Casimir effect,
theory and experiments

Serge Reynaud & Astrid Lambrecht

Laboratoire Kastler Brossel

ENS, UPMC, CNRS, Paris

www.lkb.ens.fr

www.lkb.upmc.fr

Thanks to M.-T. Jaekel (LPTENS Paris),

I. Cavero-Pelaez, A. Canaguier-Durand,

R. Guérout, J. Lussange, G. Dufour (LKB),

P.A. Maia Neto (UF Rio de Janeiro),

G.-L. Ingold (U. Augsburg), D.A.R. Dalvit,

R. Behunin, F. Intravaia, Y. Zeng (Los Alamos),

E. Fischbach, R. Decca (IUPUI Indianapolis),

C. Genet, T. Ebbesen, P. Samori (Strasbourg),

A. Liscio (Bologna), G. Palasantzas (Groningen),

V. Nesvizhevski (ILL), A. Voronin (Lebedev),

and discussions in the CASIMIR network …

casimir-network.org

A short history of quantum fluctuations

➢ 1900 : Law for blackbody radiation energy per mode (Planck)

$$\overline{E} = \overline{n}_\omega \hbar\omega \quad , \qquad \overline{n}_\omega = \frac{1}{e^{\hbar\omega/k_{\mathrm{B}}T} - 1}$$

➢ 1905 : Derivation of this law from energy quanta (Einstein)

➢ 1912 : Introduction of zero-point fluctuations (*zpf*) for matter (Planck)

$$\overline{E} = \overline{n}_\omega \hbar\omega + \frac{1}{2}\hbar\omega$$

➢ 1913 : First correct demonstration of *zpf* (Einstein and Stern)

$$\left(\overline{n}_\omega + \frac{1}{2}\right)\hbar\omega = k_{\mathrm{B}}T + O\left(\frac{1}{T}\right) \quad , \qquad T \to \infty$$

➢ 1914 : Prediction of effects of *zpf* on X-ray diffraction (Debye)
➢ 1917 : Quantum transitions between stationary states (Einstein)
➢ 1924 : Quantum statistics for "bosons" (Bose and Einstein)
➢ 1924 : Observation of effects of *zpf* in vibration spectra (Mulliken)

A short history of quantum fluctuations

➢ 1925-… : Quantum Mechanics confirms the existence of vacuum fluctuations (Heisenberg, Dirac and many others)
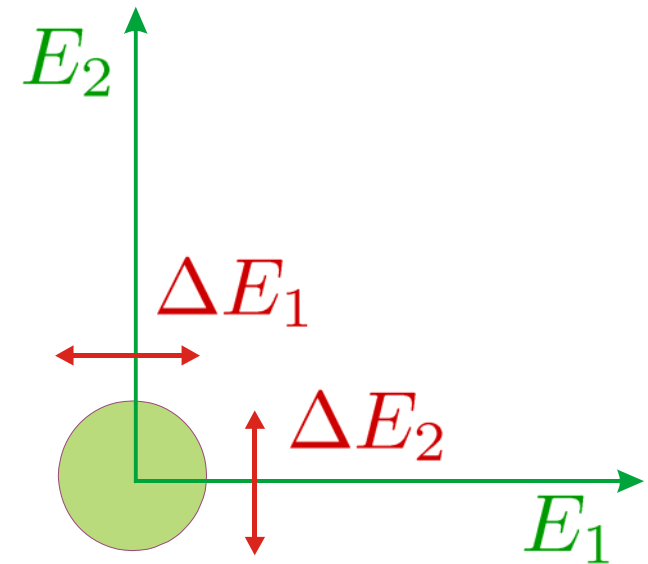
➢ Quantum electromagnetic field

  ➢ Each mode = an harmonic oscillator

$$E = E_1 \cos(\omega t) + E_2 \sin(\omega t)$$

$$\Delta E_1 \Delta E_2 \geq \epsilon_\omega^2$$

  ➢ Vacuum = ground state for all modes

  ➢ Fluctuation energy per mode $\frac{1}{2}\hbar\omega$

➢ 1945-… : Atomic, Nuclear and Particle Physics study the effects of vacuum fluctuations in microphysics

➢ 1960-… : Laser and Quantum Optics study the properties and consequences of electromagnetic vacuum fluctuations

P.W. Milonni, *The quantum vacuum* (Academic, 1994)

The puzzle of vacuum energy

➢ 1916 : *zp* fluctuations for the electromagnetic fields lead to a **BIG** problem for vacuum energy (Nernst)

$$e = \sum_{\text{modes}} \overline{n}\hbar\omega + \sum_{\text{modes}}^{\omega_{\text{max}}} \frac{\hbar\omega}{2} = \frac{\pi^2 \left(k_{\text{B}}T\right)^4}{15\left(\hbar c\right)^3} + \frac{\left(\hbar\omega_{\text{max}}\right)^4}{8\pi\left(\hbar c\right)^3}$$

From conservative estimations of the energy density in vacuum…

Bound on vacuum energy density in solar system $\longrightarrow$ $\dfrac{e^{\text{observ}}}{e^{\text{calcul}}} \sim 10^{-40}$

Cutoff at the energy in accelerators (TeV) $\longrightarrow$

…to the largest ever discrepancy between theory and experiment !

Now measured cosmic vacuum energy density $\longrightarrow$ $\dfrac{e^{\text{observ}}}{e^{\text{calcul}}} \sim 10^{-120}$

Cutoff at the Planck energy $\longrightarrow$

The puzzle of vacuum energy

➤ Standard position for a large part of the 20th century

[For the fields,] *« it should be noted that it is more consistent, in contrast to the material oscillator, not to introduce a zero-point energy of ½ h ν per degree of freedom.*

For, on the one hand, the latter would give rise to an infinitely large energy per unit volume due to the infinite number of degrees of freedom, on the other hand, it would be in principle unobservable since nor can it be emitted, absorbed or scattered and hence, cannot be contained within walls and, as is evident from experience, neither does it produce any gravitational field. »

"Wellenmechanik", W. Pauli (1933); translation by C.P. Enz (1974)

➤ Problem not yet solved, leads to many ideas, for example

When setting the cutoff to fit the cosmic vacuum energy density (dark energy), one finds a length scale λ=85µm below which gravity could be affected

Search for scale dependent modifications of the gravity force law

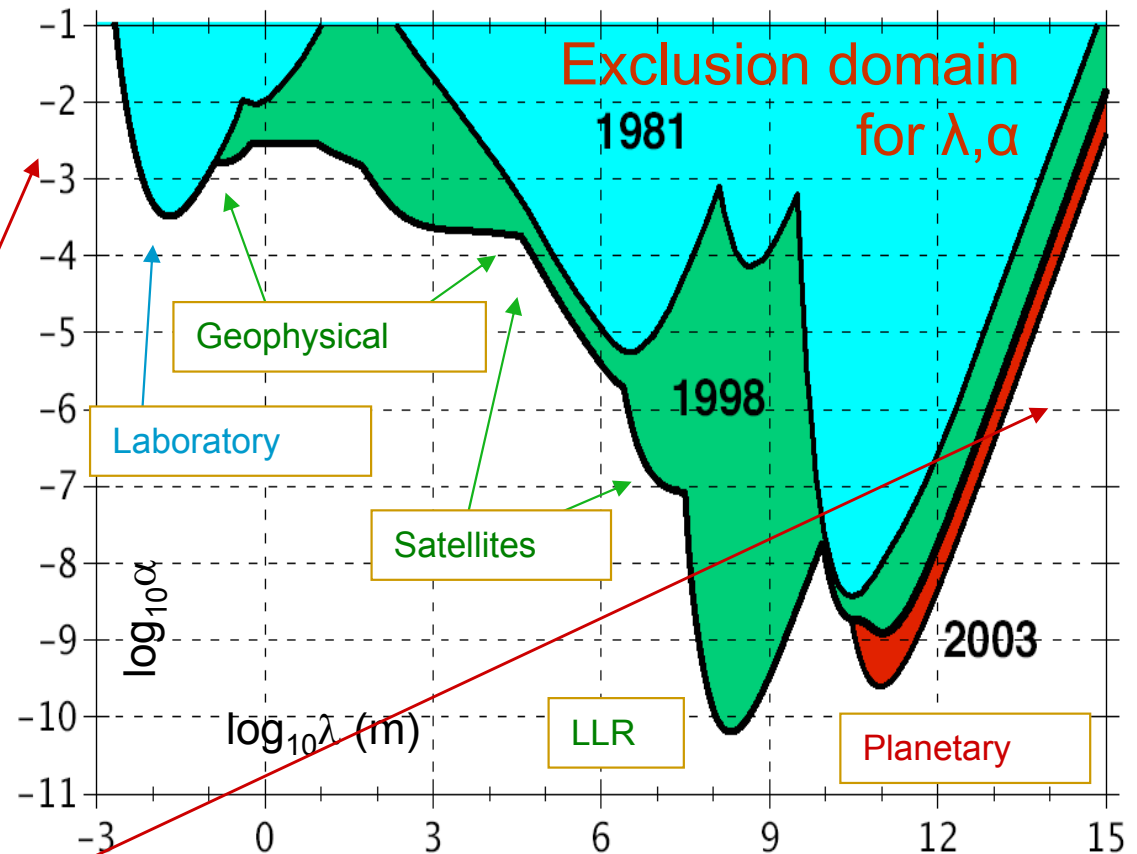> Exclusion plot for deviations with a generic Yukawa form

$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-\frac{r}{\lambda}}\right)$$

Windows remain open for deviations at short ranges

$\lambda < 1\,\mathrm{mm}$

or long ranges

$\lambda > 10^{16}\,\mathrm{m}$

Exclusion domain for λ,α

1981

Geophysical

Laboratory

1998

Satellites

$\log_{10}\alpha$

$\log_{10}\lambda$ (m)

LLR

2003

Planetary

Courtesy : J. Coy, E. Fischbach, R. Hellings, C. Talmadge & E. M. Standish (2003) ; see M.T. Jaekel & S. Reynaud IJMP **A20** (2005)

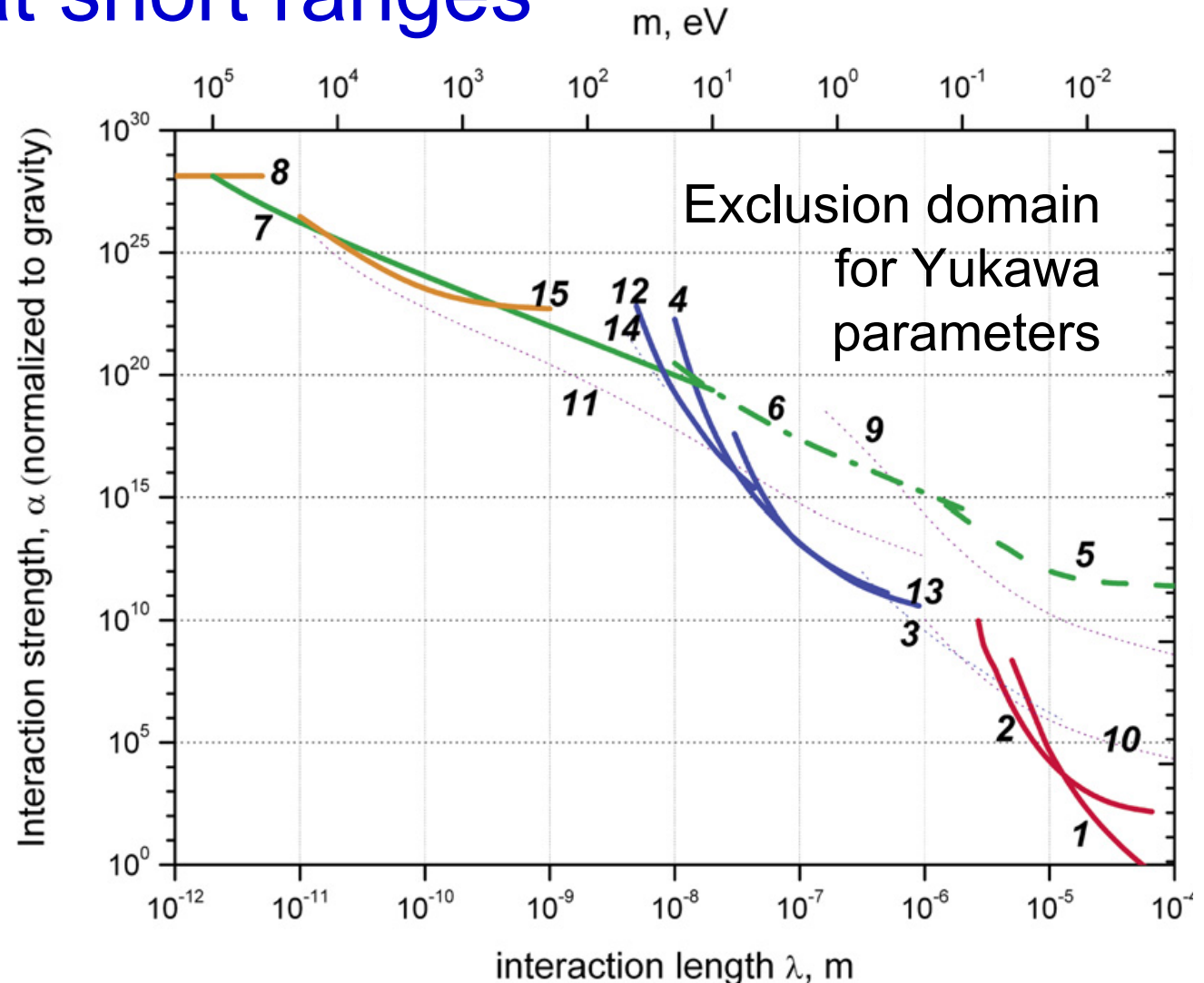*The Search for Non-Newtonian Gravity,* E. Fischbach & C. Talmadge (1998)

Testing gravity at short ranges

- Short range gravity with torsion pendulum (Eotwash experiments)

$$\alpha = 1 \quad \rightarrow \quad \lambda < 56\mu m$$

95% confidence level

- From the mm down to the pm range
  - Eotwash experiments
  - Casimir experiments
  - Neutron physics
  - Exotic atoms

Exclusion domain for Yukawa parameters

The Casimir effect

Vacuum resists when being confined within walls :
a universal effect depending only on $\hbar$, c, and geometry

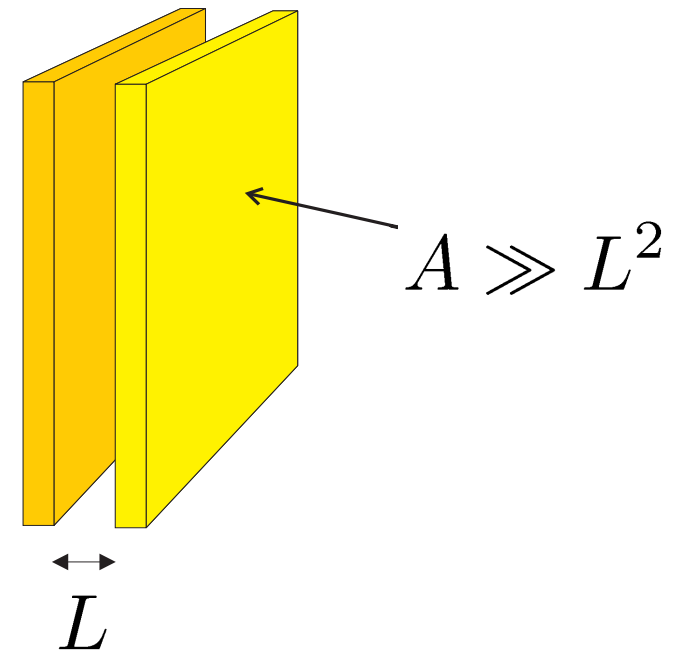$$F_{\text{Cas}} = -\frac{d\mathcal{E}_{\text{Cas}}}{dL} \ , \quad \mathcal{E}_{\text{Cas}} = -\frac{\hbar c \pi^2 A}{720 L^3}$$

$A \gg L^2$

➢ Ideal formula written for
  ➢ Parallel plane mirrors
  ➢ Perfect reflection
 ➢ Null temperature

L

➢ Attractive force = negative pressure

$$F_{\text{Cas}} = P_{\text{Cas}} A \ , \quad P_{\text{Cas}} = -\frac{\hbar c \pi^2}{240 L^4}$$

$|P_{\text{Cas}}| \sim 1\text{mPa}$
at $L = 1\mu\text{m}$

H.B.G. Casimir, Proc. K. Ned. Akad. Wet. (Phys.) **51** (1948) 79

The Casimir effect (real case)

- **Real mirrors not perfectly reflecting**
  - Force depends on non universal properties
    of the material plates used in the experiments

- **Experiments performed at room temperature**
  - Effect of thermal field fluctuations to be added
    to that of vacuum fluctuations

- **Effects of geometry and surface physics**
  - Plane-sphere geometry used in
    recent precise experiments
  - Surfaces not ideal :
   roughness, contamination,
    electrostatic patches …

$$R \gg L$$

$$\overleftrightarrow{L}$$
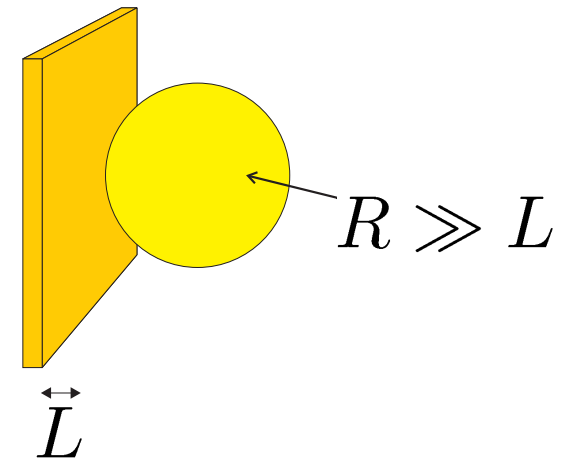
Casimir force and Quantum Optics

➢ **Many ways to calculate the Casimir effect**

➢ **« Quantum Optics » approach**

    ➢ Quantum and thermal field fluctuations pervade empty space

    ➢ They exert radiation pressure on mirrors

    ➢ Force = pressure balance between inner and outer sides of the mirrors

➢ **« Scattering theory »**

    ➢ Mirrors = scattering amplitudes depending on frequency, incidence, polarization

    ➢ Solves the high-frequency problem

    ➢ Gives results for real mirrors

    ➢ Can be extended to other geometries
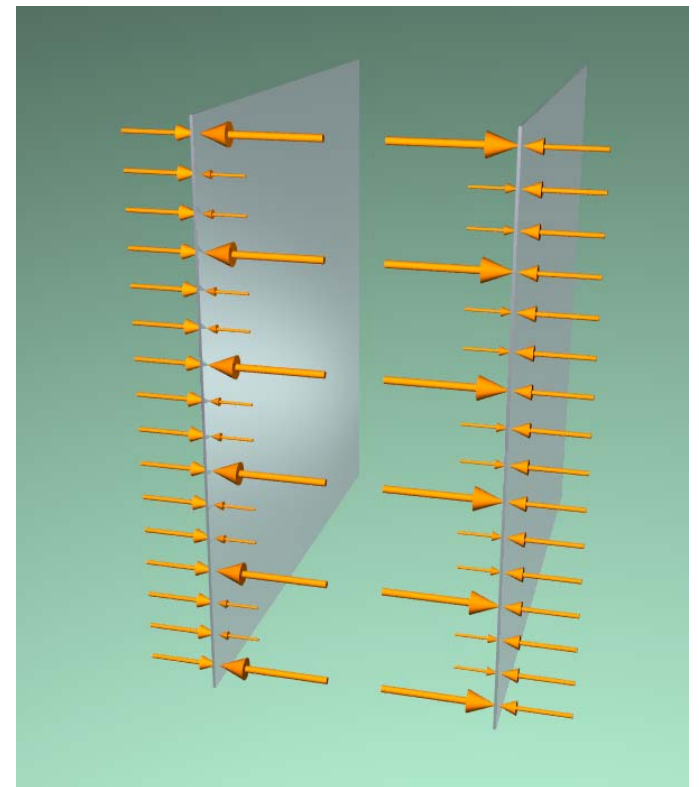
A simple derivation of the Casimir effect

➢ Quantum field theory in 1d space (2d space-time)

    ➢ Two counterpropagating scalar fields

    ➢ Mirrors are point scatterers

M$_1$

φ_{in}^{+} φ_{out}^{+}

φ_{out}^{-} φ_{in}^{-}

➢ A mirror M$_1$ at position q_1 couples the two fields counter-propagating on the 1d line

➢ The properties of the mirror M$_1$ are described by a scattering matrix S_1 which

    ➢ preserves frequency (in the static problem)

    ➢ contains a reflection amplitude r_1 , a transmission amplitude t_1 and phases which depend on the position of the mirror q_1

M. Jaekel & S. Reynaud, J. Physique **I-1** (1991) 1395
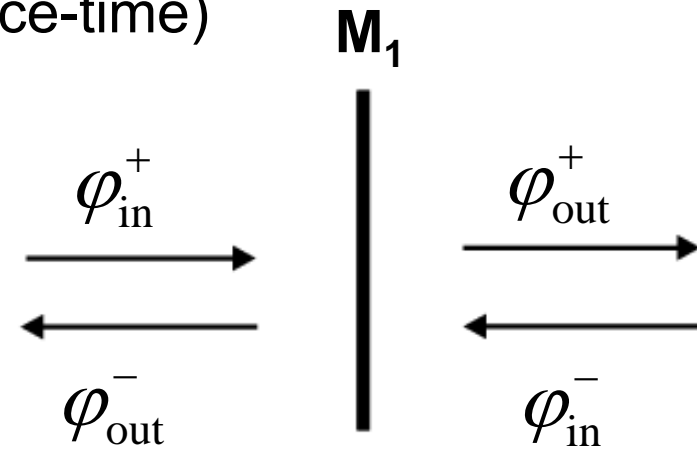
Two mirrors form a Fabry-Perot cavity

➢ All properties of the fields can be deduced from the elementary matrices S_1 and S_2

$$\varphi_{\text{in}}^{+} \qquad \varphi_{\text{cav}}^{+} \qquad \varphi_{\text{out}}^{+}$$

$$\varphi_{\text{out}}^{-} \qquad \varphi_{\text{cav}}^{-} \qquad \varphi_{\text{in}}^{-}$$

$$L$$

➢ In particular :

  ➢ The outer energies are the same as in the absence of the cavity (unitarity)

  ➢ The inner energies are enhanced for resonant modes, decreased for non-resonant modes

➢ Cavity QED language :

  ➢ The density of states (DOS) is modified by cavity confinement

$$g = \frac{\text{DOS}_{\text{inside}}}{\text{DOS}_{\text{outside}}} = \frac{1 - \left| re^{2ikL} \right|^2}{\left| 1 - re^{2ikL} \right|^2}$$

$$r \equiv r_1 r_2$$

g

1

k
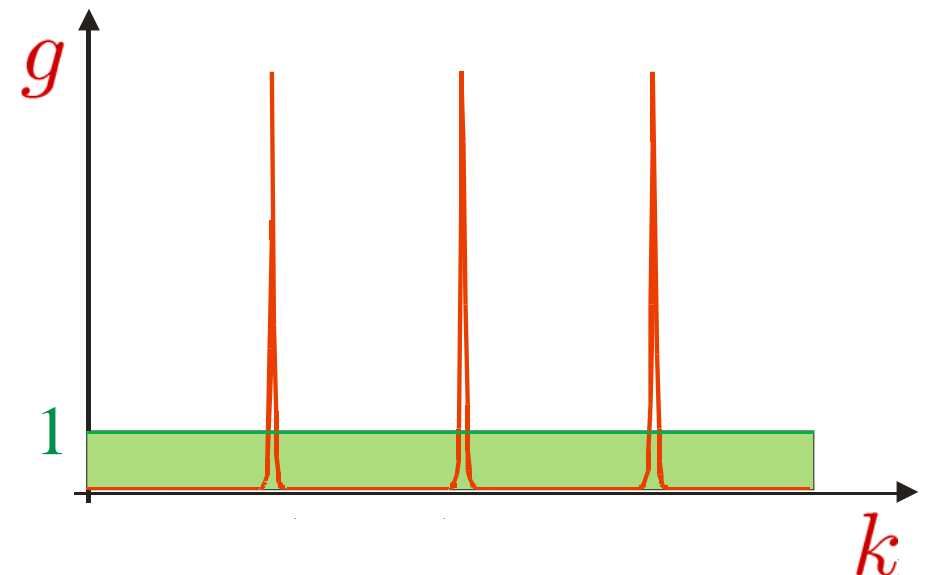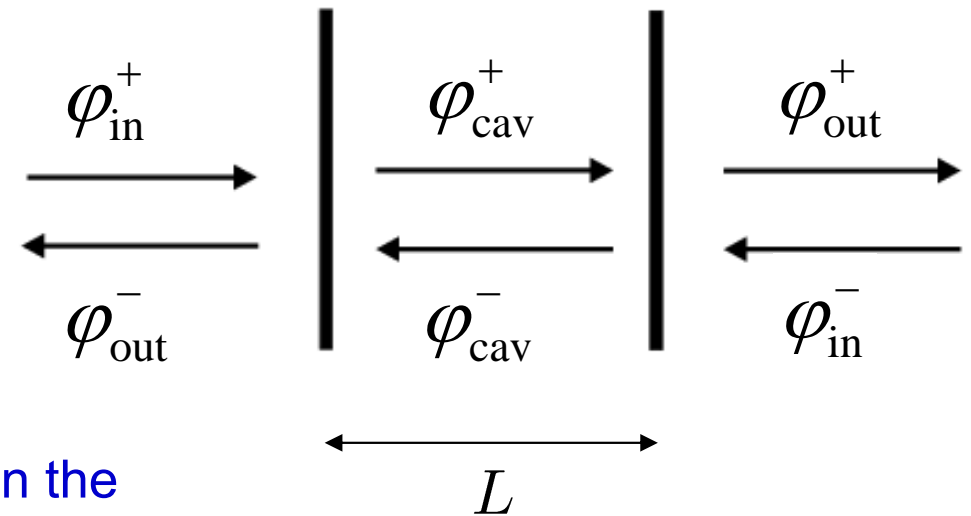
Casimir radiation pressure

➤ The Casimir force is the sum over all field modes of the difference between inner and outer radiation pressures

$$F = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi c} \, 2\hbar\omega N(\omega) \, (g(\omega) - 1) \,\Big]$$

Cavity confinement effect

Field fluctuation energy in the counter-propagating modes at frequency ω

$$2\hbar\omega N = 2\hbar\omega \left(\frac{1}{2} + \overline{n}(\omega) \right) = \frac{\hbar\omega}{\tanh \frac{\hbar\omega}{2k_\mathrm{B}T}} \,\Big]$$

Planck law including vacuum contribution

➤ Using the causality properties of the scattering amplitudes, and the transparency of mirrors at high-frequencies, the Casimir free energy can be written as a sum over Matsubara frequencies

$$F = -\frac{\partial \mathcal{F}(L, T)}{\partial L}$$

Two plane mirrors in 3d space

- Electromagnetic fields in 3d space with parallel mirrors

  - Static and specular scattering preserves frequency ω, transverse wavevector **k**, polarization p

  - reflection amplitudes depend on these quantum numbers

- Most of the derivation identical to the simpler 1d case, some elements to be treated with greater care

  - effect of dissipation and associated fluctuations

  - contribution of evanescent modes

- Free energy obtained as a Matsubara sum

$$\mathrm{Tr}\,d \equiv A \sum_p \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} d^p_{\mathbf{k}}$$

$$\xi_n = n\frac{2\pi k_{\mathrm{B}}T}{\hbar} \;,\; \kappa_n = \sqrt{\mathbf{k}^2 + \frac{\xi_n^2}{c^2}}$$

$$\mathcal{F} = k_{\mathrm{B}}T \sum_n{}' \mathrm{Tr}\ln d[i\xi_n]$$

$$d^p_{\mathbf{k}}[i\xi_n] \equiv 1 - r^p_{\mathbf{k}}[i\xi_n]e^{-2\kappa_n L}$$

- Pressure $\quad P = \dfrac{F}{A} = -\dfrac{\partial \mathcal{F}(L,T)}{A\partial L}$

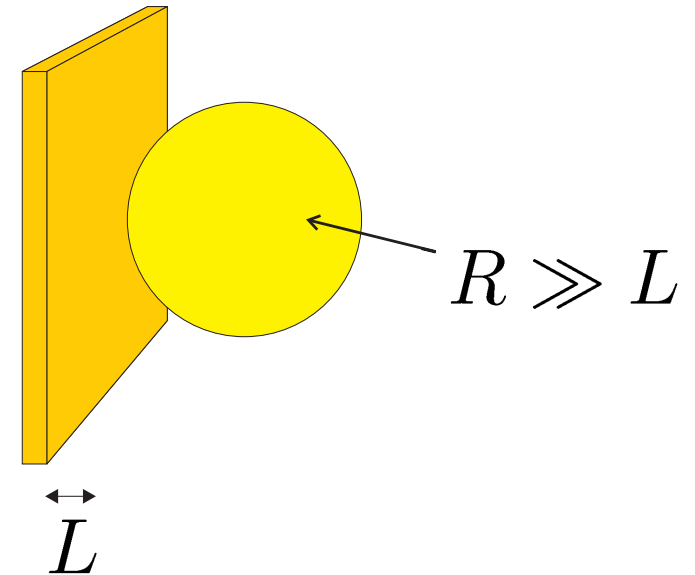$$r \equiv r_1 r_2$$

The plane-sphere geometry

- ➤ Force between a plane and a large sphere is usually computed using the "Proximity Force Approximation" (PFA)

  - ➤ Integrating the (plane-plane) pressure over the distribution of local inter-plate distance

$$F_{\mathrm{PFA}} = \int_L^\infty \mathrm{d}A \, P(L')$$

- ❑ **For a plane and a large sphere**

$$F_{\mathrm{PFA}} = 2\pi R \int_L^\infty \mathrm{d}L' \, P(L')$$

$$G_{\mathrm{PFA}} \equiv \frac{\partial F_{\mathrm{PFA}}}{\partial L} = 2\pi R P(L)$$

$$R \gg L$$

$$\overleftrightarrow{L}$$

L

- ➤ PFA is not a theorem !
- ➤ It is an approximation valid for large spheres
- ➤ Exact calculations now available "beyond PFA"

Models for the reflection amplitudes

- ➤ "Lifshitz formula" recovered for
  - ➤ bulk mirror described by a linear and local dielectric function
  - ➤ Fresnel laws for reflection

- ➤ Ideal Casimir formula recovered for $r \rightarrow 1$ and $T \rightarrow 0$

I.E. Dzyaloshinskii,
E.M. Lifshitz & L.P. Pitaevskii,
Sov. Phys. Usp. **4** (1961) 153

J. Schwinger,
L.L. de Raad & K.A. Milton,
Ann. Physics **115** (1978) 1

- ➤ The scattering formula allows one to accommodate more general cases for the reflection amplitudes
  - ➤ finite thickness, multilayer structure
  - ➤ non isotropic response, chiral materials
  - ➤ non local dielectric response
  - ➤ microscopic models of optical response …

- ➤ It has been extended to more general geometries

A. Lambrecht, P. Maia Neto & S. Reynaud, New J. Physics **8** (2006) 243

Models for metallic mirrors

- Simple models for the (reduced) dielectric function for metals

  $$\varepsilon[i\xi] = \bar{\varepsilon}[i\xi] + \frac{\sigma[i\xi]}{\xi}$$

  - bound electrons (inter-band transitions)
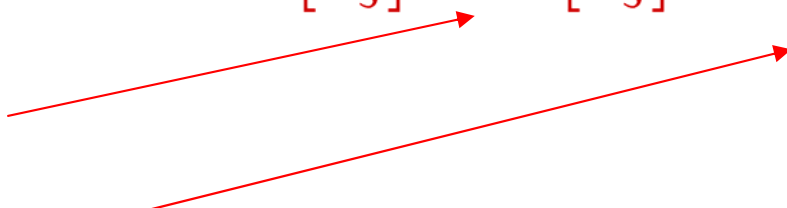
  - conduction electrons
    - determined by (reduced) conductivity σ

  - model for conductivity

    $$\sigma[i\xi] = \frac{\omega_P^2}{\xi + \gamma}$$

    - plasma frequency ω_P
    - Drude relaxation parameter γ

- Drude parameters related to the density of conduction electrons and to the static conductivity

  $$\omega_P^2 = \frac{nq^2}{\varepsilon_0 m^*}$$

  - finite conductivity σ_0 ⇔ non null γ

  $$\sigma_0 = \frac{\omega_P^2}{\gamma}$$

Pressure between metallic mirrors at T≠0

➢ Pressure variation wrt ideal Casimir formula

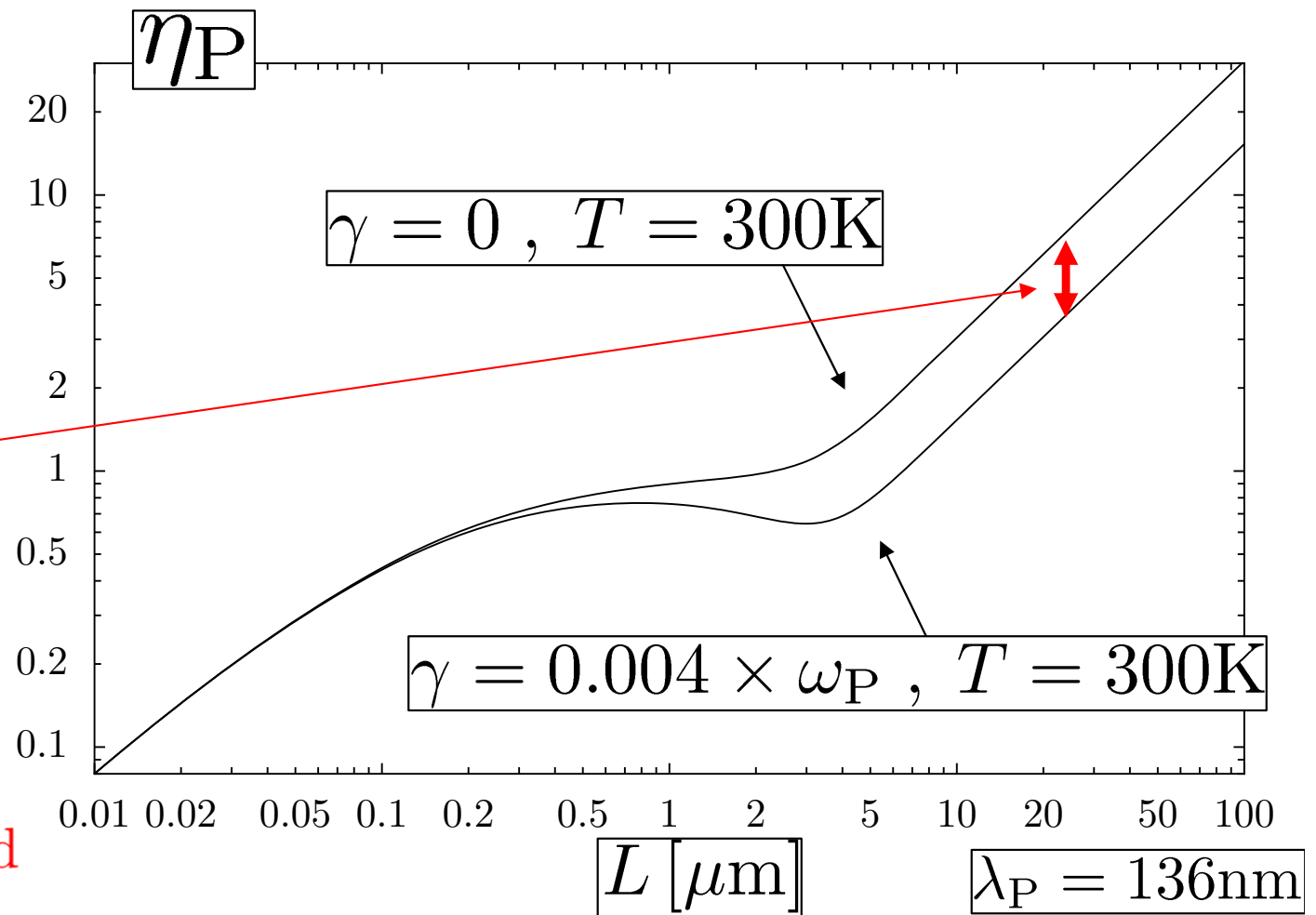$$\eta_{\mathrm{P}} = \frac{P}{P_{\mathrm{Cas}}}$$

$$P_{\mathrm{Cas}} = -\frac{\hbar c \pi^2}{240 L^4}$$

➢ small losses lead to a large factor 2 at large distance
(high temperature limit)

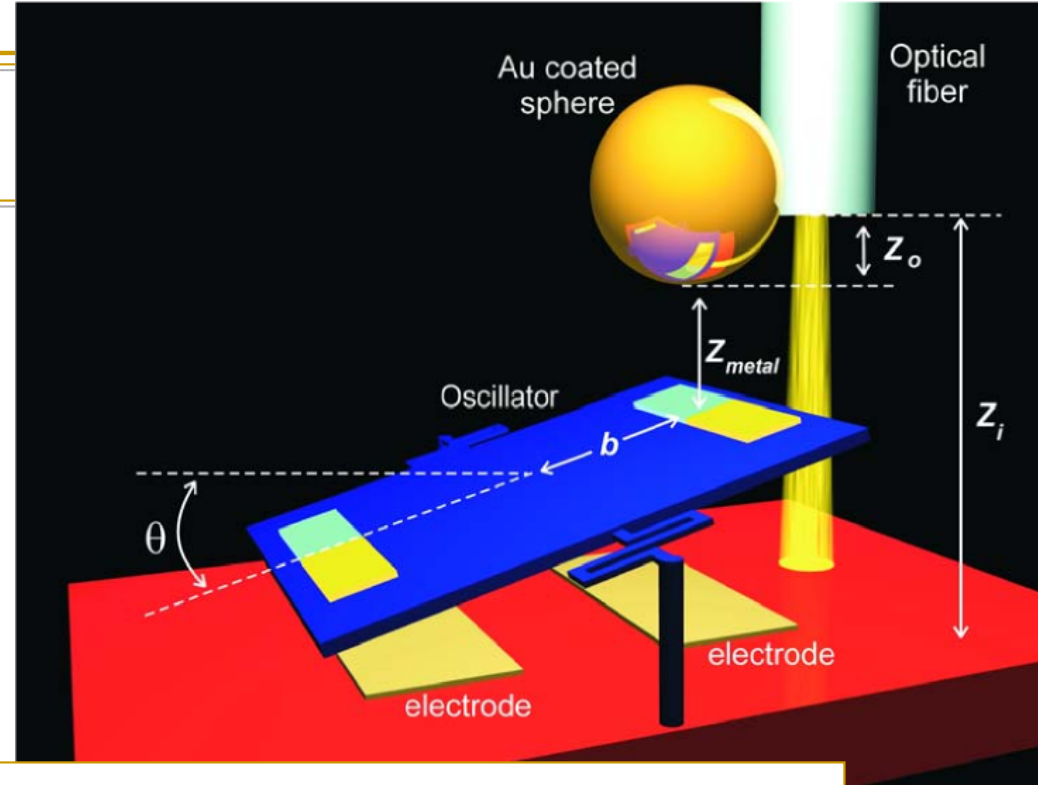$\bar{\varepsilon} = 1$ for this plot

Drude parameters : Gold

η_{P}

$\gamma = 0 \,,\; T = 300\mathrm{K}$

$\gamma = 0.004 \times \omega_{\mathrm{P}} \,,\; T = 300\mathrm{K}$

$L \,[\mu\mathrm{m}]$

$\lambda_{\mathrm{P}} = 136\mathrm{nm}$

Casimir experiments

- Recent precise experiments : dynamic measurements of the resonance frequency of a microresonator

- Shift of the resonance gives the gradient of Casimir force, *ie* the plane-plane pressure

Courtesy R.S. Decca
(Indiana U – Purdue U Indianapolis)
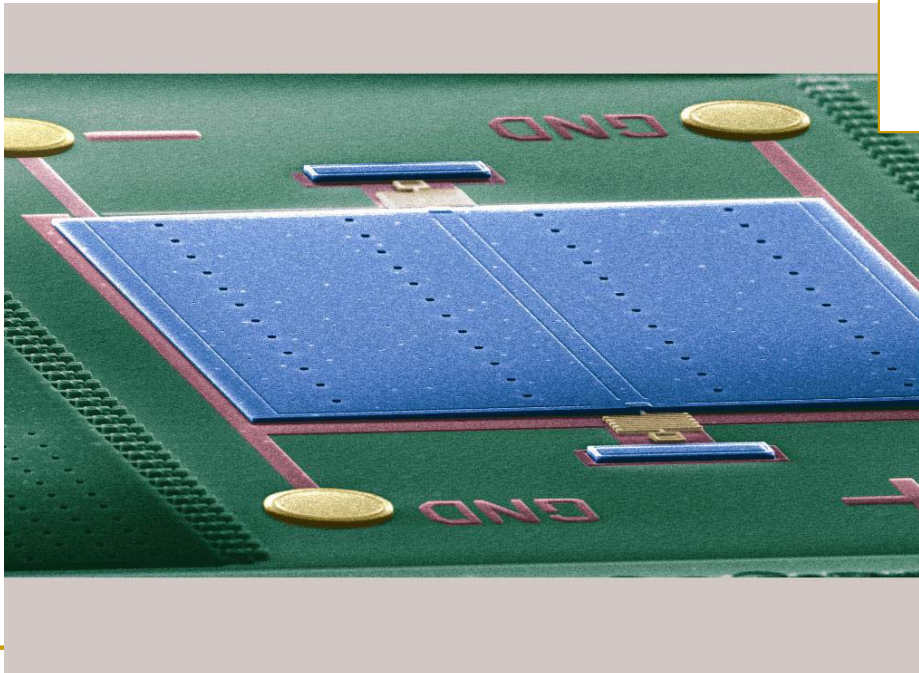
Sphere Radius: R = 150 μm

Micro Torsion Oscillator Size:

500 μm × 500 μm × 3.5 μm

Distances: L = 0.16 - 0.75 μm

➡ An unexpected result !!
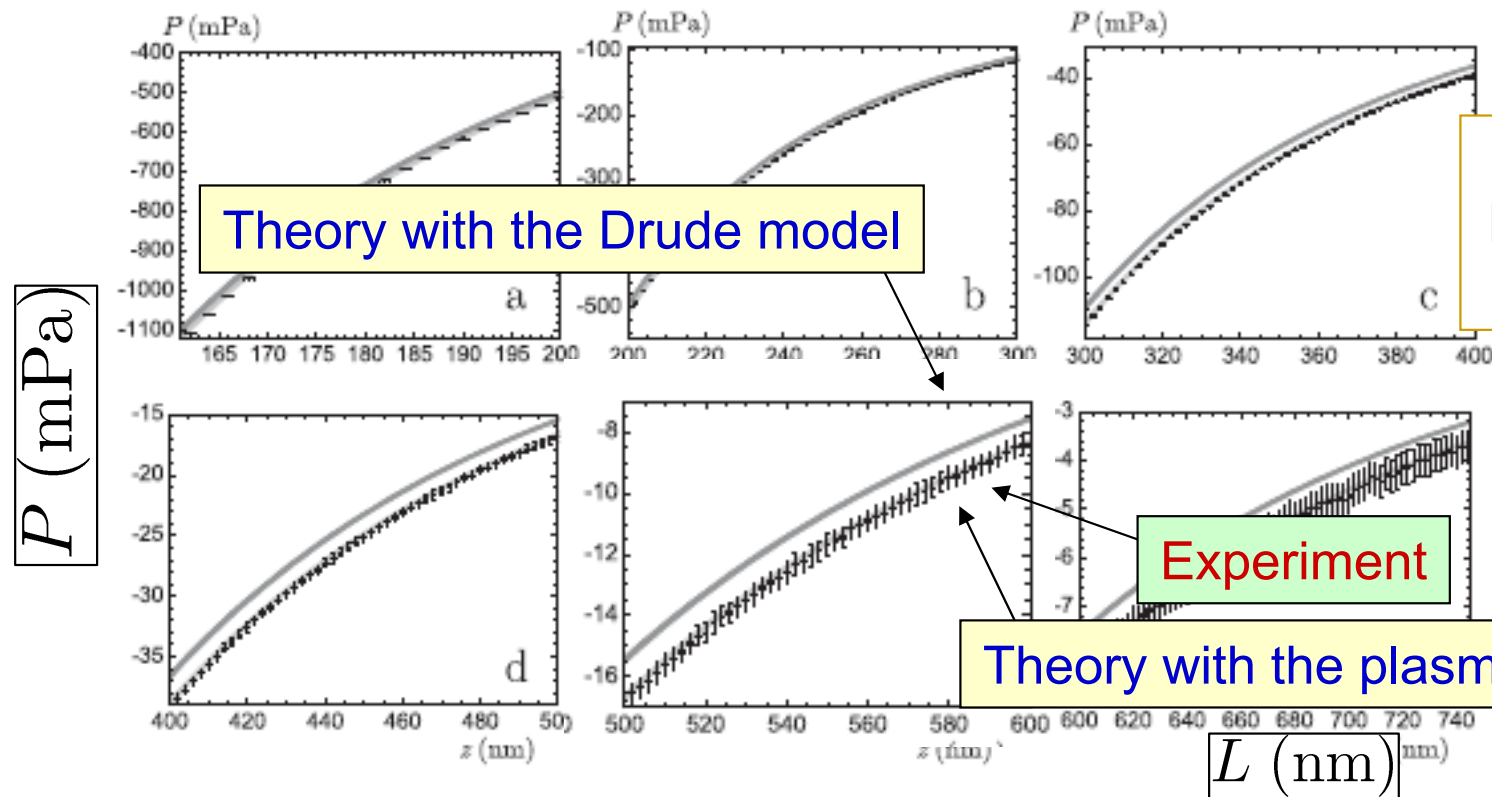
R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

Casimir experiments …

Purdue (and Riverside) measurements favor the lossless plasma model and thus deviate from theory with dissipation accounted for

BRIEF REPORTS   PHYSICAL REVIEW D 75, 077101 (2007)

Theory with the Drude model

Courtesy
R.S. Decca *et al*
(IUPUI)

$T = 300K$

Experiment

Theory with the plasma model

P (mPa)

L (nm)

FIG. 1. Experimental data for the Casimir pressure as a function of separation z. Absolute errors are shown by black crosses in different separation regions (a–f). The light- and dark-gray bands represent the theoretical predictions of the impedance and Drude model approaches, respectively. The vertical width of the bands is equal to the theoretical error, and all crosses are shown in true scale.

R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

Casimir experiments ...

- Lamoreaux group @ Yale

  - torsion-pendulum experiment

  - larger radius: R = 156 mm

  - larger distances: L = 0.7 - 7 μm
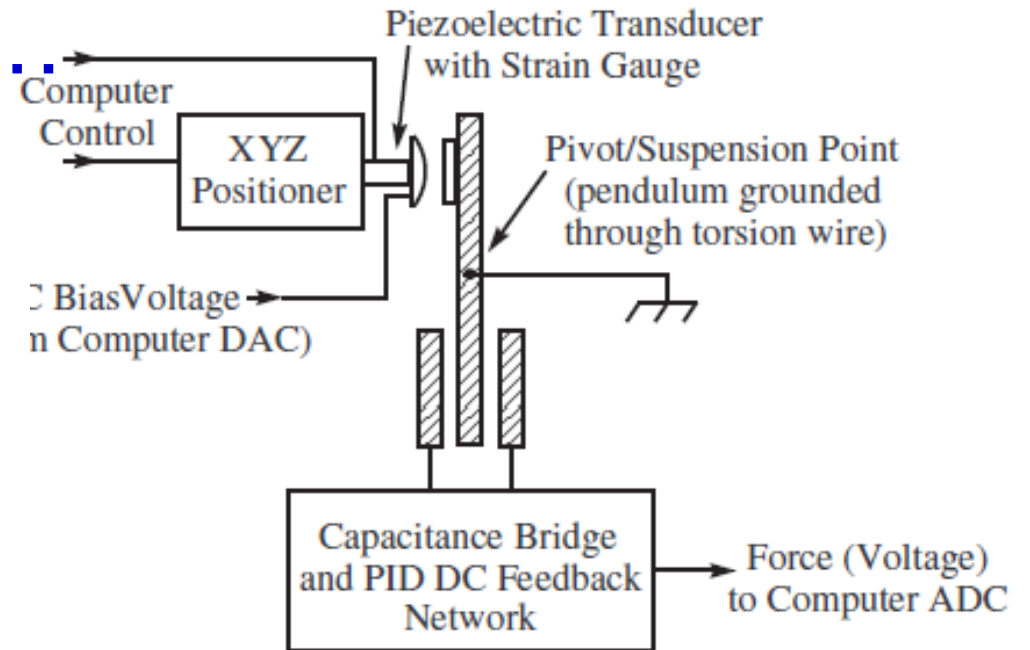
A.O. Sushkov, W.J. Kim, D.A.R. Dalvit, S.K. Lamoreaux, Nature Phys. (6 Feb 2011)

- Thermal contribution seen at large distances (where it is large)

- Results favor the Drude model after subtraction of
  a large contribution of the electrostatic patch effect (see below)

- Results of different experiments point to different models

- Some experiments disagree with the preferred theoretical model

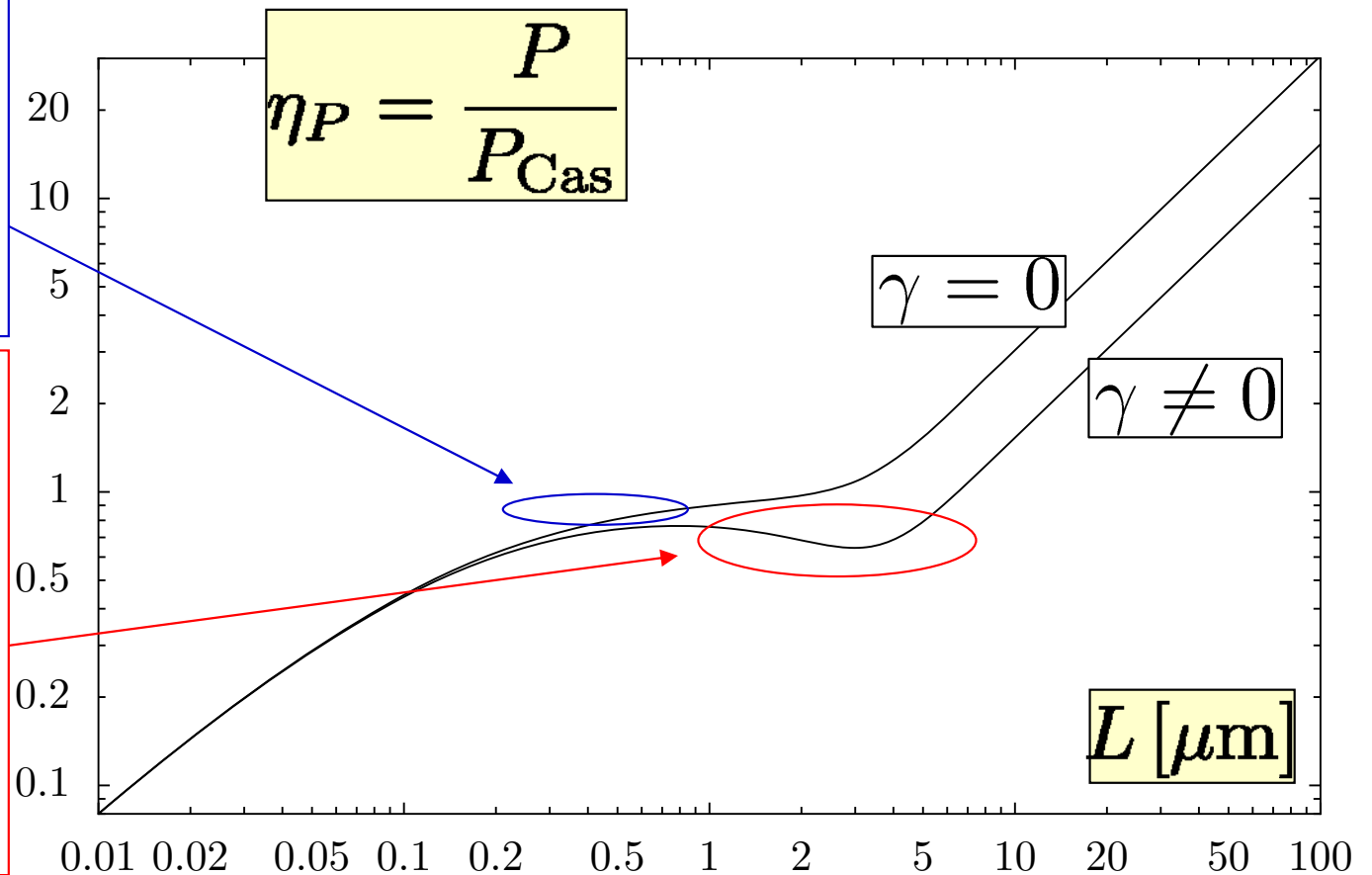K. Milton, *News & Views* Nature Physics (6 Feb 2011)

Casimir experiments and theory

R.S. Decca, D. Lopez, E. Fischbach *et al*, Phys. Rev. **D75** (2007) 077101

G.L. Klimchitskaya, U. Mohideen, V.M. Mostepanenko, R.M.P. **81** (2009) 1827

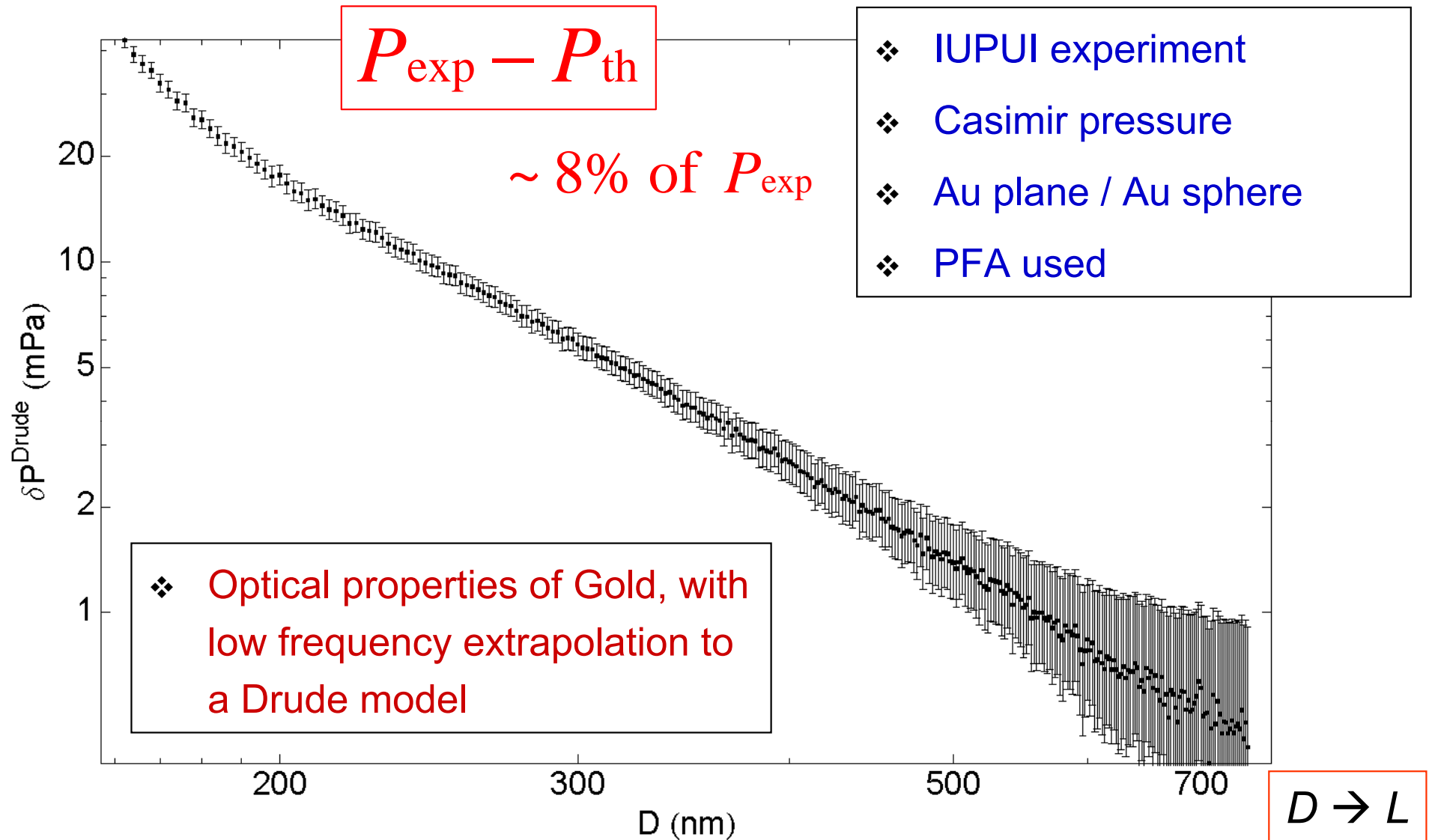- IUPUI & UCR experiments at L<0.7µm favor γ=0 (patches neglected)

- Yale experiment at 0.7µm<L<7µm favors γ≠0 after subtraction of a large contribution of patches

$$\eta_P = \frac{P}{P_{\text{Cas}}}$$

$\gamma = 0$

$\gamma \neq 0$

$L\,[\mu\text{m}]$

A.O. Sushkov, W.J. Kim, D.A.R. Dalvit, S.K. Lamoreaux, Nature Phys. (6 Feb 2011)

D. Garcia-Sanchez, K.Y. Fong, H. Bhaskaran, S. Lamoreaux, PRL (10 Jul 2012)
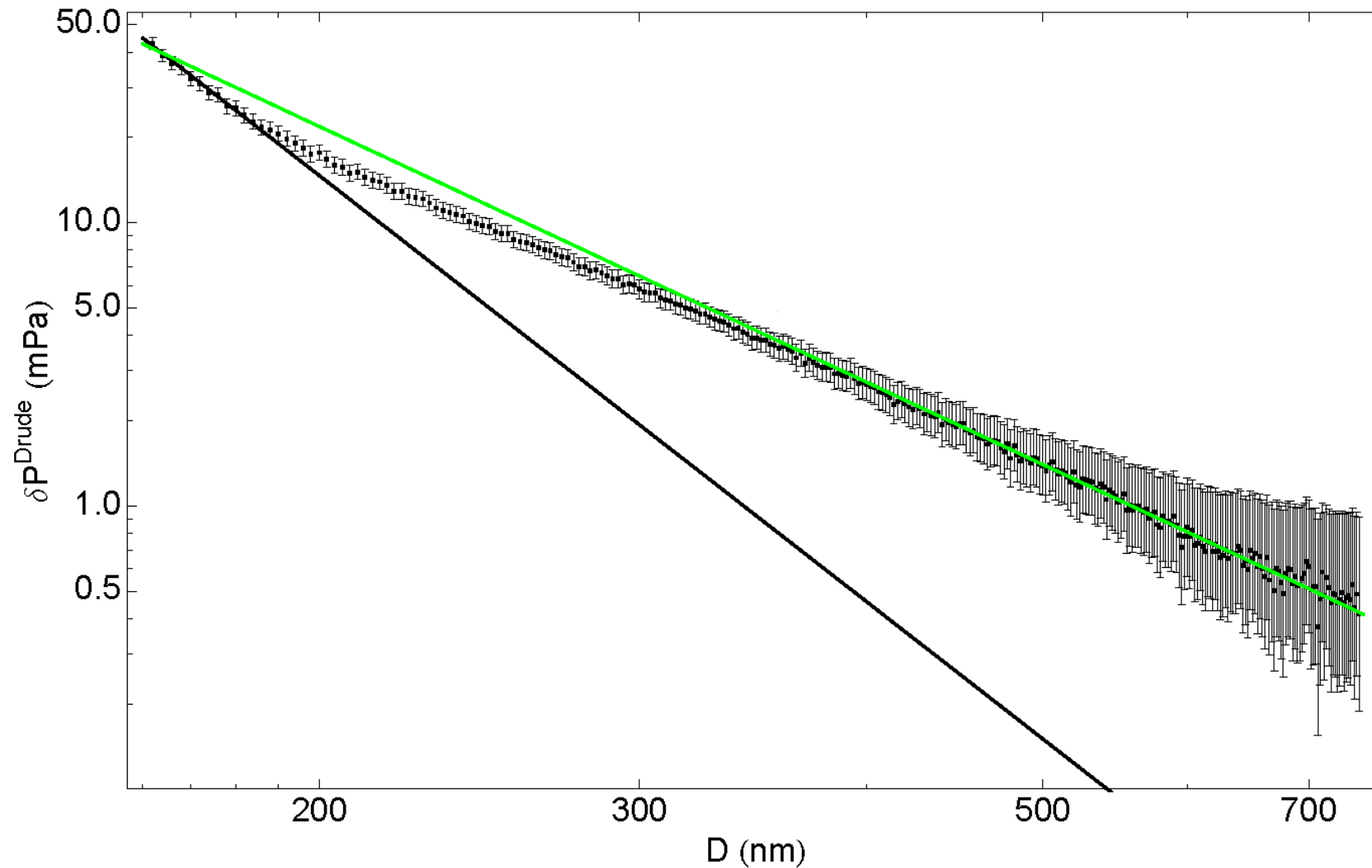
Deviation experiment / theory …

$$P_{\text{exp}} - P_{\text{th}}$$

$\sim 8\%$ of P_{exp}

- ❖ IUPUI experiment
- ❖ Casimir pressure
- ❖ Au plane / Au sphere
- ❖ PFA used

- ❖ Optical properties of Gold, with low frequency extrapolation to a Drude model

$D \rightarrow L$

Experimental data kindly provided by R. Decca (IUPUI)

Theoretical pressure calculated by R. Behunin, D. Dalvit, F. Intravaia (LANL)

Deviation experiment / theory ...

$D \rightarrow L$

The difference does not look like a Yukawa law ...

But it looks like a combination of power laws !

What can this difference mean ?

- ➤ A discrepancy between theory and (some) experiments …

  … which needs an explanation !

- ➤ New forces ???

- ➤ Artifact in the experiments ??

- ➤ Inaccuracy in the theoretical evaluations ?

  - ➤ A problem with vacuum energy ???

  - ➤ A problem with theoretical formula ??

  - ➤ A problem with the description of dissipation for metals ?   maybe …

- ➤ Systematic effects misrepresented in the analysis ?

  - ➤ The corrections beyond PFA ???   } { Now calculated

  - ➤ The contribution of plate roughness ???   Seems unlikely

  - ➤ The contribution of electrostatic patches ?   ← the main suspect !

  - ➤ Something else ??   always possible …

The patch effect

➤ **Surfaces of metallic plates are not equipotentials**

  ➤ Real surfaces are made of crystallites

  ➤ Crystallites correspond to ≠ crystallographic orientations and ≠ work functions

➤ **For ultraclean surfaces** ➜
  (ultra-high vacuum, ultra-low temperature)

  ➤ Patch pattern is related to topography

  ➤ AFM, KPFM, EBSD maps are directly related

➤ **Contamination affects the patch potentials**

  ➤ enlarges patch sizes and smoothes voltages

➤ **Patch effect has been known for decades to be a limitation for precision measurements**

  ➤ Free fall of antimatter, gravity tests, surface physics, experiments with cold atoms or ion traps …

Topo

Range : 9.6 nm

2 µm

(a)

KPFM

Range : 42 meV

H

M

L

(b)

EBSD

<111>

<100>

<110>
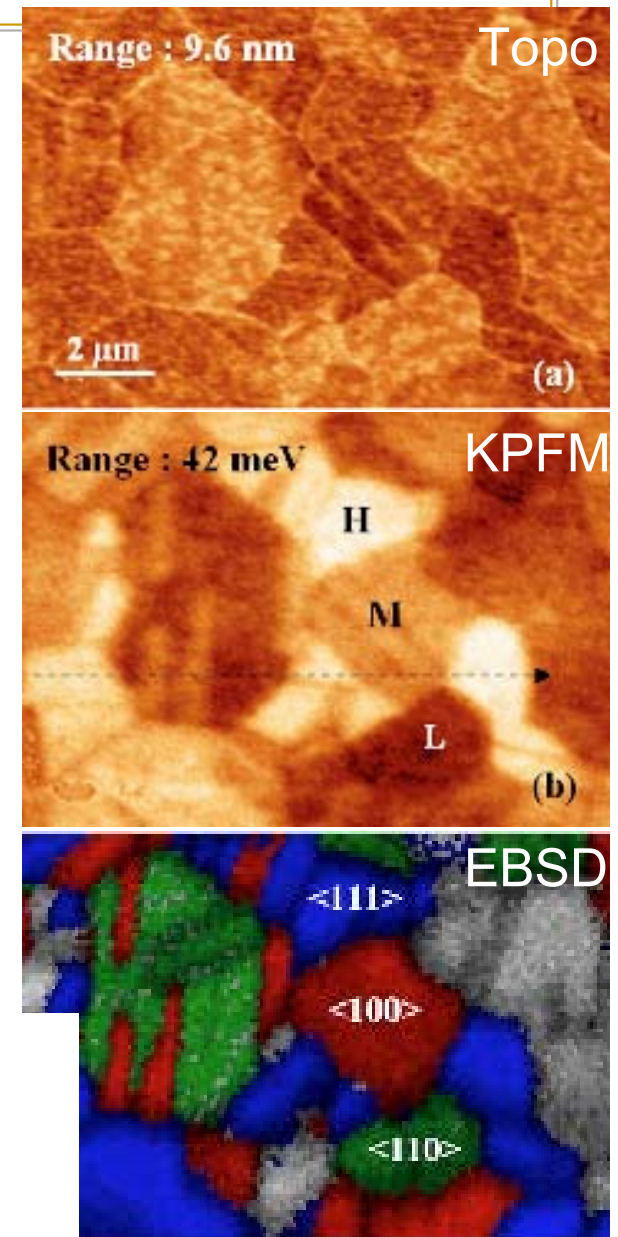
Modeling the patches

- The pressure (between two planes) due to electrostatic patches can be computed by solving the Poisson equation

$$ P = \frac{\varepsilon_o}{4\pi} \int_0^\infty \frac{\mathrm{d}k\ k^3}{\sinh^2(kL)} \left\{ C_{11}[k] + C_{22}[k] - 2C_{12}[k]\cosh(kL) \right\} $$

- It depends on the spectra describing the correlations of the patch voltages

$$ C_{ij}[\mathbf{k}] = \int \mathrm{d}^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} C_{ij}(\mathbf{r}) $$

- The spectra had not been measured in Casimir experiments up to now

$$ C_{ij}(\mathbf{r}) = \langle V_i(\mathbf{r}) V_j(\mathbf{0}) \rangle $$
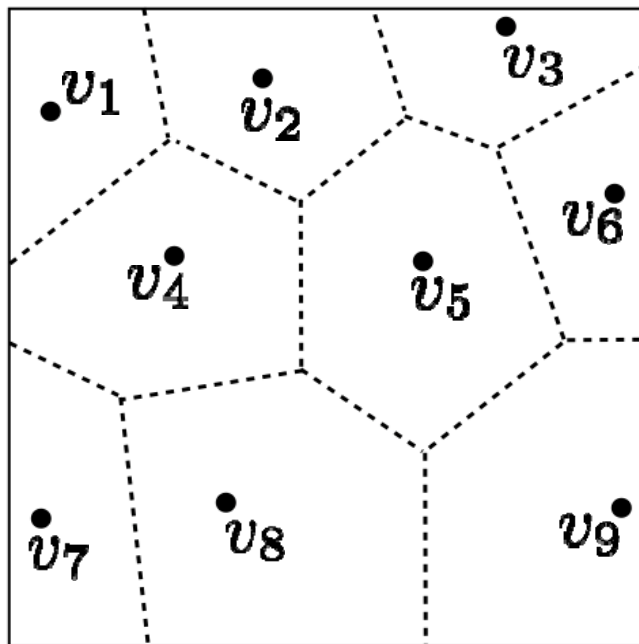
- In the commonly used model, the spectrum is supposed to have sharp cutoffs at low and high-k
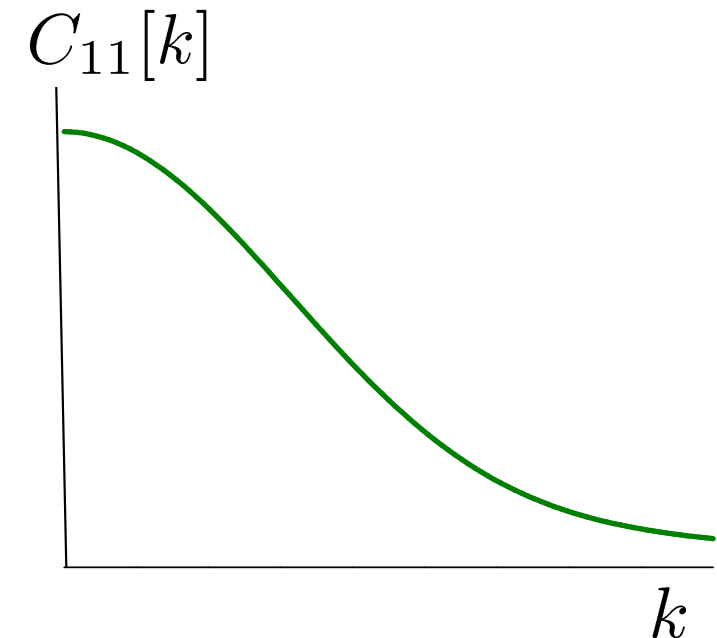- This is a very poor representation of the patches

k

k_{min} k_{max}

C.C. Speake & C. Trenkel PRL (2003) ; R.S. Decca *et al* (2005)
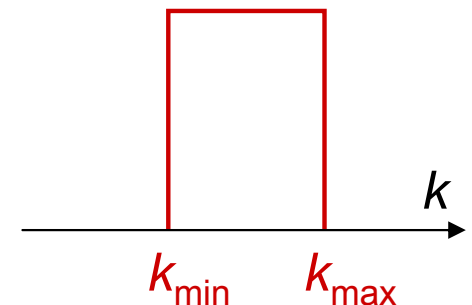
Modeling the patches ...
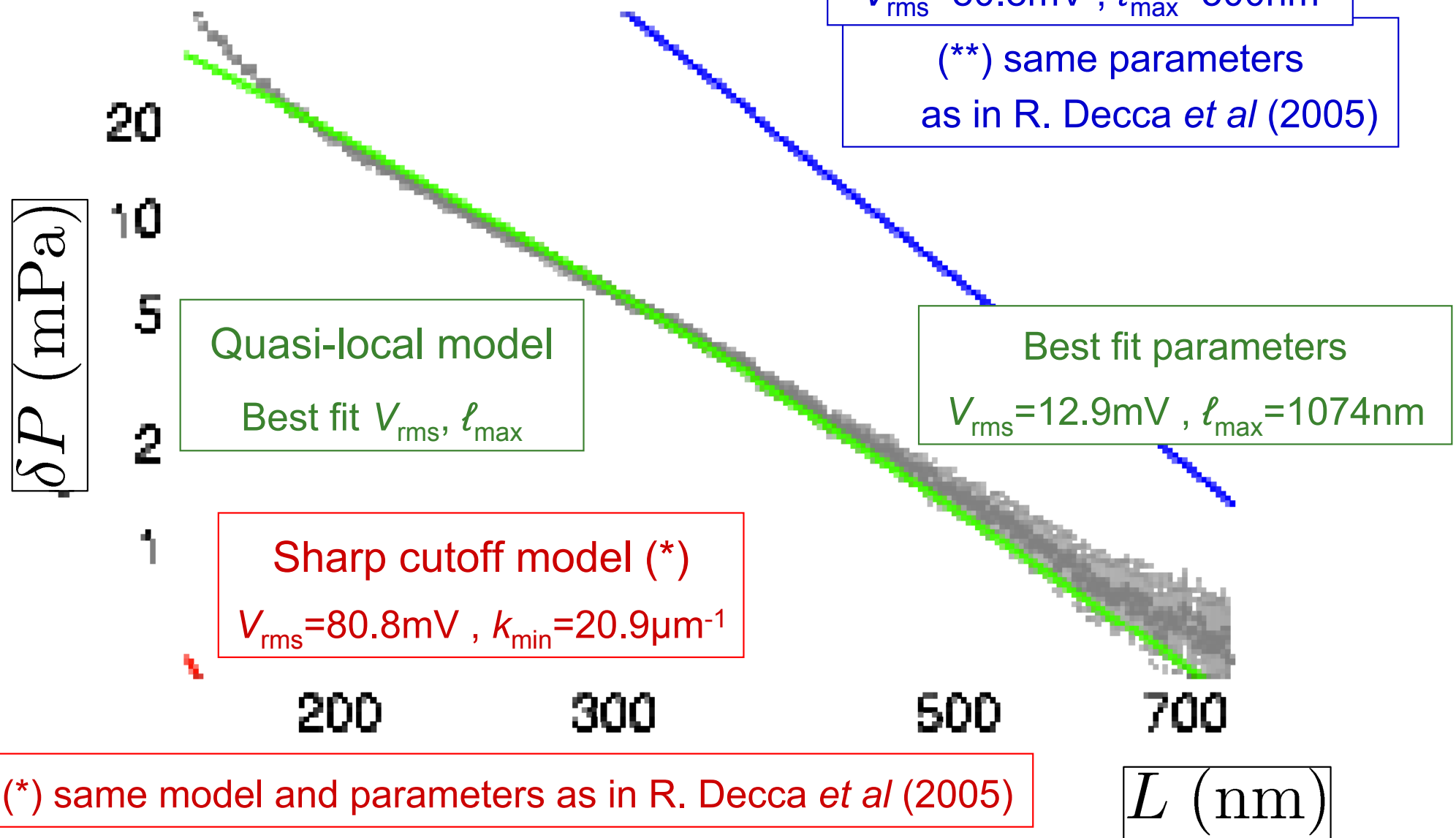
➢ A "quasi-local" representation of patches

tessellation of sample surface and random assignment of the voltage on each patch

🔶 This produces a smooth spectrum (no cutoff)

➢ Similar models used to study the effect of patches in ion traps

- R. Dubessy, T. Coudreau, L. Guidoni, PRA **80** (2009) 031402
- D.A. Hite, Y. Colombe, A.C. Wilson et al, PRL **109** (2012) 103001

Modeling the patches …

Quasi-local model

Best fit V_{rms}, ℓ_{max}

Best fit parameters

V_{rms}=12.9mV , ℓ_{max}=1074nm

Sharp cutoff model (*)

V_{rms}=80.8mV , k_{min}=20.9μm^{-1}

(*) same model and parameters as in R. Decca *et al* (2005)

δP (mPa)

20
10
5
2
1

200   300   500   700

L (nm)

R.O. Behunin, F. Intravaia, D.A.R. Dalvit, P.A. Maia Neto & SR, PRA (2012)

Provisional conclusions

❖ Casimir effect is verified but there is still room for improvement

❖ Puzzle : some experiments favor the lossless plasma model !

⊚ Maybe due to the contribution of electrostatic patches

  ➢ differences between IUPUI data and Drude model predictions can be fitted by the quasi-local model for electrostatic patches

  ➢ parameters obtained from the best fit are not compatible with the identification of patches as crystallites; they are compatible with a contamination of the metallic surfaces (V_{rms}~10mV , ℓ_{max}~1µm)

⊚ Next steps

  ➢ characterize real patches with Kelvin Probe Force Microscopy

    ➢ ongoing with ISIS Strasbourg and ISOF Bologna

  ➢ deduce the force in the plane-sphere geometry

    ➢ ongoing with LANL Los Alamos

  ➢ compare with knowledge from other domains

    ➢ surface physics, cold atoms and ion traps

R.O. Behunin *et al*,
PRA **86** (2012) 052509