Particle Physics: The Standard Model

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1. The Standard Model of Particle Physics: Overview
   - Kinematics
   - s channel and t channel
   - Cross section and total width
   - Description of an unstable particle

The Course Philosophy

- Emphasis of the course is on the phenomenology
- We will discuss experimental aspects but more important is the interpretation of measurements
- In an ideal world: construct theory and apply it
- Real (course) world: theory and application in parallel
- Build the theory knowledge to put the experiments into perspective
- Natural units: $\hbar = c = 1 \rightarrow \hbar c = 197.3 \mathrm{MeV} \cdot \mathrm{fm}$

Matter = fermions (Spin-$\frac{1}{2}$ particles):

- Electrons with two spin orientations: L and R
- Neutrinos (L)
- Quarks L and R (proton=uud, neutron=udd)
- Three families = heavier copies of the first family

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

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Interactions $=$ bosons (Spin–0 or –1 particles):

- Electromagnetism: Spin–1 massless
- Strong interaction (p=uud): Spin–1 massless
- Weak interaction: Spin–1 massive
- Masses: Spin–0 massive

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Properties: Electric Charge

- Fractional charges not observed in nature
- Strong interaction: uud, udd

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Properties: Color charge

- Sum of colors (RGB) white
- R+G+B= (qqq =baryon)
- Color+anti-color= White ($q\bar{q}$ =meson)
- Gluon carries color+anti-color
- 8 different gluons (not 9)

$$\begin{array}{cccc} C & \left(\begin{array}{c} u_L \\ d_L \end{array}\right) & \left(\begin{array}{c} c_L \\ s_L \end{array}\right) & \left(\begin{array}{c} t_L \\ b_L \end{array}\right) \\ C & & & \\ - & \left(\begin{array}{c} \nu_{e_L} \\ e_L \end{array}\right) & \left(\begin{array}{c} \nu_{\mu_L} \\ \mu_L \end{array}\right) & \left(\begin{array}{c} \nu_{\tau_L} \\ \tau_L \end{array}\right) \\ - & & & \\ C & u_R & c_R & t_R \\ C & d_R & s_R & b_R \\ - & e_R & \mu_R & \tau_R \\ - & \gamma & & \\ C+\bar{C}' & g & & \\ - & W^{\pm}, Z^{\circ} & & \\ - & H & & \end{array}$$

Rule of thumb for interactions

Interaction	Carrier	Relative strength
Gravitation	Graviton (G)	10^{-40}
Weak	Weak Bosons ($W^{\pm}$,Z°)	10^{-7}
Electromagnetic	Photon (γ)	10^{-2}
Strong	Gluon (g)	1

- Forget about Gravitation in particle physics problems
- The course and problem solving sessions will lead us to understand how the model describes the interactions and their strength.

$$\mathbf{a} = (E_a, \vec{p}_a) = (p_0, p_1, p_2, p_3)$$
$$E_a \cdot E_a - \vec{p}_a \cdot \vec{p}_a = m_a^2$$
$$g^{\mu\nu} p_\mu p_\nu = m_a^2$$

$$g^{\mu\mu} = (1, -1, -1, -1)$$
for $\mu \neq \nu : g^{\mu\nu} = 0$

Conservation of E and $\vec{p}$

$$\mathbf{a} + \mathbf{b} = \mathbf{c} + \mathbf{d}$$

therefore

$$\mathbf{a} - \mathbf{c} = \mathbf{d} - \mathbf{b}$$

Mandelstam Variables

$$a + b \rightarrow c + d$$
$$s = (\mathbf{a} + \mathbf{b})^2$$
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Theorem

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Proof.

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$$\begin{aligned} s &= \mathbf{a}^2+\mathbf{b}^2+2\cdot\mathbf{a}\cdot\mathbf{b} \\ &= m_a^2+m_b^2+2(E_a\cdot E_b-\vec{p}_a\cdot\vec{p}_b) \\ &= 2(E_a\cdot E_b-\vec{p}_a\cdot\vec{p}_b) \\ &= 2(E_a\cdot E_a+\vec{p}_a\cdot\vec{p}_a) \\ &= 2(E_a^2+E_a^2) \\ &= 4E_a^2 \\ t &= -2(E_a\cdot E_c-\vec{p}_a\cdot\vec{p}_c) \\ u &= -2(E_a\cdot E_d-\vec{p}_a\cdot\vec{p}_d) \\ &= -2(E_a\cdot E_c+\vec{p}_a\cdot\vec{p}_c) \end{aligned}$$

□

Proof.

$$
\begin{aligned}
t+u &= -2(2\cdot E_a\cdot E_c)\\
&= -2(2\cdot E_a\cdot E_a)\\
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&= 0
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2 particle reaction → 2 independent variables!

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Crossing relationship

$a+b\rightarrow c+d$	$a+\bar{c}\rightarrow \bar{b}+d$		
$s=(\mathbf{a}+\mathbf{b})^2$	$s'=(\mathbf{a}+\bar{\mathbf{c}})^2$	$=(\mathbf{a}-\mathbf{c})^2$	$=t$
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- Calculate a process as function of s,t,u
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Kinematics and Crossing and the - in Problem Solving

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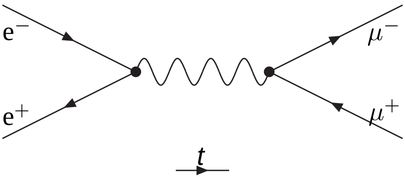
Kinematics and Crossing and the - in Problem Solving

s–channel: annihiliation

$$e^+ + e^- \rightarrow \gamma \rightarrow \mu^+\mu^-$$



$$\begin{aligned} \mathbf{q}_\gamma &= \mathbf{p}_{e^-} + \mathbf{p}_{e^+} \\ s &= \mathbf{q}_\gamma^2 \\ (CM) &= (E_{e^-} + E_{e^+})^2 \\ &> 0 \end{aligned}$$

the photon is massive (virtual)
time-like

t–channel: scattering

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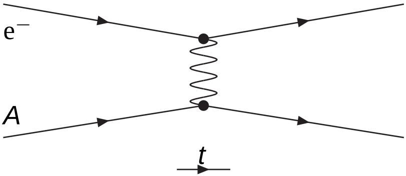

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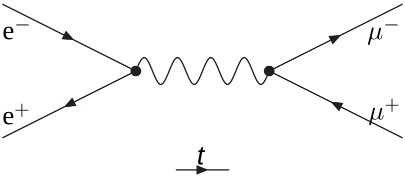

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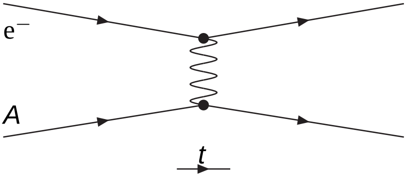

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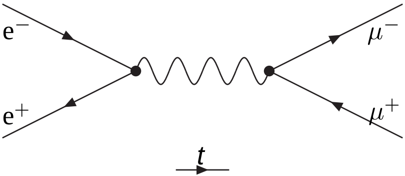

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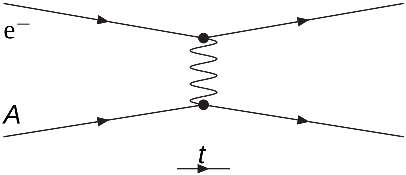

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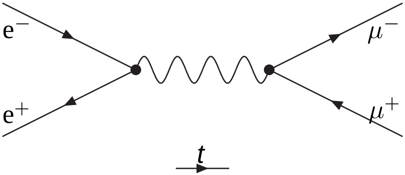

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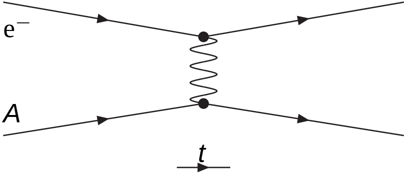

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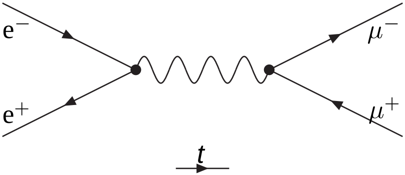

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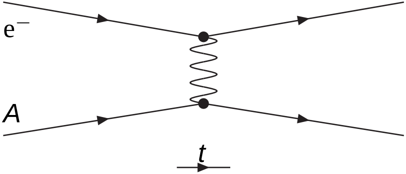

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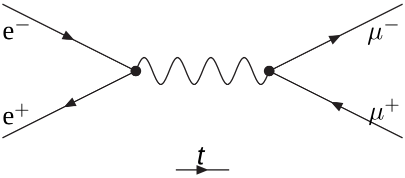

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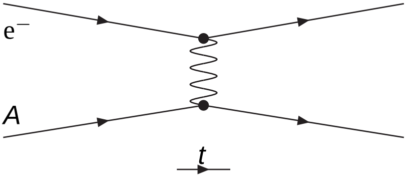

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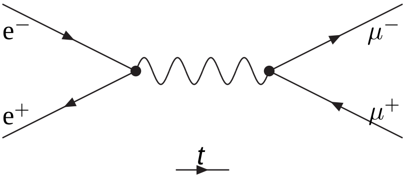

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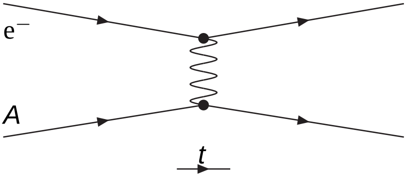

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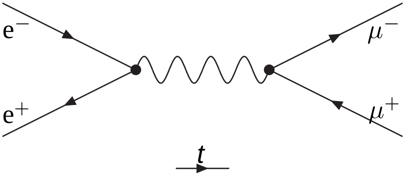

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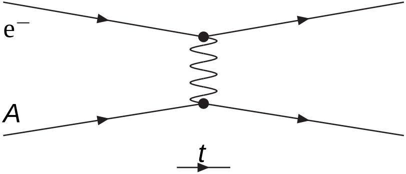

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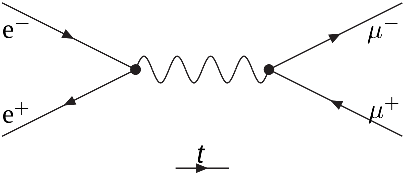

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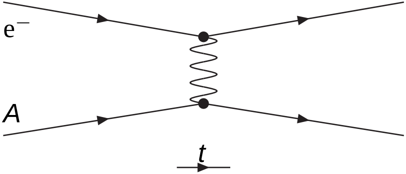

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Cross Section

- The cross section σ is the ratio of the transition rate and the flux of incoming particles.
- Its unit is cm^2
- $1\mathrm{b} = 10^{-24}\mathrm{cm}^2$ (puts barn in perspective, doesn't it?)

Two ingredients:

- the interaction transforming initial state $|i\rangle$ to a final state $\langle f|$ of m particles with four-vectors $\mathbf{p_i'}$
- kinematics (including Lorentz-Invariant phase space element)

$$d\sigma = \frac{1}{2S_{12}} \prod_{i=1}^{m} \frac{d^3 p_i'}{(2\pi)^3 2E_i'^0} (2\pi)^4 \delta(\mathbf{p_1'} + \ldots + \mathbf{p_m'} - \mathbf{p_1} - \mathbf{p_2}) |\mathcal{M}|^2$$

with (originating from flux) $S_{12} = \sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}$

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- kinematics (including Lorentz-Invariant phase space element)

$$d\sigma = \frac{1}{2S_{12}} \prod_{i=1}^{m} \frac{d^3 p'_i}{(2\pi)^3 2E'^0_i} (2\pi)^4 \delta(\mathbf{p}'_\mathbf{1} + \ldots + \mathbf{p}'_\mathbf{m} - \mathbf{p_1} - \mathbf{p_2}) |\mathcal{M}|^2$$

with (originating from flux) $S_{12} = \sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}$

Total Width or Decay Rate

- Total width is the inverse of the lifetime of the particle
- unit: energy, e.g., GeV.
- Closely related, but not identical to the cross section

$$\mathrm{d}\Gamma = \frac{1}{2E}\prod_{i=1}^{m}\frac{\mathrm{d}^3p_i'}{(2\pi)^3 2E_i'^0}\delta(\mathbf{p_1'}+...+\mathbf{p_m'}-\mathbf{p_1})|\mathcal{M}|^2$$

For the decay of an unpolarized particle of mass M into two particles (in the CM frame $\vec{p}_1' = -\vec{p}_2'$):

$$\mathrm{d}\Gamma = \frac{1}{32\pi^2}\frac{|\vec{p}_1'|}{M^2}|\mathcal{M}|^2\mathrm{d}\Omega$$

where Ω is the solid angle with $\mathrm{d}\Omega = \mathrm{d}\phi\mathrm{d}\cos\theta$

Cross section and total width for a final state with 2 particles

Cross section $2 \to 2$ reaction with four massless particles:

$$\mathrm{d}\sigma = \frac{1}{64\pi^2} \frac{|\mathcal{M}|^2}{s} \mathrm{d}\Omega$$

Width of a massive particle ($\sqrt{s} = M$) decaying to two massless particles in the final state $|\vec{p}_1'| = \sqrt{s}/2$:

$$\mathrm{d}\Gamma = \frac{1}{64\pi^2} \frac{|\mathcal{M}|^2}{\sqrt{s}} \mathrm{d}\Omega$$

Study of the phase space in Problem Solving with applications to 2-body and 3-body reactions.

- Particles: plane waves
  $\psi(\vec{x},t) \sim \exp -im_0 t$
- $m_0 \rightarrow m_0 - i\Gamma/2$

$$\begin{aligned} N(t) &= N_0 \cdot \exp -t/\tau \\ \Gamma &= 1/\tau \end{aligned}$$

Fourrier transform to momentum space:

$$\begin{aligned} A &\sim \frac{1}{(m-m_0)+i\Gamma/2} \\ |A|^2 &\sim \frac{1}{(m-m_0)^2+\Gamma^2/4} \end{aligned}$$

Γ: full width half maximum
Similarity to classical mechanics: resonance

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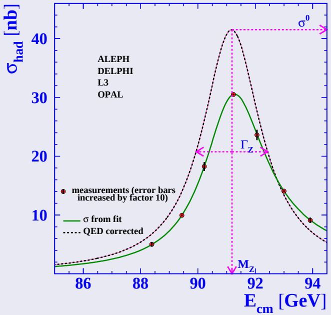
Example $e^-e^+ \rightarrow Z^\circ \rightarrow q\bar{q}$



lifetime too short to be measured directly: measure mass via decay products $q\bar{q}$
cross section measurement

- Particles: plane waves $\psi(\vec{x}, t) \sim \exp -im_0 t$
- $m_0 \to m_0 - i\Gamma/2$

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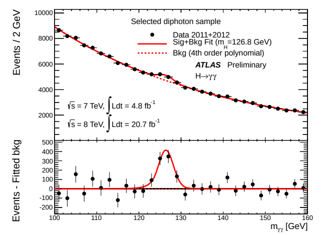
Example $pp \to H \to \gamma\gamma$ (EW 2013)



Beware: the width here has nothing to do with $\Gamma \sim 5\mathrm{MeV}$!
The experimental resolution is the origin (error propagation):

$$\begin{aligned} m_H &= \sqrt{(\mathbf{p_1}^\gamma + \mathbf{p_2}^\gamma)^2} \\ &= \sqrt{2E_1^\gamma E_2^\gamma (1 - \cos\theta)} \end{aligned}$$

Suppose that we have two (and exactly two) possible decays for the particle *a*:

$$\begin{aligned} a &\rightarrow b + c \\ a &\rightarrow d + e \end{aligned}$$

then:

$$\Gamma = \Gamma_{bc} + \Gamma_{de}$$

If a particle of a given mass can decay to more final states than another one with the same mass, it will have a shorter lifetime

Branching ratio

$\mathcal{B}(\mathrm{a} \rightarrow \mathrm{b} + \mathrm{c}) = \Gamma_{bc}/\Gamma$
The branching ratio: Of N decays of particle a, a fraction $\mathcal{B}$ will be the final state with the particles b and c. Γ_{bc} is a partial width of particle a.

Remember: for the calculation Γ ALL final states (partial widths) have to be considered.

What do we know?

- Names of particles
- Kinematic description of interactions
- Defintion of cross section and decay width

What is next?

- Electromagnetic interactions (QED)
- Strong interaction (QCD)
- Electroweak interactions