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Finite Theories after the discovery of a Higgs-like boson at the LHC

Standard Model very successful
$\rightarrow$ low energy accessible part of a (more) fundamental Theory of Elementary Particles.

BUT with

ad hoc Higgs sector

ad hoc Yukawa couplings

$\rightarrow$ free parameters (>20)

Renormalisation $\longrightarrow$ free parameters

Traditional way of reducing the number of parameters

SYMMETRY

Celebrated example: GUTs

e.g. minimal $SU(5)$ → testable $\sin^2\theta_W$; successful m_τ / m_b

However more SYMMETRY ($e.g.$ $SO(10)$, $E(6)$, $E(7)$, E_8) does not lead necessarily to more predictions for the SM parameters.

Extreme case: Superstring Ths

On the other hand

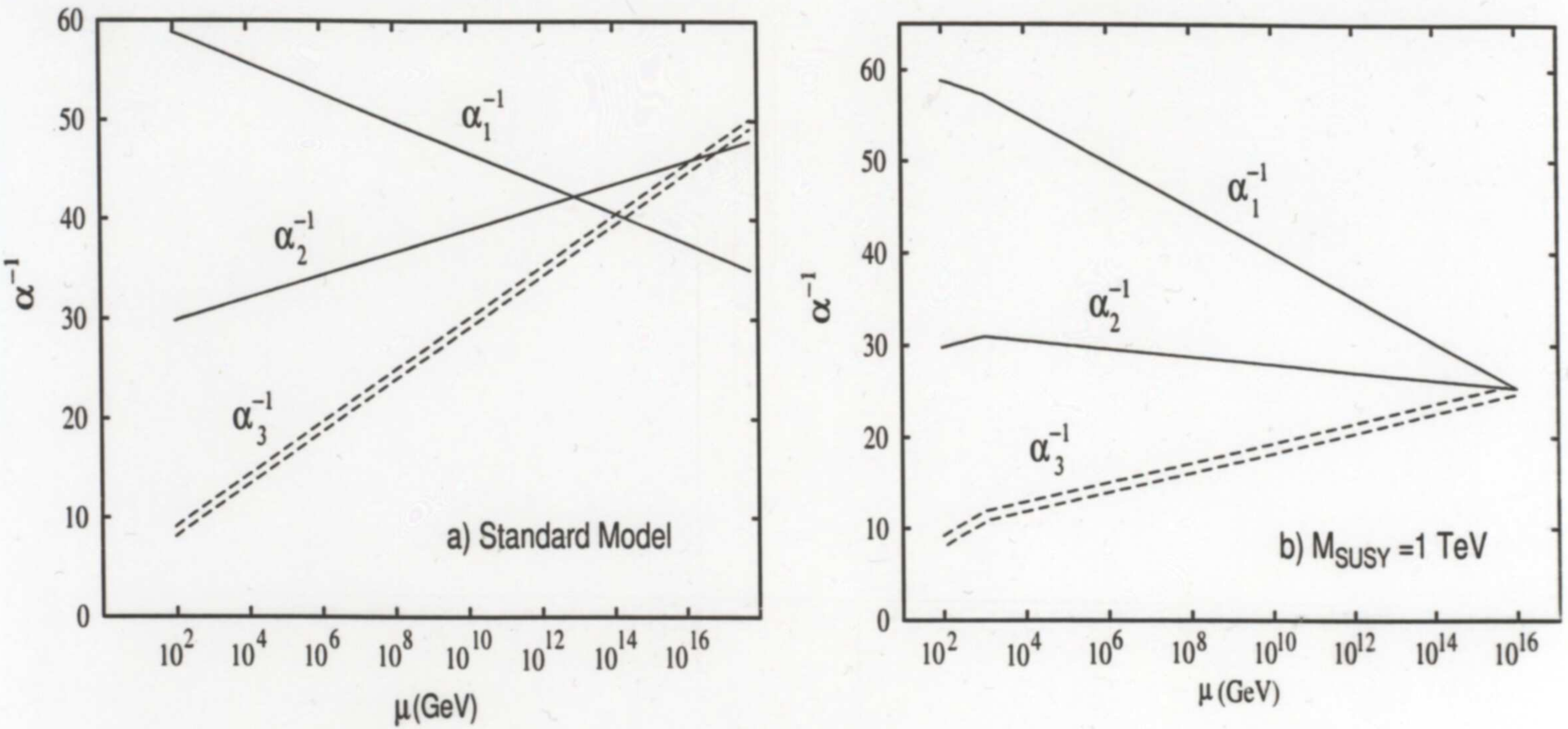
LEP data $\rightsquigarrow$ $N=1$ $SU(5)$

$\cancel{N=1}$ $\cancel{SU(5)}$ $\longrightarrow$ MSSM

MSSM best candidate for Physics Beyond SM

But with > 100 ! free parameters mostly in its SSB sector.

- Cures problem of quadratic divergencies of the SM (hierarchy problem)
- Restricts the Higgs sector leading to approximate prediction of the Higgs mass



α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
a) Standard Model
μ (GeV)
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
b) M_{SUSY} =1 TeV
μ (GeV)

- SM with two - Higgs doublets

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \left(m_3^2 H_1 H_2 + h.c. \right)$$

$$+ \frac{1}{2} \lambda_1 \left(H_1^+ H_1 \right)^2 + \frac{1}{2} \lambda_2 \left(H_2^+ H_2 \right)^2$$

$$+ \lambda_3 \left(H_1^+ H_1 \right) \left(H_2^+ H_2 \right) + \lambda_4 \left(H_1 H_2 \right) \left(H_1^+ H_2^+ \right)$$

$$+ \left\{ \frac{1}{2} \lambda_5 \left(H_1 H_2 \right)^2 + \left[\lambda_6 \left(H_1^+ H_1 \right) + \lambda_7 \left(H_1^+ H_2^+ \right) \right] \left(H_1 H_2 \right) + h.c. \right\}$$

Supesymmetry provides tree level relations among couplings

$$\lambda_1 = \lambda_2 = \frac{1}{4} \left(g^2 + g'^2 \right)$$

$$\lambda_3 = \frac{1}{4} \left(g^2 - g'^2 \right) , \quad \lambda_4 = -\frac{1}{4} g^2$$

$$\lambda_5 = \lambda_6 = \lambda_7 = 0$$

With $v_1 = \langle Re\, H_1^0 \rangle$, $v_2 = \langle Re\, H_2^0 \rangle$

and $v_1^2 + v_2^2 = (246\ GeV)^2$, $\frac{v_2}{v_1} \equiv \tan\beta$

$\Longrightarrow h^0, H^0, H^\pm, A^0$

At tree level

$$M^2_{h^0,H^0} = \frac{1}{2}\left\{ M_A^2 + M_Z^2 \mp \left[(M_A^2 + M_Z^2)^2 - 4 M_A^2 M_Z^2 \cos^2 2\beta \right]^{1/2} \right\}$$

$$M^2_{H^\pm} = M_W^2 + M_A^2$$

$$\Rightarrow \begin{cases} M_{h^0} < M_Z \, |\cos 2\beta| \\ M_{H^0} > M_Z \\ M_{H^\pm} > M_W \end{cases}$$

Radiative corrections

$$M^2_{h^0} \simeq M_Z^2 \cos^2 2\beta + \frac{3 g^2 m_t^4}{16\pi^2 M_W^2} \log \frac{\tilde{m}^2_{t_1} \tilde{m}^2_{t_2}}{m_t^4}$$

- Finite Unified Theories (from Quantum Reduction of Couplings)
- Higher Dimensional Unified Theories and Coset Space Dimensional Reduction (Classical Reduction of Couplings)
- Fuzzy Extra Dimensions and Renormalizable Unified Theories

Quantum Reduction of Couplings

Consider a GUT with

g – gauge coupling

g_i – other couplings (Yukawas, self-couplings)

Any relation among the couplings

$$\Phi(g, g_1, \dots) = \text{const}$$

which is RGI should satisfy

$$\frac{d}{dt}\Phi = 0 \, , \quad t = \ln\mu$$

$$\frac{d}{dt}\Phi = \frac{\partial \Phi}{\partial g}\underbrace{\frac{dg}{dt}}_{\beta_g} + \sum_i \frac{\partial \Phi}{\partial g_i}\underbrace{\frac{dg_i}{dt}}_{\beta_i} = 0$$

which is equivalent to

$$\frac{dg}{\beta_g} = \frac{dg_1}{\beta_1} = \frac{dg_2}{\beta_2} = \dots$$ characteristic system

$$\Longrightarrow \quad \beta_g \frac{dg_i}{dg} = \beta_i$$ Reduction eqs Oehme Zimmermann

Demand power series solution to the REs

$$g_i = \sum_{n=0} \rho_i^{(n+1)} g^{2n+1}$$

Remarkably, uniqueness of these solutions can be decided already at 1-loop!

Assume

$$\beta_i = \frac{1}{16\pi^2}\left[\sum_{j,k,l} \beta_i^{(1)jkl} g_j g_k g_l + \sum_{j\neq g} \beta_i^{(1)j} g_i g^2\right] + \cdots$$

$$\beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots$$

Assume $\rho_i^{(n)}$, $n \leq r$ have been uniquely determined

To obtain $\rho_i^{(r+1)}$, insert g_i in REs and collect terms of $O\left(g^{2r+1}\right)$

$$\rightsquigarrow \sum_{l \neq g} M(r)_i^{\,l} \rho_l^{(r+1)} = \text{lower order quantities Known by assumption}$$

where

$$M(r)_i^{\,l} = 3 \sum_{j,k \neq g} b_i^{(1)jkl} \rho_j^{(1)} \rho_k^{(1)} + b_i^{(1)l} - (2r+1) b_g^{(1)} \delta_i^l$$

$$0 = \sum_{j,k,l \neq g} b_i^{(1)jkl} \rho_j^{(1)} \rho_k^{(1)} \rho_l^{(1)} + \sum_{l \neq g} b_i^{(1)l} \rho_l^{(1)} - b_g^{(1)} \rho_i^{(1)}$$

$\Longrightarrow$ for a given set of $\rho_i^{(1)}$, the $\rho_i^{(n)}$ for all $n > 1$ can be uniquely determined if

$$\det M(n)_i^{\,l} \neq 0$$

for all n

Kubo et al.

Consider an $SU(N)$ (non-susy) theory with

$\phi^i(N)$, $\hat{\phi}_i(\bar{N})$ - complex scalars

$\psi^i(N)$, $\hat{\psi}_i(\bar{N})$ - Weyl spinors

λ^a $(a = 1, \dots, N^2-1)$ - "

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\sqrt{2}\left[g_Y \bar{\psi} \lambda^a T^a \phi - \hat{g}_Y \bar{\hat{\psi}} \lambda^a T^a \hat{\phi} + \text{h.c.}\right] - V(\phi, \hat{\phi}),$$

$$V(\phi, \hat{\phi}) = \frac{1}{4}\lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4}\lambda_2 (\hat{\phi}_i \hat{\phi}^{*i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \hat{\phi}^{*j}) + \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*j})$$

Searching for power series solution of the R.E.s we find

$$g_Y = \hat{g}_Y = g;\ \lambda_1 = \lambda_2 = \frac{N-1}{N} g^2;\ \lambda_3 = \frac{1}{2N} g^2;\ \lambda_4 = -\frac{1}{2} g^2$$

i.e. SUSY

$N=1$ gauge theories

Consider a chiral, anomaly free $N=1$ globally supersymmetric gauge th. based on a group G with gauge coupling g.

Superpotential

$$W = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} C_{ijk} \phi^i \phi^j \phi^k$$

m_{ij}, C_{ijk} - gauge invariant tensors

ϕ^i - matter fields transforming as an ir. rep. R_i of G.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2} \phi^j\,, \quad m^0_{ij} = Z^{i'j'}_{ij}\, m_{i'j'}\,, \quad C^0_{ijk} = Z^{i'j'k'}_{ijk}\, C_{i'j'k'}$$

$N=1$ non-renormalization thm ensures absence of mass and cubic-int-term infinities

$$Z^{ijk}_{i'j'k'}\, Z^{1/2\, i'}_{i''}\, Z^{1/2\, j'}_{j''}\, Z^{1/2\, k'}_{k''} = \delta^i_{(i''}\, \delta^j_{j''}\, \delta^k_{k'')}$$

$$Z^{ij}_{i'j'}\, Z^{1/2\, i'}_{i''}\, Z^{1/2\, j'}_{j''} = \delta^i_{(i''}\, \delta^j_{j'')}$$

(In the background field method)

$$Z_g Z_V^{1/2} = 1$$

$\rightarrow$ Only surviving infinities are Z^i_j (Z_V) i.e. one infinity for each field.

The 1-loop β-function of the gauge coupling is

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$\ell(R_i)$ - Dynkin index of R_i

$C_2(G)$ - quadratic Casimir of the adjoint rep.

β-functions of C_{ijk}, by virtue of the non-renormalization thm, are related with the anomalous dim. matrix γ_i^j of ϕ^i

$$\beta_{ijk}^{(1)} = \frac{dC_{ijk}}{dt} = C_{ijl}\,\gamma_k^l + C_{ikl}\,\gamma_j^l + C_{jkl}\,\gamma_i^l$$

$$\gamma_i^{(1)j} = Z^{-\frac{1}{2}}{}_i^k \frac{d}{dt} Z^{\frac{1}{2}}{}_k^j$$

$$= \frac{1}{32\pi^2}\left[C^{jkl}C_{ikl} - 2g^2 C_2(R_i)\,\delta_i^j\right]$$

$C_2(R_i)$ - quadratic Casimir of R_i

$C^{ijk} = C^*_{ijk}$

$$b_g^{(2)} = \frac{1}{(16\pi^2)^2}\, 2\, g^5 \left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{g^3}{r}\, C_2(R_i)\left[C^{jkl} C_{ikl} - 2g^2 C_2(R_i)\delta^j_i\right]$$

$$r : \operatorname{tr}\delta^{ab}$$

Parkes, West, Jones
Mezincescu, Yau
Machacek, Vaughn

$$\gamma^{(2)i}_{\ \ j} = \frac{1}{(16\pi^2)^2}\, 2\, g^4\, C_2(R_i)\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{1}{2}\left[C^{ikl} C_{jkm} + 2g^2 (R^a)^i_m (R^a)^l_j\right]$$

$$\cdot \left[C^{mpq} C_{lpq} - 2\delta^m_l\, g^2 C_2(R_i)\right]$$

$$b_g^{NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_i \ell(R_i)(1 - 2\gamma_i) - 3C_2(G)}{1 - g^2 C_2(G)/8\pi^2}\right]$$

Novikov – Shifman – Vainstein – Zakharov

Finite Unification

Old days...

...divergences are "hidden under the carpet" (Dirac, Lects on Q.F.T., '64)

Recent years...

...divergences reflect existence of a higher scale where new degrees of freedom are excited.

Not just artifacts of pert. th.

However the presence of quadratic divergences means that physics at one scale are very sensitive to unkown physics at higher scales.

⇝ SUSY ths which are free of quadratic divergences in spite of any experimental evidence...

⇝ Natural to expect that beyond unification scale the theory should be completely finite.

- $N = 4$ ⇝ finite to all orders in pert.
- $N = 2$ ⇝ only 1-loop contributions to β-function. Possible to arrange the spectrum so that theory is finite.

Multiplicities for massless irreducible reps with maximal helicity 1

Spin \ N	1	1	2	2	4
1	—	1	—	1	1
1/2	1	1	2	2	4
0	2	—	4	2	6

$$N=2 \ : \ b(g) = \frac{2g^3}{(4\pi)^2}\left(\sum_i T(R_i) - C_2(G)\right)$$

e.g. $SU(N)$ with $2N$ fundamental reps $\longrightarrow b(g) = 0$

$SU(5)$: $p(5+\bar{5})$; $q(10+\overline{10})$; $r(15+\overline{15})$
with $p+3q+7r=10$

$SO(10)$: $p(10+\overline{10})$; $q(16+\overline{16})$
with $p+2q=8$

E_6 : $4(27+\overline{27})$

Finite Unified Theories

$N=1$

- 1-loop finiteness conditions

  $b_g^{(1)} = 0$

  $\gamma^{(1)i}_{\ \ j} = 0$ - anomalous dimensions of all chiral superfields

- Exists complete classification of all chiral $N=1$ models with $b_g^{(1)} = 0$ Hamidi - Patera - Schwarz Jiang - Zhou

- 1-loop finiteness $\longrightarrow$ 2-loop finiteness Parkes-West Jones Mezincescu

.... Exist simple criteria that guarantee all loop finiteness (vanishing of all-loop beta functions)

Lucchesi-Piguet
Sibold

Ermushev
Kazakov
Tarasov

Leigh-Strassler

- All-loop finite SU(5) $\Longrightarrow$ top quark mass ✓

Kapetanakis
Mondragon
Z
'92

~~Susy~~ sector

Jones
Mezincescu
Yao

- 1-loop finiteness conds (require in particular universal soft ~~susy~~ scalar masses $(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j$)

·· 1-loop finiteness

→ 2-loop finiteness Jack Jones

Reduction of couplings

- Extension of method in SSB sector + application in min susy SU(5) Kubo Mondragon 2

·· 1-loop sum rule for soft scalar masses in non-finite susy ths. Kawamura Kobayashi Kubo

··· 2-loop sum rule for soft scalar masses in finite ths. Kobayashi Kubo Mondragon 2

* All-loop RGI relations in finite and non-finite ths Yamada Hisano, Shifman Kazakov Jack, Jones, Pickering

** All-loop sum rule for soft scalar masses in finite and non-finite ths

Kobayashi
Kubo
Z

•• SU(5) FUTs

Kobayashi
Kubo
Mondragon
Z

• Prediction of s-spectrum in terms of few parameters starting from several hundreds GeV.

•• The LSP is neutralino ✓ (see e.g. Kazakov et. al. Yoshioka)

••• Radiative E-W breaking ✓ (see e.g. Brignole Ibanez, Munoz)

•••• No funny colour, charge ✓ (see e.g. Casas et. al)

* Prediction of Higgs masses

Lightest ~ 118 - 129 GeV

Similar results also for min susy SU(5)

Consider a chiral, anomaly free, $N=1$ gauge theory with group G.

The superpotential is

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$\left.\begin{array}{l} Y^{ijk} \\ \mu^{ij} \end{array}\right\}$ gauge invariant

Yukawa couplings

Φ_i – matter superfields in irreducible reps of G

Necessary and sufficient conditions for $N=1$ 1-loop finiteness

- Vanishing of $b_g^{(1)}$ implies

$$\sum_i \ell(R_i) = 3\, C_2(G)$$

$\ell(R_i)$ – Dynkin index of R_i

$C_2(G)$ – Quadratic Casimir of G (adjoint)

$\Rightarrow$ Selection of the field content (representations) of the theory

CHIRAL TWO-LOOP-FINITE SUPERSYMMETRIC THEORIES ☆

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Any globally supersymmetric theory in four dimensions that is one-loop finite is automatically (at least) two-loop finite. We classify all such theories that are chiral and have a simple gauge group.

One of the less satisfying aspects of GUTs ("grand-unified theories") and super GUTs is that they contain many arbitrary parameters. One principle that could serve to limit the number of parameters is the requirement of finiteness. This possibility has been raised with the discovery that there are large classes of supersymmetric gauge theories that are free from ultraviolet divergences at all orders of perturbation theory [‡1]. The theories for which this has been established are all $N = 4$ and some [2] $N = 2$ super Yang–Mills theories. Finiteness allows some parameters to be introduced via mass and soft supersymmetry-breaking terms, but it relates all the dimensionless couplings. Unfortunately, all $N = 2$ or $N = 4$ theories are nonchiral ("vector-like") and do not appear suited to the construction of a realistic model. Recently, the possibility has been raised that certain $N = 1$ theories could also be finite [3,4]. A theory containing Yang–Mills and chiral superfields is ultraviolet finite at loop provided that certain conditions (described below) restricting the representations and couplings of the chiral superfields are satisfied. It has been proved by direct calculation [3] and by considerations involving the chiral anomaly [4] that these conditions ensure two-loop finiteness as well, without any additional restrictions. It is an open question whether any of the $N = 1$ theories of this class are finite beyond two loops. The purpose of this letter is to list all chiral solutions of the one-loop conditions that are based on a simple gauge group.

Consider a globally supersymmetric $N = 1$ theory in four dimensions with a simple Yang–Mills group G. In addition to the gauge superfield, it can contain chiral superfields in an arbitrary representation R of G with irreducible components R_i:

$$\mathrm{R} = \bigoplus_i \mathrm{R}_i . \tag{1}$$

Our task is to find the possible choices of R and associated couplings that ensure one-loop finiteness. We only consider chiral theories ($\mathrm{R} \neq \overline{\mathrm{R}}$). This restricts G to those groups that have complex representations, namely $\mathrm{SU}(n)$ with $n \geqslant 3$, $\mathrm{SO}(4k + 2)$ with $k \geqslant 2$, and E_6. Cancellation of the gauge-current anomaly

$$A(\mathrm{R}) = \sum_i A(\mathrm{R}_i) = 0 \tag{2}$$

is also imposed, since it is a necessary requirement for a consistent quantum theory. The anomaly condition is nontrivial only for $\mathrm{SU}(n)$.

There are two additional conditions required by one-loop finiteness [3,4]. The first is the one-loop finiteness of the gauge-field self energy. The condition is

$$l(\mathrm{R}) = \sum_i l(\mathrm{R}_i) = 3C_2(\mathrm{G}) , \tag{3}$$

where $l(\mathrm{R}_i)$ is the "index" of R_i [5] and $C_2(\mathrm{G})$ is

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[‡1] For a review see ref. [1].

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the eigenvalue of the second-order Casimir operator (which coincides with the index of the adjoint representation). Since indices are always positive (except for singlets, which are excluded), eq. (3) already limits R to a finite number of possibilities for a given group G.

The second condition is the one-loop finiteness of the chiral superfield self-energy. In terms of coefficients d describing the cubic self-coupling of the chiral superfields in the superpotential, the condition is

$$\sum_{\substack{b,c\\j,k}} d^{ijk}_{abc}\bar{d}^{i'jk}_{a'bc} = 2g^2\delta_{aa'}\delta_{ii'}C_2(\mathrm{R}_i) . \tag{4}$$

The subscripts a, b, c label components of the representations $\mathrm{R}_i, R_j, \mathrm{R}_k$.

The only irreducible representations that can occur in R are ones whose indices do not exceed $3C_2(\mathrm{G})$. Singlets (with nonzero couplings) are excluded by (4). All relevant representations of the groups with complex representations are listed in table 1. It also gives the indices and anomalies, normalized to be unity for the fundamental representation. Complex-conjugate representations, which have the same index and opposite anomaly, are not shown.

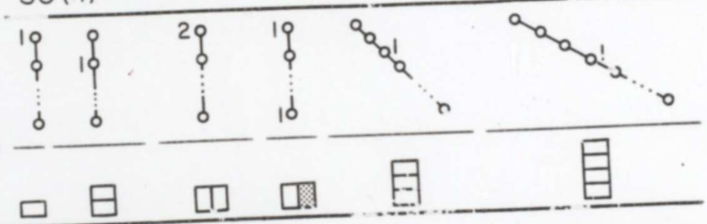
In seeking solutions to eqs. (2)–(4) it is convenient to consider first the trace of (4) given by summing over $a = a'$ and the m_α values of $i = i'$ for which $\mathrm{R}_i = \mathrm{R}_\alpha$. This results in conditions of the form

$$\sum_{\beta\gamma} |C_{\alpha\beta\gamma}|^2 = m_\alpha l(\mathrm{R}_\alpha) . \tag{5}$$

Eq. (5) is weaker than (4), but it is useful for quickly eliminating many candidates from the list of admissible R's. Detailed examination of (4) then eliminates

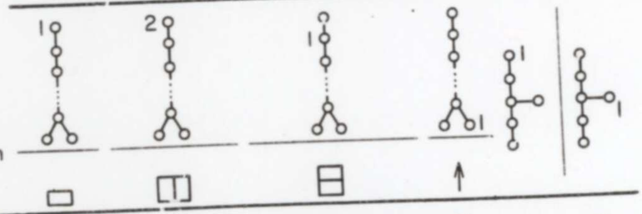
Table 1
Properties of relevant irreducible representations

SU(n)



Representation						
Dimension	n	n(n-1)/2	n(n+1)/2	n^2-1	n(n-1)(n-2)/6	n(n-1)(n-2)(n-3)/24
Index	1	n-2	n+2	2n	(n-2)(n-3)/2	(n-2)(n-3)(n-4)/6
Anomaly	1	n-4	n+4	0	(n-3)(n-6)/2	(n-3)(n-4)(n-8)/6

O(4k+2) | E(6)



	O(4k+2)				E(6)	
Representation						
Dimension	4k+2	2(k+1)(4k+1)	(4k+2)(4k+1)/2	2^{2k}	27	78
Index	1	4k+4	4k	2^{2k-3}	1	4

Table 2
Multiplicities m_α of the irreducible components R_α of R for all the solutions.

Irrep	27	$\overline{27}$	Comments
E(6)	n	$12 - n$	$7 \leqslant n \leqslant 12$

Irrep	10	54	45	16	$\overline{16}$	Comments
SO(10)	8	0	0	n	$8 - n$	$5 \leqslant n \leqslant 8$
	2	1	1	1	0	
	$12 - 2m - 2n$	1	0	n	m	$n + m \leqslant 4$, $n > m$

Irrep	$\square$	$\overline{\square}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\square\square$	$\overline{\square\square}$	Adj
SU(n)	$2n - 4$	$2n + 4$	0	1	1	0	0
$n \geqslant 7$	$n - 4$	$n + 4$	0	1	1	0	1

Irrep	8	$\overline{8}$	28	70	63
SU(8)	1	5	1	1	1

Irrep	3	$\overline{3}$	6	8
SU(3)	3	10	1	0
	0	7	1	1

Irrep	4	$\overline{4}$	15	6	10
SU(4)	0	8	1	1	1
	4	12	0	1	1

	Irrep	5	$\overline{5}$	10	$\overline{10}$	15	$\overline{15}$	24
	SU(5)	3	14	2	0	1	0	0
		6	14	0	1	1	0	0
		5	7	4	2	0	0	0
		5	10	5	0	0	0	0
I		6	9	4	1	0	0	0
		7	8	3	2	0	0	0
		8	10	3	1	0	0	0
		1	2	1	0	1	1	1
		1	9	0	1	1	0	1
		2	3	3	2	0	0	1
		3	5	3	1	0	0	1
II		4	7	3	0	0	0	1
		5	6	2	1	0	0	1
		6	8	2	0	0	0	1
		8	9	1	0	0	0	1
		3	4	1	0	0	0	2

	Irrep	6	$\overline{6}$	15	$\overline{15}$	21	$\overline{21}$	20	35
	SU(6)	0	16	3	0	1	0	0	0
Sibold el. al.		8	16	0	1	1	0	0	0
		0	4	5	3	0	0	0	0
		3	5	4	3	0	0	0	0
		0	12	6	0	0	0	0	0
		2	10	5	1	0	0	0	0
		4	8	4	2	0	0	0	0
		8	12	3	1	0	0	0	0
		0	2	1	0	0	0	3	1
		0	4	2	0	0	0	2	1
		3	5	1	0	0	0	2	1
		0	6	3	0	0	0	1	1
		2	4	2	1	0	0	1	1
		3	7	2	0	0	0	1	1
		6	8	1	0	0	0	1	1
		1	3	1	0	1	1	0	1
		2	10	0	1	1	0	0	1
		1	3	3	2	0	0	0	1
		0	8	4	0	0	0	0	1
		2	6	3	1	0	0	0	1
		3	9	3	0	0	0	0	1
		5	7	2	1	0	0	0	1
		6	10	2	0	0	0	0	1
		9	11	1	0	0	0	0	1
		0	4	2	0	0	0	0	2
		3	5	1	0	0	0	0	2
		0	2	1	0	0	0	1	2

some [1 for SU(5) and 11 for SU(6)] of the class allowed by (2), (3), and (5). The complete list of complex representations satisfying (2), (3) and (4) is given in table 2. The conjugate representations, which are also solutions, are not tabulated. In most cases the couplings are uniquely determined (up to a change of basis), but in a few cases there are free parameters or discrete alternatives. Note that there are no solutions for SO($4k + 2$) with $k > 2$.

Scanning the tables for potentially realistic schemes,

•• Vanishing of $\gamma^{(1)i}_{j}$ implies

$$Y^{ikl}\, Y_{jkl} = 2\,\delta^i_j\, g^2 C_2(R_i)$$

Yukawa gauge

$C_2(R_i)$ - quadratic Casimir of R_i

$Y^{ijk} = (Y_{ijk})^*$

$\Longrightarrow$ Yukawa and gauge couplings are related.

Note • μ^{ij} are not restricted

•• Appearance of $U(1)$ is incopatible with 1^{st} condt.

••• 2^{nd} condt forbids the presence of singlets with nonvanishing couplings.

∴∴ $\Longrightarrow$ ~~Susy~~ by G-invariant soft terms

* 1-loop finiteness condts necessary and sufficient to guarantee 2-loop finiteness

* 1-loop finiteness condts ensure that $b_g^{(3)}$ in 3-loops vanishes but in general $\gamma^{(3)}$ does not.

Grisaru - Milewski - Zanon

Parkes - West

What happens in higher loops?

So far 1-loop finiteness condts (on γs) are telling us

$$Y^{ijk} = Y^{ijk}(g)$$

$$b_Y^{(1)ijk} = 0$$

** Necessary and sufficient condts for vanishing b_g and b_{ijk} to all orders

Lucchesi
Piguet
Sibold

1. $b_g^{(1)} = 0$
2. $\gamma^{(1)i}{}_j = 0$
3. $b_Y^{ijk} = b_g \dfrac{dY^{ijk}}{dg}$

admit power series solution which in lowest order is a solution of condt 2.

3. $\to$ 3'. There exist solutions to $\gamma^{(1)i}{}_j = 0$ of the form

$$Y^{ijk} = \rho^{ijk} g, \quad \rho^{ijk} \text{ - complex}$$

3. $\to$ 4. These solutions are isolated and non-degenerate considered as solutions of $b_Y^{(1)ijk} = 0$

Recall

R-invariance, axial anomaly

In massless $N=1$ ths

$U(1)$ chiral transformation R:

$$A_\mu \to A_\mu \,, \quad \lambda \to e^{-i\alpha}\lambda \,,$$

$$\phi \to e^{-i\frac{2}{3}\alpha}\phi \,, \quad \psi \to e^{i\frac{1}{3}\alpha}\psi \,, \quad \ldots$$

$$\lambda_D = \begin{pmatrix} \lambda \\ \bar{\lambda} \end{pmatrix} \longrightarrow e^{i\alpha\gamma_5}\lambda_D$$

Noether current $J^\mu_R = \bar{\lambda}_D \gamma^\mu \gamma^5 \lambda_D + \cdots$

$$\rightsquigarrow \partial_\mu J^\mu_R = r\left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots\right)$$

$$r = b_g^{(1)} \,!$$

Only 1-loop contributions due to non-renormalization thm.

Adler, Bardeen, Jackiw, Pi, Shei, Zee

Supercurrent

$$\mathcal{J} \equiv \left\{ J'^{\mu}_{R} \; , \; Q^{\mu}_{\alpha} \; , \; T^{\mu}_{\nu} \right\} , \cdots$$

vector super multiplet

associated to R-invariance — associated to susy — associated to translation inv.

Ferrara + Zumino

supercurrent is represented as vector superfield

$$V_\mu(x,\theta,\bar\theta) = R_\mu(x) - i\theta^\alpha Q_{\mu\alpha}(x) + i\bar\theta_{\dot\alpha}\bar Q^{\dot\alpha}_\mu(x) - 2(\theta\sigma^\nu\bar\theta)T_{\mu\nu}(x) + \cdots$$

- $J'^{\mu}_{R} \neq J^{\mu}_{R}$

∴ $J'^{\mu}_{R} = J^{\mu}_{R} + O(\hbar)$

In addition

Clark Piquet Sibold

$$S = \{ b_g F^{\mu\nu}F_{\mu\nu} + \cdots , \; b_g \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \cdots ,$$

Super trace anomaly — trace anomaly of T^{μ}_{ν} — anomaly of R-current

$$b_g \lambda^\beta \sigma^{\mu\nu}_{\alpha\beta}F_{\mu\nu} + \cdots , \; \cdots \}$$

chiral super multiplet

trace anomaly of susy current

There is a relation, whose structure is independent from the renormalization scheme, although individual coefficients (except the 1-loop values of b-functions) may be scheme dependent

$$r = b_g (1 + x_g) + b_{ijk} \, x^{ijk} - \gamma_A \, r^A$$

radiative corrections

linear combinations of anomalous dims

unrenormalized coefficients of anomalies associated to chiral inv. of superpotential

Thm: If (i) no gauge anomaly
(ii) $b^{(1)}(g) = 0$ i.e. no R-current anomaly
(iii) $\gamma^{(1)}{}^i_j = 0$ implies also $r^A = 0$
(iv) exist solutions to $\gamma^{(1)} = 0$ of the form $c_{ijk} = \rho_{ijk} \, g$, ρ_{ijk} - complex
(v) these solutions are isolated + non-degenerate

when considered as solutions of $\beta^{(1)}_{ijk} = 0$.

- Then each of the solutions can be uniquely extended to a formal power series in g, and the $N=1$ Y-M models depend on the single coupling constant g with a β-function vanishing to all orders.

Proof: Inserting $\beta_{ijk} = \beta_g \dfrac{d f_{ijk}}{dg}$ in the identity and taking into account the vanishing of r, r^A

$$\rightsquigarrow \quad 0 = \beta_g \left(1 + O(\hbar) \right)$$

Its solution (as formal power series in $\hbar$) is: $\beta_g = 0$

and $\beta_{ijk} = 0$ too. //

2-loop RGEs for SSB parameters

Martin - Vaughn - Yamada - Jack - Jones '94

Consider $N=1$ gauge thy with

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

and SSB terms

$$-\mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j$$

$$+ \frac{1}{2} (m^2)^i_j \phi^{*i} \phi_j + \frac{1}{2} M \lambda\lambda + \text{h.c.}$$

- 1-loop finiteness conditions

$$h^{ijk} = -M Y^{ijk}$$

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j \qquad \text{universality}$$

in addition to $\beta_g^{(1)} = \gamma^{(1)i}_j = 0$

- - 1-loop finiteness

$\rightarrow$ 2-loop finiteness

Assuming

- $b_g^{(1)} = \gamma^{(1)j}_{i} = 0$
- the reduction eq

$$b_Y^{ijk} = b_g \, dY^{ijk}/dg$$

admits power series solution

$$Y^{ijk} = g \sum_{n=0} \rho_{(n)}^{ijk} g^{2n}$$

- $(m^2)^i_j = m_j^2 \delta^i_j$

$$\rightsquigarrow (m_i^2 + m_j^2 + m_k^2)/MM^* = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)}$$

for i,j,k with $\rho_{(0)}^{ijk} \neq 0$

where $\Delta^{(2)} = -2 \sum_l \left[(m_l^2/MM^*) - \frac{1}{3} \right] \ell(R_l)$

- $\Delta^{(2)} = 0$ for $N=4$ with 5Tr cond
- $\Delta^{(2)} = 0$ for the $N=1$, $SU(5)$ FUTs!

The SU(5) finite model

Kapetanakis, Mondragon, Z

Kobayashi, Kubo, Mondragon, Z

Content

$$3(\bar{5}+10) + 4(\overset{H_\alpha}{5}+\overset{\bar{H}_\alpha}{\bar{5}}) + 24$$

fermion supermultiplets

scalar supermultiplets

Jones-Raby
Hamidi-Schwarz
Quiros et.al
Kazakov
Babu-Enkhba
Gogoladze

$$\rightsquigarrow W = \sum_{i=1}^{3}\left[\frac{1}{2} g_i^u\, 10_i 10_i H_i + g_i^d\, 10_i \bar{5}_i \bar{H}_i\right]$$
$$+ g_{23}^u\, 10_2 10_3 H_4 + g_{23}^d\, 10_2 \bar{5}_3 \bar{H}_4 + g_{32}^d\, 10_3 \bar{5}_2 \bar{H}_4$$
$$+ \sum_{\alpha=1}^{4} g_\alpha^f H_\alpha 24 \bar{H}_\alpha + g^\lambda/3\,(24)^3$$

(with enhanced discrete symmetry after reduction of couplings)

We find

$$b_g^{(1)} = 0$$

$$b_{i\alpha}^{u(1)} = \frac{1}{16\pi^2}\Big[-\frac{96}{5}g^2 + \sum_{b=1}^{4}(g_{ib}^u)^2 + 3\sum_{j=1}^{3}(g_{ja}^u)^2$$

$$+ \frac{24}{5}(g_\alpha^f)^2 + 4\sum_{b=1}^{4}(g_{ib}^d)^2\Big]g_{i\alpha}^u$$

$$b_{i\alpha}^{d(1)} = \frac{1}{16\pi^2}\Big[-\frac{84}{5}g^2 + 3\sum_{b=1}^{4}(g_{ib}^u) + \frac{24}{5}(g_\alpha^f)$$

$$+ 4\sum_{j=1}^{3}(g_{j\alpha}^d)^2 + 6\sum_{b=1}^{4}(g_{ib}^d)^2\Big]g_{i\alpha}^d$$

$$b^{\lambda(1)} = \frac{1}{16\pi^2}\Big[-30g^2 + \frac{63}{5}(g^\lambda)^2 + 3\sum_{\alpha=1}^{4}(g_\alpha^f)^2\Big]g^\lambda$$

$$b_\alpha^{f(1)} = \frac{1}{16\pi^2}\Big[-\frac{98}{5}g^2 + 3\sum_{i=1}^{3}(g_{i\alpha}^u)^2$$

$$+ 4\sum_{i=1}^{3}(g_{i\alpha}^d)^2 + \frac{48}{5}(g_\alpha^f)^2$$

$$+ \sum_{b=1}^{4}(g_b^f)^2 + \frac{21}{5}(g^\lambda)^2\Big]g_\alpha^f$$

Considering g as the primary coupling, we solve the Red. Eqs.

$$\beta_g = \beta_a \frac{dg}{dg_a}$$

requiring power series ansatz.

$$\Rightarrow (g^u_{ii})^2 = \frac{8}{5} g^2 + \cdots, \quad (g^d_{ii})^2 = \frac{6}{5} g^2 + \cdots$$

$$(g^\lambda)^2 = \frac{15}{7} g^2 + \cdots, \quad (g^f_4)^2 = g^2, \quad (g^f_\alpha)^2 = 0 + \cdots \; (\alpha = 1,2,3)$$

Higher order terms can be uniquely determined.

$\Rightarrow$ All 1-loop β-functions vanish

Moreover

All 1-loop anomalous dimensions of chiral fields vanish.

$$\gamma^{(1)}_{10i} = \frac{1}{16\pi^2}\left[-\frac{36}{5}g^2 + 3\sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + 2\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{5}i} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{H\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 3\sum_{i=1}^{3}\left(g^u_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{\bar{H}\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{i=1}^{3}\left(g^d_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{24} = \frac{1}{16\pi^2}\left[-\frac{10}{5}g^2 + \sum_{\alpha=1}^{4}\left(g^f_\alpha\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\right]$$

$\Rightarrow$ Necessary and sufficient conds for finiteness to all orders are satisfied

- SU(5) breaks down to the standard model due to <24>

- Use the freedom in mass parameters to obtain only a pair of Higgs fields light, acquiring v.e.v.

- Get rid of unwanted triplets rotating the Higgs sector (after a fine tuning)

  see Quiros et. al., Kazakov et. al
  Yioshioka

- Adding soft terms we can achieve SUSY breaking.

1) Gauge Couplings Unification

$\sin^2\theta_W$, α_{em} $\rightsquigarrow$ $\alpha_3(M_Z)$ Marciano+Senjanovic ⋮ Amaldi et. al.

2) Bottom-Tau Yukawa Unif.

SU(5)-type

$\rightsquigarrow$ $m_t \sim 100-200$ GeV Barger et. al. Carena et. al.

*3) Top-Bottom-Tau Yuk Unif.

$h_t^2 = \frac{4}{3} h_{b,\tau}^2$ in SU(5)-FUT

Similar to SO(10) Ananthanarayan et. al. Barger et. al.

$\rightsquigarrow$ $m_t \sim 160-200$ GeV Carena et. al.

*4) Gauge-Top-Bottom-Tau Unif.

e.g. FUT-SU(5): $h_t^2 = \frac{8}{5} g_U^2$; $h_{b,\tau}^2 = \frac{6}{5} g_U^2$

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	54.1	2.2×10^{16}	5.3	183
500	0.122	54.2	1.9×10^{16}	5.3	183
10^3	0.120	54.3	1.5×10^{16}	5.2	184

FUTA

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
800	0.120	48.2	1.5×10^{16}	5.4	174
10^3	0.119	48.2	1.4×10^{16}	5.4	174
1.2×10^3	0.118	48.2	1.3×10^{16}	5.4	174

FUTB

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	M_{GUT} [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	47.9	2.2×10^{16}	5.5	178
500	0.122	47.8	1.8×10^{16}	5.4	178
1000	0.119	47.7	1.5×10^{16}	5.4	178

MIN SU(5)

The predictions for the three models for different M_s

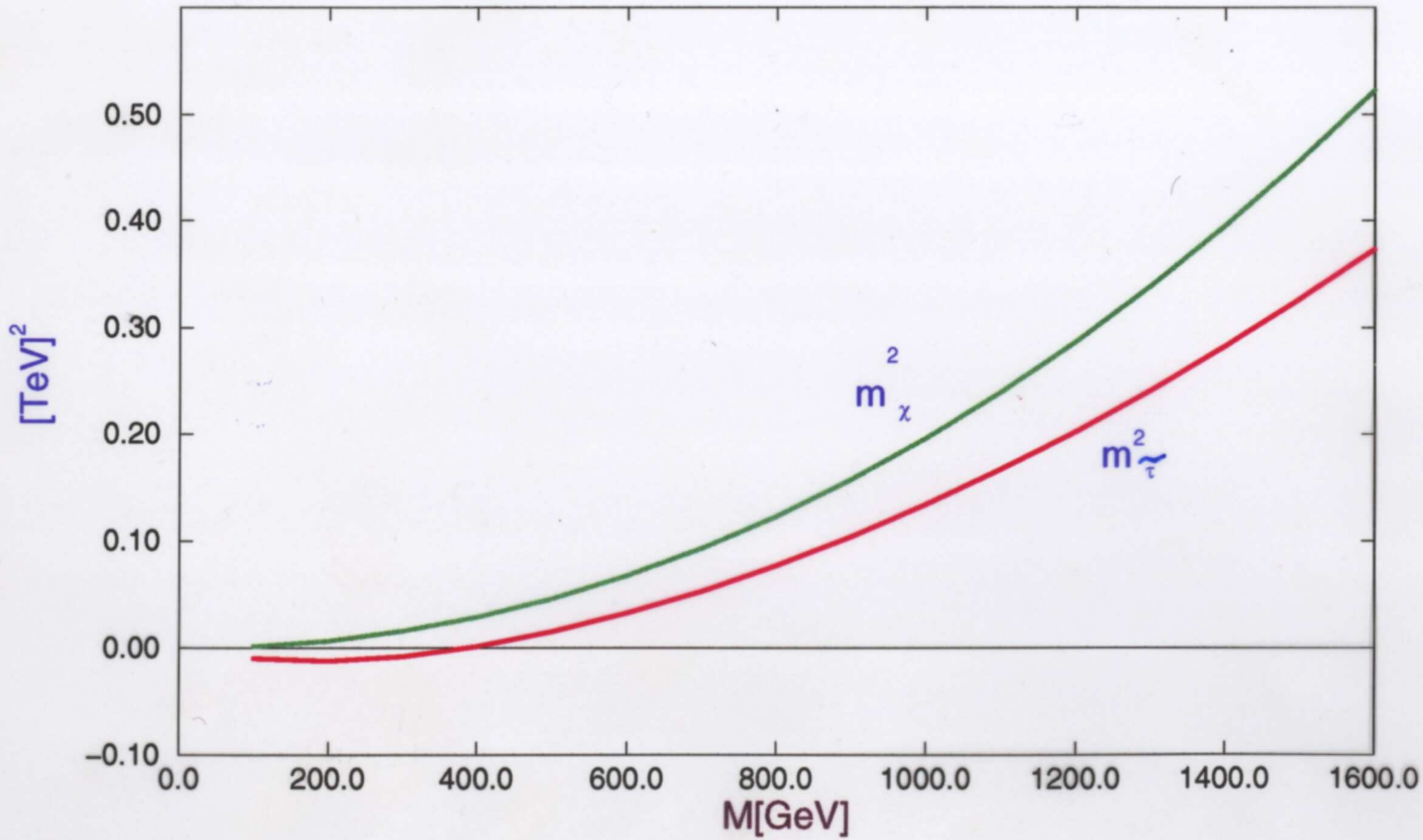
With theoretical corrections and uncertainties 8

$\sim 4\%$

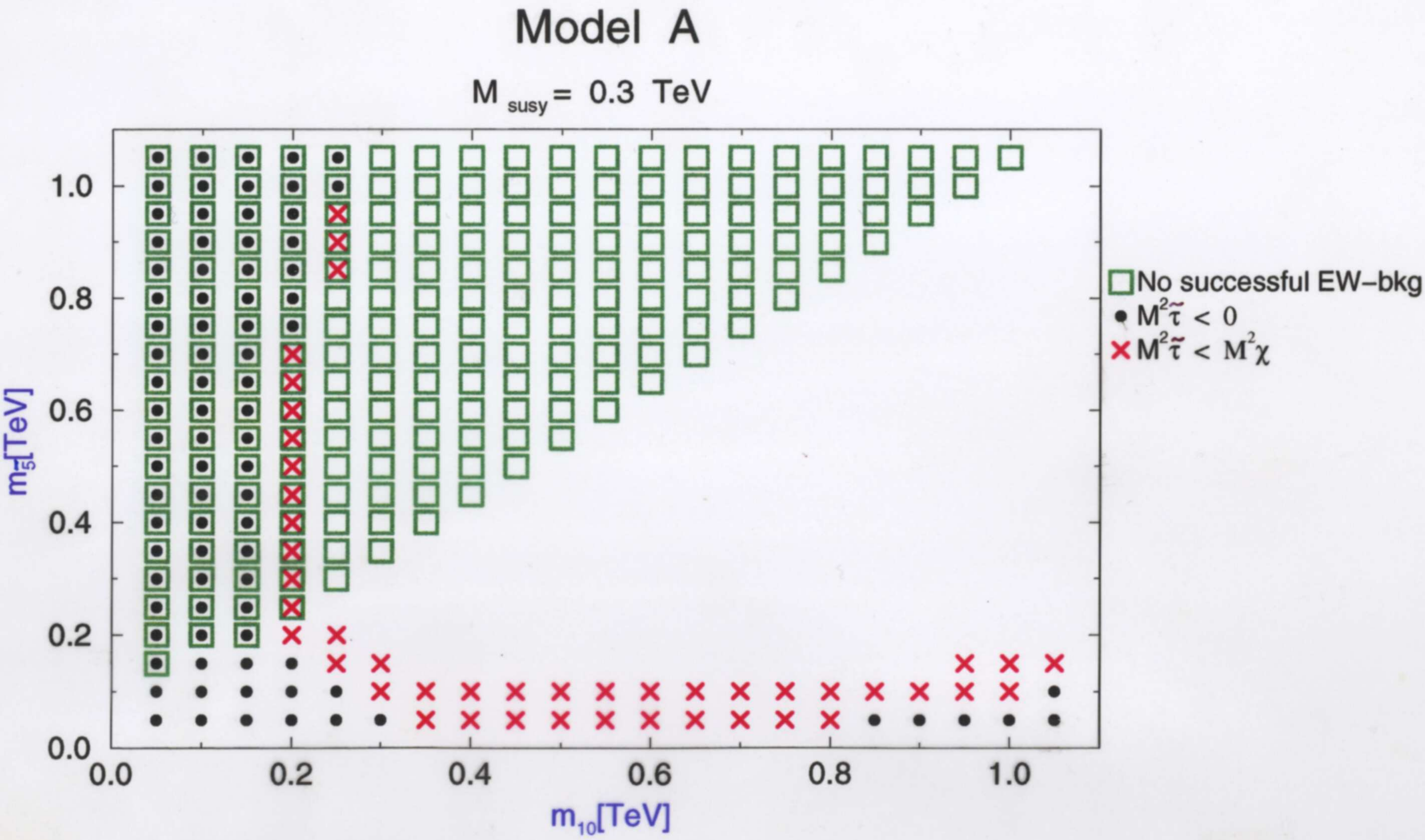
$M_t = 173.8 \pm 5$ GeV ; 178.0 ± 4.3 GeV
CDF + D0

Model A

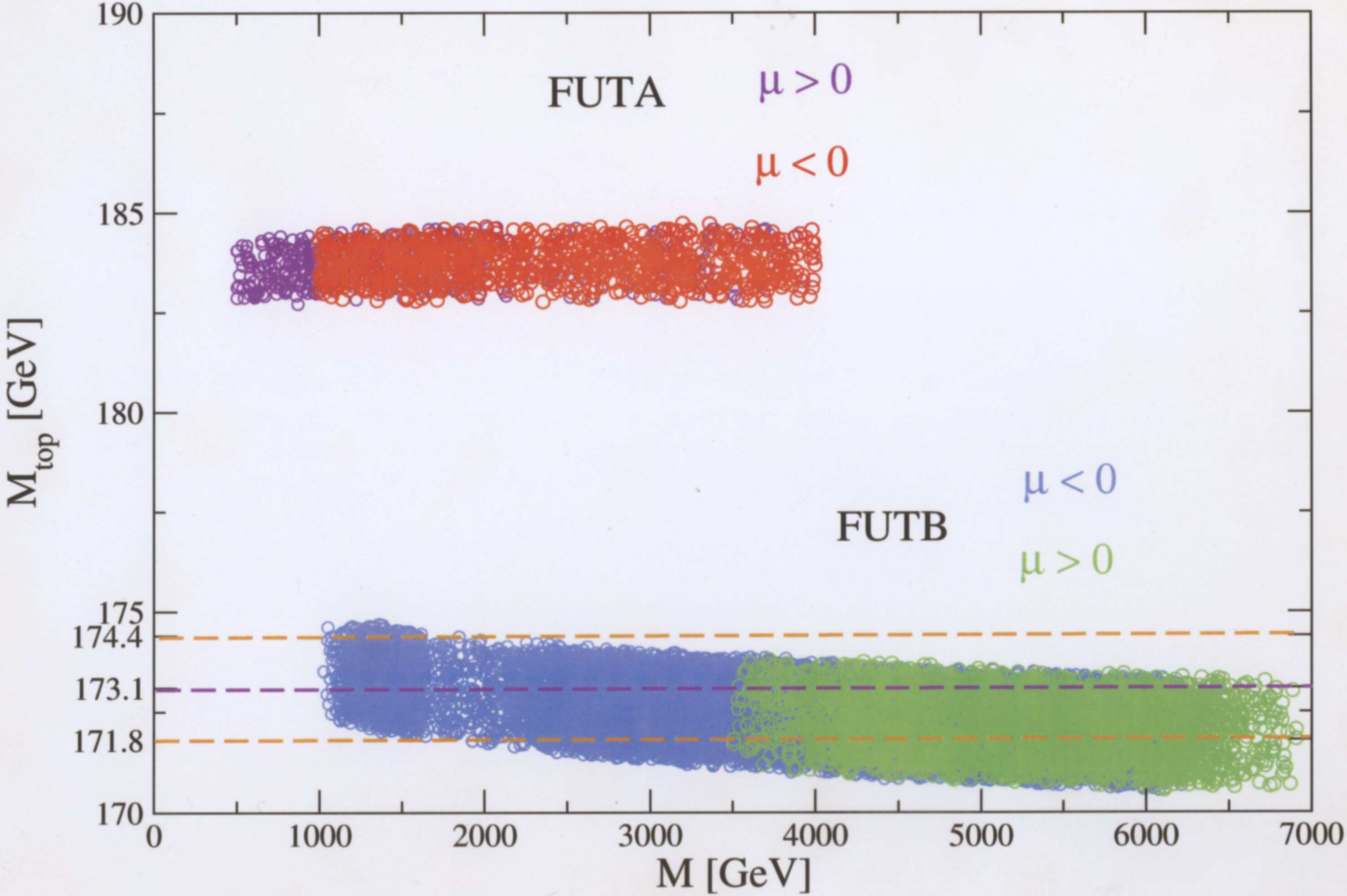
Similar behaviour holds for Model B too



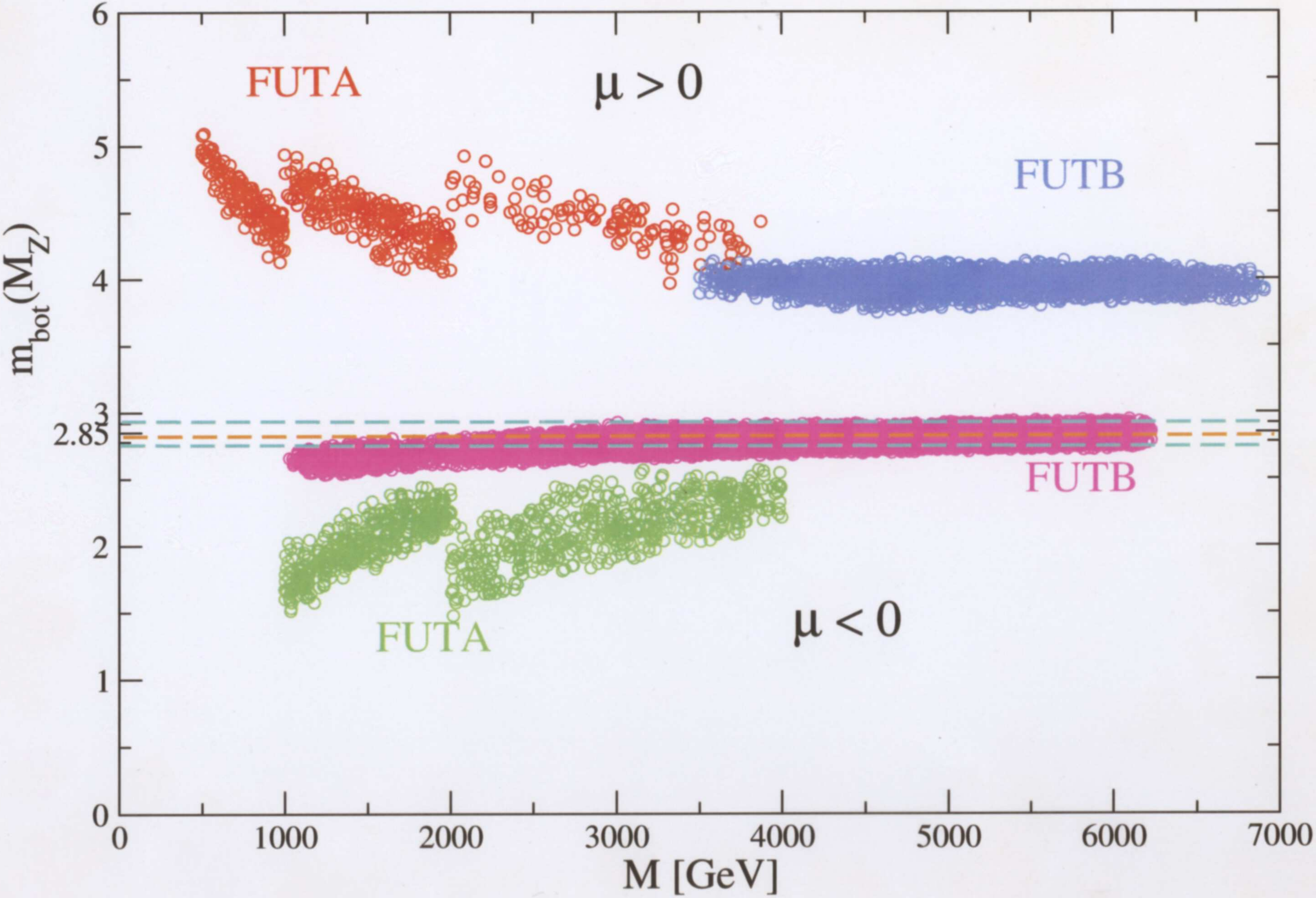
$m^2_{\tilde\tau}$ and m^2_χ for the universal choice of soft scalar masses



The empty region yields a neutralino as LSP

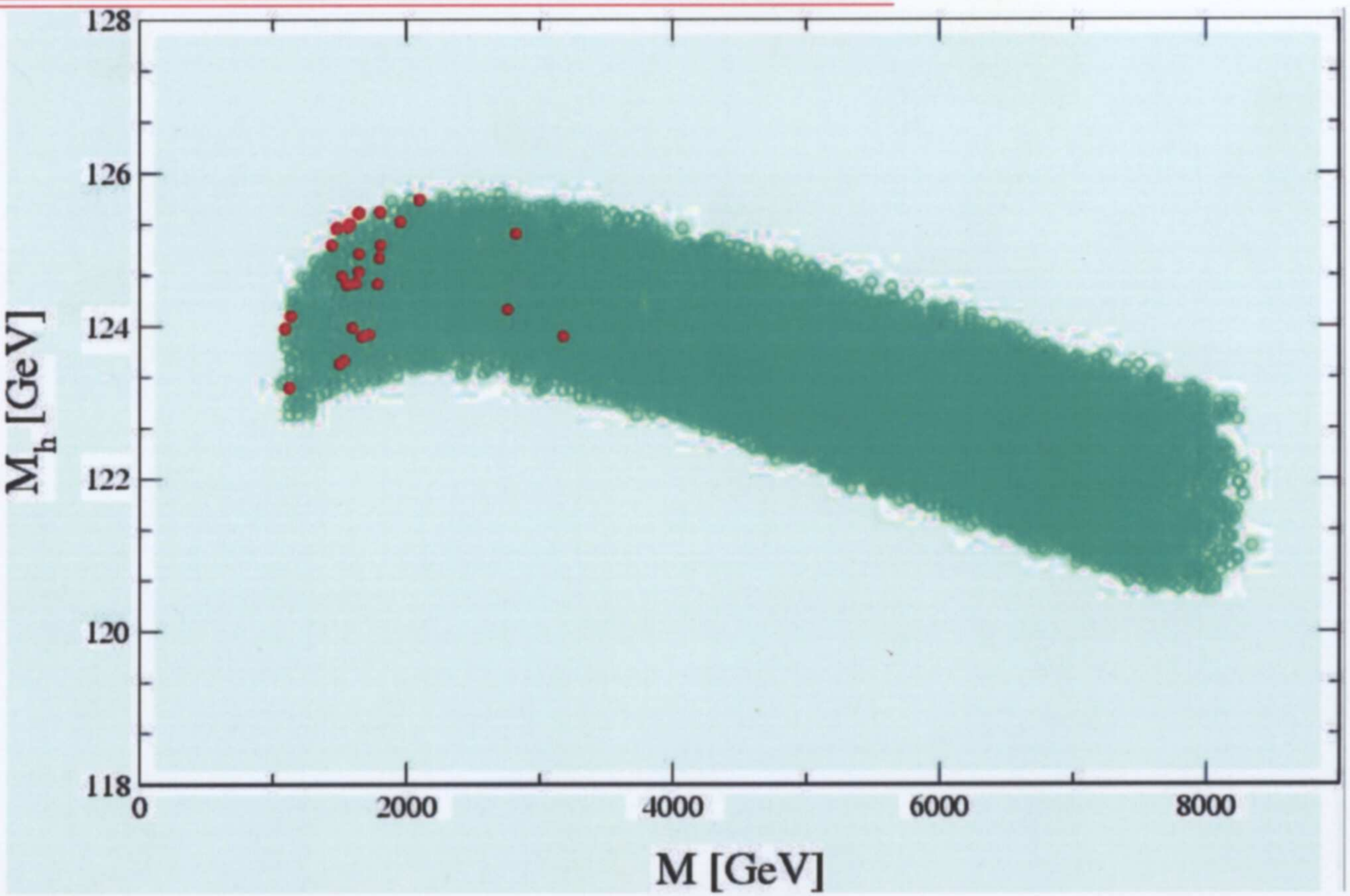


FUTA
μ > 0
μ < 0
FUTB
μ < 0
μ > 0
190
185
180
175
174.4
173.1
171.8
170
0
1000
2000
3000
4000
5000
6000
7000
M [GeV]
M_top [GeV]



FUTA
$\mu > 0$
FUTB
FUTB
FUTA
$\mu < 0$
$m_{bot}(M_Z)$
2.85
M [GeV]
0
1000
2000
3000
4000
5000
6000
7000
0
1
2
3
4
5
6

3D) Predictions for the light Higgs boson



green: consistent with B physics constraints

red: agreement with (loose) CDM bound

$$118 \text{ GeV} \leq M_h \leq 129 \text{ GeV} \quad \text{(incl. theor. unc.)}$$

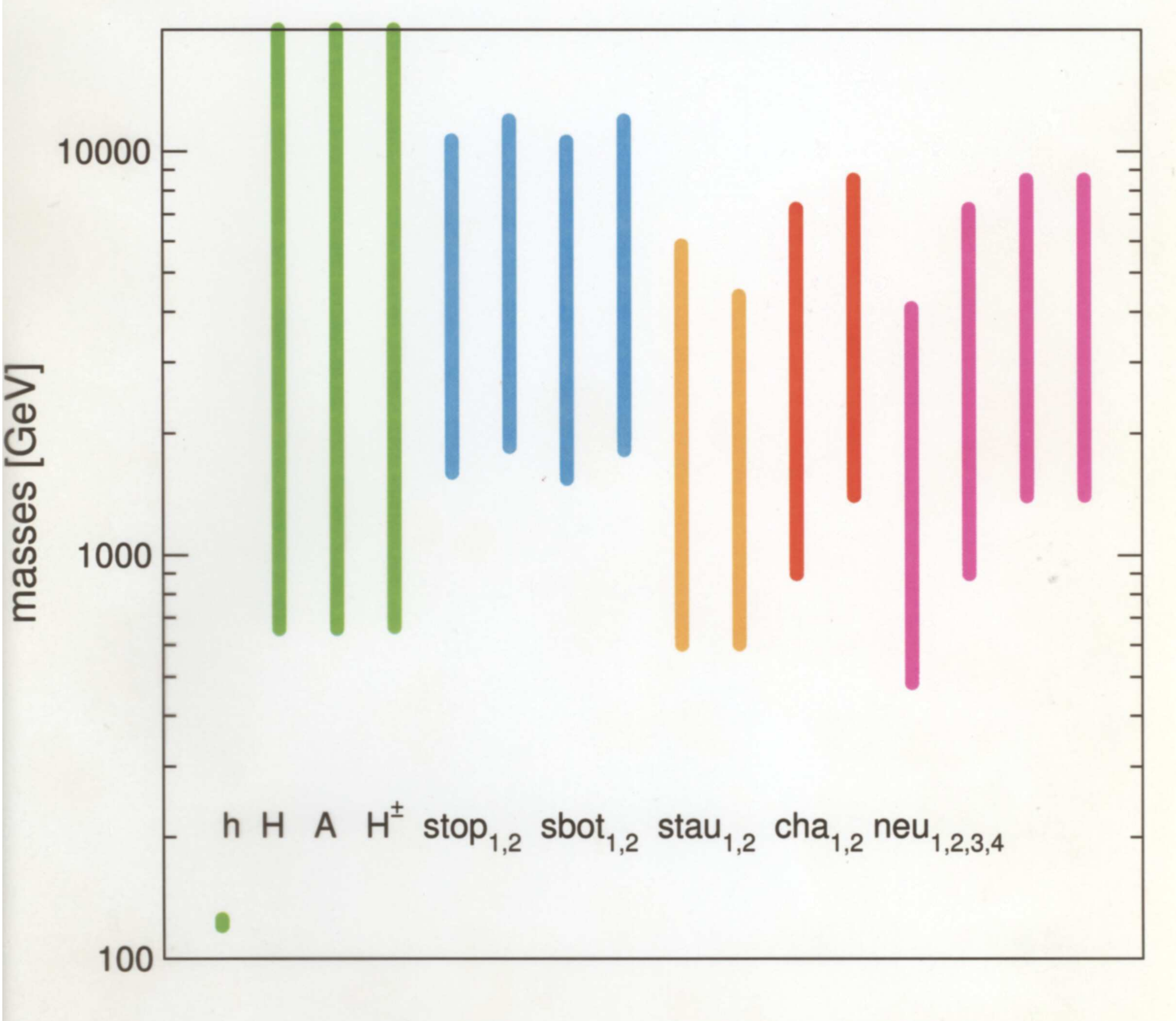
⇒ "easy" to find for LHC (but "only" SM-like . . .)

Typical mass spectrum for FUTB- :

m_t	172	$\overline{m_b}(M_Z)$	2.7
$\tan\beta =$	46	α_s	0.116
$m_{\tilde{\chi}_1^0}$	796	$m_{\tilde{\tau}_2}$	1268
$m_{\tilde{\chi}_2^0}$	1462	$m_{\tilde{\nu}_3}$	1575
$m_{\tilde{\chi}_3^0}$	2048	μ	-2046
$m_{\tilde{\chi}_4^0}$	2052	B	4722
$m_{\tilde{\chi}_1^\pm}$	1462	M_A	870
$m_{\tilde{\chi}_2^\pm}$	2052	$M_{H^\pm}$	875
$m_{\tilde{t}_1}$	2478	M_H	869
$m_{\tilde{t}_2}$	2804	M_h	124
$m_{\tilde{b}_1}$	2513	M_1	796
$m_{\tilde{b}_2}$	2783	M_2	1467
$m_{\tilde{\tau}_1}$	798	M_3	3655

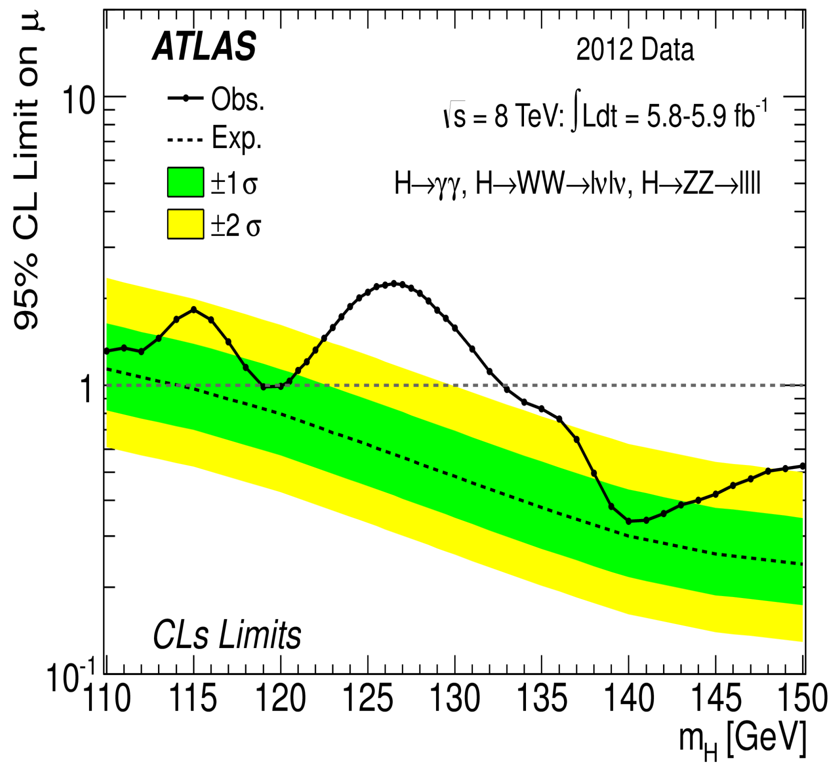
M1	580 GeV
M2	1077 GeV
Mgluino	2754 GeV
Stop1	1876 GeV
Stop2	2146 GeV
Sbot1	1849 GeV
Sbot2	2117 GeV
Mstau1	635 GeV
Mstau2	867 GeV
Char1	1072 GeV
Char2	1597 GeV
Neu1	579 GeV
Neu2	1072 GeV
Neu3	1591 GeV
Neu4	1596 GeV
Mh	123.1 GeV
MH	679 GeV
MA	680 GeV
$\mathrm{MH}^{\pm}$	685 GeV
Mtop	172.2 GeV
Mbot(M_Z)	2.71 GeV

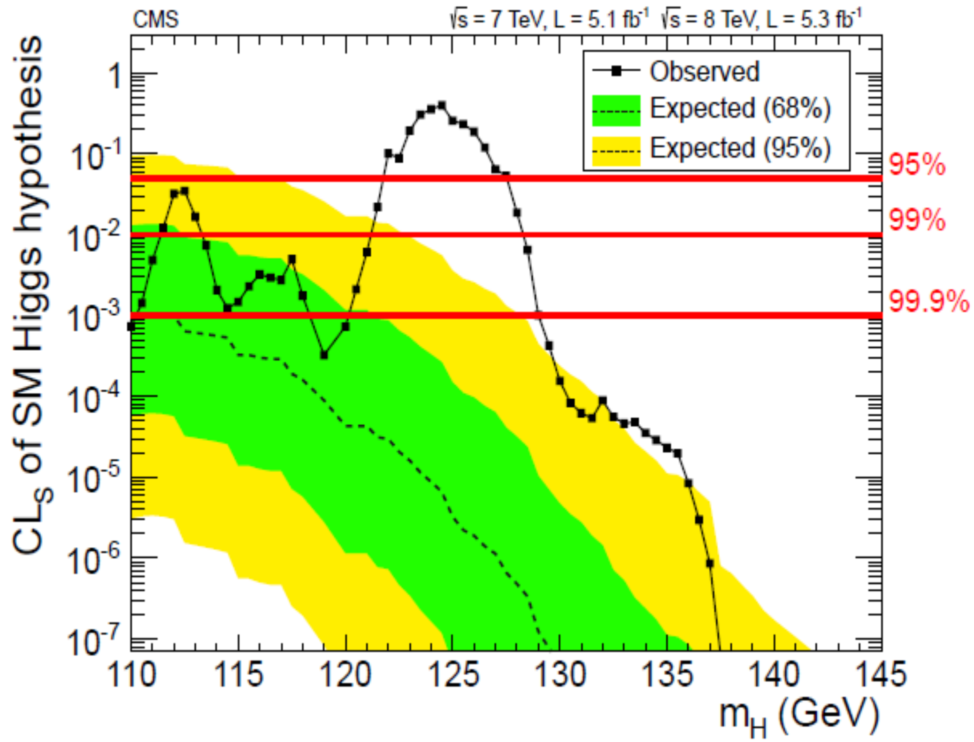
FUTB, $\mu < 0$



masses [GeV]
10000
1000
100
h H A H±
stop1,2
sbot1,2
stau1,2
cha1,2
neu1,2,3,4

It is where the SM predicts it should be



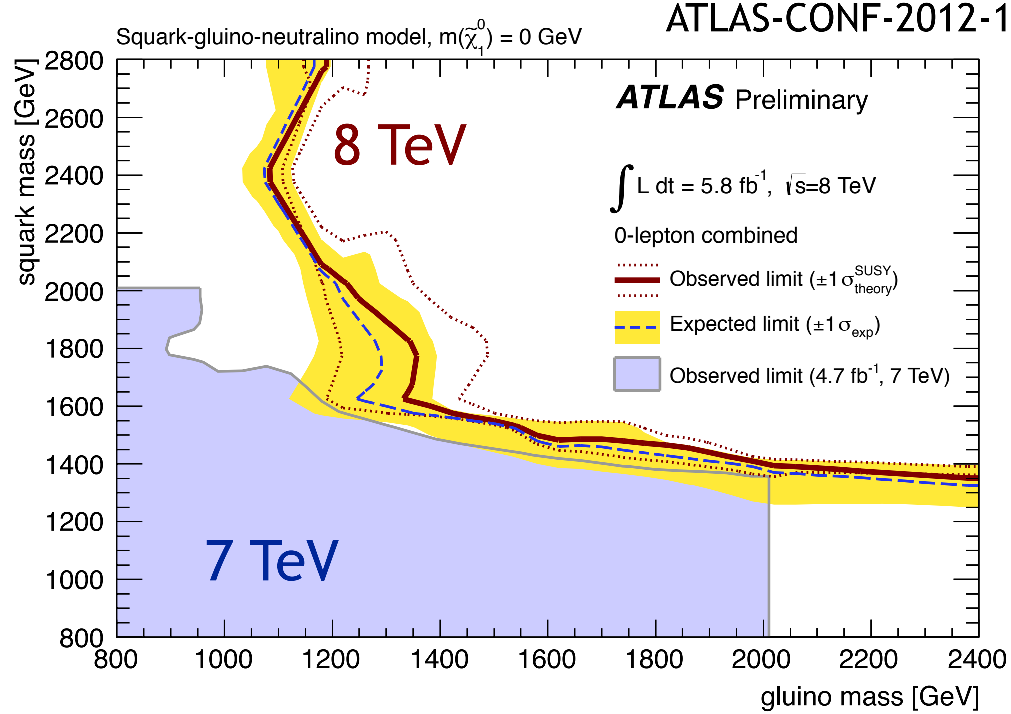


$$M_H = 126 \pm 0.4\,(stat.) \pm 0.4\,(syst.)\;\mathrm{GeV}$$

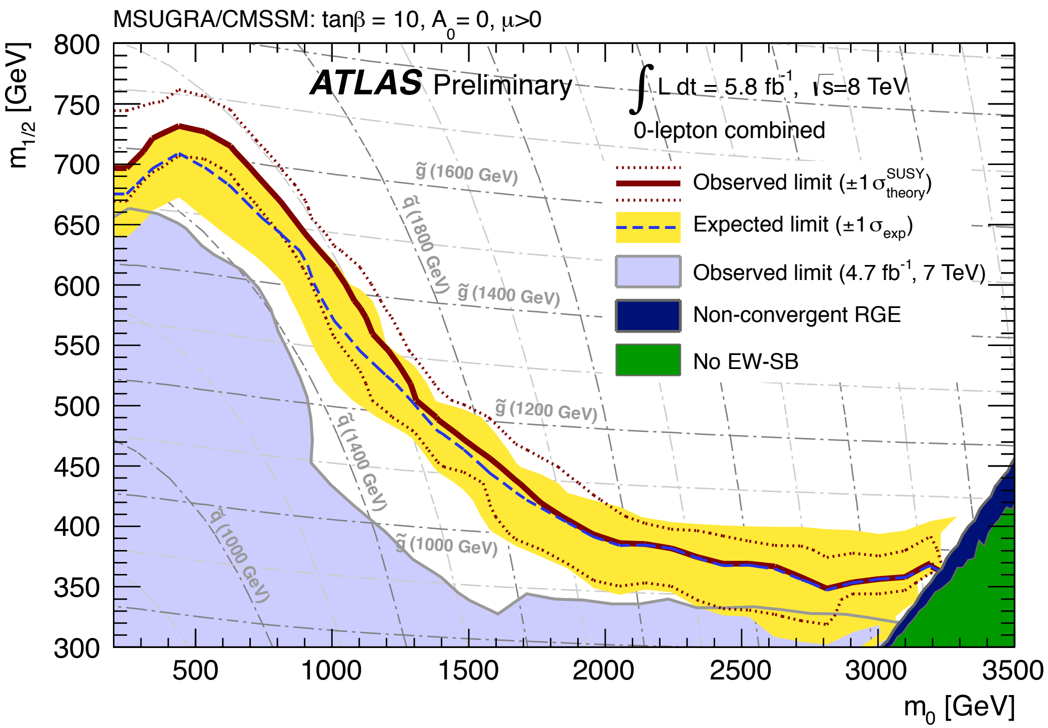
$$M_H = 125.3 \pm 0.4\,(stat.) \pm 0.5\,(syst.)\;\mathrm{GeV}$$

Jets+MET results

- Exclusions in the squark-gluino mass plane for a simplified SUSY model

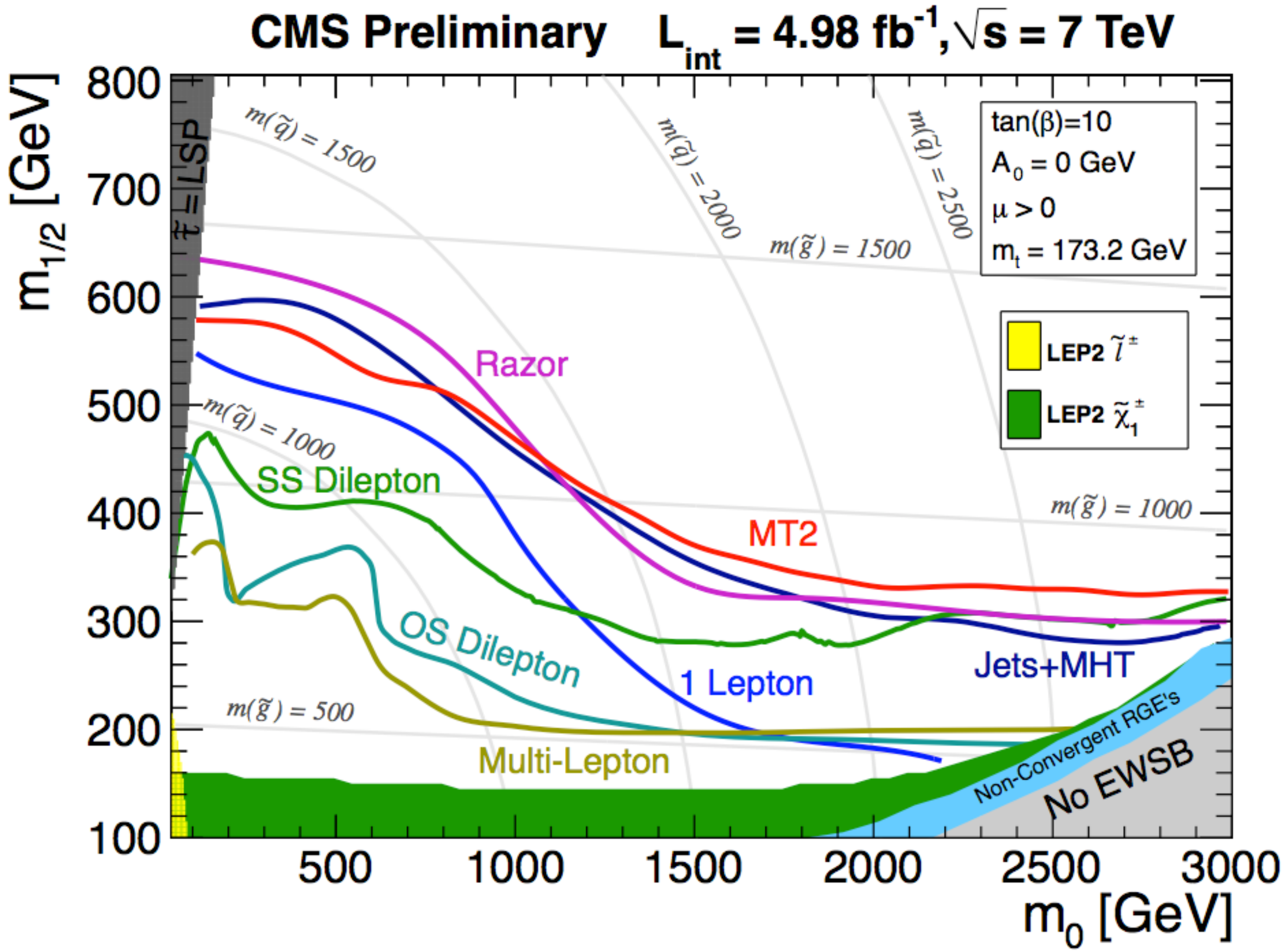


Limits stable up to ~200 GeV mass LSP



- CMSSM ($m_{1/2}$, m_0) plane: equal mass squarks and gluinos excluded below 1500 GeV

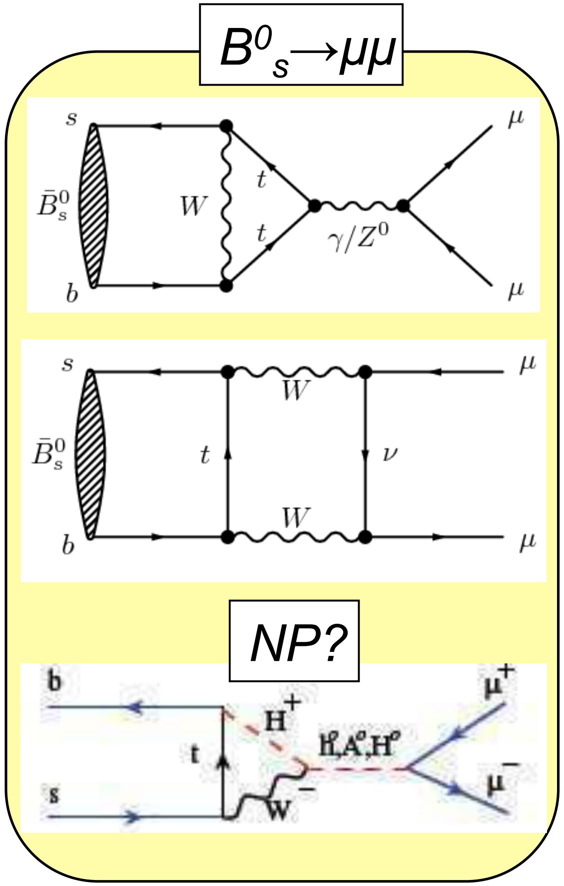
No SUSY (so far).



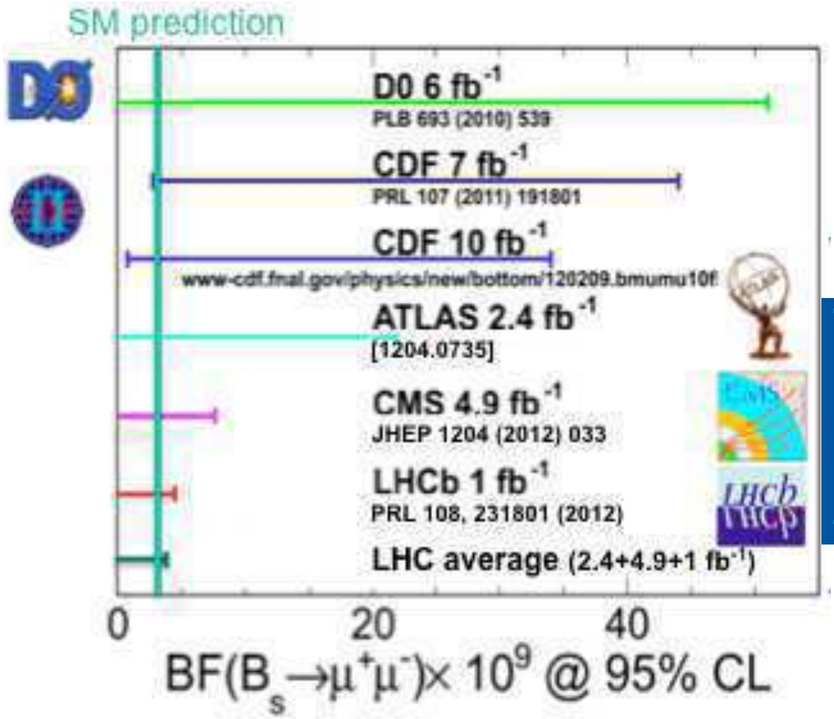
At least within constrained MSS models.

Search for NP in $B_{s(d)} \to \mu^+\mu^-$

- Highly suppressed in SM - FCNC plus helicity $(m_\mu/M_B)^2$ - and well predicted
  - $BR(B_s \to \mu^+\mu^-) = 3.2 \pm 0.03 \ 10^{-9}$
  - $BR(B_d \to \mu^+\mu^-) = 0.11 \pm 0.01 \ 10^{-9}$
    - A.J.Buras et al: arXiv: 1208.0934
- Sensitive to NP
  - Could be strongly enhanced in SUSY
  - In MSSM scales like $\sim\tan^6\beta \to$
- Limit or measurement of $B_{s,d} \to \mu^+\mu^-$ will strongly constraint parameter space



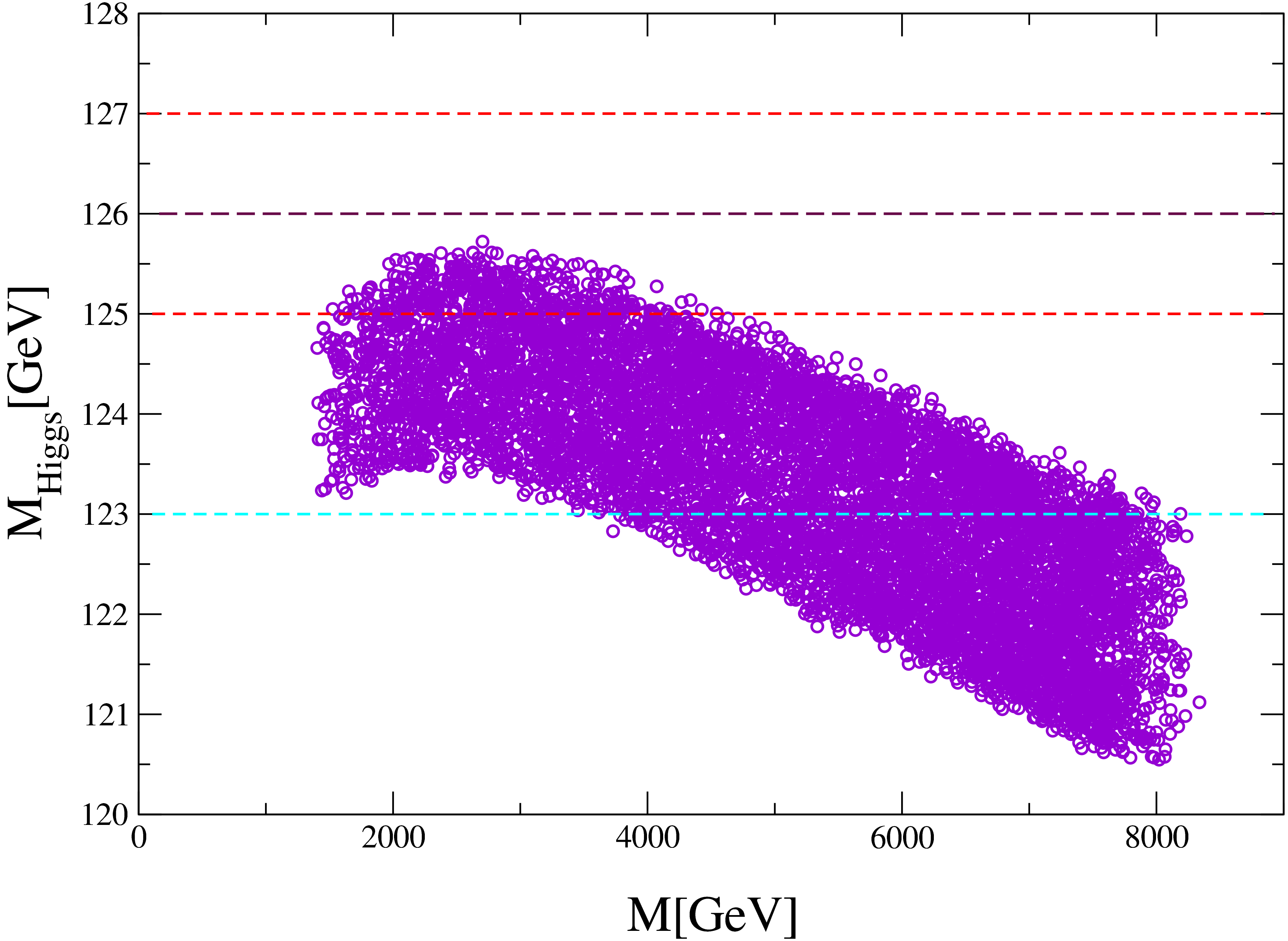
$B_{s(d)} \to \mu^+\mu^-$: summary of exp results



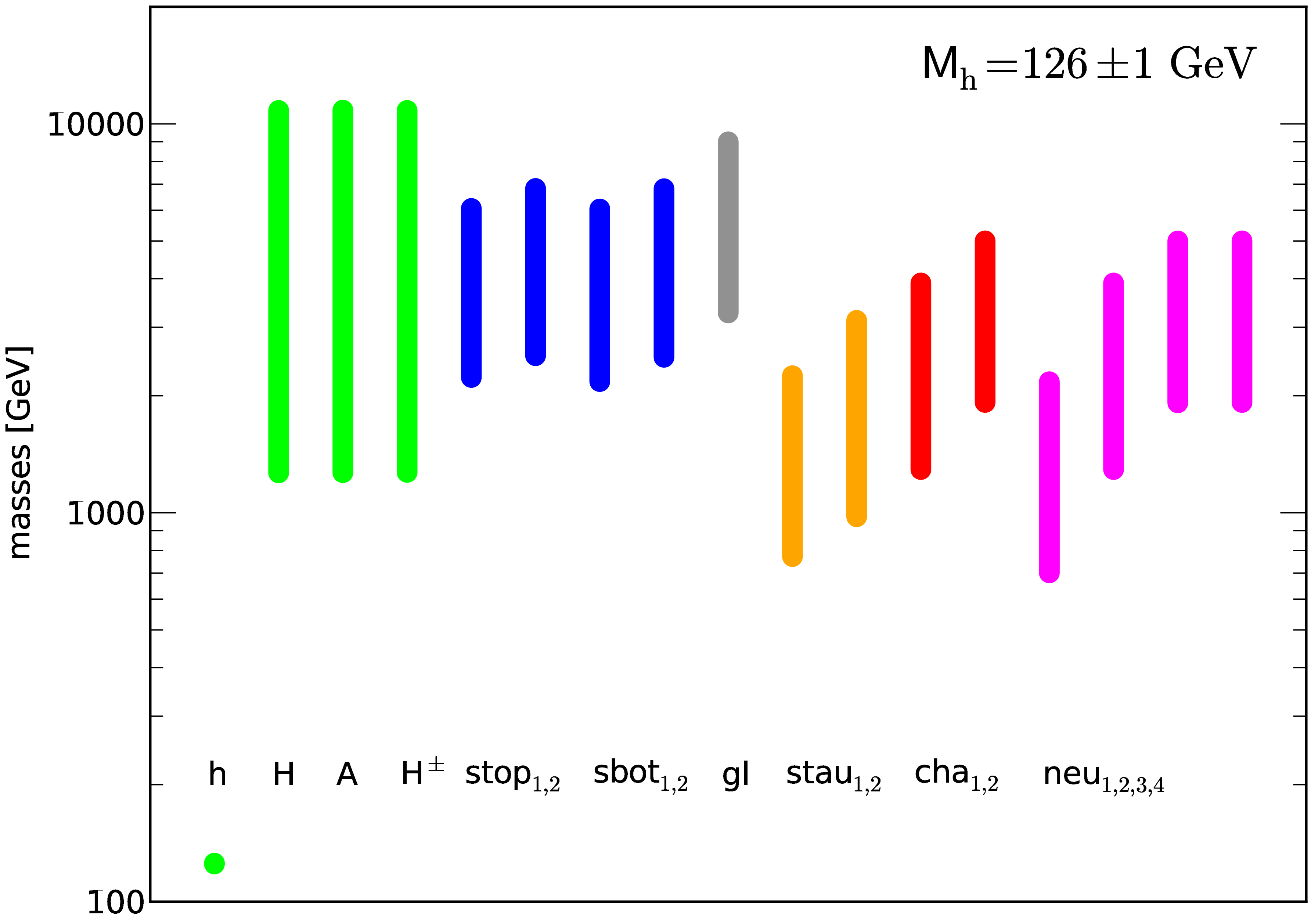
ATLAS/CMS/LHCb combined @95%CL

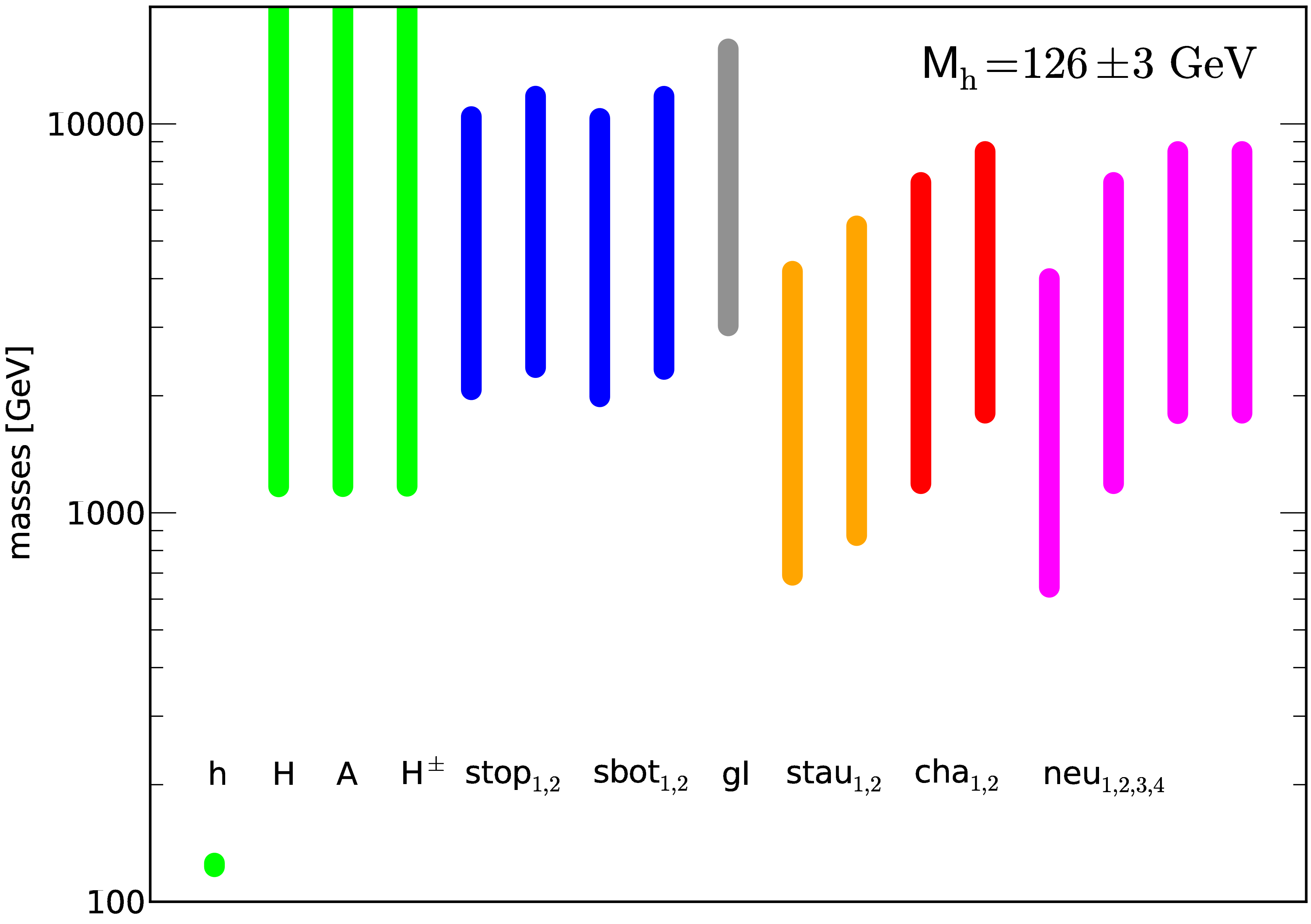
- **BR($B_s \to \mu^+\mu^-$) < 4.2 10^{-9}**
  - Slight excess of events over background, compatible with a SM signal within 1σ
- **BR($B_d \to \mu^+\mu^-$) < 8.1 10^{-10}**

LHCb-CONF-2012-017, also as CMS-PAS-BPH, ATLAS-CONF



128
127
126
125
124
123
122
121
120
0
2000
4000
6000
8000
M_{Higgs}[GeV]
M[GeV]


$M_h = 126 \pm 1$ GeV
masses [GeV]
10000
1000
100
h
H
A
$H^{\pm}$
$\text{stop}_{1,2}$
$\text{sbot}_{1,2}$
gl
$\text{stau}_{1,2}$
$\text{cha}_{1,2}$
$\text{neu}_{1,2,3,4}$



$M_h = 126 \pm 3$ GeV
10000
1000
100
masses [GeV]
h
H
A
$H^{\pm}$
$stop_{1,2}$
$sbot_{1,2}$
gl
$stau_{1,2}$
$cha_{1,2}$
$neu_{1,2,3,4}$