A strategy to compute $\bar{B} \to D^{**} \ell \, \bar{\nu}$ at finite mass on the lattice

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Introduction - Motivation

- Ultimate goal :
  decay rates of $\bar{B} \to D^{**}\ell\,\bar{\nu}$ channels in Lattice QCD with "real life" quarks

- Current goal : calculation of transition amplitudes of the type

$$\left\langle D^{**}(p_{D^{**}}\,(,\varepsilon))\,\middle|\,V_\mu\,\middle|\,\bar{B}(p_B)\right\rangle \quad \text{and} \quad \left\langle D^{**}(p_{D^{**}}\,(,\varepsilon))\,\middle|\,A_\mu\,\middle|\,\bar{B}(p_B)\right\rangle$$

Collaboration with the lattice group of the *LPSC in Grenoble* and the *LPT in Orsay*, within the *European Twisted Mass Collaboration*

Sommaire

Sommaire

Sommaire

Sommaire

"Beasties" to be delt with

D^{**} states considered

1. $\boxed{J^P = O^+ \quad \text{or} \quad {}^{2S+1}L_J = {}^3P_0}$ scalar state (D_0^*)

   (belongs to the 1/2$^+$ multiplet in the infinite mass limit)

2. $\boxed{J^P = 2^+ \quad \text{or} \quad {}^{2S+1}L_J = {}^3P_2}$ tensor state (D_2^*)

   (belongs to the 3/2$^+$ multiplet in the infinite mass limit)

Current structures

1. Axial current

$$\boxed{A^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi}$$

2. Vector current

$$\boxed{V^\mu = \bar{\psi}\gamma^\mu\psi}$$

Form factors definition

$^3P_0$ state

$$\left\langle {}^3P_0(p_{D^{**}}) \,\middle|\, V_\mu \,\middle|\, \bar{B}(p_B)\right\rangle = 0$$

$$\left\langle {}^3P_0(p_{D^{**}}) \,\middle|\, A_\mu \,\middle|\, \bar{B}(p_B)\right\rangle = \boxed{\tilde{u}_+}\,(p_B + p_{D^{**}})_\mu + \boxed{\tilde{u}_-}\,(p_B - p_{D^{**}})_\mu$$

$^3P_2$ state

$$\left\langle {}^3P_2(p_{D^{**}},\lambda) \,\middle|\, V_\mu \,\middle|\, B(p_B)\right\rangle = i\,\boxed{\tilde{h}}\,\epsilon_{\mu\rho\sigma\tau}\,\varepsilon^{\rho\alpha *}_{(p_{D^{**}},\lambda)}\,p_{B\alpha}\,(p_B + p_{D^{**}})^\sigma\,(p_B - p_{D^{**}})^\tau$$

$$\left\langle {}^3P_2(p_{D^{**}},\lambda) \,\middle|\, A_\mu \,\middle|\, B(p_B)\right\rangle = \boxed{\tilde{k}}\,\varepsilon^{*\,(p_{D^{**}},\lambda)}_{\mu\rho}\,p_B^\rho + \left(\varepsilon^{*\,(p_{D^{**}},\lambda)}_{\alpha\beta}\,p_B^\alpha p_B^\beta\right)\left[\boxed{\tilde{b}_+}\,(p_B + p_{D^{**}})_\mu + \boxed{\tilde{b}_-}\,(p_B - p_{D^{**}})_\mu\right]$$

$\Longrightarrow$ 6 form factors: $\underbrace{\tilde{u}_+,\ \tilde{u}_-}_{^3P_0}$ and $\underbrace{\tilde{h},\ \tilde{k},\ \tilde{b}_+,\ \tilde{b}_-}_{^3P_2}$

Form factors definition

Relation to the Isgur-Wise τ_j functions when $m_Q \to \infty$

- ${}^3P_0$ *state*:

$$\boxed{\tilde{u}_+ = \frac{1 - r_{D_0^*}}{\sqrt{r_{D_0^*}}}\,\tau_{1/2}} \qquad \boxed{\tilde{u}_- = -\frac{1 + r_{D_0^*}}{\sqrt{r_{D_0^*}}}\,\tau_{1/2}} \qquad (m_{D_0^*} = r_{D_0^*}\, m_B)$$

- ${}^3P_2$ *state*:

$$\boxed{\tilde{h} = \frac{\sqrt{3}}{2}\,\frac{1}{m_B^2\,\sqrt{r_{D_2^*}}}\,\tau_{3/2}} \qquad \boxed{\tilde{k} = \sqrt{3}\,\sqrt{r_{D_2^*}}\,(1+w)\,\tau_{3/2}}$$

$$\boxed{\tilde{b}_+ = -\frac{\sqrt{3}}{2}\,\frac{1}{m_B^2\,\sqrt{r_{D_2^*}}}\,\tau_{3/2}} \qquad \boxed{\tilde{b}_- = \frac{\sqrt{3}}{2}\,\frac{1}{m_B^2\,\sqrt{r_{D_2^*}}}\,\tau_{3/2}}$$

$$(m_{D_2^*} = r_{D_2^*}\, m_B \qquad \text{and} \qquad m_B\, m_{D_2^*}\, w = p_B \cdot p_{D_2^*})$$

Generalities

in order to get the decay rates, we need the form factors

BUT

in order to get the form factors, we need the transition amplitudes

SO. . .

Sommaire

Kinematics

1. D^{**} rest frame $\quad p_{D^{**}}(m_{D^{**}}, \vec{0}) \quad$ (natural units)

   $\Longrightarrow$ simplifications : e.g. spin 2 polarization tensor $\varepsilon^{\mu\nu}(\vec{0}, \lambda)$

$$\varepsilon^{\mu\nu}_{(\pm 2)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm i & 0 \\ 0 & \pm i & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \varepsilon^{\mu\nu}_{(\pm 1)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mp 1 \\ 0 & 0 & 0 & -i \\ 0 & \mp 1 & -i & 0 \end{pmatrix} \qquad \varepsilon^{\mu\nu}_{(0)} = \frac{1}{\sqrt{6}}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

2. $\bar{B}$ kinematics : particular choice $\quad p_B^{\mu} = (E_B, p, p, p)$

   $\Longrightarrow$ "simple formulæ" to extract ALL the form factors

Example : with $\mathscr{T}^A_{\mu(\lambda)} \overset{\text{def.}}{=} \langle {}^3P_2(\lambda) \,|\, A_\mu \,|\, B(p_B) \rangle$

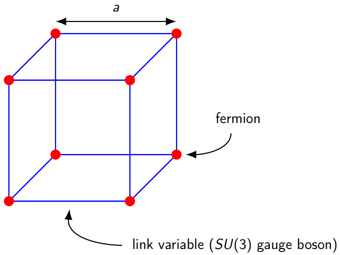
$$\boxed{\tilde{k}} = -\frac{\sqrt{6}}{p}\,\mathscr{T}^A_{1(0)} = -\frac{\sqrt{6}}{p}\,\mathscr{T}^A_{2(0)} = \frac{\sqrt{6}}{2\,p}\,\mathscr{T}^A_{3(0)} = \frac{1}{p}\left[\mathscr{T}^A_{1(+2)} + \mathscr{T}^A_{1(-2)}\right] = -\frac{1}{p}\left[\mathscr{T}^A_{2(+2)} + \mathscr{T}^A_{2(-2)}\right]$$
$$= \frac{1+i}{p}\left[i\,\mathscr{T}^A_{1(+1)} + \mathscr{T}^A_{1(-1)}\right] = -\frac{1+i}{p}\left[i\,\mathscr{T}^A_{2(+1)} + \mathscr{T}^A_{2(-1)}\right]$$

Such relations exist for the other form factors

What is the "Lattice"

Discretization

continuum of space-time coordinates in infinite volume $\Longleftrightarrow$ lattice of space-time coordinates in a finite volume



Important numbers

a : spacing of the lattice
L : spatial length
T : time length

What is the "Lattice"

By discretizing the QCD action, it is possible to :

- describe gluons : computation of *gauge configurations*
  (a configuration = a set of all the gauge links of a lattice)
- describe fermions : choice of a proper fermionic action
- compute green functions :

$$\big(\text{ green function }\big) \;=\; \left(\begin{array}{c}\text{statistical average over}\\ \text{gauge configurations}\\ \text{(canonical ensemble average)}\end{array}\right)$$

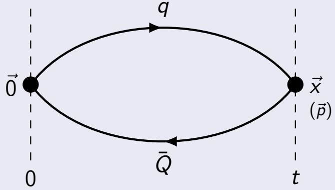
- go back to the continuum : *e.g.* limit $a \to 0$, limit $V \to \infty$

Spectroscopy of the D^{**}

Why? To calculate the mass of the $^3P_0$ and the $^3P_2$ states

How?

Consider a meson M which:



- is created at a point $(0, \vec{0})$
- propagates to the point $(t, \vec{x})$
- is destroyed at $(t, \vec{x})$

$\Longrightarrow$ two-point correlation function $\mathscr{C}_M^{(2)}(t, \vec{p})$

Spectroscopy of the D^{**}

What is $\mathscr{C}^{(2)}$?

vacuum expectation value of *interpolating fields* $\mathscr{O}$ (meson creation operator) :

$$\boxed{\begin{array}{c} \mathscr{O}(t) = \bar{\psi}_Q(t,\vec{x}_Q)\, \mathscr{P}_t(\vec{x}_Q,\vec{x}_q)\, \Gamma\, \psi_q(t,\vec{x}_q) \\ \text{where} \left| \begin{array}{r} \mathscr{P}_t(\vec{x}_Q,\vec{x}_q)\text{: combination of gauge links} \\ \Gamma\text{: Dirac matrices} \end{array} \right. \end{array}}$$

Extraction of the mass m of the meson M

"long time" behaviour of the 2-point correlation function at $\vec{p}=\vec{0}$:

$$\mathscr{C}^{(2)}_M(t,\vec{0}) \overset{\text{def.}}{=} \sum_{\text{positions}} \left\langle \mathscr{O}^\dagger(t)\mathscr{O}(0) \right\rangle \xrightarrow[t \gg 0]{} \mathscr{Z}_M \exp(-m\, t)$$

where $\mathscr{Z}_M \equiv \left|\langle 0 \,|\, \mathscr{O} \,|\, M(\vec{p})\rangle\right|^2$

Spectroscopy of the D^{**} : interpolating fields

$^3P_0$ state : *local* interpolating field (easy case)

$$\boxed{\mathcal{O}(t) = \bar{\psi}_c(t, \vec{x})\, \psi_q(t, \vec{x})}$$

$^3P_2$ state : *non local* interpolating field

$\leadsto$ Question : how to locate on the lattice a state with $J^P = 2^+$?

$\leadsto$ Answer : look at the symmetries !!

Spectroscopy of the D^{**} : a touch of group theory

Fundamental idea

$$\boxed{\text{physical object}} \xleftrightarrow[\text{with}]{\text{identified}} \boxed{\text{member of a basis of a conveniently chosen irreducible representation (IR) of the symmetry group of the system}}$$

Lattice case

- symmetry : O_h group
- IR's : "only" 10 $\underbrace{A_1^+, A_1^-, A_2^+, A_2^-}_{\text{IR dim 1}}$ $\underbrace{E^+, E^-}_{\text{IR dim 2}}$ $\underbrace{T_1^+, T_1^-, T_2^+, T_2^-}_{\text{IR dim 3}}$ ($\pm \rightsquigarrow$ parity)

Main trick

lattice state →

$$|\psi_R\rangle = \sum_{J,m} c^R_{J,m} \, |\psi\rangle_{J,m}$$

← spin J continuum state

$\in$ IR of the group O_h

$\in$ IR of J spin in the continuum (denoted $D^{(J)}$)

$$\boxed{\text{spin } J \text{ contributes to } |\psi_R\rangle \iff |\psi_R\rangle \in D^{(J)}\downarrow O_h}$$

(subduced representation of $D^{(J)}$ in O_h)

Spectroscopy of the D^{**} : a touch of group theory

Correspondance table

J	$D^{(J)} \downarrow O_h$
0	$A_1^{\pm}$
1	$T_1^{\pm}$
2	$E^{\pm} \oplus T_2^{\pm}$
etc	etc

$\Rightarrow$ 2^+ state : work with E^+ and T_2^+

Solution for "$\mathscr{P}_t(\vec{x}_Q, \vec{x}_q)\,\Gamma$"

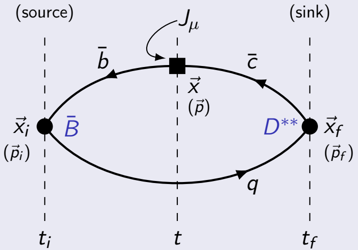
possible combination of link variables and dirac matrices that transform according to the IR E^+ and T_2^+ :

$$E^+ \begin{cases} \gamma_1 D_1 + \gamma_2 D_2 - 2\gamma_3 D_3 & (0) \\ \gamma_1 D_1 - \gamma 2_D 2 & (+2) + (-2) \end{cases} \qquad T_2^+ \begin{cases} \gamma_2 D_3 + \gamma_3 D_2 & (+1) + (-1) \\ \gamma_1 D_3 + \gamma_3 D_1 & (+1) - (-1) \\ \gamma_1 D_2 + \gamma_2 D_1 & (+2) - (-2) \end{cases}$$

D : covariant derivative on the lattice

Transition amplitudes

How ?



Object used :

three-point correlation function
$\mathscr{C}^{(3)}(t, t_i, t_f;\, \vec{p}_i, \vec{p}_f)$

$\mathscr{C}^{(3)}$ Definition

$$\mathscr{C}^{(3)}_{BJ_\mu D^{**}}(t, t_i, t_f;\, \vec{p}_i, \vec{p}_f) = \sum_{\vec{x}_i, \vec{x}, \vec{x}_f} \left\langle \mathscr{O}^{\dagger}_{D^{**}}(t_f, \vec{x}_f)\, J_\mu(t, \vec{x})\, \mathscr{O}_B(t_i, \vec{x}_i) \right\rangle \cdot e^{i(\vec{x}-\vec{x}_f)\cdot\vec{p}_f} \cdot e^{-i(\vec{x}-\vec{x}_i)\cdot\vec{p}_i}$$

Transition amplitudes

Transition amplitudes

ratio $$R(t) \overset{\text{def.}}{=} \boxed{\frac{\mathscr{C}^{(3)}_{BJD^{**}}(t, t_i, t_f;\ \vec{p}_i, \vec{p}_f)}{\mathscr{C}^{(2)}_{B}(t, t_i;\ \vec{p}_i)\,\mathscr{C}^{(2)}_{D^{**}}(t, t_f;\ \vec{p}_f)}\sqrt{\mathscr{Z}_B}\sqrt{\mathscr{Z}_{D^{**}}}}$$

$$\xrightarrow[t_f \gg t \gg t_i]{} \quad \left\langle D^{**}(\vec{p}_f)\,\middle|\, J_\mu \,\middle|\, \bar{B}(\vec{p}_i)\right\rangle$$

(remember that $\mathscr{Z}_X$ comes from $\mathscr{C}^{(2)}_X$)

Technicalities

A few examples of issues we are faced with. . .

1. *B* meson : too big to fit inside the lattice
   $\Rightarrow$ different m_b and extrapolation to physical mass
2. Twisted mass fermions : possible mix of 0^+ and 0^- states
   $\Rightarrow$ disentanglement required (*GEVP method*)
3. Renormalization : renormalization constants for the axial and vector current
4. Behaviour at small $\vec{p}$: many quantities $\rightarrow$ 0 when $\vec{p} = \vec{0}$
   $\Rightarrow$ go to higher impulsions but increase in noise !!

Sommaire

Conclusions

- Theoretical viewpoint : *everything can be calculated :*
  - general expressions for $B \to D^{**}$ transition amplitudes on the lattice
  - formula giving each form factor in terms of those transition amplitudes
  - decay rates
- Computational viewpoint : *very delicate computations*
  - isolation of excited states
  - possible increase in noise when going to high $\vec{p}$
  - high statistics requirement