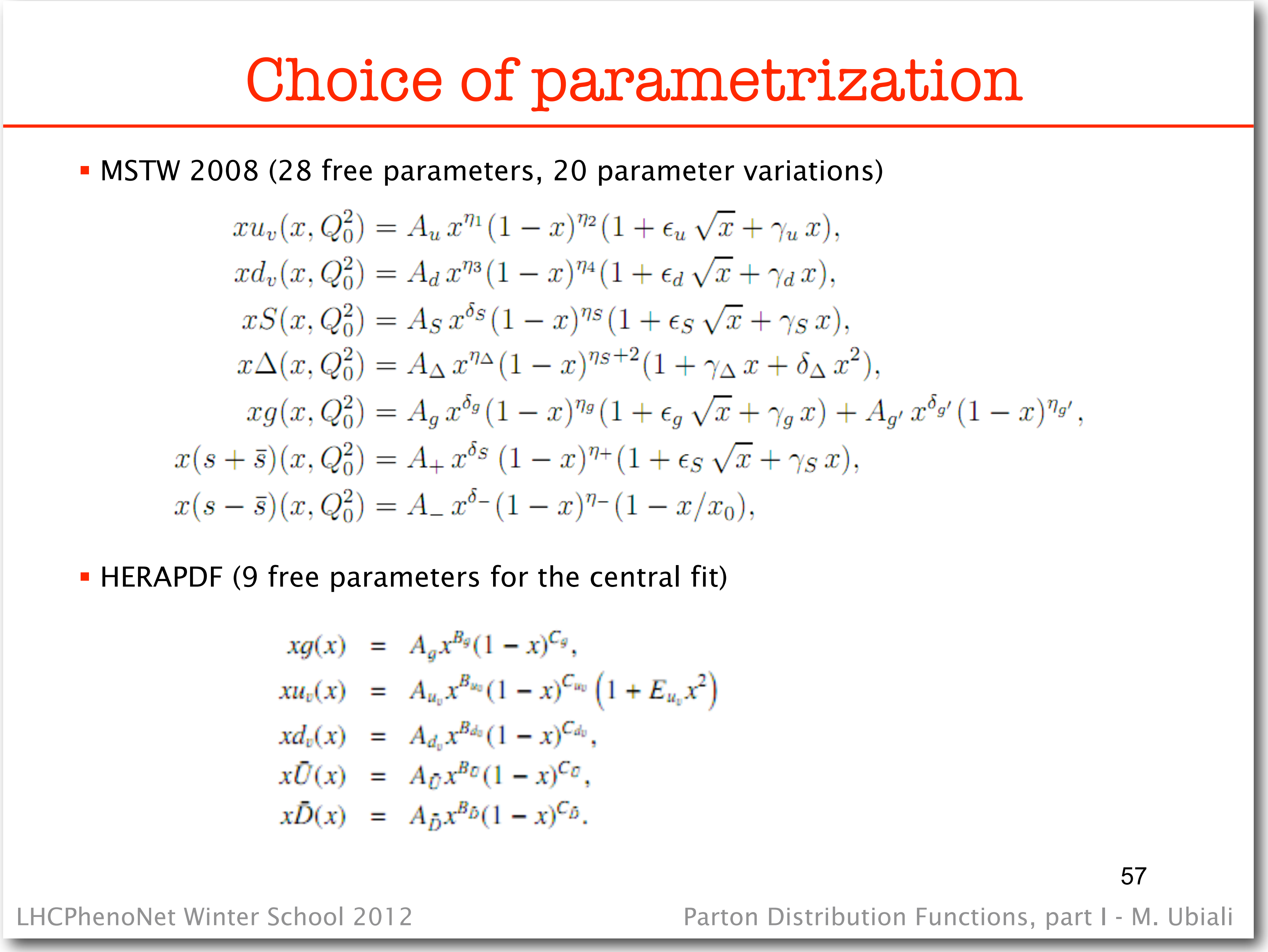
Parton Distribution Functions, part I - M. Ubiali

Slide from M. Ubiali

PDFs: choices



Choice of parametrization

- MSTW 2008 (28 free parameters, 20 parameter variations)

$$
\begin{aligned}
xu_v(x,Q_0^2) &= A_u\, x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\sqrt{x}+\gamma_u\, x),\\
xd_v(x,Q_0^2) &= A_d\, x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\sqrt{x}+\gamma_d\, x),\\
xS(x,Q_0^2) &= A_S\, x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\sqrt{x}+\gamma_S\, x),\\
x\Delta(x,Q_0^2) &= A_\Delta\, x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta\, x+\delta_\Delta\, x^2),\\
xg(x,Q_0^2) &= A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\sqrt{x}+\gamma_g\, x)+A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}},\\
x(s+\bar{s})(x,Q_0^2) &= A_+\, x^{\delta_S}\,(1-x)^{\eta_+}(1+\epsilon_S\sqrt{x}+\gamma_S\, x),\\
x(s-\bar{s})(x,Q_0^2) &= A_-\, x^{\delta_-}(1-x)^{\eta_-}(1-x/x_0),
\end{aligned}
$$

- HERAPDF (9 free parameters for the central fit)

$$
\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g},\\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right)\\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}},\\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}},\\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}
$$

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Parton Distribution Functions, part I - M. Ubiali

Slide from M. Ubiali

PDFs: choices

Choice of parametrization

An alternative approach: Neural Networks

- Each neuron receives input from neurons in preceding layer.
- Activation determined by weights and thresholds according to a non linear function:

$$\xi_i = g\left(\sum_j \omega_{ij}\xi_j - \theta_i\right), \qquad g(x) = \frac{1}{1+e^{-x}}$$

- NN are non-linear statistical tools
- Any continuous function can be approximated with a neural network with one internal layer and a non-linear activation function
- They are just another basis of functions!

1 - 2 - 1

$$\xi_1^{(3)}(\xi_1^{(1)}) = \frac{1}{1+e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1+e^{\theta_1^{(2)} - \xi_1^{(1)}\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1+e^{\theta_2^{(2)} - \xi_1^{(1)}\omega_{21}^{(1)}}}}}$$

- Provide a parametrization which is redundant and robust against variations

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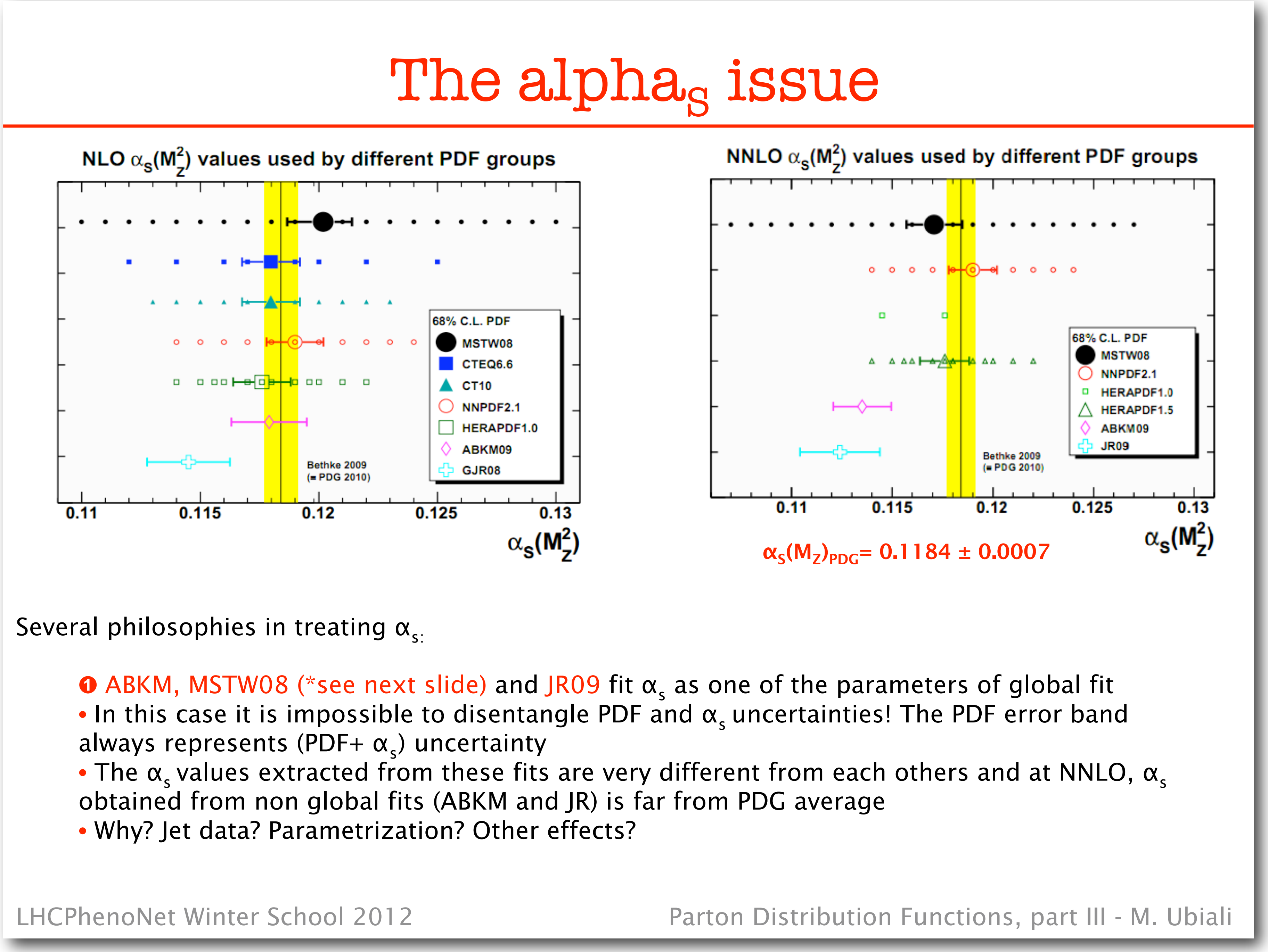
PDFs: choices



The alpha$_s$ issue

$\alpha_S(M_Z)_{PDG}$= 0.1184 ± 0.0007

Several philosophies in treating $\alpha_{s:}$

❶ ABKM, MSTW08 (*see next slide) and JR09 fit α_s as one of the parameters of global fit
- In this case it is impossible to disentangle PDF and α_s uncertainties! The PDF error band always represents (PDF+ α_s) uncertainty
- The α_s values extracted from these fits are very different from each others and at NNLO, α_s obtained from non global fits (ABKM and JR) is far from PDG average
- Why? Jet data? Parametrization? Other effects?

LHCPhenoNet Winter School 2012 Parton Distribution Functions, part III - M. Ubiali

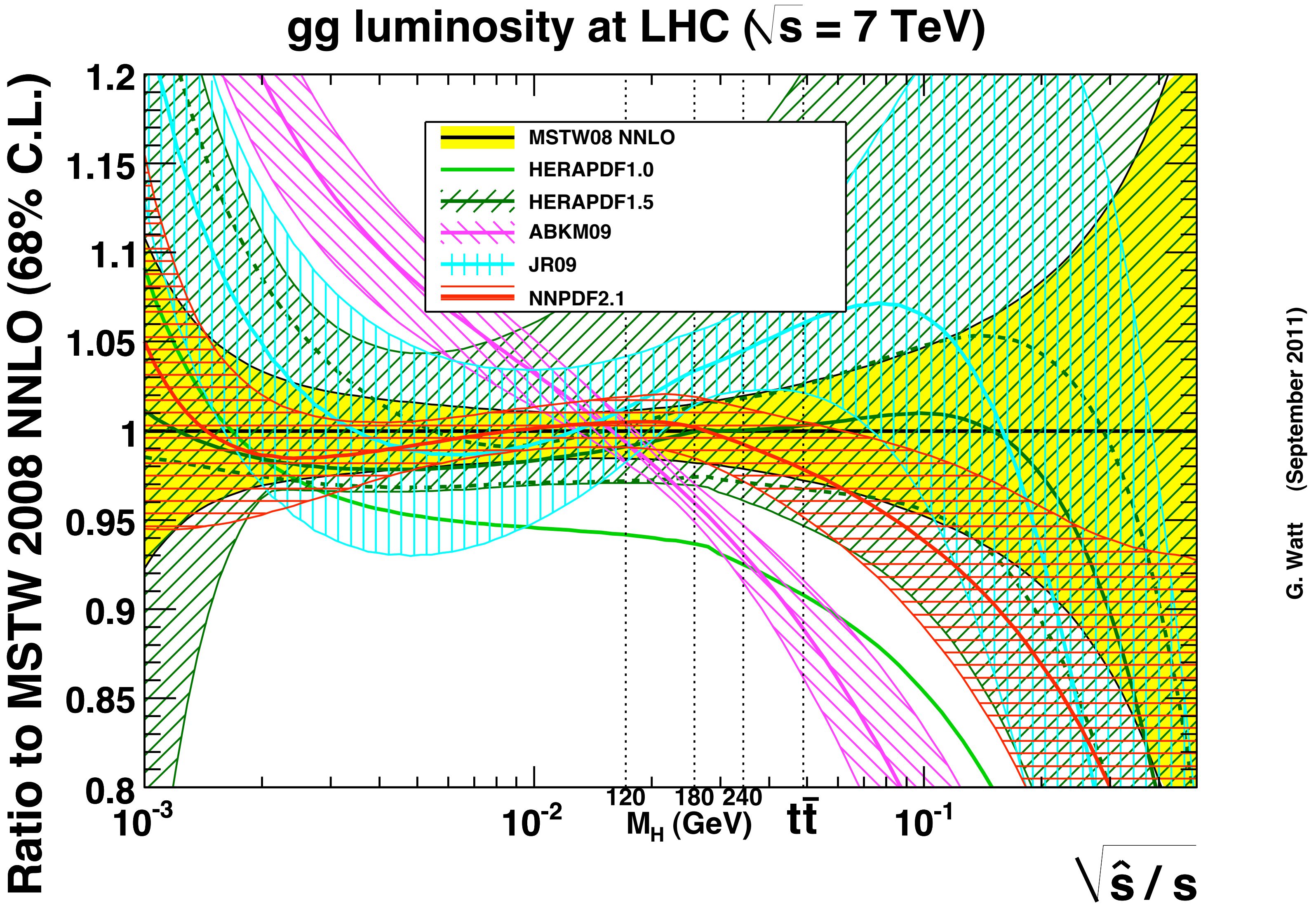
PDFs: state of the art

The most commonly used PDF sets (MSTW, CTEQ, NNPDF) use

- global fits to many data sets
- NNLO evolution
- proper matching at heavy quark thresholds
- external α_s, or many sets provided
- error estimate

The resulting PDF sets are in fairly good agreement

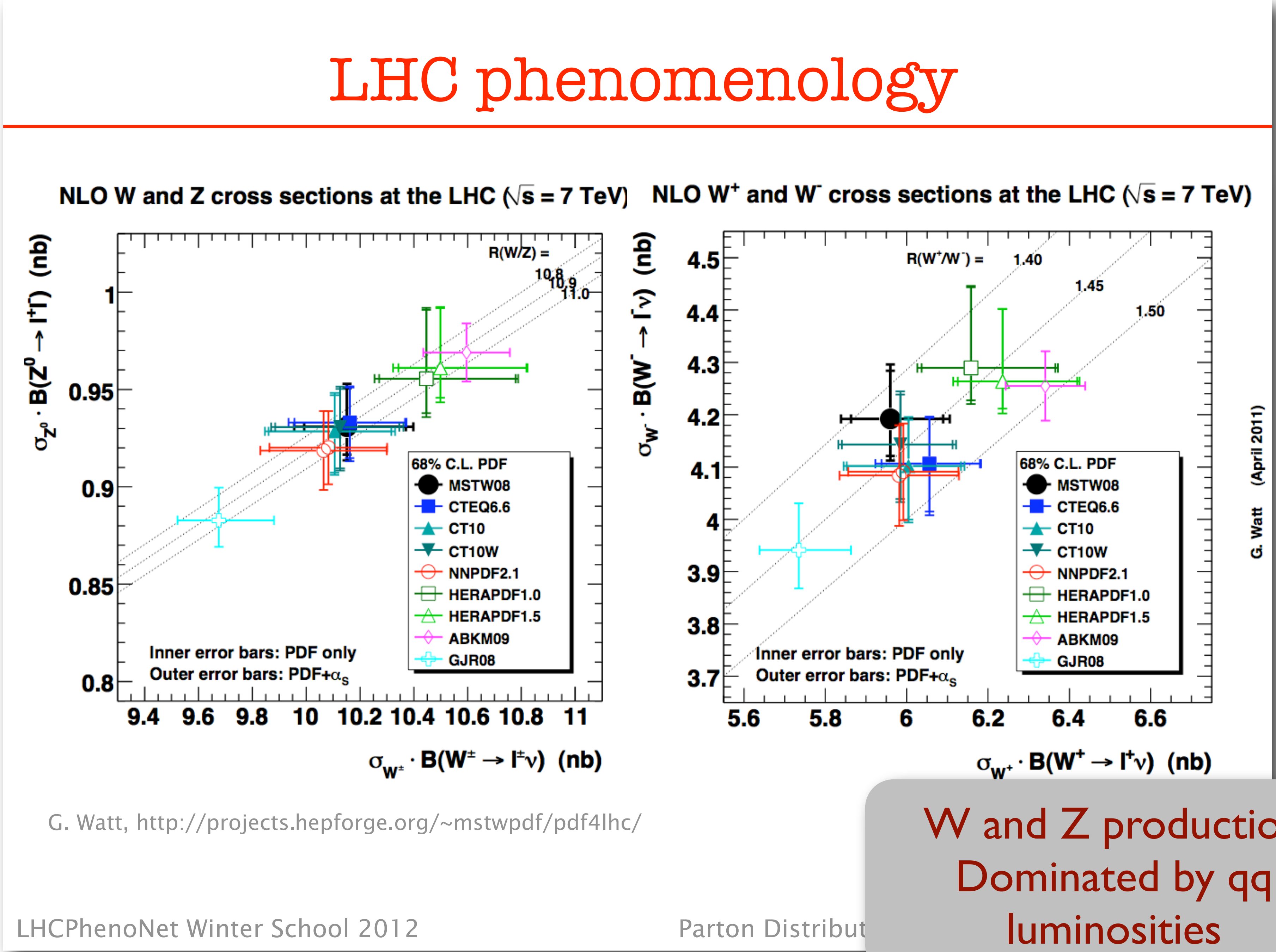
Comparison between PDFs



Two global fits (MSTW and NNPDF) show the best agreement within O(5-10%) uncertainty at the 68% CL level

Slide from M. Ubiali

Comparison between PDFs



LHC phenomenology

NLO W and Z cross sections at the LHC ($\sqrt{s}$ = 7 TeV)

NLO W^+ and W^- cross sections at the LHC ($\sqrt{s}$ = 7 TeV)

G. Watt, http://projects.hepforge.org/~mstwpdf/pdf4lhc/

LHCPhenoNet Winter School 2012

Parton Distribut

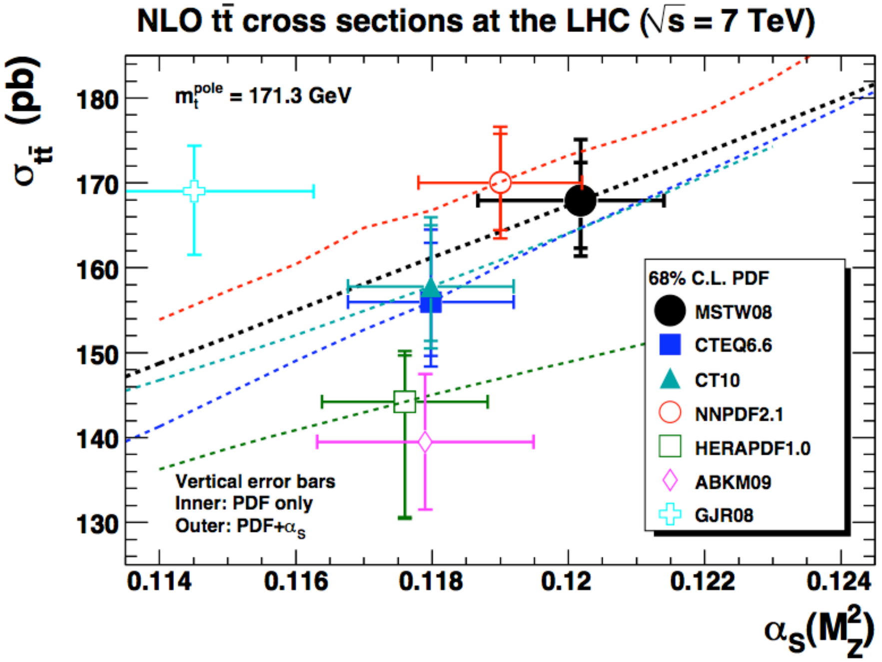
W and Z production
Dominated by qq
luminosities

Slide from M. Ubiali

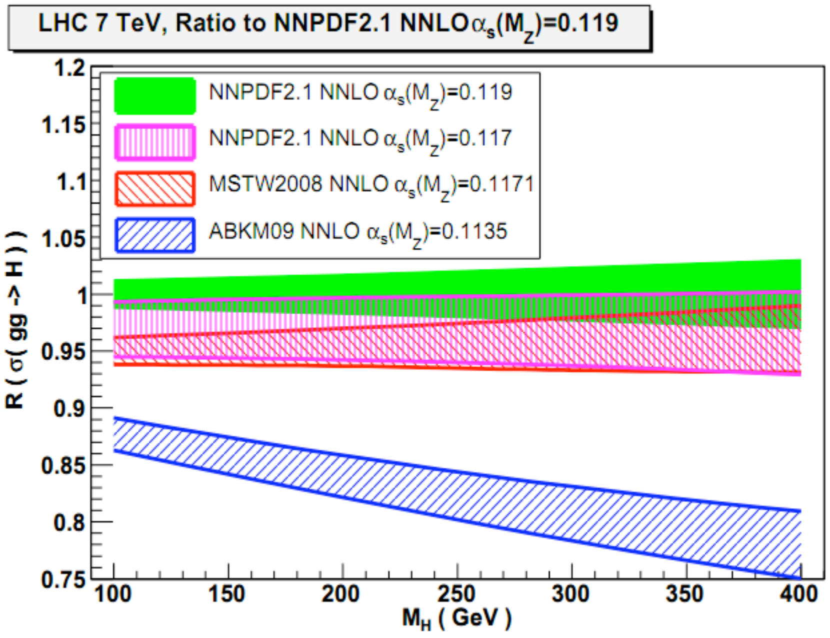
Comparison between PDFs

LHC phenomenology

G. Watt, http://projects.hepforge.org/~mstwpdf/pdf4lhc/



Ball et al, arXiv:1110.2483



LHC precise measurements will soon discriminate among PDF sets and provide stronger constraints

ttbar production
Dominated by gg
luminositiy

Theory uncertainty in PDFs

The 'error bands' returned by the PDF sets only include the experimental uncertainties of the data used in the fits. What is the theoretical uncertainty?

The kernels used in the evolution can be written as a series expansion in $\alpha_s(\mu_R)$

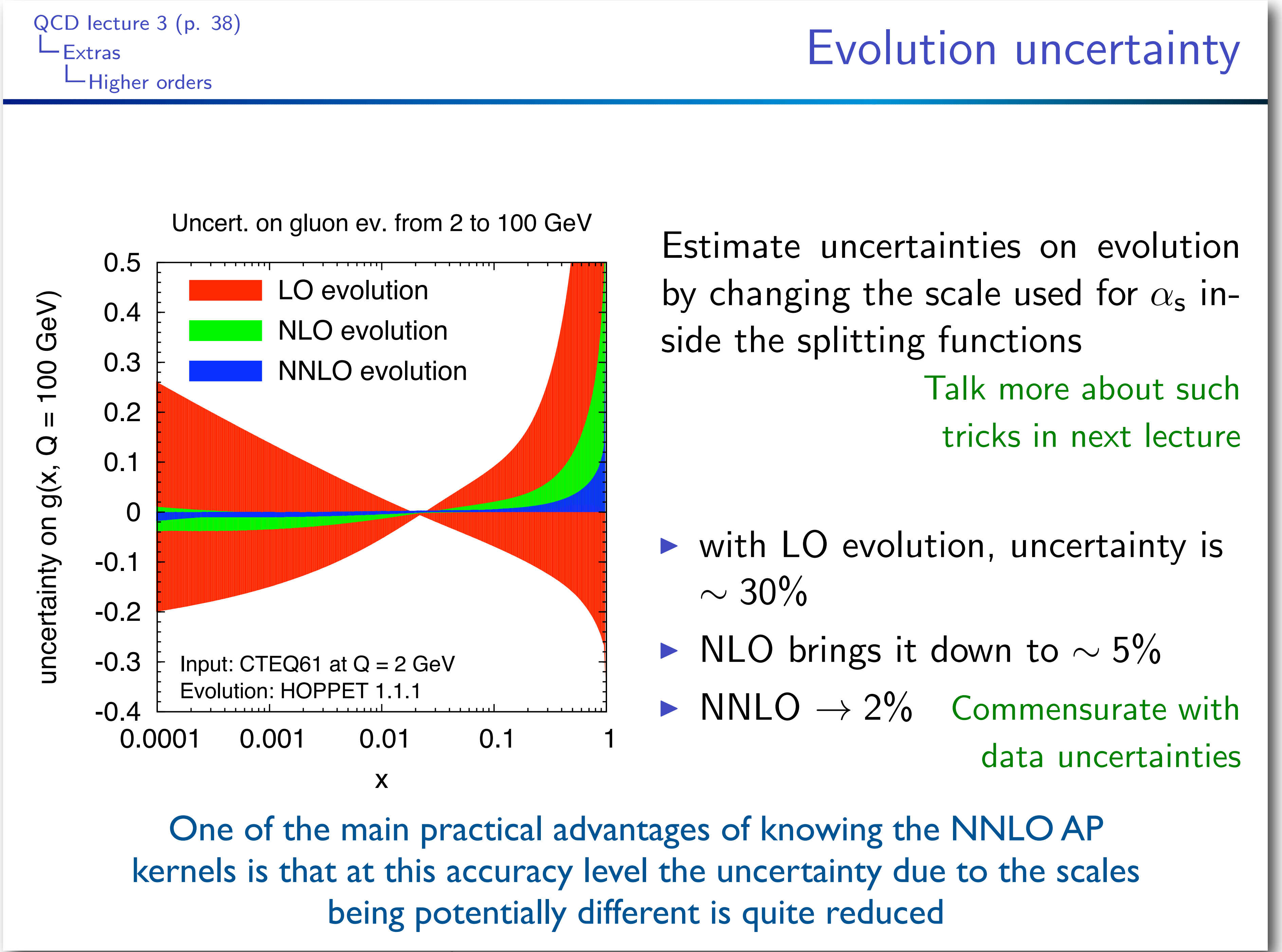
$$P(z,\mu_F) = \left(\frac{\alpha_s(\mu_R)}{2\pi}\right) P^{(0)}(z) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 P^{(1)}(z,\mu_R/\mu_F) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^3 P^{(2)}(z,\mu_R/\mu_F$$

Usually one takes $\mu_R = \mu_F$, and promptly forgets that he may do otherwise.

Taking the two scales different is an uncertainty in the evolution of the PDFs.

Slide from
G. Salam

DGLAP: uncertainy of evolution



QCD lecture 3 (p. 38)
└Extras
 └Higher orders

Evolution uncertainty

Estimate uncertainties on evolution by changing the scale used for α_s inside the splitting functions

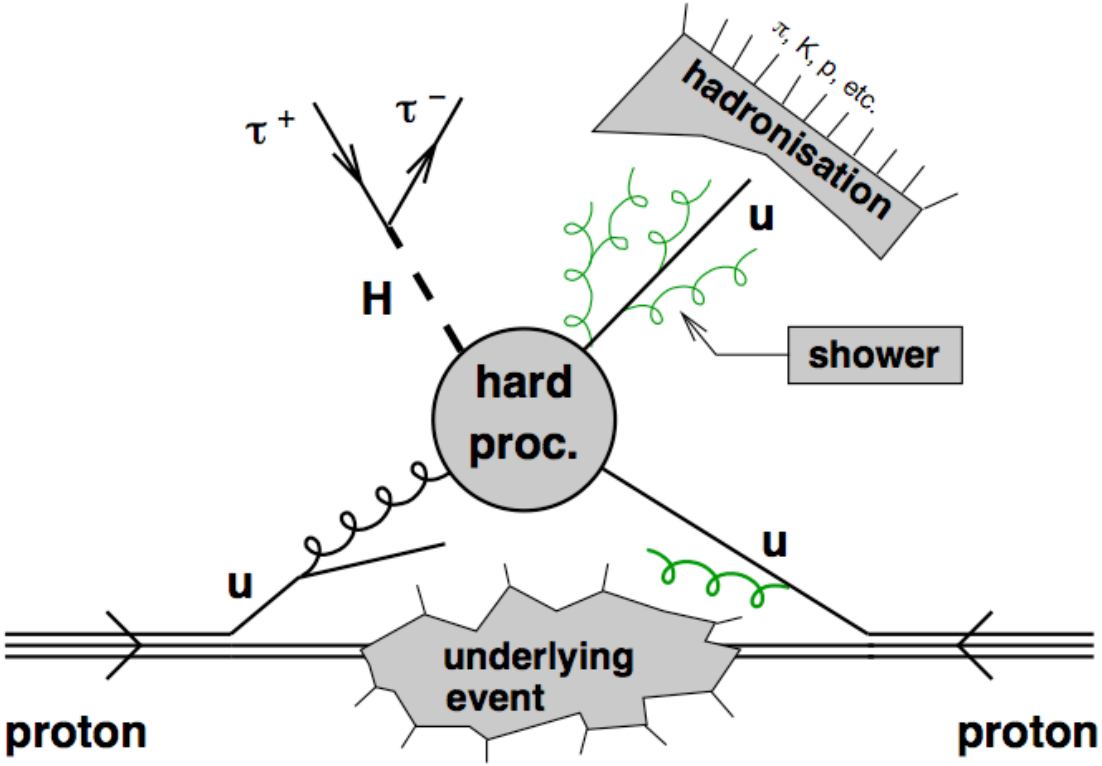
Talk more about such tricks in next lecture

- with LO evolution, uncertainty is $\sim 30\%$
- NLO brings it down to $\sim 5\%$
- NNLO $\rightarrow 2\%$

Commensurate with data uncertainties

One of the main practical advantages of knowing the NNLO AP kernels is that at this accuracy level the uncertainty due to the scales being potentially different is quite reduced

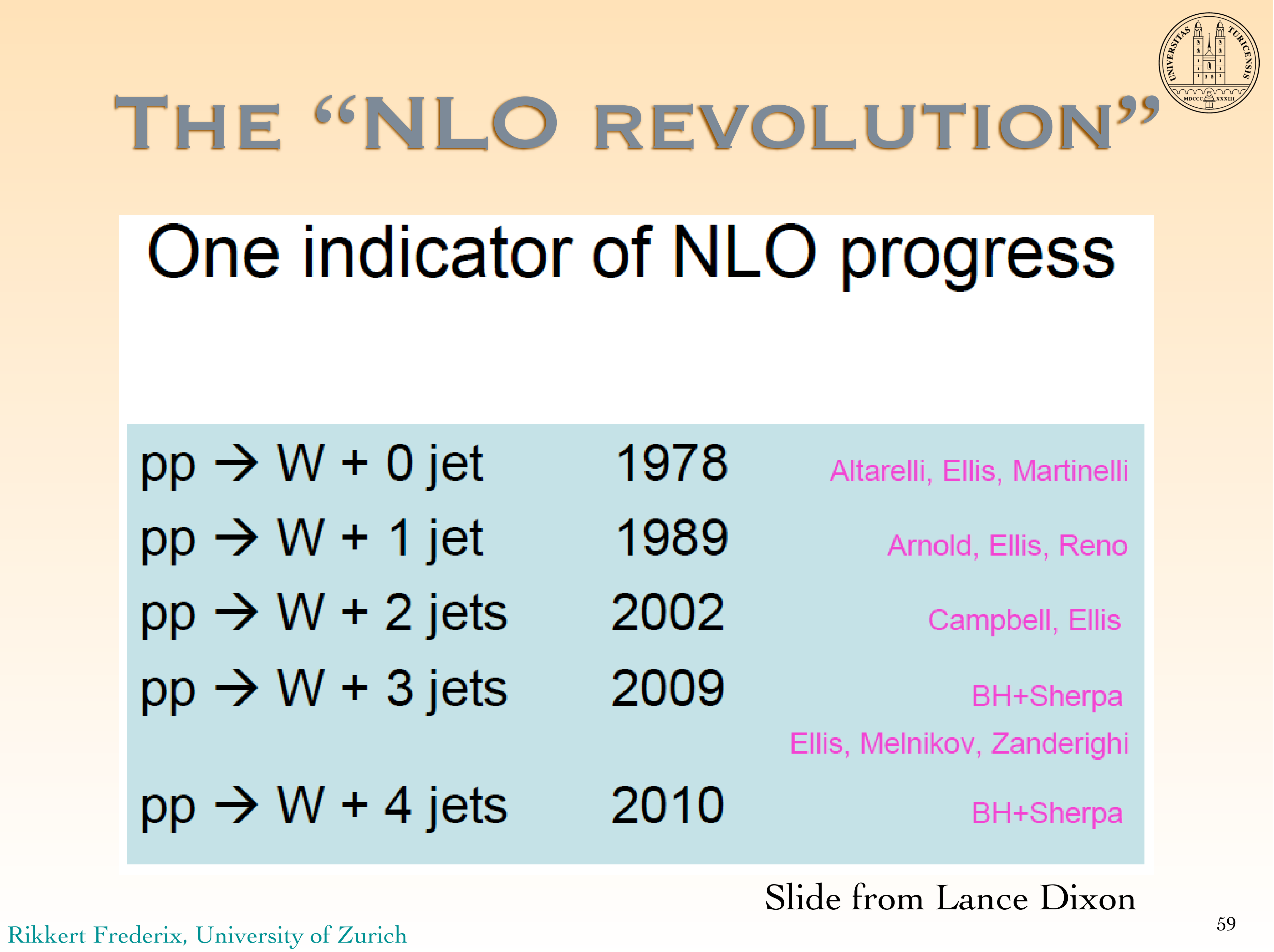
Ingredients and tools



- PDFs
- Hard scattering
- Final state tools

Slide from R. Frederix

What can we calculate



THE "NLO REVOLUTION"

One indicator of NLO progress

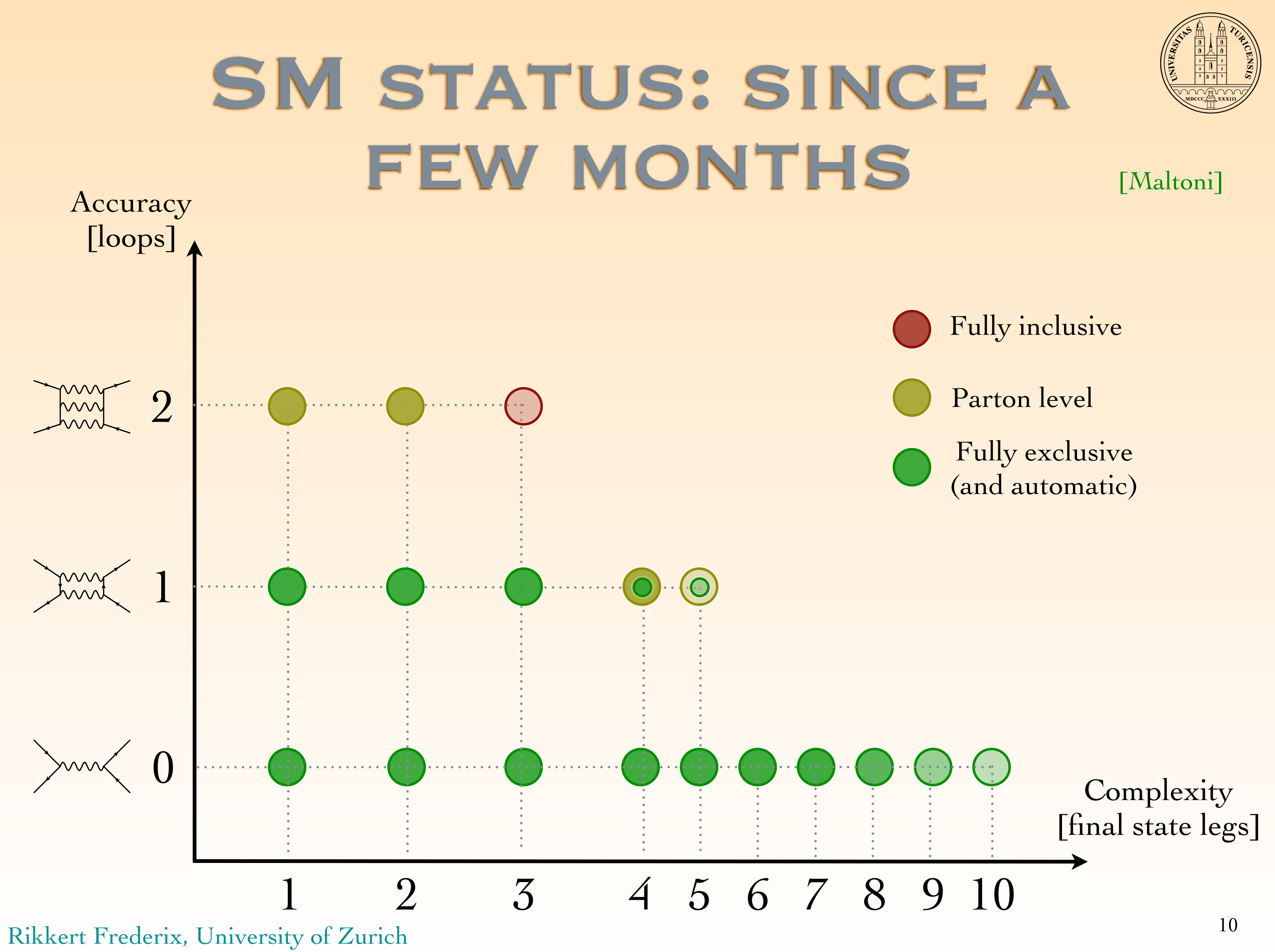
pp → W + 0 jet	1978	Altarelli, Ellis, Martinelli
pp → W + 1 jet	1989	Arnold, Ellis, Reno
pp → W + 2 jets	2002	Campbell, Ellis
pp → W + 3 jets	2009	BH+Sherpa Ellis, Melnikov, Zanderighi
pp → W + 4 jets	2010	BH+Sherpa

Slide from Lance Dixon

Rikkert Frederix, University of Zurich

Slide from R. Frederix

What can we calculate



SM STATUS: SINCE A FEW MONTHS

[Maltoni]

Rikkert Frederix, University of Zurich

10

Tools for the hard scattering

Can be divided in

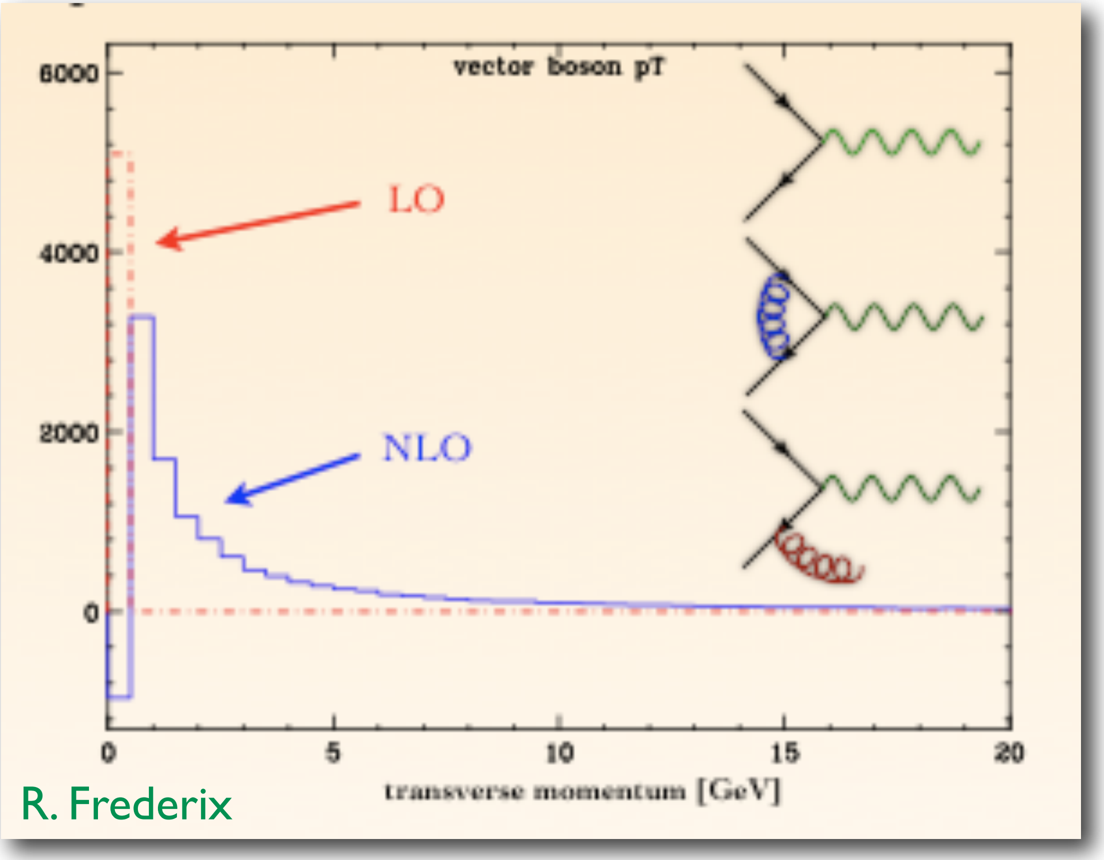
- **Integrators**
  - evaluate the (differential) cross section by integrating the calculation over the phase space, yielding (partly) inclusive quantities
  - Produce weighted events (the weight being the value of the cross section)
  - Calculations exist at LO, NLO, NNLO

- **Generators**
  - generate fully exclusive configurations
  - Events are unweighted (i.e. produced with the frequency nature would produce them)
  - Easy at LO, get complicated when dealing with higher orders

It's easy to say 'NLO'...

Even if a calculation yields an NLO-accurate result for a quantity, not all distributions that can be returned by the same code have necessarily NLO accuracy

Example: vector boson production in Drell-Yan



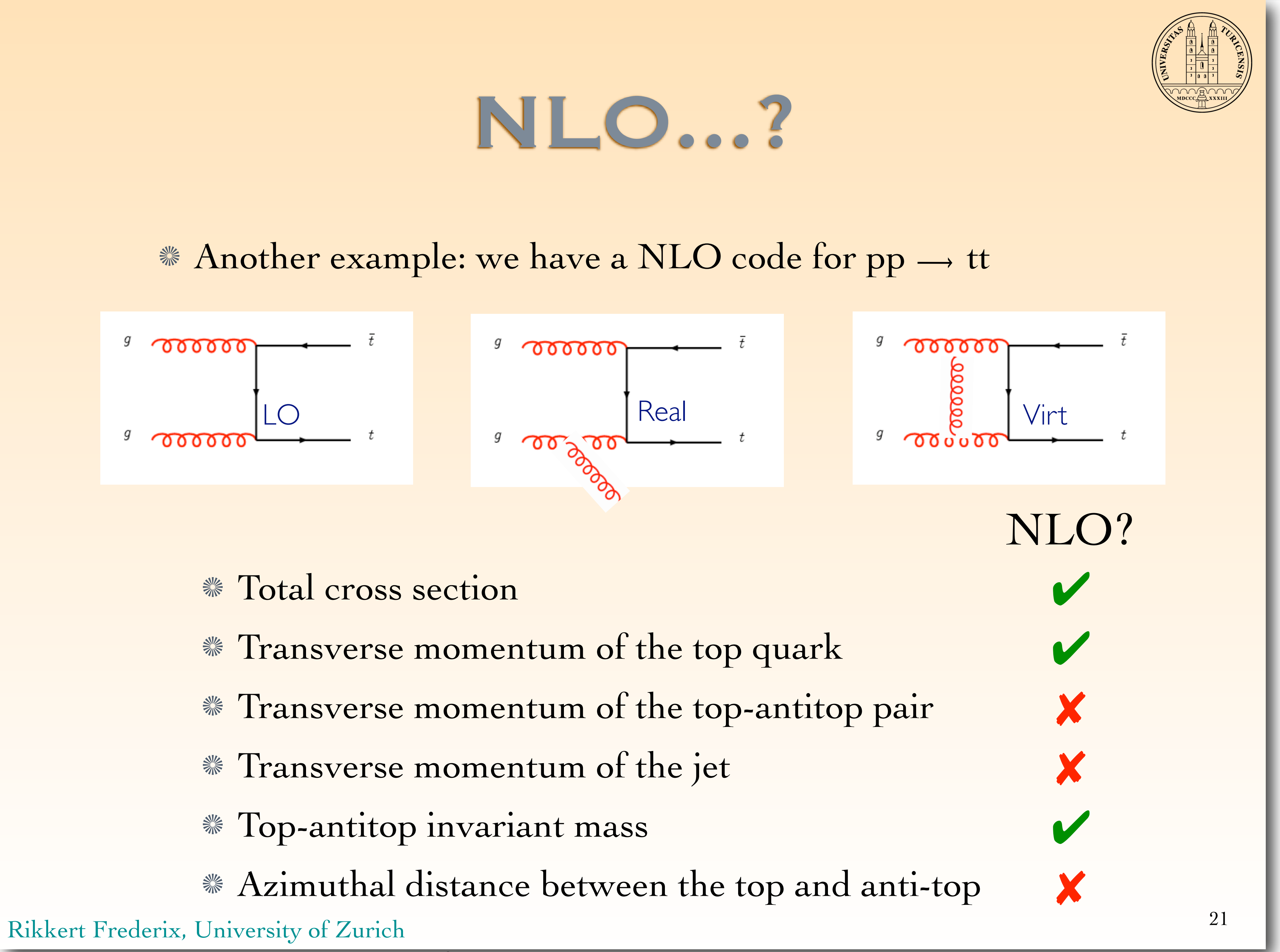
R. Frederix

- At $O(\alpha_s^0)$, the total rate is LO, the p_T is always zero
- at $O(\alpha_s^1)$ (1 gluon emission + virtual) the total rate is NLO, but the p_T distribution is only LO

You only get NLO when you calculate something that was not trivially zero at the lower order

Slide from R. Frederix

It’s easy to say ‘NLO’...



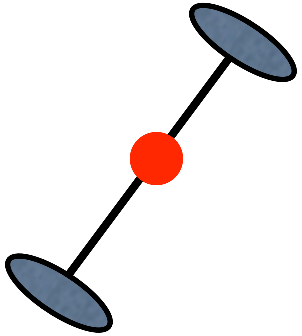
NLO...?

- Another example: we have a NLO code for pp ⟶ tt

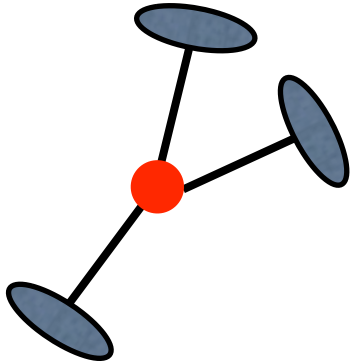
	NLO?
Total cross section	✓
Transverse momentum of the top quark	✓
Transverse momentum of the top-antitop pair	✗
Transverse momentum of the jet	✗
Top-antitop invariant mass	✓
Azimuthal distance between the top and anti-top	✗

It’s easy to say ‘NLO’...

Yet another example: jet production

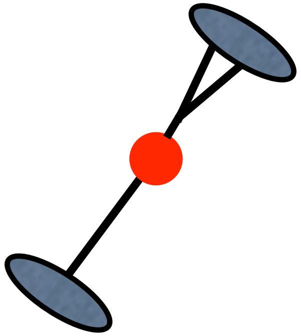


2 partons, 2 jets: LO



3 partons, 3 jets: LO

[Well separated hard jets. Real corrections only needed]



3 partons, 2 jets: **N**LO

The jet has internal structure

[Partons can become collinear and soft. **Virtual** corrections needed]

Fixed order calculation

Born

$$d\sigma^{Born} = B(\Phi_B)d\Phi_B$$

NLO

$$d\sigma^{NLO} = [B(\Phi_B) + V(\Phi_B)]\, d\Phi_B + R(\Phi_R)d\Phi_R$$

$$d\Phi_R = d\Phi_B\, d\Phi_{rad}$$

$$d\Phi_{rad} = d\cos\theta\, dE\, d\phi$$

Problem:
$V(\Phi_B)$ and $\int R d\Phi_R$ are divergent

Subtraction terms

An observable *O* is
infrared and collinear safe if

$$O(\Phi_{\mathrm{R}}(\Phi_{\mathrm{B}},\Phi_{\mathrm{rad}})) \to O(\Phi_{\mathrm{B}})$$

Soft or collinear limit

One can then write

This integration performed analytically

$$\langle O\rangle = \int \left[B(\Phi_B) + V(\Phi_B) + \int C(\Phi_R) d\Phi_{rad} \right] O(\Phi_B) d\Phi_B$$

$$+ \left[R(\Phi_R) O(\phi_R) - C(\Phi_R) O(\Phi_B) \right] d\Phi_R$$

Separately finite

This (or a similar) cancellation will always be implicit in all subsequent equations

Sudakov form factor

Factorisation

$$d\sigma_{n+1}(\Phi_{n+1}) = \mathcal{P}(\Phi_{\mathrm{rad}})\, d\sigma_n(\Phi_n)\, d\Phi_{\mathrm{rad}}$$

Emission probability

$$\mathcal{P}(\Phi_{\mathrm{rad}})\, d\Phi_{\mathrm{rad}} \approx \frac{\alpha_S(q)}{\pi} \frac{dq}{q} P(z,\phi)\, dz \frac{d\phi}{2\pi}$$

Sudakov form factor = probability of **no emission**

from large scale q_1 to smaller scale q_2

$$\boxed{\Delta_S(q_1, q_2) = \exp\left[-\int_{q_2}^{q_1} \frac{\alpha_S(q)}{\pi} \frac{dq}{q} \int_{z_0}^{1} P(z)\, dz \right]}$$

Parton Shower MC

Based on the **iterative emission of radiation** described in the **soft-collinear limit**

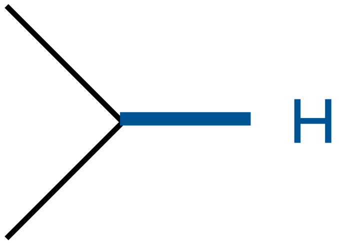
$$d\sigma^{(MC)}(\Phi_R)d\Phi_R = B(\Phi_B)d\Phi_B\mathcal{P}(\Phi_{rad})d\Phi_{rad}$$

Pros: soft-collinear radiation is resummed to all orders in pQCD

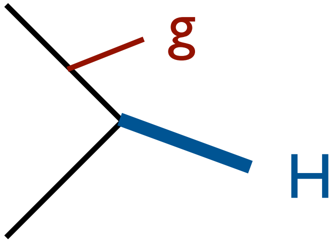
Cons: hard large-angle radiation is missing

Overall accuracy will be leading log (LL) for the radiation, and leading order (i.e. Born) for the integrated cross sections

PS example: Higgs plus radiation



Leading order.
No radiation, Higgs $p_T = 0$



With emission of radiation
Higgs $p_T \neq 0$

Description of hardest emission in PS MC (either event is generated)

$$\frac{d\sigma^{(MC)}}{dy\, dp_T} = \frac{d\sigma^{(B)}}{dy}\delta(p_T)\Delta(Q_0) + \Delta(p_T)\frac{d\sigma^{(MC)}}{dy\, dp_T}$$

$$\Delta(p_T) = \exp\left[-\int_{p_T}^{Q}\frac{\frac{d\sigma^{(MC)}}{dy\, dp'_T}}{\frac{d\sigma^{(B)}}{dy}}dp'_T\right]$$

Sudakov form factor

x-sect for
no emission

prob. of
no emission
(down to the
PS cutoff)

prob. of
no emission
down to p_T

x-sect for
emission at p_T,
as described by the MC

Shower unitarity

It holds

$$\int_0^Q \left[\delta(p_{\mathrm{T}})\Delta(Q_0) + \frac{\Delta(p_{\mathrm{T}}) \frac{\mathrm{d}\sigma^{(\mathrm{MC})}}{dy\mathrm{d}p_{\mathrm{T}}}}{\frac{\mathrm{d}\sigma^{(\mathrm{B})}}{\mathrm{d}y}} \right] \mathrm{d}p_{\mathrm{T}} = \Delta(Q_0) + \int_{Q_0}^Q \frac{\mathrm{d}\Delta(p_{\mathrm{T}})}{\mathrm{d}p_{\mathrm{T}}} \mathrm{d}p_{\mathrm{T}} = \Delta(Q) = 1$$

Shower unitarity

so that

$$\int_0^Q \mathrm{d}p_{\mathrm{T}} \frac{\mathrm{d}\sigma^{(\mathrm{MC})}}{dy\mathrm{d}p_{\mathrm{T}}} = \int_0^Q \left[\delta(p_{\mathrm{T}})\Delta(Q_0) + \frac{\Delta(p_{\mathrm{T}}) \frac{\mathrm{d}\sigma^{(\mathrm{MC})}}{dy\mathrm{d}p_{\mathrm{T}}}}{\frac{\mathrm{d}\sigma^{(\mathrm{B})}}{\mathrm{d}y}} \right] \mathrm{d}p_{\mathrm{T}} = \frac{\mathrm{d}\sigma^{(\mathrm{B})}}{\mathrm{d}y}$$

A parton shower MC correctly reproduces the Born cross section for integrated quantities

PS MC in different notation

Writing the real cross section as described by the Monte Carlo (i.e. with the parton shower) simply as R^{MC}, we can rewrite

$$d\sigma^{MC} = B d\Phi_B \left[\Delta_{MC}(Q_0) + \Delta_{MC}(p_T) \frac{R^{MC}}{B} d\Phi_{rad} \right]$$

with $$\Delta_{MC}(p_T) = \exp\left[- \int_{p_T} \frac{R^{MC}}{B} d\Phi_{rad} \right]$$

as our Master Formula for a Parton Shower Monte Carlo.

Thanks to the shower unitarity, it holds

$$\Rightarrow \int d\sigma^{MC} = \int B d\Phi_B = \sigma^{LO}$$

Matrix Element corrections

In a PS Monte Carlo $R^{(MC)}(\Phi_R) = B(\Phi_B)\mathcal{P}(\Phi_{rad})$

soft-collinear approximation

Replace the MC description of radiation with the **correct** one: $\mathcal{P}(\Phi_{rad}) \longrightarrow \frac{R}{B}$

The Sudakov becomes

$$\Delta(p_\mathrm{T}) = \exp\left[-\int_{p_\mathrm{T}}^{Q} \frac{\frac{\mathrm{d}\sigma^{(\mathrm{MC})}}{\mathrm{d}y\,\mathrm{d}p'_\mathrm{T}}}{\frac{\mathrm{d}\sigma^{(\mathrm{B})}}{\mathrm{d}y}}\mathrm{d}p'_\mathrm{T}\right] \longrightarrow \Delta_R(p_T) = \exp\left[-\int \frac{R}{B}\Theta(k_T(\Phi_R) - p_T)d\Phi_{rad}\right]$$

and the x-sect formula for the hardest emission

$$d\sigma^{MEC} = Bd\Phi_B\left[\Delta_R(Q_0) + \Delta_R(p_T)\frac{R}{B}d\Phi_{rad}\right]$$

Conventions for Sudakov form factor

$$\Delta_{\mathrm{S}}(q_1, q_2) = \exp\left[-\int_{q_2}^{q_1} \frac{\alpha_{\mathrm{S}}(q)}{\pi}\frac{\mathrm{d}q}{q}\int_{z_0}^{1} P(z)\,\mathrm{d}z\right]$$

Full expression,with details of soft-collinear radiation probability

$$\Delta(p_{\mathrm{T}}) = \exp\left[-\int_{p_{\mathrm{T}}}^{Q} \frac{\frac{\mathrm{d}\sigma^{(\mathrm{MC})}}{\mathrm{d}y\,\mathrm{d}p'_{\mathrm{T}}}}{\frac{\mathrm{d}\sigma^{(\mathrm{B})}}{\mathrm{d}y}}\mathrm{d}p'_{\mathrm{T}}\right]$$

Dropped upper limit, taken implicitly to be the hard scale Q

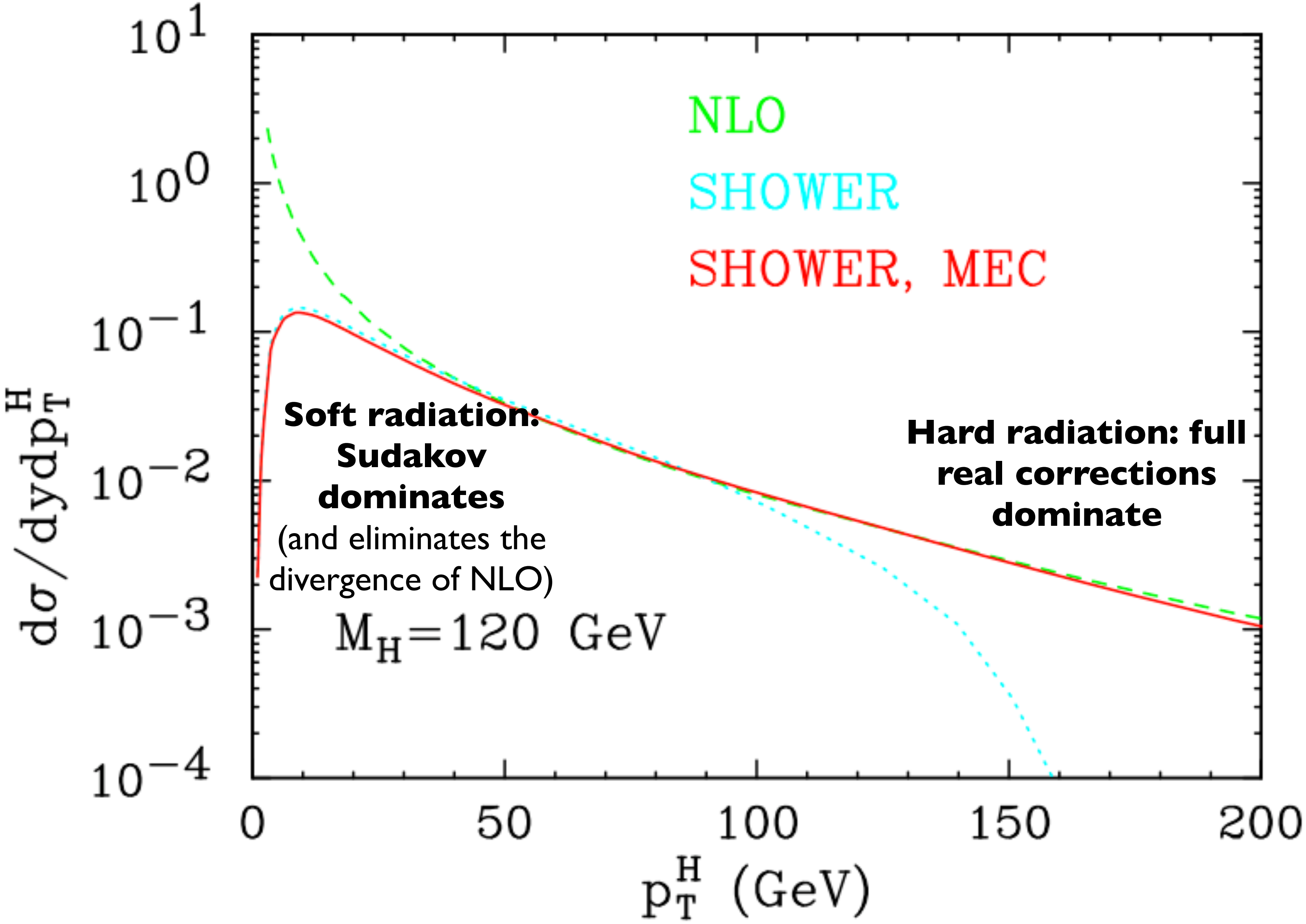
$$\Delta_R(p_T) = \exp\left[-\int \frac{R}{B}\Theta(k_T(\Phi_R) - p_T)d\Phi_{rad}\right]$$

Introduced suffix (R in this case) to indicate expression used to described radiation

$$\Delta_R(p_T) = \exp\left[-\int_{p_T} \frac{R}{B}d\Phi_{rad}\right]$$

Integration boundaries only implicitly indicated

Matrix Element corrections



Soft radiation: Sudakov dominates (and eliminates the divergence of NLO)

Hard radiation: full real corrections dominate

Beyond PS MC

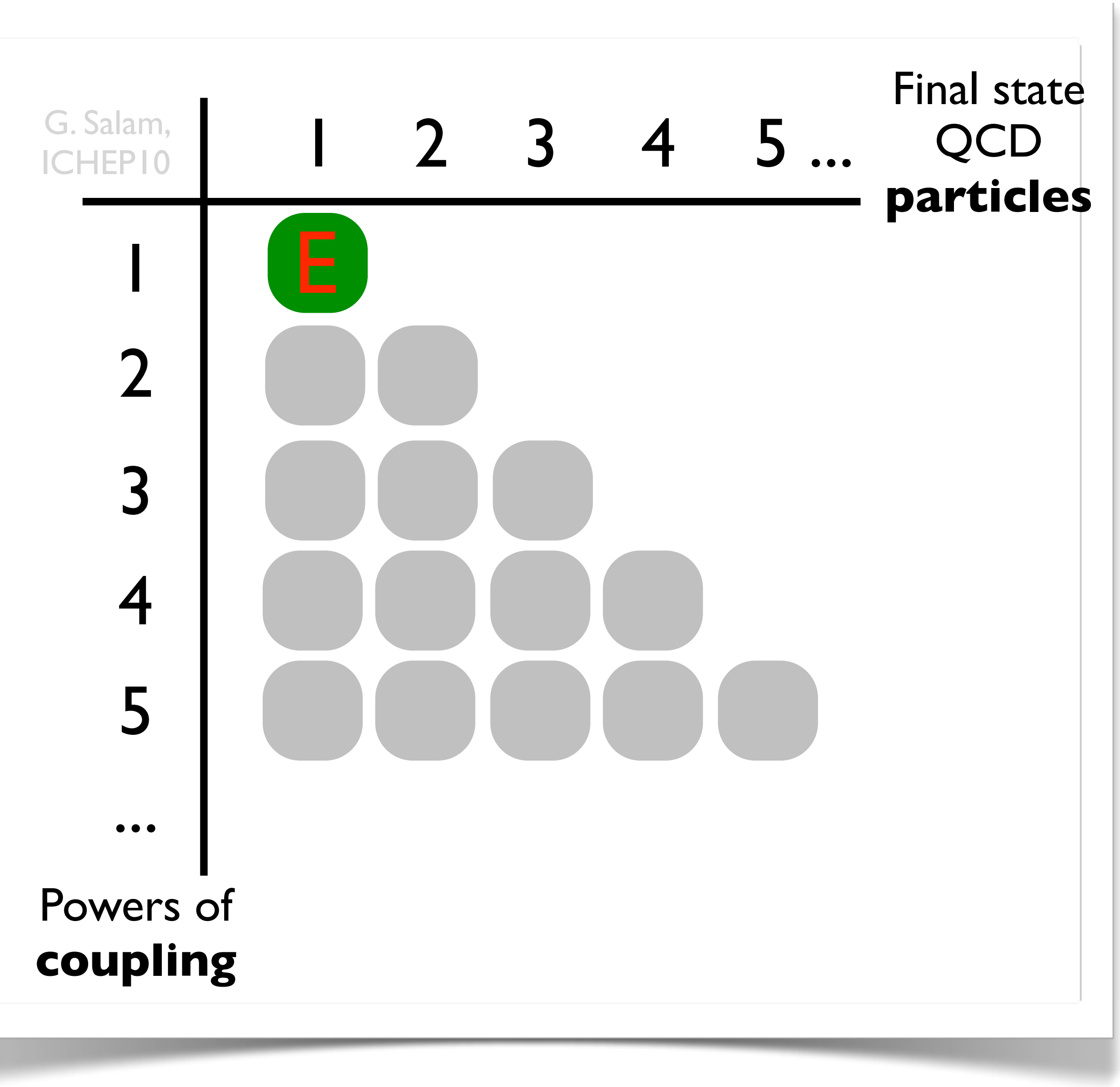
We wish to go beyond a Parton Shower (+MEC) Monte Carlo, so that

- we can successfully interface **matrix elements for multi-parton production with a parton shower**

- we can successfully interface a **parton shower with a NLO calculation**

The quest for exactness

E exact ▨ PS

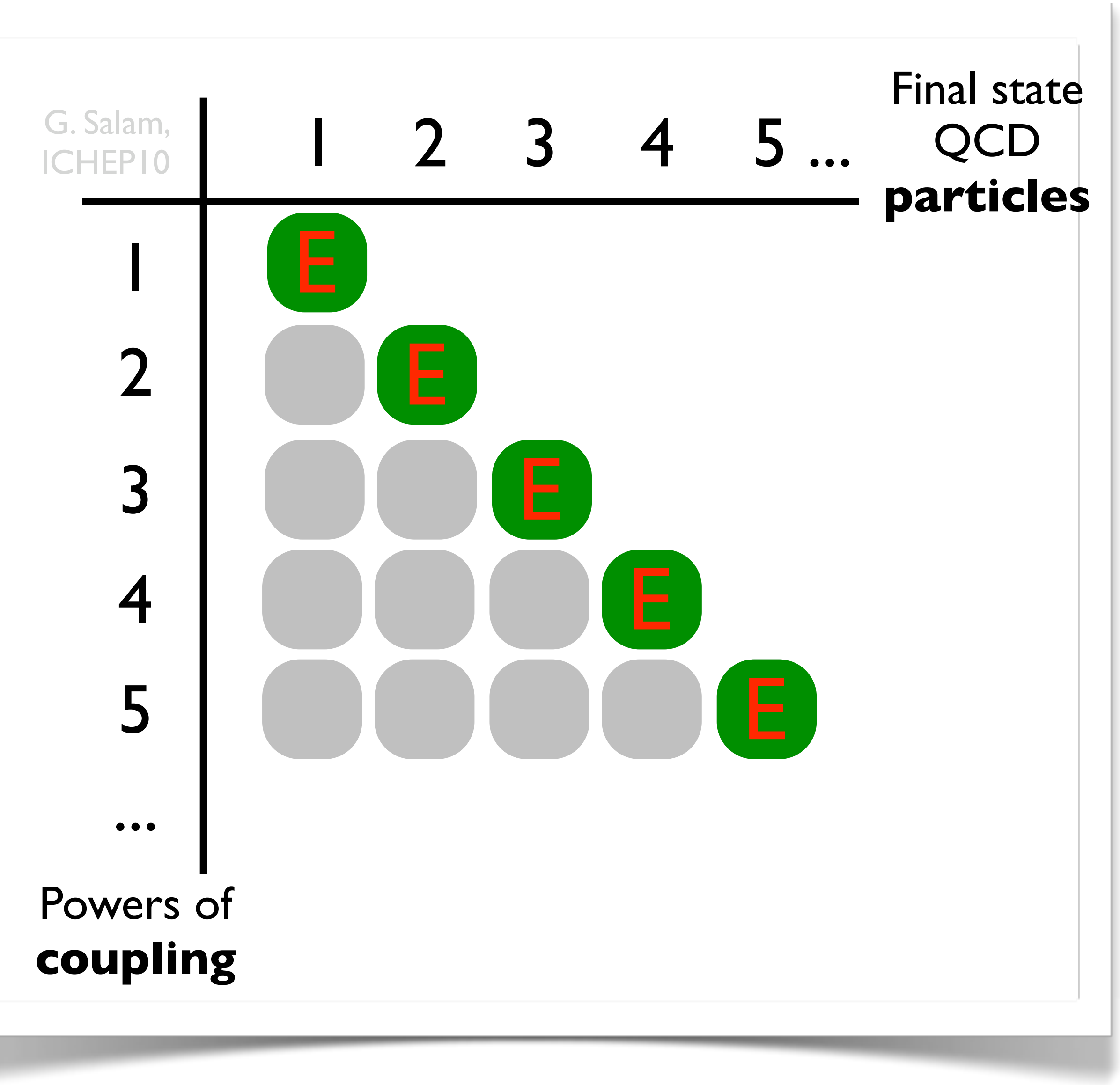


Parton shower (PS+MEC) Montecarlo (PYTHIA, HERWIG…)

Matching

The quest for exactness

E exact ▨ PS



G. Salam, ICHEP10

Powers of **coupling** \ Final state QCD **particles**	1	2	3	4	5 …
1	E				
2	▨	E			
3	▨	▨	E		
4	▨	▨	▨	E	
5	▨	▨	▨	▨	E
…					

Parton shower (PS+MEC)

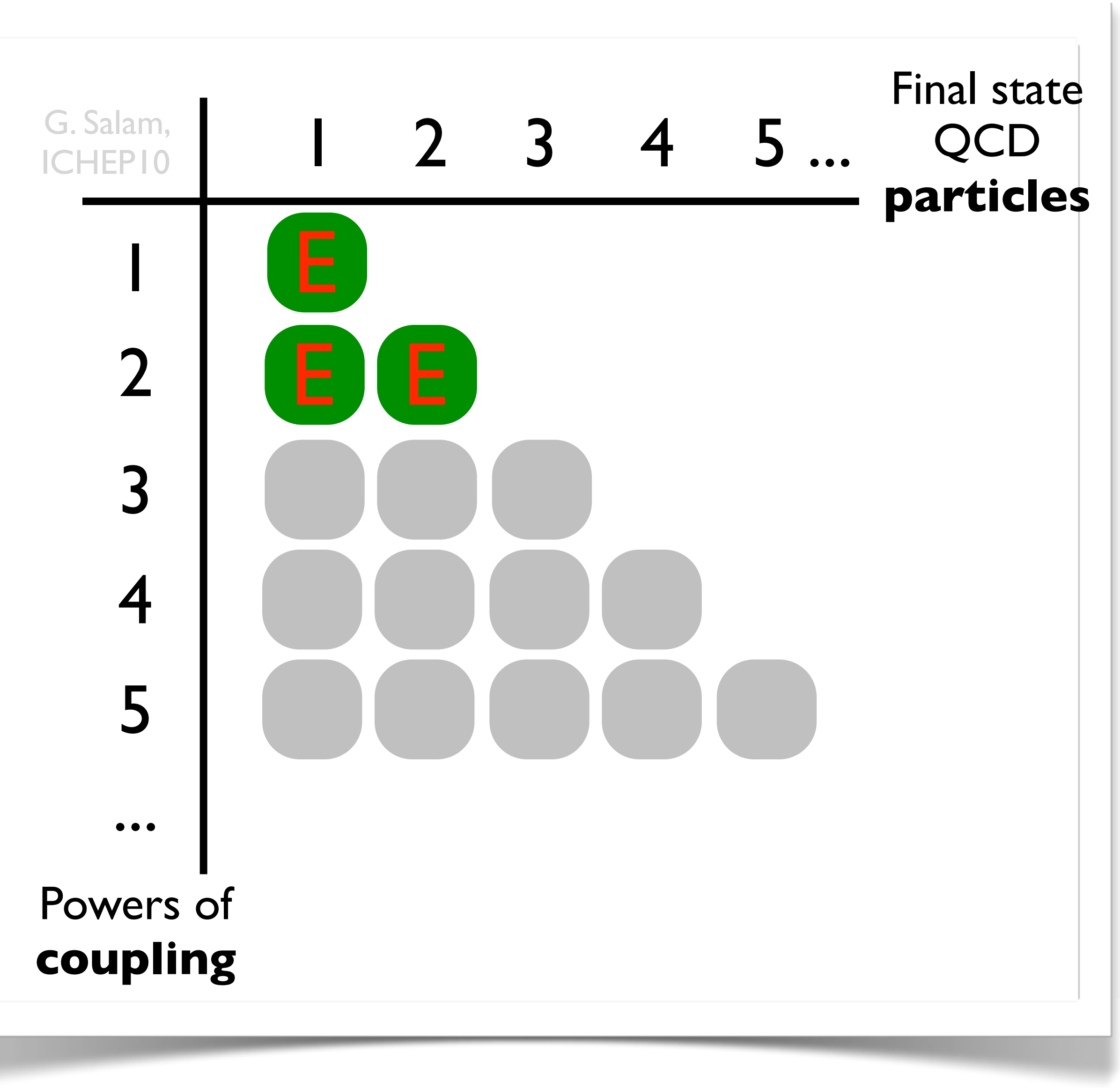
Montecarlo (PYTHIA, HERWIG…)

PS + Matrix Element (ME)

(using CKKW/MLM)

The quest for exactness

E exact ▨ PS



G. Salam, ICHEP10

Powers of **coupling** \ Final state QCD **particles**	1	2	3	4	5 ...
1	E				
2	E	E			
3	▨	▨	▨		
4	▨	▨	▨	▨	
5	▨	▨	▨	▨	▨
...					

Parton shower (PS+MEC)
Montecarlo (PYTHIA, HERWIG…)

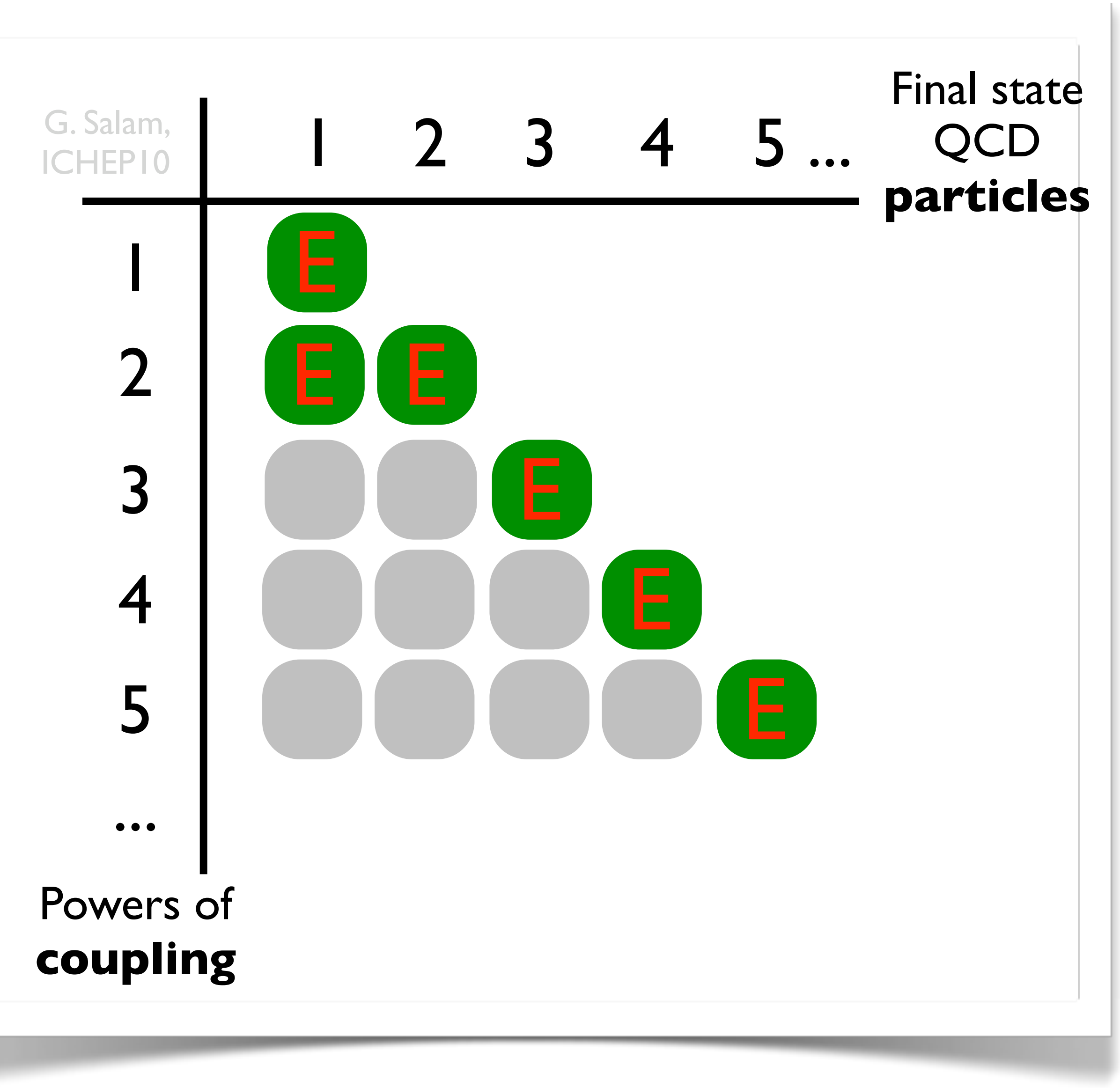
PS + Matrix Element (ME)
(using CKKW/MLM)

PS + NLO
(MC@NLO, POWHEG)

Matching

The quest for exactness

E exact ▨ PS



G. Salam, ICHEP10

Final state QCD **particles**

	1	2	3	4	5 ...
1	E				
2	E	E			
3	▨	▨	E		
4	▨	▨	▨	E	
5	▨	▨	▨	▨	E
...					

Powers of **coupling**

Parton shower (PS+MEC)
Montecarlo (PYTHIA, HERWIG…)

PS + Matrix Element (ME)
(using CKKW/MLM)

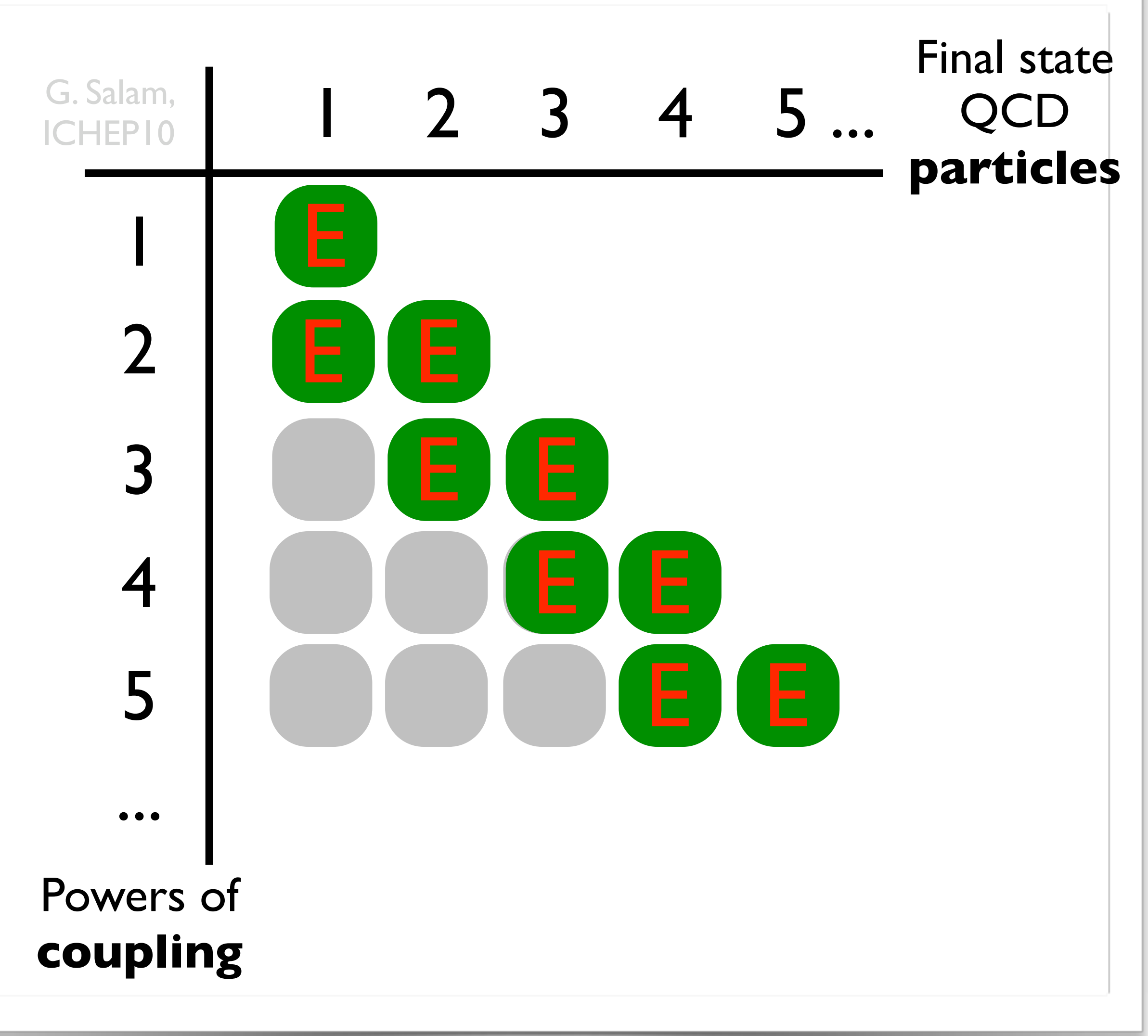
PS + NLO
(MC@NLO, POWHEG)

PS + NLO + ME
(MENLOPS)
[Hamilton, Nason '10]

Matching

The quest for exactness

🟩 E exact ▨ PS



G. Salam, ICHEP10

Final state QCD **particles**

	1	2	3	4	5 …
1	E				
2	E	E			
3	▨	E	E		
4	▨	▨	E	E	
5	▨	▨	▨	E	E
…					

Powers of **coupling**

Parton shower (PS+MEC)
Montecarlo (PYTHIA, HERWIG…)

PS + Matrix Element (ME)
(using CKKW/MLM)

PS + NLO
(MC@NLO, POWHEG)

PS + NLO + ME
(MENLOPS)
[Hamilton, Nason '10]

The future
PS + NLO + ME$_{\mathrm{NLO}}$
(aMC@NLO)

Beyond PS MC

We wish to go beyond a Parton Shower (+MEC) Monte Carlo, so that

- we can successfully interface **matrix elements for multi-parton production with a parton shower**

- we can successfully interface a **parton shower with a NLO calculation**

MCs at NLO

Matrix-element corrected shower Monte Carlos still have leading order accuracy for the total rates

$$d\sigma^{MEC} = Bd\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T)\frac{R}{B}d\Phi_{rad}\right] \quad \text{and} \quad \Delta_R(Q_0) + \int \Delta_R(p_T)\frac{R}{B}d\Phi_{rad} = 1$$

$$\Rightarrow \int d\sigma^{MEC} = \int Bd\Phi_B = \sigma^{LO}$$

We want to do better, and **merge** PS and NLO, so that

$$\int d\sigma^{PS+NLO} = \int (B+V)d\Phi_B + \int Rd\Phi_R = \sigma^{NLO}$$

MCs at NLO

Existing ‘MonteCarlos at NLO’:

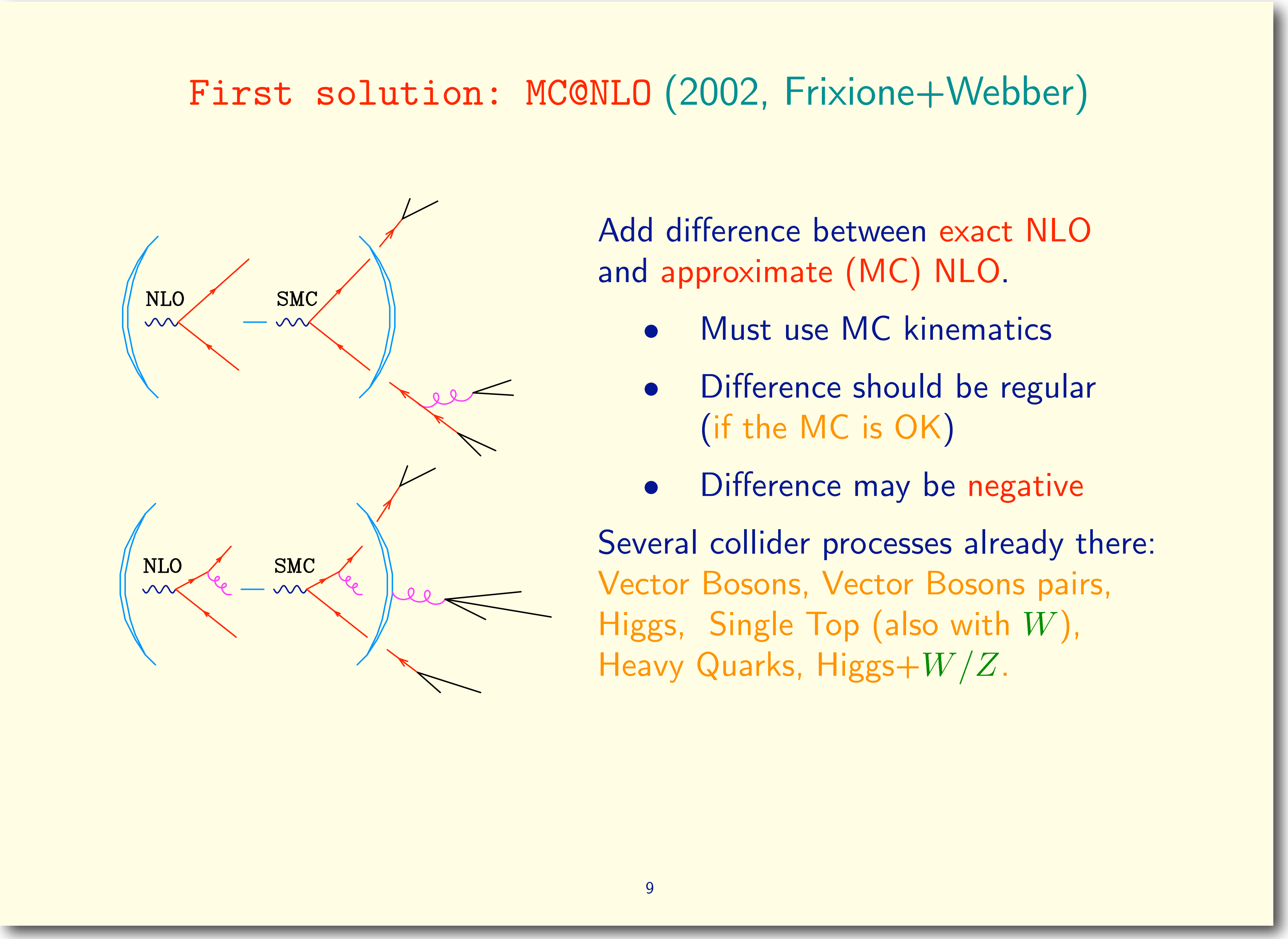
- MC@NLO [Frixione and Webber, 2002]
- POWHEG [Nason, 2004]

NB. MC@NLO is a code, POWHEG is a method

Evolving into (semi)automated forms:

- The POWHEG BOX [powhegbox.mib.infn.it 2010]
- POWHEL (HELAC + POWHEG BOX) [Trocsanyi et al 2012]
- aMC@NLO [amcatnlo.cern.ch 2011]

Slide from P. Nason



First solution: MC@NLO (2002, Frixione+Webber)

Add difference between exact NLO and approximate (MC) NLO.

- Must use MC kinematics
- Difference should be regular (if the MC is OK)
- Difference may be negative

Several collider processes already there: Vector Bosons, Vector Bosons pairs, Higgs, Single Top (also with W), Heavy Quarks, Higgs+W/Z.

MC@NLO

Idea: remove from the NLO the terms that are already generated by the parton shower (NB. MC-specific)

$$d\sigma^{MC@NLO} = \bar{B}_{MC} d\Phi_B \underbrace{\left[\Delta_{MC}(Q_0) + \Delta_{MC}(p_T)\frac{R^{MC}}{B} d\Phi_{rad}\right]}_{\text{MC shower}} + \underbrace{[R - R^{MC}] d\Phi_R}_{\text{'hard' event}}$$

$$\underbrace{\bar{B}_{MC}}_{\text{'soft' event}} = B + \left[V + \int R^{MC} d\Phi_{rad}\right]$$

It is easy to see that, as desired,

$$\int d\sigma^{MC@NLO} = \int d\sigma^{NLO}$$

POWHEG

Idea: generated hardest radiation first, then pass event to MC for generation of subsequent, softer radiation

$$d\sigma^{POWHEG} = \bar{B} d\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

$$\bar{B} = B + \left[V + \int R \, d\Phi_{rad} \right]$$

NLO x-sect

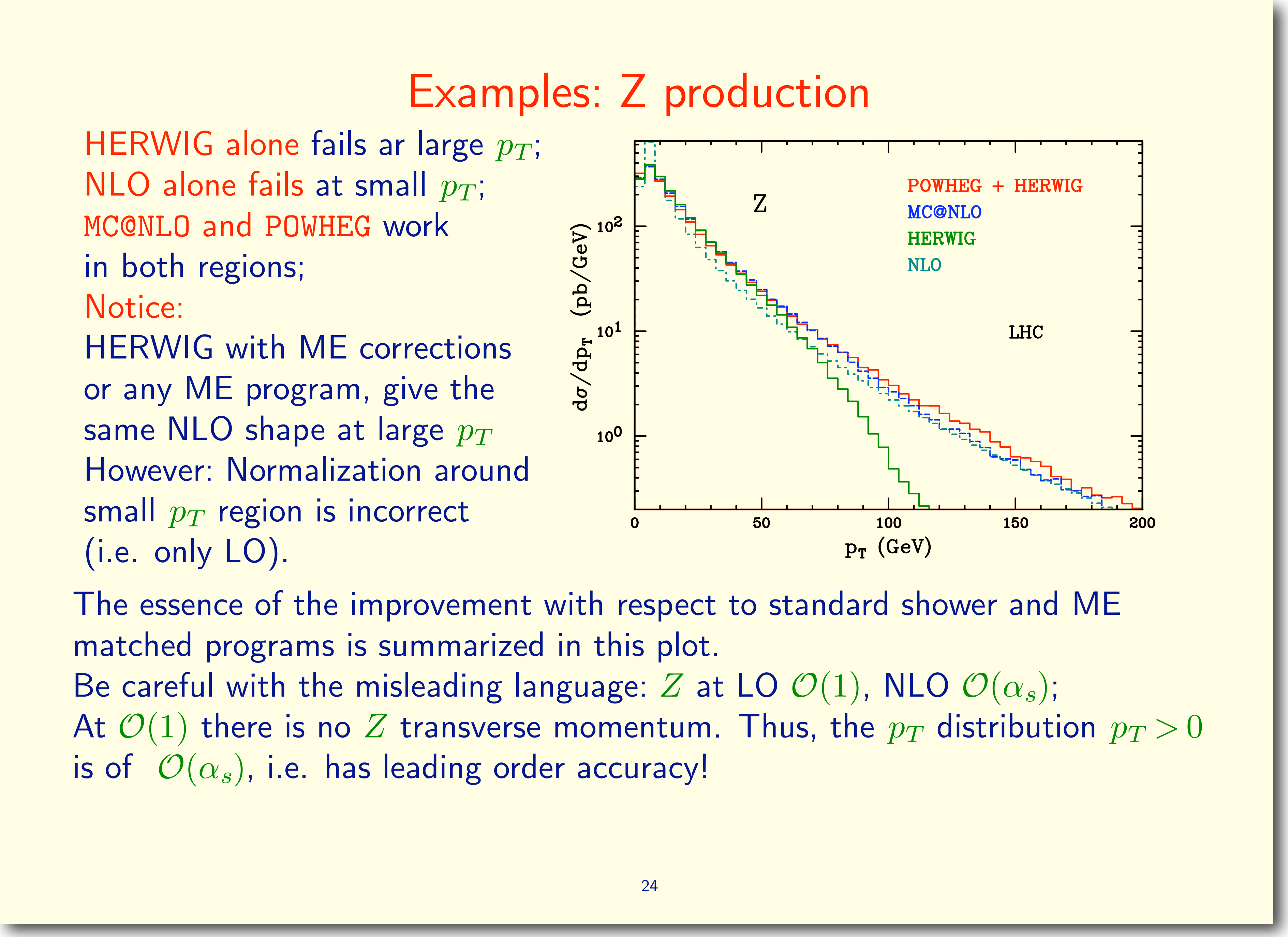
MC shower

It is easy to see that, as desired,

$$\int d\sigma^{POWHEG} = \int d\sigma^{NLO}$$

Slide from
P. Nason

NLO v. MC@NLO v. POWHEG



Examples: Z production

HERWIG alone fails ar large p_T;
NLO alone fails at small p_T;
`MC@NLO` and `POWHEG` work
in both regions;
Notice:
HERWIG with ME corrections
or any ME program, give the
same NLO shape at large p_T
However: Normalization around
small p_T region is incorrect
(i.e. only LO).

The essence of the improvement with respect to standard shower and ME matched programs is summarized in this plot.
Be careful with the misleading language: Z at LO $\mathcal{O}(1)$, NLO $\mathcal{O}(\alpha_s)$;
At $\mathcal{O}(1)$ there is no Z transverse momentum. Thus, the p_T distribution $p_T > 0$ is of $\mathcal{O}(\alpha_s)$, i.e. has leading order accuracy!

MC@NLO v. POWHEG

The two methods are largely equivalent.

They do, however, have separate **pros** and **cons**.

MC@NLO

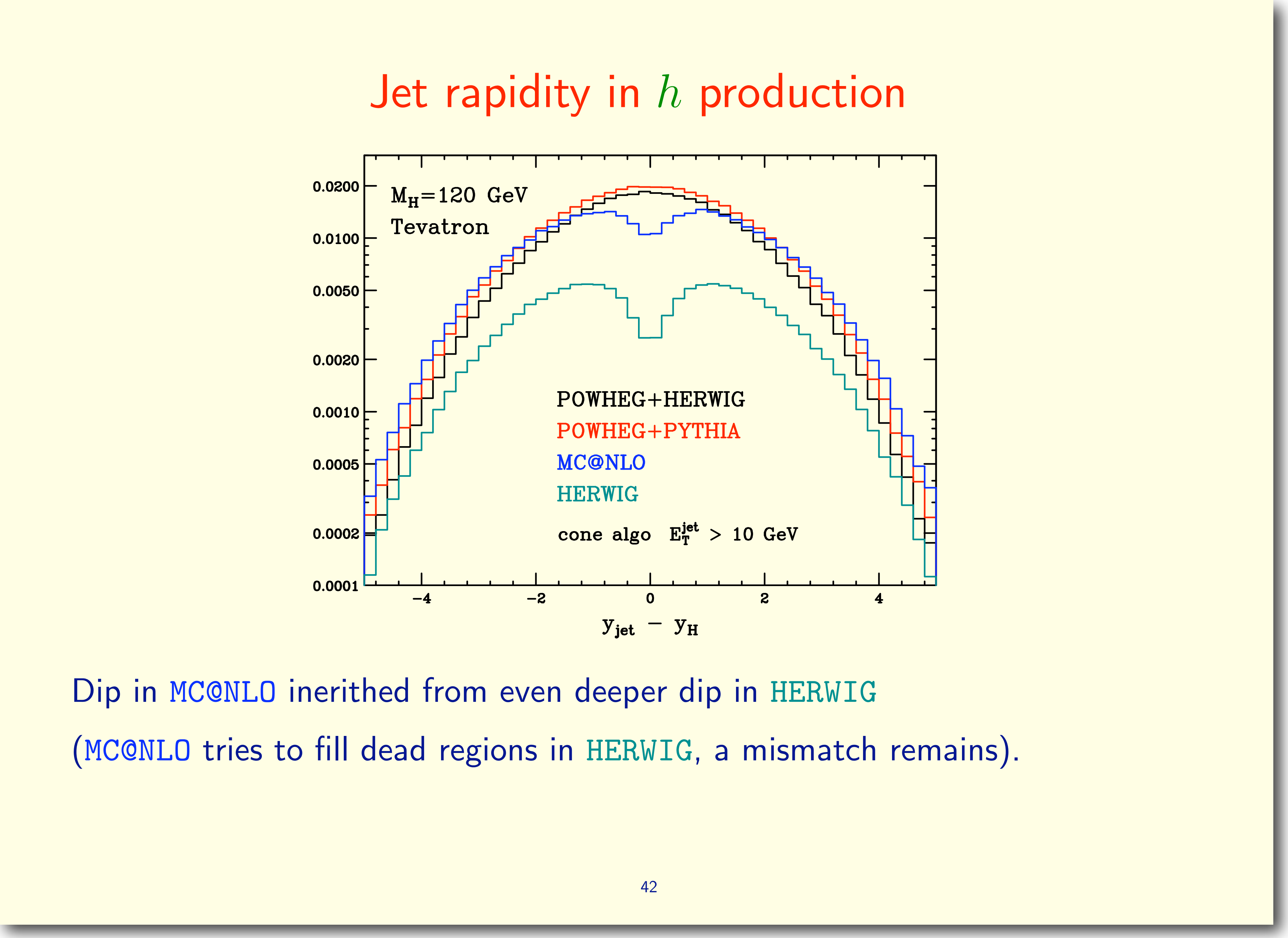
- can have negative weights
- needs specific implementation for each PS MonteCarlo (but now exists for both HERWIG and PYTHIA)
- ‘rapidity dip’ in some distributions
- Distributions from NLO codes rigorously reproduced
- fully automated in aMC@NLO

POWHEG

- weights always positive
- interfaces naturally to any PS MonteCarlo
- can generate large (NNLO) K-factors in some distributions (but a practical solution is available)
- not yet fully automated (but the POWEG BOX is a step in this direction, and it is being exploited by POWHEL in this direction)

Slide from P. Nason

The MC@NLO dip



Jet rapidity in h production

Dip in `MC@NLO` inerithed from even deeper dip in `HERWIG`

(`MC@NLO` tries to fill dead regions in `HERWIG`, a mismatch remains).

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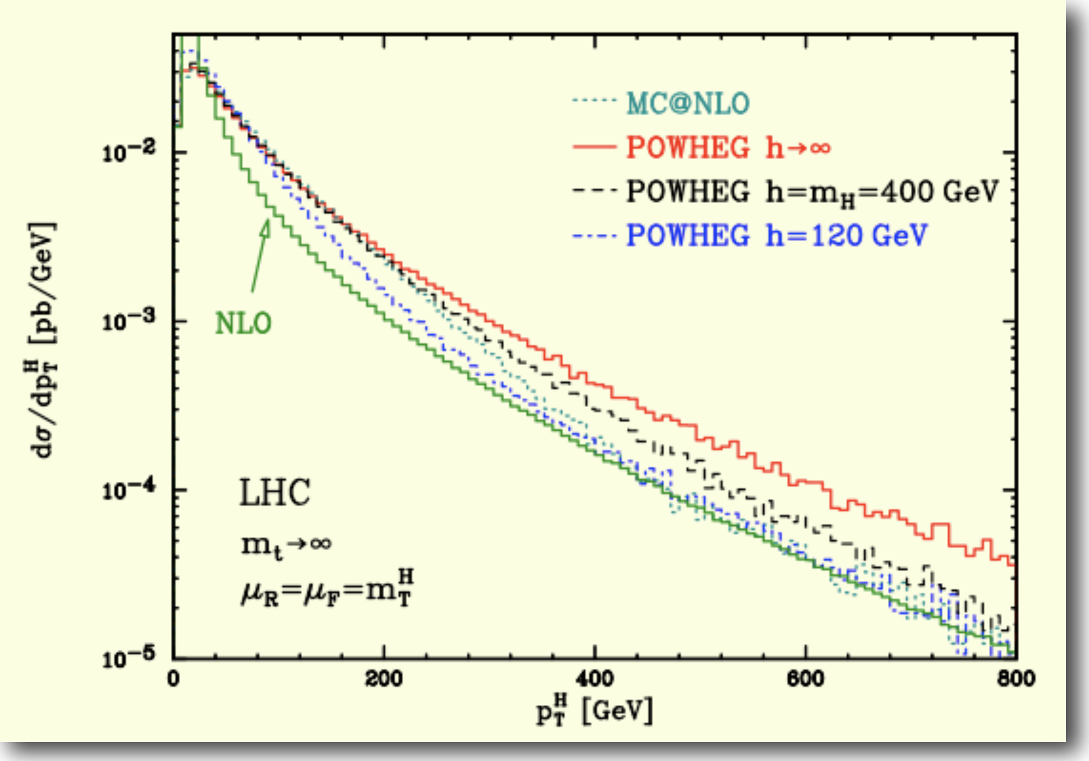
Large p_T enhancement in POWHEG

The 'naive' formulation for POWHEG is

$$d\sigma^{POWHEG} = \bar{B} d\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T) \frac{R}{B} d\Phi_{rad} \right]$$

In this form $\bar{B} d\Phi_B$ provides the NLO K-factor (order 1+ $O(\alpha_s)$) , but also associates it to large p_T radiation, where the calculation is already $O(\alpha_s)$ (but only LO accuracy).



This generates an effective (but not necessarily correct) $O(\alpha_s^2)$ term (i.e. NNLO for the total cross section)

OK because beyond nominal accuracy, but one may feel uncomfortable with such large numerical factors

Modified POWHEG

The ‘problem’ with the naive POWHEG comes from the hard radiation being enhanced by spurious higher orders. In order to suppress this effect, we split

$$R = R^S + R^F \qquad R^S \equiv \frac{h^2}{h^2+p_T^2} R \qquad R^F \equiv \frac{p_T^2}{h^2+p_T^2} R$$

Contains singularities

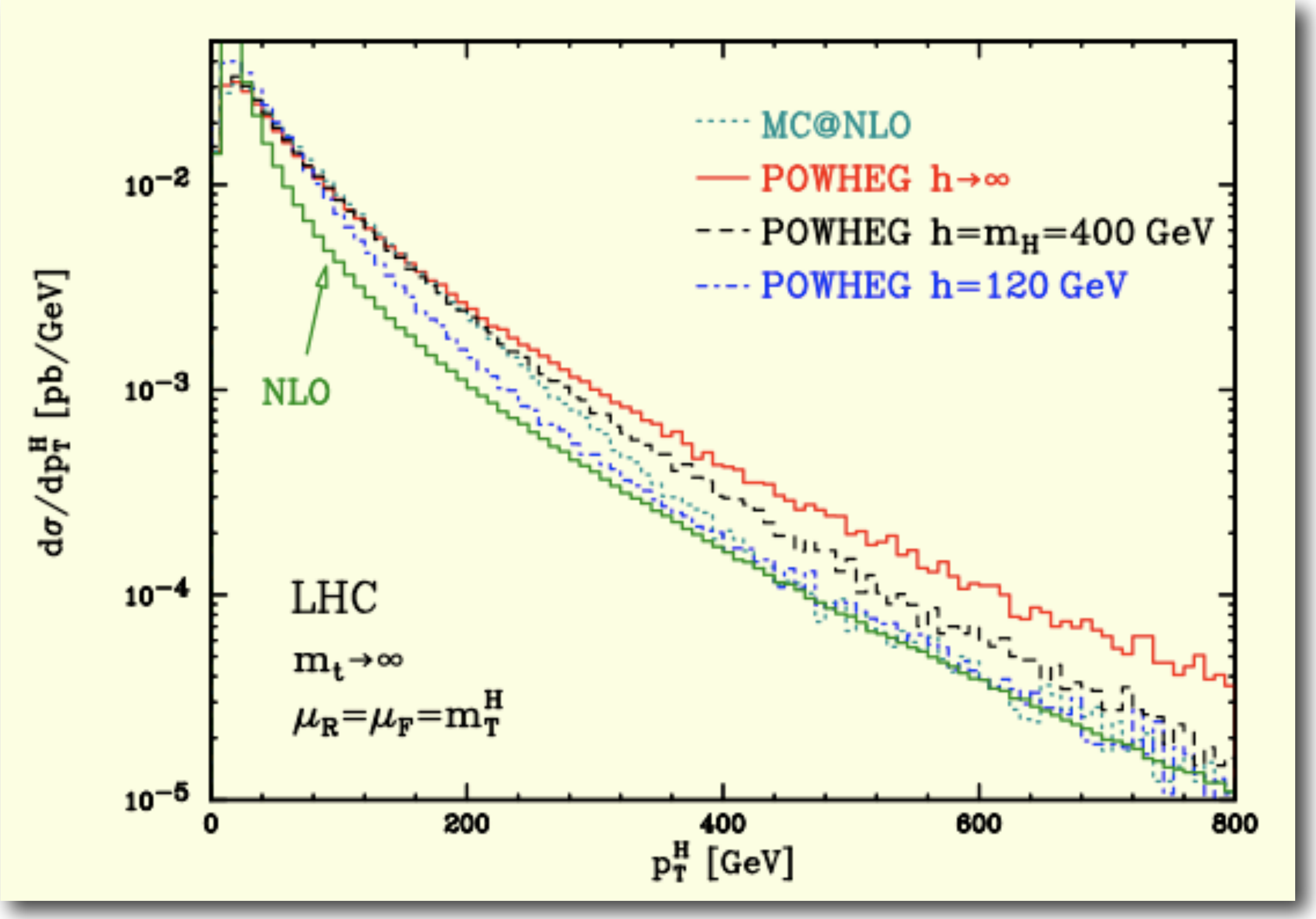
Regular in small p$_T$ region

$$d\sigma^{POWHEG} = \bar{B}^S d\Phi_B \left[\Delta_S(Q_0) + \Delta_S(p_T)\frac{R^S}{B} d\Phi_{rad}\right] + R^F d\Phi_R$$

$$\bar{B}^S = B + \left[V + \int R^S \, d\Phi_{rad}\right] \qquad \Delta_S(p_T) = \exp\left[-\int_{p_T} \frac{R^S}{B} d\Phi_{rad}\right]$$

Modified POWHEG

In the h→∞ limit the exact NLO result is recovered



Comparisons

$$d\sigma^{MC} = Bd\Phi_B \left[\Delta(Q_0) + \Delta(p_T)\frac{R^{MC}}{B}d\Phi_{rad}\right]$$

$$d\sigma^{MEC} = Bd\Phi_B \left[\Delta_R(Q_0) + \Delta_R(p_T)\frac{R}{B}d\Phi_{rad}\right]$$

$$d\sigma^{NLO} = [B+V]\,d\Phi_B + Rd\Phi_R$$

$$d\sigma^{MC@NLO} = \bar{B}_{MC}d\Phi_B \left[\Delta_{MC}(Q_0) + \Delta_{MC}(p_T)\frac{R^{MC}}{B}d\Phi_{rad}\right] + [R - R^{MC}]d\Phi_R$$

$$d\sigma^{POWHEG} = \bar{B}^S d\Phi_B \left[\Delta_S(Q_0) + \Delta_S(p_T)\frac{R^S}{B}d\Phi_{rad}\right] + R^F d\Phi_R$$

POWHEG approaches MC@NLO if $R^S \rightarrow R^{MC}$

Beyond PS MC

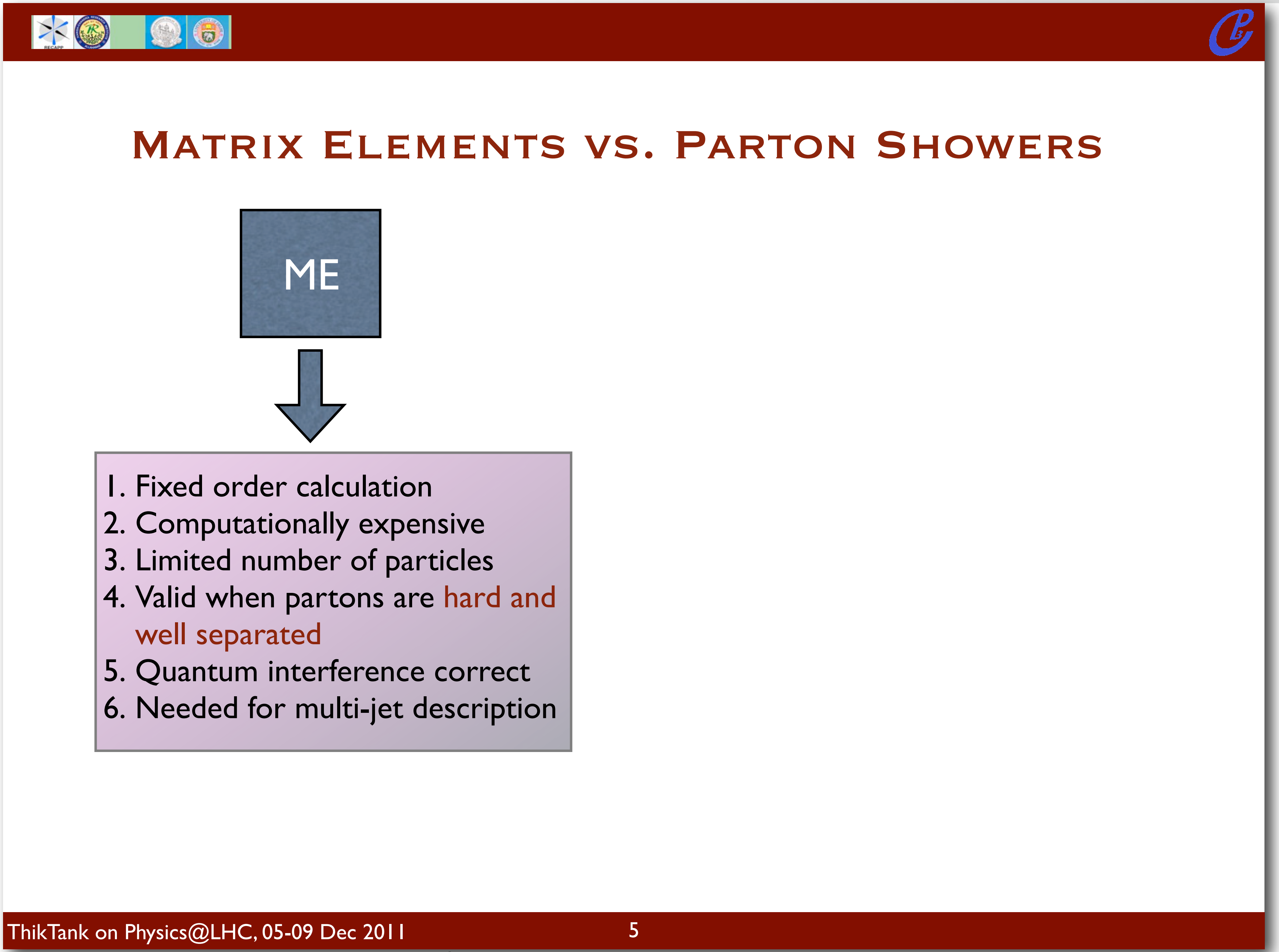
We wish to go beyond a Parton Shower (+MEC) Monte Carlo, so that

- we can successfully interface **matrix elements for multi-parton production with a parton shower**

- we can successfully interface a **parton shower with a NLO calculation**

Slide from F. Maltoni

Why ME+PS



MATRIX ELEMENTS VS. PARTON SHOWERS

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

ThikTank on Physics@LHC, 05-09 Dec 2011

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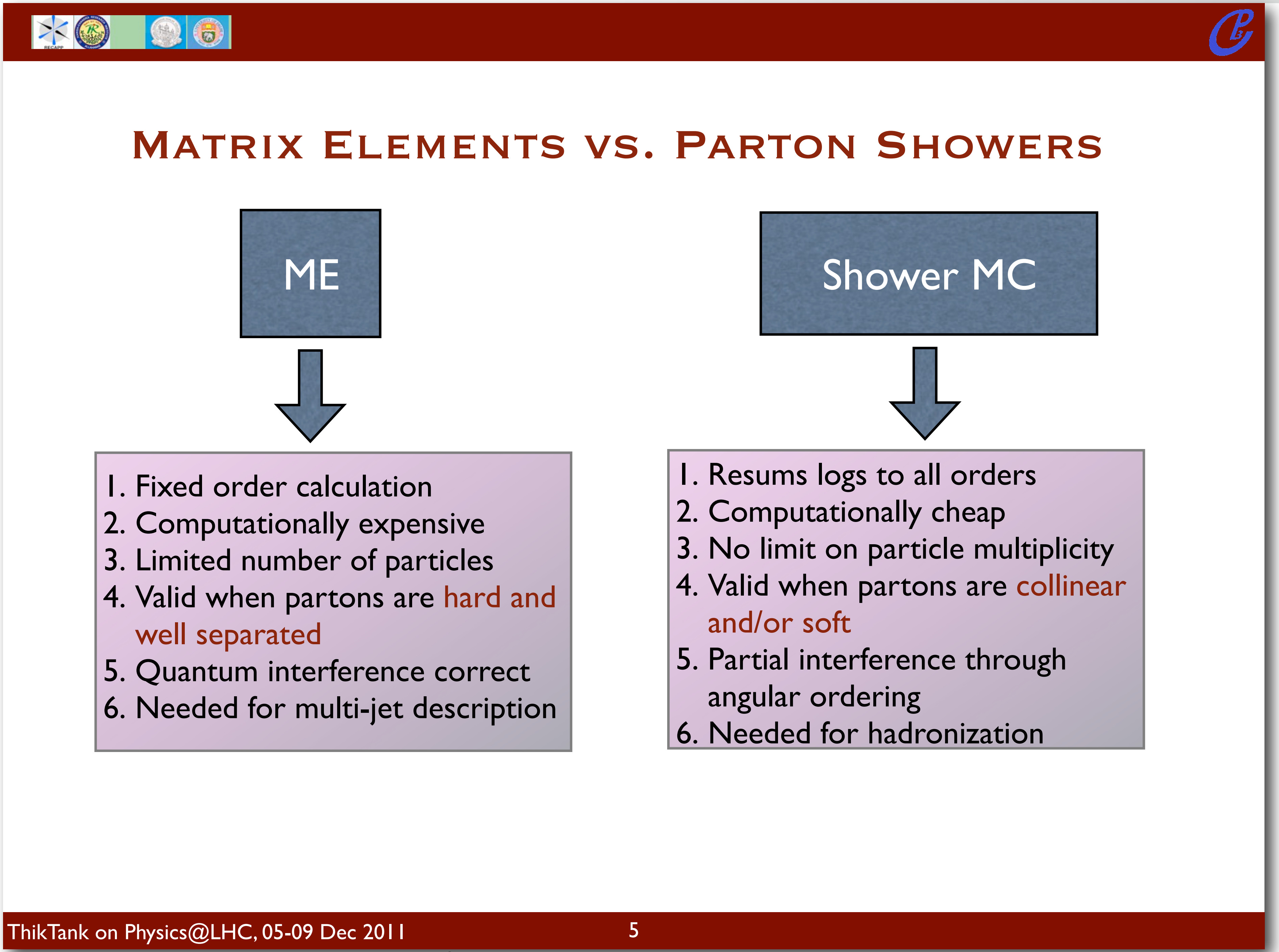
Why ME+PS

Slide from F. Maltoni



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Shower MC

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
6. Needed for hadronization

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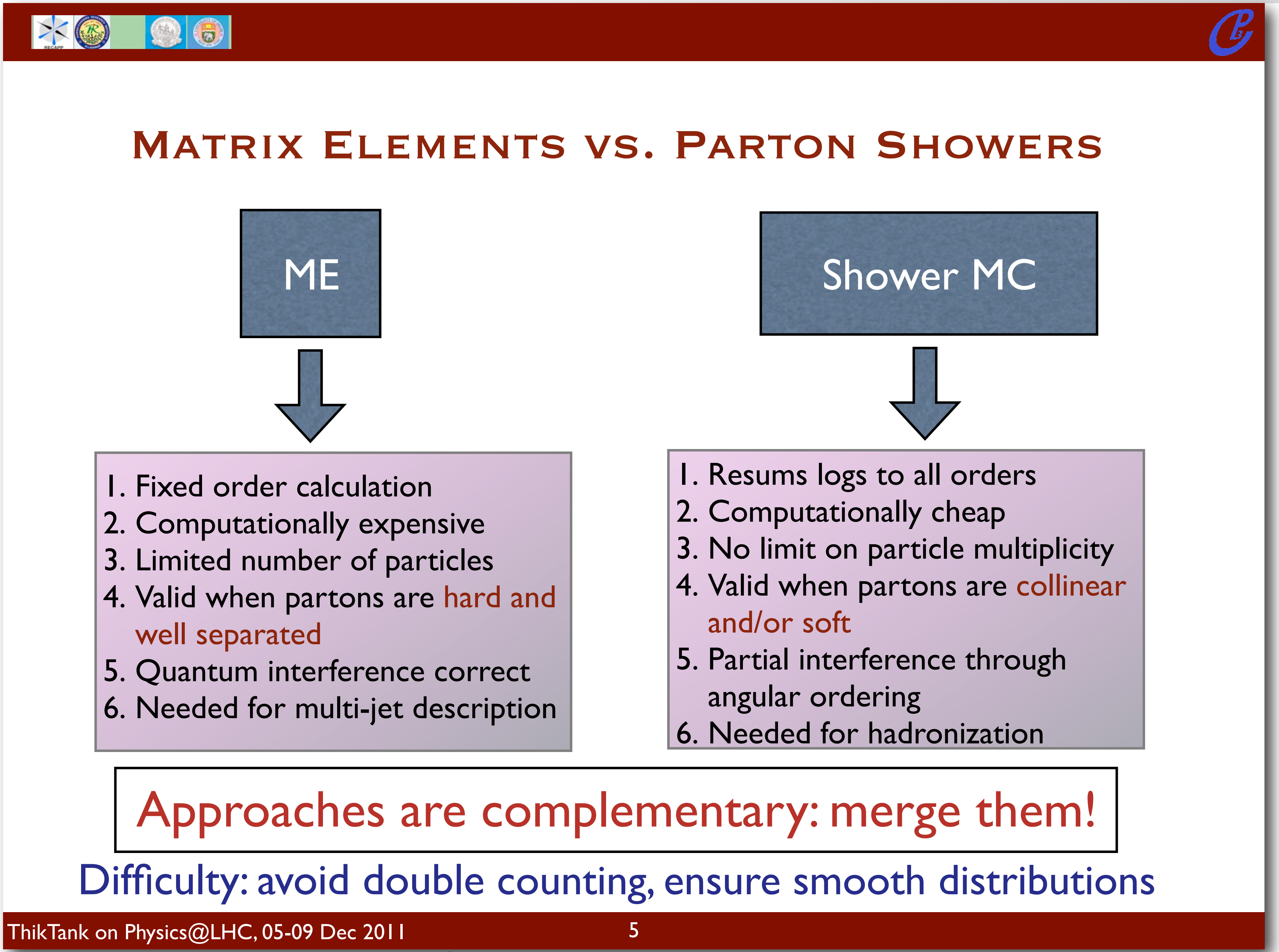
Slide from F. Maltoni

Why ME+PS



Matrix Elements vs. Parton Showers

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Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

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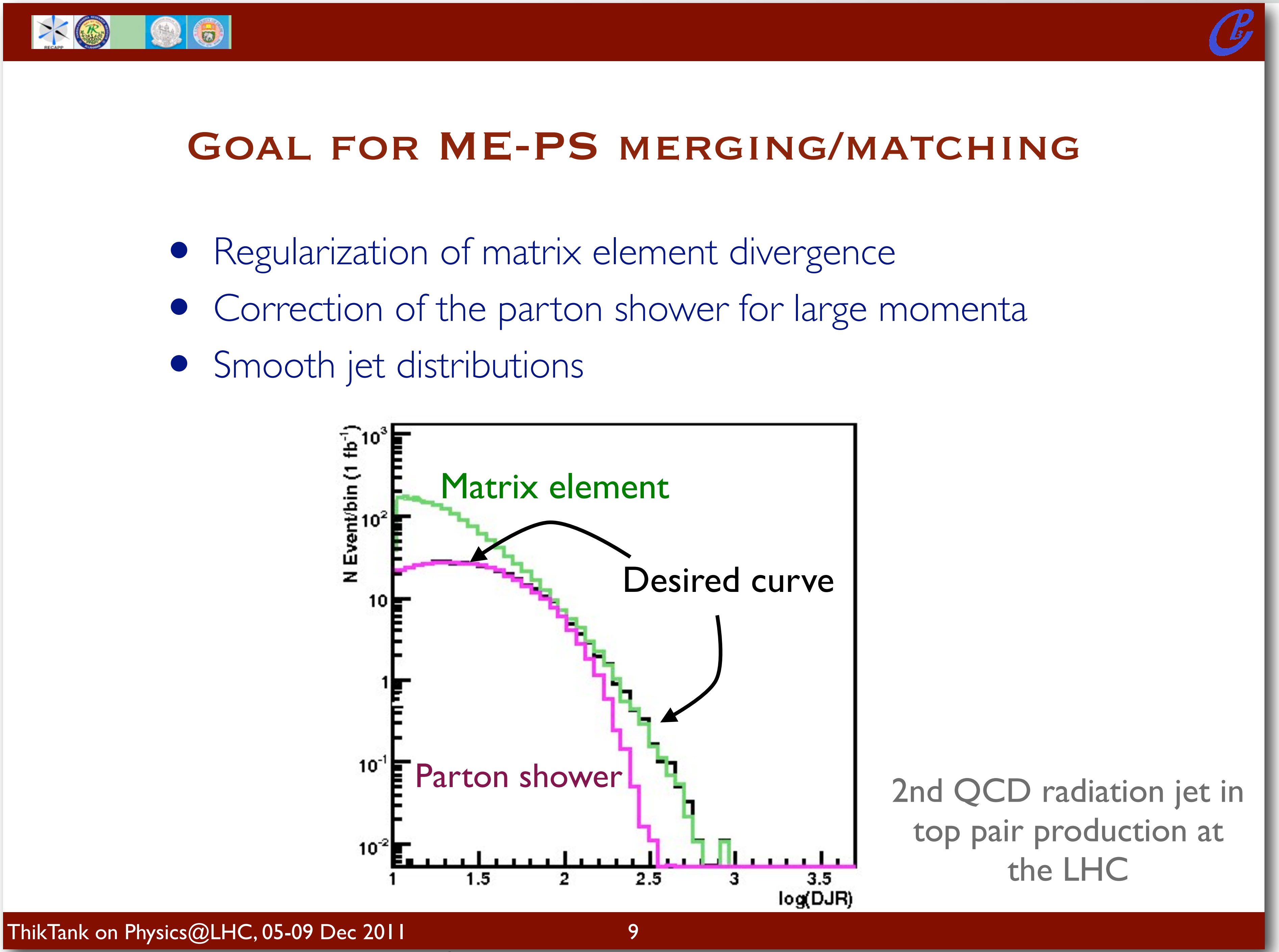
Slide from F. Maltoni

Why ME+PS



GOAL FOR ME-PS MERGING/MATCHING

- Regularization of matrix element divergence
- Correction of the parton shower for large momenta
- Smooth jet distributions

2nd QCD radiation jet in top pair production at the LHC

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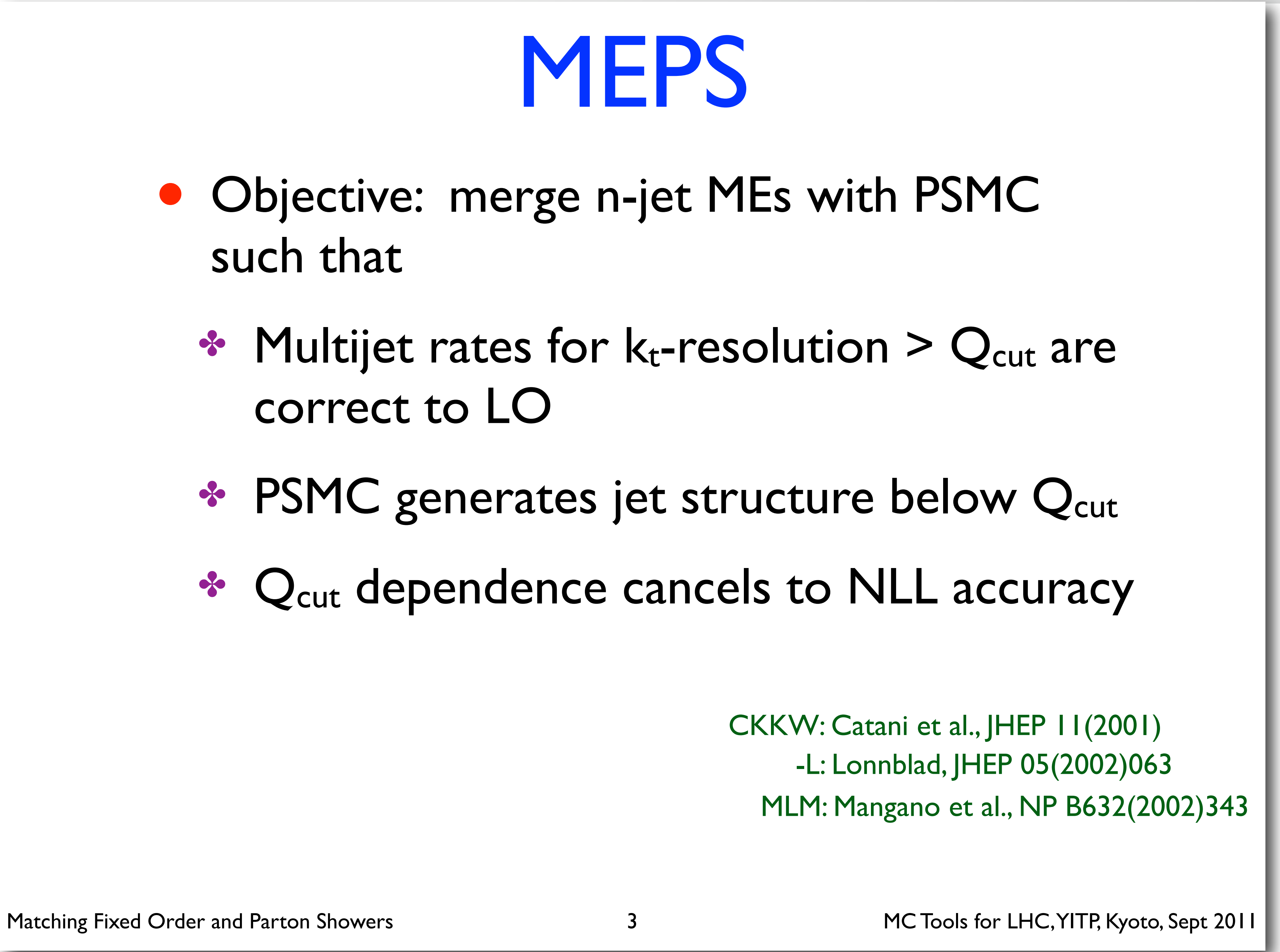
Slide from B.Webber

Why ME+PS



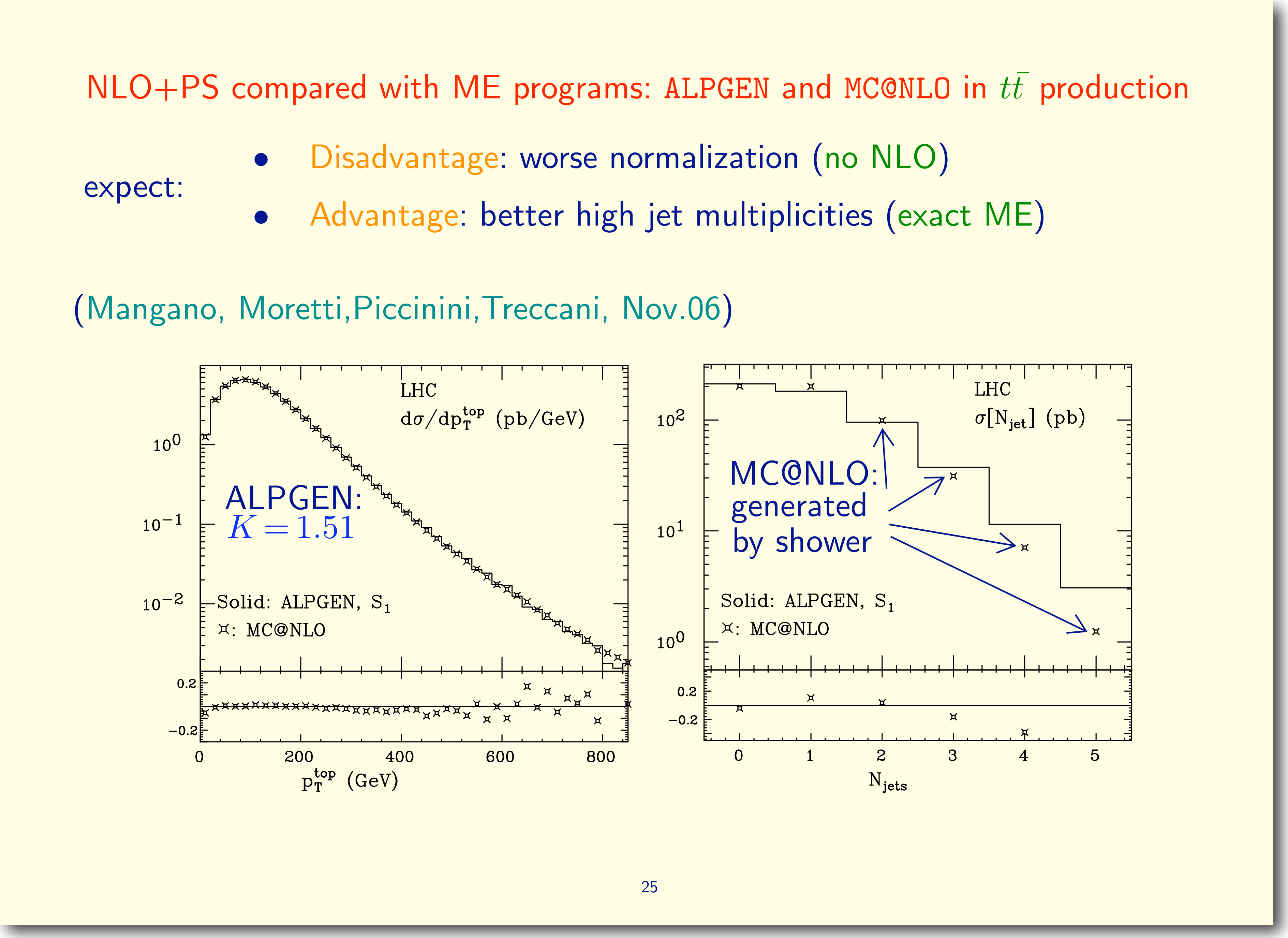
MEPS

- Objective: merge n-jet MEs with PSMC such that
  - Multijet rates for k_t-resolution > Q_{cut} are correct to LO
  - PSMC generates jet structure below Q_{cut}
  - Q_{cut} dependence cancels to NLL accuracy

CKKW: Catani et al., JHEP 11(2001)
-L: Lonnblad, JHEP 05(2002)063
MLM: Mangano et al., NP B632(2002)343

Slide from P. Nason

MCs at NLO v. ME+PS



NLO+PS compared with ME programs: `ALPGEN` and `MC@NLO` in $t\bar{t}$ production

expect:

- Disadvantage: worse normalization (no NLO)
- Advantage: better high jet multiplicities (exact ME)

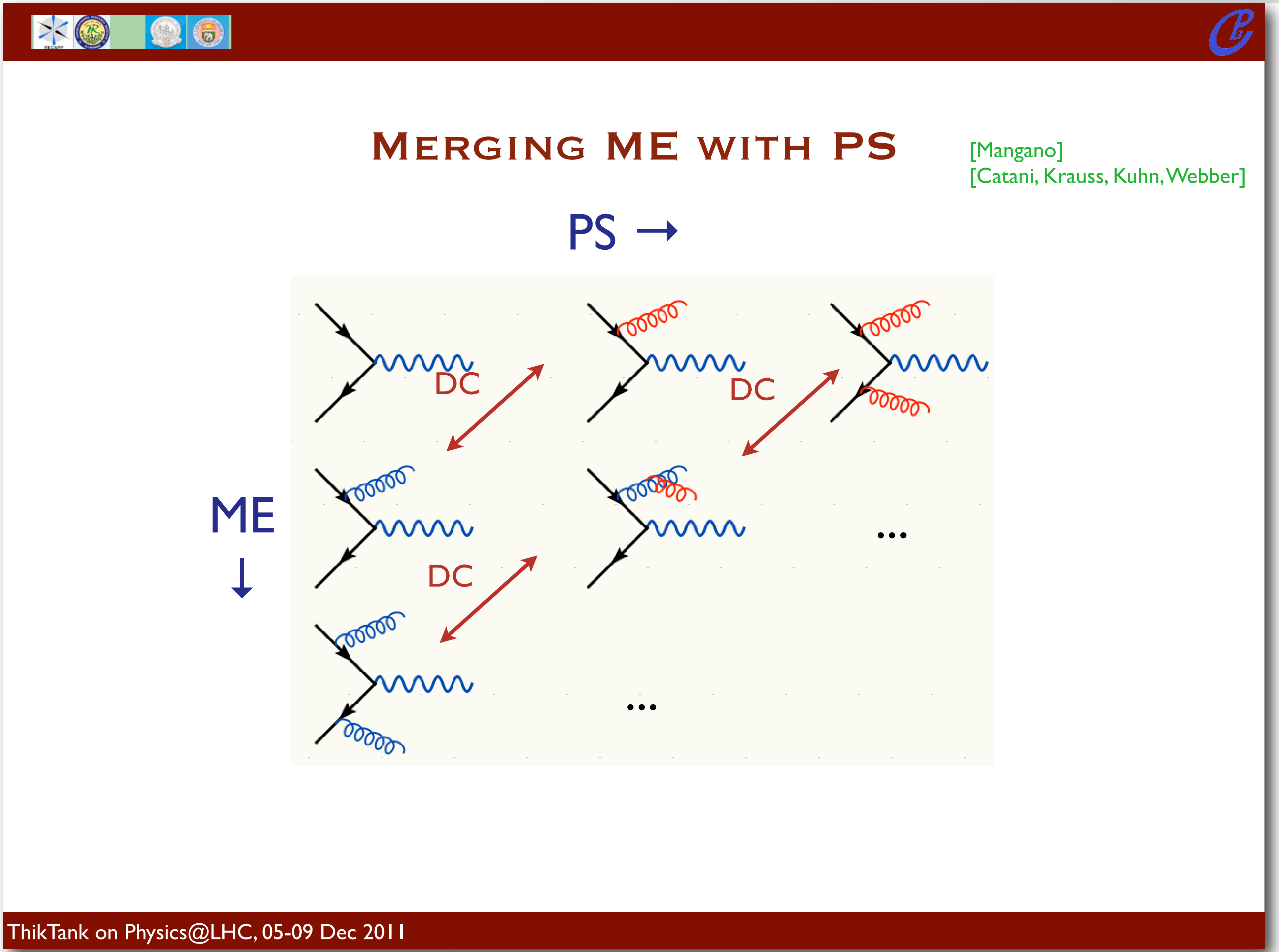
(Mangano, Moretti,Piccinini,Treccani, Nov.06)

ALPGEN: $K = 1.51$

MC@NLO: generated by shower

Slide from F. Maltoni

Double counting



MERGING ME WITH PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]

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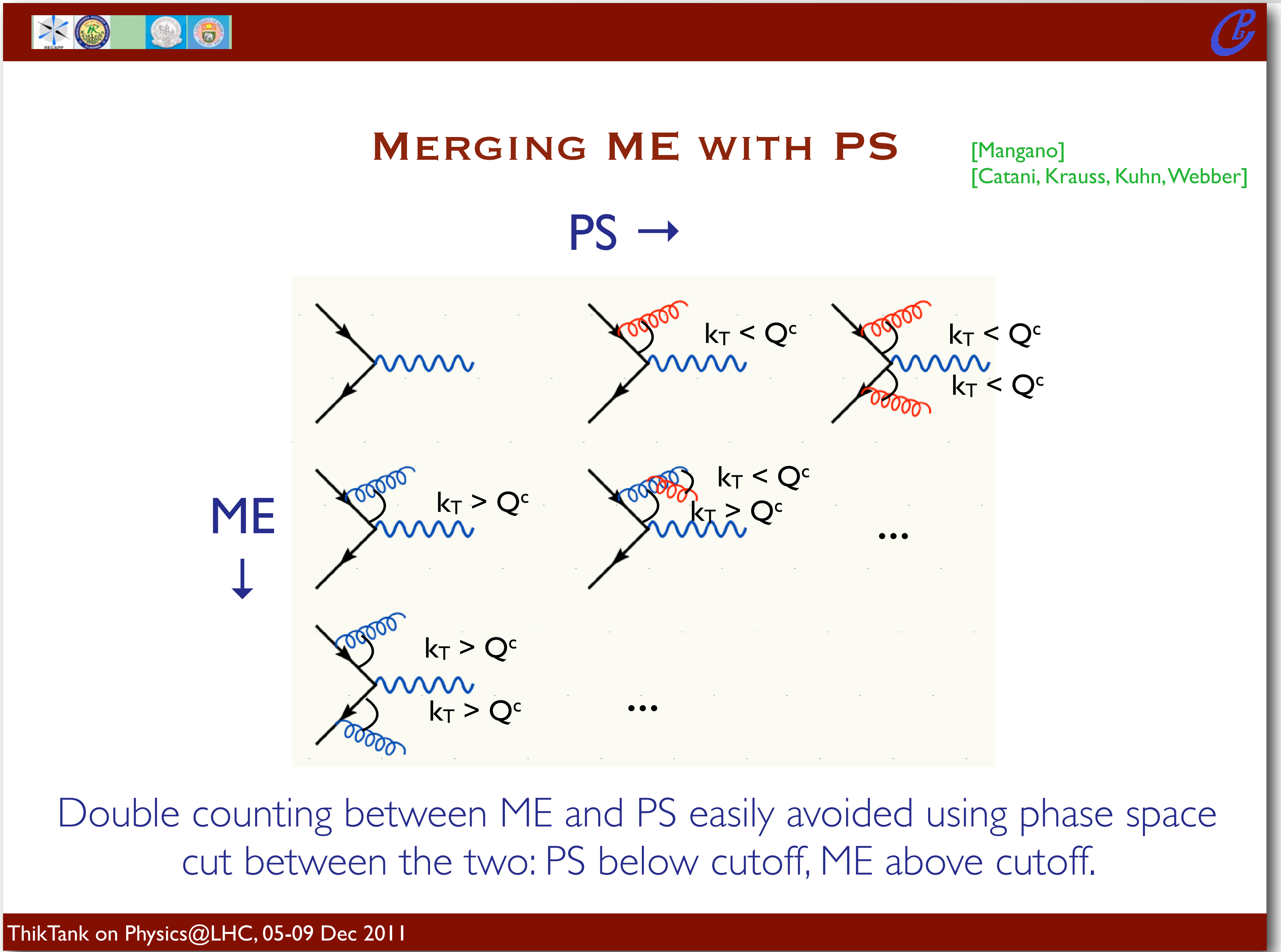
Slide from F. Maltoni

Double counting



MERGING ME WITH PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]

Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

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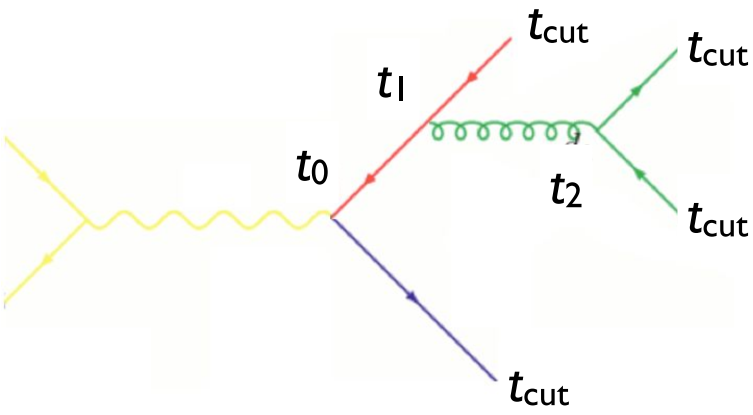
MERGING ME WITH PS

- So double counting no problem, but what about getting smooth distributions that are independent of the precise value of Q^c?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Let's take another look at the PS!

Slide from F. Maltoni

MERGING ME WITH PS



Slide from F. Maltoni

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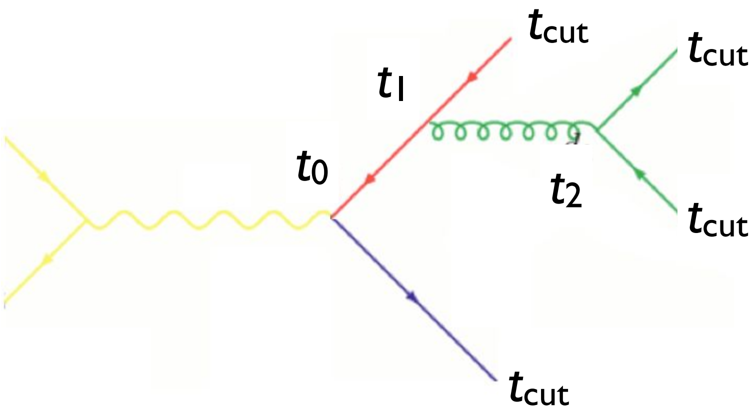
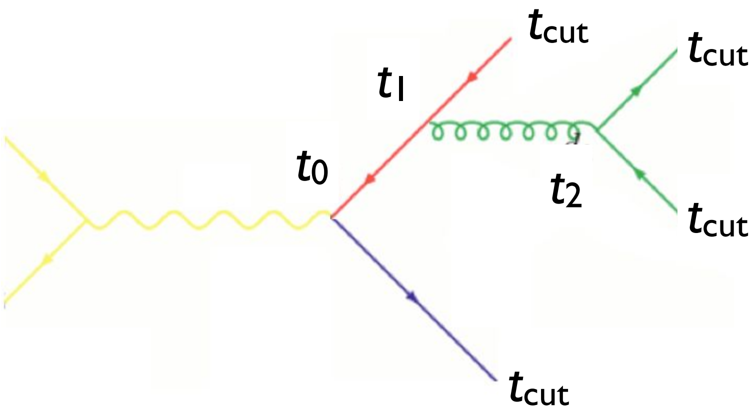


- How does the PS generate the configuration above?

Slide from F. Maltoni

MERGING ME WITH PS

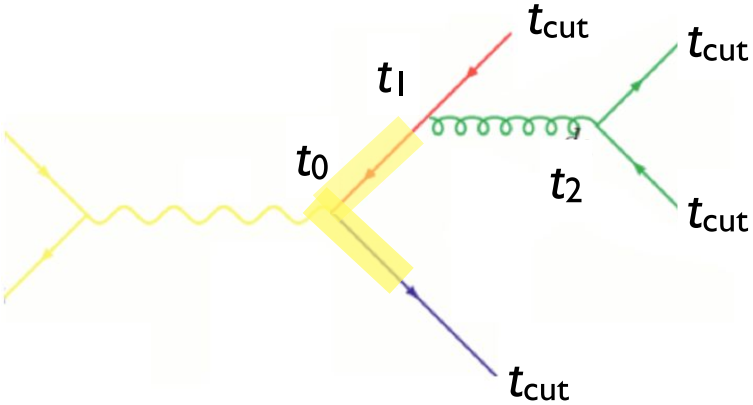


- How does the PS generate the configuration above?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(t_1,t_0))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

Slide from F. Maltoni

MERGING ME WITH PS



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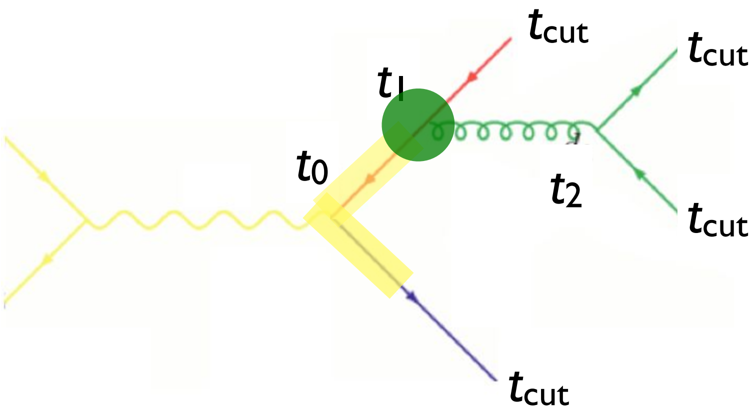

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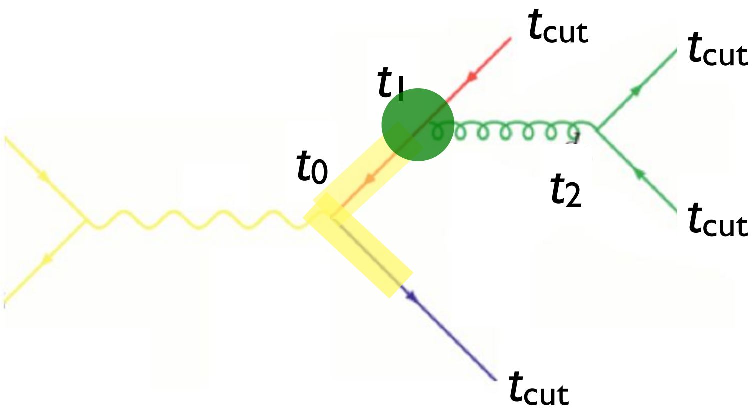

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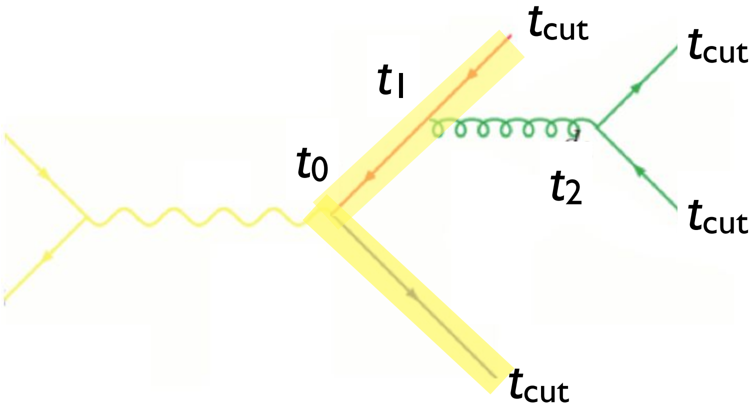
and for the whole tree

$$(\Delta_q(t_{\text{cut}}, t_0))^2 \Delta_g(t_2, t_1) (\Delta_q(_{\text{cut}}, t_2))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Slide from F. Maltoni

MERGING ME WITH PS



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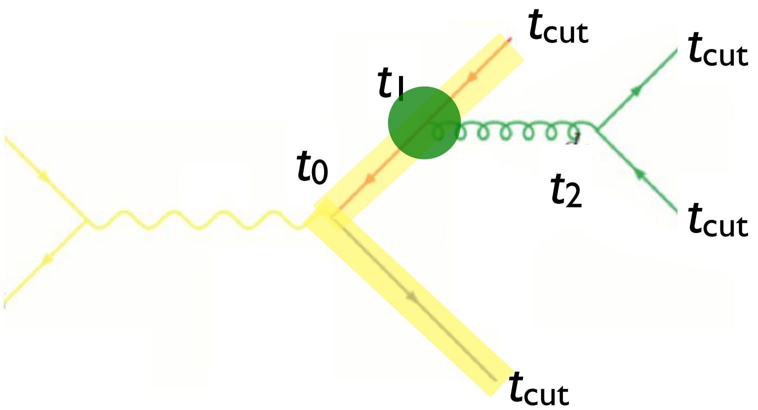

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Slide from F. Maltoni

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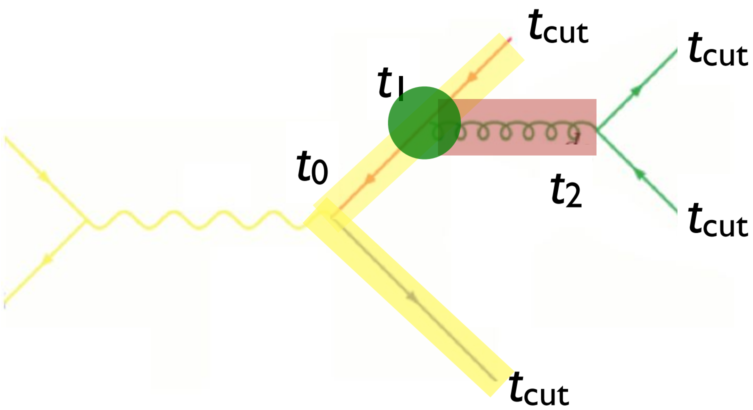

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Slide from F. Maltoni

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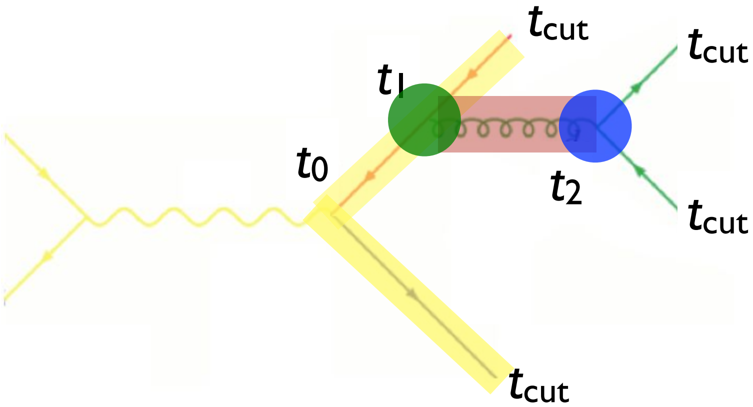

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Slide from F. Maltoni

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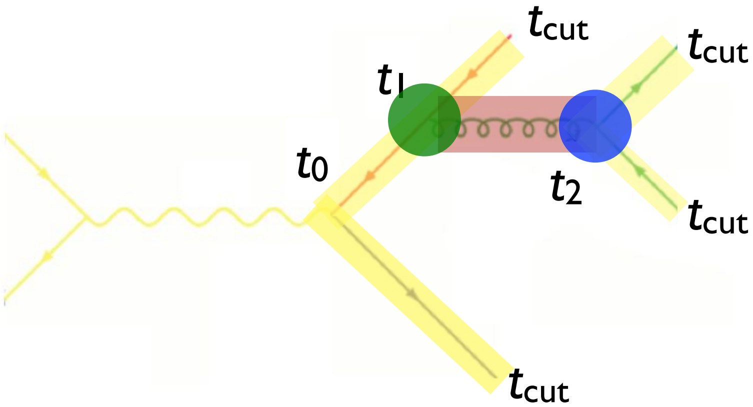

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Slide from F. Maltoni

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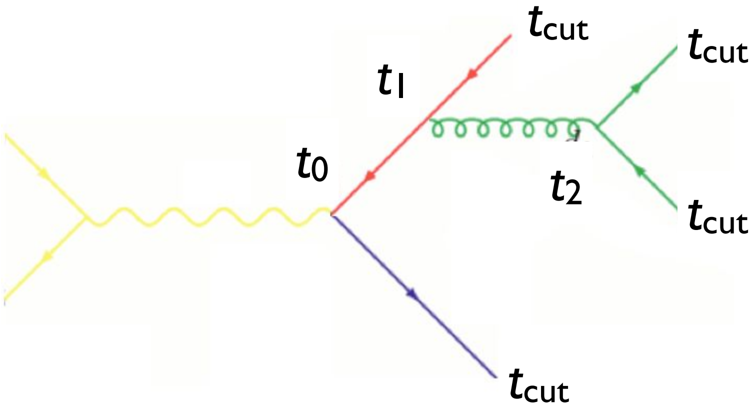

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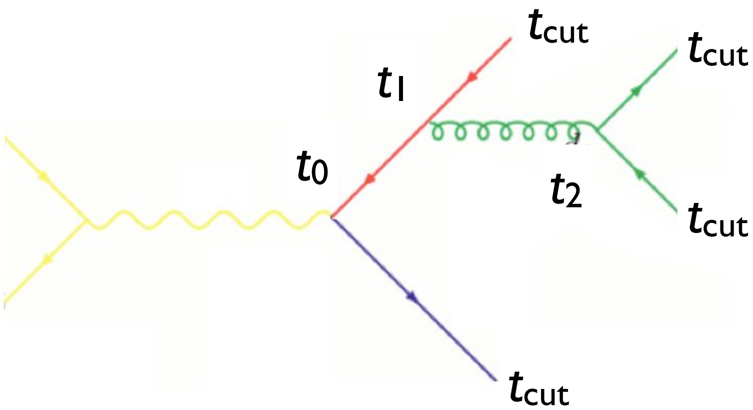

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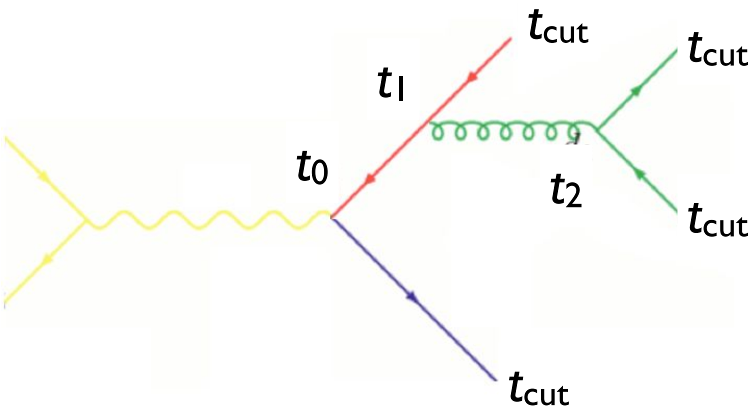
Corresponds to the matrix element
BUT with **α**$_s$ evaluated at the scale of each splitting

Slide from F. Maltoni

Merging ME with PS



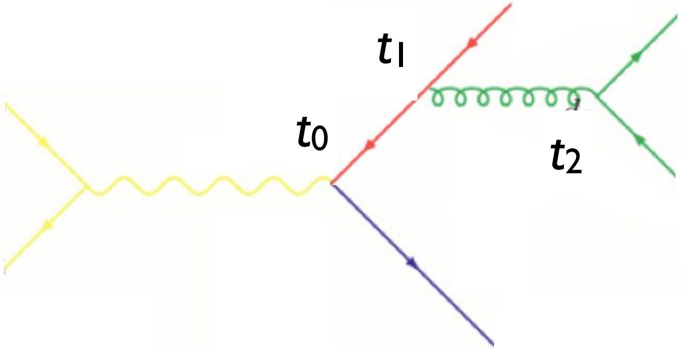
$$\boxed{(\Delta_q(t_{\rm cut},t_0))^2\Delta_g(t_2,t_1)(\Delta_q({}_{\rm cut},t_2))^2}\boxed{\frac{\alpha_s(t_1)}{2\pi}P_{gq}(z)\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')}$$

Corresponds to the matrix element
BUT with α_s evaluated at the scale of each splitting

Sudakov suppression due to disallowing additional radiation above the scale $t_{\rm cut}$

Slide from F. Maltoni

MERGING ME WITH PS



$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

- To get an equivalent treatment of the corresponding matrix element, do as follows:

1. Cluster the event using some clustering algorithm
   - this gives us a corresponding "parton shower history"
2. Reweight **α_s** in each clustering vertex with the clustering scale
$$|\mathcal{M}|^2 \to |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(t_0)} \frac{\alpha_s(t_2)}{\alpha_s(t_0)}$$
3. Use some algorithm to apply the equivalent Sudakov suppression $(\Delta_q(t_{\mathrm{cut}}, t_0))^2 \Delta_g(t_2, t_1)(\Delta_q(_{\mathrm{cut}}, t_2))^2$

Slide from F. Maltoni

ME+PS matching methods

ME+PS matching methods

- CKKW [Catani, Krauss, Kuhn, Webber, 2001]
- CKKW-L [Lonnblad, 2002]
- MLM [Mangano, 2002]

Slide from B. Webber

Why ME+PS

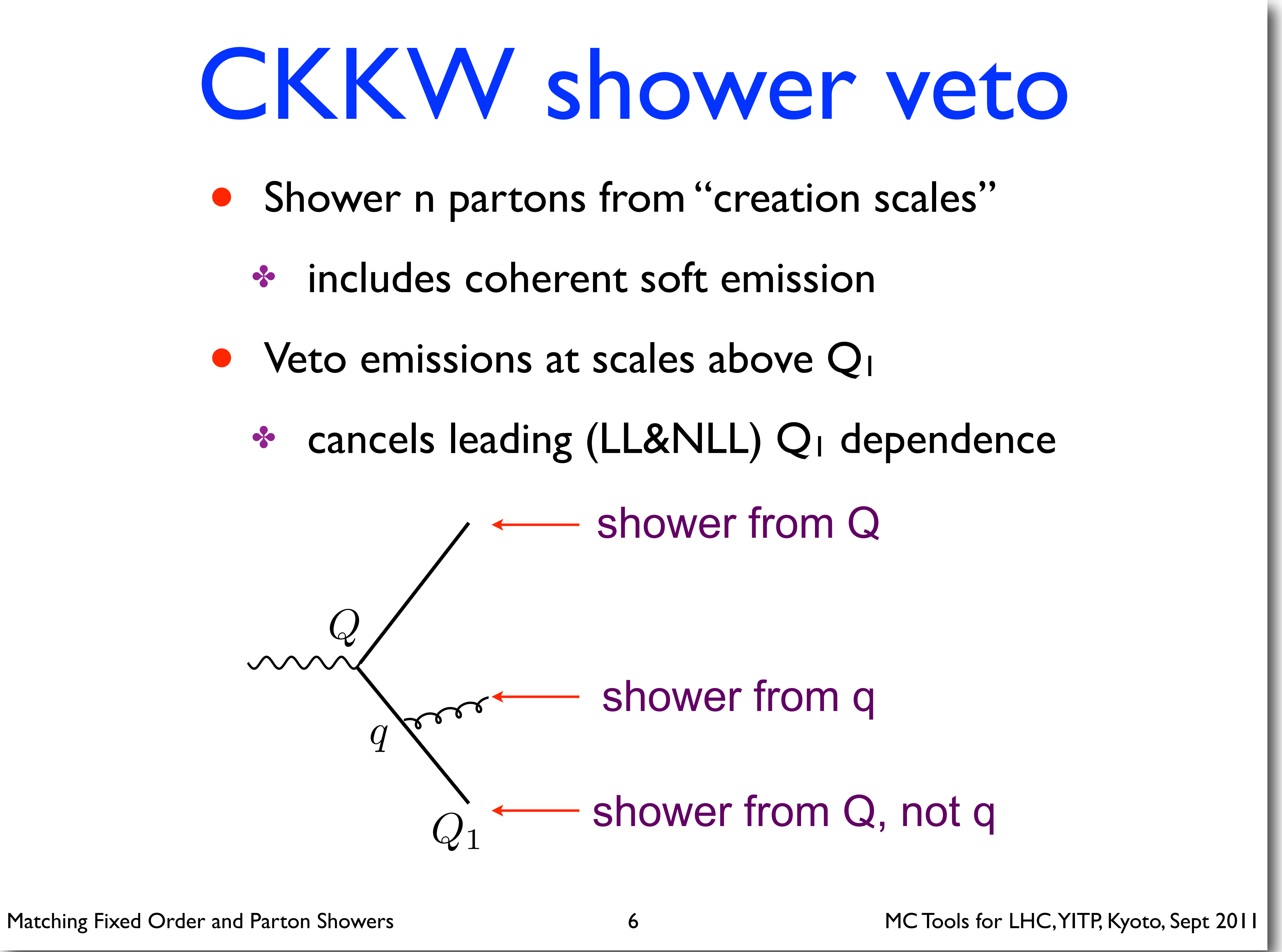
CKKW reweighting

- Choose n according to $R_n(Q, Q_1)$ (LO)
  - use $[\alpha_S(Q_1)]^n$
- Use exact LO ME to generate n partons
- Construct "equivalent shower history"
  - preferably using k_T-type algorithm
- Weight vertex at scale q by $\alpha_S(q)/\alpha_S(Q_1) < 1$
- Weight parton of type i from Q_j to Q_k by

$$\Delta_i(Q_j, Q_1)/\Delta_i(Q_k, Q_1)$$

Slide from B.Webber

Why ME+PS



CKKW shower veto

- Shower n partons from "creation scales"
  - includes coherent soft emission
- Veto emissions at scales above Q_1
  - cancels leading (LL&NLL) Q_1 dependence

shower from Q

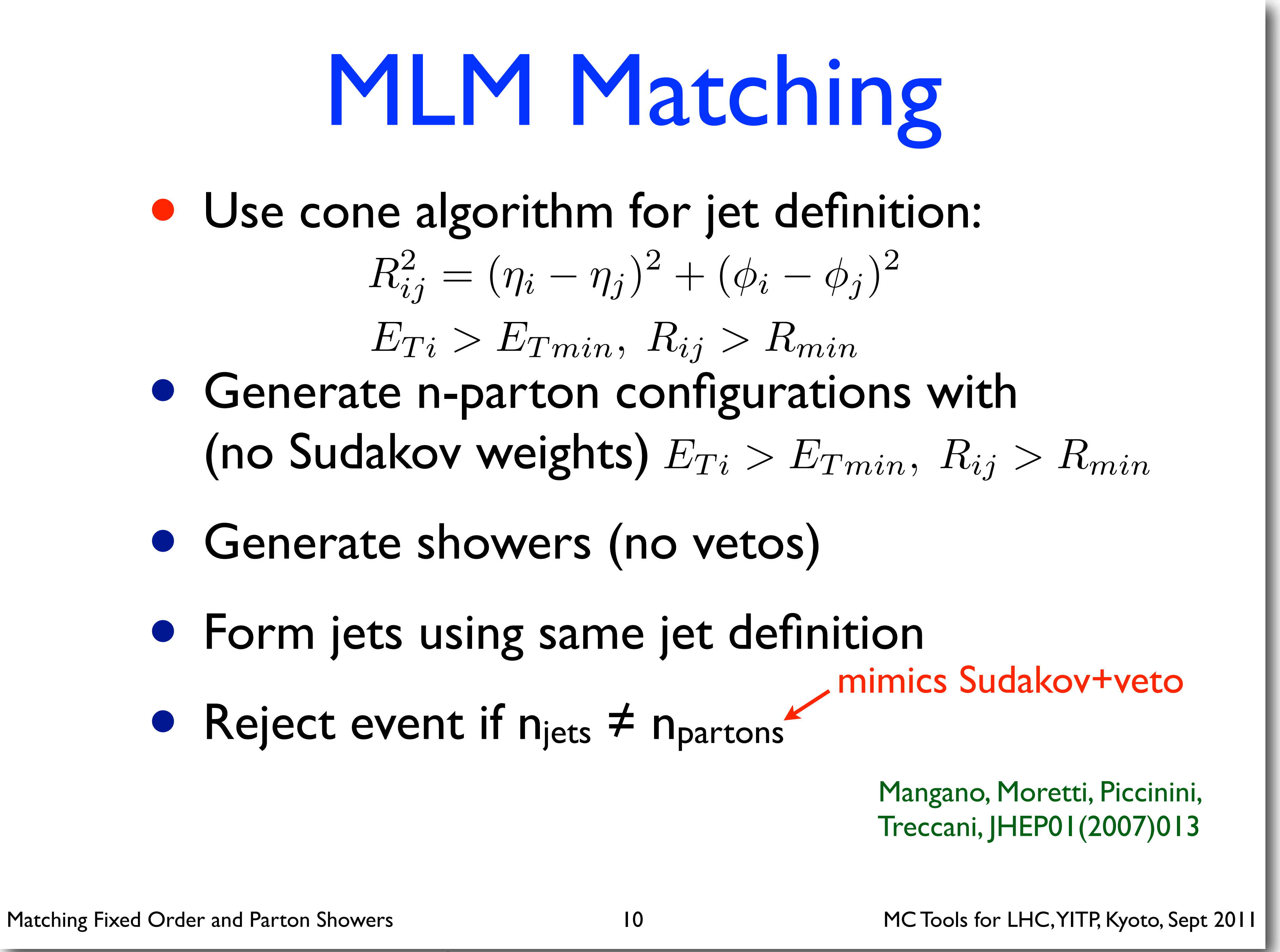
shower from q

shower from Q, not q

Matching Fixed Order and Parton Showers 6 MC Tools for LHC, YITP, Kyoto, Sept 2011

Slide from B.Webber

Why ME+PS



MLM Matching

- Use cone algorithm for jet definition:

$$R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$$

$$E_{Ti} > E_{Tmin}, \; R_{ij} > R_{min}$$

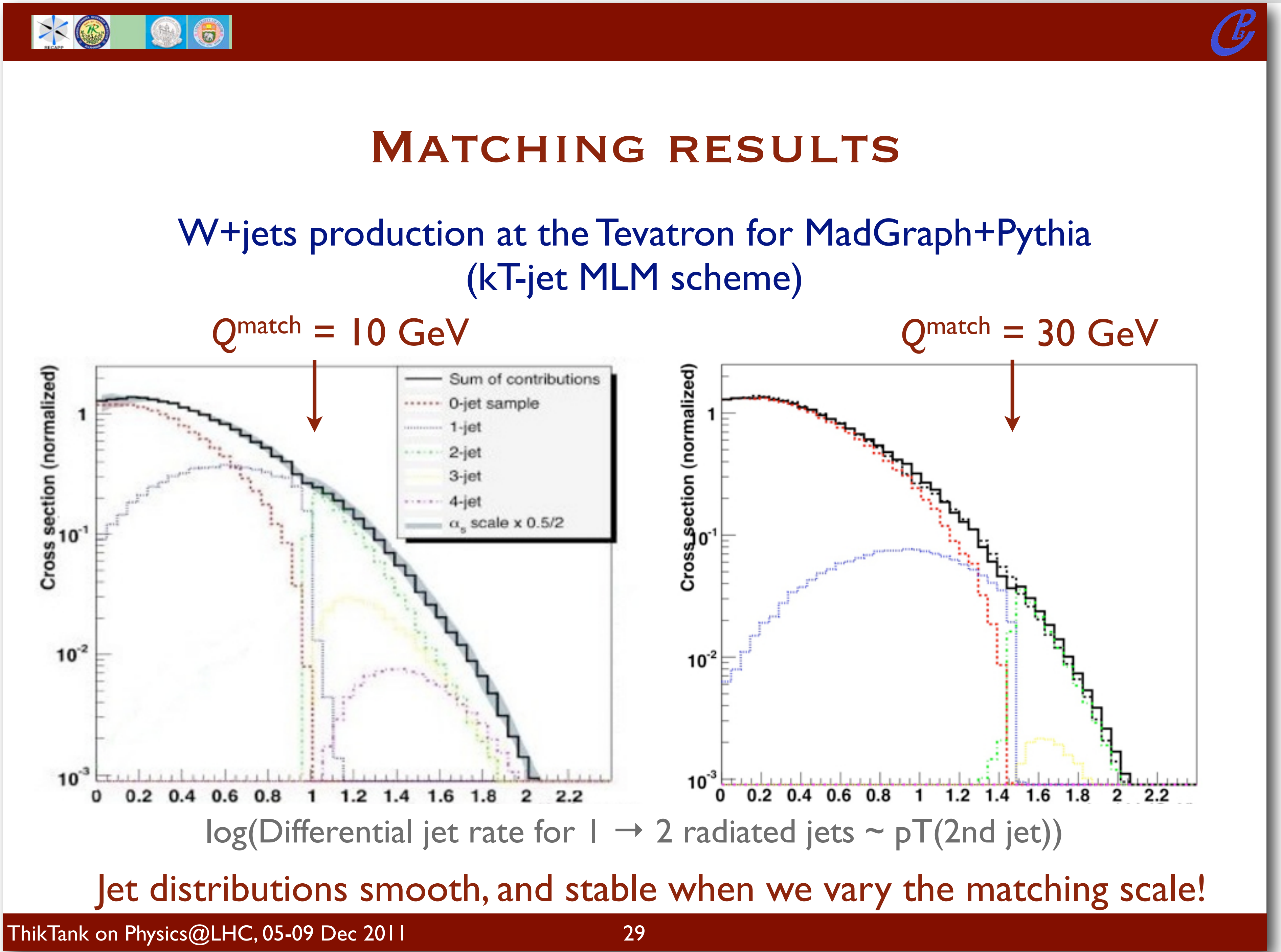
- Generate n-parton configurations with (no Sudakov weights) $E_{Ti} > E_{Tmin}, \; R_{ij} > R_{min}$
- Generate showers (no vetos)
- Form jets using same jet definition
- Reject event if $n_{\text{jets}} \neq n_{\text{partons}}$ ← mimics Sudakov+veto

Mangano, Moretti, Piccinini, Treccani, JHEP01(2007)013

Matching Fixed Order and Parton Showers 10 MC Tools for LHC, YITP, Kyoto, Sept 2011

Slide from F. Maltoni

Example of PS+ME matching



MATCHING RESULTS

W+jets production at the Tevatron for MadGraph+Pythia
(kT-jet MLM scheme)

Q^{match} = 10 GeV

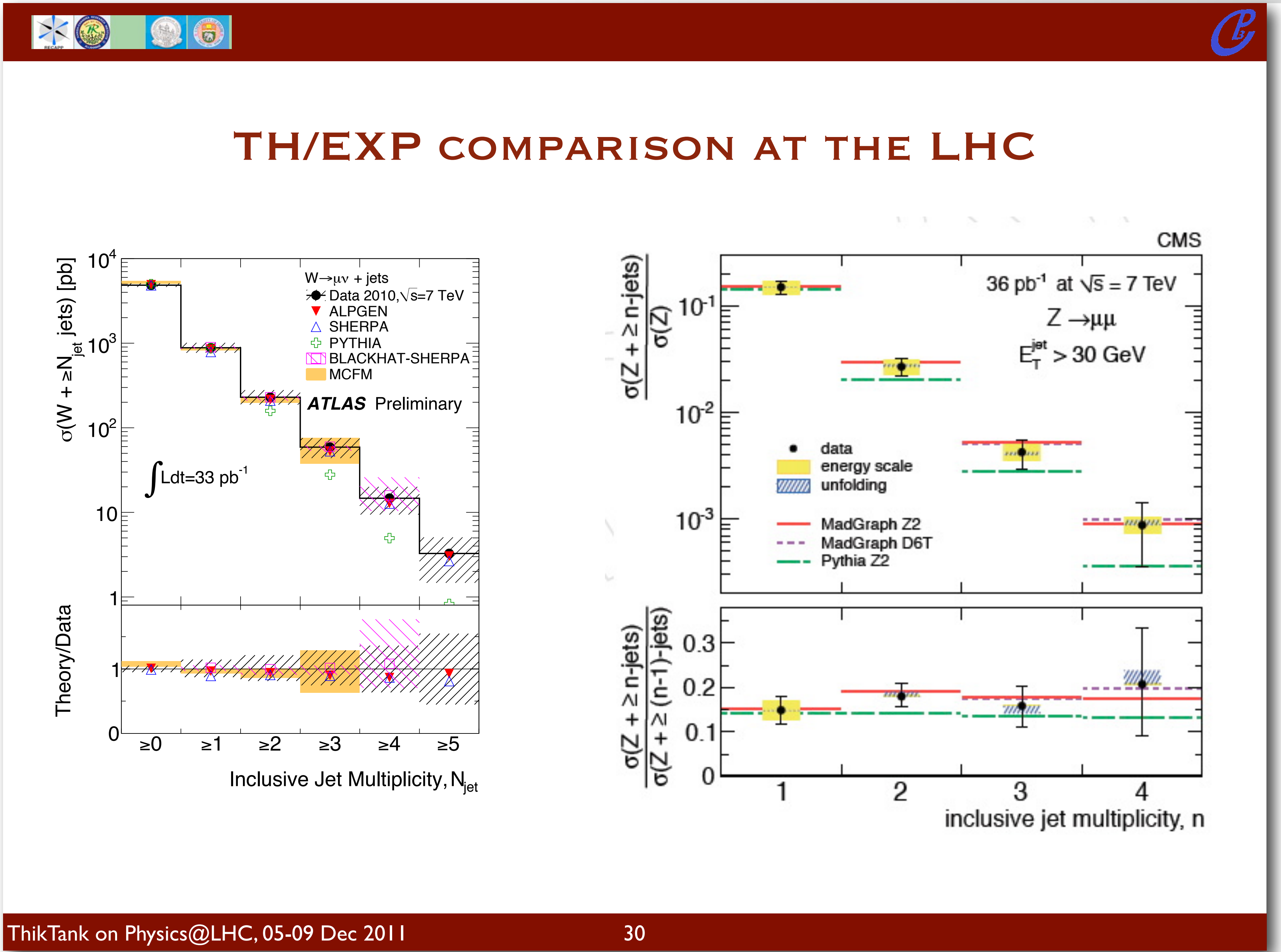
Q^{match} = 30 GeV

log(Differential jet rate for 1 → 2 radiated jets ~ pT(2nd jet))

Jet distributions smooth, and stable when we vary the matching scale!

Slide from F. Maltoni

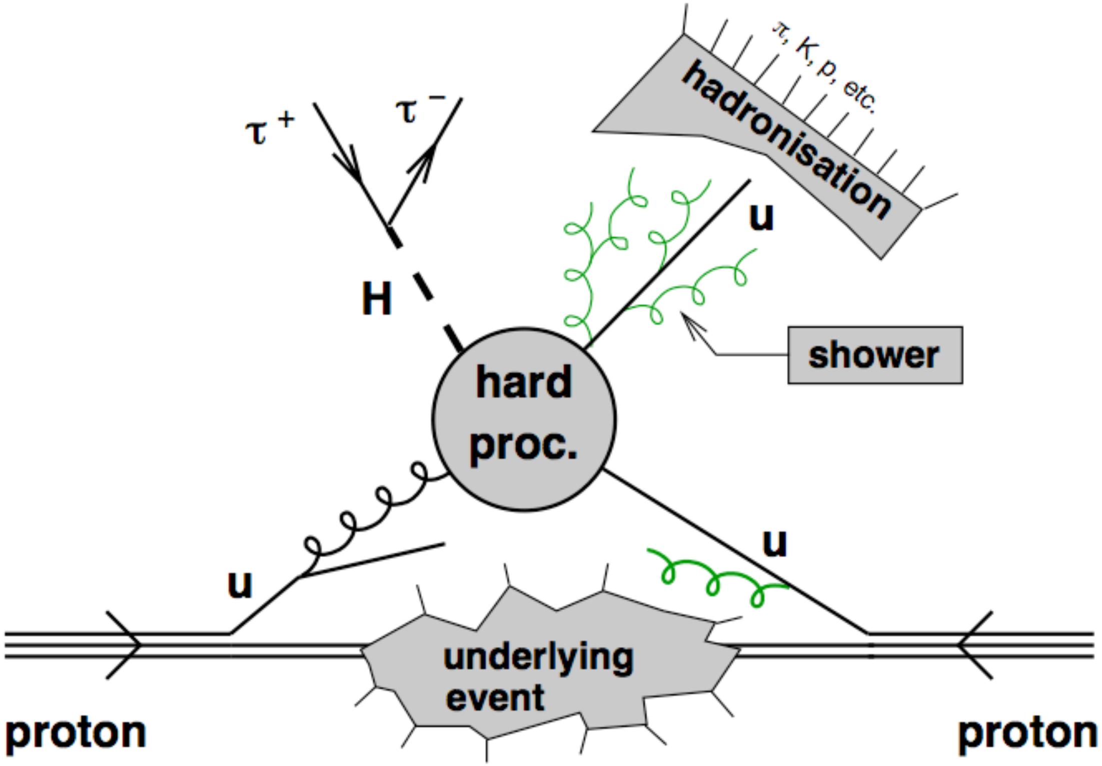
Example of PS+ME matching



TH/EXP COMPARISON AT THE LHC

ThikTank on Physics@LHC, 05-09 Dec 2011 30

Ingredients and tools

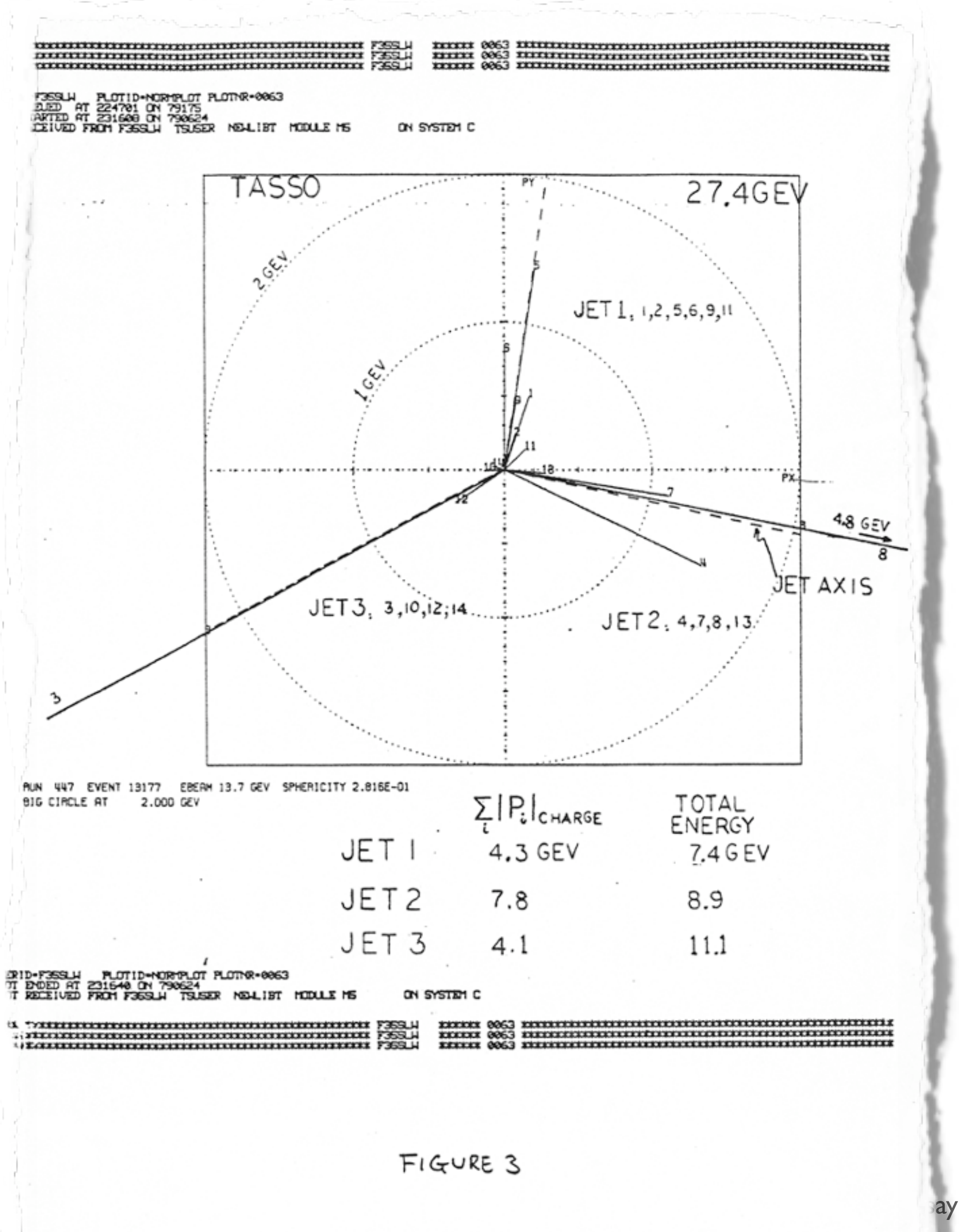


- PDFs
- Hard scattering
- Final state tools

Gluon ‘discovery’

1979:

Three-jet events observed by TASSO, JADE, MARK J and PLUTO at PETRA in e^+e^- collisions at 27.4 GeV



```
F35SLW    PLOTID=NORMPLOT PLOTNR=0063
EUED  AT 224701 ON 79175
ARTED AT 231608 ON 790624
CEIVED FROM F35SLW  TSUSER  NEWLIBT  MODULE M5       ON SYSTEM C
```

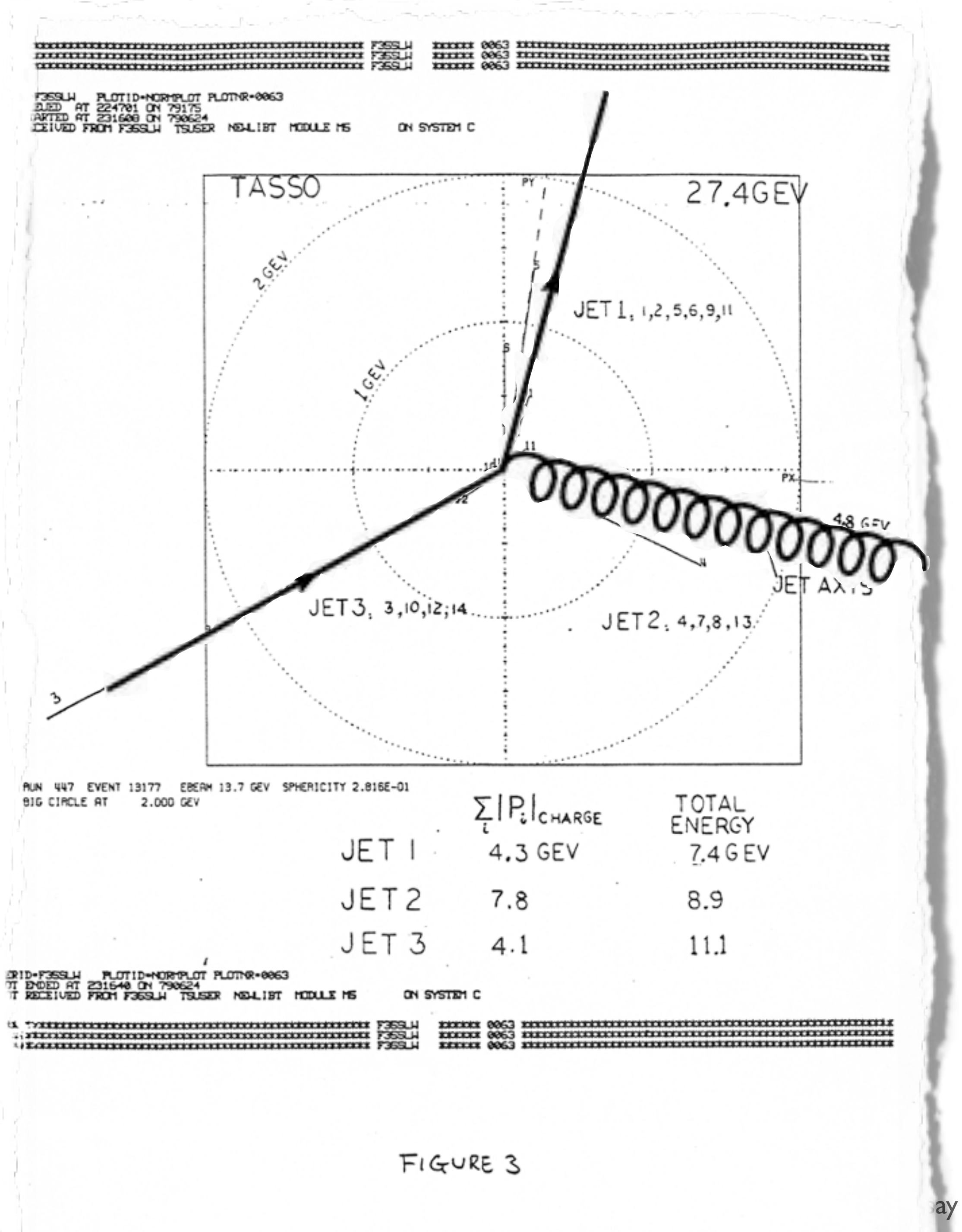
```
RUN  447  EVENT 13177   EBEAM 13.7 GEV  SPHERICITY 2.816E-01
BIG CIRCLE AT     2.000 GEV
```

	$\sum_i \lvert P_i \rvert_{\text{CHARGE}}$	TOTAL ENERGY
JET 1	4.3 GEV	7.4 GEV
JET 2	7.8	8.9
JET 3	4.1	11.1

```
ERID=F35SLW   PLOTID=NORMPLOT PLOTNR=0063
JT ENDED AT 231640 ON 790624
JT RECEIVED FROM F35SLW  TSUSER  NEWLIBT  MODULE M5       ON SYSTEM C
```

FIGURE 3

Gluon ‘discovery’



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T ENDED AT 231640 ON 790624
T RECEIVED FROM F35SLW  TSUSER  NEWLIBT  MODULE M5       ON SYSTEM C
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FIGURE 3

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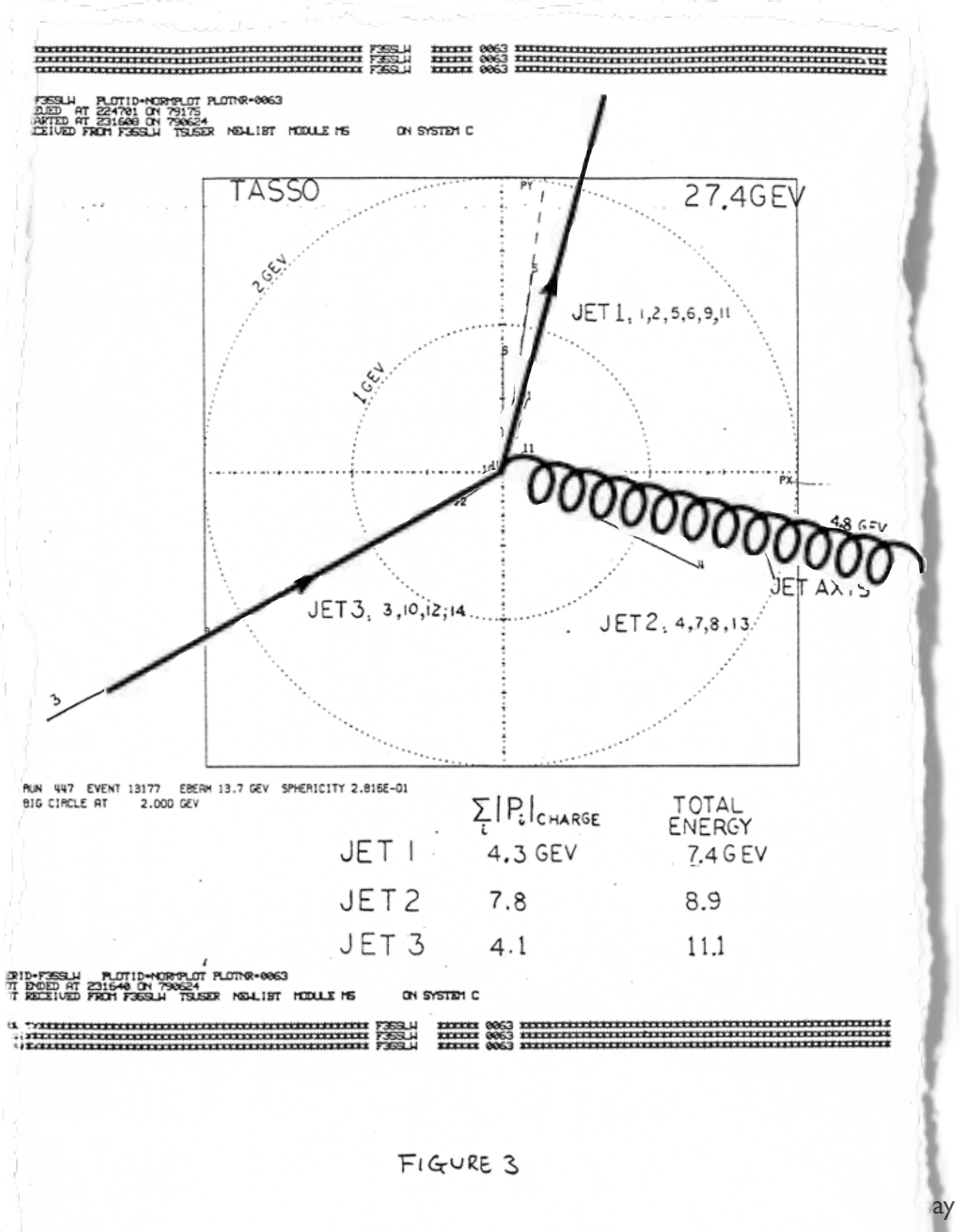
Interpretation:
large angle emission of a hard gluon

Gluon ‘discovery’



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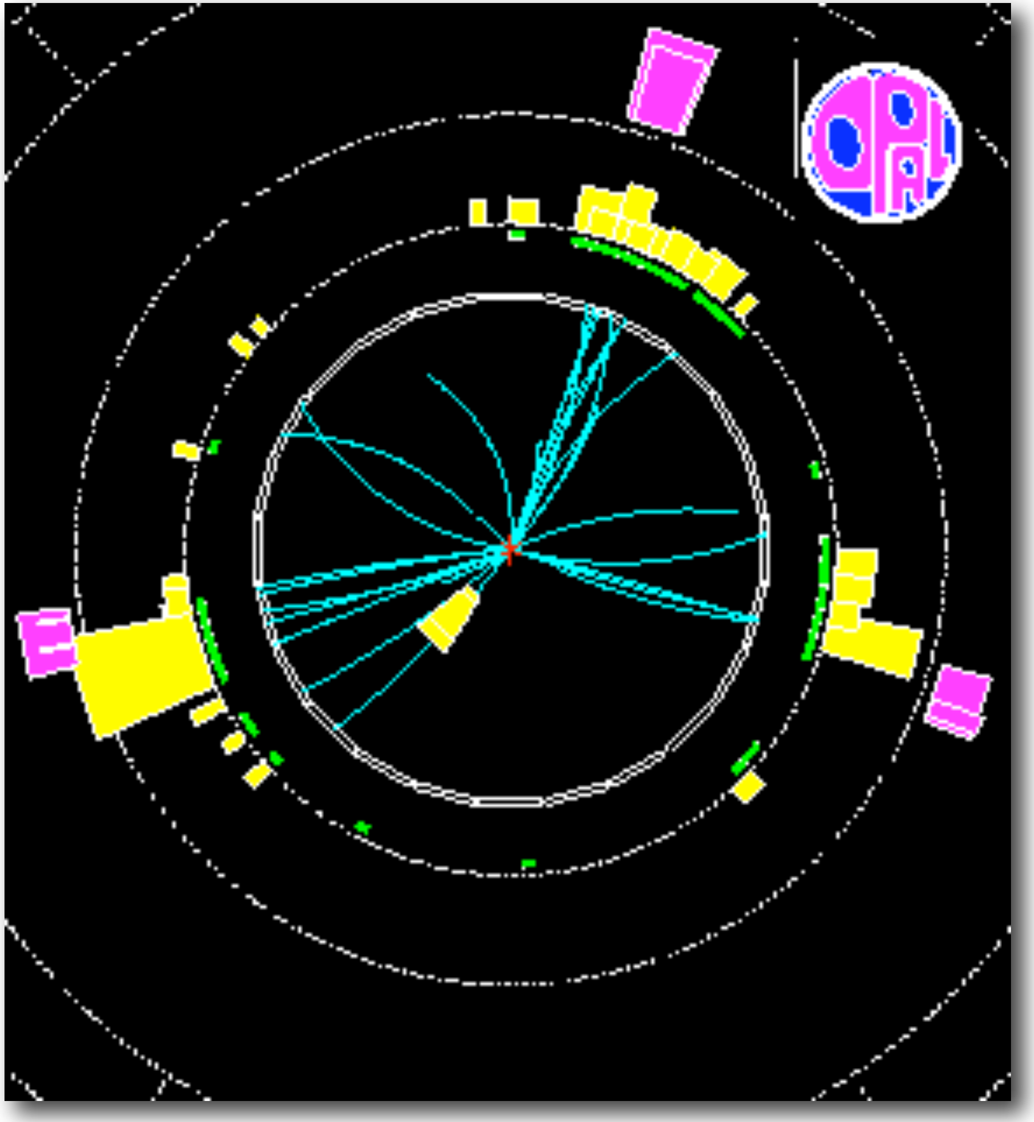
Three-jet events observed by TASSO, JADE, MARK J and PLUTO at PETRA in e^+e^- collisions at 27.4 GeV

Interpretation:
large angle emission of a hard gluon

Jets viewed as a proxy to the initial partons

Why jets



From PETRA to LEP

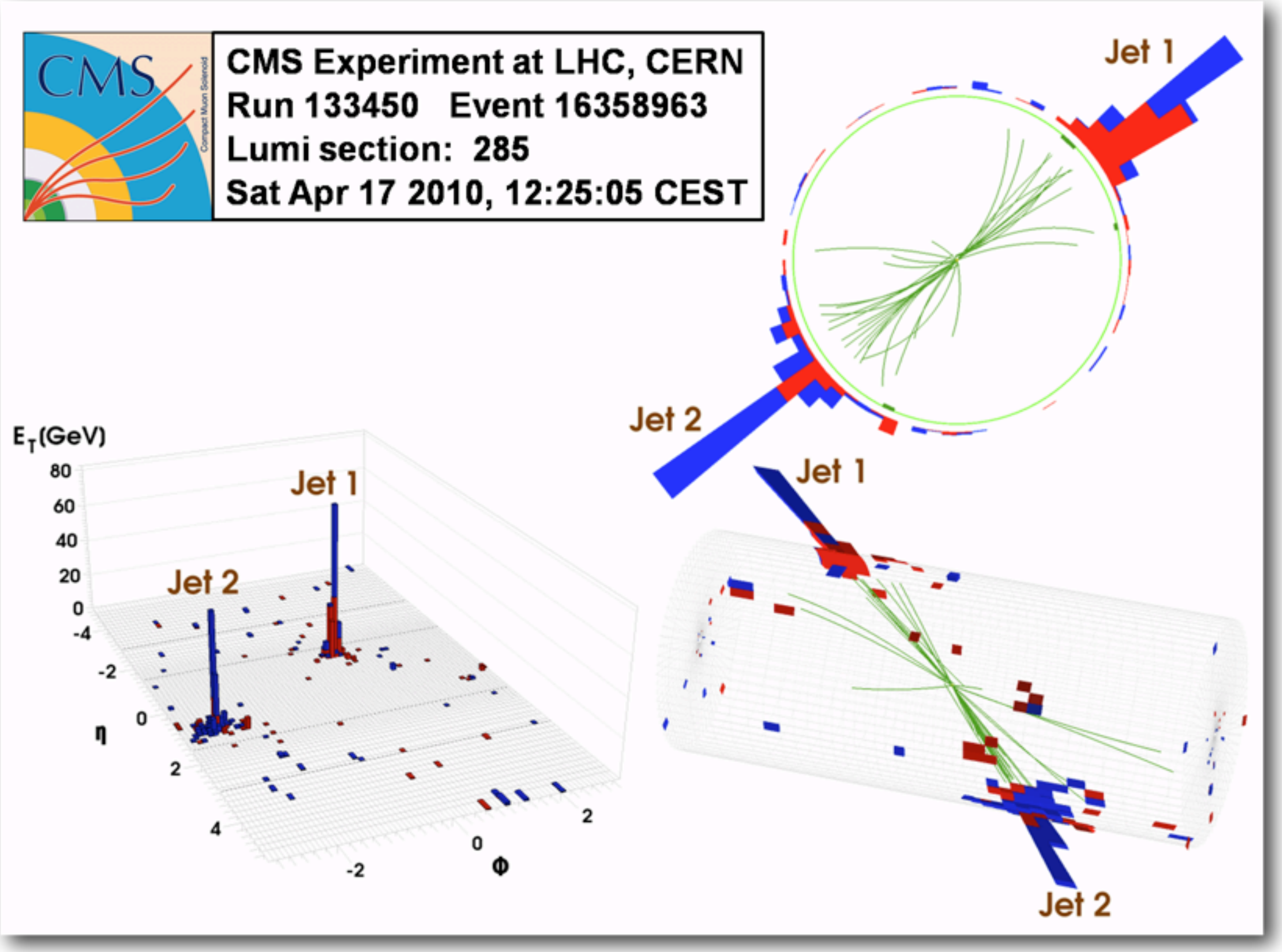
A **jet** is something that happens in high energy events: **a collimated bunch of hadrons flying roughly in the same direction**

We could eyeball the collimated bunches, but it becomes impractical with millions of events

The classification of particles into jets is best done using a **clustering algorithm**

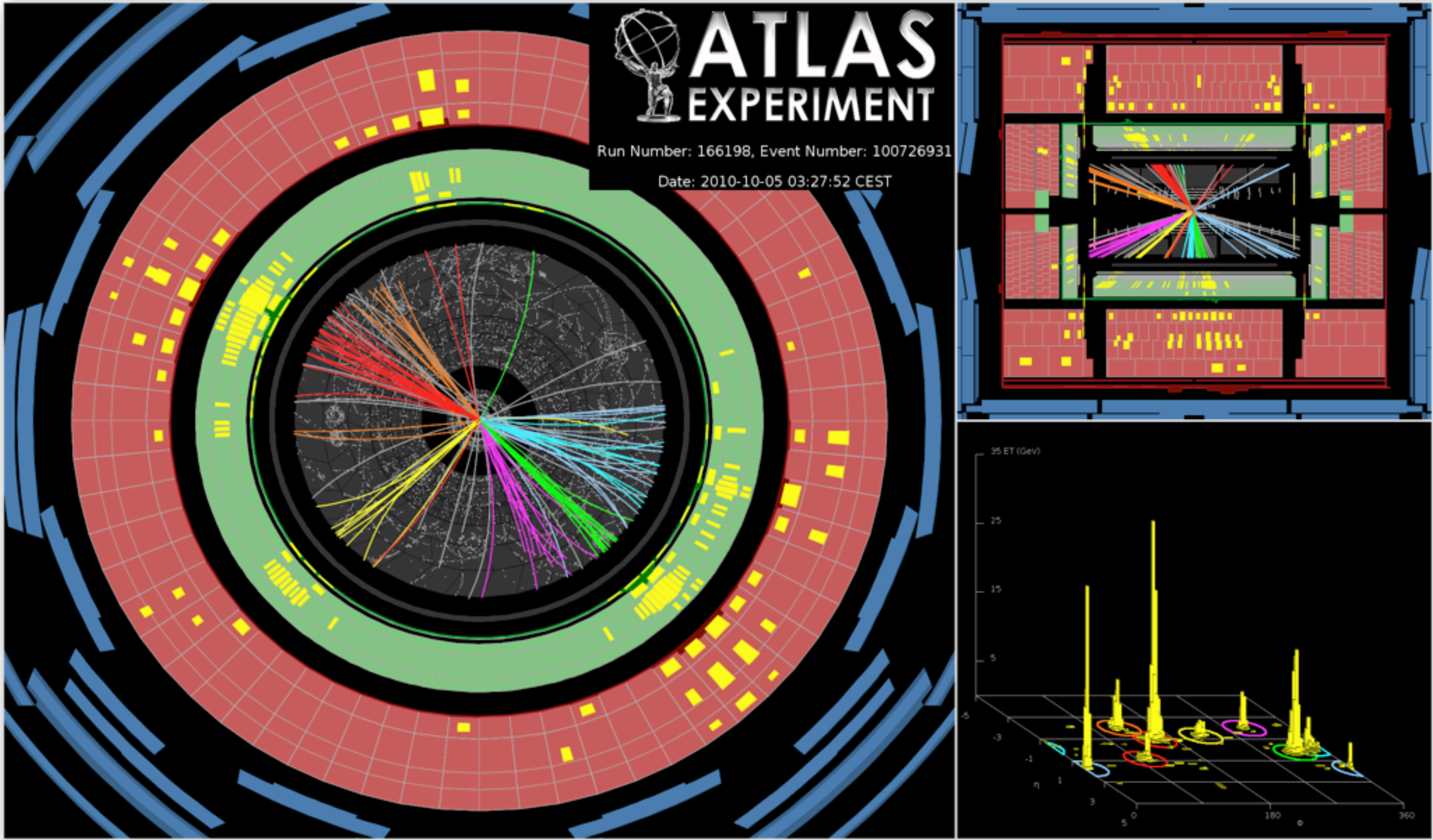
Jets @ LHC

A few decades after PETRA and LEP, the event displays got prettier, but jets are still pretty much the same



Dijet event from CMS

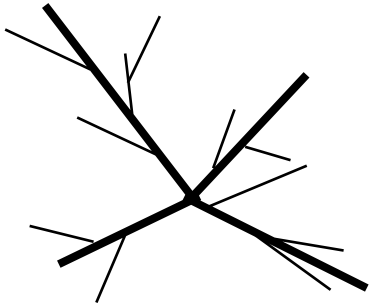
Jets @ LHC



8(!) jets event from ATLAS

Taming reality

Multileg + PS



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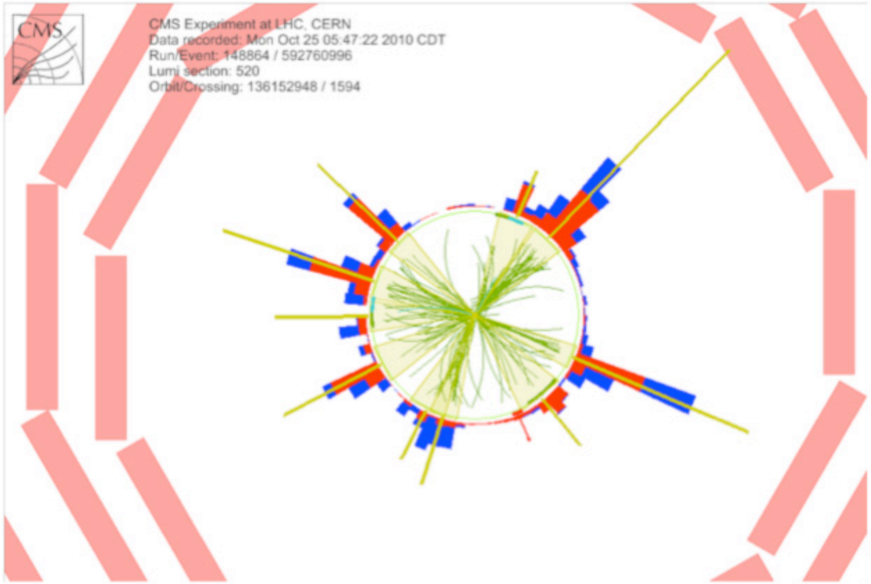
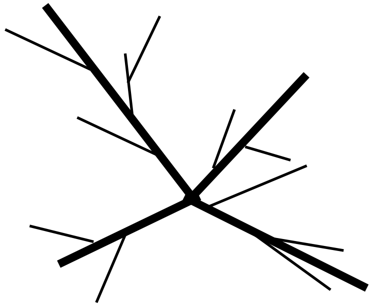


QCD predictions

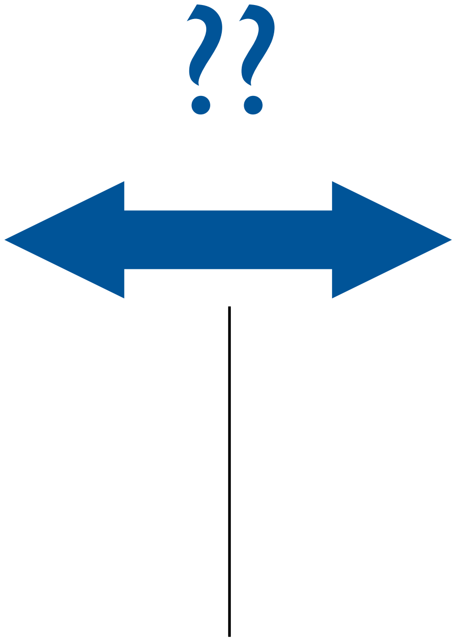
Real data

Taming reality

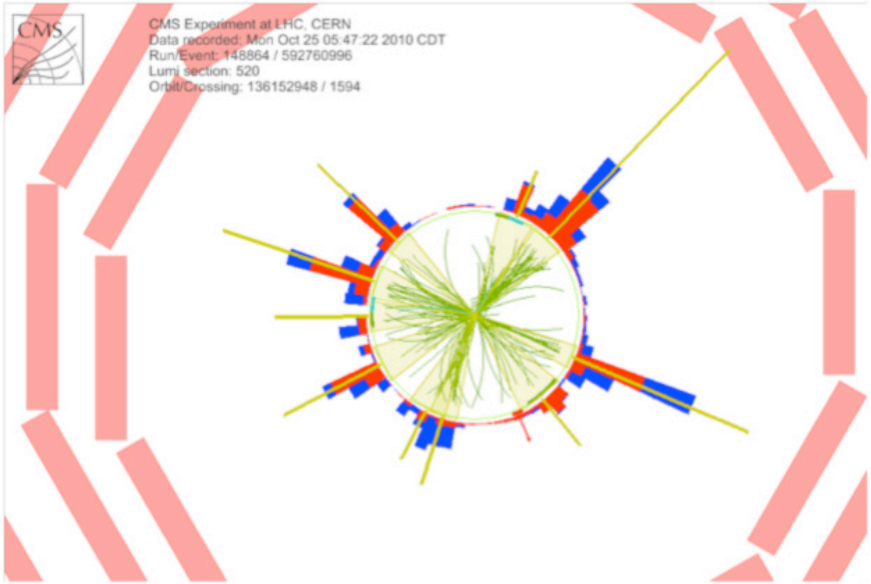
Multileg + PS



QCD predictions



Jets



Real data

One purpose of a ‘jet clustering’ algorithm is to **reduce the complexity** of the final state, simplifying many hadrons to **simpler objects** that one can hope to **calculate**

Jets in physics

- While we could take almost any clustering algorithm and, with a reasonable distance, use it to construct jets, i.e. clusters of hadrons, the result may not be particularly useful.
  We must also be guided by physics, so that
  - *the procedure leads to calculable results → infrared and collinear safety*
  - *the results are robust with respect to dynamics that we cannot calculate in detail → resiliency to hadronisation effects*

This puts strong constraints on the distances and algorithms that we can use

IRC safety

An observable is **infrared and collinear safe** if, in the limit of a **collinear splitting**, or the **emission of an infinitely soft** particle, the observable remains **unchanged:**

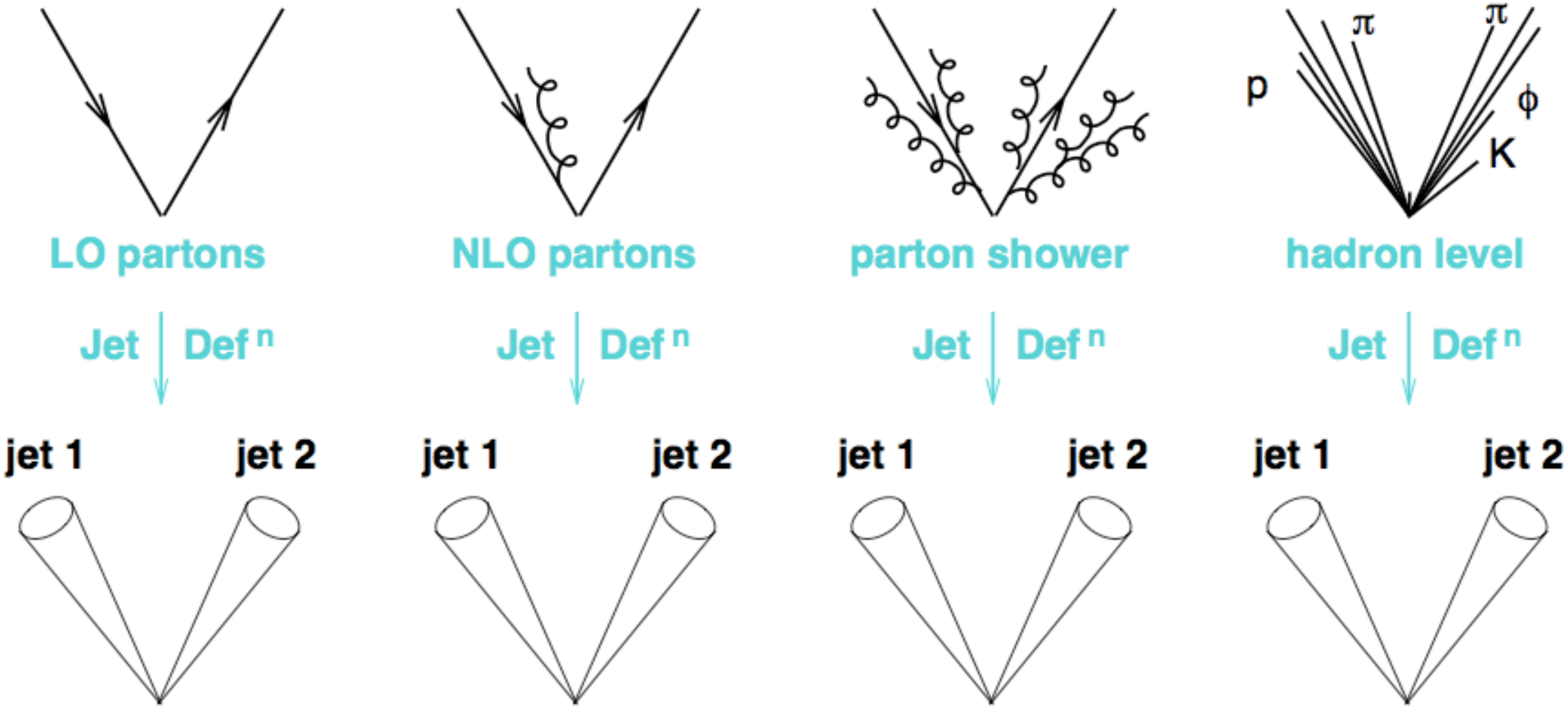
$$O(X;p_1,\ldots,p_n,p_{n+1}\to 0)\to O(X;p_1,\ldots,p_n)$$

$$O(X;p_1,\ldots,p_n\parallel p_{n+1})\to O(X;p_1,\ldots,p_n+p_{n+1})$$

If we wish to be able to calculate a jet rate in perturbative QCD the jet algorithm that we use must be IRC safe:
soft emissions and collinear splittings must not change the hard jets

Jets as proxies

A good jet definition should be resilient to QCD effects



NB. ‘Resiliency’ does not mean ‘total insensitivity’

A ‘hadron jet’ is **not** a parton

Most definitions will give very similar results (especially for inclusive observables), but it is important to be aware of potential differences, and not to compare apples with oranges.

Jets can serve two purposes

- They can be ***observables***, that one can measure and calculate
- They can be ***tools***, that one can employ to extract specific properties of the final state

Jet Definition

A **jet algorithm**

$$\{p_i\} \rightarrow \{j_k\}$$

particles,
calo cells, ...

jets

+

its **parameters** (e.g. R)

+

a **recombination scheme**

=

a **Jet Definition**

Two main classes of jet algorithms

- **Sequential recombination algorithms**

  Bottom-up approach: combine particles starting from **closest ones**

  How? Choose a **distance measure**, iterate recombination until few objects left, call them jets

  Works because of mapping closeness ⇔ QCD divergence

  Examples: Jade, k_t, Cambridge/Aachen, anti-k_t, …..

- **Cone algorithms**

  Top-down approach: find coarse regions of energy flow.

  How? Find **stable cones** (i.e. their axis coincides with sum of momenta of particles in it)

  Works because QCD only modifies energy flow on small scales

  Examples: JetClu, MidPoint, ATLAS cone, CMS cone, SISCone…...

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→ hierarchical clustering

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→ partitional clustering

Recombination algorithms

- First introduced in e^+e^- collisions in the ’80s
- Typically they work by calculating a **‘distance’** between particles, and then recombine them pairwise according to a given order, until some condition is met (e.g. no particles are left, or the distance crosses a given threshold)

IRC safety can usually be seen to be trivially guaranteed

JADE algorithm

distance:

$$y_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{Q^2}$$

- Find the minimum y_{min} of all y_{ij}
- If y_{min} is below some jet resolution threshold y_{cut}, recombine i and j into a single new particle ('pseudojet'), and repeat
- If no $y_{min} < y_{cut}$ are left, all remaining particles are jets

Problem of this particular algorithm:
two soft particles emitted at large angle get easily recombined into a single jet:
counterintuitive and perturbatively troublesome

e^+e^- k_t (Durham) algorithm

[Catani, Dokshitzer, Olsson, Turnock, Webber '91]

Identical to JADE,
but with distance:

$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$$

In the collinear limit, the numerator reduces to the **relative transverse momentum** (squared) of the two particles, hence the name of the algorithm

The use of the min() avoids the problem of recombination of back-to-back particles present in JADE: a soft and a hard particle close in angle are 'closer' than two soft ones at large angle

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One key feature of the k_t algorithm is its relation to the structure of QCD divergences:

$$\frac{dP_{k\to ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j)\theta_{ij}}$$

The k_t algorithm inverts the QCD branching sequence (the pair which is recombined first is the one with the largest probability to have branched)

k_t algorithm in hadron collisions

(Inclusive and longitudinally invariant version)

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2} \qquad d_{iB} = p_{ti}^2$$

- Calculate the distances between the particles: $\mathbf{d_{ij}}$
- Calculate the beam distances: $\mathbf{d_{iB}}$
- Combine particles with **smallest distance** d_{ij} or, if d_{iB} is smallest, call it a jet
- Find again smallest distance and repeat procedure until no particles are left (this stopping criterion leads to the *inclusive* version of the k_t algorithm)

- Given N particles this is, naively, an $O(N^3)$ algorithm: calculate N^2 distances, repeat for all N iterations. 1 second to cluster 1000 particles: too slow for practical use.
- An NlnN implementation exists: 1ms for 1000 particles. Can even use it in the trigger.

The k_t algorithm and its siblings

One can generalise the k_t distance measure:

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta y^2 + \Delta\phi^2}{R^2} \qquad d_{iB} = k_{ti}^{2p}$$

p = 1 k_t algorithm

S. Catani, Y. Dokshitzer, M. Seymour and B. Webber, Nucl. Phys. B406 (1993) 187
S.D. Ellis and D.E. Soper, Phys. Rev. D48 (1993) 3160

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M. Wobisch and T. Wengler, hep-ph/9907280

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Y. Dokshitzer, G. Leder, S.Moretti and B. Webber, JHEP 08 (1997) 001
M.Wobisch and T.Wengler, hep-ph/9907280

p = -1 **anti-k_t algorithm**

MC, G. Salam and G. Soyez, arXiv:0802.1189

NB: in anti-kt pairs with a **hard** particle will cluster first: if no other hard particles are close by, the algorithm will give **perfect cones**

Quite ironically, a sequential recombination algorithm is the 'perfect' cone algorithm

IRC safe algorithms

k_t	SR $d_{ij} = \min(k_{ti}^2, k_{tj}^2)\Delta R_{ij}^2/R^2$ hierarchical in rel p_t	Catani et al '91 Ellis, Soper '93	NlnN
Cambridge/ Aachen	SR $d_{ij} = \Delta R_{ij}^2/R^2$ hierarchical in angle	Dokshitzer et al '97 Wengler, Wobish '98	NlnN
anti-k_t	SR $d_{ij} = \min(k_{ti}^{-2}, k_{tj}^{-2})\Delta R_{ij}^2/R^2$ gives perfectly conical hard jets	MC, Salam, Soyez '08 (Delsart, Loch)	$N^{3/2}$
SISCone	Seedless iterative cone with split-merge gives 'economical' jets	Salam, Soyez '07	$N^2 lnN$

'second-generation' algorithms

All are available in FastJet, http://fastjet.fr

(As well as many IRC unsafe ones)

Summary

- Impressive progress in calculations, tools and ingredients has come together to allow seamless and accurate simulation of very complex processes
- In these lecture I only scratched the surface, and left out huge parts altogether:
  - *NNLO calculations*
  - *Improvements to parton shower in event generators*
  - *Improvements to description of Underlying Event*
  - *Jet substructure techniques for boosted particles*
  - .....
- All this (and further improvements) will hopefully pay off even more in LHC searches for new (unexpected?) physics

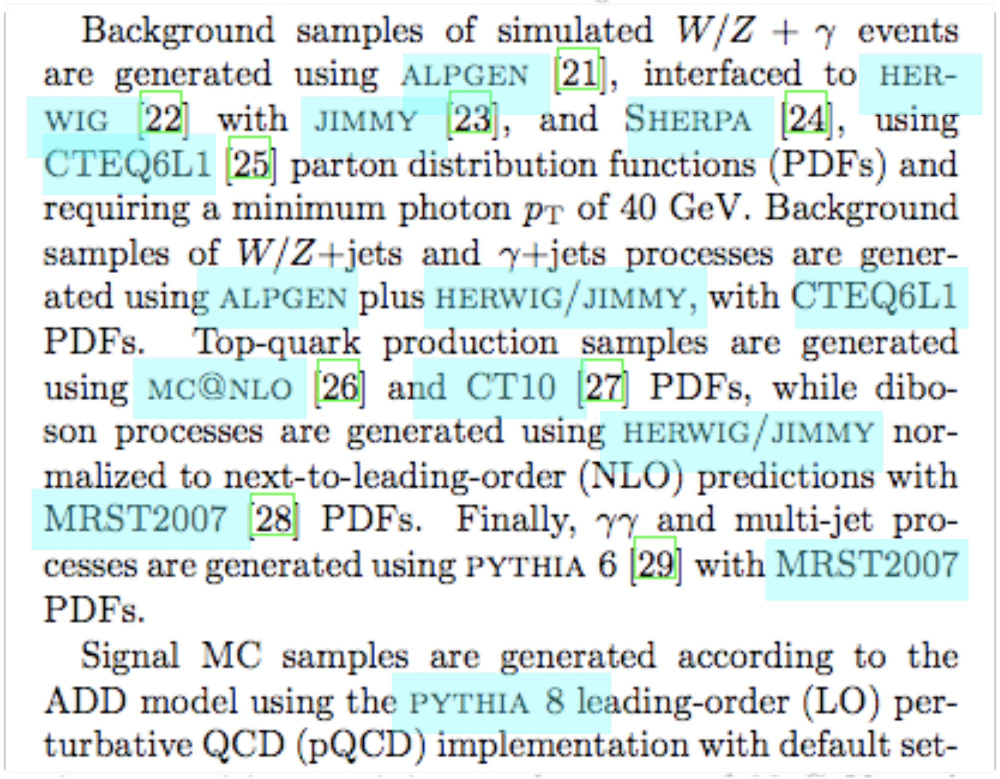
The pervasiveness of tools

Search for exotic physics...

Search for dark matter candidates and large extra dimensions in events with a photon and missing transverse momentum in pp collision data at $\sqrt{s} = 7$ TeV with the ATLAS detector

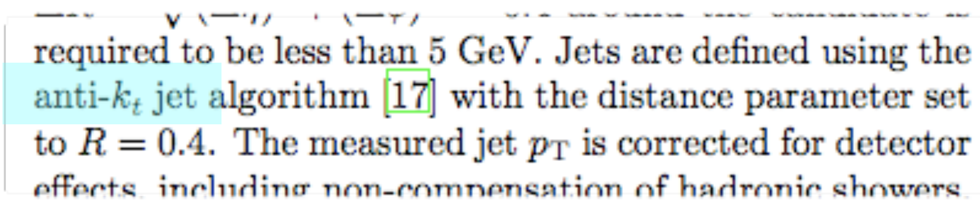
arXiv:1209.4625

Today!

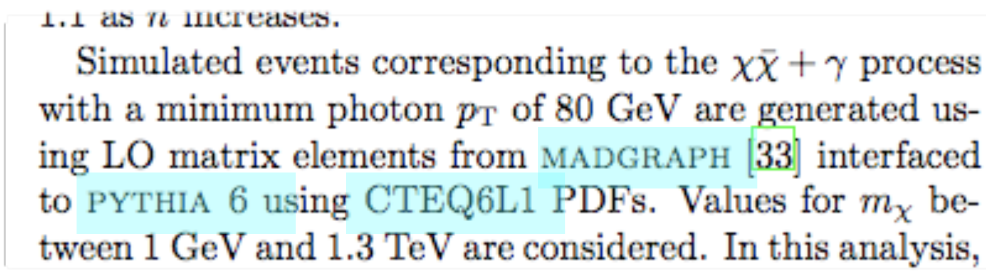


Background samples of simulated $W/Z + \gamma$ events are generated using ALPGEN [21], interfaced to HERWIG [22] with JIMMY [23], and SHERPA [24], using CTEQ6L1 [25] parton distribution functions (PDFs) and requiring a minimum photon p_T of 40 GeV. Background samples of W/Z+jets and γ+jets processes are generated using ALPGEN plus HERWIG/JIMMY, with CTEQ6L1 PDFs. Top-quark production samples are generated using MC@NLO [26] and CT10 [27] PDFs, while diboson processes are generated using HERWIG/JIMMY normalized to next-to-leading-order (NLO) predictions with MRST2007 [28] PDFs. Finally, $\gamma\gamma$ and multi-jet processes are generated using PYTHIA 6 [29] with MRST2007 PDFs.

Signal MC samples are generated according to the ADD model using the PYTHIA 8 leading-order (LO) perturbative QCD (pQCD) implementation with default set-



required to be less than 5 GeV. Jets are defined using the anti-k_t jet algorithm [17] with the distance parameter set to $R = 0.4$. The measured jet p_T is corrected for detector effects, including non-compensation of hadronic showers



1.1 as n increases.

Simulated events corresponding to the $\chi\bar{\chi} + \gamma$ process with a minimum photon p_T of 80 GeV are generated using LO matrix elements from MADGRAPH [33] interfaced to PYTHIA 6 using CTEQ6L1 PDFs. Values for m_χ between 1 GeV and 1.3 TeV are considered. In this analysis,

... with extensive use of very unexotic (QCD) tools