$B \to D^{**}$ at infinite heavy mass

Workshop on B decay into D^{**} an related issues, Paris

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Introduction

- Consider semileptonic decays of B mesons (B, B^*) into orbitally excited P wave D mesons (D^{**}):

  $$B^{(*)} \quad \rightarrow \quad D^{**} \, l \, \nu.$$

- Precise knowledge of the corresponding branching fractions important, e.g. to reduce the systematic uncertainty in the measurements of the CKM matrix element $|V_{cb}|$.
- There is a persistent conflict ("$1/2$ versus $3/2$ puzzle") between theory and experiment:
  - Experiment favors the decay into "$1/2$ P wave D^{**}'s".
  - Theory favors the decay into "$3/2$ P wave D^{**}'s".
  - Lattice calculations can help to resolve this conflict.

Outline

- Heavy-light mesons.
- The $1/2$ versus $3/2$ puzzle:
  - Experimental side.
  - Theory side.
  - Possible explanations to resolve the puzzle.
- Lattice computation of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$:
  - Simulation setup, static and light quark propagators.
  - Static-light meson creation operators.
  - Static-light meson masses.
  - 2-point functions, ground state norms.
  - 3-point functions, Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$.
  - Extrapolation to the u/d quark mass.
- Conclusions.

Heavy-light mesons

- Heavy-light meson: a meson made from a heavy quark (b, c) and a light quark (u, d), i.e. $B = \{\bar{b}u\,,\,\bar{b}d\}$, $D = \{\bar{c}u\,,\,\bar{c}d\}$.
- Static limit, i.e. $m_b, m_c \rightarrow \infty$:
  - No interactions involving the static quark spin.
  - Classify states according to parity $\mathcal{P}$ and total angular momentum of the light cloud (light quarks and gluons) j.
- m_b, m_c finite, but heavy:
  - Classify states according to parity $\mathcal{P}$ and total angular momentum J.
  - Although j is not a "true quantum number" anymore, it is still an approximate quantum number $\rightarrow$ notation D_J^j.
  - $D^{**} = \{D_0^*\,,\,D_1'\,,\,D_1\,,\,D_2^*\}$.

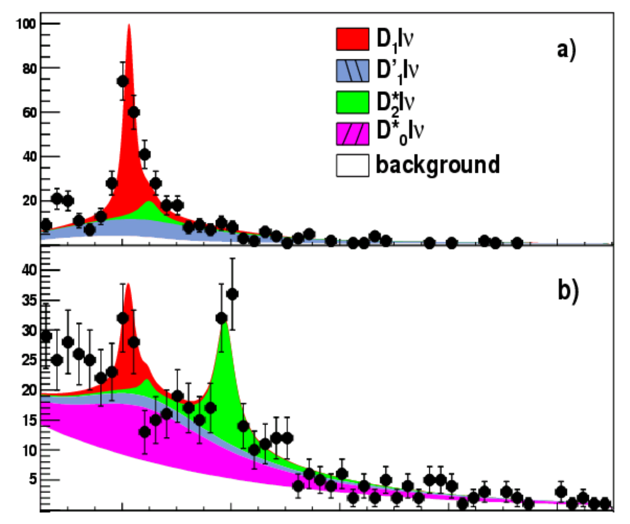
$j^{\mathcal{P}}$	$J^{\mathcal{P}}$
$(1/2)^- \equiv S$	$0^- \equiv B, D$
	$1^- \equiv B^*, D^*$
$(1/2)^+ \equiv P_-$	$0^+ \equiv D_0^* \equiv D_0^{1/2}$
	$1^+ \equiv D_1' \equiv D_1^{1/2}$
$(3/2)^+ \equiv P_+$	$1^+ \equiv D_1 \equiv D_1^{3/2}$
	$2^+ \equiv D_2^* \equiv D_2^{3/2}$

$1/2$ **versus** $3/2$**: experimental side (1)**

- Consider the semileptonic decay $B \rightarrow X_c \, l \, \nu$.
- Experiments, which have studied this decay: ALEPH, BaBar, BELLE, CDF, DELPHI, DØ.
- What is X_c?
  - $\approx 75\%$ D and D^*, i.e. S wave states (agreement with theory).
  - $\approx 10\%$ $D_1^{3/2}$ and $D_2^{3/2}$, i.e. $j = 3/2$ P wave states (agreement with theory).
  - For the remaining $\approx 15\%$ the situation is not clear:
    - A natural candidate would be $D_0^{1/2}$ and $D_1^{1/2}$, i.e. $j = 1/2$ P wave states.
    - This would imply $\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu) > \Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)$, which is in conflict with theory.
    - This conflict between experiment and theory is called the "$1/2$ versus $3/2$ puzzle".

$1/2$ versus $3/2$: experimental side (2)

- Example plot from BaBar/SLAC:
  - Horizontal axis: $m(D^{(*)}\pi) - m(D^{(*)})$ in GeV/c^2.
  - Vertical axis: $\mathrm{events}/(20\,\mathrm{MeV}/c^2)$.
  - Simultaneous fit of four probability distribution functions (D_0^*, D_1', D_1, D_2^*) to $m(D^{(*)}\pi) - m(D^{(*)})$ data:
    - **a)** $B^- \rightarrow D^{*+}\pi^- l^- \bar{\nu}_l$.
    - **b)** $B^- \rightarrow D^{+}\pi^- l^- \bar{\nu}_l$.



  - Two states (D_1 and D_2^*, i.e. the $j = 3/2$ P wave states) have small widths and can "clearly" be identified.
  - Two states (D_0^* and D_1', i.e. the $j = 1/2$ P wave states) have very large widths.

[B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **101**, 261802 (2008) [arXiv:0808.0528 [hep-ex]]]

$1/2$ versus $3/2$: theory side (1)

- Static limit, i.e. $m_b, m_c \to \infty$.
- Parameterization of the matrix elements relevant for decays $B \to X_c \, l \, \nu$ by a small set of form factors (Isgur-Wise functions) due to heavy quark symmetry. [N. Isgur and M. B. Wise, Phys. Rev. D **43**, 819 (1991)]
- In particular for $B \to D^{**} \, l \, \nu$,

$$\begin{aligned} \langle D_0^{1/2}(v')|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle \quad & \propto \quad \tau_{1/2}(w)(v-v')_\mu \\ \langle D_2^{3/2}(v',\epsilon)|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle \quad & \propto \quad \tau_{3/2}(w)\Big((w+1)\epsilon^*_{\mu\alpha}v^\alpha - \epsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\nu\Big). \end{aligned}$$

where $w = v'v \geq 1$.

$1/2$ versus $3/2$: theory side (2)

- Relation to decay rates:

$$\begin{aligned}\frac{d\Gamma(B \rightarrow D_J^{1/2} l\nu)}{dw} & \propto & G_F^2 |V_{cb}|^2 K_J^{1/2}(w) \Big|\tau_{1/2}(w)\Big|^2 \quad , \quad J = 0, 1 \\ \frac{d\Gamma(B \rightarrow D_J^{3/2} l\nu)}{dw} & \propto & G_F^2 |V_{cb}|^2 K_J^{3/2}(w) \Big|\tau_{3/2}(w)\Big|^2 \quad , \quad J = 1, 2,\end{aligned}$$

where K_J^j are analytically known kinematical factors, e.g.

$$\begin{aligned}K_0^{1/2}(w) & = & 4r^3 (w^2 - 1)^{3/2} (1 - r)^2 \\ K_1^{1/2}(w) & = & 4r^3 (w - 1)(w^2 - 1)^{1/2} \Big((w - 1)(1 + r)^2 + 4w(1 + r^2 - 2rw)\Big) \\ \ldots\end{aligned}$$

with $r = m(D)/m(B)$.

$1/2$ versus $3/2$: theory side (3)

- By means of OPE a couple of sum rules have been derived in the static limit:
  - Most prominent sum rule in this context: Uraltsev sum rule,

    $$\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 - \left|\tau_{1/2}^{(n)}(1)\right|^2 = \frac{1}{4}$$

    ($\tau_{1/2} \equiv \tau_{1/2}^{(0)}$ and $\tau_{3/2} \equiv \tau_{3/2}^{(0)}$; the sum is over all $1/2$ and $3/2$ P wave meson states respectively).

    [N. Uraltsev, Phys. Lett. B **501**, 86 (2001) [arXiv:hep-ph/0011124]]
  - From experience with sum rules one expects approximate saturation from the ground states, i.e.

    $$\left|\tau_{3/2}^{(0)}(1)\right|^2 - \left|\tau_{1/2}^{(0)}(1)\right|^2 \approx \frac{1}{4},$$

    which implies $|\tau_{1/2}(1)| < |\tau_{3/2}(1)|$. This strongly suggests $\Gamma(B \rightarrow D_{0,1}^{1/2}\, l\, \nu) < \Gamma(B \rightarrow D_{1,2}^{3/2}\, l\, \nu)$, which is in conflict with experiment.

$1/2$ versus $3/2$: theory side (4)

- Phenomenological models:
  - $|\tau_{1/2}(1)| < |\tau_{3/2}(1)|$ and $\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu) < \Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)$, which is in "conflict" with experiment.

    [V. Morenas, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **56** 5668 (1997) [arXiv:hep-ph/9706265]]

    [D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B **434**, 365 (1998) [arXiv:hep-ph/9805423]]

    [...]
  - Same qualitative picture also beyond the static limit, i.e. for finite m_b and m_c.

    [D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **61**, 014016 (2000) [arXiv:hep-ph/9906415]]

$1/2$ versus $3/2$: possible explanations (1)

- **Experiment:**

  (A) The signal for the remaining 15% of X_c is rather vague; therefore, only a small part might be $D_{0,1}^{1/2}$.

- **OPE:**

  - Sum rules might not be saturated by the ground states.

  (B) Sum rules hold in the static limit and might change for finite quark masses.

  (C) Sum rules make statements about $\tau_{1/2}(w = 1)$ and $\tau_{3/2}(w = 1)$; to obtain decay rates, however, one has to integrate over w.

- **Phenomenological models:**

  - Models might give a wrong answer.

- **Most probable scenario:** a combination of (A), (B) and (C).

  [N. Uraltsev, arXiv:hep-ph/0409125]

$1/2$ versus $3/2$: possible explanations (2)

- A lattice calculation of $\tau_{1/2}$ and $\tau_{3/2}$ could shed some light on this puzzle.
- Exploratory quenched lattice study confirmed the theory side:
  $\tau_{1/2}(1) = 0.38(4)$, $\tau_{3/2}(1) = 0.53(8)$.
  [D. Becirevic *et al.*, Phys. Lett. B **609**, 298 (2005) [arXiv:hep-lat/0406031]]
- In the following I will report about the first unquenched lattice calculation of $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$.
  [B. Blossier, M. Wagner and O. Pene [ETM Collaboration], JHEP **0906**, 022 (2009) [arXiv:0903.2298 [hep-lat]]]

Lattice calculation of $\tau_{1/2}$ and $\tau_{3/2}$ (1)

- The "Isgur-Wise relations"

$$\begin{aligned} \langle D_0^{1/2}(v')|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle \quad &\propto \quad \tau_{1/2}(w)(v-v')_\mu \\ \langle D_2^{3/2}(v',\epsilon)|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle \quad &\propto \quad \tau_{3/2}(w)\Big((w+1)\epsilon^*_{\mu\alpha}v^\alpha - \epsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\nu\Big). \end{aligned}$$

  are note directly useful to compute $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$.

- They can be rewritten in the following form, which is directly accessible to a lattice calculation:

$$\begin{aligned} \langle D_0^{1/2}(v)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle \quad &= \quad -ig_{jk}\Big(m(D_0^{1/2}) - m(B)\Big)\tau_{1/2}(1) \\ \langle D_2^{3/2}(v,\epsilon)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle \quad &= \quad +i\sqrt{3}\epsilon_{jk}\Big(m(D_2^{3/2}) - m(B)\Big)\tau_{3/2}(1). \end{aligned}$$

  [A. K. Leibovich, Z. Ligeti, I. W. Stewart and M. B. Wise, Phys. Rev. D **57**, 308 (1998) [arXiv:hep-ph/9705467]]

Lattice calculation of $\tau_{1/2}$ and $\tau_{3/2}$ (2)

- We compute

$$\langle D_0^{1/2}(v)|\bar{c}\gamma_5\gamma_j D_k b|B(v)\rangle \quad = \quad -ig_{jk}\Big(m(D_0^{1/2}) - m(B)\Big)\tau_{1/2}(1)$$

via

$$\tau_{1/2}(1) \quad = \lim_{t_0-t_1\rightarrow\infty\,,\,t_1-t_2\rightarrow\infty} \tau_{1/2,\textrm{effective}}(t_0-t_1,t_1-t_2)$$

$$\tau_{1/2,\textrm{effective}}(t_0-t_1,t_1-t_2) \quad =$$

$$= \quad \frac{1}{Z_\mathcal{D}}\left|\frac{N(P_-)\quad N(S)\quad \Big\langle\Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger\ (\bar{Q}\gamma_5\gamma_3 D_3 Q)(t_1)\ \mathcal{O}^{(S)}(t_2)\Big\rangle}{\Big(m(P_-)-m(S)\Big)\quad \Big\langle\Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger\mathcal{O}^{(P_-)}(t_1)\Big\rangle\quad \Big\langle\Big(\mathcal{O}^{(S)}(t_1)\Big)^\dagger\mathcal{O}^{(S)}(t_2)\Big\rangle}\right| .$$

- We need:
  - Static-light meson creation operators $\mathcal{O}^{(S)}$, $\mathcal{O}^{(P_-)}$, $\mathcal{O}^{(P_+)}$.
  - Static-light meson masses $m(S)$, $m(P_-)$ and $m(P_+)$.
  - 2-point and 3-point functions (and norms $N(S)$, $N(P_-)$, $N(P_+)$).

Simulation setup (1)

- Lattice volume: $L^3 \times T = 24^3 \times 48$.
- Gauge action: tree-level Symanzik improved,

$$S_\mathrm{G}[U] \ = \\ = \ \frac{\beta}{6}\bigg(b_0 \sum_{x,\mu\neq\nu} \mathrm{Tr}\Big(1 - P^{1\times 1}(x;\mu,\nu)\Big) + b_1 \sum_{x,\mu\neq\nu} \mathrm{Tr}\Big(1 - P^{1\times 2}(x;\mu,\nu)\Big)\bigg),$$

$b_0 = 1 - 8b_1$, $b_1 = -1/12$.

- Gauge coupling $\beta = 3.9$ corresponds to $a = 0.0855\,\mathrm{fm}$.

Simulation setup (2)

- Fermionic action: Wilson twisted mass, $N_f = 2$ degenerate flavors,

$$S_\mathrm{F}[\chi, \bar{\chi}, U] \quad = \quad a^4 \sum_x \bar{\chi}(x) \Big(D_\mathrm{W} + i \mu_\mathrm{q} \gamma_5 \tau_3 \Big) \chi(x)$$

$$D_\mathrm{W} \quad = \quad \frac{1}{2} \Big(\gamma_\mu (\nabla_\mu + \nabla_\mu^*) - a \nabla_\mu^* \nabla_\mu \Big) + m_0$$

(m_0: untwisted mass; μ_q: twisted mass; τ_3: third Pauli matrix acting in flavor space).

- Relation between the physical basis ψ and the twisted basis χ (in the continuum):

$$\psi \quad = \quad \frac{1}{\sqrt{2}} \Big(\cos(\omega/2) + i \sin(\omega/2) \gamma_5 \tau_3 \Big) \chi$$

$$\bar{\psi} \quad = \quad \frac{1}{\sqrt{2}} \bar{\chi} \Big(\cos(\omega/2) + i \sin(\omega/2) \gamma_5 \tau_3 \Big)$$

(ω: twist angle; $\omega = \pi/2$: maximal twist).

Simulation setup (3)

- Untwisted mass m_0, tuned to maximal twist ($\kappa = 1/(8+2m_0) = 0.160856$) $\rightarrow$ "automatic $\mathcal{O}(a)$ improvement of physical quantities".

μ_q	m_PS in MeV	number of gauge configurations
0.0040	$314(2)$	1400
0.0064	$391(1)$	1450
0.0085	$448(1)$	1350

Static and light quark propagators

- Static quark propagators:

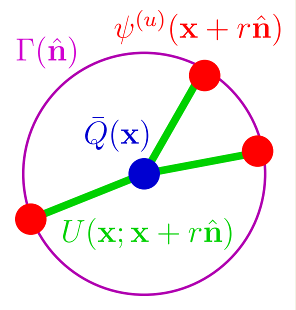
$$\Big\langle Q(x)\bar{Q}(y)\Big\rangle_{Q,\bar{Q}} \quad = \quad = \quad \delta^{(3)}(\mathbf{x}-\mathbf{y})U^{(\mathrm{HYP2})}(x;y)\bigg(\Theta(y_0-x_0)\frac{1-\gamma_0}{2}+\Theta(x_0-y_0)\frac{1+\gamma_0}{2}\bigg).$$

  - Essentially Wilson lines in time direction.
  - HYP2 static action to improve the signal-to-noise ratio.

- Light quark propagators:

  - Stochastic timeslice propagators.

Static-light meson creation operat



- In the continuum, physical basis:

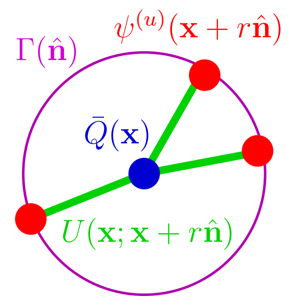
$$\mathcal{O}^{(\Gamma)}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x};\mathbf{x}+r\hat{\mathbf{n}}) \psi^{(u)}(\mathbf{x}+r\hat{\mathbf{n}}).$$

  - $\bar{Q}(\mathbf{x})$ creates an infinitely heavy i.e. static antiquark at position $\mathbf{x}$.
  - $\psi^{(u)}(\mathbf{x}+r\hat{\mathbf{n}})$ creates a light quark at position $\mathbf{x}+r\hat{\mathbf{n}}$ separated by a distance d from the static antiquark.
  - The spatial parallel transporter

$$U(\mathbf{x};\mathbf{x}+d\hat{\mathbf{n}}) \quad = \quad P\left\{\exp\left(+i\int_{\mathbf{x}}^{\mathbf{x}+d\hat{\mathbf{n}}} dz_j\, A_j(\mathbf{z})\right)\right\}$$

    connects the antiquark and the quark in a gauge invariant way via gluons.
  - The integration over the unit sphere $\int d\hat{\mathbf{n}}$ combined with a suitable weight factor $\Gamma(\hat{\mathbf{n}})$ yields well defined total angular momentum J and parity $\mathcal{P}$ ($\Gamma(\hat{\mathbf{n}})$ is a combination of spherical harmonics [→ angular momentum] and γ-matrices [→ spin]; Wigner-Eckart theorem).

Static-light meson creation operat



- In the continuum, physical basis:

$$\mathcal{O}^{(\Gamma)}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \int d\hat{\mathbf{n}}\, \Gamma(\hat{\mathbf{n}}) U(\mathbf{x};\mathbf{x}+r\hat{\mathbf{n}}) \psi^{(u)}(\mathbf{x}+r\hat{\mathbf{n}}).$$

- List of operators (J: total angular momentum; j: total angular momentum of the light cloud; $\mathcal{P}$: parity):

$\Gamma(\hat{\mathbf{n}})$	$J^{\mathcal{P}}$	$j^{\mathcal{P}}$	$\mathrm{O_h}$	lattice $j^{\mathcal{P}}$	notation
γ_5	0^-	$(1/2)^-$	A_1	$(1/2)^-$, $(7/2)^-$, ...	S
1	0^+	$(1/2)^+$		$(1/2)^+$, $(7/2)^+$, ...	P_-
$\gamma_1\hat{n}_1 - \gamma_2\hat{n}_2$ (cyclic)	2^+	$(3/2)^+$	E	$(3/2)^+$, $(5/2)^+$, ...	P_+
$\gamma_5(\gamma_1\hat{n}_1 - \gamma_2\hat{n}_2)$ (cyclic)	2^-	$(3/2)^-$		$(3/2)^-$, $(5/2)^-$, ...	$D_\pm$

- On the lattice, twisted basis:

$$\mathcal{O}^{(\Gamma)}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \sum_{\mathbf{n}=\pm\hat{\mathbf{e}}_1,\pm\hat{\mathbf{e}}_2,\pm\hat{\mathbf{e}}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x};\mathbf{x}+r\mathbf{n}) \chi^{(u)}(\mathbf{x}+r\mathbf{n}).$$

Static-light meson creation operators (3)

- On the lattice, twisted basis:

$$\mathcal{O}^{(\Gamma)}(\mathbf{x}) \quad = \quad \bar{Q}(\mathbf{x}) \sum_{\mathbf{n}=\pm\hat{\mathbf{e}}_1,\pm\hat{\mathbf{e}}_2,\pm\hat{\mathbf{e}}_3} \Gamma(\hat{\mathbf{n}}) U(\mathbf{x};\mathbf{x}+r\mathbf{n}) \chi^{(u)}(\mathbf{x}+r\mathbf{n}).$$

  - Due to the twisted basis each operator creates both a $\mathcal{P} = +$ part and a $\mathcal{P} = -$ part (e.g. $\bar{Q}\gamma_5\chi \approx (\bar{Q}\gamma_5\psi - i\bar{Q}\psi)/\sqrt{2}$).
  - Smearing techniques to optimize the ground state overlaps:
    - APE smearing for spatial links U.
    - Gaussian smearing for light quark fields $\chi^{(u)}$.

Static-light meson masses (1)

- Consider 2×2 correlation matrices:

$$\mathcal{C}_{JK}(t) \quad = \quad \left\langle \left(\mathcal{O}^{(\Gamma_J)}(t)\right)^\dagger \mathcal{O}^{(\Gamma_K)}(0) \right\rangle.$$

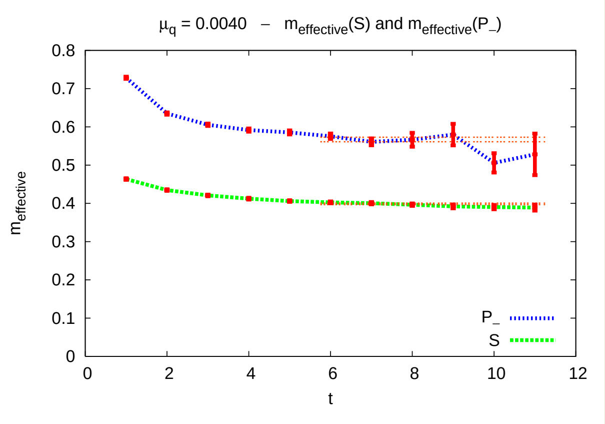
  - For S and P_-, $\Gamma_J \in \{\gamma_5\,,\,1\}$.
  - For P_+, $\Gamma_J \in \{\gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2\,,\ \gamma_5(\gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2)\}$.

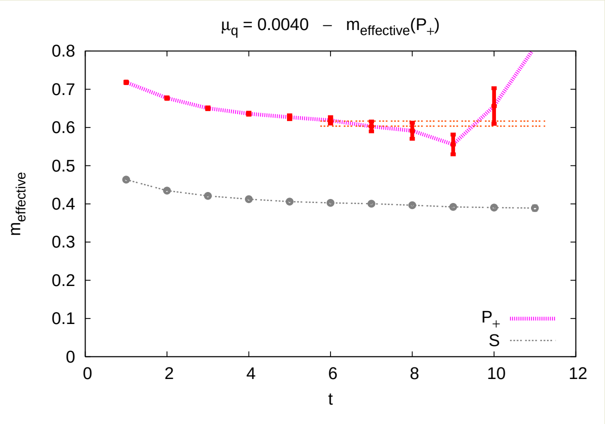
- Solve a generalized eigenvalue problem:

$$\mathcal{C}_{JK}(t) v_K^{(n)}(t) \quad = \quad \mathcal{C}_{JK}(t_0) v_K^{(n)}(t) \lambda^{(n)}(t, t_0).$$

- Determine static-light meson masses from effective mass plateaus:

$$m^{(n)}_{\textrm{effective}}(t) \quad = \quad \ln\left(\frac{\lambda^{(n)}(t, t_0)}{\lambda^{(n)}(t+1, t_0)}\right).$$





Static-light meson masses (2)

- The generalized eigenvalue problem,

$$\mathcal{C}_{JK}(t) v_K^{(n)}(t) \quad = \quad \mathcal{C}_{JK}(t_0) v_K^{(n)}(t) \lambda^{(n)}(t, t_0),$$

also yields appropriate linear combinations of twisted basis meson creation operators with well defined parity:

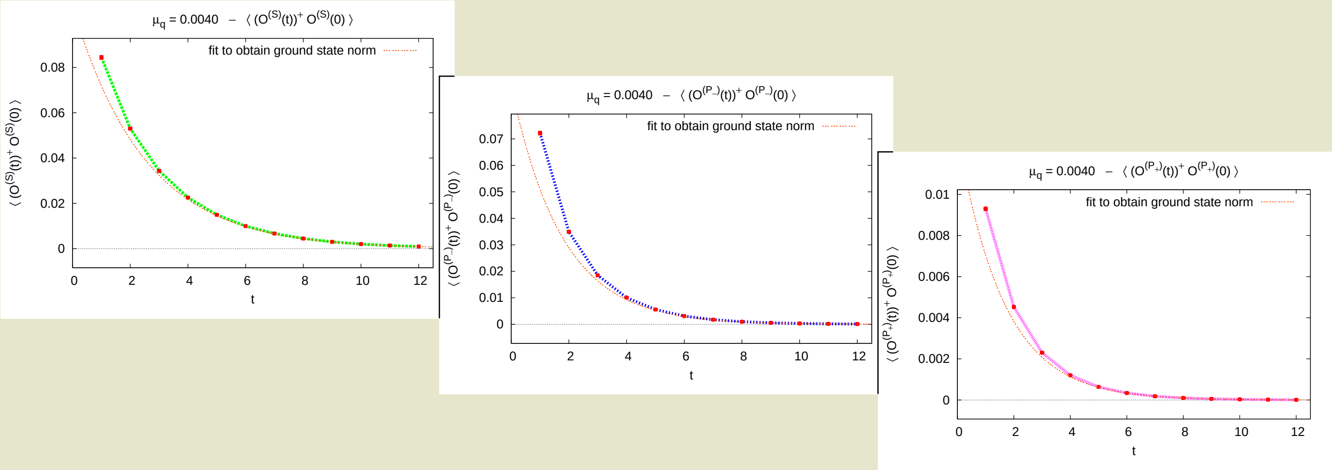
$$\begin{aligned}
\mathcal{O}^{(S)} &= v_{\gamma_5}^{(S)}(t) \mathcal{O}^{(\gamma_5)} + v_1^{(S)}(t) \mathcal{O}^{(1)} \\
\mathcal{O}^{(P_-)} &= v_{\gamma_5}^{(P_-)}(t) \mathcal{O}^{(\gamma_5)} + v_1^{(P_-)}(t) \mathcal{O}^{(1)} \\
\mathcal{O}^{(P_+)} &= v_{\gamma_x \hat{n}_x - \gamma_y \hat{n}_y}^{(P_+)}(t) \mathcal{O}^{(\gamma_x \hat{n}_x - \gamma_y \hat{n}_y)} + v_{\gamma_5 (\gamma_x \hat{n}_x - \gamma_y \hat{n}_y)}^{(P_+)}(t) \mathcal{O}^{(\gamma_5 (\gamma_x \hat{n}_x - \gamma_y \hat{n}_y))}.
\end{aligned}$$

2-point functions, ground state norms

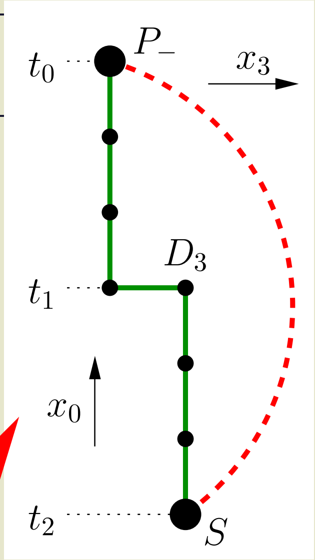
- 2-point functions are now straightforward to compute:

$$\left\langle \Big(\mathcal{O}^{(S)}(t)\Big)^\dagger \mathcal{O}^{(S)}(0) \right\rangle \quad , \quad \left\langle \Big(\mathcal{O}^{(P_-)}(t)\Big)^\dagger \mathcal{O}^{(P_-)}(0) \right\rangle \quad , \quad \left\langle \Big(\mathcal{O}^{(P_+)}(t)\Big)^\dagger \mathcal{O}^{(P_+)}(0) \right\rangle .$$

- Ground state norms $N(S)$, $N(P_-)$ and $N(P_+)$ by fitting exponentials at large temporal separations, e.g. $N(S)^2 e^{-mt}$ to $\langle (\mathcal{O}^{(S)}(t))^\dagger \mathcal{O}^{(S)}(0)\rangle$.



3-point functions, $\tau_{1/2}$ and $\tau_{3/2}$



- Compute the Isgur-Wise function

$$\tau_{1/2}(1) \quad = \quad \left|\frac{\langle P_- | \bar{Q}\gamma_5\gamma_3 D_3 Q | S\rangle}{m(P_-) - m(S)}\right|$$

via "effective form factors":

$$\tau_{1/2}(1) \quad = \quad \lim_{t_0 - t_1 \to \infty\, ,\, t_1 - t_2 \to \infty} \tau_{1/2,\text{effective}}(t_0 - t_1, t_1 - t_2)$$

$$\tau_{1/2,\text{effective}}(t_0 - t_1, t_1 - t_2) \quad =$$

$$= \quad \frac{1}{Z_{\mathcal{D}}}\left|\frac{N(P_-) \quad N(S) \quad \Big\langle \Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger \ (\bar{Q}\gamma_5\gamma_3 D_3 Q)(t_1) \ \mathcal{O}^{(S)}(t_2)\Big\rangle}{\Big(m(P_-) - m(S)\Big) \ \Big\langle \Big(\mathcal{O}^{(P_-)}(t_0)\Big)^\dagger \mathcal{O}^{(P_-)}(t_1)\Big\rangle \ \Big\langle \Big(\mathcal{O}^{(S)}(t_1)\Big)^\dagger \mathcal{O}^{(S)}(t_2)\Big\rangle}\right| .$$

- $\tau_{3/2}(1)$ analogously: replace

$$P_- \quad \to \quad P_+ \quad , \quad \gamma_3 D_3 \quad \to \quad \frac{\gamma_5(\gamma_1 D_1 - \gamma_2 D_2)}{\sqrt{6}} .$$

3-point functions, $\tau_{1/2}$ and $\tau_{3/2}$ (2)

- Z_D in

$$\tau_{1/2,\text{effective}}(t_0 - t_1, t_1 - t_2) \quad =$$

$$= \quad \frac{1}{Z_\mathcal{D}} \left| \frac{N(P_-) \quad N(S) \quad \left\langle \left(\mathcal{O}^{(P_-)}(t_0)\right)^\dagger \ (\bar{Q}\gamma_5\gamma_3 D_3 Q)(t_1) \ \mathcal{O}^{(S)}(t_2) \right\rangle}{\left(m(P_-) - m(S)\right) \ \left\langle \left(\mathcal{O}^{(P_-)}(t_0)\right)^\dagger \mathcal{O}^{(P_-)}(t_1) \right\rangle \ \left\langle \left(\mathcal{O}^{(S)}(t_1)\right)^\dagger \mathcal{O}^{(S)}(t_2) \right\rangle} \right| .$$

is the renormalization constant of the heavy-heavy current $\bar{Q}\gamma_5\gamma_3 D_3 Q$, i.e.

$$(\bar{Q}\gamma_5\gamma_3 D_3 Q)^R \quad = \quad \frac{(\bar{Q}\gamma_5\gamma_3 D_3 Q)^B}{Z_D} ,$$

to first order in perturbation theory.

  - Analytical formulae long and "complicated".
  - Tree-level Symanzik improved gauge action, HYP2 static action: $Z_D = 0.976$.

3-point functions, $\tau_{1/2}$ an

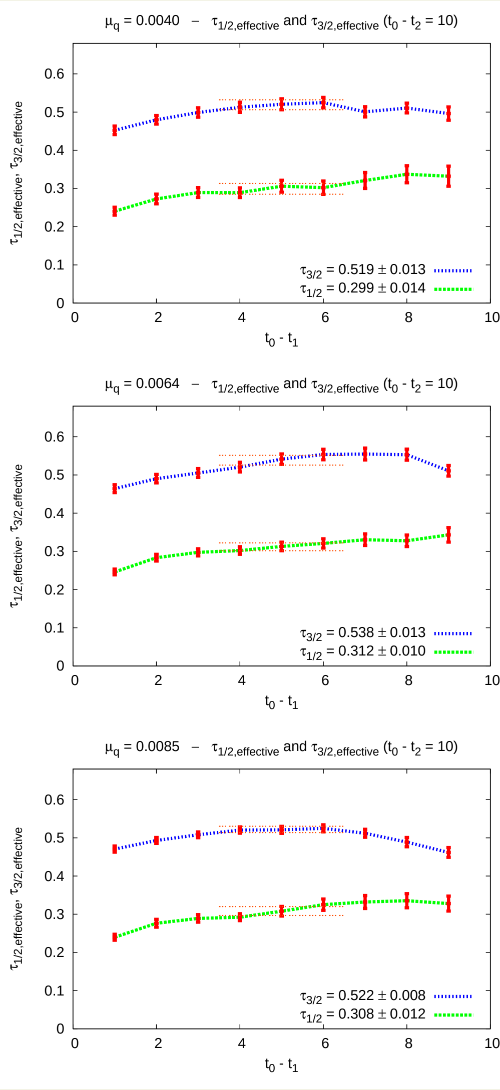
- Results for various light quark masses:

	$t_0 - t_2 = 10$		
μ_{q}	$\tau_{1/2}(1)$	$\tau_{3/2}(1)$	$(\tau_{3/2})^2 - (\tau_{1/2})^2$
0.0040	$0.299(14)$	$0.519(13)$	$0.180(16)$
0.0064	$0.312(10)$	$0.538(13)$	$0.193(13)$
0.0085	$0.308(12)$	$0.522(8)$	$0.177(9)$

- The Uraltsev sum rule,

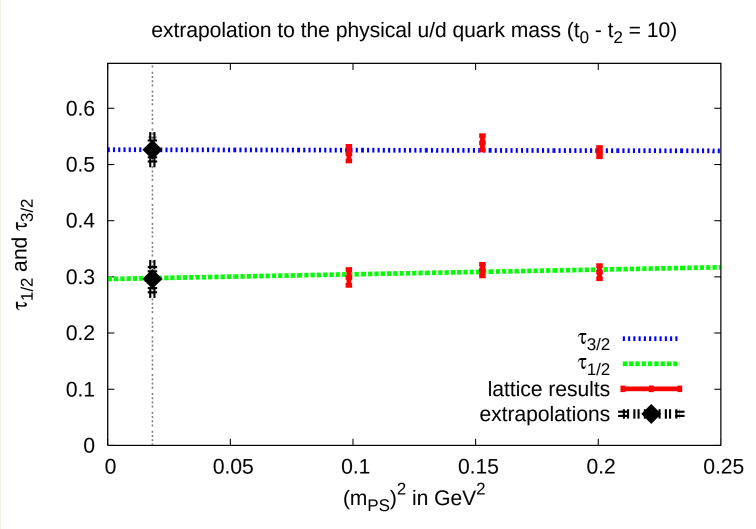
$$\sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 - \left|\tau_{1/2}^{(n)}(1)\right|^2 \quad = \quad \frac{1}{4},$$

is almost fulfilled by the ground state contributions $\tau_{1/2}^{(0)}(1) \equiv \tau_{1/2}(1)$ and $\tau_{3/2}^{(0)}(1) \equiv \tau_{3/2}(1)$.



Extrapolation to the u/d quark mass

- Linear extrapolation in $(m_{\mathrm{PS}})^2$ to the u/d quark mass $m_{\mathrm{PS}} = 135\,\mathrm{MeV}$:
  - $\tau_{1/2} = 0.296(26)$.
  - $\tau_{3/2} = 0.526(23)$.



Conclusions

- First dynamical lattice computation of the Isgur-Wise functions $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$:
  - $\tau_{1/2}(1) = 0.296(26)$, $\tau_{3/2}(1) = 0.526(23)$.
  - This indicates $\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu) < \Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)$ in the static limit.
  - Expectation from sum rules confirmed:
    - ∗ Uraltsev sum rule is approximately fulfilled by the ground states.
    - ∗ $\tau_{1/2}(1) \ll \tau_{3/2}(1)$.
    - ∗ Numerical values in agreement with sum rule expectation.
  - Phenomenological models qualitatively and quantitatively confirmed.
  - Experiment:
    - ∗ Fair agreement with the experimentally measured $\tau_{3/2}(1) \approx 0.75$.
    - ∗ No agreement with the experimentally measured $\tau_{1/2}(1) \approx 1.28$.

      [D. Liventsev *et al.* [Belle Collaboration], Phys. Rev. D **77**, 091503 (2008) [arXiv:0711.3252 [hep-ex]]]