SAPIENZA UNIVERSITÀ DI ROMA and INFN Istituto Nazionale di Fisica Nucleare

INDIRECT SEARCH OF EXOTIC MESONS: $B \to J/\psi$ + All

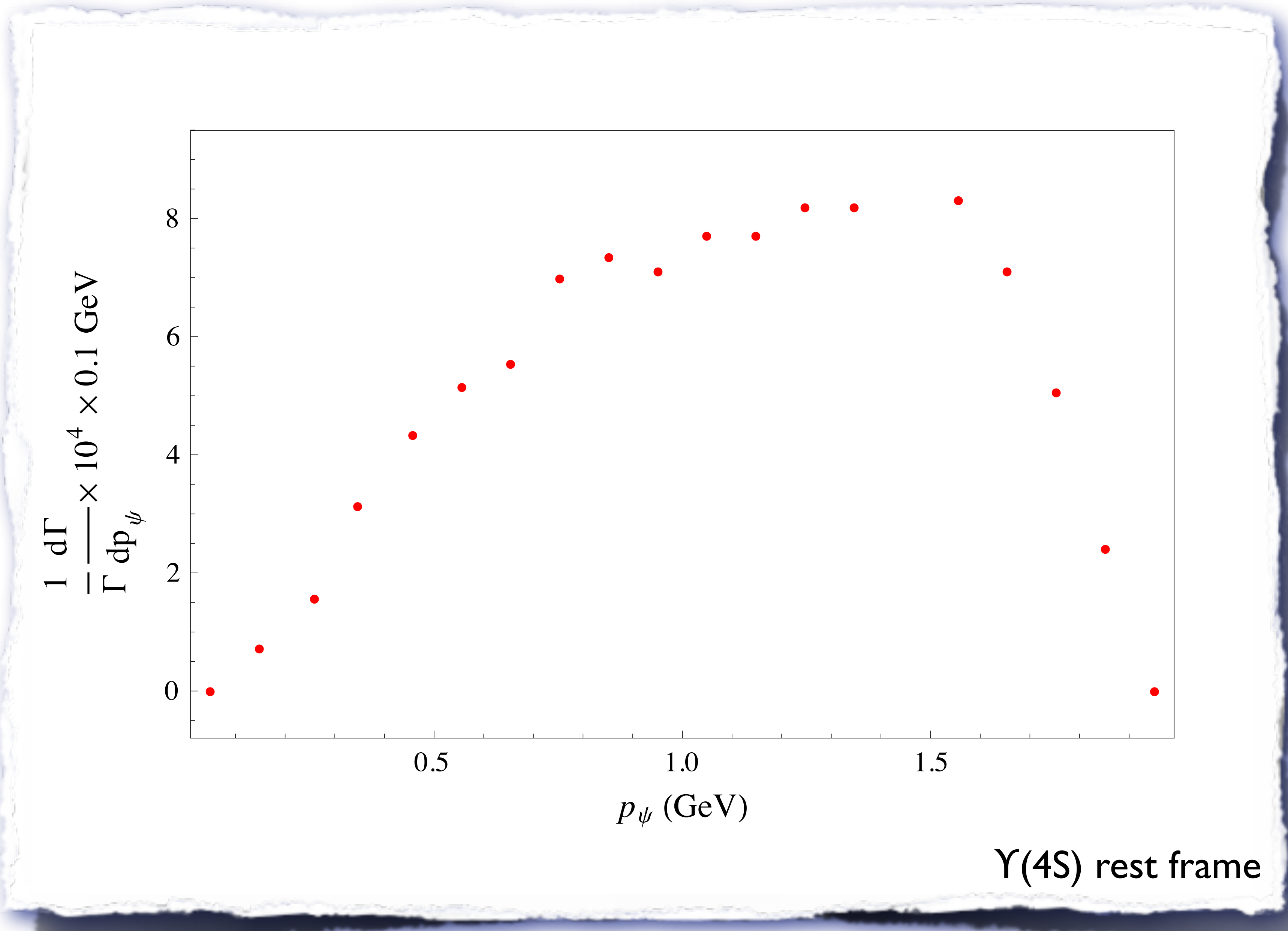
CHIARA SABELLI

Based on **Phys.Rev. D83 (2011) 114029** with
TJ Burns, F Piccinini, AD Polosa and V Prosperi

B → J/ψ + All

:: BaBar, Phys. Rev. D67,032002 ::

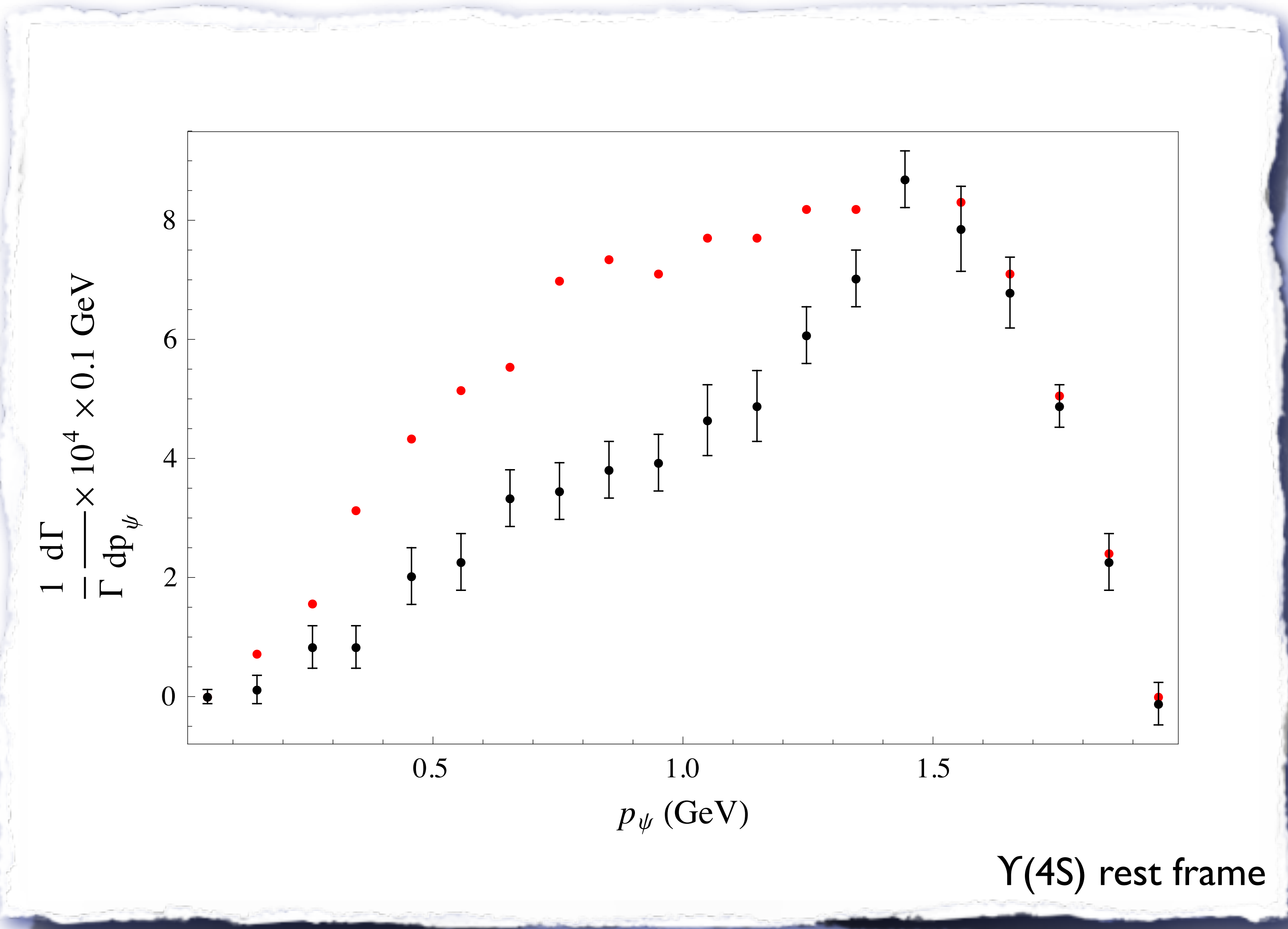
- In e^+e^- collisions at $\sqrt{s} \sim m_{\Upsilon(4S)}$ with 20.3 fb^{-1} they measure: B → J/ψ + All.
- In B → J/ψ + All there is a feed-down from $\chi_{c1,2} \to J/\psi\, \gamma$ and $\psi(2S) \to J/\psi\, \pi^+ \pi^-$.



B → J/ψ + All : direct

:: BaBar, Phys. Rev. D67,032002 ::

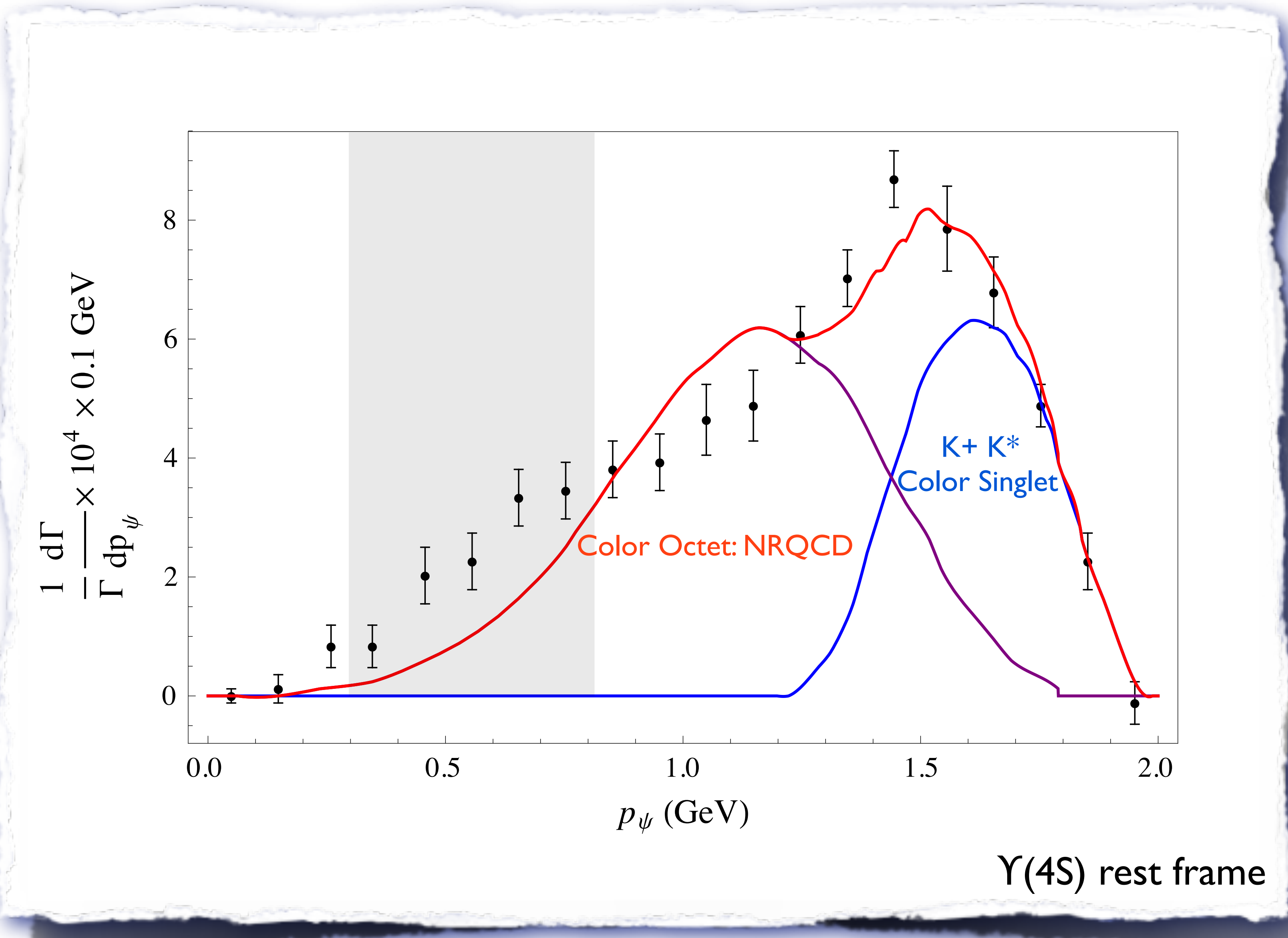
- Subtracting the feed-down from $\chi_{c1,2} \to J/\psi\,\gamma$ and $\psi(2S) \to J/\psi\,\pi^+\pi^-$, they obtain the p^* decay distribution of J/ψ produced directly in B decays.



B → J/ψ + All : direct

:: BaBar, Phys. Rev. D67,032002 ::

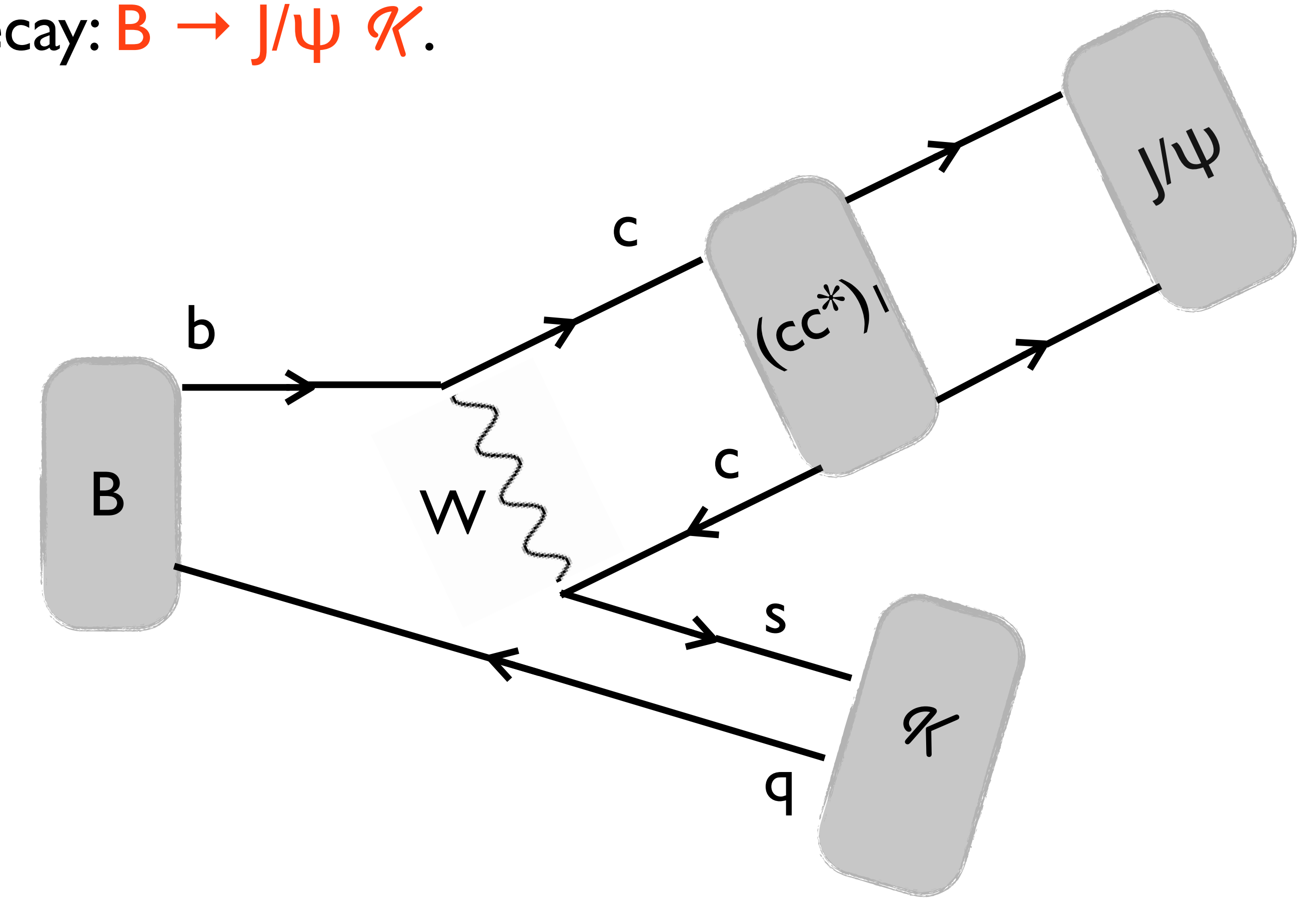
- Subtracting the feed-down from $\chi_{c1,2} \to J/\psi\ \gamma$ and $\psi(2S) \to J/\psi\ \pi^+\pi^-$, they obtain the p^* decay distribution of J/ψ produced directly in B decays.
- Theoretical predictions reveal an excess at low p_ψ.



B → J/ψ + $\mathcal{K}$

:: K. Nakamura *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010) ::

- If the cc* pair is produced in color singlet configuration one has a two body decay: B → J/ψ $\mathcal{K}$.



B^+ DECAY MODES	Fraction (Γ_i/Γ)
Charmonium modes	
$J/\psi(1S)K^+$	$(1.007\pm0.035)\times10^{-3}$
$J/\psi(1S)K^*(892)^+$	$(1.43\pm0.08)\times10^{-3}$
$J/\psi(1S)K(1270)^+$	$(1.8\pm0.5)\times10^{-3}$

$B \to J/\psi + \mathcal{K}$

:: Belle, Phys. Rev. D83,032005 ::

PHYSICAL REVIEW D 83, 032005 (2011)

Study of the $K^+\pi^+\pi^-$ final state in $B^+ \to J/\psi K^+\pi^+\pi^-$ and $B^+ \to \psi' K^+\pi^+\pi^-$

(The Belle Collaboration)

J_1	Submode	Decay fraction
	Nonresonant $K^+\pi^+\pi^-$	$0.152 \pm 0.013 \pm 0.028$
1^+	$K_1(1270) \to K^*(892)\pi$	$0.232 \pm 0.017 \pm 0.058$
	$K_1(1270) \to K\rho$	$0.383 \pm 0.016 \pm 0.036$
	$K_1(1270) \to K\omega$	$0.0045 \pm 0.0017 \pm 0.0014$
	$K_1(1270) \to K_0^*(1430)\pi$	$0.0157 \pm 0.0052 \pm 0.0049$
	$K_1(1400) \to K^*(892)\pi$	$0.223 \pm 0.026 \pm 0.036$
1^-	$K^*(1410) \to K^*(892)\pi$	$0.047 \pm 0.016 \pm 0.015$
2^+	$K_2^*(1430) \to K^*(892)\pi$	$0.088 \pm 0.011 \pm 0.011$
	$K_2^*(1430) \to K\rho$	0.0233 (fixed)
	$K_2^*(1430) \to K\omega$	0.00036 (fixed)
	$K_2^*(1980) \to K^*(892)\pi$	$0.0739 \pm 0.0073 \pm 0.0095$
	$K_2^*(1980) \to K\rho$	$0.0613 \pm 0.0058 \pm 0.0059$
2^-	$K(1600) \to K^*(892)\pi$	$0.0187 \pm 0.0058 \pm 0.0050$
	$K(1600) \to K\rho$	$0.0424 \pm 0.0062 \pm 0.0110$
	$K_2(1770) \to K^*(892)\pi$	$0.0164 \pm 0.0055 \pm 0.0061$
	$K_2(1770) \to K_2^*(1430)\pi$	$0.0100 \pm 0.0028 \pm 0.0020$
	$K_2(1770) \to Kf_2(1270)$	$0.0124 \pm 0.0033 \pm 0.0022$
	$K_2(1770) \to Kf_0(980)$	$0.0034 \pm 0.0017 \pm 0.0011$

TABLE V. Fitted parameters of the signal function for $B^+ \to J/\psi K^+\pi^+\pi^-$, along with the corresponding decay fractions.

$$B^+ \to \mathcal{K}_j J/\psi \to \mathcal{R}_i J/\psi \to J/\psi\, K^+\pi^+\pi^-$$

$$\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi \to \mathcal{R}_i J/\psi \to J/\psi\, K^+\pi^+\pi^-) = \mathcal{B}_{\rm tot} f_i^j$$

B → J/ψ + $\mathcal{K}$

:: Burns, Piccinini, Polosa, Prosperi, Sabelli, arXiv:1104.1781::

- From the fractions we compute the two body branching ratios

$\mathcal{K}_j$	$m_{\mathcal{K}_j}$ (GeV)	$\Gamma_{\mathcal{K}_j}$ (GeV)	$\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi) \times 10^5$	
K	0.494	–	95.0 ± 3.6	*
K^*	0.892	0.050	137.0 ± 7.8	*
$K_1(1270)$	1.270	0.090	144.0 ± 29.3	
$K_1(1400)$	1.403	0.174	25.1 ± 5.7	
$K^*(1410)$	1.414	0.232	$> 5.1 \pm 2.4$ and $< 11.8 \pm 5.7$	
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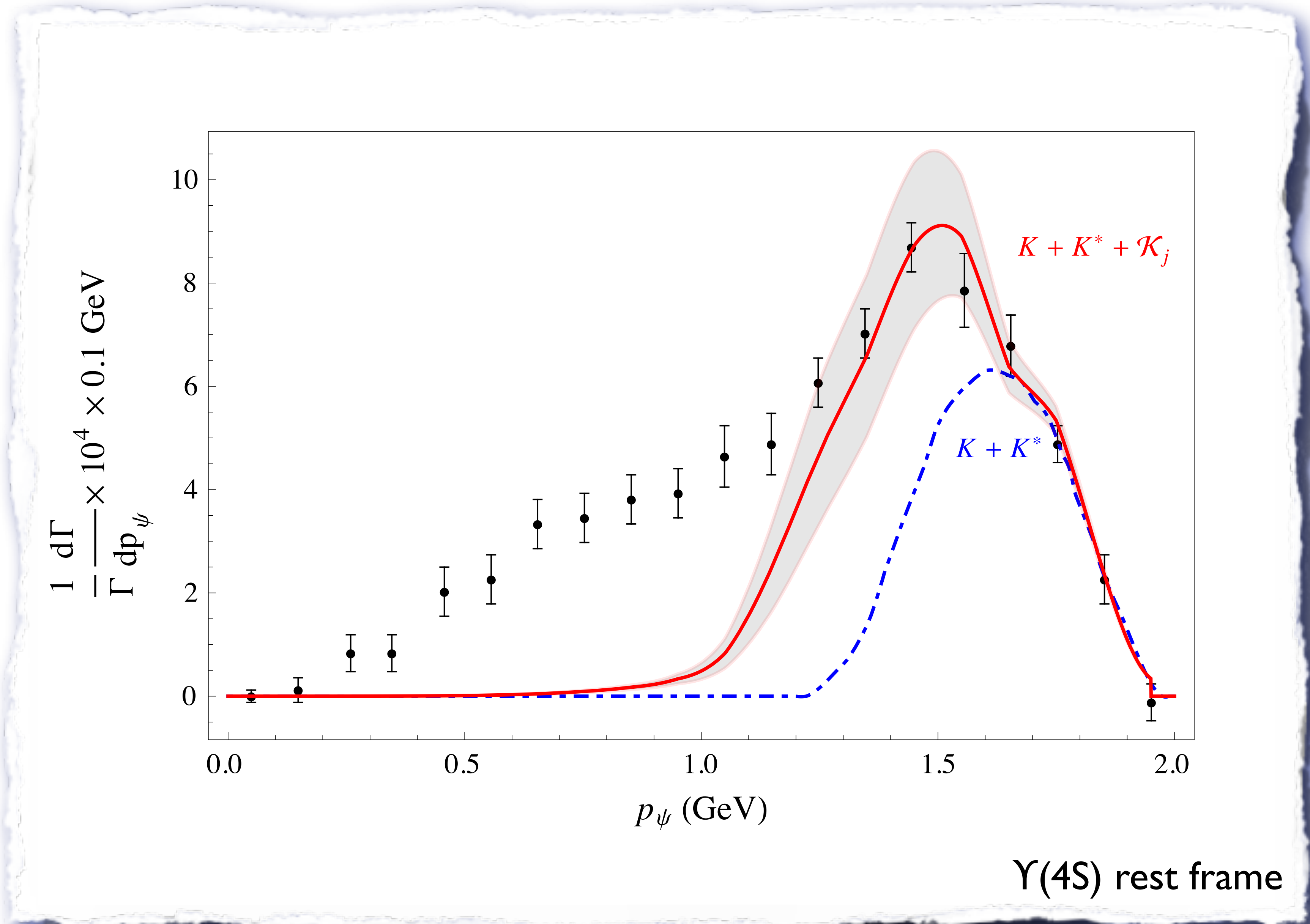
$K_2(1980)$	1.973	0.373	$> 15.2 \pm 2.5$	

* http://hfag.phys.ntu.edu.tw/b2charm/index.html

B → J/ψ + 𝒦

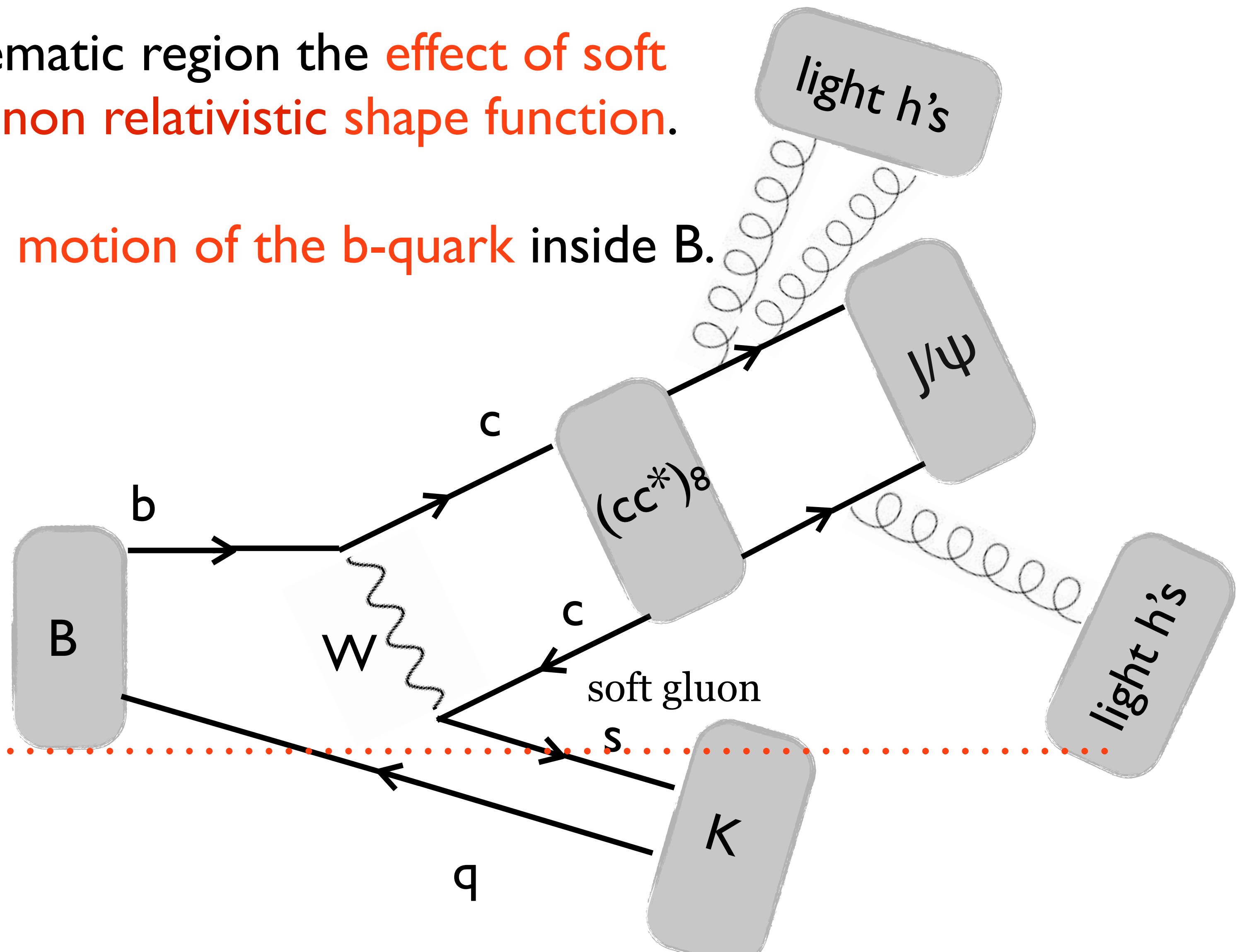
- Two body contributions accounts for the high p_ψ region: we found good agreement for $p_\psi > 1.2$ GeV.



Color Octet Contribution

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- If the cc^* pair is produced in color octet configuration one has a multi body decay.
- NRQCD matrix elements describe the fragmentation $(cc^*)_8 \to J/\psi$.
- Near the extreme endpoint of the kinematic region the effect of soft gluon emission can be modelled with a non relativistic shape function.
- ACCMM model accounts for the Fermi motion of the b-quark inside B.



Hypothesis:
no interaction between the hard part and soft part of the process.

Color Octet Contribution

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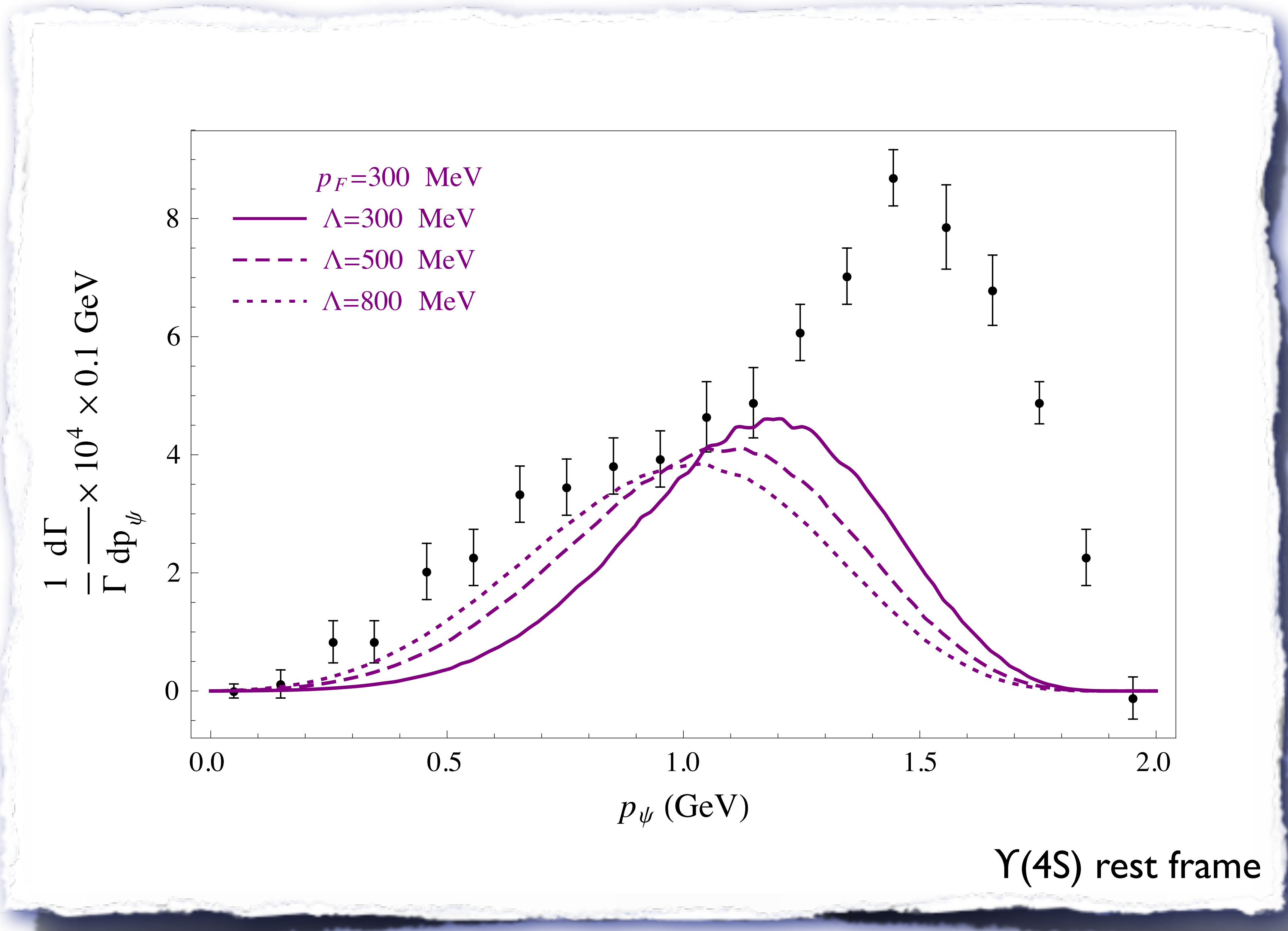
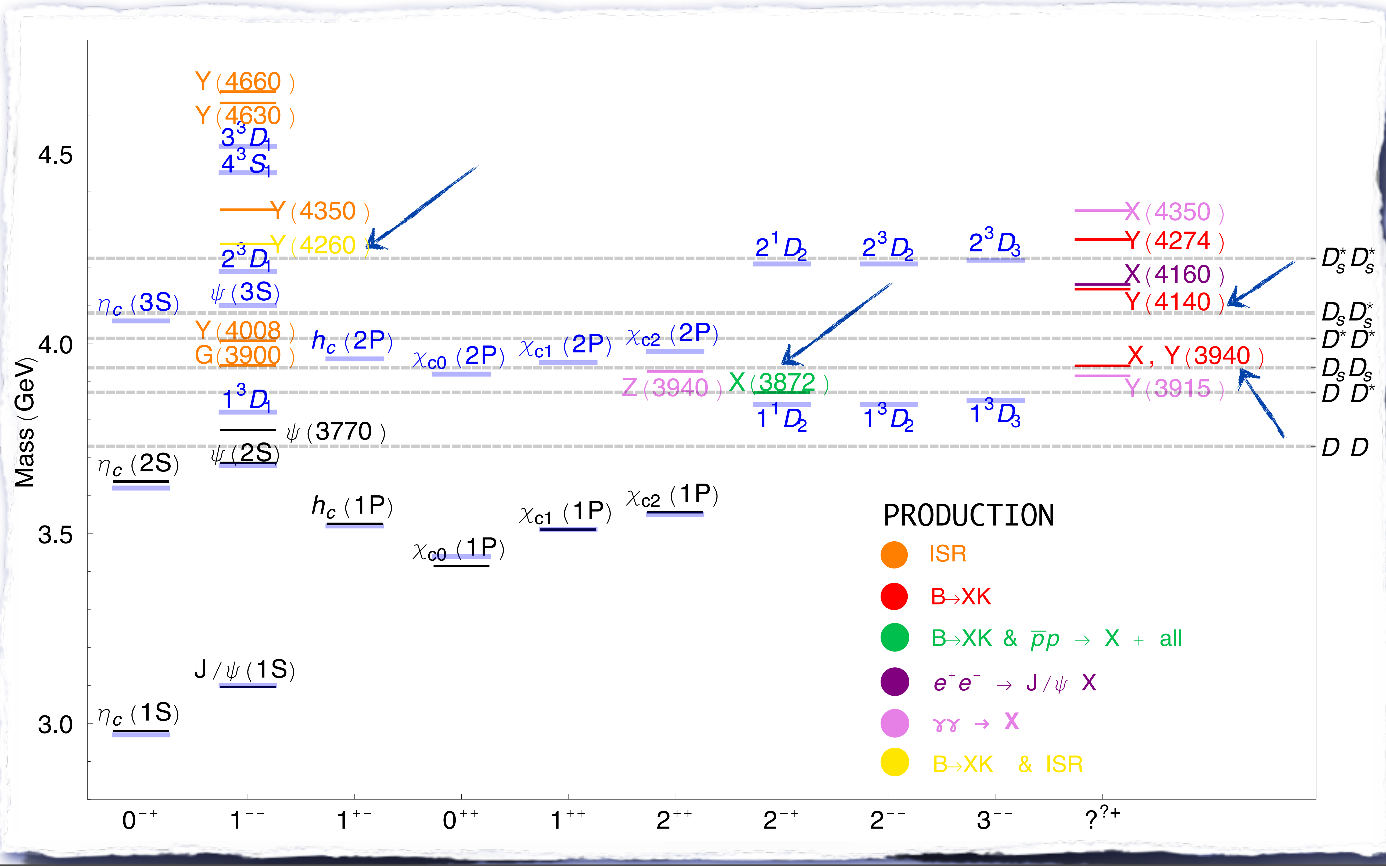
- Two main parameters to model the color octet contribution:
  $\Lambda_{QCD} \in [200,450]$MeV :the characteristic energy-momentum scale of the soft gluons;
  $p_F \in [300,450]$MeV : Fermi momentum of the b-quark inside the B-meson.



Which XYZ contribute to B → J/ψ + All ?



B → $\mathcal{K}\mathcal{X}$ → $\mathcal{K}$ J/ψ + light hadrons

- B → K(500) $\mathcal{X}$ → K(500) J/ψ + light hadrons branching ratios are known:

$\mathcal{X}_j$	$m_{\mathcal{X}_j}$ (GeV)	$\Gamma_{\mathcal{X}_j}$ (GeV)	Final State	$\mathcal{B}(B \to K\mathcal{X}_j \to KJ/\psi + \text{light hadrons}) \times 10^5$
$X(3872)$	3.872	0.003	$J/\psi\ \rho \to J/\psi\ \pi^+\pi^-$	0.72 ± 0.22 [A]
			$J/\psi\ \omega$	0.6 ± 0.3 [B]
$Y(3940)$	3.940	0.087	$J/\psi\ \omega$	3.70 ± 1.14 [C]
$Y(4140)$	4.140	0.012	$J/\psi\ \phi$	0.9 ± 0.4 [D]
$Y(4260)$	4.260	0.095	$J/\psi\ f_0 \to J/\psi\ \pi^+\pi^-$	2.00 ± 0.73 [C]

[A] B. Aubert et al. (BABAR), Phys. Rev. **D77**, 111101 (2008), 0803.2838.
[B] P. del Amo Sanchez et al. (BABAR), Phys. Rev. **D82**, 011101 (2010), 1005.5190.
[C] `http://hfag.phys.ntu.edu.tw/b2charm/index.html`.
[D] K. Yi and f. t. C. collaboration, PoS EPS-HEP **2009**, 2009:085,2009 (2009), 0910.3163.

- For heavy kaons $\mathcal{K}$ we deduce the coupling B-$\mathcal{K}\mathcal{X}$ from the B-K(500)$\mathcal{X}$ one:

<u>Spin 0 $\mathcal{K}$</u> $\langle \mathcal{X}(\epsilon, p)\mathcal{K}(q)|B(P)\rangle = g\,\epsilon \cdot q$

<u>Spin 1 $\mathcal{K}$</u> $\langle \mathcal{X}(\epsilon, p)\mathcal{K}(\eta, q)|B(P)\rangle = g'\,\epsilon \cdot \eta$

$g' = \Lambda\, g$

Λ some mass scale

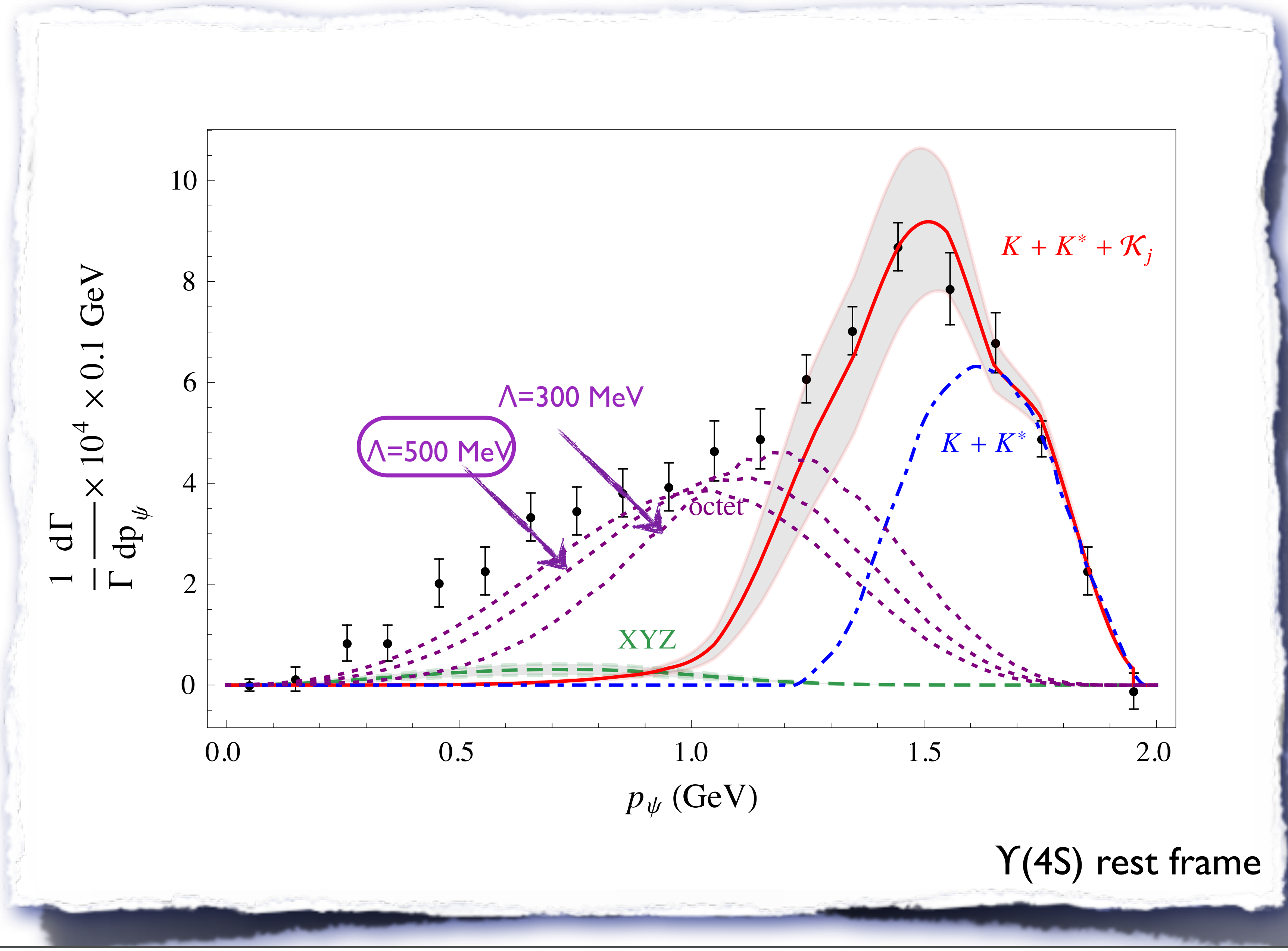
We assume

$\Lambda = m_{K(J=1)}$

taking all $\mathcal{X}$ to be Spin 1 states.

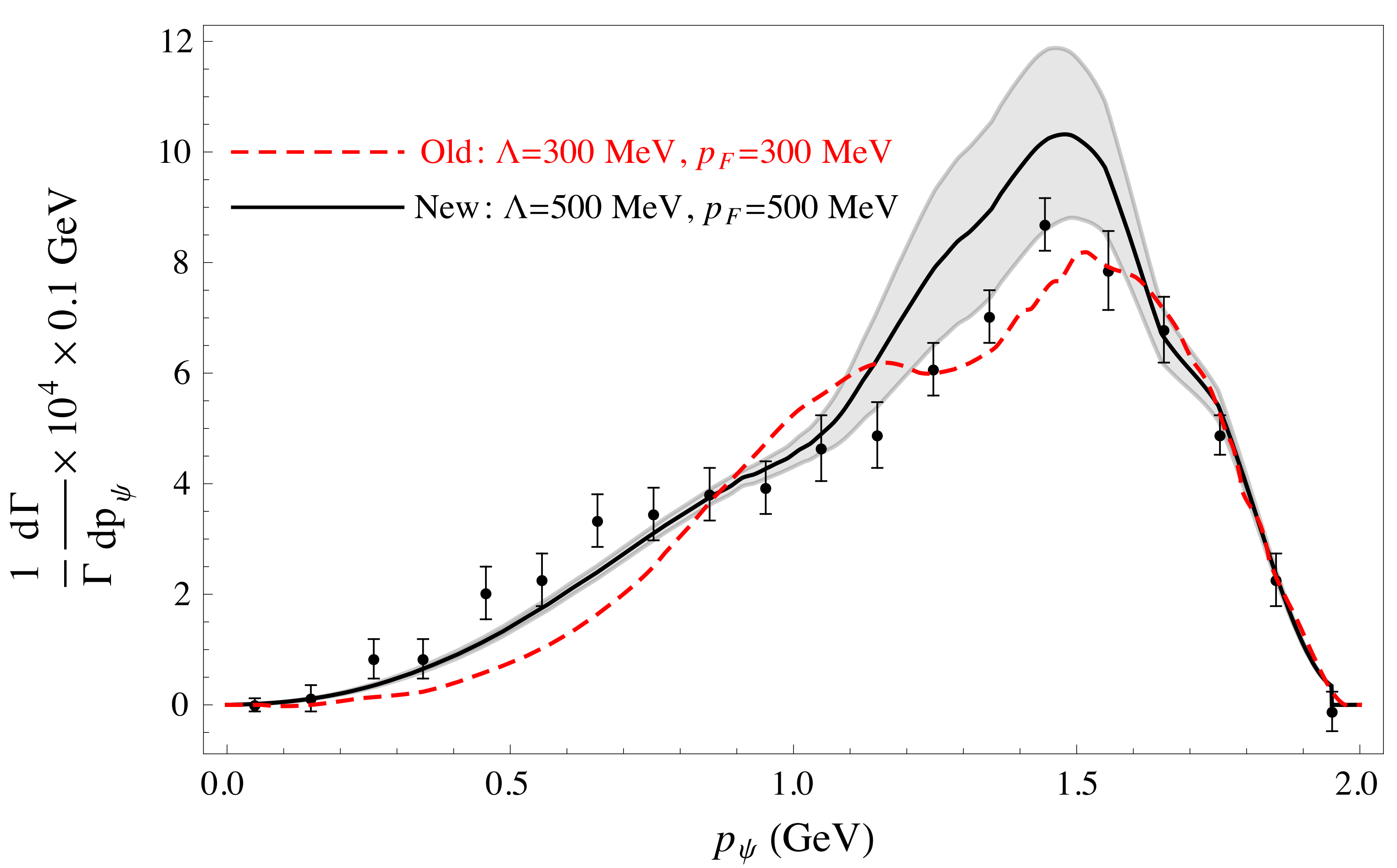
Results (1)

- We simulate the decay B → 𝒦X → 𝒦 J/ψ + light hadrons taking into account the partial decay wave.
- We fit the sum of all the contributions to data using as a free parameter the overall normalization of the color octet component.



Results (2)

- The best fit in the allowed region for the two parameters (Λ_{QCD} , p_F) is obtained choosing: Λ_{QCD} = 500 MeV and p_F = 500 MeV .

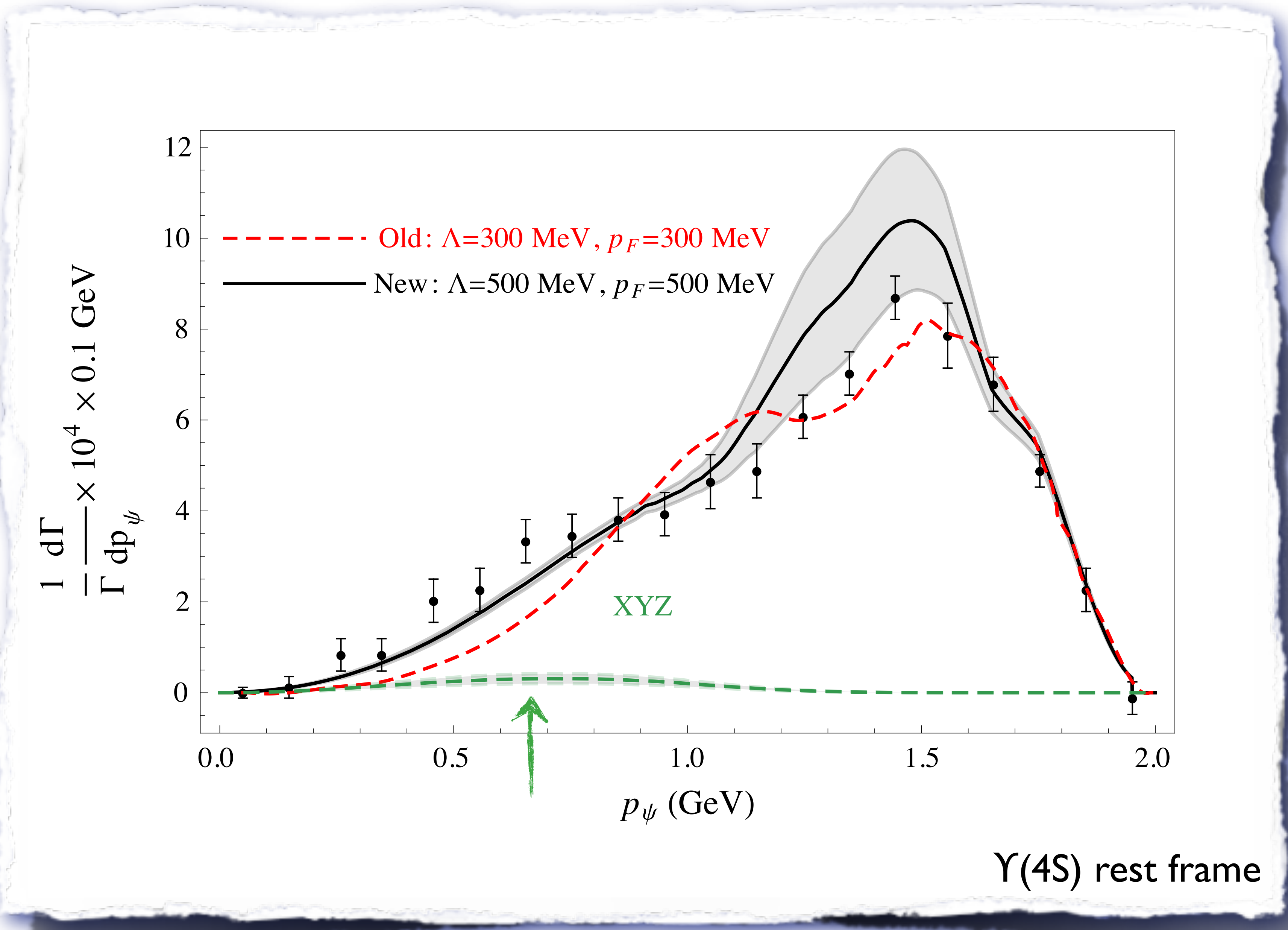


Υ(4S) rest frame

Old: χ^2=60/19
New: χ^2=22/19

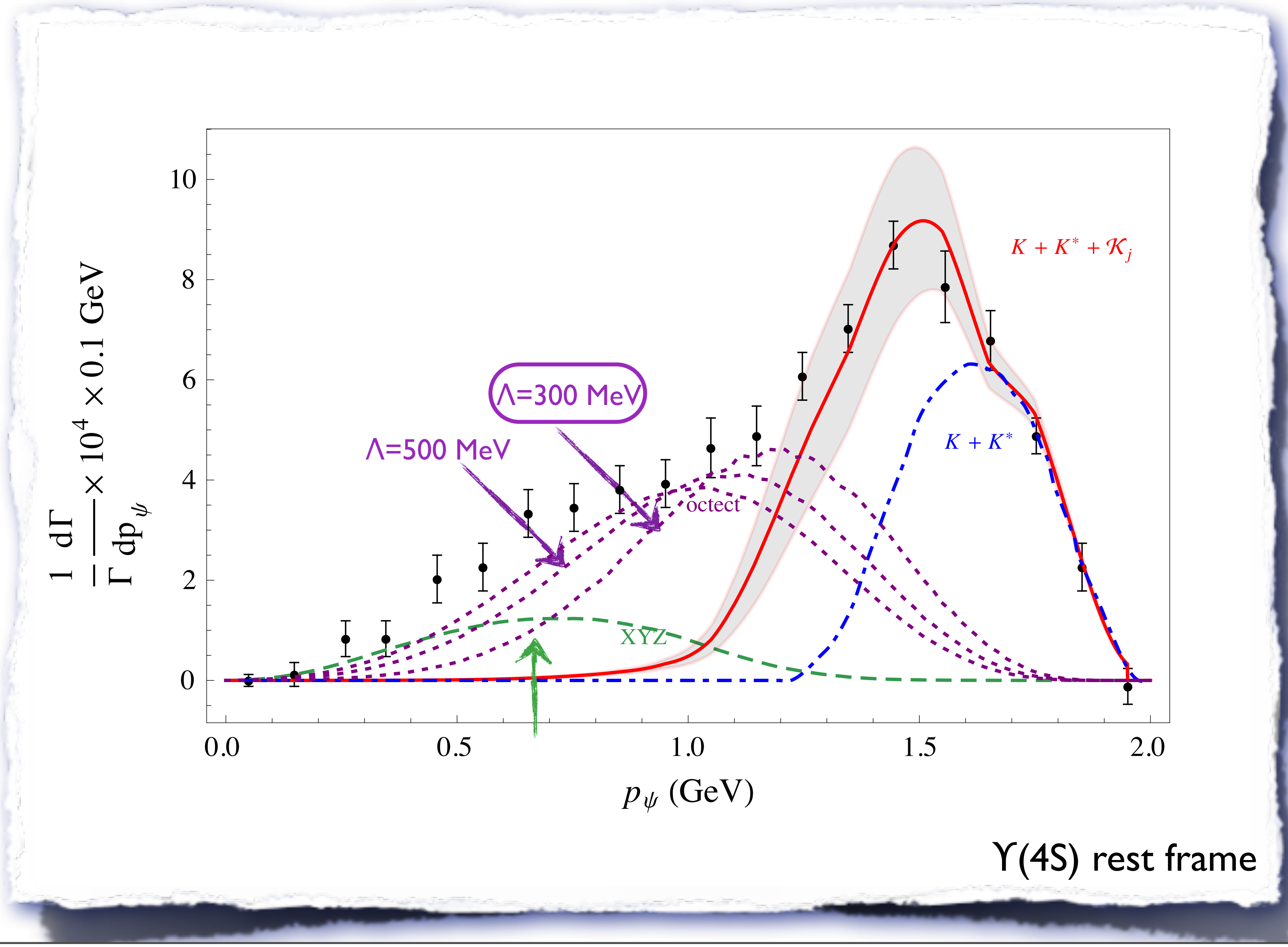
Results (2)

- The best fit in the allowed region for the two parameters (Λ_{QCD} , p_F) is obtained choosing: Λ_{QCD} = 500 MeV and p_F = 500 MeV .



Results (3)

- If the branching ratio due to XYZ turns out to be larger than the one measured (more XYZ states!) the best fit could be obtained with more reasonable parameters for the color octet component.



Back Up

$B \to J/\psi$ + All

:: BaBar, Phys. Rev. D67,032002 ::

- In e^+e^- collisions at $\sqrt{s} \sim m_{\Upsilon(4S)}$ with 20.3 fb^{-1} they measure:
  $B \to J/\psi$ + All, $B \to \psi(2S)$ + All, $B \to \chi_{c1,2}$ + All.

- $J/\psi \to e^+e^-, \mu^+\mu^-$
  $\psi(2S) \to e^+e^-, \mu^+\mu^-$ and $J/\psi\, \pi^+\pi^-$
  $\chi_{c1,2} \to J/\psi\, \gamma$

- $\mathcal{B}(J/\psi \to e^+e^-)$=5.94%
  $\mathcal{B}(J/\psi \to \mu^+\mu^-)$=5.93%

- $\mathcal{B}(\psi(2S) \to e^+e^-)$=0.765%
  $\mathcal{B}(\psi(2S) \to \mu^+\mu^-)$=0.760%
  $\mathcal{B}(\psi(2S) \to J/\psi\pi^+\pi^-)$=33.1%

- $\mathcal{B}(\chi_{c1} \to J/\psi\, \gamma)$=34.1%
  $\mathcal{B}(\chi_{c2} \to J/\psi\, \gamma)$=19.4%

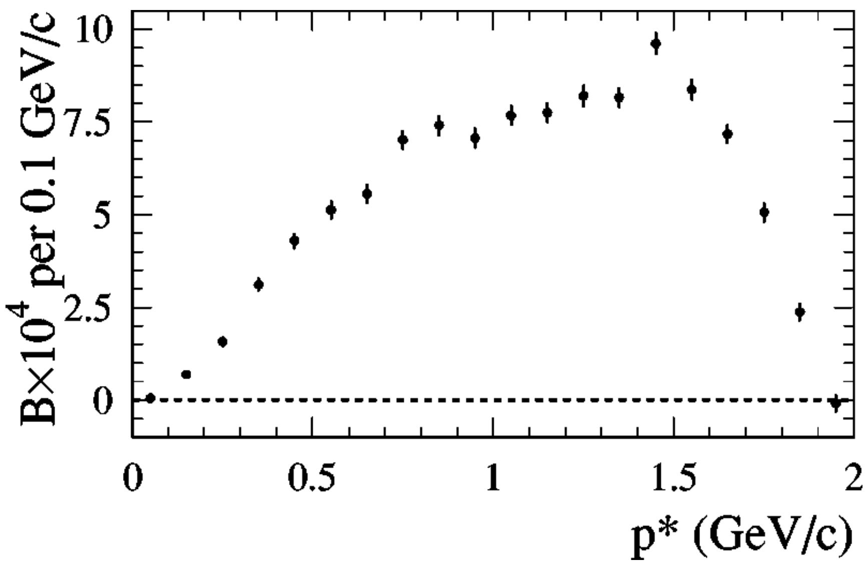
TABLE II. Summary of B branching fractions (percent) to charmonium mesons with statistical and systematic uncertainties. The direct branching fraction is also listed, where appropriate. The last column contains the world average values [15].

Meson	*Value*	*Stat*	*Sys*	*World Average*
J/ψ	1.057	$\pm$0.012	$\pm$0.040	1.15$\pm$0.06
J/ψ direct	0.740	$\pm$0.023	$\pm$0.043	0.80$\pm$0.08
χ_{c1}	0.367	$\pm$0.035	$\pm$0.044	0.36$\pm$0.05
χ_{c1} direct	0.341	$\pm$0.035	$\pm$0.042	0.33$\pm$0.05
χ_{c2}	0.210	$\pm$0.045	$\pm$0.031	0.07$\pm$0.04
χ_{c2} direct	0.190	$\pm$0.045	$\pm$0.029	-
$\psi(2S)$	0.297	$\pm$0.020	$\pm$0.020	0.35$\pm$0.05

B → J/ψ + All

:: BaBar, Phys. Rev. D67,032002 ::

Total contribution



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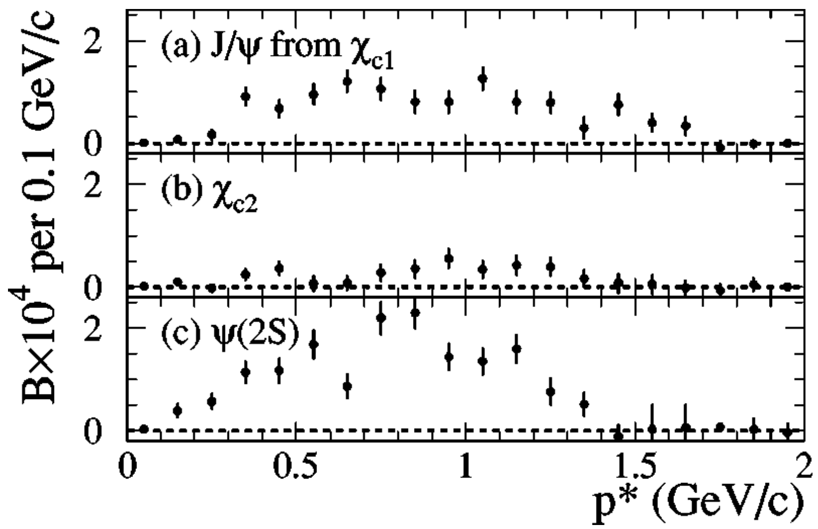
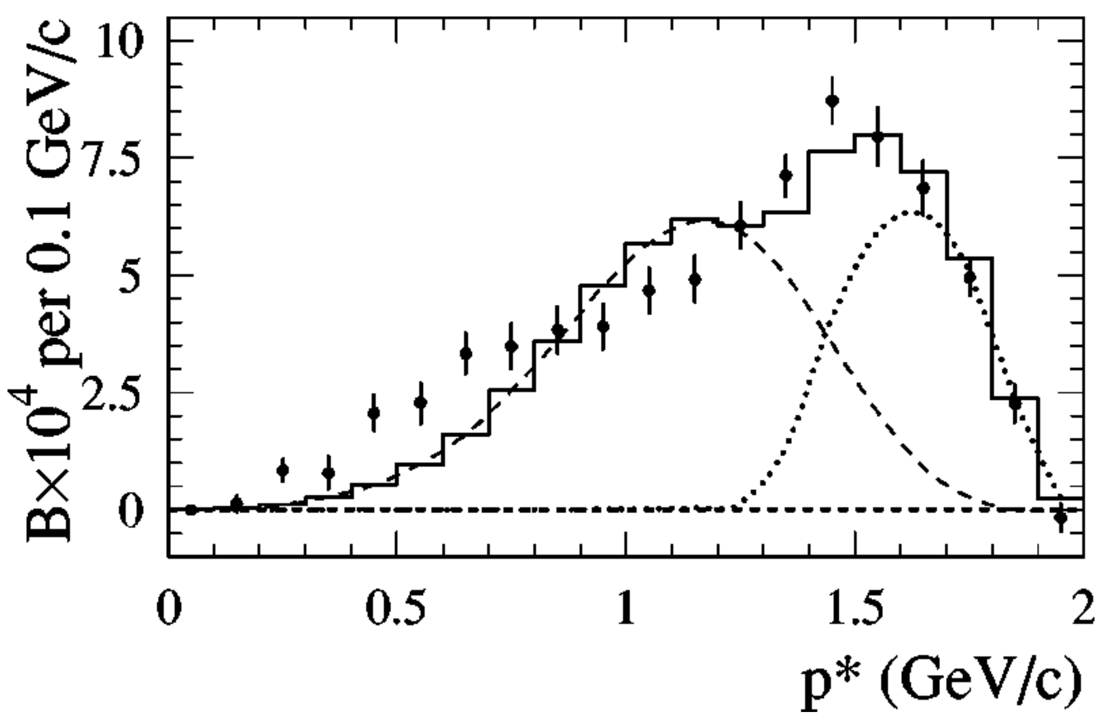
Feed-down contribution



Direct contribution



$B \to J/\psi + \mathcal{K}$

:: Belle, Phys. Rev. D83,032005 ::

- 535×10^{6} BB^{*} events ($492\ fb^{-1}$) from $e^{+}e^{-}$ collisions at $\sqrt{s} \sim m_{\Upsilon(4S)}$
- $\Delta E = E^{*}(B) - E_{beam}$ and $M_{bc} = \sqrt{(E^{2}_{beam} - P^{*2}(B))}$
  Signal region: $-3\sigma_{\Delta E} < \Delta E - \mu_{\Delta E} < 3\sigma_{\Delta E}$
  Sideband region: $-0.13\ GeV < \Delta E - \mu_{\Delta E} < -0.05\ GeV$
  & $+0.05\ GeV < \Delta E - \mu_{\Delta E} < +0.13\ GeV$
- $\mathcal{B}(B \to J/\psi\ K^{+}\pi^{+}\pi^{-}) = (7.16 \pm 0.10 \pm 0.60) \times 10^{-4}$
  $\mathcal{B}(B \to \psi(2S)\ K^{+}\pi^{+}\pi^{-}) = (4.31 \pm 0.20 \pm 0.50) \times 10^{-4}$
- The PDF is $p(\underline{x},\underline{a})$, with $\underline{x} = M^{2}(K\pi\pi), M^{2}(K\pi), M^{2}(\pi\pi)$ and $\underline{a}$=fit parameters

$$p(\vec{x};\vec{a}) = n_B \frac{p_B(\vec{x})}{\int p_B(\vec{x})d^3x} + n_S \frac{p_S(\vec{x};\vec{a})}{\int p_S(\vec{x};\vec{a})d^3x}$$

Background modelled from sideband region

Signal

$$p_S(\vec{x};\vec{a}) = \varepsilon(\vec{x})\phi(\vec{x})s(\vec{x};\vec{a})$$

$$s(\vec{x};\vec{a}) \equiv s(\vec{x};a_k)$$

$$= |a_{nr}A_{nr}(\vec{x})|^2 + \sum_{J_1}\left|\sum_{J_2} a_{J_1J_2}A_{J_1J_2}(\vec{x})\right|^2$$

B → J/ψ + 𝒦

:: Belle, Phys. Rev. D83,032005 ::

- The $K^+\pi^+\pi^-$ final state is modelled as a non resonant signal plus a superposition of initial state resonances R_1. The latter are assumed to decay through intermediate state resonances R_2

$$R_1 \to a R_2 \text{ and } R_2 \to bc$$

Signal

$$p_S(\vec{x};\vec{a}) = \varepsilon(\vec{x})\phi(\vec{x})s(\vec{x};\vec{a})$$

$$s(\vec{x};\vec{a}) \equiv s(\vec{x};a_k)$$

$$= |a_{nr}A_{nr}(\vec{x})|^2 + \sum_{J_1}\left|\sum_{J_2} a_{J_1J_2}A_{J_1J_2}(\vec{x})\right|^2$$

complex coefficients

- Since the components of the signal function are not individually normalized, a decay fraction is calculated as

$$f_k = \frac{\int \phi(\vec{x})|a_kA_k(\vec{x})|^2 d^3x}{\int \phi(\vec{x})s(\vec{x};\vec{a})d^3x}$$

B → J/ψ + $\mathcal{K}$

:: Burns, Piccinini, Polosa, Prosperi, Sabelli, arXiv:1104.1781::

- Belle measures

$$\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi \to R_i J/\psi \to J/\psi\, K^+\pi^+\pi^-) = \mathcal{B}_{\rm tot} f_i^j$$

where the intermediate resonant states are

$$\mathcal{R}_i = K\rho,\;\; K\omega,\;\; K^*\pi,\;\; K_0^*(1430)\pi,\;\; K_2^*(1430)\pi \;\text{ and }\; K f_{0,2}$$

- To extract two body branching ratios one needs to take into account isospin multiplicity

$$\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi \to R_i J/\psi \to J/\psi\, K^+\pi^+\pi^-) = \mathcal{I}_i \times \mathcal{B}(B^+ \to \mathcal{K}_j J/\psi) \times \mathcal{B}(\mathcal{K}_j \to R_i) \times \mathcal{B}(R_i \to K\pi\pi)$$

where the isospin factors are

$$\mathcal{I}(K\rho) = 1/3,\;\; \mathcal{I}(K^*\pi) = \mathcal{I}(K_0^*(1430)\pi) = 4/9,\;\; \mathcal{I}(K\omega) = 1,\;\; \mathcal{I}(K f_0) = \mathcal{I}(K f_2) = 2/3$$

so that

$$\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi) = \frac{\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi \to R_i J/\psi \to J/\psi\, K^+\pi^+\pi^-)}{\mathcal{I}_i \times \mathcal{B}(\mathcal{K}_j \to R_i) \times \mathcal{B}(R_i \to K\pi\pi)} = \frac{\mathcal{B}_{\rm tot} f_i^j}{\mathcal{I}_i \times \mathcal{B}(\mathcal{K}_j \to R_i) \times \mathcal{B}(R_i \to K\pi\pi)}$$

B → J/ψ + $\mathcal{K}$

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- Interference effects among different heavy kaons $\mathcal{K}_j$ have been neglected, so that one needs to rescale the two body branching ratios by some factor.

$$\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi \to \mathcal{R}_i J/\psi \to J/\psi\ K^+\pi^+\pi^-) = \mathcal{B}_{\rm tot} \tilde{f}_i^j$$

$$\tilde{f}_i^j = C \times \left(1 - \frac{\Gamma_j}{m_j}\right) f_i^j \qquad \mathcal{B}_{\rm tot} = (71.6 \pm 1 \pm 6) \times 10^{-5}$$

$\mathcal{K}_j$	$m_{\mathcal{K}_j}$ (GeV)	$\Gamma_{\mathcal{K}_j}$ (GeV)	$\mathcal{B}(B^+ \to \mathcal{K}_j J/\psi) \times 10^5$	
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B → $\mathcal{K}\,\mathcal{X}$ → $\mathcal{K}$ J/ψ + light hadrons

- B → K(500) $\mathcal{X}$ → K(500) J/ψ + light hadrons branching ratios are known:

$\mathcal{X}_j$	$m_{\mathcal{X}_j}$ (GeV)	$\Gamma_{\mathcal{X}_j}$ (GeV)	Final State	$\mathcal{B}(B \to K\mathcal{X}_j \to K J/\psi + \text{light hadrons}) \times 10^5$
$X(3872)$	3.872	0.003	$J/\psi\ \rho \to J/\psi\ \pi^+\pi^-$	0.72 ± 0.22 [A]
			$J/\psi\ \omega$	0.6 ± 0.3 [B]
$Y(3940)$	3.940	0.087	$J/\psi\ \omega$	3.70 ± 1.14 [C]
$Y(4140)$	4.140	0.012	$J/\psi\ \phi$	0.9 ± 0.4 [D]
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- For heavy kaons $\mathcal{K}$ we deduce the coupling B-$\mathcal{K}\,\mathcal{X}$ from the B-K(500)$\mathcal{X}$ one:

Spin 0 $\mathcal{K}$ $\langle \mathcal{X}(\epsilon,p)\mathcal{K}(q)|B(P)\rangle = g\, \epsilon \cdot q$

Spin 1 $\mathcal{K}$ $\langle \mathcal{X}(\epsilon,p)\mathcal{K}(\eta,q)|B(P)\rangle = g'\, \epsilon \cdot \eta$

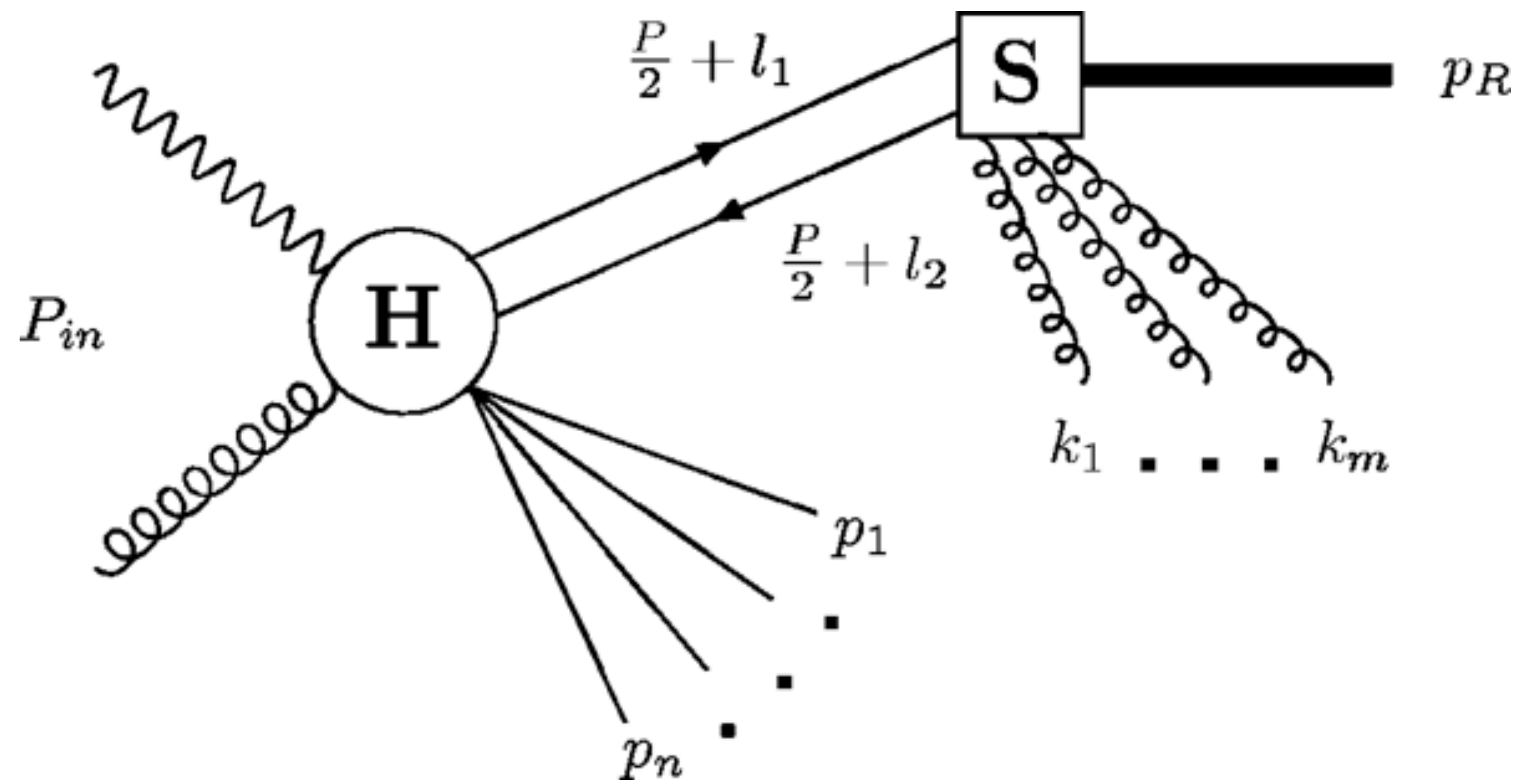
$g' = \Lambda\, g$

Λ some mass scale

From $\mathcal{B}(B \to K^* X(3872)) \times \mathcal{B}(X(3872) \to J/\psi\ \pi^+\pi^-) < 0.34 \times 10^{-5}$
we deduce Λ > 600 MeV ≈ m_{K^*} and thus we assume Λ = m_{K1},
taking all $\mathcal{X}$ to be Spin 1 states.

Color Octet Contribution

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::



- At leading order in the non-relativistic expansion the cc^* pair has to be produced in a color singlet $^3S_1$ state.
- At relative order $v^4 \approx 1/15$ in the non-relativistic expansion, J/ψ can also be produced through cc^* in $^1S_0{}^{(8)}$, $^3P_J{}^{(8)}$, $^3S_1{}^{(8)}$ color octet states
- The short-distance structure of the ΔB=1 weak effective Hamiltonian favors the production of color octet cc^* pairs in the $b \rightarrow cc^*q$ transition

Factorization

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- The hard and soft part of the process can be factorized

$$(2\pi)^3 2p_R^0 \frac{d\sigma}{d^3p_R} \equiv \sum_n \int \frac{d^4 l}{(2\pi)^4} \hat{\sigma}(c\bar{c}[n])(l) F_n(l)$$

$$\text{flux} \times \int dPS[p_i](2\pi)^4 \delta\left(P + l + \sum_i p_i - P_{in}\right) \times H_n(P_{in}, P, l, p_i)$$

hard process cross section

$$F_n(l) = \int \frac{dk^2}{2\pi} \frac{d^3 k}{(2\pi)^3 2k^0} (2\pi)^4 \times \delta(p_R + k - P - l) \Phi_n(k; p_R, P)$$

shape function

- The distribution can be written as an integral over the energy and invariant mass of the soft radiated system:

$$(2\pi)^3 2p_R^0 \frac{d\sigma}{d^3p_R} = \sum_n \int_0^{\alpha\beta} \frac{dk^2}{2\pi} \int_{(\alpha^2+k^2)/(2\alpha)}^{(\beta^2+k^2)/(2\beta)} dk_0 \times \text{flux} \times \bar{H}_n(P_{in}, P, l, p_X) \frac{1}{4\pi(\beta-\alpha)} \Phi_n(k; p_R, P)$$

Shape Function

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- The color octet configurations which contribute to B→J/ψ + All are

$$n = {}^1S_0^{(8)}, {}^3P_0^{(8)}, {}^3S_1^{(8)}$$

- The shape function is related to the NRQCD matrix elements as

$$\int \frac{d^4 l}{(2\pi)^4} F_n(l) = \frac{1}{(2\pi)^3} \int_0^\infty dk^2 \int_{\sqrt{k^2}}^\infty dk_0 \times \sqrt{k_0^2 - k^2}\, \Phi_n(k; p_R, P) = \langle \mathcal{O}_n^{J/\psi} \rangle,$$

- An ansatz for the shape function is

$$\Phi_n(k; p_R, P) = a_n |\mathbf{k}|^{b_n} \exp(-k_0^2/\Lambda_n^2) k^2 \exp(-k^2/\Lambda_n^2)$$

which is exact in the Coulombic limit. The exponential cutoff reflects the expectations that the typical energy and invariant mass of the radiated system is of order $\Lambda_n \approx m_c v^2$

Shape Function

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- The decay distribution in the rest frame of the cc* pair is

$$\frac{d\hat{\Gamma}}{d\hat{E}_R} = \frac{|\hat{p}_R|}{4\pi^2} \sum_n \int_0^{\alpha\beta} \frac{dk^2}{2\pi} \int_{(\alpha^2+k^2)/(2\alpha)}^{(\beta^2+k^2)/(2\beta)} dk_0 \times \frac{1}{2m_b} H_n(m_b, M_{c\bar{c}}(k)) \frac{M_R}{8\pi m_b |\hat{p}_R|} \Phi_n(k)$$

where

$$M_{c\bar{c}}^2(k) = (p+l)^2 = (p_R+k)^2 = M_R^2 + 2M_R k_0 + k^2$$

- To normalize the shape function one uses

$$\langle \mathcal{O}_8^{J/\psi}(^3S_1)\rangle = (0.5\text{–}1.0)\times 10^{-2}\ \text{GeV}^3$$

$$M_k^{J/\psi}(^1S_0^{(8)}, {}^3P_0^{(8)}) = \langle \mathcal{O}_8^{J/\psi}(^1S_0)\rangle + \frac{k}{m_c^2}\langle \mathcal{O}_8^{J/\psi}(^3P_0)\rangle \qquad M_{3.1}^{J/\psi}(^1S_0^{(8)}, {}^3P_0^{(8)}) = (1.0\text{–}2.0)\times 10^{-2}\ \text{GeV}^3$$

Fermi Motion

:: Beneke, Schuler, Wolf, Phys.Rev. D62 (2000) 034004 ::

- The b quark is moving inside the B meson at rest with a momentum p according to some distribution with a width of few hundred MeV. The cloud of gluons and light quarks is treated as spectator.

$$\Phi_{ACM}(p) = \frac{4}{\sqrt{\pi} p_F^3} \exp(-p^2/p_F^2)$$

- One needs thus to consider a floating b-mass

$$m_b^2(p) = M_B^2 + m_{sp}^2 - 2 M_B \sqrt{m_{sp}^2 + p^2}$$

- To obtain the distribution in the B rest frame

$$\frac{d\Gamma}{dE_R} = \int_{\max\{0, p_-\}}^{p_+} dp\, p^2 \Phi_{ACM}(p) \frac{m_b^2(p)}{2 p E_b(p)} \times \int_{\hat{E}_R^{\min}(p)}^{\hat{E}_R^{\max}(p)} \frac{d\hat{E}_R}{\hat{E}_R} \frac{d\hat{\Gamma}}{d\hat{E}_R}.$$