Lattice result on $J/\psi \to \eta_c\gamma$, $h_c \to \eta_c\gamma$

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Based on "*Lattice QCD study of the radiative decays $J/\psi \to \eta_c\gamma$ and $h_c \to \eta_c\gamma$*"
D.Becirevic and F.Sanfilippo, arXiv:1206.1445, JHEP 1301 (2013) 028

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Summary

$J/\psi \to \eta_c \gamma$ radiative decay

1. $\Gamma\left(J/\psi \to \eta_c \gamma\right)$ experimental situation
2. Theoretical puzzle
3. Lattice computation

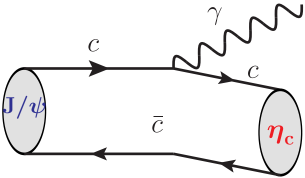
Two interesting tests of SM

1. $\Delta_c = M_{J/\psi} - M_{\eta_c}$ hyperfine splitting
2. $J/\psi \to e^+ e^-$ decay constant

$h_c \to \eta_c \gamma$ radiative decay

1. $\Gamma\left(h_c \to \eta_c \gamma\right)$ lattice determination
2. Prediction for Γ_{h_c}

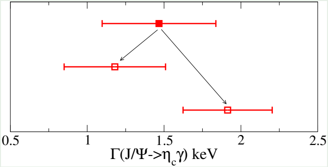
$J/\psi \to \eta_c \gamma$ radiative decay



Current experimental situation is **unclear**

$\Gamma\left(J/\psi \to \eta_c \gamma\right)_{\mathrm{PDG}} = \mathbf{(1.58 \pm 0.37)\,keV}$:

- Crystal Ball ('86): $\mathbf{(1.18 \pm 0.33)\,keV}$
- CLEO ('09): $\mathbf{(1.91 \pm 0.28 \pm 0.03)\,keV}$



PDG heavily influenced by Crystal Ball

- KEDR (arXiv:1002.2071): $\mathbf{(2.17 \pm 0.14 \pm 0.37)\,keV}$ (preliminary) final result expected this year
- BESIII will hopefully clarify the situation

$J/\psi \to \eta_c\gamma$ radiative decay

Theoretical predictions are **inconclusive**

- Dispersive bound from $\Gamma(\eta_c \to 2\gamma)$:
  $\Gamma(J/\psi \to \eta_c\gamma) < \mathbf{3.2\,keV}$ [M.A. Shifman, Z. Phys. C 6 ('80)]
- Two QCD sum rule calculations gave two different results:
  - $\sim \mathbf{(1.7 \pm 0.4)\ keV}$ [A.Y. Khodjamirian, Sov. J. Nucl. Phys. 39 ('84)]
  - $\sim \mathbf{(2.6 \pm 0.5)\ keV}$ [Beilin and Radyushkin, Nucl. Phys. B 260 ('85)]
- QCD eff. theory
  - $\mathbf{(1.5 \pm 1.0)\ keV}$ [N.Brambilla et al, PRD73 ('06)]
  - $\mathbf{(2.14 \pm 0.40)\ keV}$ [A.Pineda, J.Segovia, arXiv:1302.3528]
- Potential Quark Models:
  - $\sim \mathbf{3.3\,keV}$ [M.B Voloshin, Prog.Part.Nucl.Phys. 61 ('07)]
  - $\sim \mathbf{2.85\,keV}$ [E. Eichten et al., RMP80 ('08)]

Lattice QCD computations

- Quenched and single lattice spacing: $\mathbf{2.35(10)\,keV}$[J.J Dudek et al., PRD 73 ('06)]
- Unquenched but still single lattice spacing: $\mathbf{2.77\,(5)\ keV}$[Chen et. al, PRD 84 ('11)]

Both results obtained at large negative q^2's , then extrapolated to $q^2 = 0$

Lattice QCD

Desired features

Continuum: Several lattice spacings to take continuum limit

Renormalization: Non perturbative

Momentum: Work directly at $q^2 = 0$ to avoid the q^2 extrapolation

Unquenching: Include 2 physical light, strange and charm dynamical quarks

What we currently have...

Continuum: 4 different lattice spacings ($a \in [0.054; 0.100]\ \mathrm{fm}$)

Renormalization: Non perturbative (RI-MOM)

Momentum: Work at $q^2 = 0$ using twisted boundary conditions

Unquenching: Only 2 dynamical light quarks ($M_\pi \in [280; 500]\ \mathrm{MeV}$)

- Wilson regularization of QCD with twisted mass term (tmQCD)
- QCD gauge field configurations produced by ETM collaboration

Form factor computation

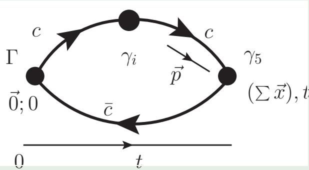
$$\Gamma\left(J/\psi \to \eta_c \gamma\right) \propto \left|\langle \eta_c \left| J^{em} \right| J/\psi \rangle\right|^2_{q^2=0}$$

Three points functions

$$C^{(3)}_{ij}(t) = \langle \mathrm{Tr}\left[S_c(y;0)\gamma_i S_c(0,x)\gamma_j S_c^{\vec{p}}(x,y)\gamma_5\right]\rangle$$

at intermediate times:

$$C^{(3)}_{ij}(t) \underset{0\ll t\ll T}{\simeq} Z_{J\psi} Z_{\eta_c} \exp\left[\left(E_{\eta_c} - M_{J/\psi}\right)t\right] \langle \eta_c | J^{em}_j | (J/\psi)_i \rangle$$



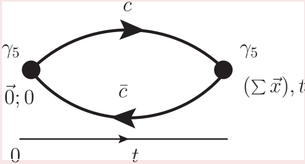
Two points functions

$$C^{(2)}(t) = \langle \mathrm{Tr}\left[S_c(0,0;\vec{x},t)\gamma_5 S_c(\vec{x},t;\vec{0},0)\gamma_5\right]\rangle =$$

at large times:

$$\underset{t\to\infty}{\simeq} Z_{\eta_c}{}^2 \exp(-M_{\eta_c} t)$$

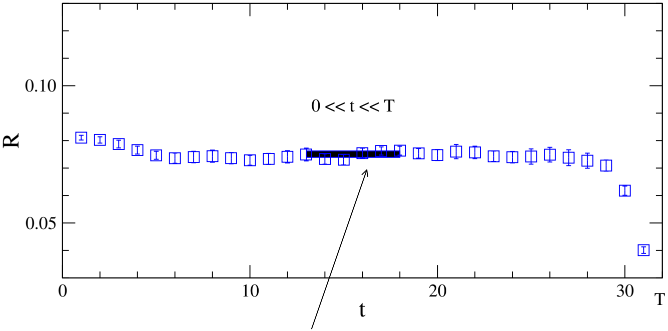
and similarly for $Z_{J/\psi}$, $M_{J/\psi}$.



Matrix element $\langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$ (example)

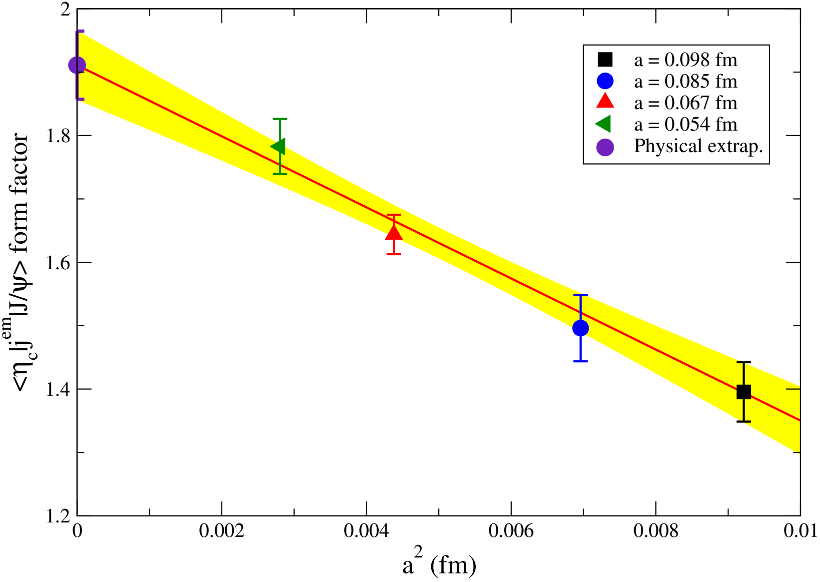
Determine Z and M from 2 points functions and combine with $C^{(3)}$:

$$R(t) \equiv \frac{C_{ij}^{(3)}(t)}{Z_{J\psi} Z_{\eta_c} \exp\left[\left(E_{\eta_c} - M_{J/\psi}\right)t\right]} \underset{0 \ll t \ll T}{\simeq} \langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$$

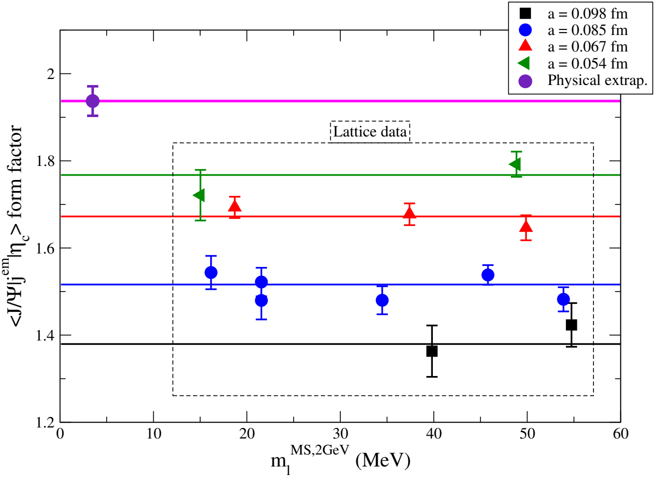


$\langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$ on the plateau of $R(t)$

Continuum extrapolation of $\langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$



Dependence on light quark mass



Insensitive to variation of the light sea quark mass m_ℓ^{sea} (expected because $m_c^{val} \gg m_\ell^{sea}$)

Expected insensitivity to the dynamical strange quark ($m_c \gg m_s \gg m_\ell^{sea}$)

Numerical result

Our final result

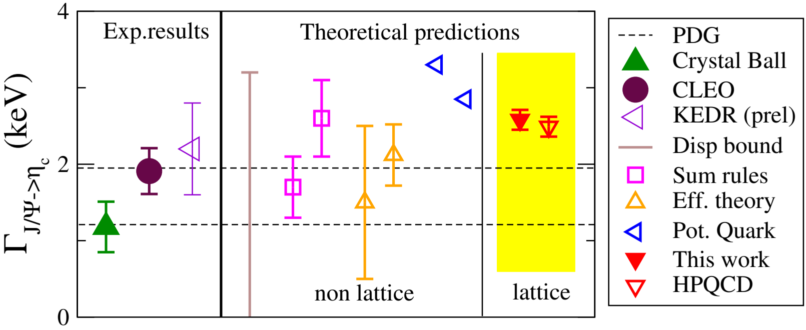
- Putting everything together we get: $\Gamma(J/\psi \to \eta_c\gamma) = 2.58\,(13)\ \mathrm{keV}$
- Our value is clearly:
  - larger than Crystal Ball('86) 1.18(33) keV
  - compatible with CLEO('09) 1.91(30) keV, and KEDR('10) 2.17(40) keV

Recent development

- Recently HPQCD collab. (PRD86 (2012) 094501) reported result:
  - using Staggered quarks (HISQ regularization)
  - including also dynamical strange quark
- They show that $\langle \eta_c | J_j^{em} | (J/\psi)_i \rangle$ does not depend on m_s^{sea}
- Excellent agreement with our result in the continuum limit:

$$\Gamma_{J/\psi \to \eta_c\gamma} = 2.49\,(19)\ \mathrm{keV}$$

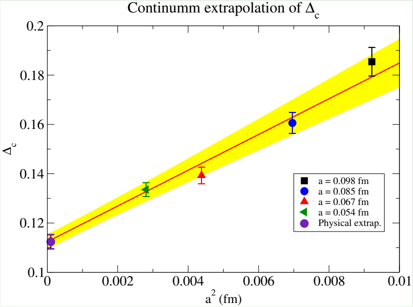
Is the $J/\psi \to \eta_c\gamma$ puzzle solved?

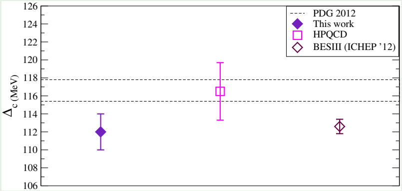


- Two different lattice approaches give the same result in the continuum
- On the theory side the problem is (almost) fully solved
- This becomes a precision test of QCD
- The experimental situation needs to be clarified

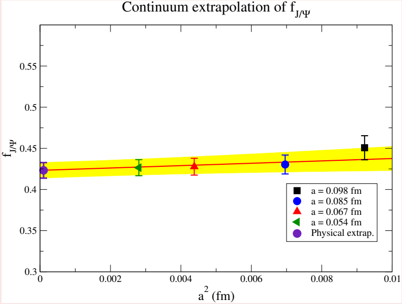
Other two precise tests of SM

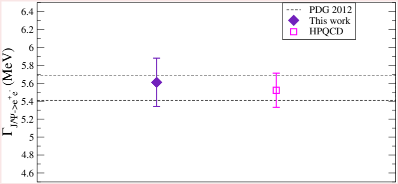
Hyperfine splitting $\Delta_c = M_{J/\Psi} - M_{\eta_c}$





$J/\Psi \to e^+e^-$ decay constant

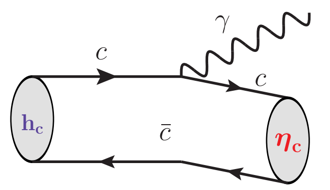




$h_c \to \eta_c \gamma$ radiative decay

h_c: charmonium $J^{PC} = 1^{+-}$ state

- h_c only recently observed at CLEO (2005)
- $\mathrm{Br}\,(h_c \to \eta_c \gamma) = 53\,(7)\,\%$ at BESIII 2010

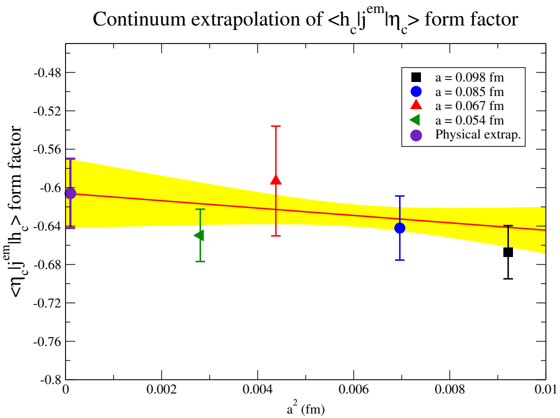


Open questions on the initial state

- Lifetime of h_c not measured yet
- No QCD based estimate: no QCDSR, no effective theory

As before, we can provide the first unquenched LQCD result in the continuum: $\Gamma\,(h_c \to \eta_c \gamma)$ is a prediction!

$h_c \to \eta_c \gamma$ radiative decay



We get:

$\Gamma(h_c \to \eta_c \gamma) = 0.72(5)\,\mathrm{MeV}.$

BESIII measured:

$\mathrm{Br}(h_c \to \eta_c \gamma) = (53 \pm 7)\,\%$

We can predict h_c lifetime:

$$\Gamma_{h_c} = \frac{\Gamma(h_c \to \eta_c \gamma)}{\mathrm{Br}(h_c \to \eta_c \gamma)}$$

$1.37(23)\,\mathrm{MeV}.$

Very recently (Confinement '12) BESIII presented preliminary results for Γ_{h_c}:

$$\Gamma_{h_c}^{incl} = 0.73(45)(28)\,\mathrm{MeV}, \qquad \Gamma_{h_c}^{excl} = 0.70(28)(22)\,\mathrm{MeV}$$

Conclusions

Main message from Lattice QCD side

First full determination of $J/\psi \to \eta_c\gamma$ and $h_c \to \eta_c\gamma$ form factors:

- high statistics unquenched simulations
- continuum extrapolation under control
- non-perturbatively renormalized

Results

- **Assessed** theoretical estimate of $\Gamma\,(J/\psi \to \eta_c\gamma)$
- Provided a **prediction for h_c lifetime**

Future perspective

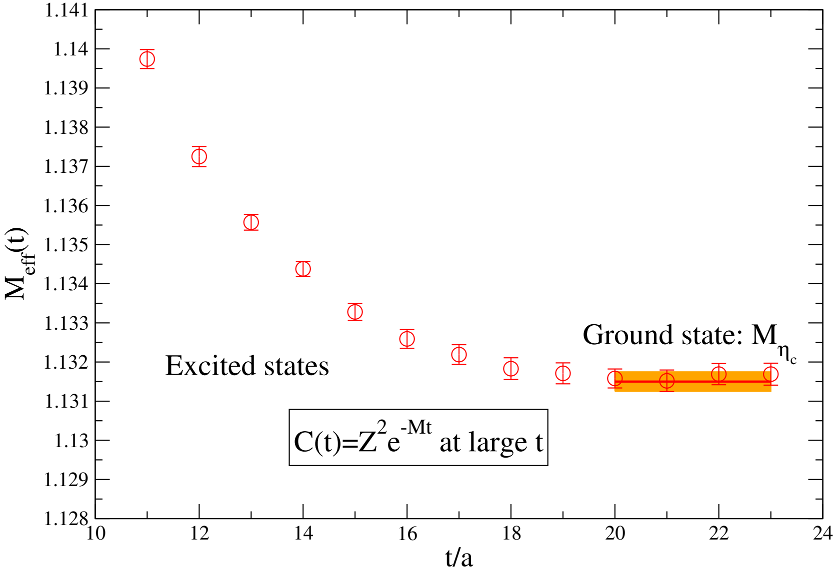
- Check insensitivity to the dynamical charm quark with lattice tmQCD
- Currently working on $\psi' \to \eta_c\gamma$ and $\boldsymbol{\eta_c' \to J/\psi\gamma}$

- New measurement of $\Gamma\,(J/\psi \to \eta_c\gamma)$ indispensable
- Improve the measurement of Γ_{h_c}

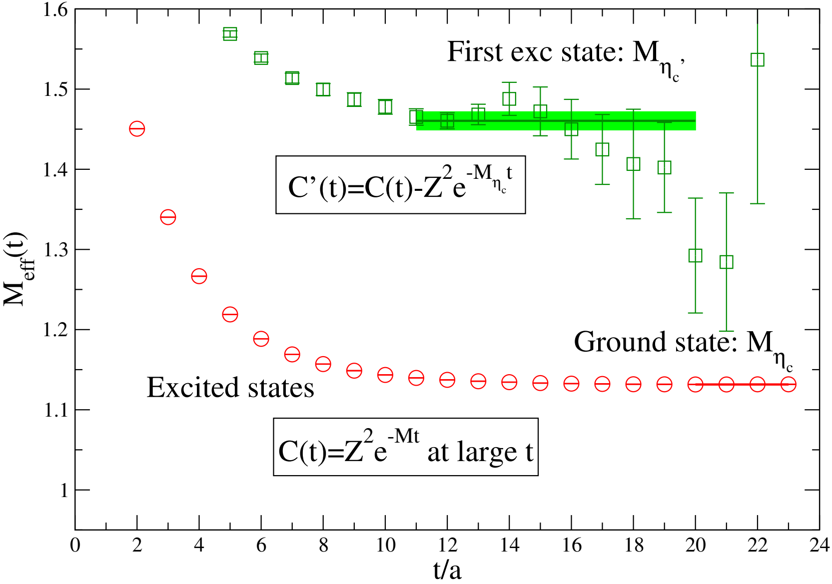
Backup

slides

$C(t)$ effective mass



$C'(t)$ effective mass



Can we do charm physics on current lattices?

Some back of the envelop calculation

- Lattice spacings: $a \sim 0.050 \div 0.100$ fm, $1/a \sim 2 \div 4$ GeV
- Charmed meson mass: $M_{D^\pm} = 1.87$ GeV, $M_{J/\Psi} = 3.1$ GeV

To study charm physics on such lattices seem questionable but...

Some deeper calculation

- In the free theory the cut off is given by $p_{max} = \pi/a \sim 6 \div 12$ GeV!
- Seems to be almost good also to study b quark...

Cutoff of interacting theory is unknown: only actual computations can teach us

How to keep the situation under control?

Having 4 different lattice spacing, and $\mathcal{O}(a)$ improved theory allows:

- to drop coarsest lattice spacing and check for stability of $a \to 0$ limit
- to assess the convergence $\propto a^2$ to the continuum limit: $\Phi^{latt} = \Phi^{cont.} + \Phi' a^2$

Determination of the charm quark mass

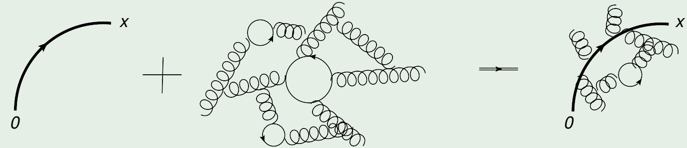
Wick contraction

$$C(\tau) = \sum_{\vec{x}} \left\langle O^{\dagger}(\vec{x},\tau)\, O\left(\vec{0},0\right) \right\rangle \underset{Wick}{=} \mathrm{Tr}\left[\Gamma S_l\left(\vec{x},\tau;\vec{0},0\right)\Gamma S_c\left(\vec{0},0;\vec{x},\tau\right)\right]$$

Quark propagator calculation

Solving Dirac equation on gauge background provides full quark propagator

$$D_q\,(\mathrm{y,x}) \cdot S_q\,(x,0) = \delta_{y,0} \qquad D_q = \left(\tfrac{1}{2\kappa} + K[U]\right)\mathbf{1} + i m_q \gamma_5 \tau_3$$

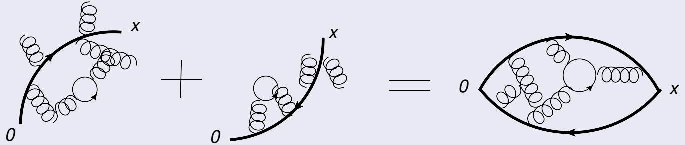


In practice

- D operator and the propagator S are large matrices ($\mathcal{O}\left(10^9\right)$ lines)
- Solving Dirac equation requires large amount of CPU resources

Two points correlation function computation

Combine 2 propagators with suitable Dirac structures



Effective mass

Where ground state dominate, correlation decays exponentially:

$$C(\tau) \underset{\tau\to\infty}{\longrightarrow} Z_D^2 e^{-M_D \tau}$$

The derivative of its logarithm (effective mass) is a constant:

$$M_{eff}(\tau) \equiv -\partial_\tau \log C(\tau) \underset{\tau\to\infty}{\longrightarrow} M_D$$

- Deviation from constant behavior of $M_D(\tau)$ reveals excited states
- Derivative to be computed numerically: $\partial_\tau = f(\tau+1) - f(\tau)$