Excited heavy-quark mesons and the heavy quark expansion

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HQE in QCD

The $1/m_Q$ expansion in QCD can be constructed for a few important cases where we now know quite a bit about its dynamics-driven terms

The key information is provided by the Small Velocity heavy quark sum rules, including spin sum rules established for relativistic bound-states of QCD

Allowed to predict values of $\overline{\Lambda}$, μ_π^2, ρ_D^3, ... based on one number:
the hyperfine mass splitting $M_{B^*} - M_B \approx 47\,\text{MeV}$

The most precision applications have been done for inclusive decays

A similar analysis has been extended motivated by the formfactor $F(0)$ in $B \to D^* \ell\nu$ near zero recoil

Gambino, Mannel, N.U. arXiv:1004.2859 [hep-ph]
arXiv:1206.2296 [hep-ph]

Model-independent treatment of heavy mesons

The status report (72 pages...)

Arrived at three apparently isolated, yet linked through the HQE, observations for heavy meson phenomenology

- Large negative *overlap* corrections to $F(0)$ driving it down to $F(0)\approx 0.86$

- Large nonlocal correlators of $\bar{Q}\vec{\pi}^2 Q$ and $\bar{Q}\vec{\sigma}\vec{B}Q$ in B mesons from the hyperfine splitting ΔM^2 in B vs. D

  The enhanced negative corrections in $F(0)$ are related to the 'discrepancy' in the hyperfine splitting ratio between charm and beauty mesons

- Enhanced inclusive yield of *radials* and 'D-waves' in $b\to c\,\ell\nu$

  Resolve '$\frac{1}{2}>\frac{3}{2}$' paradox ?
  Account for the missing semileptonic channels
  Predict significance of the $\frac{3}{2}^+$ 'D-wave'

- As a byproduct we find significant corrections to the ground-state factorization; relevant for precision inclusive decays

a number of theoretical results *en route*

Brief synopsis

$$F_{D^*} = \sqrt{\xi_A^{\rm pert} - \Delta_{\rm power} - w_{\rm inel}}$$

QM:

$$F_{D^*} = \sqrt{\xi_A^{\rm pert}} - \langle \delta J^A \rangle - \delta \langle \Psi^\dagger_{D^*} \Psi_B \rangle$$

Le Yaouanc et al. 1994

$$\sqrt{\xi_A^{\rm pert}} \simeq 0.98 \quad \text{at } \mu \simeq 0.8\,\text{GeV}, \qquad -\Delta_{\frac{1}{m_Q^2}} - \Delta_{\frac{1}{m_Q^3}} \simeq -0.13$$

$$F_{D^*} \leq 0.92 \quad \text{– upper bound}$$

Would expect formfactor about 0.92 *if no overlap deficit were there*

$w_{\rm inel}$ – *wavefunction overlap deficit* – is more significant than expected

N.U. hep-ph/0312001

Relate it to the hyperfine splitting in B and D: $w_{\rm inel} \gtrsim 0.14$

$$F_{D^*} \approx 0.86 \quad \text{– prediction}$$

GMU 2010, 2012

Wavefunction overlap deficit $\Longleftrightarrow$ yield of excited states (near zero recoil)

Required to turn the upper bound for F_{D^*} into an estimate, $\sim 15\%$

Model-independent analysis:

$$\mathrm{d}w_{\rm inel}(\omega) = \left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)^2 \frac{\rho_p^{(\frac{1}{2}^+)}(\omega)}{\omega^2} + \left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)\left(\frac{1}{3m_c}+\frac{1}{m_b}\right)\frac{\rho_{pg}^{(\frac{1}{2}^+)}(\omega)}{\omega^2} + \frac{1}{4}\left(\frac{1}{3m_c}+\frac{1}{m_b}\right)^2 \frac{\rho_g^{(\frac{1}{2}^+)}(\omega)}{\omega^2} + \frac{1}{6m_c^2}\frac{\rho_g^{(\frac{3}{2}^+)}(\omega)}{\omega^2}$$

- The four spectral densities $\rho^{(+)}(\omega)$ form a positive set
- Factorization properties
- The $1/m_Q$ correction to the hyperfine splitting is expressed through

$$\rho_{\rm hp} = \int \frac{\mathrm{d}\omega}{\omega}\left(\frac{\rho_g^{(\frac{3}{2}^+)}(\omega)}{2} + 2\rho_{pg}^{(\frac{1}{2}^+)}(\omega) - \frac{2\rho_g^{(\frac{1}{2}^+)}(\omega)}{3}\right)$$

An application to excited states: consider a 'weighted probability'

$$w_{(1)}(\mu) = \sum_{\epsilon<\mu} \epsilon_n w_n \;\approx\; \epsilon_{\rm rad}\left(w_{\rm rad} + w_{\frac{3}{2}}\right)$$

$w_{(1)}$ is calculated in the OPE similar to the sum rule for $F(0)$ itself:

$$w_{(1)} = \frac{-(\rho_{\pi G}^3+\rho_A^3)}{3m_c^2} + \frac{-2\rho_{\pi\pi}^3-\rho_{\pi G}^3}{3m_c m_b} + \frac{\rho_{\pi\pi}^3+\rho_{\pi G}^3+\rho_S^3+\rho_A^3}{4}\left(\frac{1}{m_c^2}+\frac{2}{3m_c m_b}+\frac{1}{m_b^2}\right)+\mathcal{O}\left(\frac{1}{m_Q^3}\right)$$

The structure is transparent when viewed through the BPS expansion

$$w_{(1)}^{(\rm BPS)} = \frac{-(\rho_{\pi G}^3+\rho_A^3)}{3m_c^2} + \mathcal{O}\left(\frac{1}{m_c^3}\right)$$

$(\rho_{\pi G}^3+\rho_A^3)-\rho_{LS}^3$ determines ΔM^2 to order $1/m_Q$

Extract comparing B and D mesons   a technical point of the analysis

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Extract comparing B and D mesons   a technical point of the analysis

$\delta^{(2)} w_{(1)}$ is positive; $\delta^{(1)} w_{(1)}$ comes with small coefficient $1/3m_c m_b$ and the minimum is very shallow:

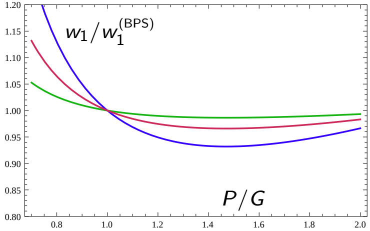
$$w_{(1)}/w_{(1)}^{(\rm BPS)} = 1 - \underbrace{(1-\nu_{3/2})\frac{m_c^2}{m_b^2}}_{<0.07} + [\ldots]^2 \qquad \nu_{3/2} > 0$$

Therefore, nearly a functional relation between $\delta\Delta M^2$ and $w_{(1)}$, $w_{\rm inel}$

$$w_{\rm inel} = \frac{0.45\,{\rm GeV}^3 + \tilde\kappa\, 0.35\,{\rm GeV}^3}{3m_c^2\,\epsilon_{\rm rad}} \simeq 14\%$$

Analysis of hyperfine splitting: $|\tilde\kappa| \lesssim 0.15$

A 6% decrease in F_{D^*}



The way to evaluate $w_{\rm inel}$ through $w_{(1)}$ in the 't Hooft model yields almost exact number

F_{D^*} summary

QCD lower bound:

$F_{D^*} < 0.92$ $\quad$ ($F_{D^*} < 0.9$ including continuum estimate)

The unbiased predicted value

$$F_{D^*} \lesssim 0.86$$

The central number has about 2% e-bars it may lower if μ_π^2 turns out larger

Central value goes down for increasing μ_π^2 yet the corrections from higher power terms also increase

Taking the central value for V_{cb} from $\Gamma_{\rm sl}(B)$ and the $B \to D^* \ell\nu$ extrapolation

$$F_{D^*} \approx 0.85$$

$B \to D\,\ell\nu$ near zero recoil

Experimentally challenging   theoretically advantageous

$$\langle D(p_2)|\bar{c}\gamma_\nu b|B(p_1)\rangle = f_+(p_1+p_2)_\nu + f_-(p_1-p_2)_\nu \qquad f_\pm \equiv f_\pm(\vec{q}^{\,2})$$

$F_+ \equiv \frac{2\sqrt{M_B M_D}}{M_B+M_D}\, f_+$ has $1/m_Q$ corrections...

$$F_+ = 1 + \left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{M_B - M_D}{M_B + M_D} - \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

$$\overline{\Lambda} = M_B - m_b, \quad \overline{\Sigma} = \ldots$$

$$\boxed{\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) = 1.04 \pm 0.01 \pm 0.01}$$

N.U. 2003

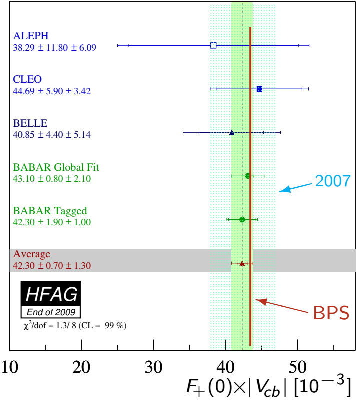
All orders in $1/m$ in 'BPS', to $1/m^2 \cdot 1/\mathrm{BPS}^2$, α_s^1

The bulk 3% is the perturbative factor, only 1% comes from power terms

Taking V_{cb} from $\Gamma_{\rm sl}(B)$:

$$F_+(0) \simeq 1.021 \pm 0.019 \pm 0.041 \pm \delta_{\rm incl}$$

Good agreement with the dynamic heavy quark expansion in QCD



Using $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ I predicted

$$|V_{cb}|F_+(0) = 43.\cancel{7}\,{\scriptstyle 5} \cdot 10^{-3}$$

There is no tension between *inclusive* and *exclusive* V_{cb}

rather a remarkable agreement

'Radials'

Estimated D^{**} contribution around 15% of the D^* at zero recoil, but

- This is only the axial-induced yield
- Without the phase-space suppression

These are 'radial' excitations ($j^P = \frac{1}{2}^+$) or 'D-wave' states ($j^P = \frac{3}{2}^+$)

Prediction: $\gtrsim 7 \div 8\%$ of $\Gamma_{\rm sl}$ over the full phase space Why?

The dominant amplitude is not the HQET one $\propto \mathbf{v}^2 \ll 1$, but $1/m_c$!

$$A(B \to D^{(n)^+}) = G_F V_{cb} \left[r\,\mathbf{v}^2 + P_c \frac{\mu_{\rm hadr}}{m_c} + P_b \frac{\mu_{\rm hadr}}{m_b} + \ldots \right]$$

Sum of the squared sum for the latter two is fixed by $w_{\rm inel}$ at zero recoil

$$\mathrm{BR}_{\text{`rad'}} \propto w_{\rm inel} = \xi_A^{\rm pert} - F_{D^*}^2 - \Delta_A$$

We can evaluate $w_{\rm inel}$ well enough and do not need a phenomenological extraction

Using a certain trick radically simplifies the 'inclusive' calculation

Complementary to the analysis by Jugeau, Le Yaouanc, Oliver, Raynal 2005

Production of particular excited mesons is unreliable: heavy quark symmetry works poorly for excited charmed states

Evaluate the decay rate as a decay of a B-‘meson’ with a spinless b-quark into the excited ‘mesons’ with a spinless c-quark (they have spin either $\frac{1}{2}$ or $\frac{3}{2}$). This automatically sums the rate over the members of a hyperfine multiplet; the sum is stable against spin symmetry violation in charm

The corresponding transition amplitude is not a HQS-invariant (it comes from the $1/m_Q$ terms), but transforms accordingly in respect to spin

The phase-space effects are automatically included (factor of 2 suppression); the assumption is only a constant amplitude, terms $\mathbf{v}^2$ are neglected in it. Its normalization is fixed by zero-recoil analysis

The total yield summed over ‘radials’ and ‘D-waves’ is well constrained

$$\frac{\Gamma_{\rm sl}(B\to\mathcal{R}^{(1)}) + \Gamma_{\rm sl}(B\to\mathcal{D}^{(1)})}{\Gamma_{\rm sl}} \gtrsim 7\%$$

The HQ piece of the amplitude is expected to increase the yield

Probably the charm mesons with $M \simeq 2.6\,\text{GeV}$ are $\frac{1}{2}^+$ and with $M \simeq 2.75\,\text{GeV}$ are $\frac{3}{2}^+$

Hyperfine splitting in D vs. B

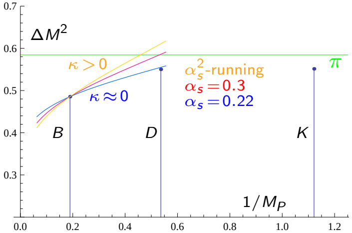
Encountered in the estimate of the negative overlap correction to $F_{D^*}(0)$
Fixes magnitude of the nonlocal correlators $\Longleftrightarrow$ inelastic amplitudes

Experiment: $M_{B^*}^2 - M_B^2 \simeq M_{D^*}^2 - M_D^2 \simeq M_{K^*}^2 - M_K^2 \simeq M_\rho^2 - M_\pi^2$

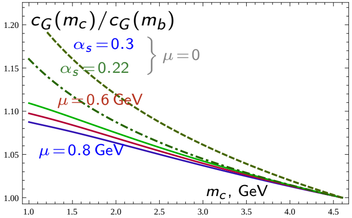
If these were exact and if perturbative corrections could be discarded

$$-(-\rho_{LS}^3 + \rho_{\pi G}^3 + \rho_A^3) \simeq 2\overline{\Lambda}\mu_G^2\,(1+\kappa)$$

Parametrizes corrections



Need precision evaluation of perturbative effects without double counting, use Wilsonian approach



The final outcome: $\kappa \approx -0.2$ and $-(\rho_{\pi G}^3 + \rho_A^3) \approx 0.45\,\text{GeV}^3$ – large!

$$\rho^3_{\pi\pi} + \rho^3_S = -(\rho^3_{\pi G} + \rho^3_A) + \int \frac{d\omega}{\omega}\left[\underbrace{\rho_p^{\frac{1}{2}^+} - 2\rho_{pg}^{\frac{1}{2}^+} + \rho_g^{\frac{1}{2}^+}}_{(\delta\,\mathrm{BPS})^2}\right] > -(\rho^3_{\pi G} + \rho^3_A)$$

Equality is attained in the BPS limit

We can use the existing precision fits to semileptonic moments to determine $\rho^3_{\pi\pi} + \rho^3_S$; the preliminary result is

$$\rho^3_{\pi\pi} + \rho^3_S \gtrsim (0.33 \pm 0.17)\,\mathrm{GeV}^3 \qquad (\text{cf. } 0.45\,\mathrm{GeV}^3)$$

Another intriguing consistency of the different independent analyses

The transition amplitudes are given by (for actual mesons use the $3D$ spin-trace formalism)

$$\langle \tfrac{1}{2}^+ | \pi_k \pi_l | \Omega_0 \rangle = \tfrac{P}{3}\,\delta_{kl}\,\chi^\dagger \Psi_0 - \tfrac{G}{6}\,\chi^\dagger \sigma_{kl} \Psi_0$$

$$\langle \tfrac{3}{2}^+ | \pi_k \pi_l | \Omega_0 \rangle = \tfrac{f}{20}(\chi_k^\dagger \sigma_l + \chi_l^\dagger \sigma_k)\Psi_0 + \tfrac{g}{4}\, i\epsilon_{klm}\,\chi_m^\dagger \Psi_0$$

BPS: $\begin{cases} P = G \\ f = g \end{cases}$

in these terms

$$\delta\rho_p^{(\frac{1}{2}^+)}(\omega) = P^2\,\delta(\omega - \epsilon_n), \quad \delta\rho_g^{(\frac{1}{2}^+)}(\omega) = G^2\,\delta(\omega - \epsilon_n), \quad \delta\rho_{pg}^{(\frac{1}{2}^+)}(\omega) = PG\,\delta(\omega - \epsilon_n)$$

$$\delta\rho_f^{(\frac{3}{2}^+)}(\omega) = f^2\,\delta(\omega - \epsilon_n), \quad \delta\rho_g^{(\frac{3}{2}^+)}(\omega) = g^2\,\delta(\omega - \epsilon_n), \quad \delta\rho_{fg}^{(\frac{3}{2}^+)}(\omega) = f\,g\,\delta(\omega - \epsilon_n)$$

A 10% of $\Gamma_{\rm sl}$ yield of 'radials' would a priori seem to do the job

Would this actually solve all the problems?

If the 'radials' with $M \approx 2.7\,$GeV had a large width, they could mimic reportedly seen wide structures at a lower mass around 2.4 GeV: in particular, the strong phase-space dependence on M would effectively shift the invariant mass of $D^*\pi(s)$ down (distort Breit-Wigner shape)

However, if the width is small, around 100 MeV, that is hardly a solution...

Are these 2.6 GeV and 2.75 GeV relatively narrow states reliably established?

Can there be a problem with the cascade decays or combinatorics in interpretation of the alleged 'wide P-waves'?

Outlook

We have looked at the continuum production. The soft $D^{(*)}\pi$ have $\frac{3}{2}^-$ and $\frac{1}{2}^-$ components equal (violate BPS), but it looks not really significant in magnitude. There are some interesting model-independent aspects to explore, they may boost things up

We are now in the era* of higher excitations, starting with 'radials' and D-waves (or exotics?). They are more significant than expected from naive nonrelativistic quark models (an evident HQ symmetry for charm may root in the proximity to BPS)

What are then our hopes to theoretically understand physics of higher excited heavy mesons?
The exact relations – 'D' Orsay sum rules' for higher multipole transitions
Le Yaouanc, Oliver, Raynal 2003

These are strong constrains! Unfortunately, so far their potential has not been fully explored

* Jugeau, Le Yaouanc, Oliver, Raynal, 2003-2005

Outlook

It appears that these per se do not fix completely the required transition amplitudes. Assuming BPS regime helps, yet not radically, freedom still remains. Nevertheless the remarkable consistency so far is observed

I anticipate that incorporating in full the D' Orsay relations will allow a qualitative classification of the dynamic structure and a far more complete quantitative description for the relevant heavy quark states, including the finite-mass effects

Of course experimental data are vital in this whole program

The question of interpreting the nature of a meson is not evident. We have found indications that the transition amplitude $\langle \frac{3}{2}^+ | \bar{Q} \vec{B}_{\rm chr} Q | \Omega_0 \rangle$ is large, yet this does not mean by itself that the state contains a non-spectator gluon component: for instance, $\langle B | \bar{b} \vec{\sigma} \vec{B}_{\rm chr} b | B \rangle$ is also large!

Summary

Comprehensive heavy quark expansion can say much about nonperturbative effects in certain cases

Inequalities or positivity properties are essential, require a physical renormalization scheme – it is available!

Heavily exploit physical behavior of the correlators in Minkowski domain

- $F_{D^*} \approx 0.86$; uncertainty about 2% at known μ_π^2, ρ_D^3, plus effect of higher-order power terms. Central value goes down for larger μ_π^2, yet higher power corrections become significant
- Large *overlap deficit* decreasing F_{D^*} by 6%

  Close to the BPS estimate
- Large nonlocal correlators $\rho^3_{...}$ from the hyperfine splittings
  'Discrepancy' in the splitting for B and D is settled
- Large inclusive yield of 'radials' plus 'D-waves', $7 \div 10\%$ of $\Gamma_{\rm sl}$
  Resolve $\frac{1}{2}$ vs. $\frac{3}{2}$ problem ?
  May provide missing semileptonic channels

This was only a single application motivated by $F_{D^*}(0)$