Particle Physics: The Standard Model

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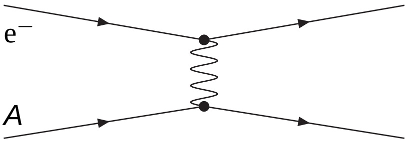
- QCD works, but are quarks real or mathematical constructions?
- use $\mathrm{e}A \rightarrow \mathrm{e}A'$
- electrons or muons pointlike probe possibly non-pointlike objects
- Fixed target experiments:
  - SLAC: 1GeV$< E <$30GeV
  - FERMILAB/CERN: $E \sim$300GeV
- Colliding beams experiment:
  - DESY (HERA): E_e =20GeV, E_p =800GeV



Process

- incoming electron $\mathbf{k}$
- outgoing electron $\mathbf{k}'$
- emission of photon $\mathbf{q} = \mathbf{k} - \mathbf{k}'$
- incoming nucleon A $\mathbf{p}$
- outgoing nucleon A' $\mathbf{p}'$

Measurements

- M: mass of the nucleon A
- E: energy of the incoming electron (ref:lab)
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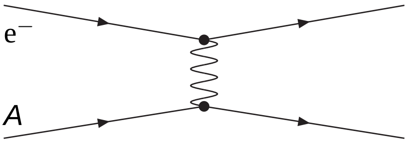
- θ: scattering angle in lab frame

k is known, k' is reconstructed from E' and θ



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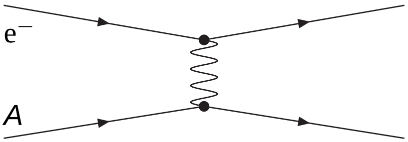

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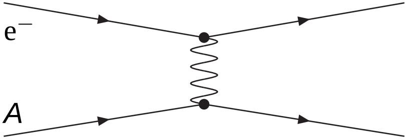

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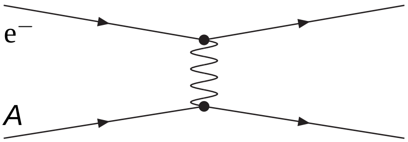

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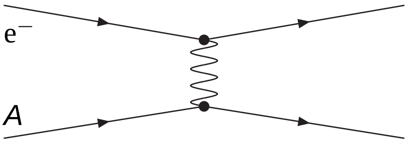

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Definitions

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- scattered electron energy: $\frac{\mathbf{p}\cdot\mathbf{k}'}{M} = \frac{E\cdot M}{M} = E'$
- $m_\gamma < 0$: $Q^2 = -\mathbf{q}^2 = 2EE'(1-\cos\theta) = 4EE'\sin^2\frac{\theta}{2}$
- energy transfer in lab system: $\nu = \frac{\mathbf{pq}}{M} = \frac{\mathbf{p}(\mathbf{k}-\mathbf{k}')}{M} = E - E'$
- Bjorken scale variable: $x = \frac{Q^2}{2M\nu}$ with $0 \le x \le 1$
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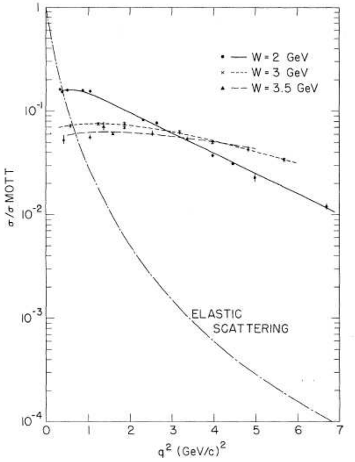

Scattering of an electron on a point-like object (proton): Mott

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{4E^2} \frac{\cos^2 \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}}$$

- Low Q^2
- $E = 7\text{GeV}$ and $E = 17\text{GeV}$
- angles: 6°, 10°
- not compatible with Mott

Dirac equation for adjoint spinor

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\begin{aligned}
i\gamma^\mu\partial_\mu\psi - m\psi &= 0 \\
-i(\gamma^\mu)^\star\partial_\mu\psi^\star - m\psi^\star &= 0 \\
-i(\partial_\mu\psi^\dagger)(\gamma^\mu)^\dagger - m\psi^\dagger &= 0 \\
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EM current conserved

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\begin{aligned}
\partial_\mu j^\mu &= \partial_\mu[-e\bar{\psi}\gamma^\mu\psi] \\
&= -e(\partial_\mu\bar{\psi})\gamma^\mu\psi - e\bar{\psi}\gamma^\mu\partial_\mu\psi && \text{Dirac} \\
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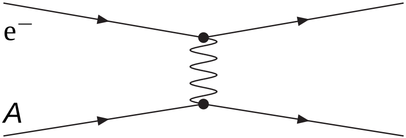

A': mass of target plus photon

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\begin{aligned}
(\mathbf{p}+\mathbf{q})^2 &= \mathbf{p}^2 + 2\cdot\mathbf{p}\cdot\mathbf{q} + \mathbf{q}^2 \\
&= M^2 + 2\cdot M\cdot\nu - Q^2 \\
M^2 &\leq M^2 + 2\cdot M\cdot\nu - Q^2
\end{aligned}
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(In)-Elastic scattering

$$2M\nu \geq Q^2$$

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\begin{aligned}
T_{fi} &= 4\pi\alpha\bar{u}(\mathbf{k}')\gamma^\mu u(\mathbf{k})\frac{1}{q^2}W_\mu \\
d\sigma &\sim L^{\mu\nu}W^{eN}_{\nu\mu} \\
L^{\mu\nu} &= k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu}(\mathbf{k}\cdot\mathbf{k}')
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- hadronic tensor: $W^{eN}_{\nu\mu} = f(W_\mu, W_\nu) = f(\mathbf{p},\mathbf{q})$
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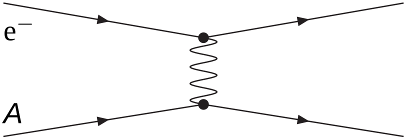

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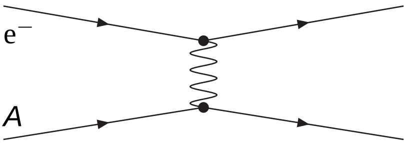

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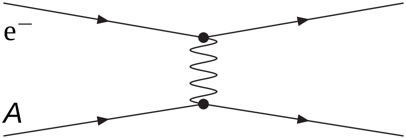

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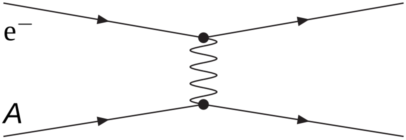

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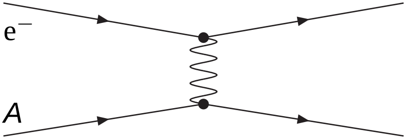

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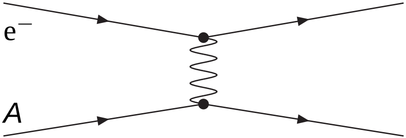

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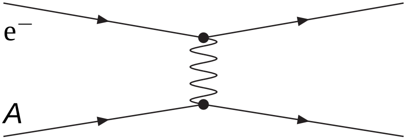

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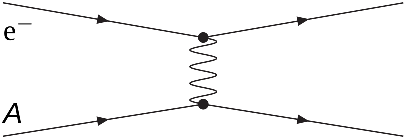

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Parton model

Bjorken: a nucleon is a beam of partons where x describes the fraction of the nucleon momentum carried by the partons

$$\begin{aligned} 2MW_1(\nu, Q^2) &= F_1(x) + \mathcal{O}(\tfrac{1}{Q^2}) \quad \textit{Magnetic SF} \\ \nu W_2(\nu, Q^2) &= F_2(x) + \mathcal{O}(\tfrac{1}{Q^2}) \quad \textit{Electromagnetic SF} \end{aligned}$$

Quarks

The functions $F_1(x)$ and $F_2(x)$ are related to the probability to find a parton (= quark):

$$\begin{aligned} 2MW_1(\nu, Q^2) &= \sum_i Q_i^2 f_i(x) \\ \nu W_2(\nu, Q^2) &= x \sum_i Q_i^2 f_i(x) + x \sum_s Q_s^2 f_s(x) \end{aligned}$$

Q_i charge of the quark, s scalars (no magnetism!)
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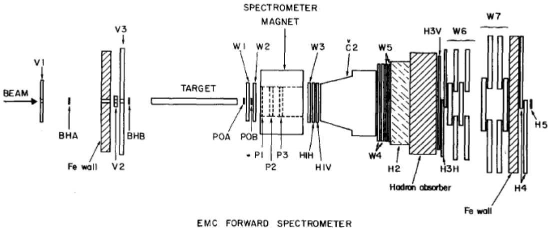
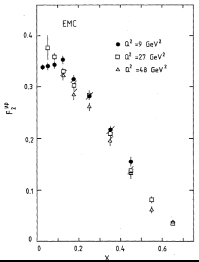


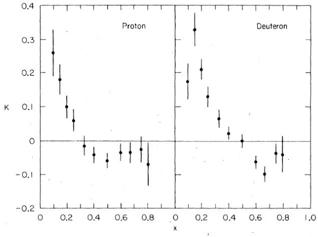


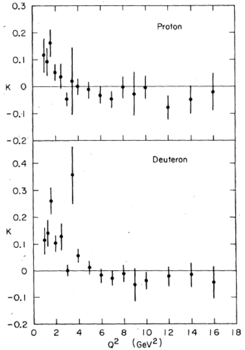
- boost (300GeV on target at rest) leads to forward detection (problem solving)
- to first order F_2 is independent of Q^2, i.e. scale invariant
- QPM works to first order

Callan-Gross 1969

$$R(x) = \frac{F_2(x) - xF_1(x)}{F_2(x)}$$

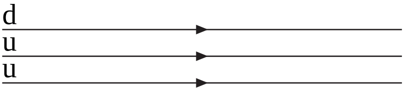
$= 0$ for Spin-$\frac{1}{2}$
$= 1$ for Spin-0
1983: $R = -0.01 \pm 0.11$

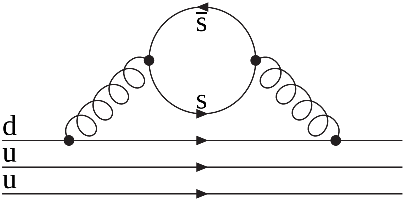




(SLAC) small x and small Q^2

Description of a Proton





- BREIT frame: ($E = E'$)
- partons without transverse momentum
- $|\mathrm{uud}\rangle$

- $|\mathrm{uud} + \mathrm{u}\bar{\mathrm{u}} + \mathrm{d}\bar{\mathrm{d}} + \mathrm{s}\bar{\mathrm{s}} + \mathrm{c}\bar{\mathrm{c}} + \ldots\rangle$
- u, d valence quarks
- s,.... see quarks
- non-zero transverse momentum and gluon

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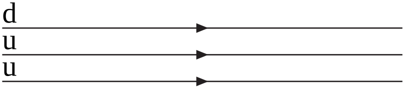

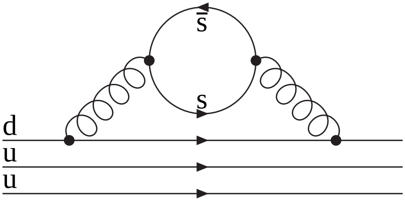

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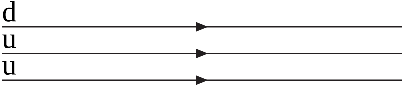

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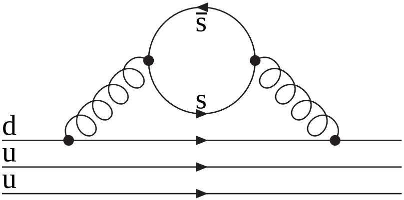

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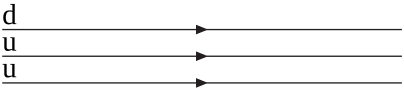

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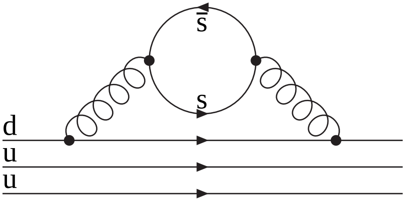

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Ansatz:

$$F_2^p = x \sum_{i=1}^{6} Q_i^2 f_i^p(x)$$
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$f_{\mathrm{u}}(x)$ etc are the parton distribution function (PDF). Not a probability because of the normalization:

$$\int f_{\mathrm{u}}^p(x) - f_{\bar{\mathrm{u}}}^p(x) dx = 2$$
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F_2^n of the neutron is then:

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F_2^n of the neutron is related to the proton via $SU(2)$-Isospin:
$U(\pi)|\mathrm{u}\rangle = |\mathrm{d}\rangle$ The probability to find a u in a proton

$$\langle \mathrm{u}|p\rangle = \langle \mathrm{u}|1_2|p\rangle = \langle \mathrm{u}|U^{-1}(\pi)U(\pi)|p\rangle = \langle \mathrm{d}|n\rangle$$

is equal to the probability to find a d in a neutron:

$$F_2^n = x[\tfrac{4}{9}(f_{\mathrm{d}}^p(x) + f_{\bar{\mathrm{d}}}^p(x)) + \tfrac{1}{9}(f_{\mathrm{u}}^p(x) + f_{\bar{\mathrm{u}}}^p(x)) + \tfrac{1}{9}(f_{\mathrm{s}}^n(x) + f_{\bar{\mathrm{s}}}^n(x))]$$

Going further:

$$\begin{aligned} f_{\mathrm{u}}^p &= f_{\mathrm{u}}^p{}_V + f_{\mathrm{u}}^p{}_S \\ f_{\mathrm{u}}^p{}_S &= f_{\mathrm{d}}^p{}_S = f_{\mathrm{s}}^p{}_S = f_{\mathrm{s}}^n{}_S \cdots \\ F_2^p - F_2^n &= \tfrac{x}{3}(f_{\mathrm{u}}^p{}_V - f_{\mathrm{d}}^p{}_V) \\ &= \tfrac{x}{3}(f_{\mathrm{u}}^p{}_V - \tfrac{1}{2} f_{\mathrm{u}}^p{}_V) \\ &= \tfrac{x}{6} f_{\mathrm{u}}^p{}_V \end{aligned}$$

F_2^n of the neutron is related to the proton via $SU(2)$-Isospin:
$U(\pi)|\mathrm{u}\rangle = |\mathrm{d}\rangle$ The probability to find a u in a proton

$$\langle \mathrm{u}|p\rangle = \langle \mathrm{u}|1_2|p\rangle = \langle \mathrm{u}|U^{-1}(\pi)U(\pi)|p\rangle = \langle \mathrm{d}|n\rangle$$

is equal to the probability to find a d in a neutron:

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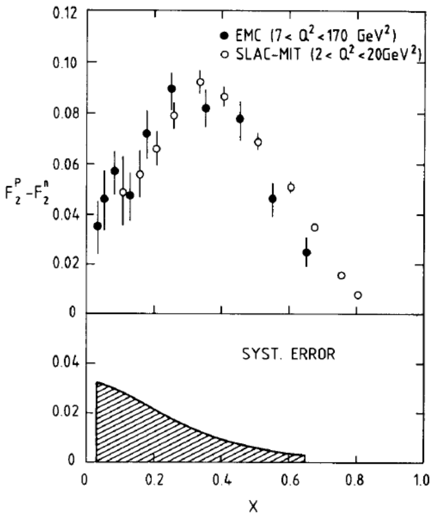
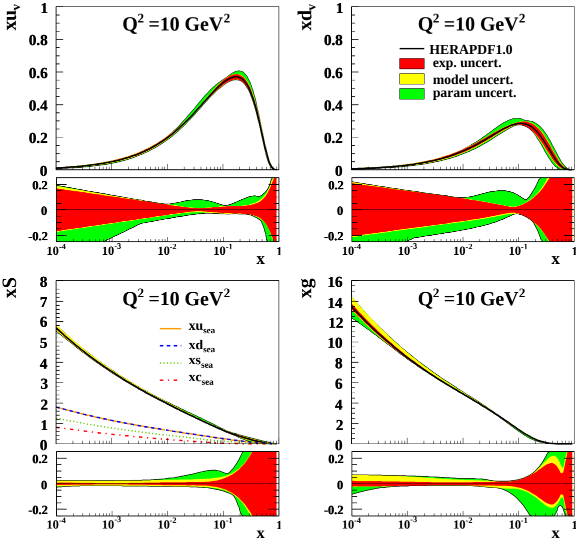


- F_2^p from $\mu + H$
- F_2^n from $\mu + D$
- correct and subtract.....
- distribution peaks at 0.3
- essentially independent of Q^2 (scaling)

H1 and ZEUS


xu_v
Q^2 =10 GeV^2
xd_v
Q^2 =10 GeV^2
HERAPDF1.0
exp. uncert.
model uncert.
param uncert.
xS
Q^2 =10 GeV^2
xu_sea
xd_sea
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xc_sea
xg
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x

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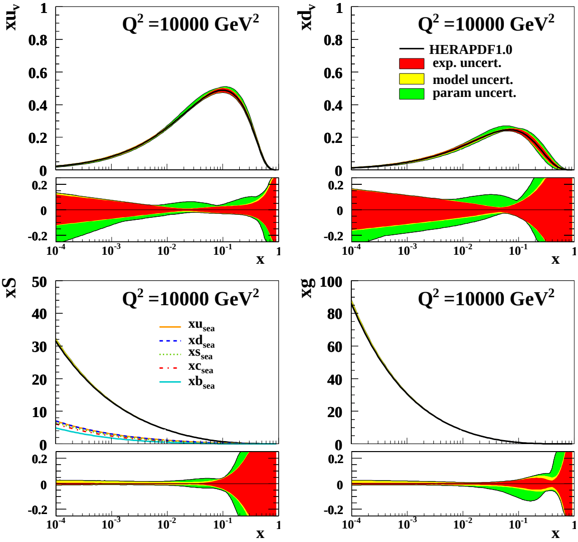
xu_v
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xS
Q^2 =10000 GeV^2
xu_sea
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xs_sea
xc_sea
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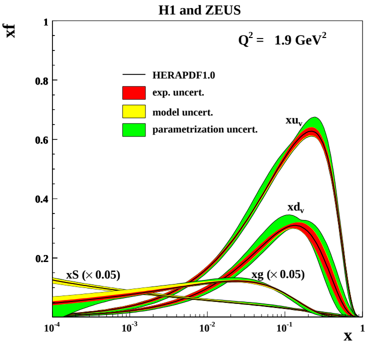
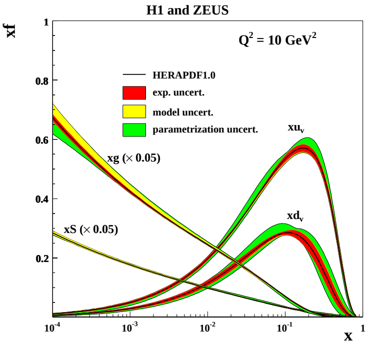




PDF Summary

- PDFs measured over a large range of x and Q^2
- small x strong increase of $xg(x)$

Parton Model Sum Rules

Baryonnumber:

$$\int_0^1 \sum_q \frac{1}{3}(f_q - f_{\bar{q}})dx = 1$$

The baryonnumber is 1.

The hadron charge:

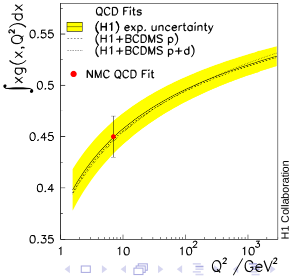
$$\int_0^1 \sum_q q_j f_q dx = Q_H$$

The momentum:

$$\int_0^1 x \sum_j f_j dx = 1 - \epsilon$$

ϵ gluon momentum (integration):

$$\int_0^1 \frac{x}{6} f_{uV}(x)dx = 0.5$$



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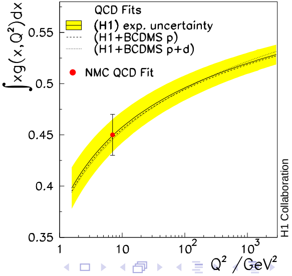

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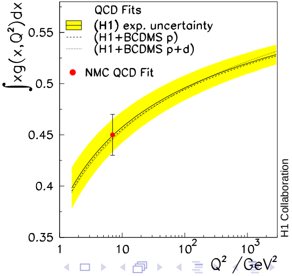
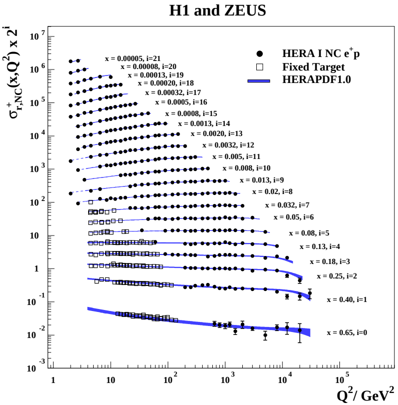


QPM → QCD

- $f_i(x) \to f_i(x, Q^2)$
- increase Q improves parton resolution
- small Q parton $\mathbf{p}$
- increase Q
  $\mathbf{p} = \mathbf{p_1} + \mathbf{p_2}$
- small x more particles $\to f_i \uparrow$
- large x less particles $\to f_i \downarrow$
- Attempt to describe evolution with Q^2



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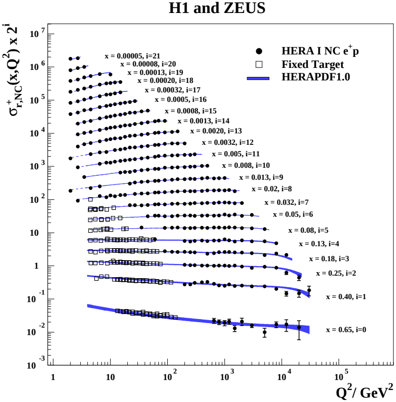

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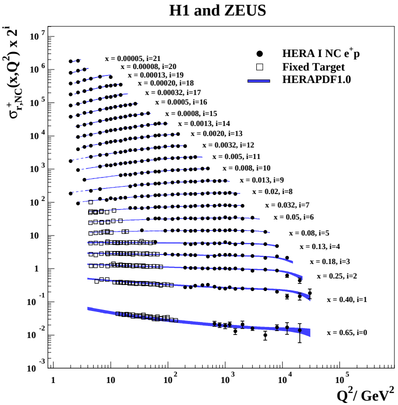

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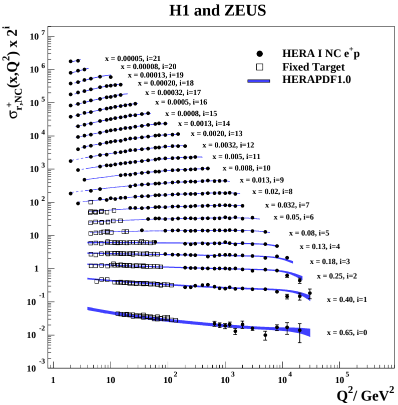

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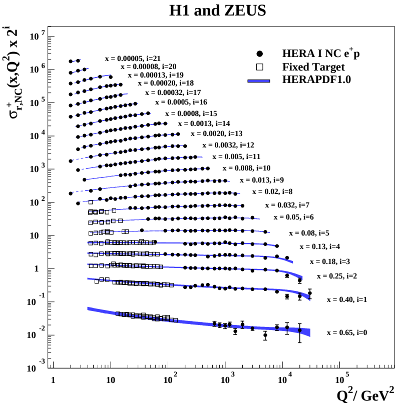

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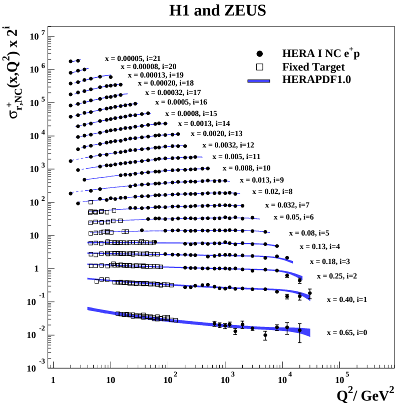

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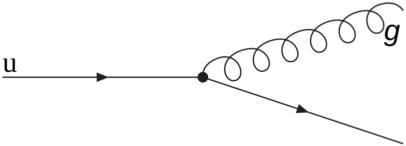
- proportional to g_S^2
- daughter momentum x smaller than mother momentum y $\rightarrow \int_x^1$
- only depends on the relative longitudinal momenta $\frac{x}{y}$

Example:

$$P_{Gq}(x) = \frac{4}{3}\frac{1+(1-x)^2}{x}$$



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Derivation of functions $\rightarrow$ QCD

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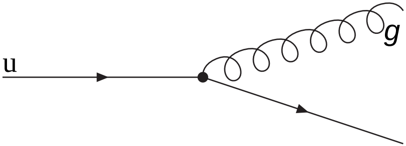

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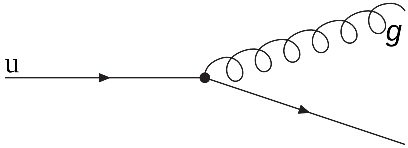

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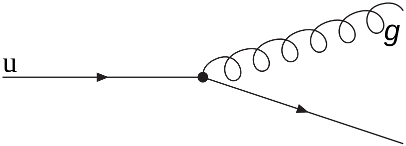

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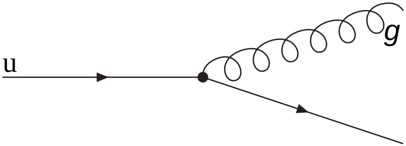

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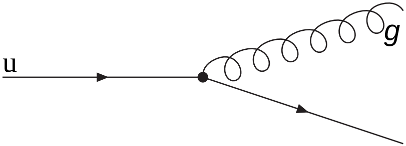

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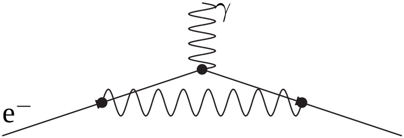
Asymptotic freedom



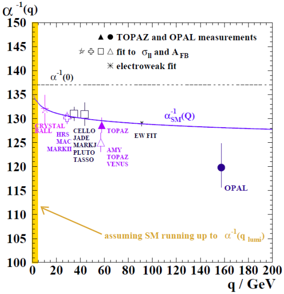
Electron in (out): $\mathbf{p}$ ($\mathbf{p}'$)
Photon: $\mathbf{l}$
inner Electron in (out): $\mathbf{p}-\mathbf{l}$, ($\mathbf{p}'-\mathbf{l}$)

$$\int d^4l \frac{1}{l^2} \frac{\not{p}-\not{l}+m}{(p-l)^2-m^2} \frac{\not{p}'-\not{l}+m}{(p'-l)^2-m^2}$$
$$\int d^4l \frac{1}{l^2} \frac{\not{p}-\not{l}+m}{p^2-m^2-2pl+l^2} \frac{\not{p}'-\not{l}+m}{p'^2-m^2-2p'l+l^2}$$
$$\int d^4l \frac{1}{l^2} \frac{\not{p}-\not{l}+m}{-2pl+l^2} \frac{\not{p}'-\not{l}+m}{-2p'l+l^2}$$

- p, p' are fixed!
- infrared catastrophy for $l \rightarrow 0$: $\frac{1}{l^2} \frac{1}{pl} \frac{1}{p'l}$
- cured by compensation real/virtual diagrams
- ultraviolet catastrophy: $l \rightarrow \infty$
- logarithmic divergence
- renormalization and regularisation

QED

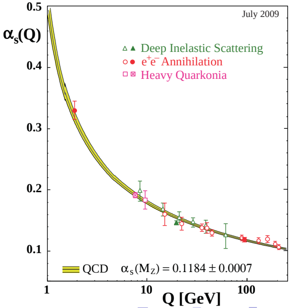
$$\alpha_{EM}(Q^2) = \frac{\alpha_{EM}}{1 - \sum_f \prod_f (Q^2)}$$

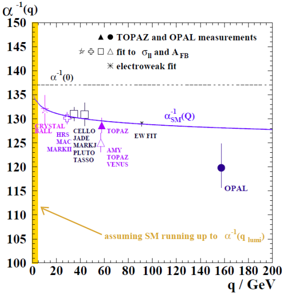


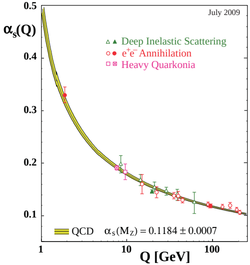
QCD

$$\alpha_S(Q^2) = \frac{12\pi}{(33 - 2N_f)\log(Q^2/\Lambda^2)}$$

with $\Lambda \sim 200\text{MeV}$, N_f number of active flavors (max 6 quarks)

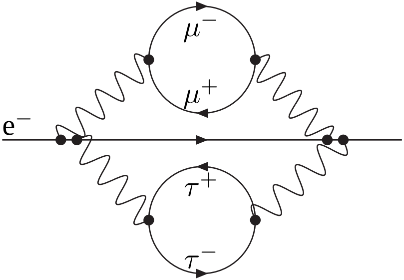


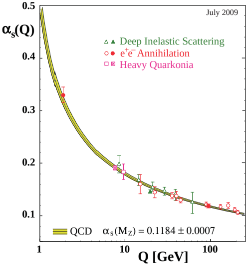




- energy ↑ → coupling ↑
- finite (1/137) at 0
- shielding like a di-electric medium

- energy ↑ → coupling ↓
- infinite at 0: bound state (non-perturbative)
- large Q^2: free quarks (pQCD)





- energy $\uparrow \rightarrow$ coupling $\uparrow$
- finite (1/137) at 0
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- $Q^2 \uparrow$ resolves bare charge

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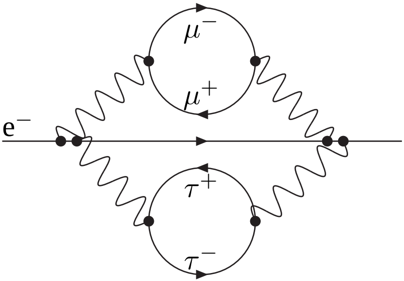

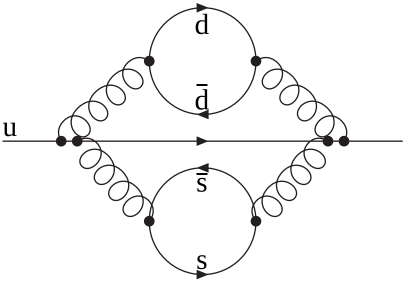

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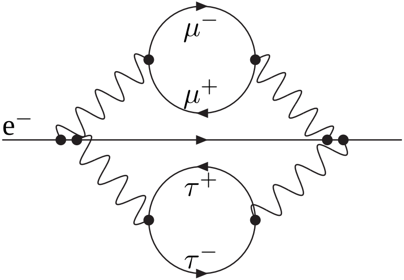
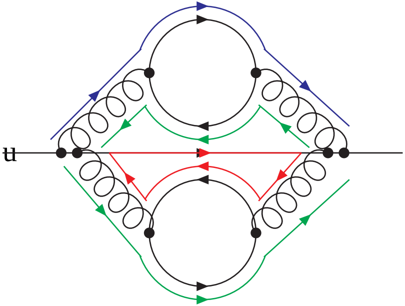
- TGV changes the shielding to anti-shielding





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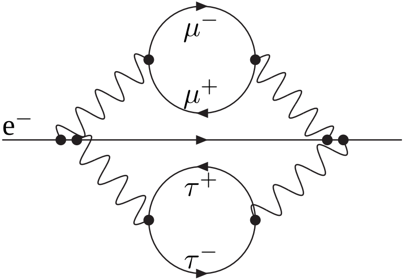

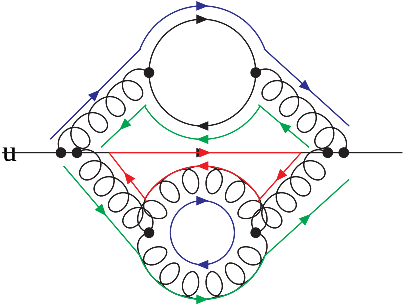

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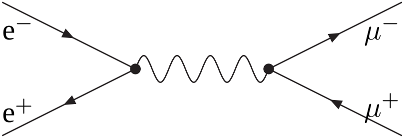

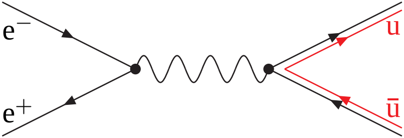

- treat quarks as free particles
- hadronization does not disturb measurement

remember: quark-photon vertex $\sim q$

$$R = \frac{\sum_i \sigma(e^-e^- \to q\bar{q})_i}{\sigma(e^-e^- \to \mu^+\mu^-)} = \sum_i q_i^2 N_C$$

The ratio is sensitive to the number of colors!

- $\sqrt{s} > 10 GeV$ $R = 3 \times (3 \times \frac{1}{3}^2 + 2 \times \frac{2}{3}^2) = \frac{11}{3}$
- $2.5 GeV < \sqrt{s} < 10 GeV$ $R = 3 \times (2 \times \frac{1}{3}^2 + 2 \times \frac{2}{3}^2) = \frac{10}{3}$
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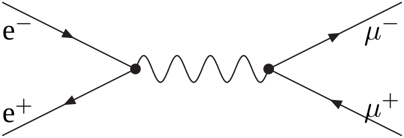

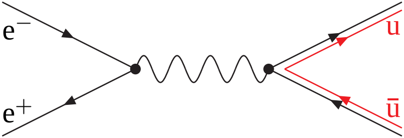

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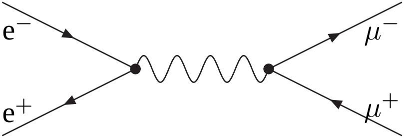

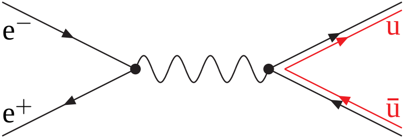

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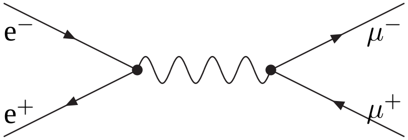

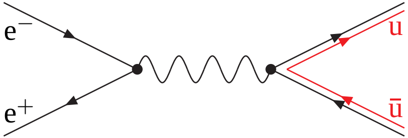

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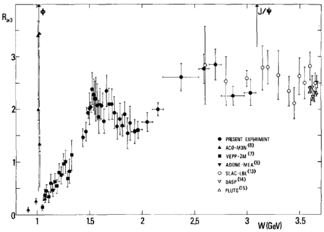
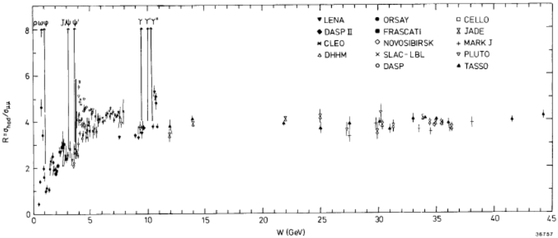
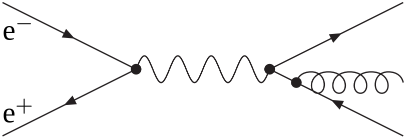


Experiments

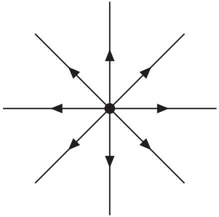
- ADONE (QED)
- PETRA (data)
- in agreement with quark counting
- resonances spoil



- Quarks and gluons are bound in hadrons
- Color is compatible with 3
- the gluon carries $\mathcal{O}(50\%)$ or the proton momentum
- $SU(3)_C$ the gluon is a spin-1 particle
- $\mathrm{e^+e^-} \rightarrow gg$ not possible
- $\mathrm{e^+e^-} \rightarrow \mathrm{q\bar{q}}g$
- LAB frame is CM frame
- Problem: Hadronization
- reconstruct jets



Start with two jets at threshold:



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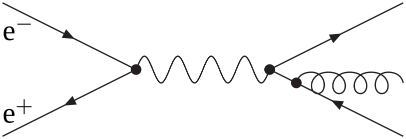


Two jets above threshold:

Transverse momentum $\sim$ 200MeV
Boost collimates

Remember infrared catastrophy:

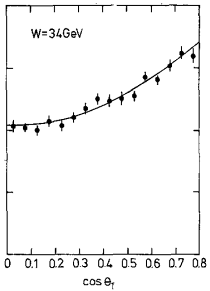
$$\sigma(e^+e^- \to q\bar{q}g) = \infty$$

Reason:

$$\mathbf{P}_q = \mathbf{P}'_q + \mathbf{K}_g$$

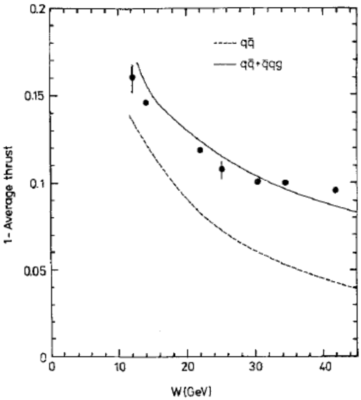
cannot distinguish left from right.
Solution: define infrared-safe observables like thrust:

$$T = \max\left(\frac{\sum_i |\mathbf{p_i} \cdot \hat{n}|}{\sum_i |\mathbf{p_i}|}\right)$$



- thrust memorizes the quark spin
- compatible with Spin-$\frac{1}{2}$

- collimated 2jets: $T = 1$
  $\rightarrow 1 - T = 0$
- $\mathrm{q\bar{q}}$: $1 - T$ should decrease to 0 as function of energy
- gluon should increase $1 - T$ (more isotropical)
- distribution compatible with $\mathrm{q\bar{q}}g$



More Jet Algorithms in Problem Solving

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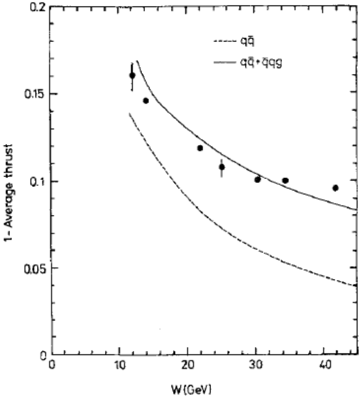

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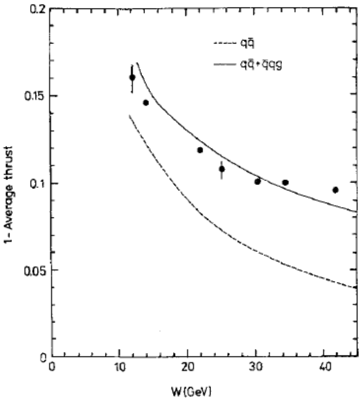

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