Particle Physics: The Standard Model

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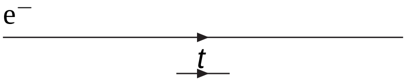
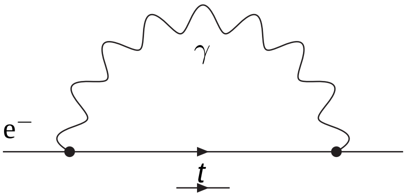
The History

- Introduction of particles ($\alpha\tau o\mu o\varsigma$)
- Particle-Wave dualism (deBroglie wave length)
- Particles are fields in a quantum field theory
- 1941: Stueckelberg proposes to interpret electron lines going back in time as positrons
- end of 1940s: Feynman, Tomonaga, Schwinger et al develop renormalization theory
- anomalous magnetic moment predicted (not today)

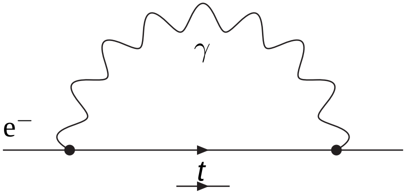
Quantum Field Theory in a nutshell



- Leading Order (LO) diagram is the simplest diagram
- The electron is on-shell ($\mathbf{p}^2 = m_e^2$), no interactions



- NLO (next-to-leading order) diagram
- Process not allowed in classical mechanics
- Heisenberg: $\Delta E \Delta t \geq 1 \rightarrow$ process allowed for reabsorption after $\Delta t \sim 1/\Delta E$



- Quantum mechanics: add all diagrams, but that would also include $N_\gamma = \infty$
- Each vertex is an interaction and each interaction has a strength ($|\mathcal{M}|^2 \sim \alpha = 1/137$)
- Perturbation theory with Sommerfeld convergence

- Construct the Lagrangian of Free Fields
- Introduce interactions via the minimal substitution scheme
- Derive Feynman rules ($\rightarrow$ courses by Adel Bilal, Pierre Binétruy, Pierre Fayet, Matteo Cacciari, Slava Ryshkov)
- Construct (ALL) Feynman diagrams of the process
- Apply Feynman rules

Some aspects are not part of these lectures, but will sketch the ideas

- Remember the particle zoo
- treat only the carrier of the interaction γ
- as well as the e

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} c_L \\ s_L \end{pmatrix} \quad \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$\begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix}$$

$$\begin{matrix} u_R & c_R & t_R \\ d_R & s_R & b_R \\ e_R & \mu_R & \tau_R \end{matrix}$$

$$\gamma$$
$$g$$
$$W^{\pm}, Z^{\circ}$$
$$H$$

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$$\begin{pmatrix} \mathrm{u_L} \\ \mathrm{d_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s_L} \end{pmatrix} \quad \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b_L} \end{pmatrix}$$

$$\begin{pmatrix} \nu_{\mathrm{e_L}} \\ \mathrm{e_L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\mu_\mathrm{L}} \\ \mu_\mathrm{L} \end{pmatrix} \quad \begin{pmatrix} \nu_{\tau_\mathrm{L}} \\ \tau_\mathrm{L} \end{pmatrix}$$

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$$\gamma$$
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The photon

MAXWELL equations:

$$\begin{aligned} \partial_\mu F^{\mu\nu}(\mathbf{x}) &= j^\nu(\mathbf{x}) \\ \epsilon^{\mu\nu\rho\sigma}\partial_\nu F_{\rho\sigma}(\mathbf{x}) &= 0 \end{aligned}$$

with the photon field tensor:

$$F^{\mu\nu}(\mathbf{x}) = \partial^\mu A^\nu(\mathbf{x}) - \partial^\nu A^\mu(\mathbf{x})$$

Fermions

The DIRAC equation:

$$(i\gamma^\mu\partial_\mu - m)\psi(\mathbf{x}) = 0$$

leading to:

$$\bar{\psi}(\mathbf{x})(i\gamma^\mu\partial_\mu - m)\psi(\mathbf{x})$$

with $\bar{\psi} = \psi^\dagger\gamma^0 = \psi^{T\star}\gamma^0$

The free Lagrangian ($\mathcal{L}_0$)

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}(\mathbf{x})F^{\mu\nu}(\mathbf{x}) + \bar{\psi}(\mathbf{x})(i\gamma^\mu\partial_\mu - m)\psi(\mathbf{x})$$

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Minimal Substitution

$$i\partial_\mu \rightarrow i\partial_\mu + eA_\mu(\mathbf{x})$$

$$\bar{\psi}(\mathbf{x})\gamma^\mu i\partial_\mu\psi(\mathbf{x})$$

$$\rightarrow \bar{\psi}(\mathbf{x})\gamma^\mu(i\partial_\mu + eA_\mu(\mathbf{x}))\psi(\mathbf{x})$$

$$= \bar{\psi}(\mathbf{x})\gamma^\mu i\partial_\mu\psi(\mathbf{x}) + e\bar{\psi}(\mathbf{x})\gamma^\mu A_\mu(\mathbf{x})\psi(\mathbf{x})$$

leads to a coupling between photon and fermion fields:

Interaction Lagrangian $\mathcal{L}'$

$$\mathcal{L}' = -j^\mu A_\mu = e\bar{\psi}(\mathbf{x})\gamma^\mu A_\mu(\mathbf{x})\psi(\mathbf{x})$$

the negative sign for $j^\mu = -e\bar{\psi}(\mathbf{x})\gamma^\mu\psi(\mathbf{x})$

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Gauge Invariance

Principle

Invariance of the Lagrangian under local $U(1)$ transformations
or: why should physics at the Elysée be different at the ENS?

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \partial_\mu \Lambda(\mathbf{x}) \\ \psi(\mathbf{x}) &\rightarrow \exp(ie\Lambda(\mathbf{x}))\psi(\mathbf{x}) \end{aligned}$$

$$\mathcal{L}_0 + \mathcal{L}' = \mathcal{L} \rightarrow \mathcal{L}$$

Local gauge invariance under a $U(1)$ gauge symmetry (1929 Weyl)
if $\Lambda \neq f(\mathbf{x})$ it is a global $U(1)$ symmetry.

$U(1)$ Gauge invariance Photon field:

Proof.

$$
\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \\
&= \partial_\mu (A_\nu + \partial_\nu \Lambda) - \partial_\nu (A_\mu + \partial_\mu \Lambda) \\
&= \partial_\mu A_\nu - \partial_\nu A_\mu + \partial_\mu \partial_\nu \Lambda - \partial_\nu \partial_\mu \Lambda \quad \partial_\mu \partial_\nu = \partial_\nu \partial_\mu \\
&= \partial_\mu A_\nu - \partial_\nu A_\mu \\
&= F_{\mu\nu}
\end{aligned}
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□

Photon field ok

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\begin{aligned}
& \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \\
\rightarrow\; & \psi^\dagger \exp(-ie\Lambda)\gamma^0(i\gamma^\mu\partial_\mu - m)(\psi \exp(ie\Lambda)) \\
=\; & \exp(-ie\Lambda)\bar{\psi}(i\gamma^\mu\partial_\mu - m)(\psi \exp(ie\Lambda)) \\
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Fermion field

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Interaction

Proof.

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\begin{aligned}
& e\bar{\psi}\gamma^{\mu}A_{\mu}\psi(\mathbf{x}) \\
= \; & e\exp(-ie\Lambda)\bar{\psi}\gamma^{\mu}(A_{\mu}+\partial_{\mu}\Lambda)\psi\exp(ie\Lambda) \\
= \; & e\bar{\psi}\gamma^{\mu}(A_{\mu}+\partial_{\mu}\Lambda)\psi \\
= \; & e\bar{\psi}\gamma^{\mu}A_{\mu}\psi + e\bar{\psi}\gamma^{\mu}(\partial_{\mu}\Lambda)\psi
\end{aligned}
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□

- Interaction term combined with fermion field ($-ie\bar{\psi}\gamma^{\mu}\partial_{\mu}\Lambda\psi$) ok
- gauge invariance of the fermion field cries for the introduction of a gauge boson!

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= \; & e \exp(-ie\Lambda)\bar{\psi}\gamma^\mu(A_\mu + \partial_\mu\Lambda)\psi \exp(ie\Lambda) \\
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= \; & e\bar{\psi}\gamma^\mu A_\mu\psi + e\bar{\psi}\gamma^\mu(\partial_\mu\Lambda)\psi
\end{aligned}
$$

□

- Interaction term combined with fermion field ($-ie\bar{\psi}\gamma^\mu\partial_\mu\Lambda\psi$) ok
- gauge invariance of the fermion field cries for the introduction of a gauge boson!

Interaction

Proof.

$$
\begin{aligned}
& e\bar{\psi}\gamma^{\mu}A_{\mu}\psi(\mathbf{x}) \\
= \; & e \exp(-ie\Lambda)\bar{\psi}\gamma^{\mu}(A_{\mu} + \partial_{\mu}\Lambda)\psi \exp(ie\Lambda) \\
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Proof.

$$
\begin{aligned}
& e\bar{\psi}\gamma^\mu A_\mu \psi(\mathbf{x}) \\
= \; & e \exp(-ie\Lambda)\bar{\psi}\gamma^\mu (A_\mu + \partial_\mu \Lambda)\psi \exp(ie\Lambda) \\
= \; & e\bar{\psi}\gamma^\mu (A_\mu + \partial_\mu \Lambda)\psi \\
= \; & e\bar{\psi}\gamma^\mu A_\mu \psi + e\bar{\psi}\gamma^\mu (\partial_\mu \Lambda)\psi
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$$

□

- Interaction term combined with fermion field ($-ie\bar{\psi}\gamma^\mu \partial_\mu \Lambda \psi$) ok
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External lines

initial state electron	$u(p)$
initial state positron	$\bar{v}(p)$
initial state photon	ϵ^{μ}
final state electron	$\bar{u}(p)$
final state positron	$v(p)$
final state photon	$\epsilon^{\mu\star}$

Internal lines and vertex

virtual photon	$\frac{-ig_{\mu\nu}}{k^2+i\epsilon}$
virtual electron	$i\frac{\not{p}+m}{p^2-m^2+i\epsilon}$
interaction (vertex)	$ie\gamma^{\mu}$

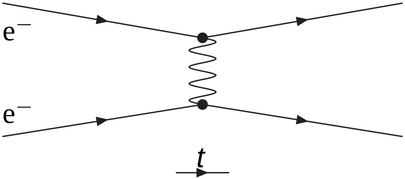
Matrix element

$$|\mathcal{M}|^2 = \sum_{fi}{}' T_{fi} T_{fi}^{\dagger}$$

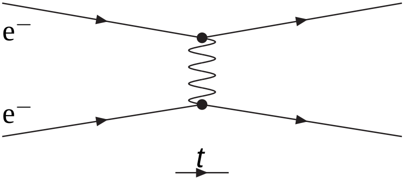
Sum over final state, average over initial state

Moeller Scattering $e^-e^- \to e^-e^-$

- Simplest diagram with initial and final state of two electrons
- conserve electric charge and momentum at each vertex
- t channel only: $C(e^- + e^-) = -2e \neq C(\gamma) = 0$
- **p** conservation at each vertex $\to$ 2 diagrams
  $q_\gamma = \mathbf{p_2} - \mathbf{p_3} \neq \mathbf{p_2} - \mathbf{p_4}$



$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \to e^-(\mathbf{p_3})e^-(\mathbf{p_4})$



$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \to e^-(\mathbf{p_4})e^-(\mathbf{p_3})$

Moeller Scattering $e^-e^- \to e^-e^-$

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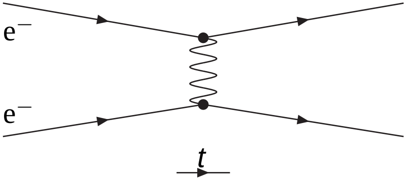
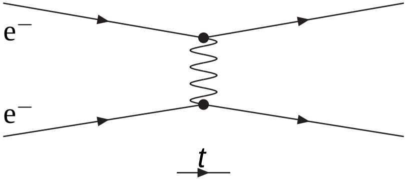


$e^-(\mathbf{p_1})e^-(\mathbf{p_2}) \to e^-(\mathbf{p_3})e^-(\mathbf{p_4})$



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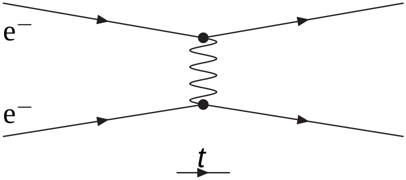

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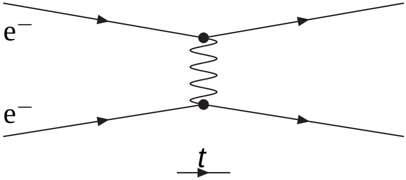

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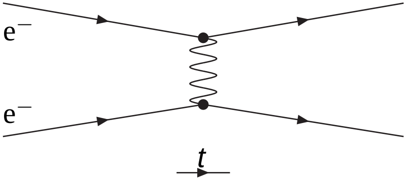

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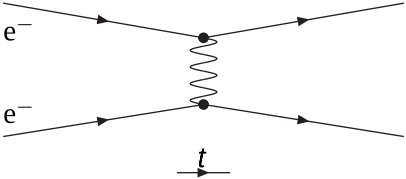

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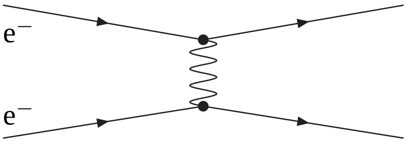
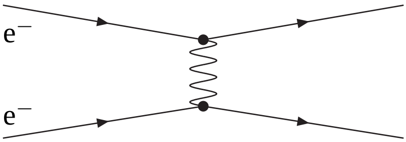


- Fermion arrow tip to end
- Interaction
- propagator (internal line)
- second graph $\mathbf{p_3} \leftrightarrow \mathbf{p_4}$
- graphs fermion permutation: $-$
- $\mathbf{k} = f(\mathbf{p_i})$

$$T_{fi} = [\bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2})$$
$$- \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]$$



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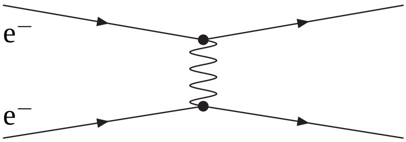
$$



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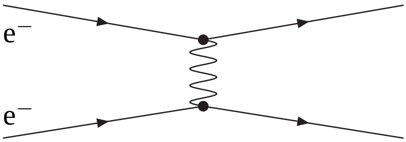

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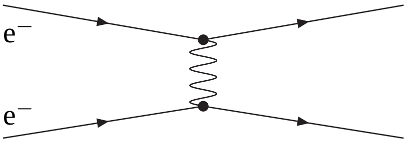

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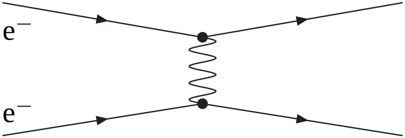

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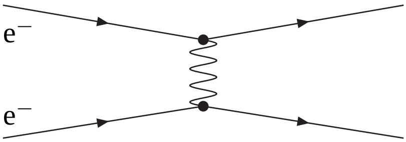

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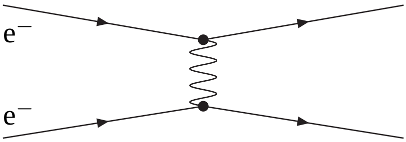

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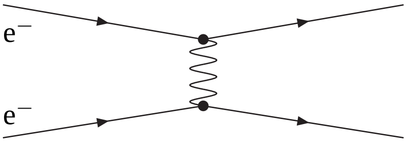

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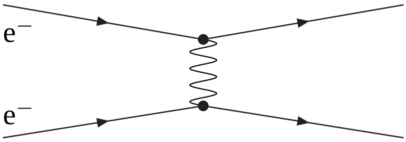
\end{aligned}
$$



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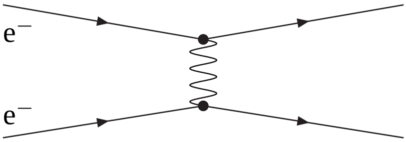

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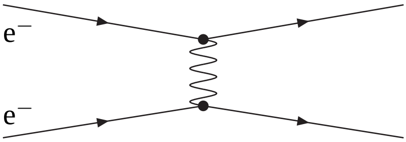

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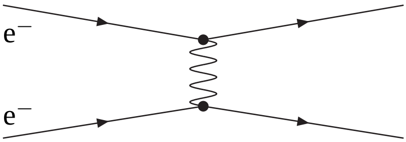

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T_{fi} = [\quad & \bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2}) \\
- \quad & \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]
\end{aligned}
$$

$$\frac{1}{i}T_{fi} = \frac{1}{i}[\bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2})$$

$$- \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})]$$

$$= e^2[\bar{u}(\mathbf{p_4})\gamma^\mu u(\mathbf{p_1})(\frac{g_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})\gamma^\nu u(\mathbf{p_2})$$

$$- \bar{u}(\mathbf{p_3})\gamma^\rho u(\mathbf{p_1})(\frac{g_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})\gamma^\sigma u(\mathbf{p_2})]$$

$$|\mathcal{M}|^2 = \sum'_{fi} T_{fi}T^\dagger_{fi}$$

$$= \frac{1}{4}\sum_{fi} T_{fi}T^\dagger_{fi}$$

Calculation to be continued in Problem Solving

$$|\mathcal{M}|^2 = \frac{64\pi^2\alpha^2}{t^2u^2}[(s-2m^2)^2(t^2+u^2)+ut(-4m^2s+12m^4+ut)]$$

$$\begin{aligned}
\frac{1}{i} T_{fi} &= \frac{1}{i}\Big[\bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})\Big(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2}\Big)\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2}) \\
&\quad - \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})\Big(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2}\Big)\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})\Big] \\
&= e^2\Big[\bar{u}(\mathbf{p_4})\gamma^\mu u(\mathbf{p_1})\Big(\frac{g_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2}\Big)\bar{u}(\mathbf{p_3})\gamma^\nu u(\mathbf{p_2}) \\
&\quad - \bar{u}(\mathbf{p_3})\gamma^\rho u(\mathbf{p_1})\Big(\frac{g_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2}\Big)\bar{u}(\mathbf{p_4})\gamma^\sigma u(\mathbf{p_2})\Big] \\
|\mathcal{M}|^2 &= \sum\nolimits'_{fi} T_{fi} T^\dagger_{fi} \\
&= \frac{1}{4}\sum\nolimits_{fi} T_{fi} T^\dagger_{fi}
\end{aligned}$$

Calculation to be continued in Problem Solving

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$$
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\frac{1}{i}T_{fi} &= \frac{1}{i}[\quad \bar{u}(\mathbf{p_4})(-ie\gamma^\mu)u(\mathbf{p_1})(\frac{-ig_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})(-ie\gamma^\nu)u(\mathbf{p_2}) \\
&\quad - \quad \bar{u}(\mathbf{p_3})(-ie\gamma^\rho)u(\mathbf{p_1})(\frac{-ig_{\rho\sigma}}{(\mathbf{p_3}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_4})(-ie\gamma^\sigma)u(\mathbf{p_2})] \\
&= e^2[\quad \bar{u}(\mathbf{p_4})\gamma^\mu u(\mathbf{p_1})(\frac{g_{\mu\nu}}{(\mathbf{p_4}-\mathbf{p_1})^2})\bar{u}(\mathbf{p_3})\gamma^\nu u(\mathbf{p_2}) \\
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|\mathcal{M}|^2 &= \sum\nolimits'_{fi} \quad T_{fi}T_{fi}^\dagger \\
&= \frac{1}{4}\sum\nolimits_{fi} \quad T_{fi}T_{fi}^\dagger
\end{aligned}
$$

Calculation to be continued in Problem Solving

$$
|\mathcal{M}|^2 = \frac{64\pi^2\alpha^2}{t^2u^2}[(s-2m^2)^2(t^2+u^2)+ut(-4m^2s+12m^4+ut)]
$$

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$$\frac{d\sigma}{d\Omega} = |\mathcal{M}|^2 \frac{1}{64\pi^2 s}$$

$0 \leq \theta \leq \pi/2$ (electrons) $m_e \approx 0$

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t &= -2\mathbf{p_1 p_3} = -2(\sqrt{s}/2\sqrt{s}/2 - s/4\cos\theta) = -s/2(1-\cos\theta) \\
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$s\frac{d\sigma}{d\Omega}$ is scale invariant: measure of the pointlikeness of a particle

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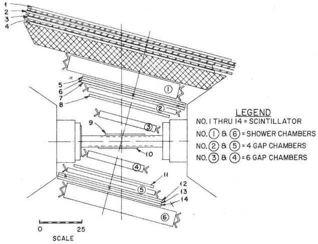


FIG. 1. Storage-ring interaction region and detector system for 556-MeV/electron scattering experiment.

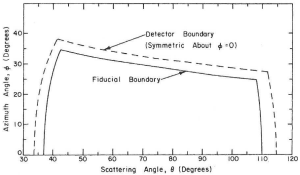


FIG. 2. Detector boundary and fiducial boundaries.

- Stanford-Princeton Storage ring
- $2e^-$ beams $\sqrt{s} = 556 MeV$

- limited detector acceptance
- differential cross section measurement and prediction

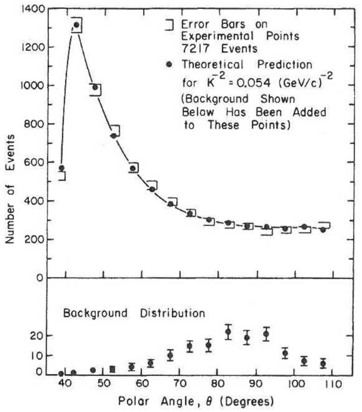
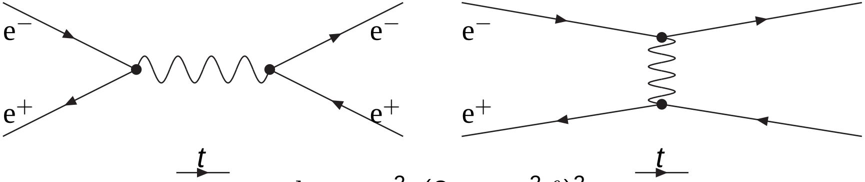


FIG. 3. Comparison of experimental result with Møller scattering modified by radiative corrections. Because the detector geometry is included, the theoretical curve is not symmetric about 90°.

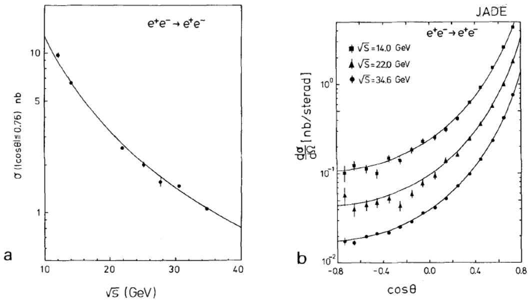
- Typical t channel $\theta = 0 \to d\sigma/d\Omega \to \infty$
- Extremely good agreement between the measurement and the theory prediction
- e^-e^- colliders discontinued (1971)

The Bhabha Process Homi Bhabha studied in the 1930s in Great Britain, worked in India afterwards

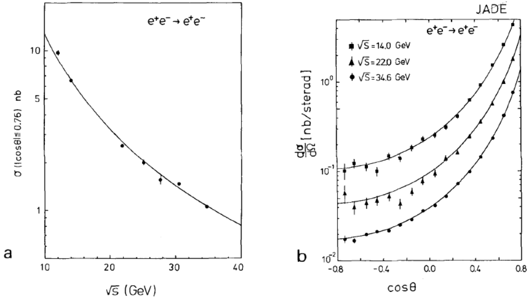


$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16s}\frac{(3+\cos^2\theta)^2}{\sin^4\frac{\theta}{2}}$$

- $0 \leq \theta \leq \pi$
- t channel: $\sim \sin^{-4}(\theta/2)$
- s channel: $\sim 1 + \cos^2\theta$



- PETRA $\mathrm{e}^+\mathrm{e}^-$ collider $\sqrt{s} \leq 35\mathrm{GeV}$
- JADE, TASSO, CELLO
- total cross section
- differential cross section



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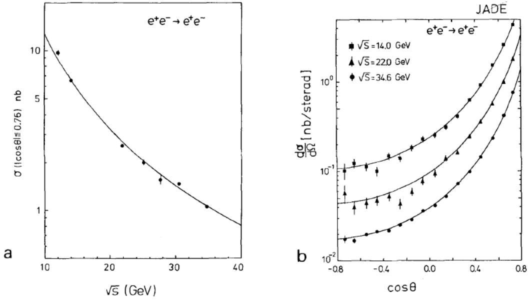

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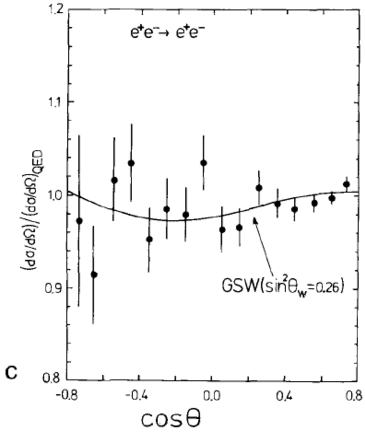


- Excellent agreement with QED
- Errors reflect statistics
- QED deviation : $s/\Lambda^2 < 5\%$ with $s = 35^2\mathrm{GeV}^2$
- $\rightarrow (\hbar c)/\Lambda = (0.197\mathrm{GeV} \cdot \mathrm{fm})/\Lambda \approx 0.13 \cdot 10^{-3}\mathrm{fm}$
- $N = \int Ldt \cdot \sigma$
- Today Bhabha is a luminosity measurement

Electrical field

- acceleration
- charge times potential difference
- typical unit: eV

Magnetic field

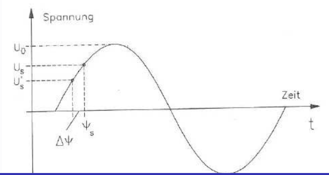
- no acceleration
- B field unit: $[B] = \frac{Vs}{m^2}$
- force on charged particle in magnetic field: $F = q\vec{v} \times \vec{B} = q\frac{p}{m}B$
- centrifugal force: $F = mv^2/r = p^2/(m \cdot r)$
- $R = p/(qBc)$ (c because of natural units)

Acceleration

- strong fields difficult to achieve (breakdown)
- accelerate successively
- linear assembly: distance between potential diffs must increase
- circular assembly: several rotations possible

Phase focussing

- particle sees nominal (not maximal) field
- early particle: less field, less acceleration
- late particle: more field, stronger acceleration



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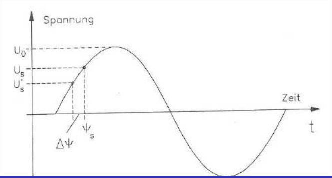

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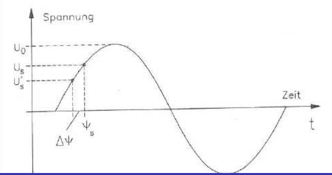

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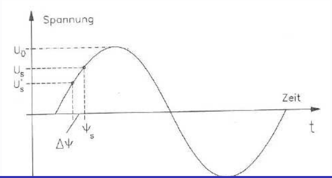

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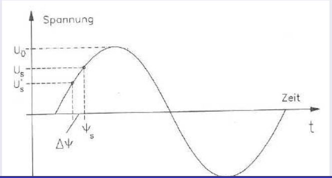

LEP/LHC

- circular tunnel 28km circumference
- electron+positron: 210GeV
  - weak field
  - strong cavities
  - energy loss per turn: 6GeV ($\sim E^4/R$)
- LHC proton-proton (14TeV)
  - strong field 10T
  - energy loss per turn: 500keV

Lepton collider cavities

- LEP: up to 10MV/m
- ILC: 35-40 MV/m
- supraconducting (T_{He})

Magnetic field LHC

$$R = \frac{7000GeV}{0.3 \cdot 10^{-9} m/s \cdot 10T \cdot 1e}$$
$$= \frac{7000GeV}{0.3 \cdot 10^{9} m/s \cdot 10Vs/m^2 \cdot 10^{-9} Ge}$$
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Instantaneous Luminosity

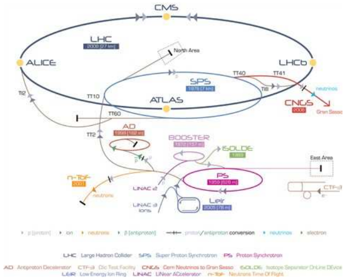
$$L \sim \frac{N^2 k_b f \gamma F}{4\pi\epsilon\beta*}$$
$$\sim \frac{(10^{11})^2 \cdot 2800 \cdot 40MHz \cdot \gamma F}{4\pi \cdot 15\mu m \cdot \beta*}$$

LHC: $10^{34} cm^{-2} s^{-1}$

Integrated Luminosity

$$N = \int Ldt \cdot \sigma$$

LHC: $25 fb^{-1}$ per experiment

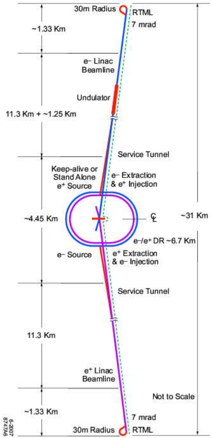


Linac	Booster	PS	SPS
50	1.4	25	450
MeV	GeV	GeV	GeV

7TeV per beam

LC the future?

- linear: no synchrotron radiation
- 40km
- polarization
- luminosity
- 250GeV to 1TeV (3TeV: CLIC)



Detection at high energies

- $a + b \to X \to$ neutral + charged
- particles long-lived wrt detector volume
- Tracker: charged particle momenta
- Calorimeter: neutral and charge particles

Tracker

- measure points in B-field
- reconstruct sagitta
- highest precision: silicon (dense, $\sim 15\mu m$)
- lower precision: TPC (gazeous)

Electromagnetic calorimeter

- $e + A \to e + \gamma + A$
- $\gamma \to e^+e^-$ etc
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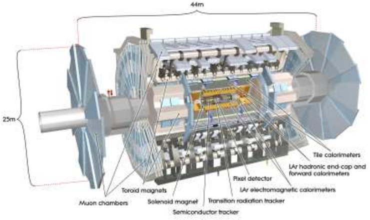
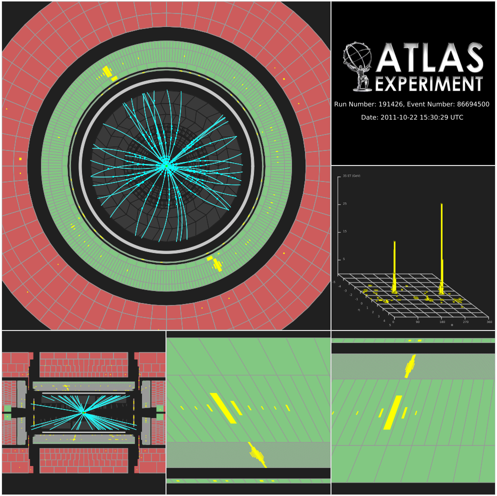


ATLAS

- Silicon tracking (100M channels 2T)
- Calorimeter (100k)
- Muon chambers (toroid)

Experimental Challenges

- bunches every 8m
- 25ns between crossings (fast readout)
- order 20 interactions per crossing
- trigger: 40MHz to 200Hz
- alignment
- calibration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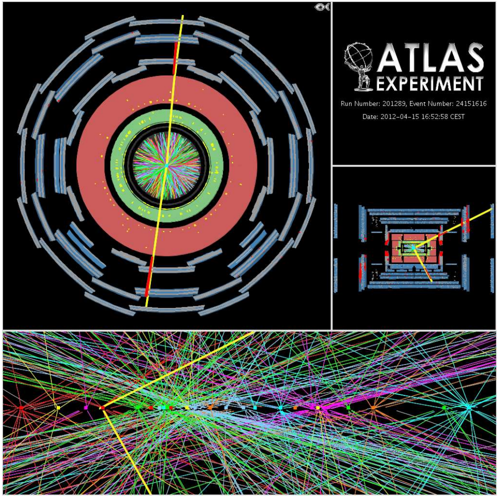
Run Number: 191426, Event Number: 86694500
Date: 2011-10-22 15:30:29 UTC


ATLAS EXPERIMENT
Run Number: 201289, Event Number: 24151616
Date: 2012-04-15 16:52:58 CEST

A calorimeter tracker for the future?