Semi-leptonic τ / D $\rightarrow$ S-wave or radial excitations

Separate S-wave
and radial excitations

Resonance shape
(BW, GS,
Blatt-Weisskopf,…)

November 27, 2012
P. Roudeau

Radial excitations ($J^P=1^-$)

State	decay
ρ(1450) **M=1465 ± 25 MeV** **Γ=400 ± 60 MeV**	ππ **4**π ππ**/4** π **= 0.37 ± 0.10**
K*(1410) **M=1414 ± 15 MeV** **Γ=232 ± 21 MeV**	**K**π **(6.6±1.3)**%

PDG, educated guesses

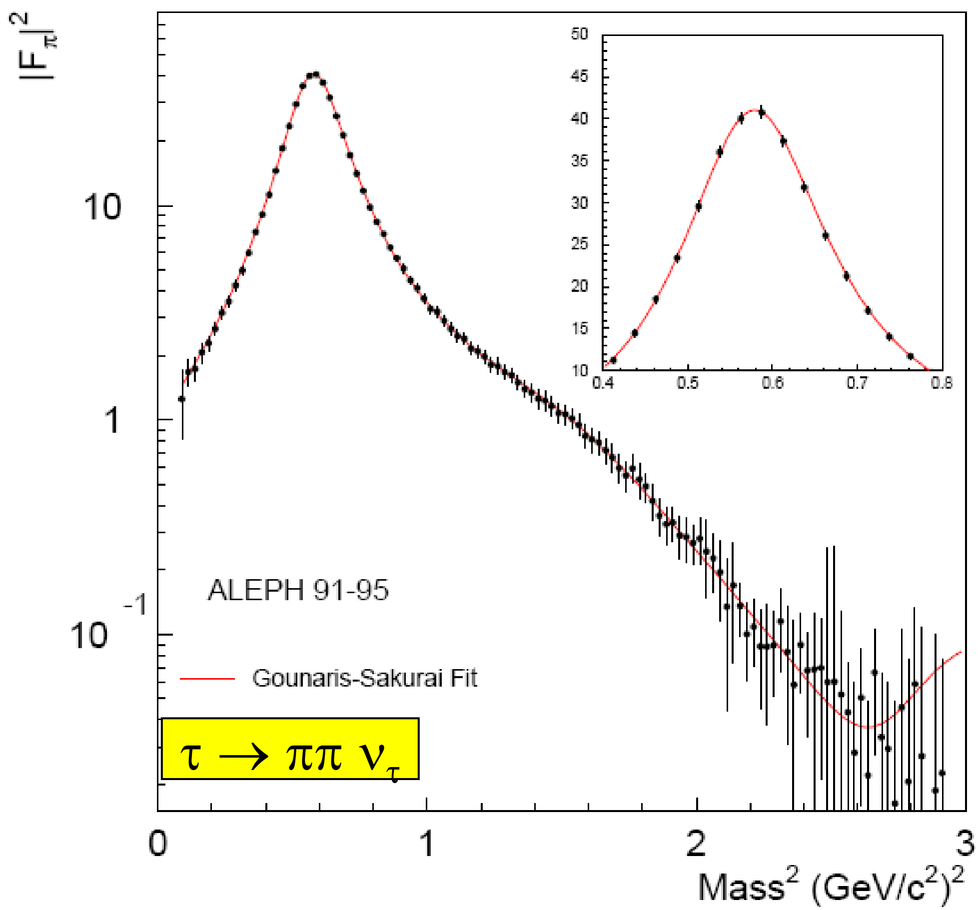
LASS (1988)

Properties of these states are not well established

$\tau \rightarrow \pi\pi\, \nu_\tau$ ... back to LEP ! (BF~ 25 %)

$$F_\pi^{I=1,0}(s) = \frac{\mathrm{BW}_{\rho(770)}(s)\,\frac{1+\delta\,\mathrm{BW}_{\omega(783)}(s)}{1+\delta} + \beta\,\mathrm{BW}_{\rho(1450)}(s) + \gamma\,\mathrm{BW}_{\rho(1700)}(s)}{1+\beta+\gamma}$$

$$\mathrm{BW}^{\mathrm{GS}}_{\rho(m_\rho)}(s) = \frac{m_\rho^2(1+d\cdot\Gamma_\rho/m_\rho)}{m_\rho^2 - s + f(s) - i\sqrt{s}\,\Gamma_\rho(s)}$$



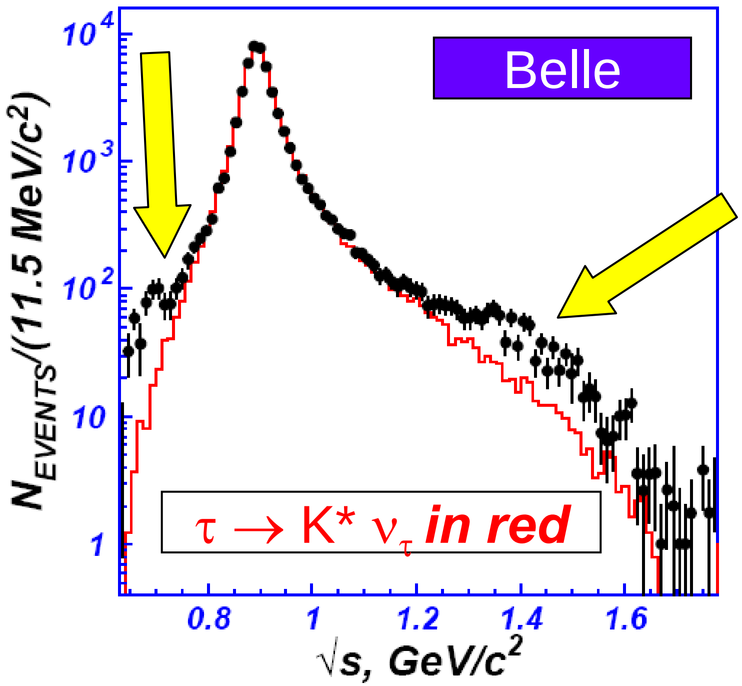
$m_{\rho^-(770)}$	775.5 ± 0.6
$m_{\rho^0(770)}$	773.1 ± 0.5
$\Gamma_{\rho^-(770)}$	148.2 ± 0.8
$\Gamma_{\rho^0(770)}$	148.0 ± 0.9
$\alpha_{\rho\omega}$	$(2.03 \pm 0.10)\ 10^{-3}$
ϕ_α	(13.0 ± 2.3)
β	0.166 ± 0.005
ϕ_β	177.8 ± 5.2
$m_{\rho(1450)}$	1409 ± 12
$\Gamma_{\rho(1450)}$	501 ± 37
γ	0.071 ± 0.006
ϕ_γ	[0]
$m_{\rho(1700)}$	1740 ± 20
$\Gamma_{\rho(1700)}$	[235]
χ^2/DF	383/326

- Related to $e^+e^- \rightarrow \pi^+\pi^-$

- QM: β, γ real neg. and pos. respectively

-BW(0)=1, no Blatt-Weisskopf factors

$\tau^- \to K^0\pi^- \nu_\tau$ … b-factories (BF~ 1 %)



$$F_V = \frac{1}{1+\beta+\chi}\left[BW_{K^*(892)}(s) + \beta BW_{K^*(1410)}(s) + \chi BW_{K^*(1680)}(s)\right]$$

$$F_S = \varkappa \frac{s}{M^2_{K_0^*(800)}} BW_{K_0^*(800)}(s) + \gamma \frac{s}{M^2_{K_0^*(1430)}} BW_{K_0^*(1430)}(s)$$

$$BW_R(s) = \frac{M_R^2}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)} \qquad \Gamma_{K^*}(s) = \Gamma_{K^*}\frac{s}{M^2_{K^*}}\frac{\sigma^3_{K\pi}(s)}{\sigma^3_{K\pi}(M^2_{K^*})}$$

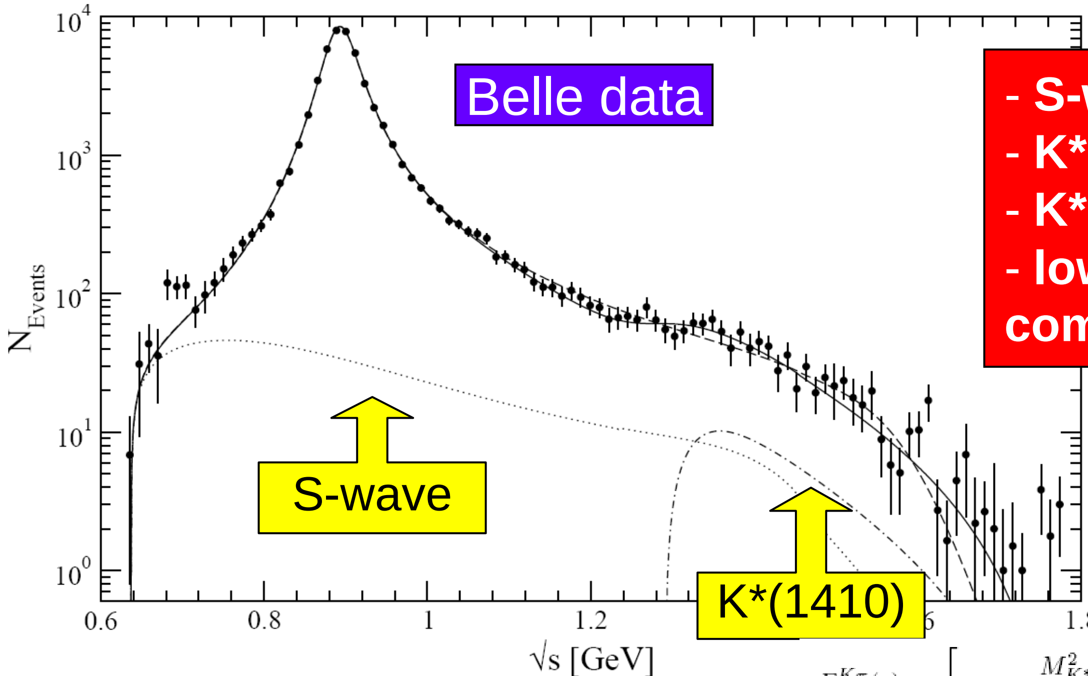
	$K^*(892)$	$K_0^*(800) + K^*(892) +$ $+K^*(1410)$
$M_{K^*(892)^-}$, MeV/c^2	895.53 ± 0.19	895.47 ± 0.20
$\Gamma_{K^*(892)^-}$, MeV	49.29 ± 0.46	46.19 ± 0.57
$\lvert\beta\rvert$		0.075 ± 0.006
$\arg(\beta)$		1.44 ± 0.15
$\lvert\chi\rvert$		
$\arg(\chi)$		
$\varkappa$		1.57 ± 0.23
χ^2/n.d.f.	448.4/87	90.2/84
$P(\chi^2)$, %	0	30

	$K_0^*(800) + K^*(892) + K_0^*(1430)$	
	solution 1	solution 2
$M_{K^*(892)^-}$, MeV/c^2	895.42 ± 0.19	895.50 ± 0.22
$\Gamma_{K^*(892)^-}$, MeV	46.14 ± 0.55	46.20 ± 0.69
$\lvert\gamma\rvert$	0.954 ± 0.081	1.92 ± 0.20
$\arg(\gamma)$	0.62 ± 0.34	4.03 ± 0.09
$\varkappa$	1.27 ± 0.22	2.28 ± 0.47
χ^2/n.d.f.	86.5/84	95.1/84
$P(\chi^2)$, %	41	19

- S-wave + (?) K*(1410) needed
- no angular analysis yet , unfortunate to separate J=0, 1 components.
- fits are questionable

$\tau^- \to K^0 \pi^- \nu_\tau$ … analyzed by theorists

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us}|^2 M_\tau^3}{32\pi^3 s} S_{\rm EW} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) q_{K\pi}^3 |F_+^{K\pi}(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi} |F_0^{K\pi}(s)|^2\right]$$



- S-wave from theory
- K*(1410) needed
- K*(1410) rate is model dependent
- low mass S-wave Kπ is large compared with $K^*_0(1430)$

$$F_+^{K\pi}(s) = \frac{F_+^{K\pi}(0)}{1+\beta}\left[BW_{K^*}(s) + \beta\, BW_{K^{*\prime}}(s)\right]$$

$$F_+^{K\pi}(s) = \left[\frac{M_{K^*}^2 + \gamma s}{M_{K^*}^2 - s - i M_{K^*}\Gamma_{K^*}(s)} - \frac{\gamma s}{M_{K^{*\prime}}^2 - s - i M_{K^{*\prime}}\Gamma_{K^{*\prime}}(s)}\right] e^{\frac{3}{2}{\rm Re}[\tilde H_{K\pi}(s) + \tilde H_{K\eta}(s)]}$$

	dashed line	full line
	BW form for $F_+^{K\pi}(s)$	Chiral form for $F_+^{K\pi}(s)$
$\bar B_{K\pi}$ ($B_{K\pi}$)	$0.423 \pm 0.012\,\%$ ($0.421\,\%$)	$0.430 \pm 0.011\,\%$ ($0.427\,\%$)
M_{K^*}	895.12 ± 0.19 MeV	895.28 ± 0.20 MeV
Γ_{K^*}	46.79 ± 0.41 MeV	47.50 ± 0.41 MeV
$M_{K^{*\prime}}$	1598 ± 25 MeV	1307 ± 17 MeV
$\Gamma_{K^{*\prime}}$	224 ± 47 MeV	206 ± 49 MeV
β, γ	-0.079 ± 0.010	-0.043 ± 0.010
χ^2/n.d.f.	$88.7/81$	$79.5/81$

PDG 2012 (LASS 1988)

1414 ± 15

232 ± 21

M. Jamin, A. Pich and J. Portolés, arXiv:0803.1786

Conclusions/remarks on τ decays

$$F_+^{K\pi}(s) = \frac{F_+^{K\pi}(0)}{1+\beta}\left[BW_{K^*}(s) + \beta\, BW_{K^{*\prime}}(s)\right]$$

β includes the difference between $B(K^* \to K\pi) \sim 1$ and $B(K^{*\prime} \to K\pi) = (6.6 \pm 1.3)\%$
or $B(\rho \to \pi\pi) \sim 1$ and $B(\rho' \to \pi\pi) = (37 \pm 10)\%$(?)

Consider $\beta_V = \beta \times \sqrt{B(V \to \mathbf{p1\ p2}) / B(V' \to \mathbf{p1\ p2})}$

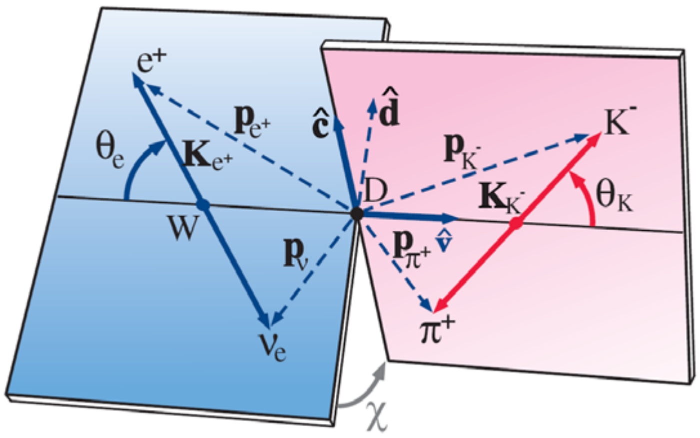
- β_ρ = 0.27+0.05-0.03
- β_{K^*} = 0.31+0.04-0.03

$B(\tau \to K^*(892)\,\nu_\tau) = 1.20 \pm 0.07\ \%$
$B(\tau \to K^*(1410)\,\nu_\tau) = 0.15\ {+0.14}\ {-0.10}\ \%$

Aleph

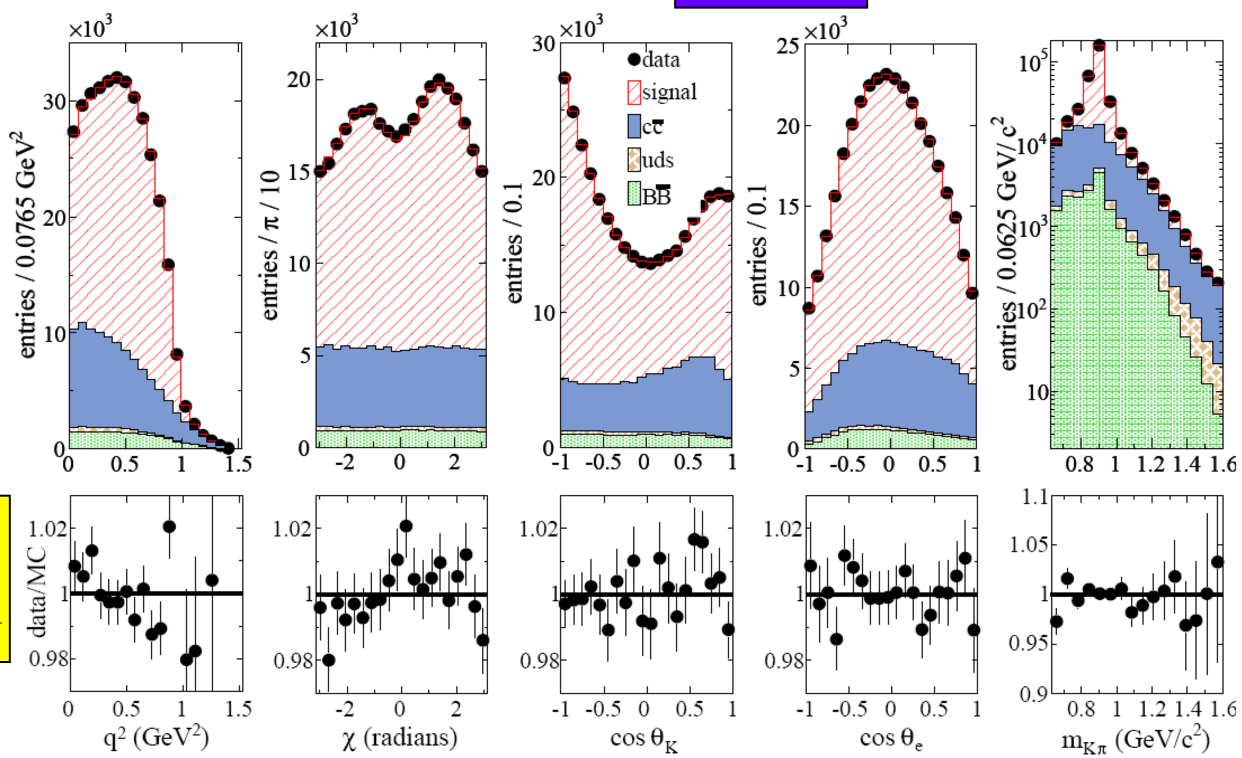
Large statistics
at b-factories remain
to be analyzed.

$D^+ \to K^-\pi^+ e^+\nu_e$



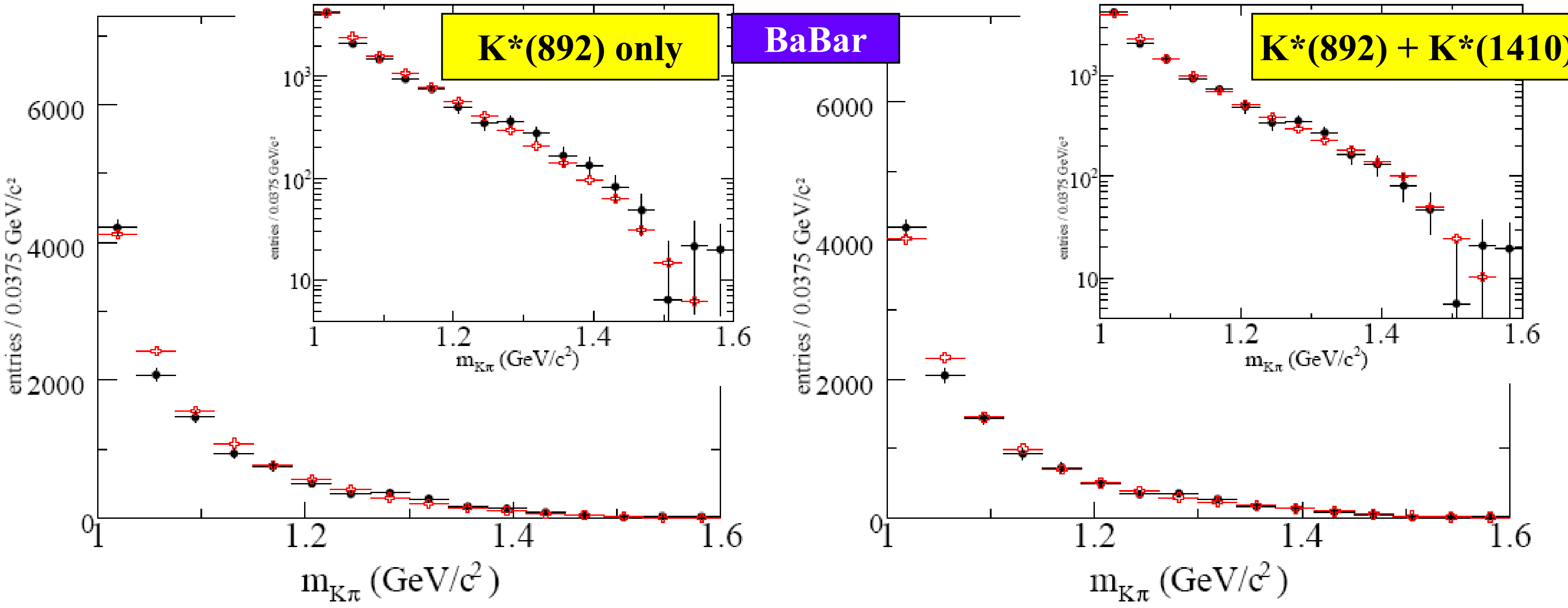
- analysis in 5D
(some projections below)

BaBar



- large statistics
- non-negligible background

$D^+ \to K^- \pi^+ e^+ \nu_e$



$$\mathcal{A}_{K^*(892)} = \frac{m_{K^*(892)} \Gamma^0_{K^*(892)} F_1(m)}{m^2_{K^*(892)} - m^2 - i m_{K^*(892)} \Gamma_{K^*(892)}(m)}$$

$$F_1(m) = \frac{p^*}{p^*_0} \frac{B(p^*)}{B(p^*_0)} \qquad B = 1/\sqrt{1 + r^2_{BW} p^{*2}}$$

- analysis in 5D

$D^+ \to K^- \pi^+ e^+ \nu_e$

BaBar

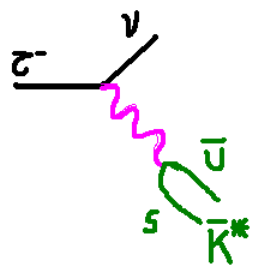
variable	$S+\overline{K}^*(892)^0$	$S+\overline{K}^*(892)^0$ $\overline{K}^*(1410)^0$
$m_{K^*(892)}$ (MeV/c^2)	894.77 ± 0.08	895.43 ± 0.21
$\Gamma^0_{K^*(892)}$ (MeV/c^2)	45.78 ± 0.23	46.48 ± 0.31
r_{BW} (GeV/c)$^{-1}$	3.71 ± 0.22	2.13 ± 0.48
m_A (GeV/c^2)	2.65 ± 0.10	2.63 ± 0.10
r_V	1.458 ± 0.016	1.463 ± 0.017
r_2	0.804 ± 0.020	0.801 ± 0.020
r_S (GeV)$^{-1}$	-0.470 ± 0.032	-0.497 ± 0.029
$r_S^{(1)}$	0.17 ± 0.08	0.14 ± 0.06
$a^{1/2}_{S,BG}$ (GeV/c)$^{-1}$	1.82 ± 0.14	2.18 ± 0.14
$b^{1/2}_{S,BG}$ (GeV/c)$^{-1}$	-1.66 ± 0.65	1.76 fixed
$r_{K^*(1410)^0}$		0.074 ± 0.016
$\delta_{K^*(1410)^0}$ (degree)		8.3 ± 13.0
r_D (GeV)$^{-1}$		
δ_D (degree)		
N_{sig}	243850 ± 699	243219 ± 713
N_{bkg}	107370 ± 593	108001 ± 613
Fit probability	4.6%	6.4%

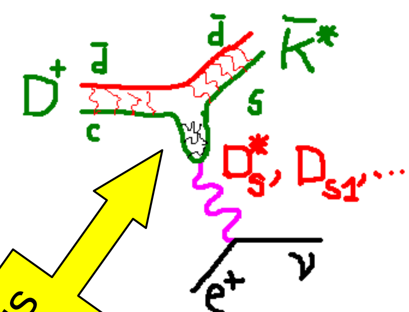
$\beta = r\, m_{K^*} / m_{K^{*\prime}}\, \Gamma_{K^{*\prime}} / \Gamma_{K^*}$

$-\beta = 0.23 \pm 0.05$(stat.)± 0.07(syst.) higher (?) than in τ (0.08 ± 0.01).

-phase ~ 0, not 180°
(consistent with LASS)

τ and D s.l. decays are different





couplings $g(D_s{}^*DK^*)$

Estimate of radial excit. prod. In B s.l. decays

$B(D^+ \to \overline{K^*}(892)\, e^+\nu_e) = 5.52 \pm 0.15\ \%$
$B(D^+ \to \overline{K^*}(1410)\, e^+\nu_e) = 0.3 \pm 0.2\ \%$
$B(D^+ \to \overline{K_1}(1270)\, e^+\nu_e) = 0.2 \pm 0.1\ \%$

Using $\beta_V = 0.3$ one may estimate (?) a 10% D*' production rate relative to the D*.

But …..

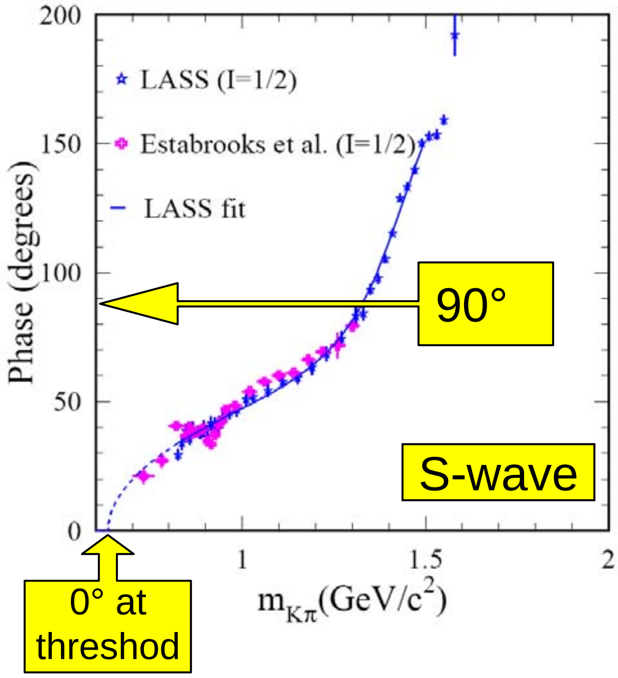
- how couplings $g(D_s^* D K^{*(')})$ compare with $g(B_c^* B D^{*(')})$
- in the $D^{(*)}\pi$ channels one needs also the values of $B(\ D^{(*)'} \to D^{(*)}\pi\ )$.

S-wave

State	decay
K_0*(800) (?) **M=682 ± 29 MeV** **Γ=547 ± 24 MeV**	**Kπ (?)**
K_0*(1430) **M=1425 ± 50 MeV** **Γ=270 ± 80 MeV**	**Kπ (93±10)%**

PDG

LASS (1988)



- LASS expt. K p → Kπ N

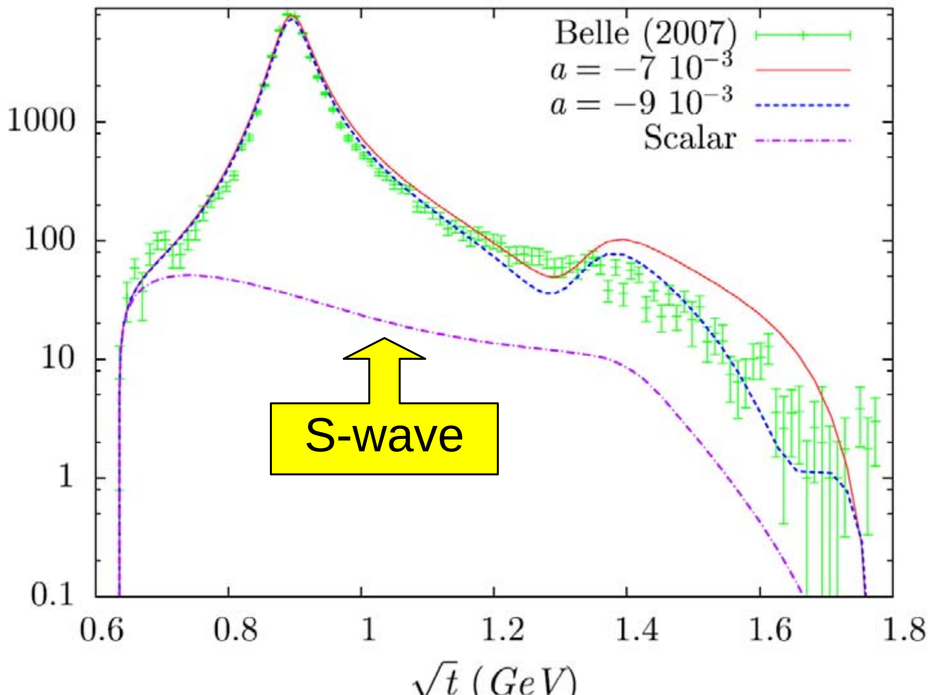
provides constraints on the phase

S-wave in τ decays

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2|V_{us}|^2 M_\tau^3}{32\pi^3 s} S_{\rm EW}\left(1-\frac{s}{M_\tau^2}\right)^2\left[\left(1+2\frac{s}{M_\tau^2}\right)q_{K\pi}^3\,|F_+^{K\pi}(s)|^2+\frac{3\Delta_{K\pi}^2}{4s}\,q_{K\pi}|F_0^{K\pi}(s)|^2\right]$$

Depends on SU(3) violation (no contribution in the ππ channel).

$$\Delta_{K\pi} \equiv M_K^2 - M_\pi^2$$



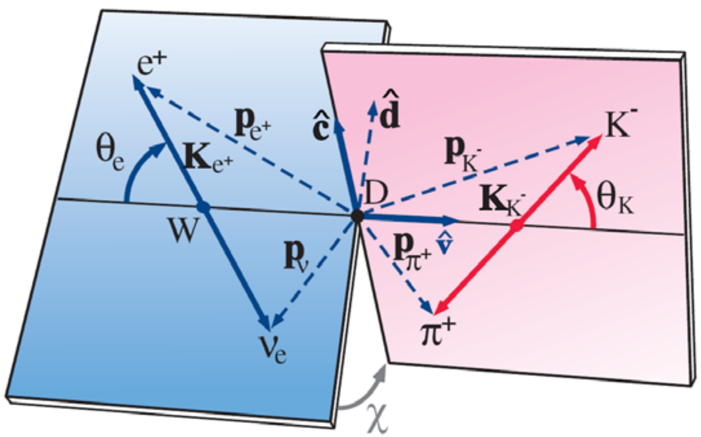
B. Moussallam,
EPJ, C53 (2008)

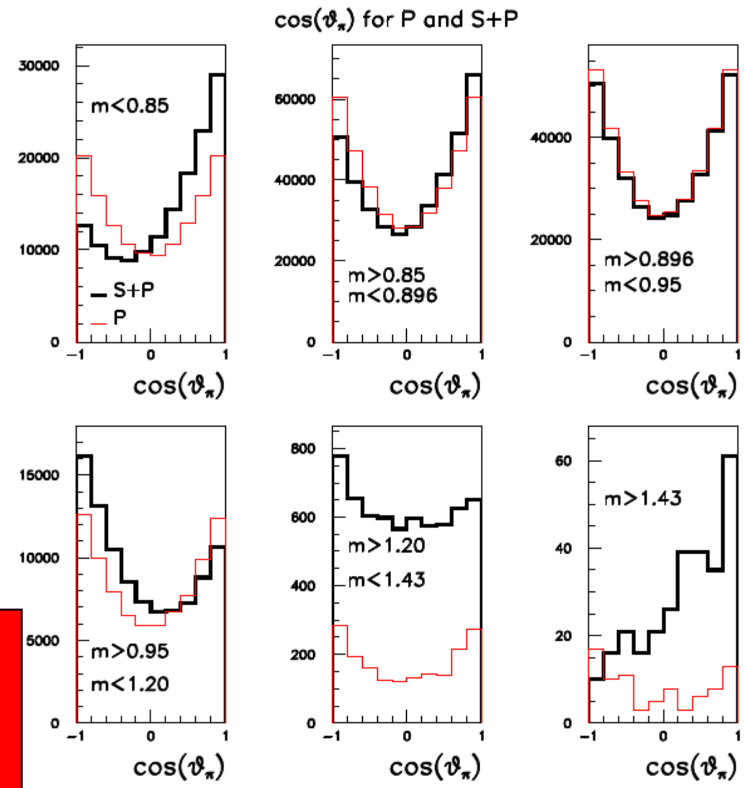
V. Bernard, D.R. Boito and E. Passemar,
arXiv:1103.4855

- missing angular analysis to have an accurate determination of the S-wave contribution. Component « fixed » from theory.
- the K^*_0(1430) is not dominant, « large » effect close to threshold.

S-wave in $D \to K\pi l \nu_l$ decays

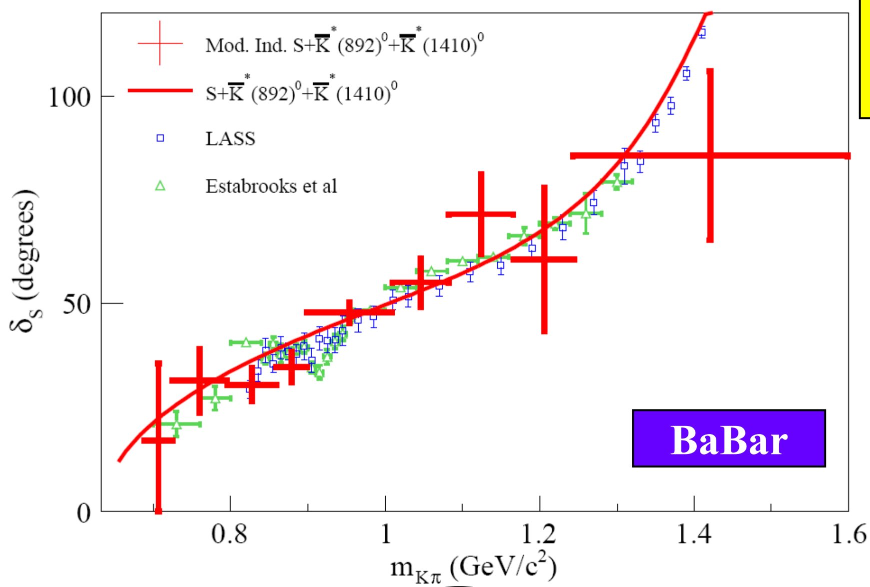
Observed first by FOCUS in $D^+ \to K\pi\, \mu^+ \nu_\mu$, accurately measured by BaBar.





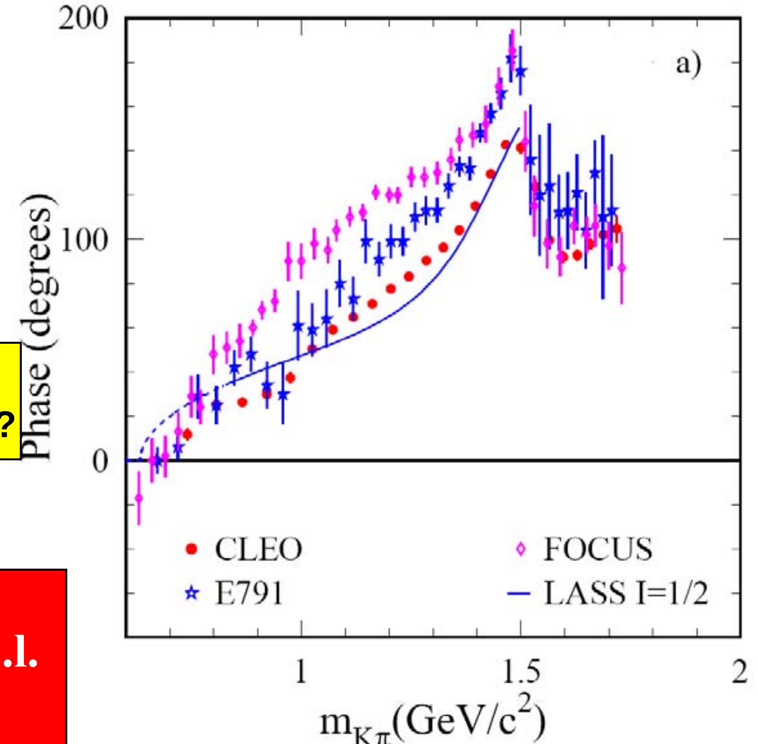
-analysis in 5D allows to separate the different angular momentum components

S-wave phase in $D \to K\pi\, l\, \nu_l$ decays



S-wave phase from $D^+ \to K^-\pi^+ e^+\nu_e$

S-wave phase from $D^+ \to K^-\pi^+ \pi^+$



$$A(s,t) = \sum_{L=0}^{L_{\max}} \Omega_L(s,t)\,\mathcal{F}_D^L(q(s))\,\mathcal{A}_L(s)$$

$$W_R = c_R \mathcal{W}_R \mathcal{F}_R^L(r_R P$$

Blatt-Weisskopf factors ?
Why neglected for S-waves?

Watson theorem: below the inelastic threshold, same hadronic phase in $K\pi$ elastic scatt. and in s.l. decays (for amplitudes of same I and L).

NO CONSTRAINT on the amplitude magnitude.

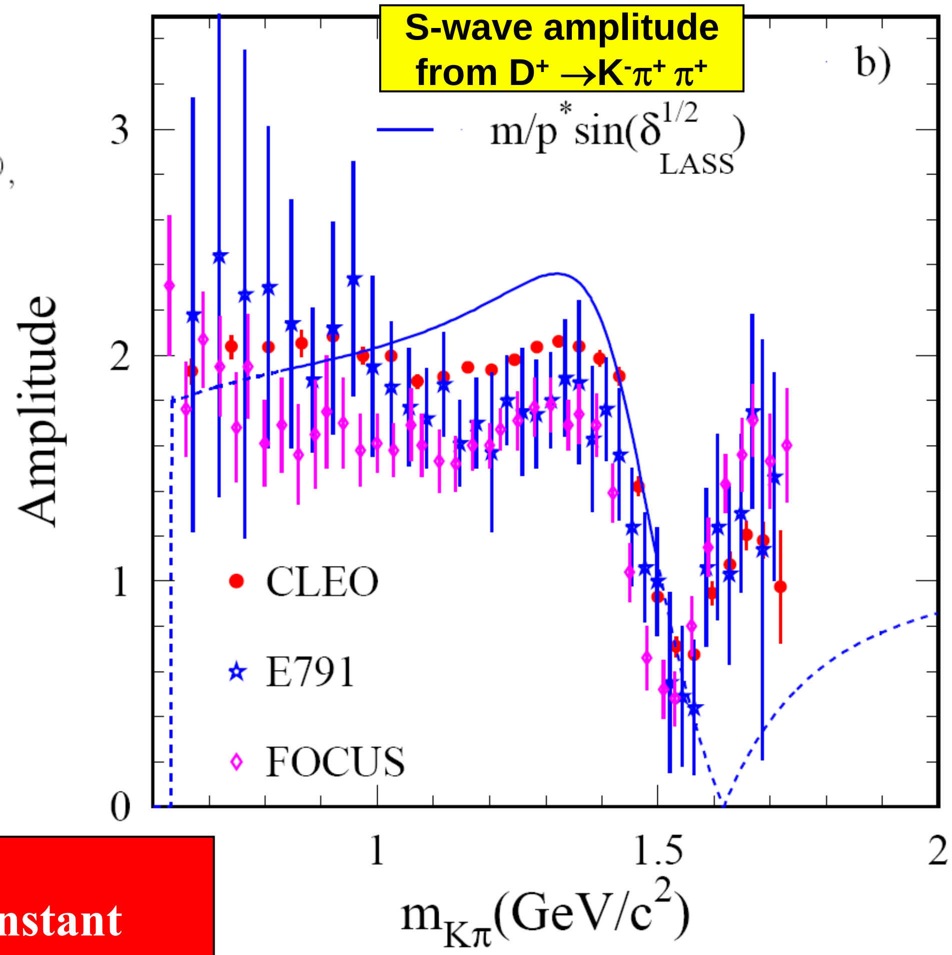
S-wave amplitude in $D \to K\pi l \nu_l$ decays

$$\mathcal{A}_S = r_S P(m) e^{i\delta_S(m)}, \quad m < 1.4 \text{ GeV}$$

$$\mathcal{A}_S = r_S P(m_{K_0^*(1430)}) \times \sqrt{\frac{(m_{K_0^*(1430)}\Gamma_{K_0^*(1430)})^2}{(m^2_{K_0^*(1430)} - m^2)^2 + (m_{K_0^*(1430)}\Gamma_{K_0^*(1430)})^2}}\, e^{i\delta_S(m)}, \quad m > 1.4 \text{ GeV}$$

$$P(m) = 1 + r_S^{(1)} \times x$$

$$x = \sqrt{\left(\frac{m}{m_K + m_\pi}\right)^2 - 1}$$



The S-wave amplitude is essentially a constant within the physical region.

Quite different from the $K_0^*(1430)$ Breit-Wigner.

$D^+ \to K^-\pi^+ e^+\nu_e$

BaBar

variable	$S + \overline{K}^*(892)^0$	$S + \overline{K}^*(892)^0$ $\overline{K}^*(1410)^0$
$m_{K^*(892)}$ (MeV/c^2)	894.77 ± 0.08	895.43 ± 0.21
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r_{BW} (GeV/c)$^{-1}$	3.71 ± 0.22	2.13 ± 0.48
m_A (GeV/c^2)	2.65 ± 0.10	2.63 ± 0.10
r_V	1.458 ± 0.016	1.463 ± 0.017
r_2	0.804 ± 0.020	0.801 ± 0.020
r_S (GeV)$^{-1}$	-0.470 ± 0.032	-0.497 ± 0.029
$r_S^{(1)}$	0.17 ± 0.08	0.14 ± 0.06

$K^{*0}(892)$ mass, width

Blatt-Weisskopf factors

	BaBar	PDG
$\mathcal{B}(D^+ \to K^-\pi^+e^+\nu_e)(\%)$	$4.00 \pm 0.03 \pm 0.04 \pm 0.09$	4.1 ± 0.6
$\mathcal{B}(D^+ \to K^-\pi^+e^+\nu_e)_{\overline{K}^*(892)^0}(\%)$	$3.77 \pm 0.04 \pm 0.05 \pm 0.09$	3.68 ± 0.21
$\mathcal{B}(D^+ \to K^-\pi^+e^+\nu_e)_{S-wave}(\%)$	$0.232 \pm 0.007 \pm 0.007 \pm 0.005$	0.21 ± 0.06

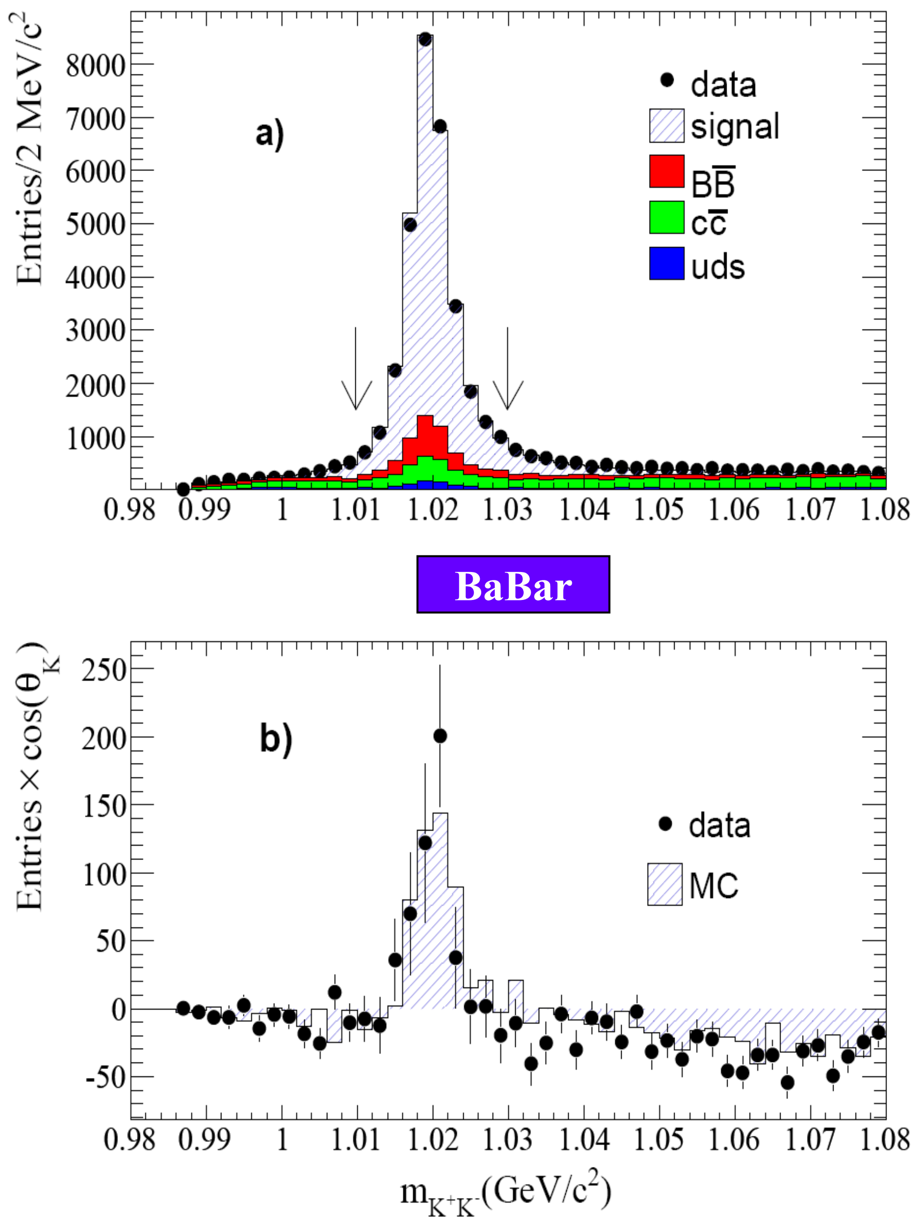
The S-wave component is about 5% of the Kpi final state.

It is dominated by low mass events and not by the $K^*_0(1430)$ (!).
It is almost impossible to separate the $K^*_0(1430)$ contribution.

$D_s^+ \to K^-K^+ e^+\nu_e$

- First measurement of the S-wave in BaBar.

- $D_s^+ \to f_0\, e^+\nu_e$, $f_0 \to \pi^+\pi^-$ meas. in CLEO



- The S-wave component is compatible with f_0 production

-It is about 5% of the φ final state (with large uncertainties).

S-wave in charm s.l. decays (summary)

-The S-wave phase can be constrained below the inelastic threshold (mainly in the Kpi channel). At threshold, constraints from chiral theory.

-The S-wave amplitude is not a simple sum of BW distributions

- $K^*_0(1430)$ or $f_0(1500)$ are not dominant.

- What is the S-wave Dpi system? Is the $D^*_0(2300)$ equivalent to the $K^*_0(1430)$?

- What are the $c\bar{q}$ states which correspond to the $K^*_0(800)$ or $f_0(980)$?

- What constraints from unitarity, analyticity can be verified by the $D\pi$ S-wave?

- Over what range is it elastic ?

Conclusions

- Radial vector excitations in τ and D s.l. decays are at the level of 5-10% (with large uncertainties) relative to vector states.

- Properties of radial excitations in $K\pi$ and $\pi\pi$ systems are still not well established

- There are large statistics available at B-factories (τ decays) which can help to improve this situation.

- the $K\pi$ S-wave contribution in D s.l. decays is accurately measured

- what is the $D\pi$ S-wave made of ?

Backup

$\tau^- \to K^0\pi^- \nu_\tau$ … analyzed by theorists

$$F_+^{K\pi}(s) = \frac{F_+^{K\pi}(0)}{1+\beta}\left[BW_{K^*}(s) + \beta\, BW_{K^{*\prime}}(s)\right]$$

Expt. (PDG 2012):
B($\tau^- \to$ ***K***$\pi\ \nu_\tau$) = **(1.277 ± 0.036)%**
B($\tau^- \to$ ***K****$\ \nu_\tau$) = **(1.20 ± 0.07)%**

Chiral pert. Theory

The channel can be related to $K\to\pi e\nu$.

$$F^{\overline{K^0}\pi^-}(Q^2) = f(0)\left[1 + \frac{1}{6}\langle r^2\rangle_V^{K\pi} Q^2 + \cdots\right]$$

$$f(0) = f_S(0) = 1$$

$$\frac{1}{1+\beta_{K^*}}\left(\frac{1}{m_{K^*}^2} + \frac{\beta_{K^*}}{m_{K^{*\prime}}^2}\right) = \frac{\langle r^2\rangle_V^{K\pi}}{6}$$

$$\langle r^2\rangle_V^{K\pi} = 0.37\,\mathrm{fm}^2 = 9.55\,\mathrm{GeV}^{-2}$$