

Charm mixing at LHCb



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Mixing of neutral mesons: idea

Behavior of Neutral Particles under Charge Conjugation

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AND

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(Received November 1, 1954)

Some properties are discussed of the θ^0 , a heavy boson that is known to decay by the process $\theta^0 \rightarrow \pi^+ + \pi^-$. According to certain schemes proposed for the interpretation of hyperons and K particles, the θ^0 possesses an antiparticle $\bar{\theta}^0$ distinct from itself. Some theoretical implications of this situation are discussed with special reference to charge conjugation invariance. The application of such invariance in familiar instances is surveyed in Sec. I. It is then shown in Sec. II that, within the framework of the tentative schemes under consideration, the θ^0 must be considered as a "particle mixture" exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that no more than half of all θ^0 's undergo the familiar decay into two pions. Some experimental consequences of this picture are mentioned.

[Phys. Rev. 97 \(1955\) 1387](#)

- Known: K^0 (θ^0) can decay to $\pi^+\pi^-$
- Hypothesis: K^0 has a distinct anti-particle \bar{K}^0
- Claim: K^0 (\bar{K}^0) is a "particle mixture" with two distinct lifetimes, each lifetime has its own set of decay modes

Mixing of neutral mesons: formalism

- Time-evolution described by Schrödinger's equation

$$i \frac{\partial}{\partial t} \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \right] \begin{pmatrix} |P^0(t)\rangle \\ |\bar{P}^0(t)\rangle \end{pmatrix}$$

- Eigenstates can have different masses and decay width

$$|P_{L,H}\rangle = p|P^0\rangle \pm q|\bar{P}^0\rangle \quad \text{where} \quad \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

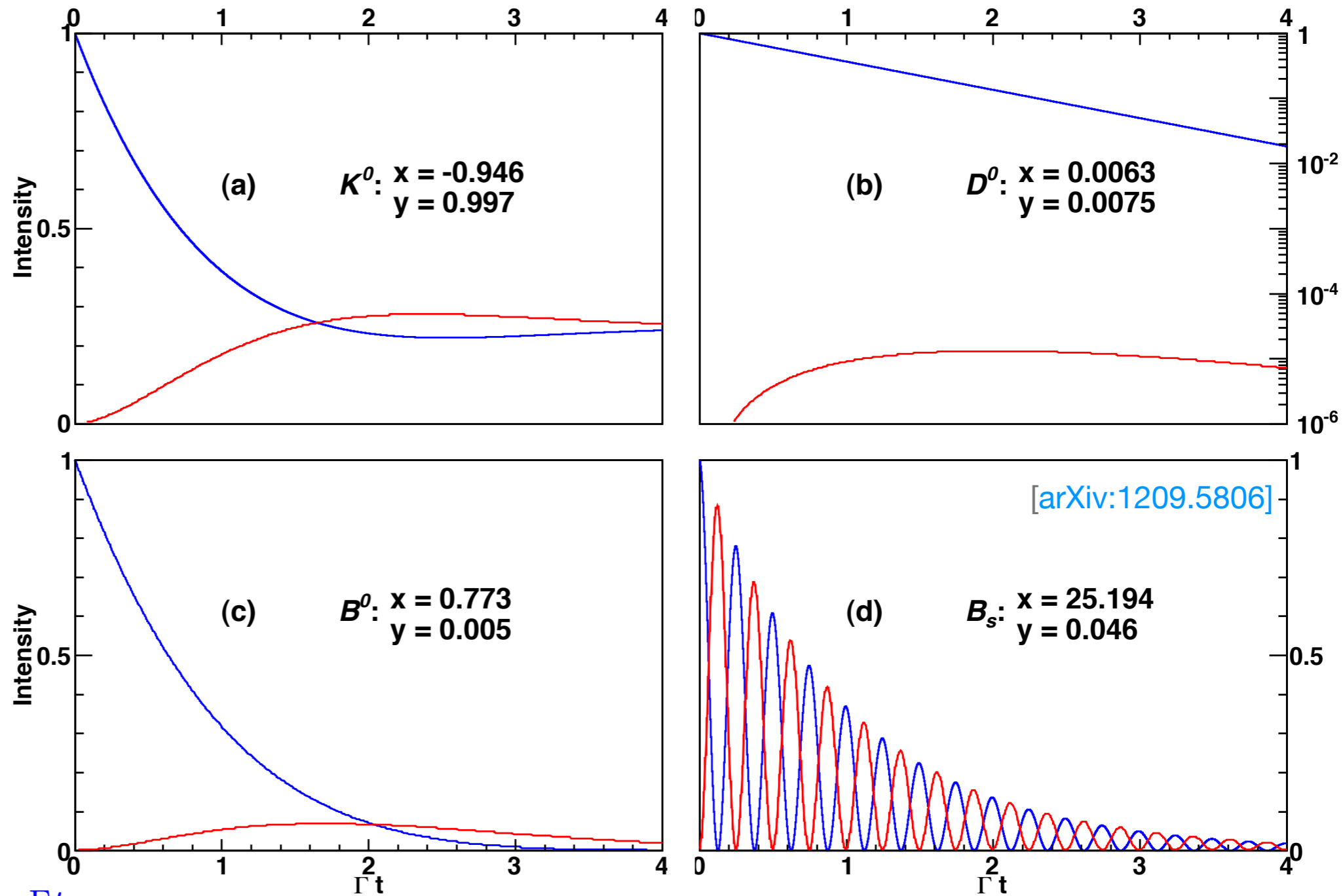
$$x = \frac{\Delta m}{\Gamma} = \frac{m_H - m_L}{(\Gamma_H + \Gamma_L)/2}, \quad y = \frac{\Delta\Gamma}{2\Gamma} = \frac{\Gamma_H - \Gamma_L}{\Gamma_H + \Gamma_L}$$

- If CP is conserved, q and p are real, i.e. $|q/p| = 1$ and $\varphi = \arg(q/p) = 0$

Mixing of neutral mesons: phenomenology

Blue line:
given a P^0 , at $t=0$,
the probability of
finding a P^0 at t

Red Line:
given a P^0 , at $t=0$,
the probability of
finding a \bar{P}^0 at t

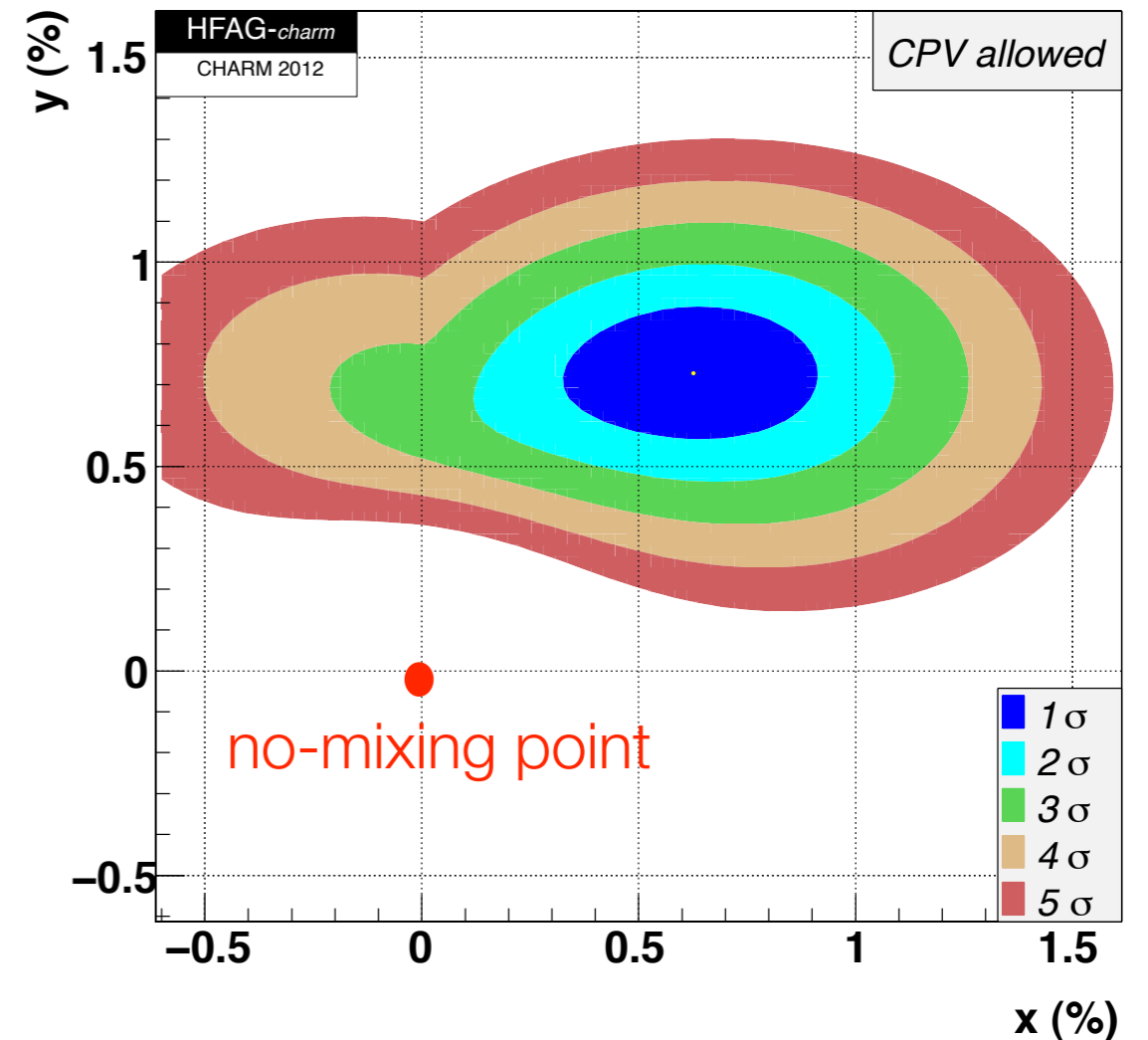


$$|\langle P^0(0) | P^0(t) \rangle|^2 \propto e^{-\Gamma t} [\cosh(y\Gamma t) + \cos(x\Gamma t)]$$

$$|\langle P^0(0) | \bar{P}^0(t) \rangle|^2 \propto e^{-\Gamma t} [\cosh(y\Gamma t) - \cos(x\Gamma t)]$$

What about charm?

- Charm mixing “known” since 2007
 - As of November 2012, no single 5σ observation
 - No-mixing excluded at $\sim 10\sigma$ when all results are combined
- So far no evidence for CP violation in charm mixing
- After years of dedicated experiments, LHCb can now probe the charm sector with unprecedented precision



Why is charm interesting?

- Low standard model rate, potentially a powerful probe for new physics

- Small contributions from box diagrams

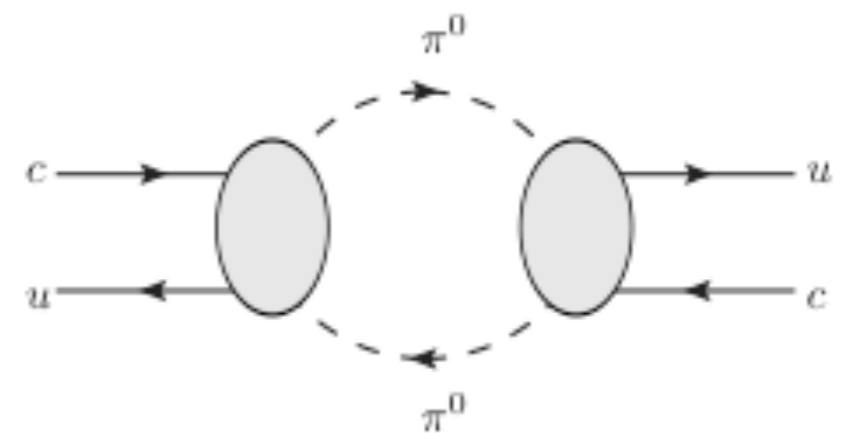
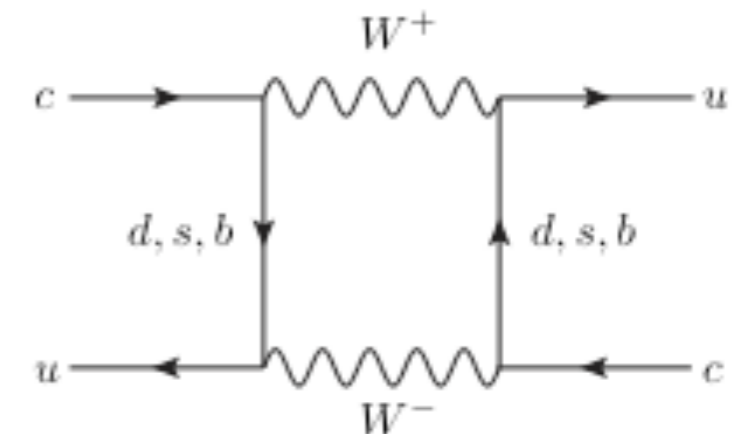
- b loop CKM suppressed $\Leftrightarrow |V_{ub}V_{cb}^*|^2 \ll 1$

- s, d loops GIM suppressed $\Leftrightarrow (m_s^2 - m_d^2)/m_W^2 \sim 0$

- Long-distance effects important (and difficult to calculate)

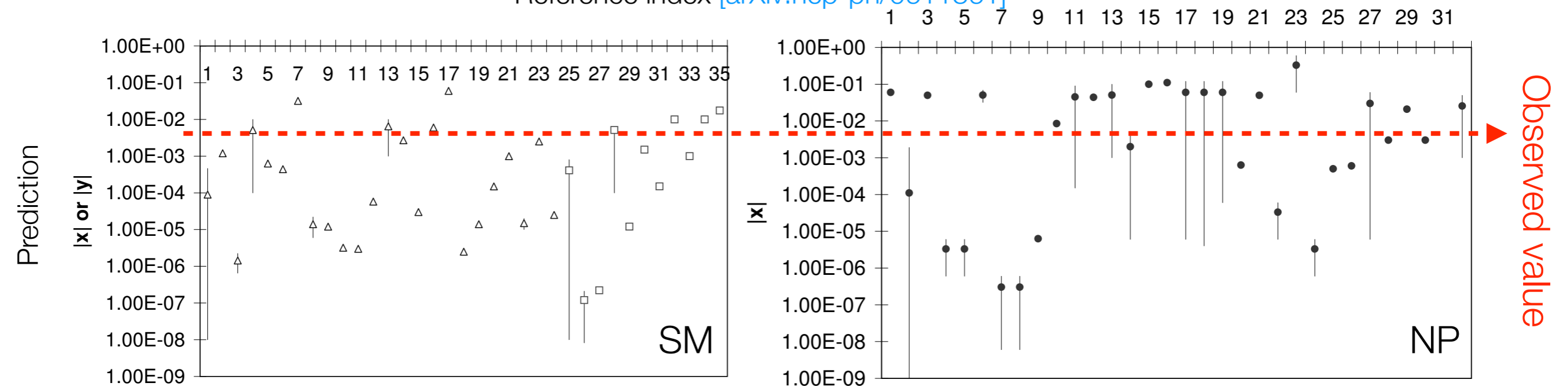
- Charm is the only up-type quark where we can look for flavor/CP violation

- In the standard model the largest flavor/violating effects appear in the down sector, no reason this should be true if new physics is present at the electroweak scale



“Charming puzzle”

Reference index [\[arXiv:hep-ph/0611361\]](https://arxiv.org/abs/hep-ph/0611361)



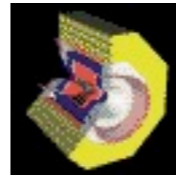
- Observed mixing rate is on the upper end of most standard model predictions
- Could be interpreted as a hint for the presence of new physics
- More precise measurements (and reliable theory calculations) are needed to clear the picture

Charm mixing: experimental status

From HFAG page:

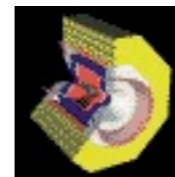
$$D^0 \rightarrow K^+ \pi^-$$

E791



$$D^0 \rightarrow h^+ h^-$$

E791



$$D^0 \rightarrow K^+ \pi^- \pi^0$$



$$D^0 \rightarrow K^+ \pi^+ 2\pi^-$$



$$D^0 \rightarrow K_S^0 h^+ h^-$$



$$D^0 \rightarrow K^+ \ell^- \nu$$

E791



$$\psi(3770) \rightarrow D^0 \bar{D}^0$$

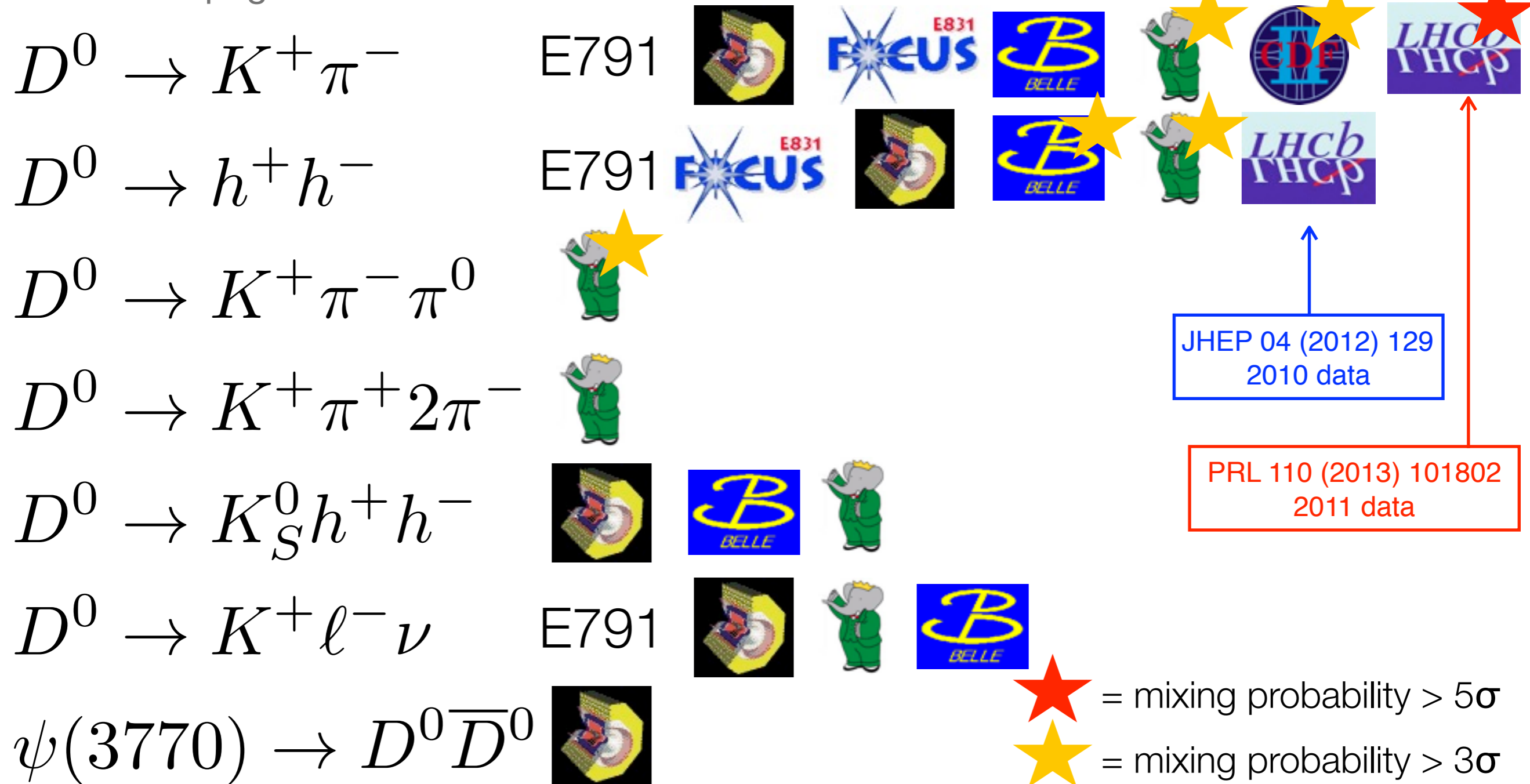


JHEP 04 (2012) 129
2010 data

★ = mixing probability $> 3\sigma$

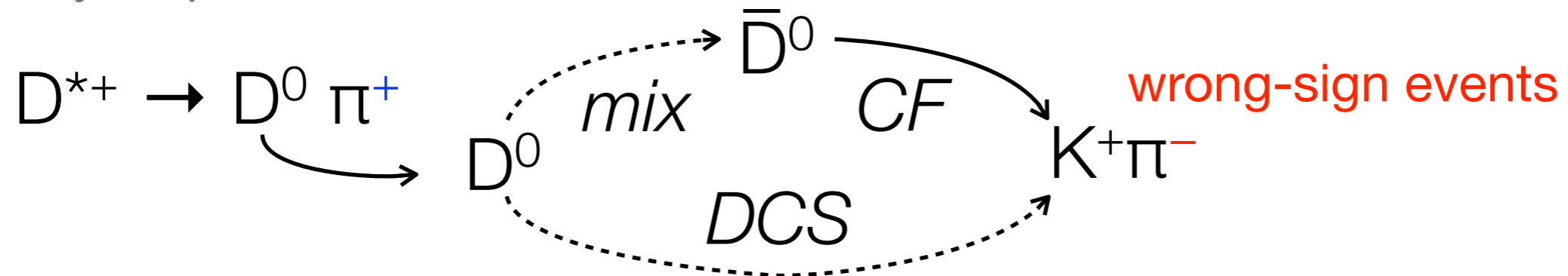
Charm mixing: experimental status

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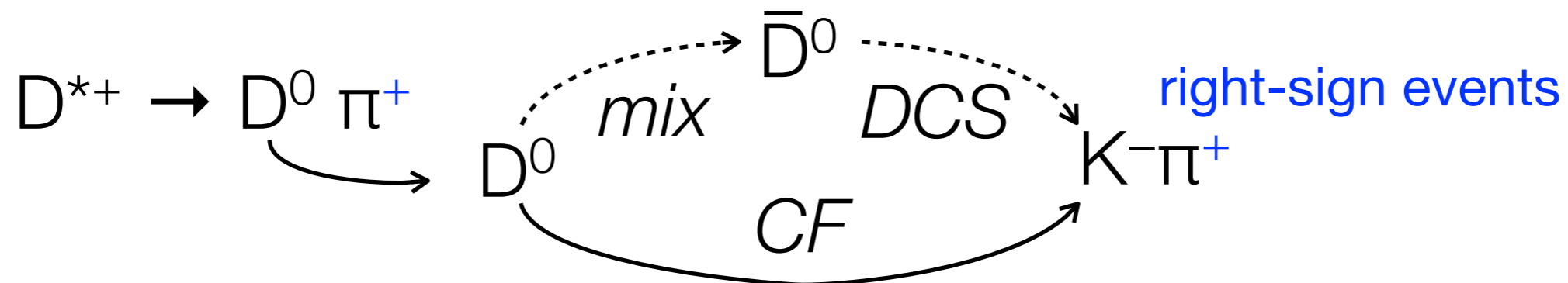


Charm mixing with $D^0 \rightarrow K^+ \pi^-$

- Exploit interference between mixing and doubly-Cabibbo-suppressed decay amplitudes



- Compare to RS events which are dominated by Cabibbo-favored amplitude

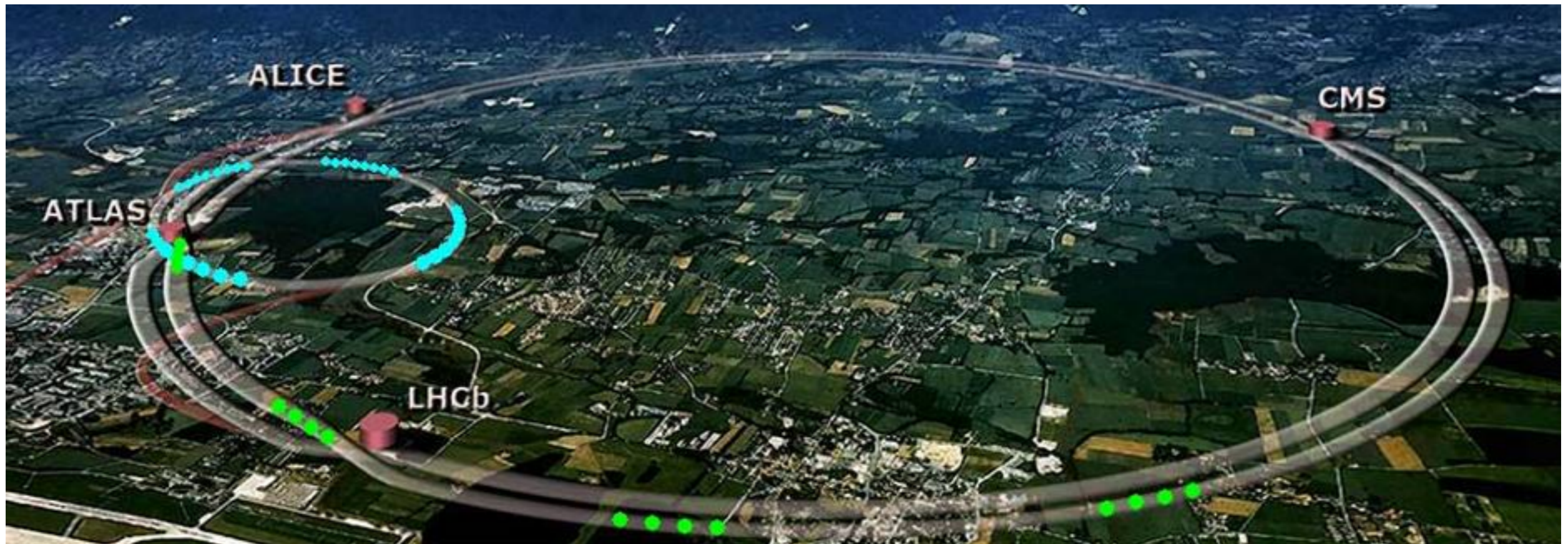


- Assuming $|x|, |y| \ll 1$ and no CPV

$$R(t) = \frac{N_{WS}(t)}{N_{RS}(t)} = R_D + \sqrt{R_D} y' t + \frac{x'^2 + y'^2}{4} t^2 \quad \begin{aligned} x' &= x \cos \delta + y \sin \delta \\ y' &= y \cos \delta - x \sin \delta \end{aligned}$$

Experimental apparatus

When “charm is more than just beauty” ...



At the LHC charm production is ~20 times more abundant than beauty:

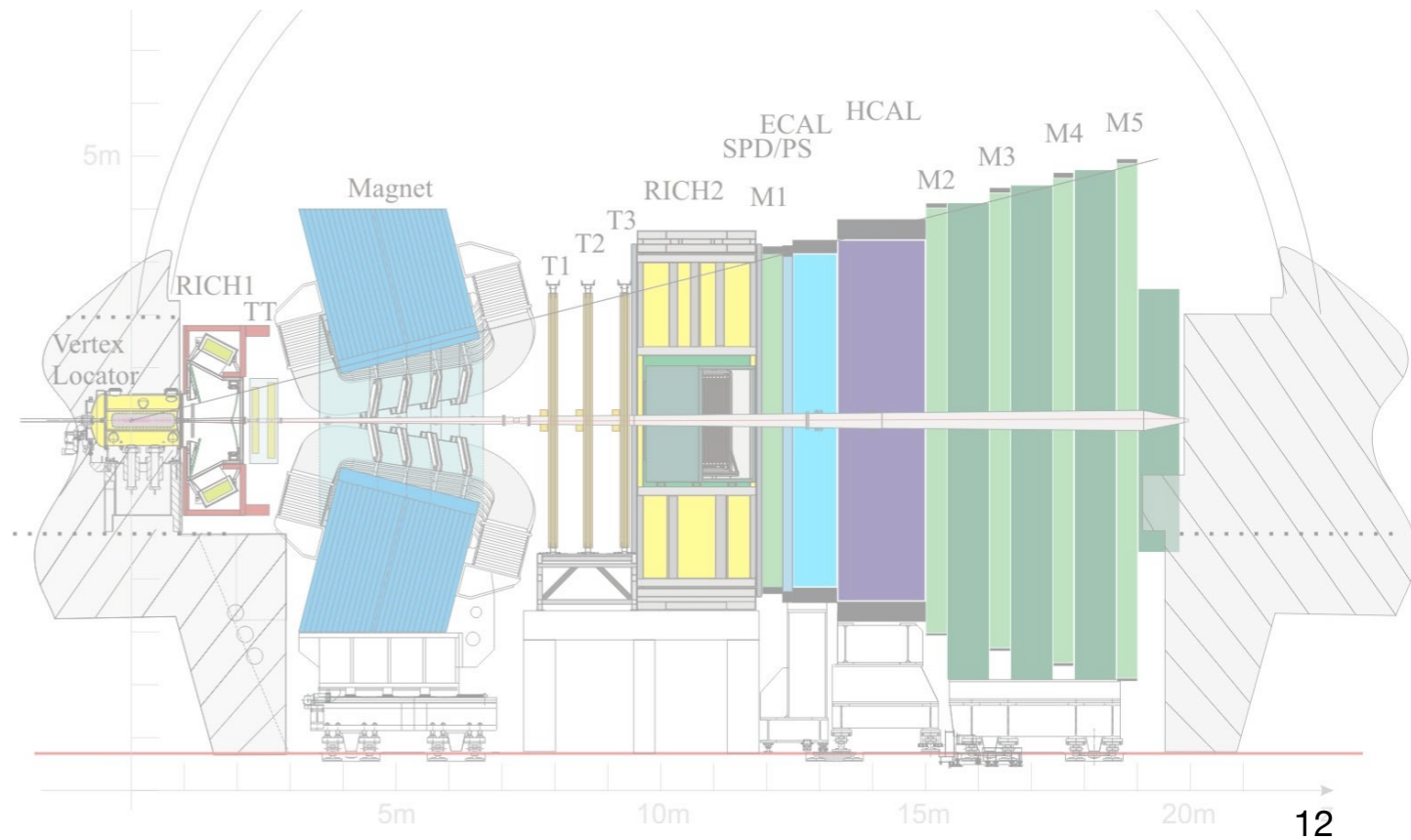
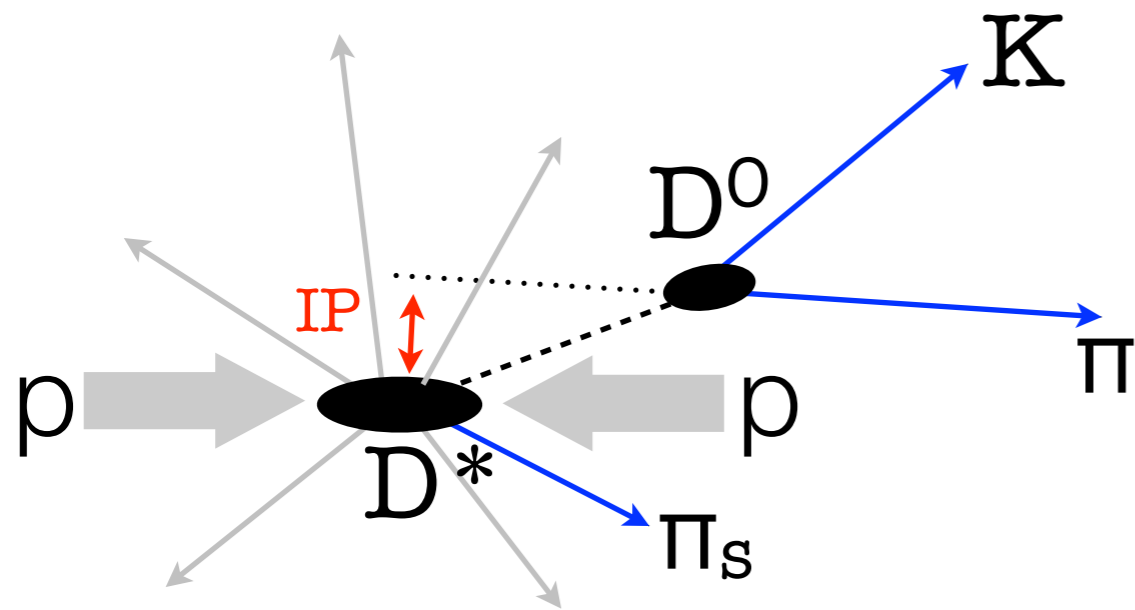
$$\begin{aligned}\sigma(pp \rightarrow c\bar{c}X) &= 1419 \pm 134 \mu\text{b}^* \\ \sigma(pp \rightarrow b\bar{b}X) &= 75 \pm 14 \mu\text{b}^{**}\end{aligned} \quad \text{@ 7 TeV and in LHCb acceptance}$$

The LHC is effectively a c-hadron factory!

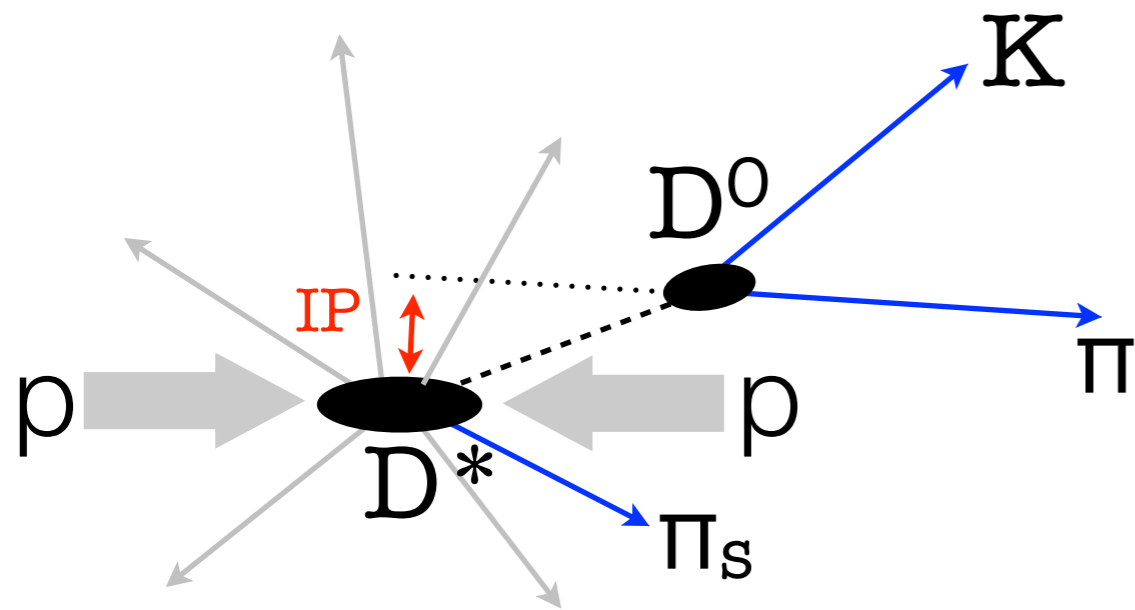
* Nucl. Phys. B 871 (2013) 1

** Phys. Lett. B 694 (2010) 209

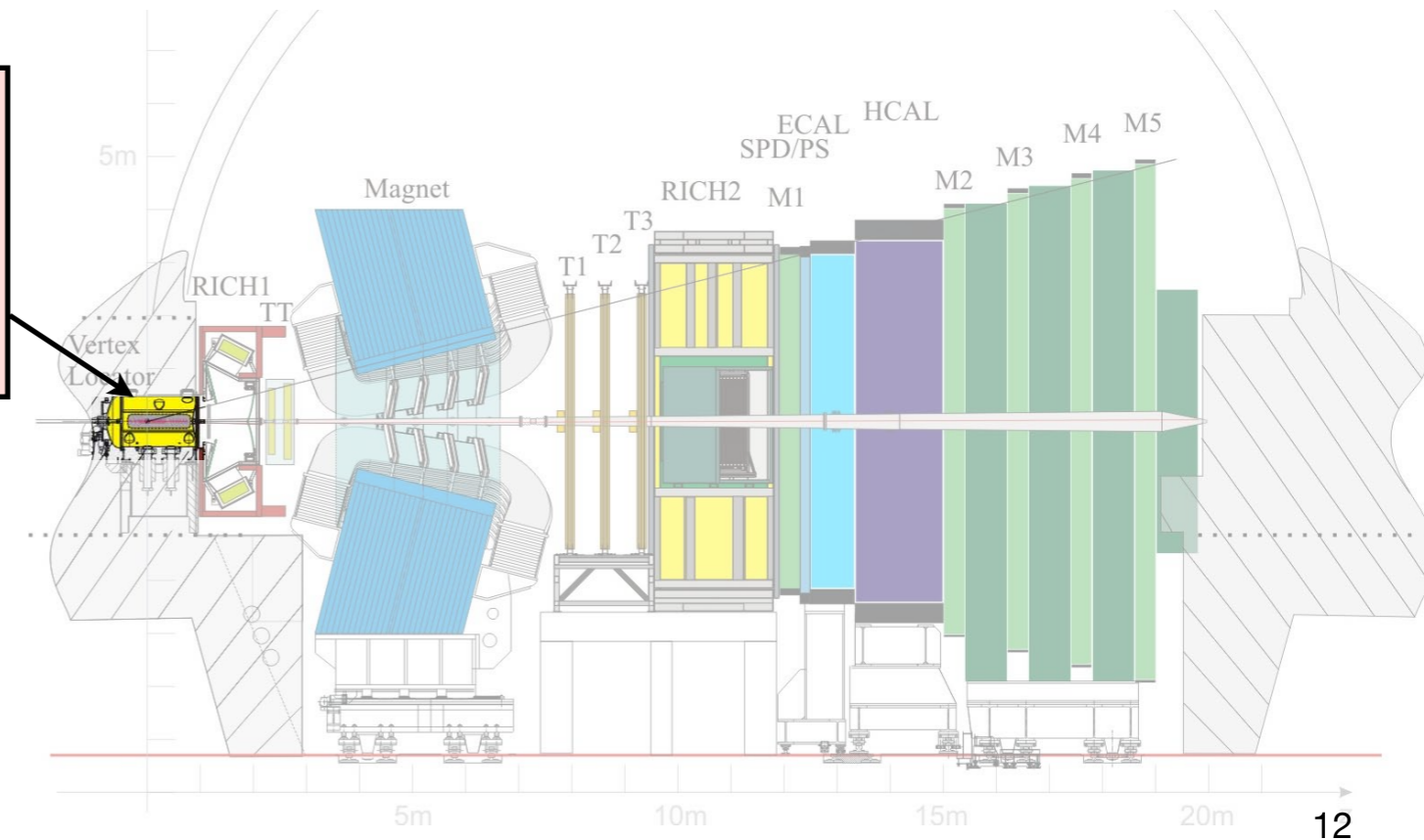
Hadronic charm decays at LHCb



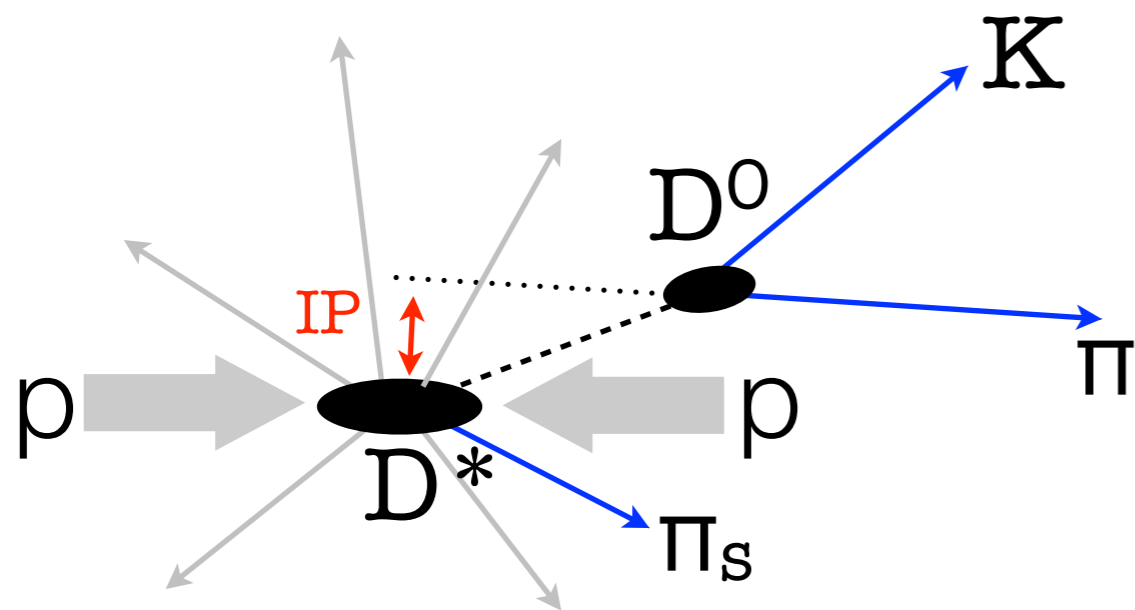
Hadronic charm decays at LHCb



Silicon Vertex Locator:
20 μm impact parameter resolution,
corresponding to $\sim 0.1\tau$ decay-time
resolution for a 2-body charm decay

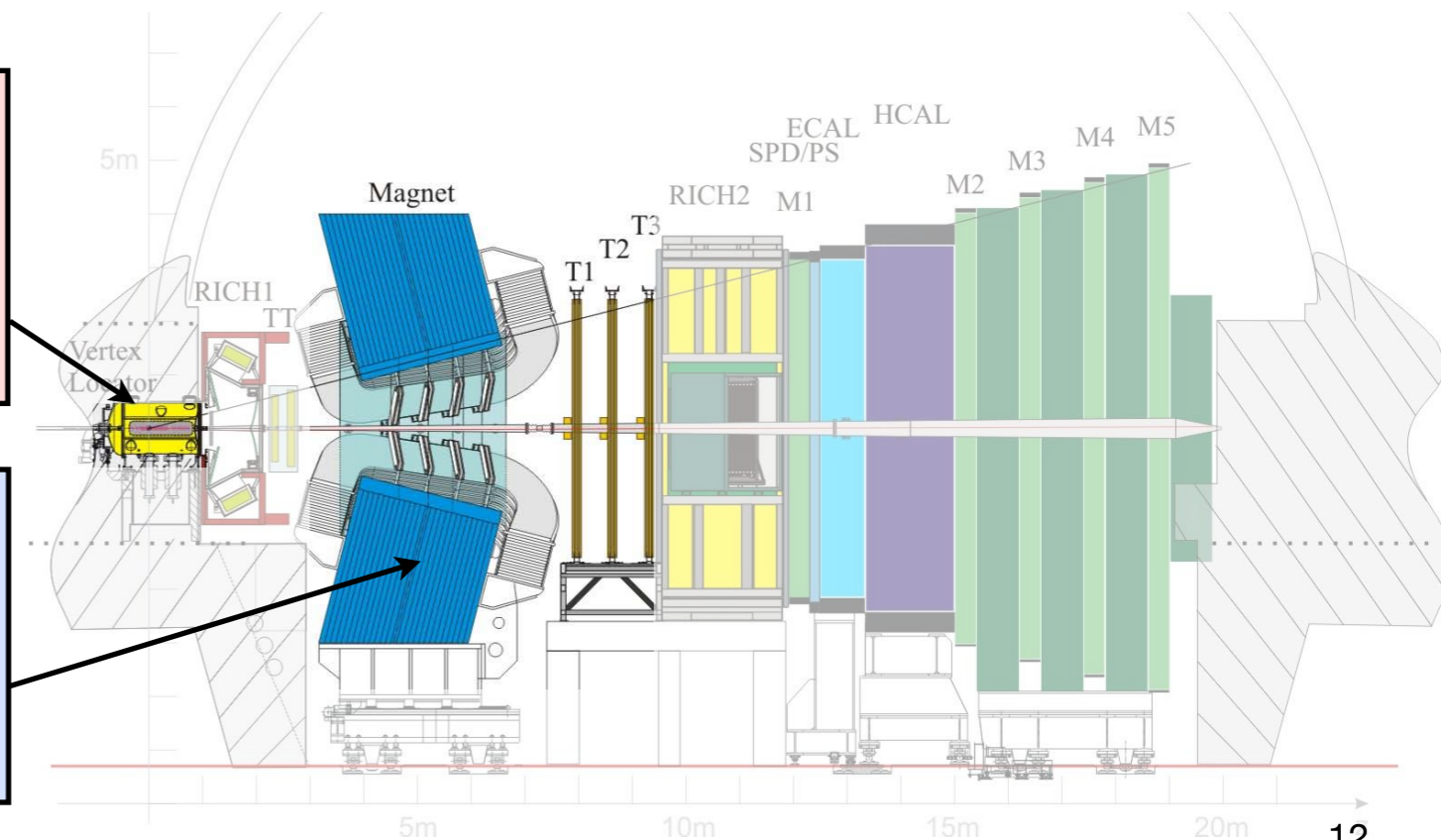


Hadronic charm decays at LHCb

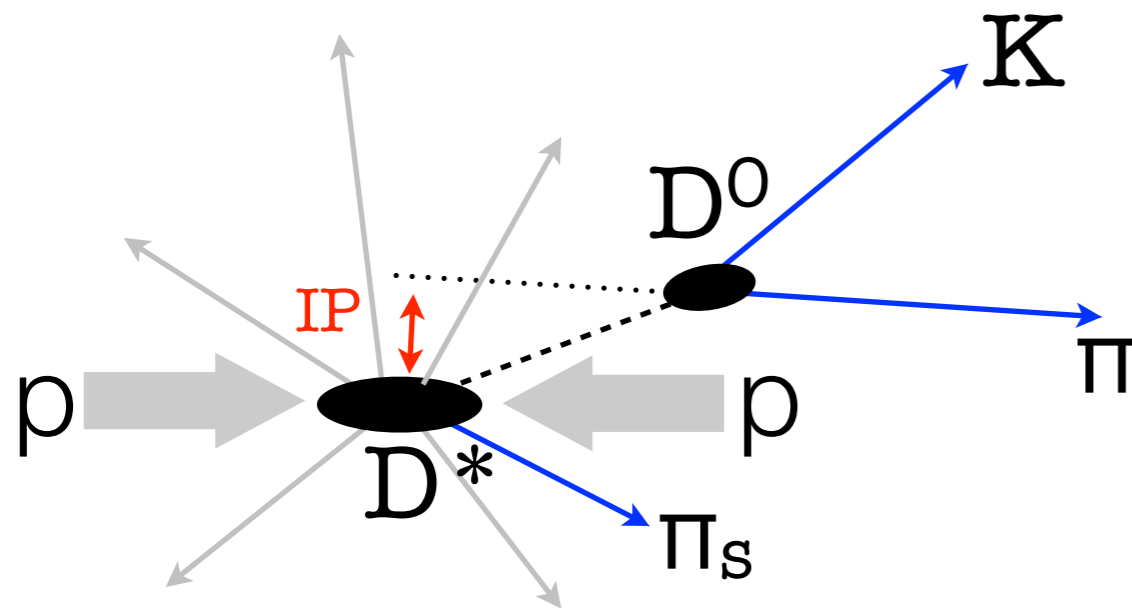


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Excellent tracking:
 $\Delta p/p = 0.4\text{-}0.6\%$ at 5-100 GeV/c,
corresponding to $\sim 8 \text{ MeV}/c^2$ mass
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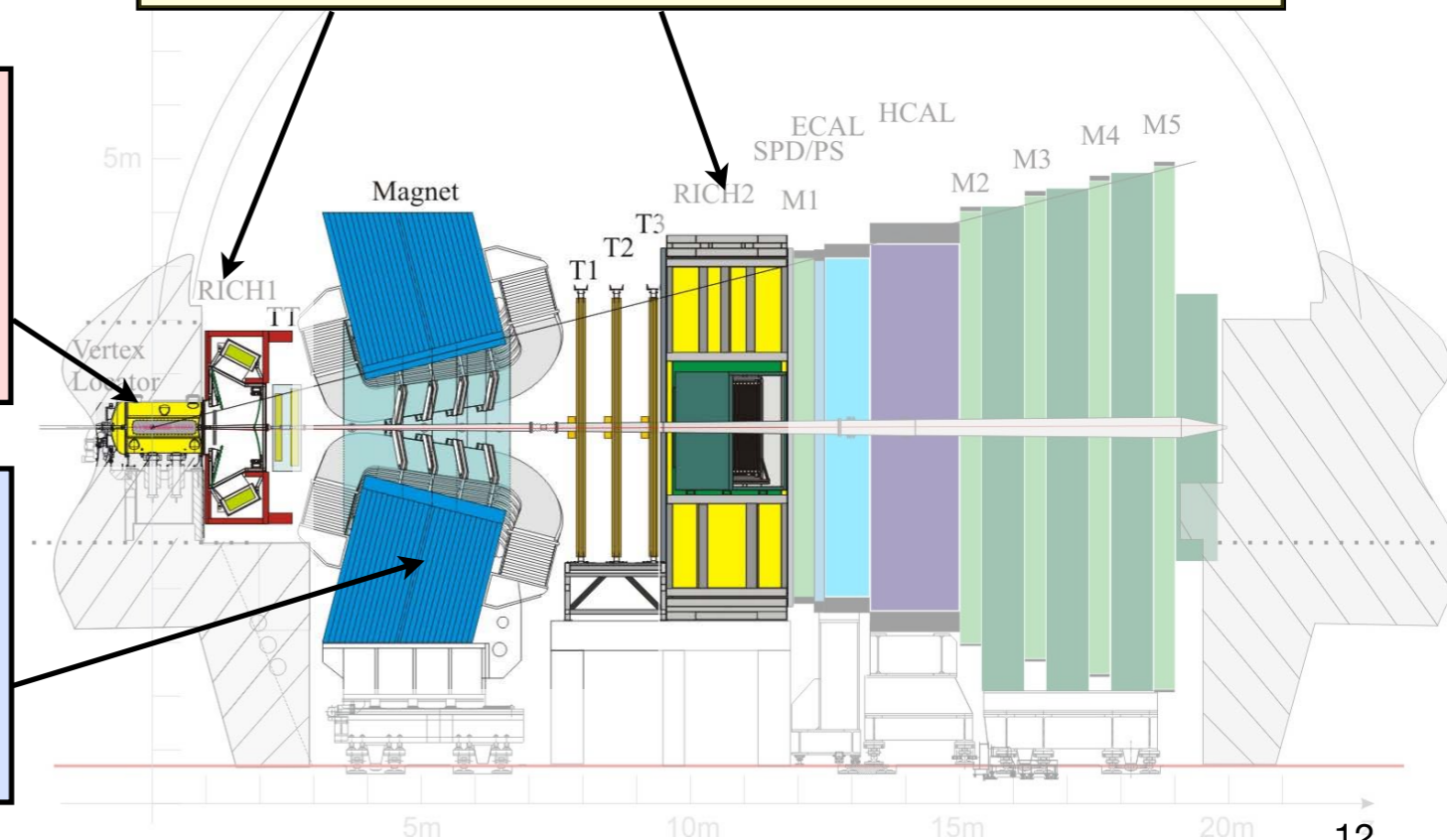
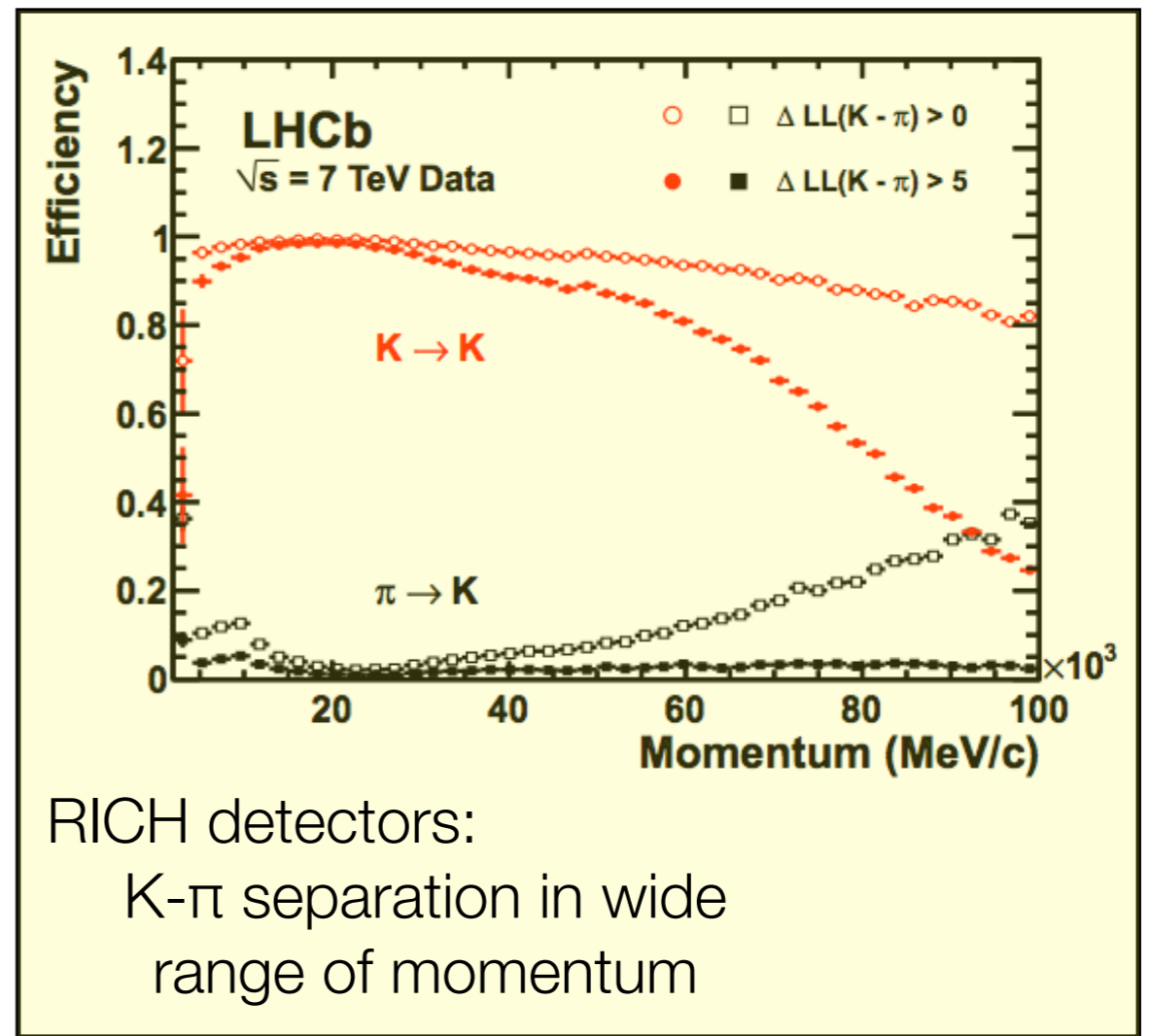


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LHCb trigger on hadronic charm decays

LHC rate ~ 15 MHz

$c\bar{c}$ fraction $\sim 10\%$

Hardware E_T trigger ~ 1 MHz

$c\bar{c}$ fraction $\sim 50\%$

No possibility of an inclusive charm trigger!

Use exclusive triggers tuned for the needs of specific analyses to deliver high signal efficiency and purity

High p_T , IP track ~ 80 kHz

efficiency $\sim 50\%$

Exclusive $D \rightarrow hh/3h/4h$ ~ 2 kHz

efficiency $\sim 50-90\%$

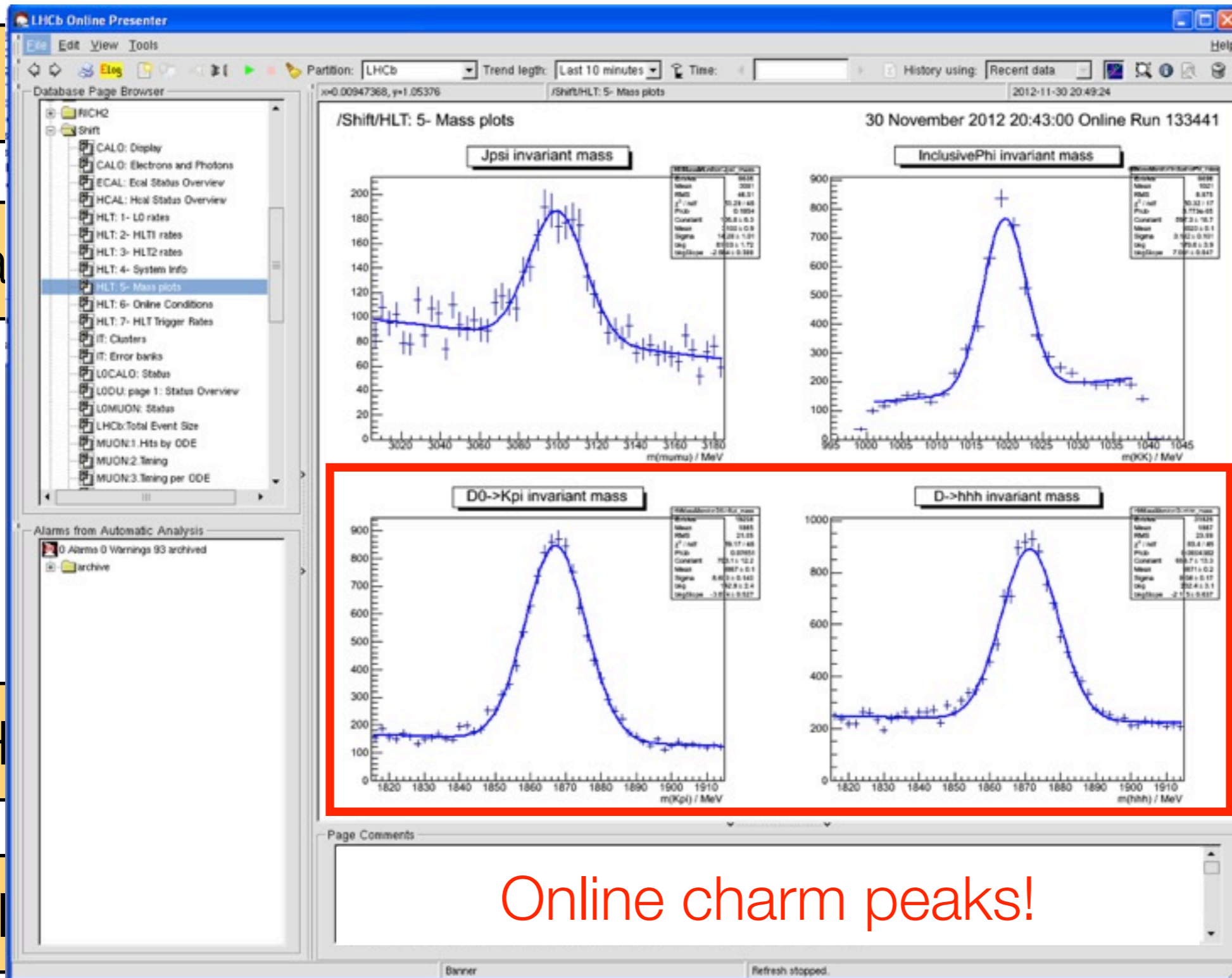
LHCb trigger on hadronic charm decays

Ha

Ha

H

Exc



0%

50%

ger!

f specific
d purity

0%

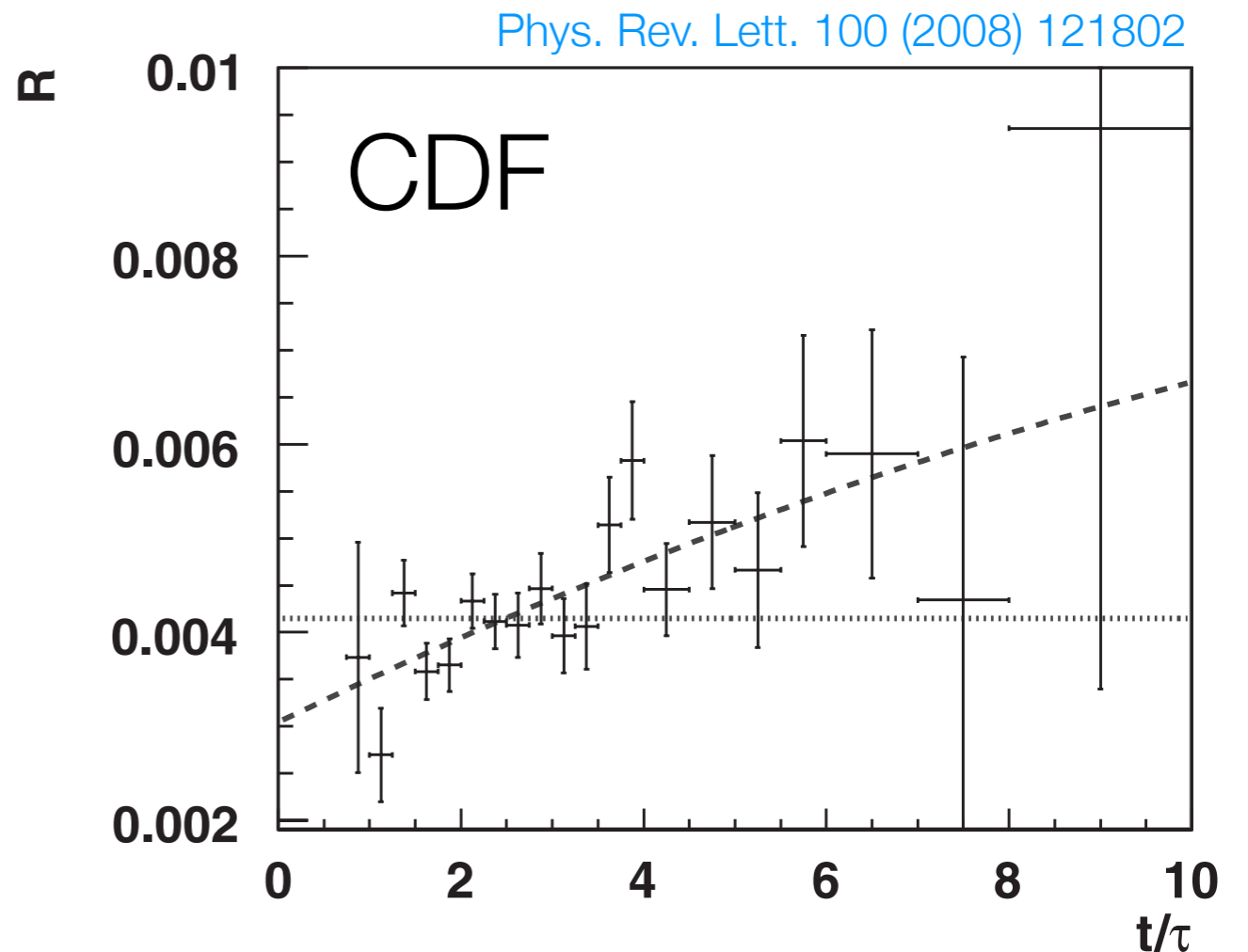
90%

Online charm peaks!

Analysis

Analysis outline

- Assume CP conservation
- Strategy similar to CDF's analysis:
 - Count WS and RS events in bins of decay time
 - separate signal from backgrounds
 - Fit the ratio of yields vs decay time
 - flat? no mixing
 - not flat? mixing



$D^* \rightarrow D^0(\rightarrow K\pi)\pi$ signal vs backgrounds

backgrounds

signal

singly
mis-ID
2-body
decays

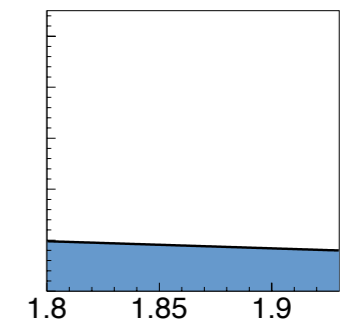
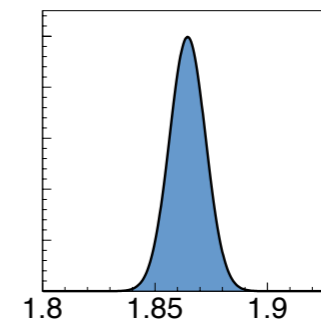
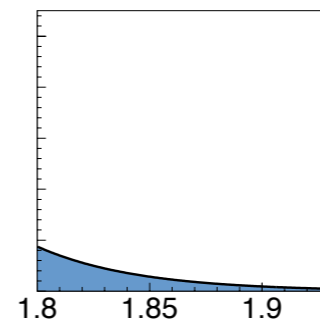
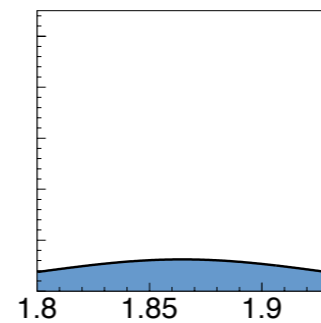
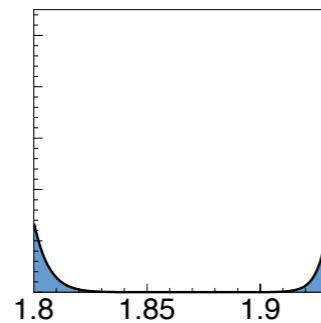
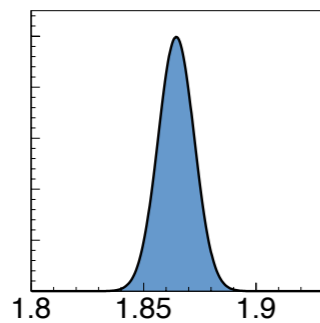
doubly
mis-ID
(swapped)
 $K\pi$ decays

multibody
decays

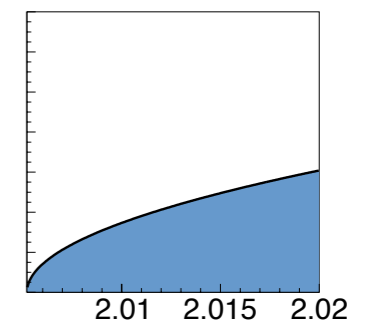
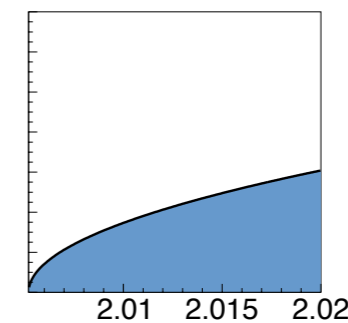
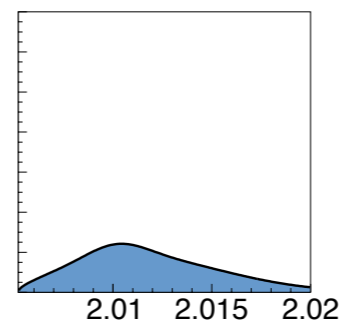
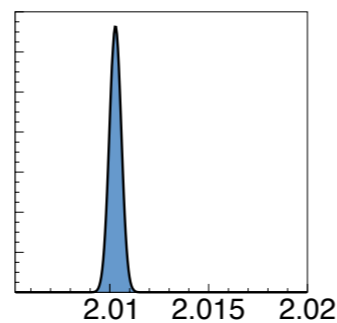
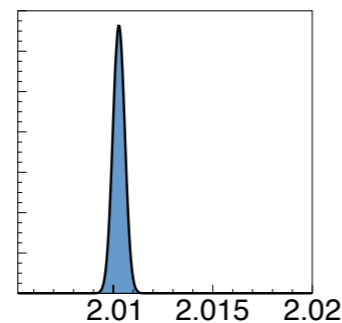
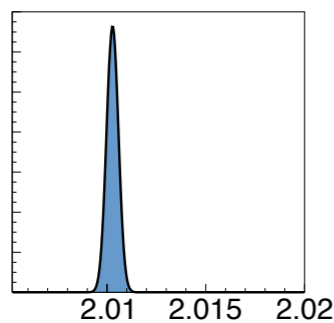
real D^0 +
random soft
pion

3 random
tracks

$K\pi$
mass



$D^0\pi_s$
mass



$D^* \rightarrow D^0(\rightarrow K\pi)\pi$ signal vs backgrounds

backgrounds

signal

singly
mis-ID
2-body
decays

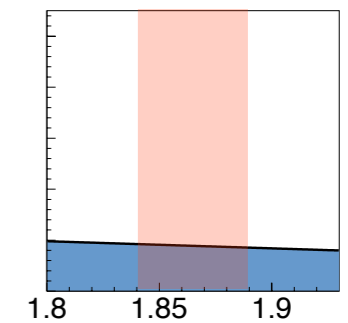
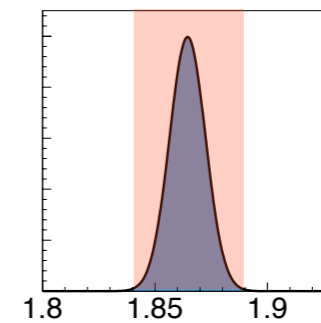
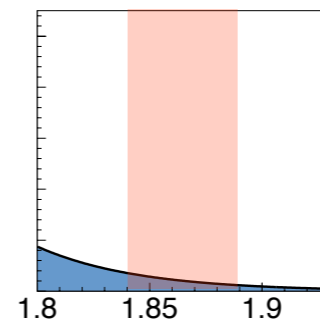
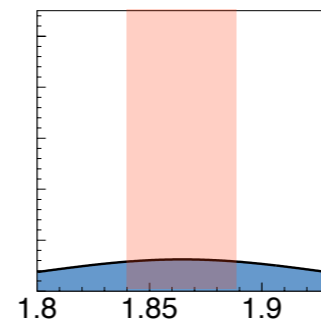
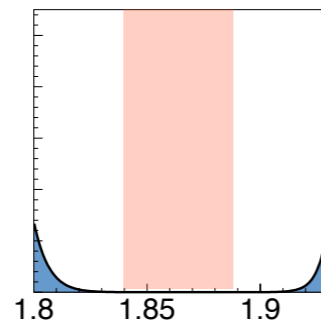
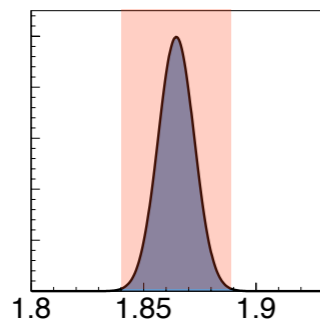
doubly
mis-ID
(swapped)
 $K\pi$ decays

multibody
decays

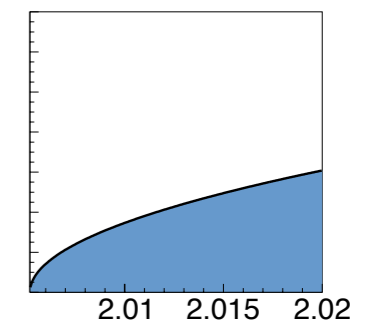
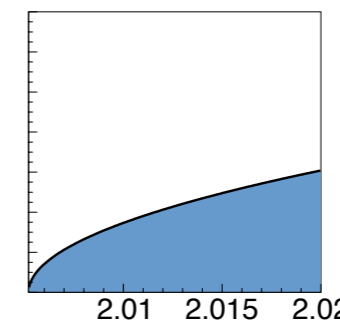
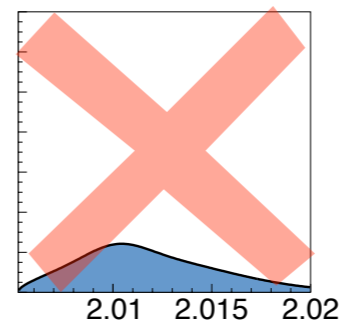
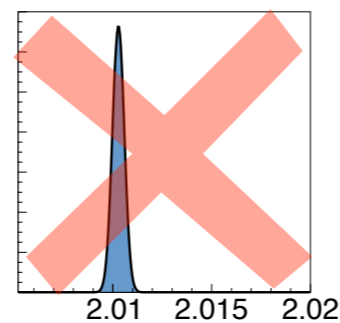
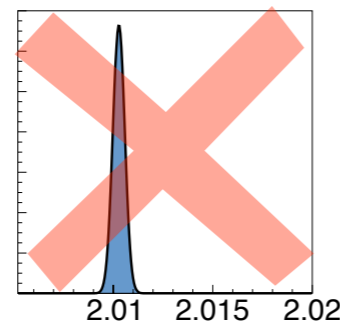
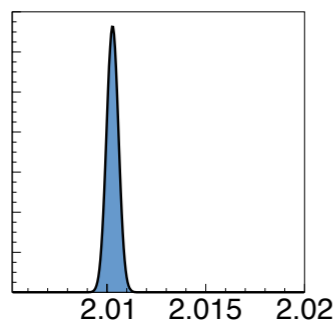
real D^0 +
random soft
pion

3 random
tracks

$K\pi$
mass

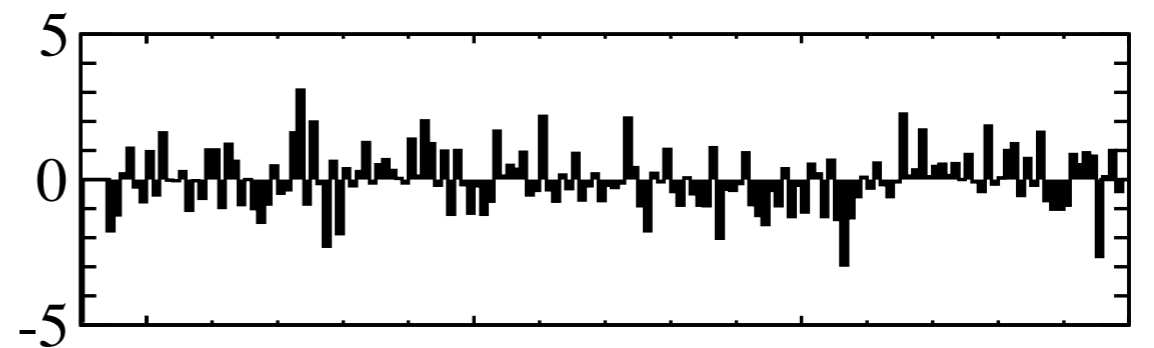
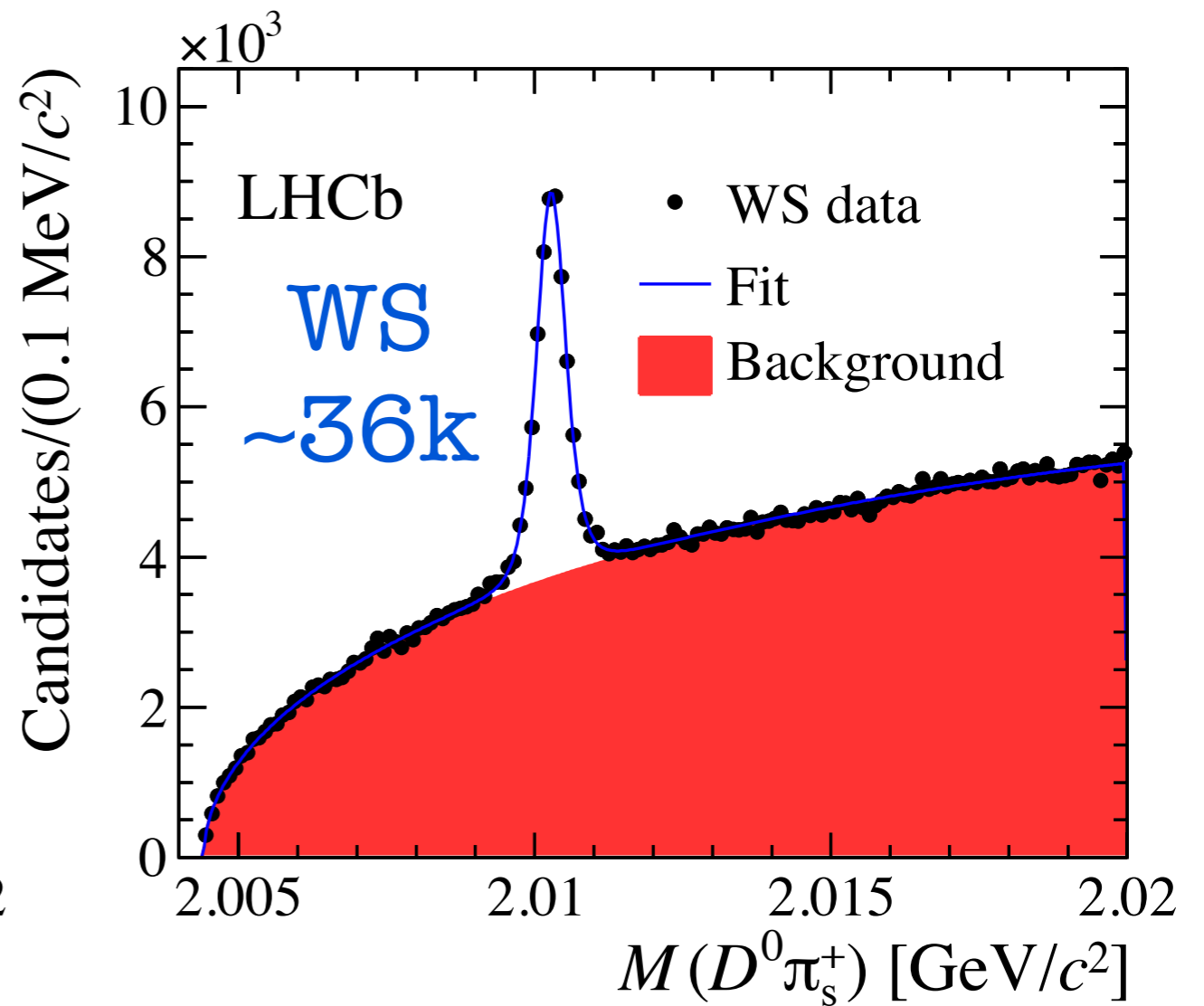
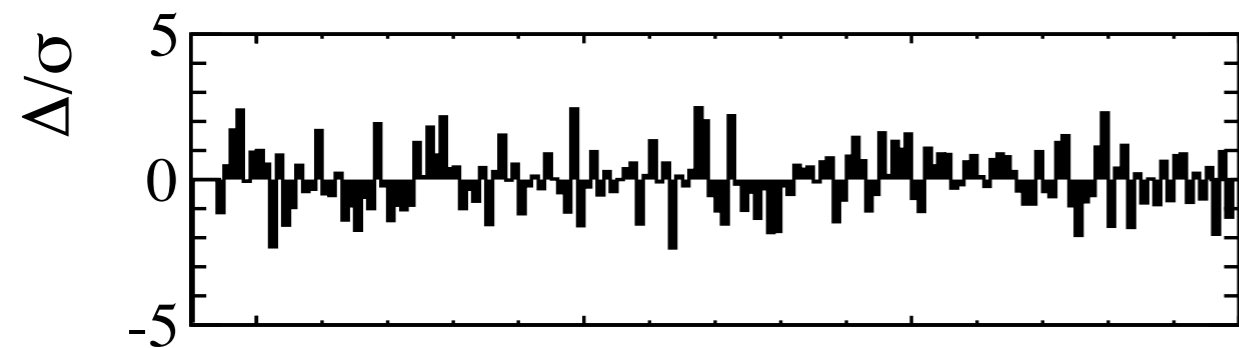
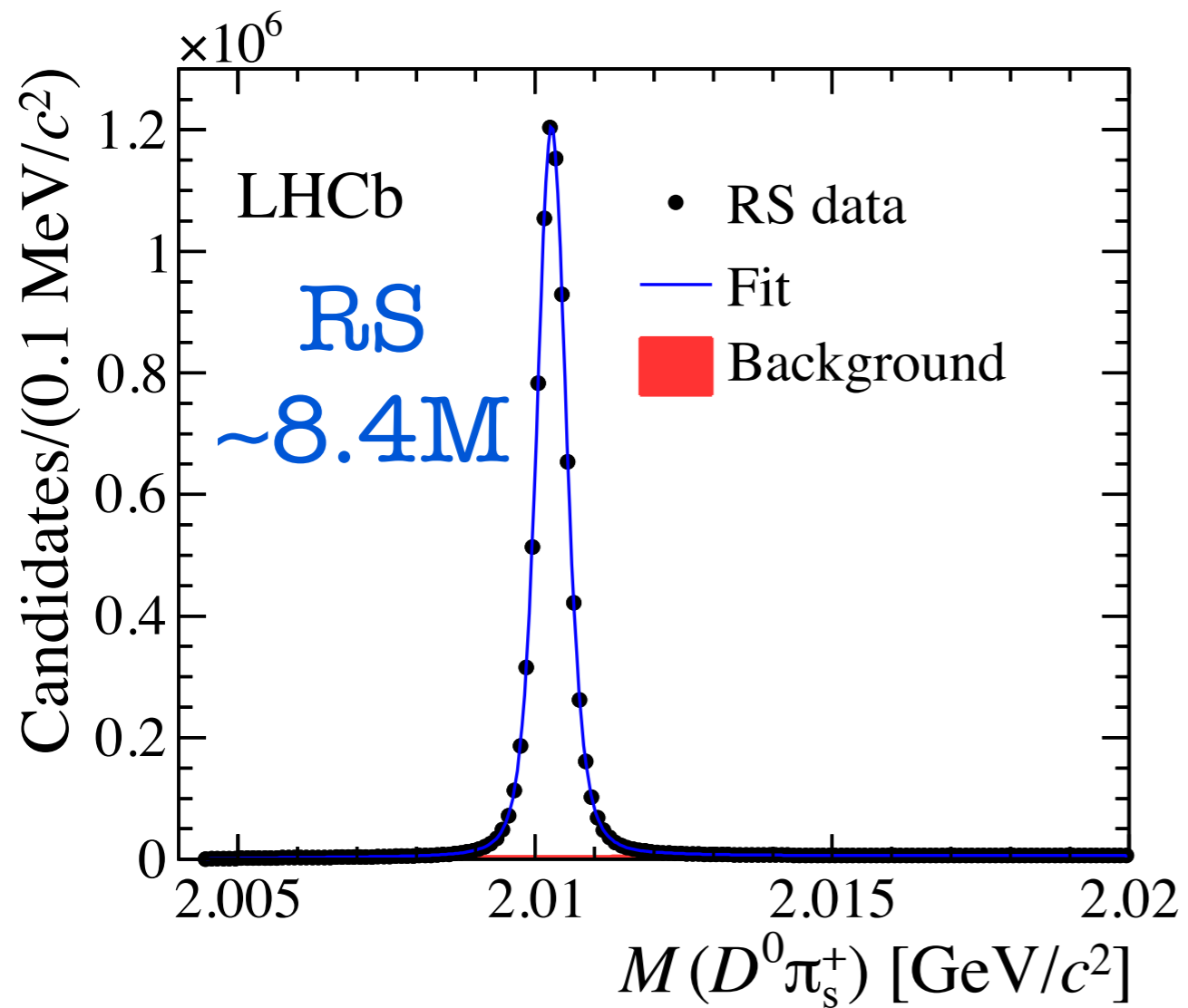


$D^0\pi_s$
mass

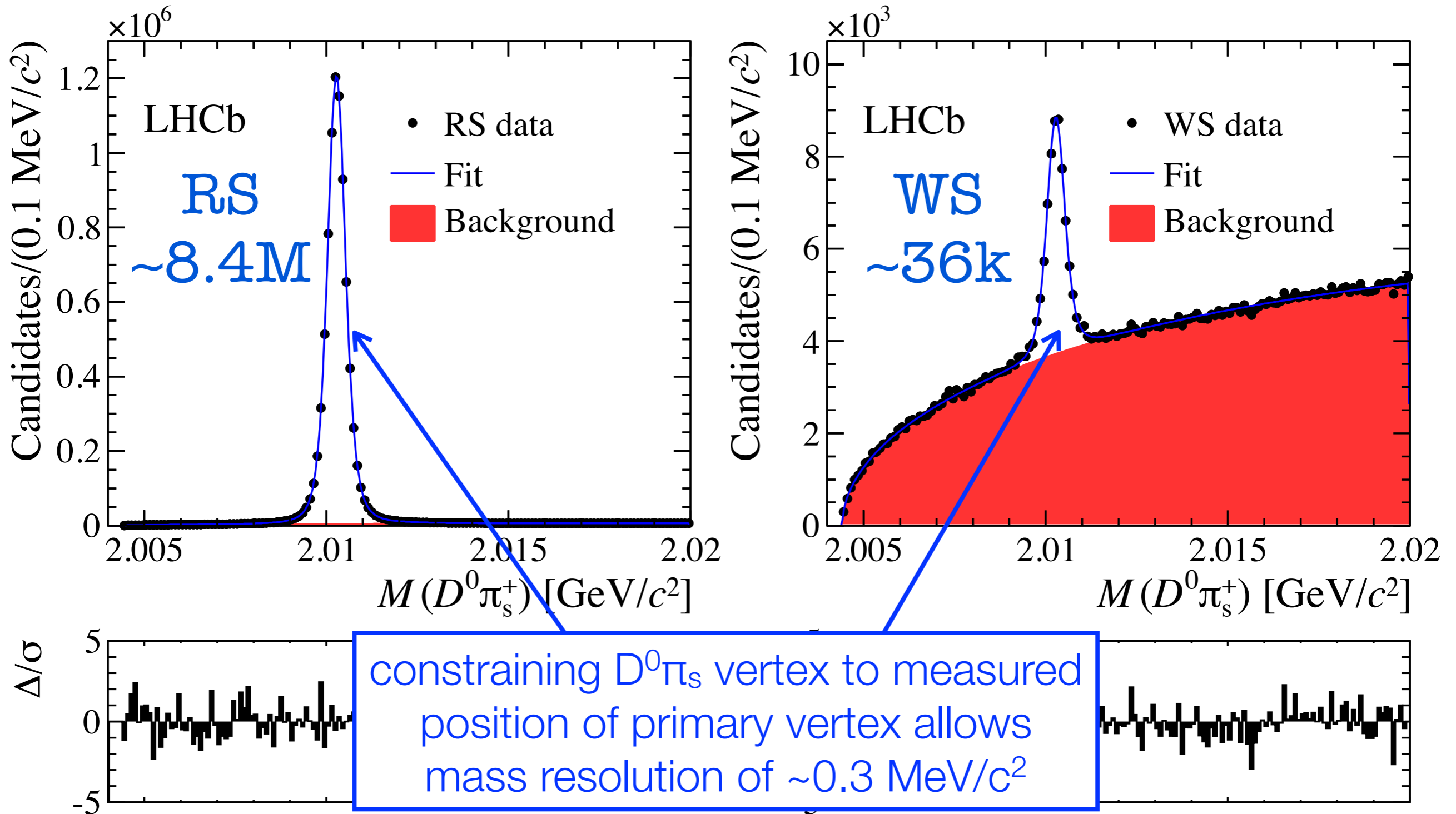


Cut tight on PID and D^0 mass to reduce physics bkg and fit $D^0\pi_s$ mass, then consider only signal and random pions in the fit

Time-integrated yields

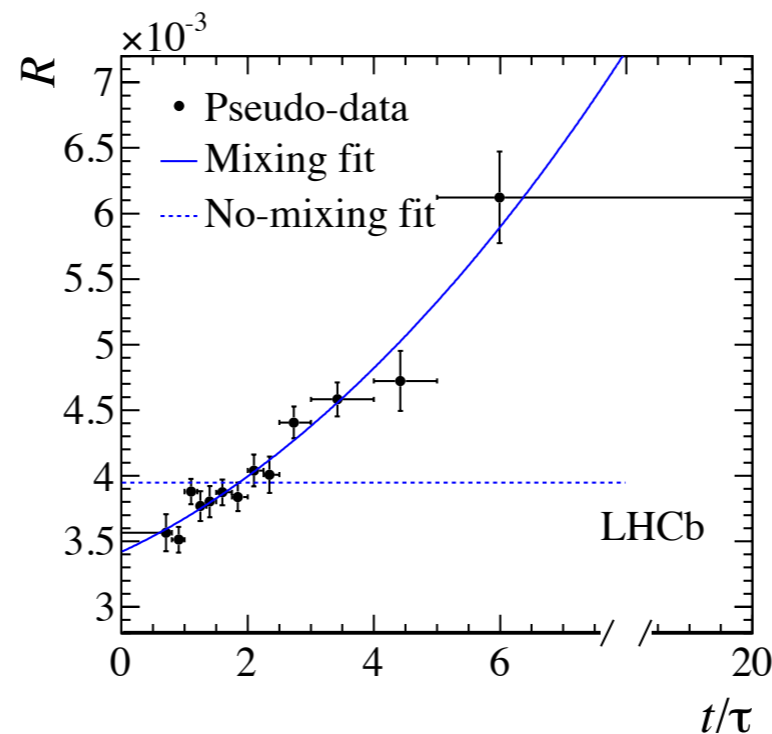
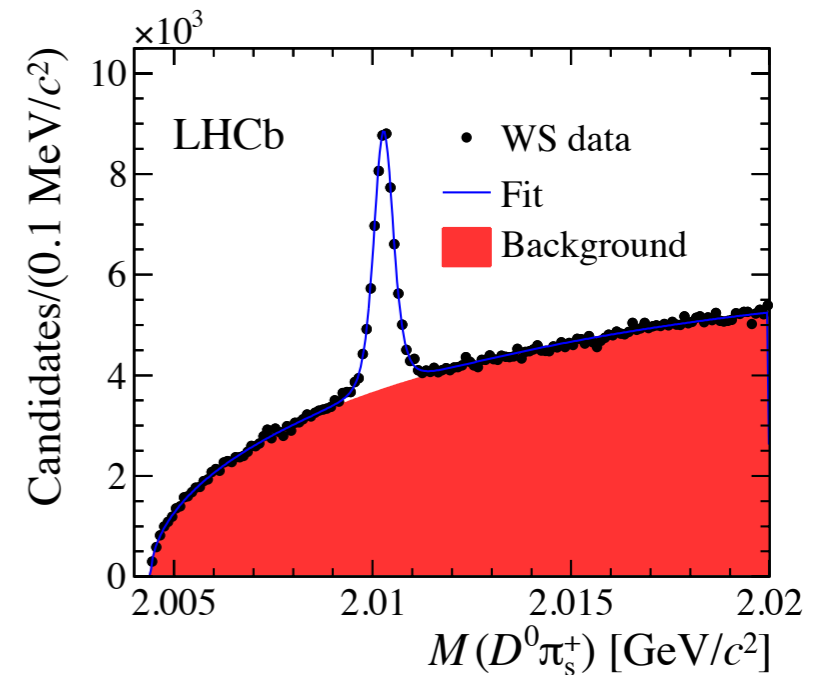
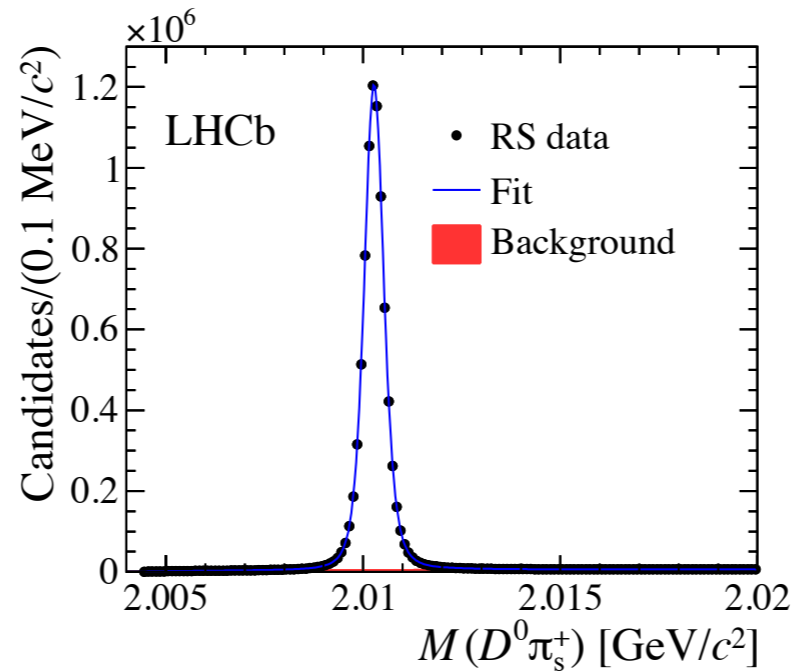


Time-integrated yields



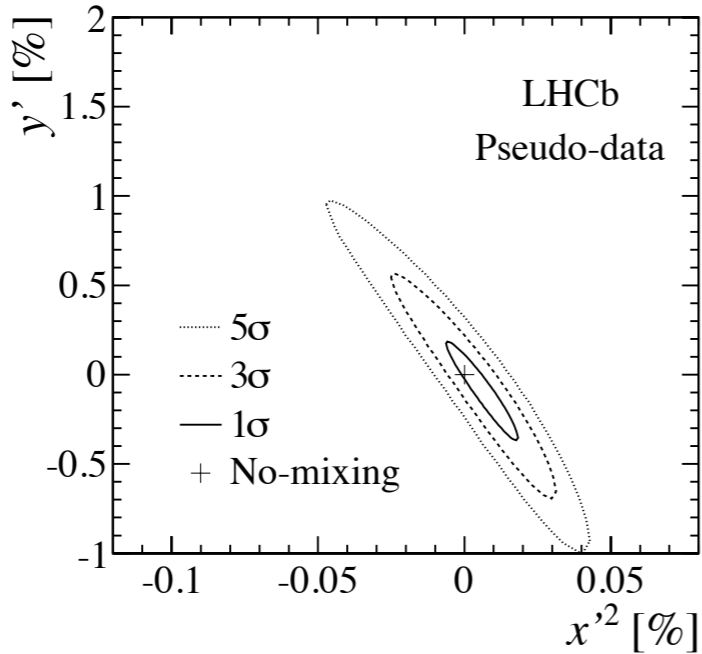
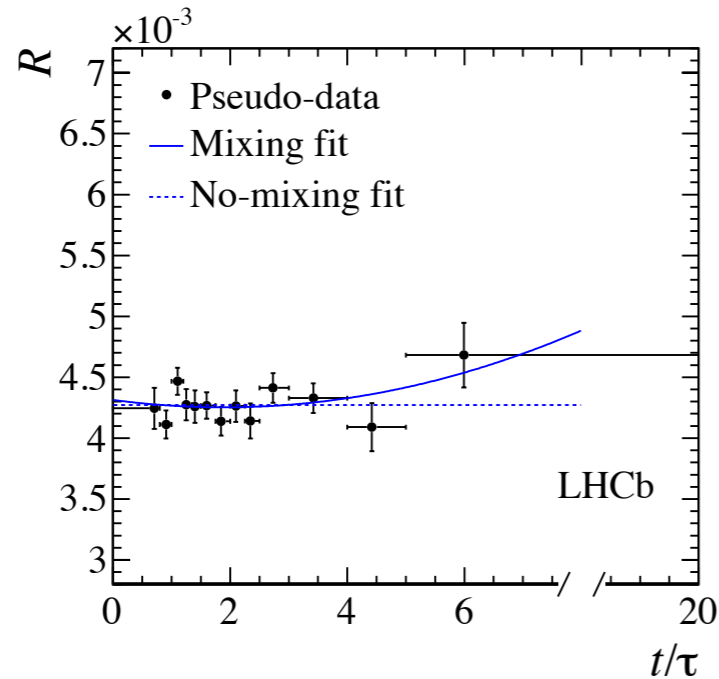
Time-dependent fit strategy

- In each decay-time bin
 1. Fit RS sample to determine shape's parameters
 2. Fit WS sample with signal shape fixed to RS and bkg shape free to float
 3. Calculate WS/RS ratio from measured yields

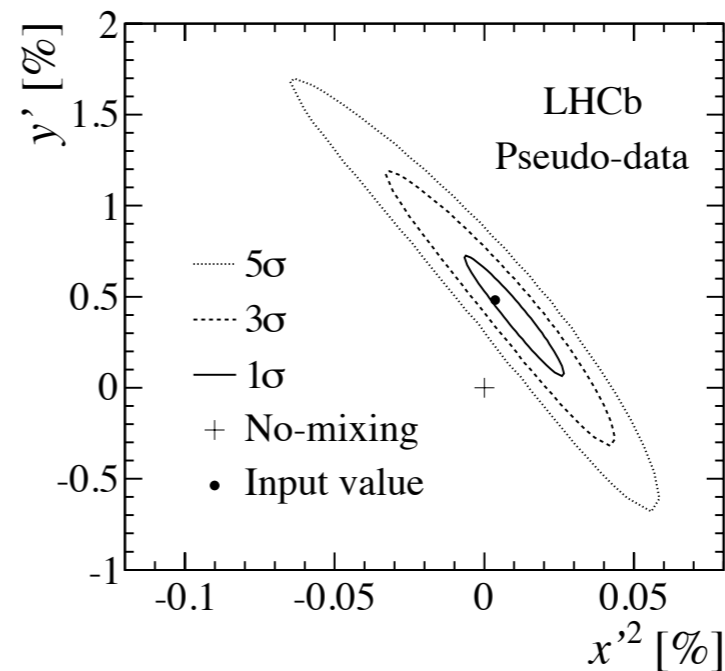
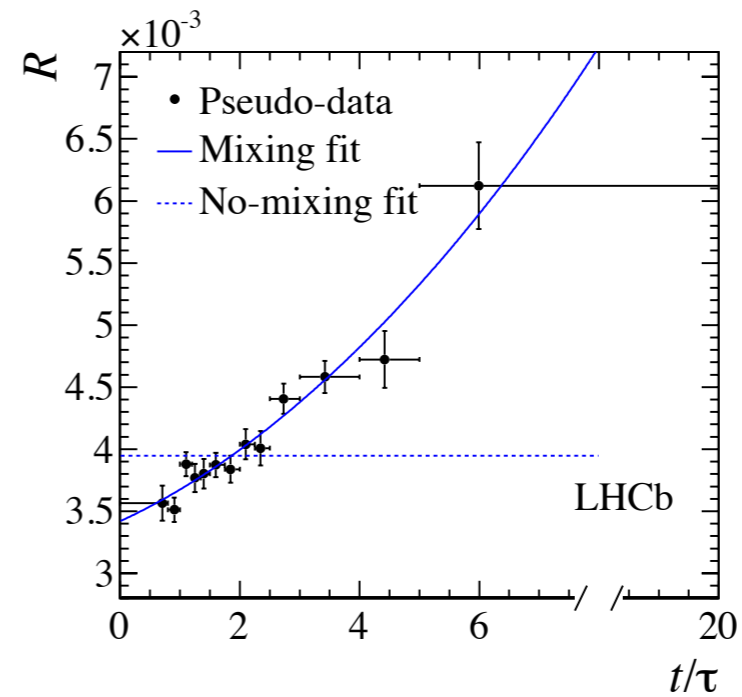


Pseudo-experiments

No mixing simulated



World average mixing simulated



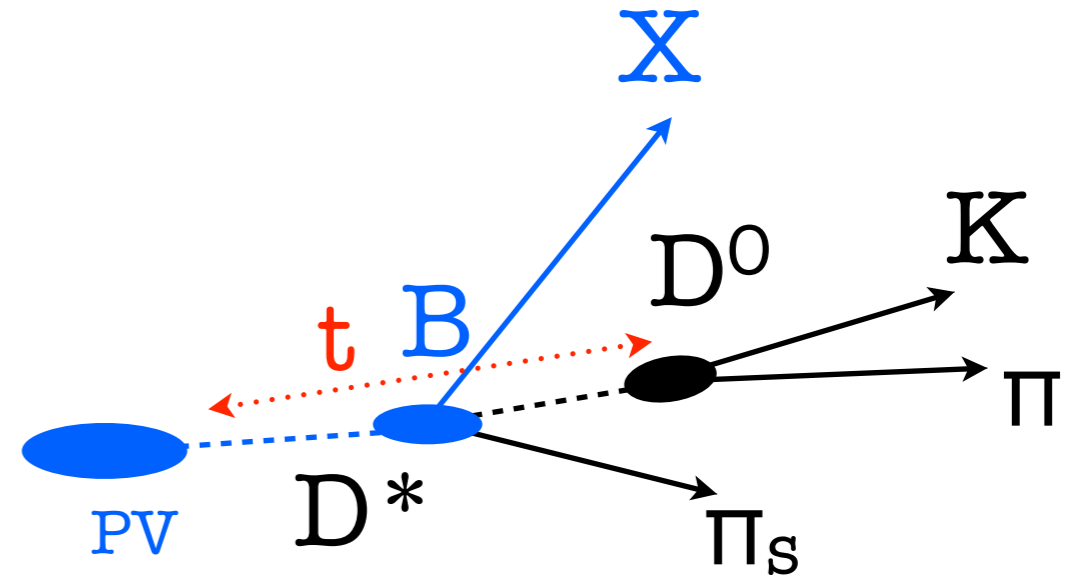
~9.7σ
expected

Measurements on pseudo-experiments indicate that the fit procedure is stable and free of any bias

Systematics

- Most systematic cancel in the ratio between WS and RS events
- The main sources of systematic uncertainty are those which could alter the observed decay-time dependence of the ratio:
 - charm mesons from b-hadron decays
 - backgrounds from mis-identified charm decays which peak in $M(D^0\pi_s)$
- These effects are expected to depend on the true value of the mixing parameters and are accounted for in the time-dependent fit

Secondary D decays



- D from B decays have wrong decay time
- Neglecting the secondary component could induce a time-dependent bias on the measured WS/RS ratio:

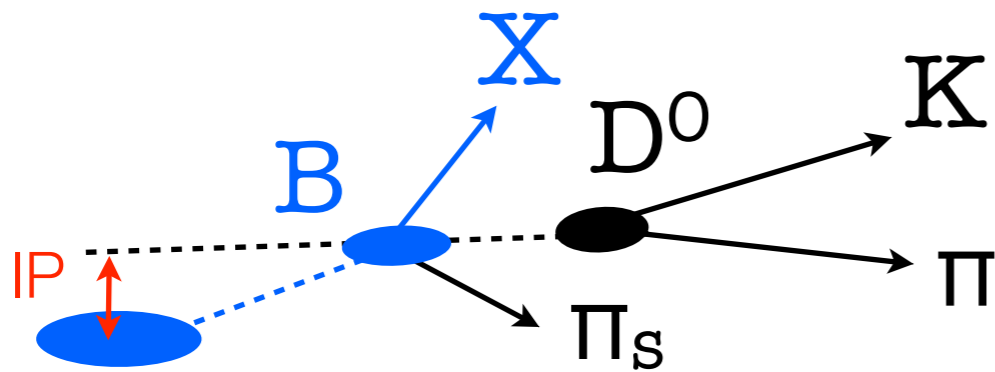
$$R^m(t) = \frac{N^{WS}(t) + N_B^{WS}(t)}{N^{RS}(t) + N_B^{RS}(t)} = R(t) \left\{ 1 - f_B^{RS}(t) \left[1 - \frac{R_B(t)}{R(t)} \right] \right\}$$

where

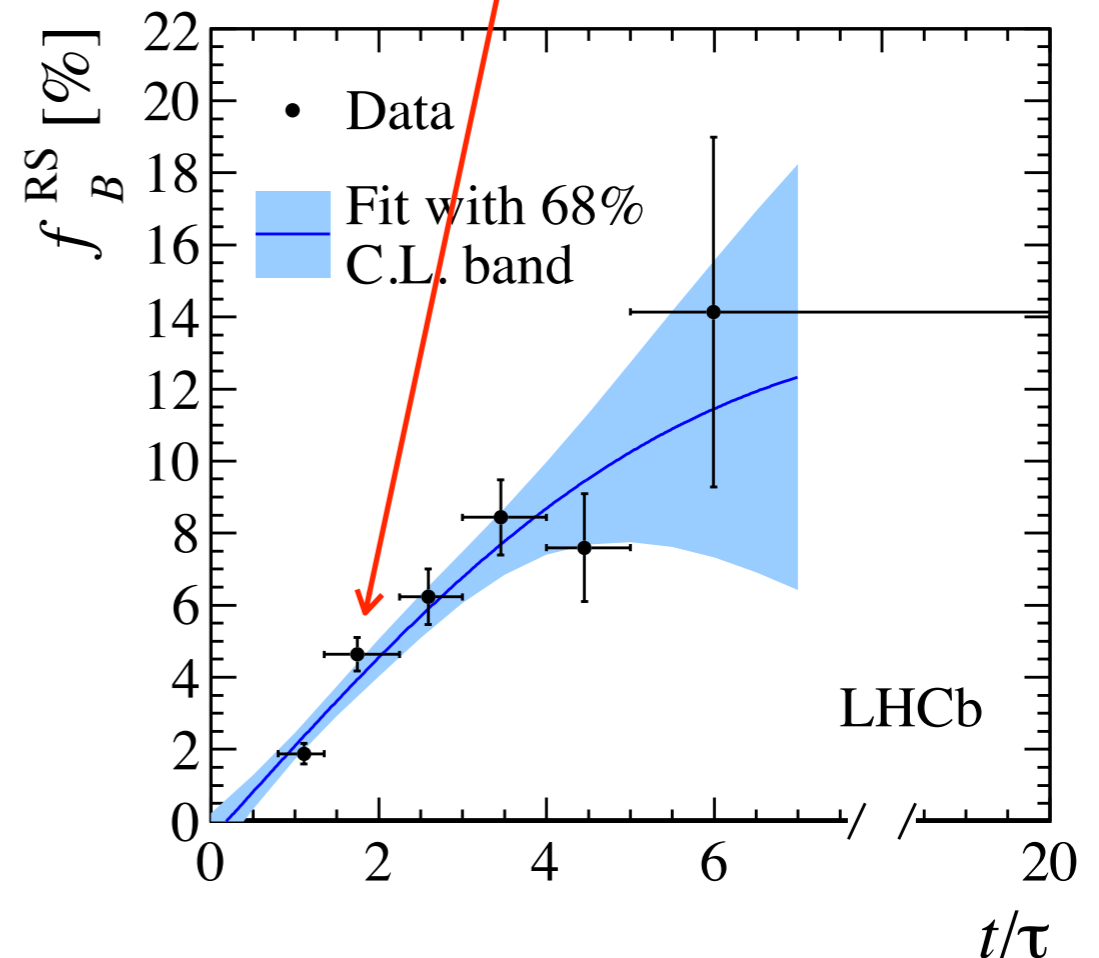
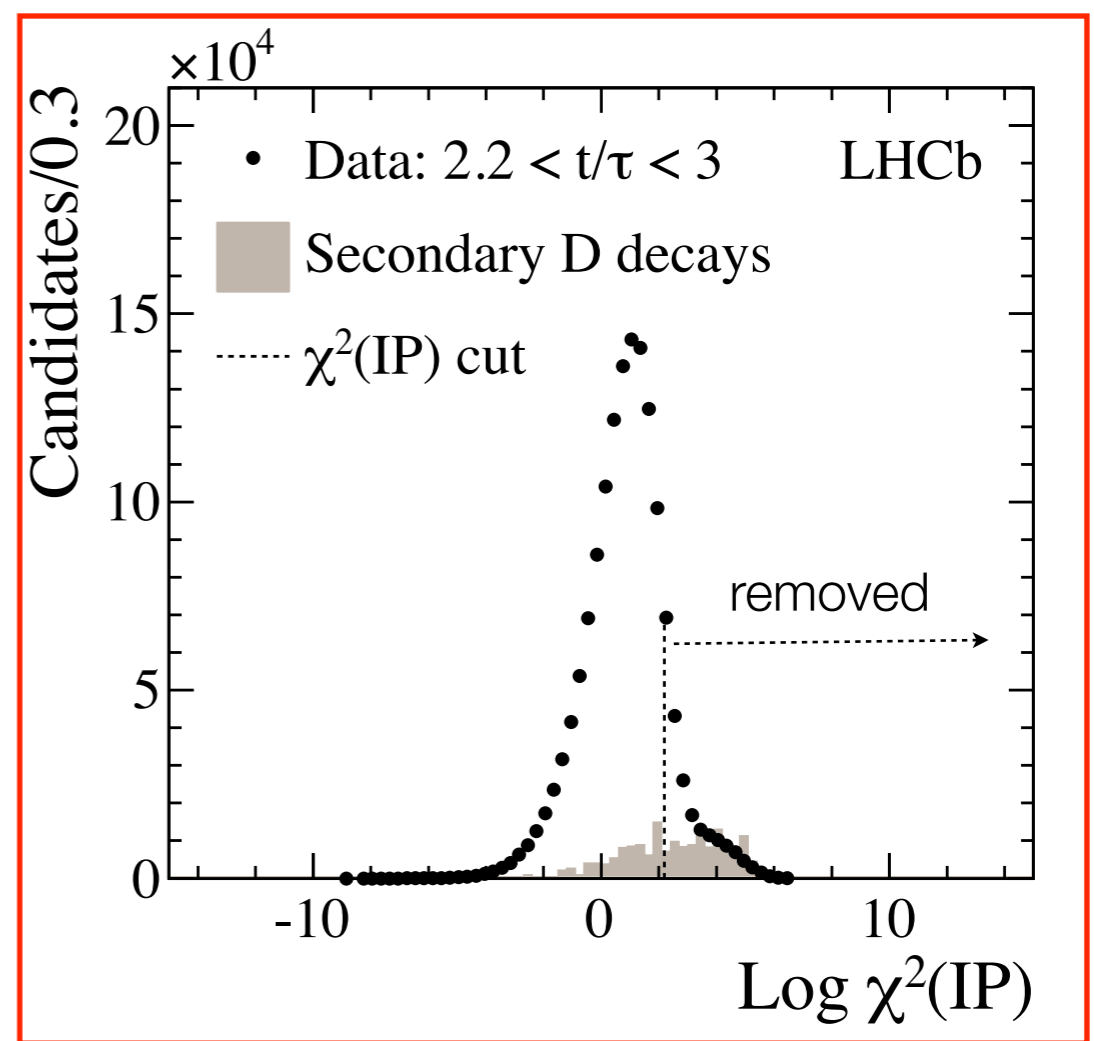
$$f_B^{RS}(t) = \frac{N_B^{RS}(t)}{N^{RS}(t) + N_B^{RS}(t)}, \quad R_B(t) = \frac{N_B^{WS}(t)}{N_B^{RS}(t)}$$

Measuring $f_B^{RS}(t)$

- $c\tau(B) \approx 450 \mu\text{m}$, D from B have non-zero impact parameter

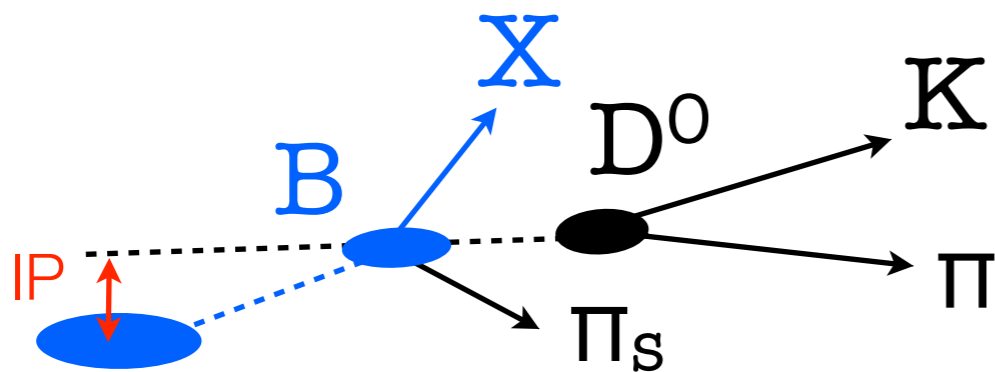


- Cut on $\chi^2(\text{IP})$ removes most of them but still $\sim 3\%$ of our candidates are likely to come from a B decay
- Fit $\log \chi^2(\text{IP})$ vs decay time and extrapolate fraction below cut
- Secondary shape estimated from events reconstructed also as $B \rightarrow D^*(3)\pi$, $B \rightarrow D^*\mu X$ or $B \rightarrow D^0\mu X$

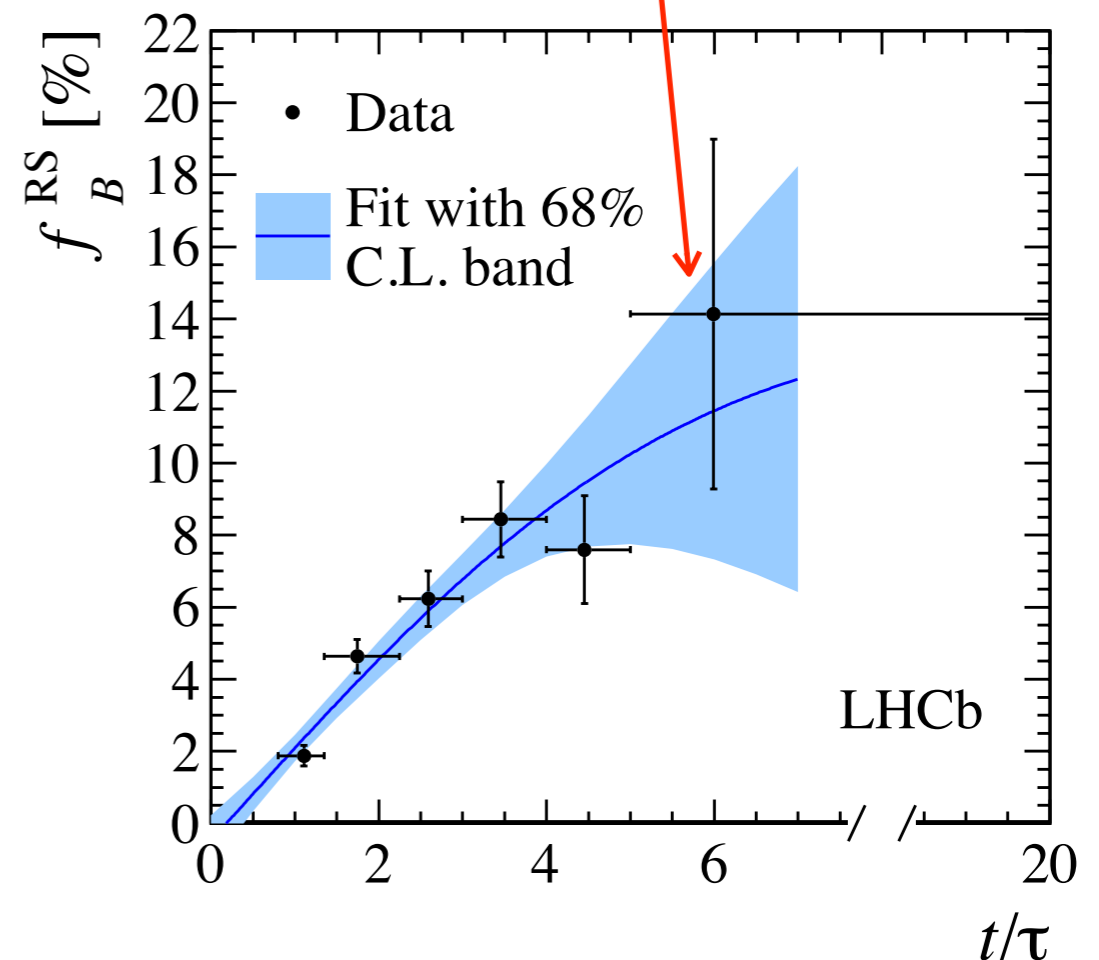
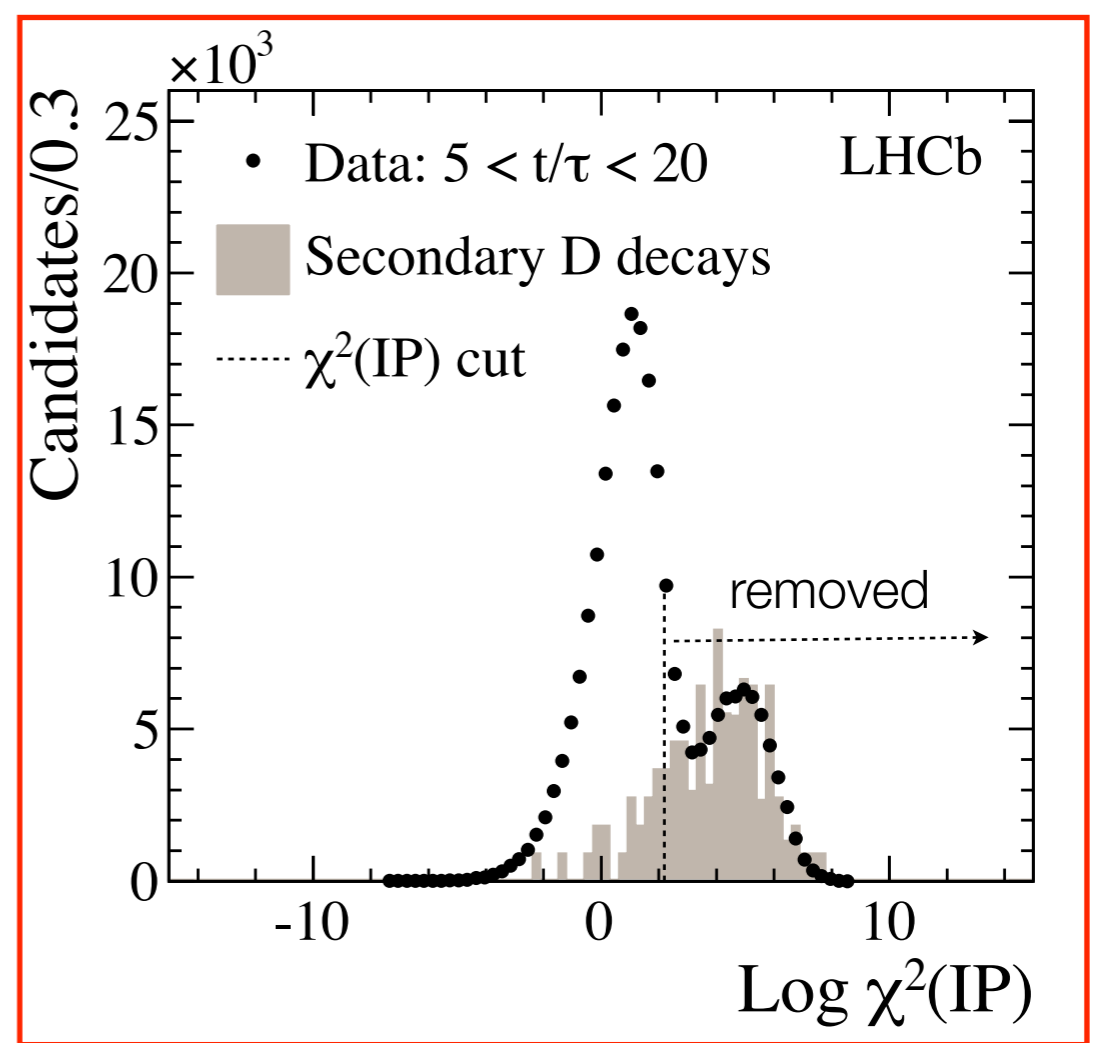


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Bias from secondary D decays

$$\Delta_B(t) = f_B^{RS}(t) \left[1 - \frac{R_B(t)}{R(t)} \right]$$

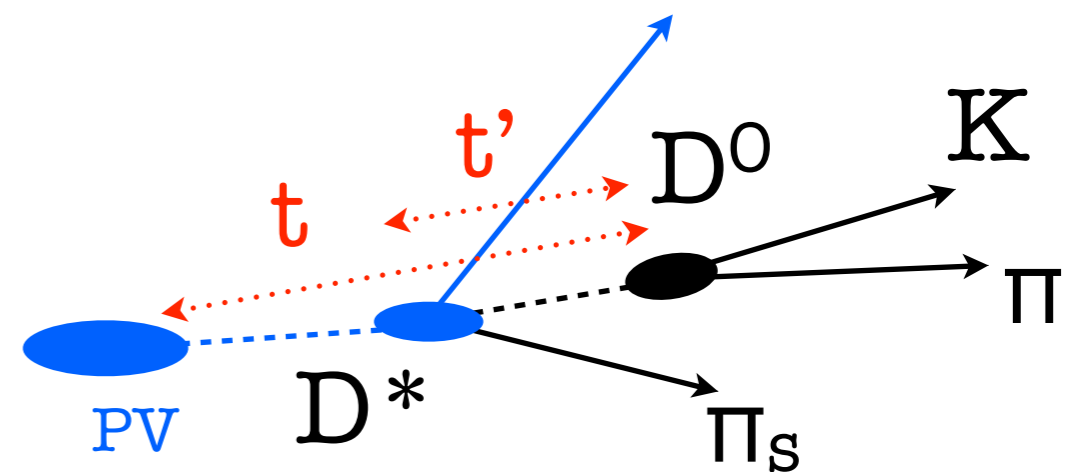
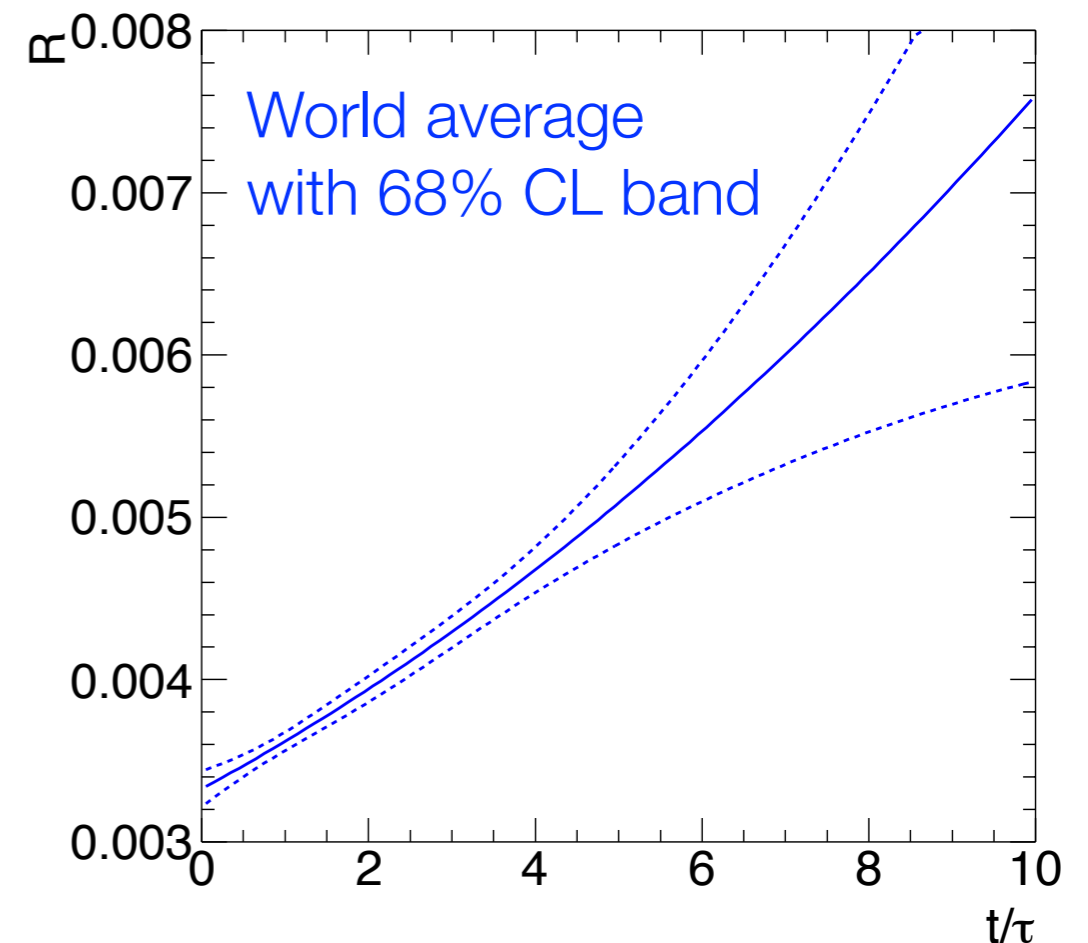
- $R(t)$ is a monotonic increasing function and for secondaries $t \geq t'$ (real decay time), then

$$R(0) = R_D \leq R_B(t) = R(t') \leq R(t)$$

- The bias is bounded by

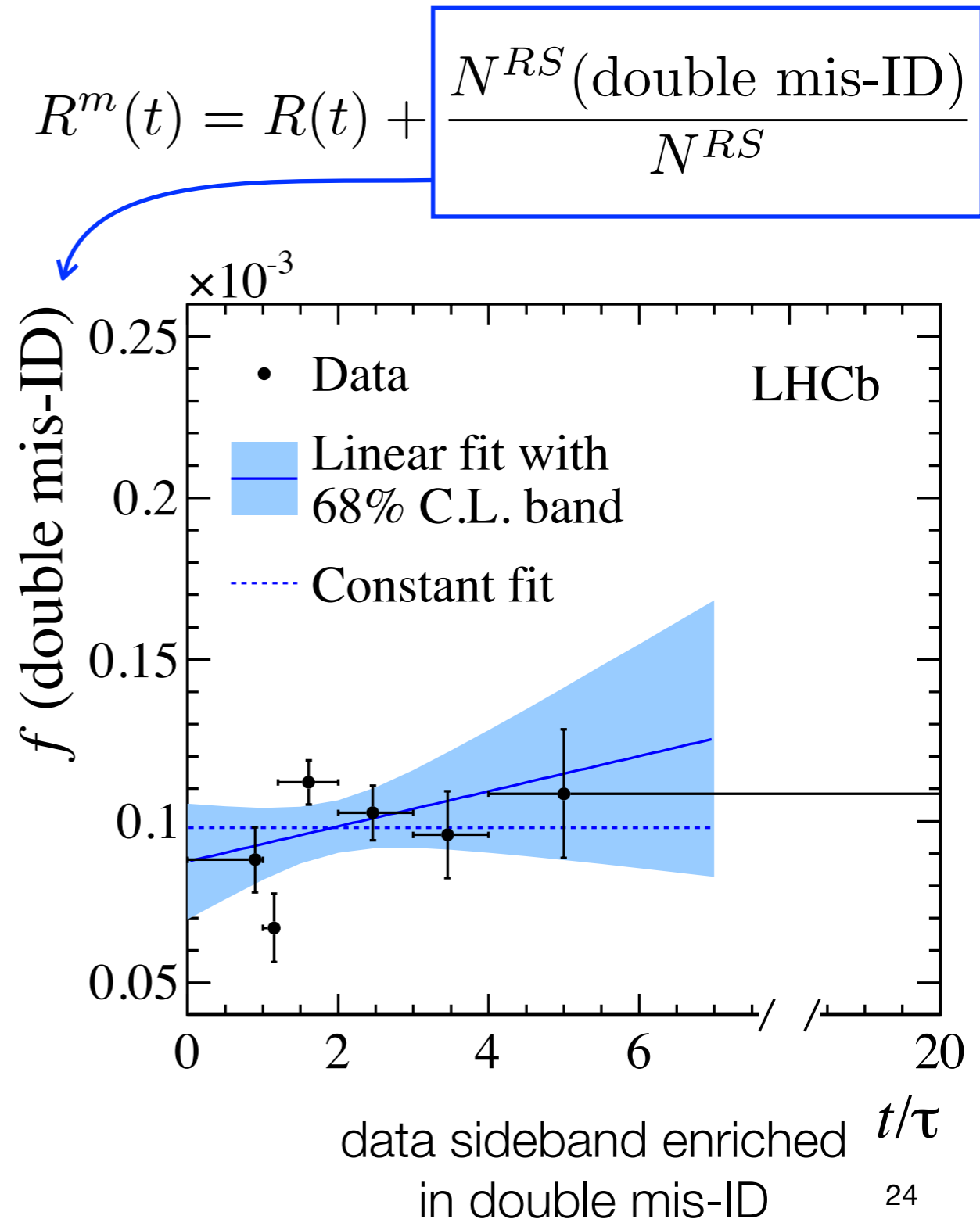
$$0 \leq \Delta_B(t) \leq f_B^{RS}(t) \left[1 - \frac{R_D}{R(t)} \right]$$

- The contamination fraction is small enough that we can assume the maximum bias in the time-dependent fit



Peaking background

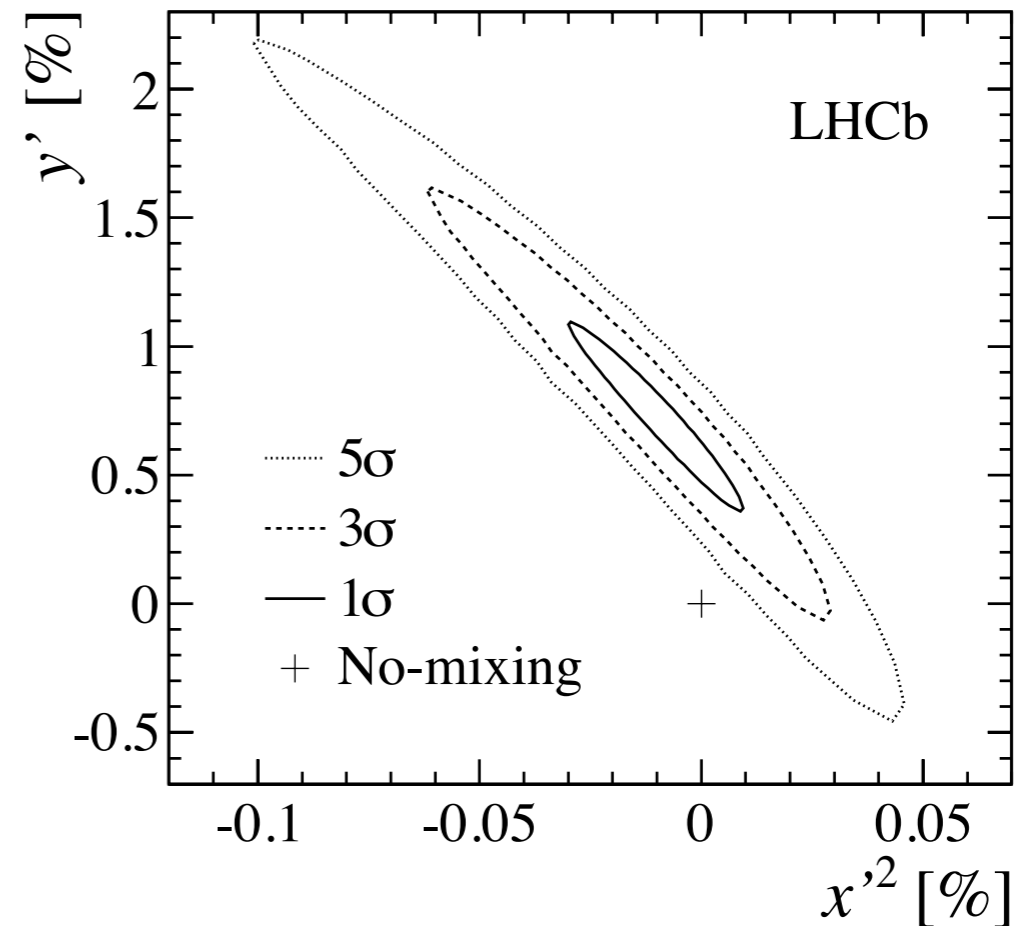
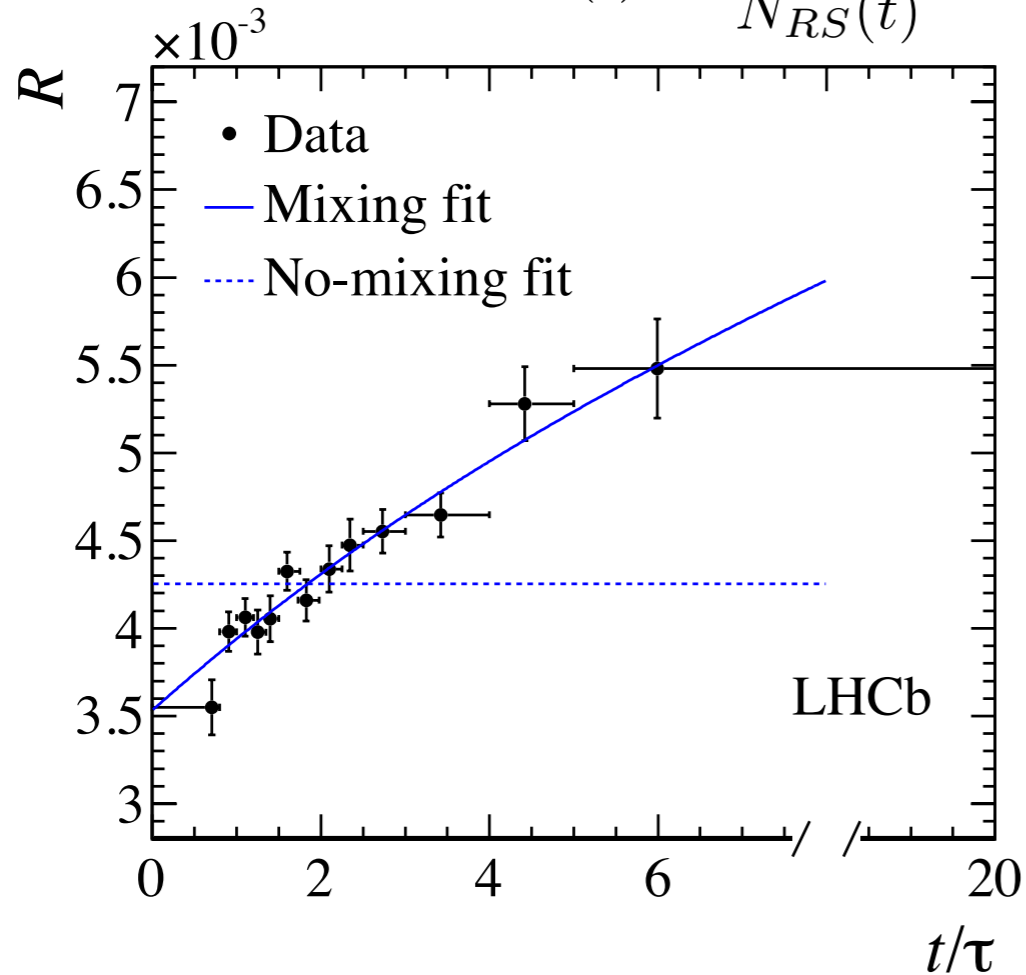
- Mass fits do not distinguish between signal and backgrounds which peak in $M(D^0\pi_s)$
- Such backgrounds are highly suppressed by tight PID cuts and reduced D^0 mass window
- Dominant residual contamination is from $(0.4 \pm 0.2)\%$ doubly mis-identified RS events in the WS sample
- (Un)observed time-dependence is included as a possible bias in the fit



Results

Results

$$R(t) = \frac{N_{WS}(t)}{N_{RS}(t)} = R_D + \sqrt{R_D}y't + \frac{x'^2 + y'^2}{4}t^2$$

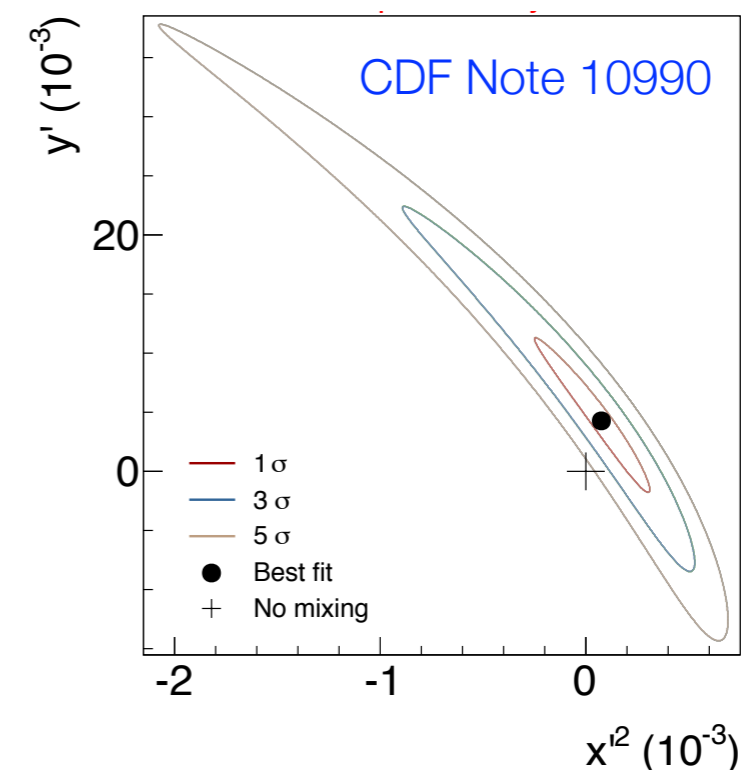
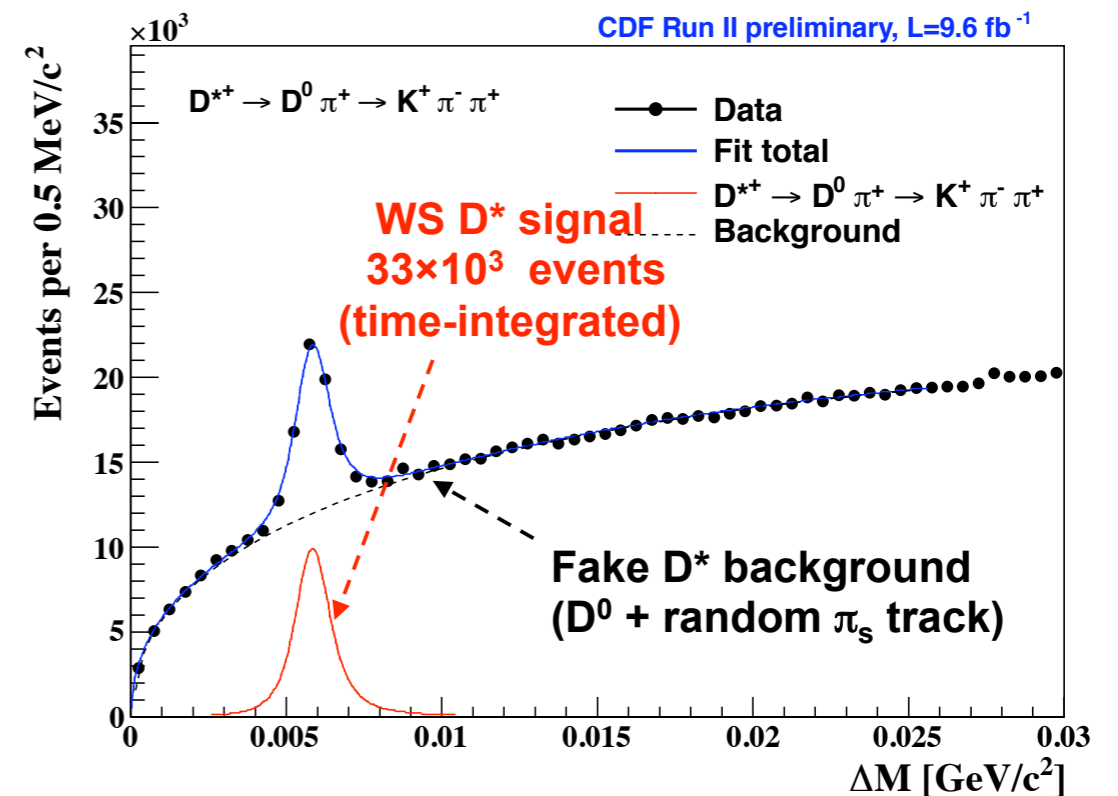


Fit type (χ^2/ndf)	Parameter	Fit result (10^{-3})	Correlation coefficient		
			R_D	y'	x'^2
Mixing (9.5/10)	R_D	3.52 ± 0.15	1	-0.954	+0.882
	y'	7.2 ± 2.4		1	-0.973
	x'^2	-0.09 ± 0.13			1
No mixing (98.1/12)	R_D	4.25 ± 0.04			

No-mixing
hypothesis
excluded at
9.1 σ

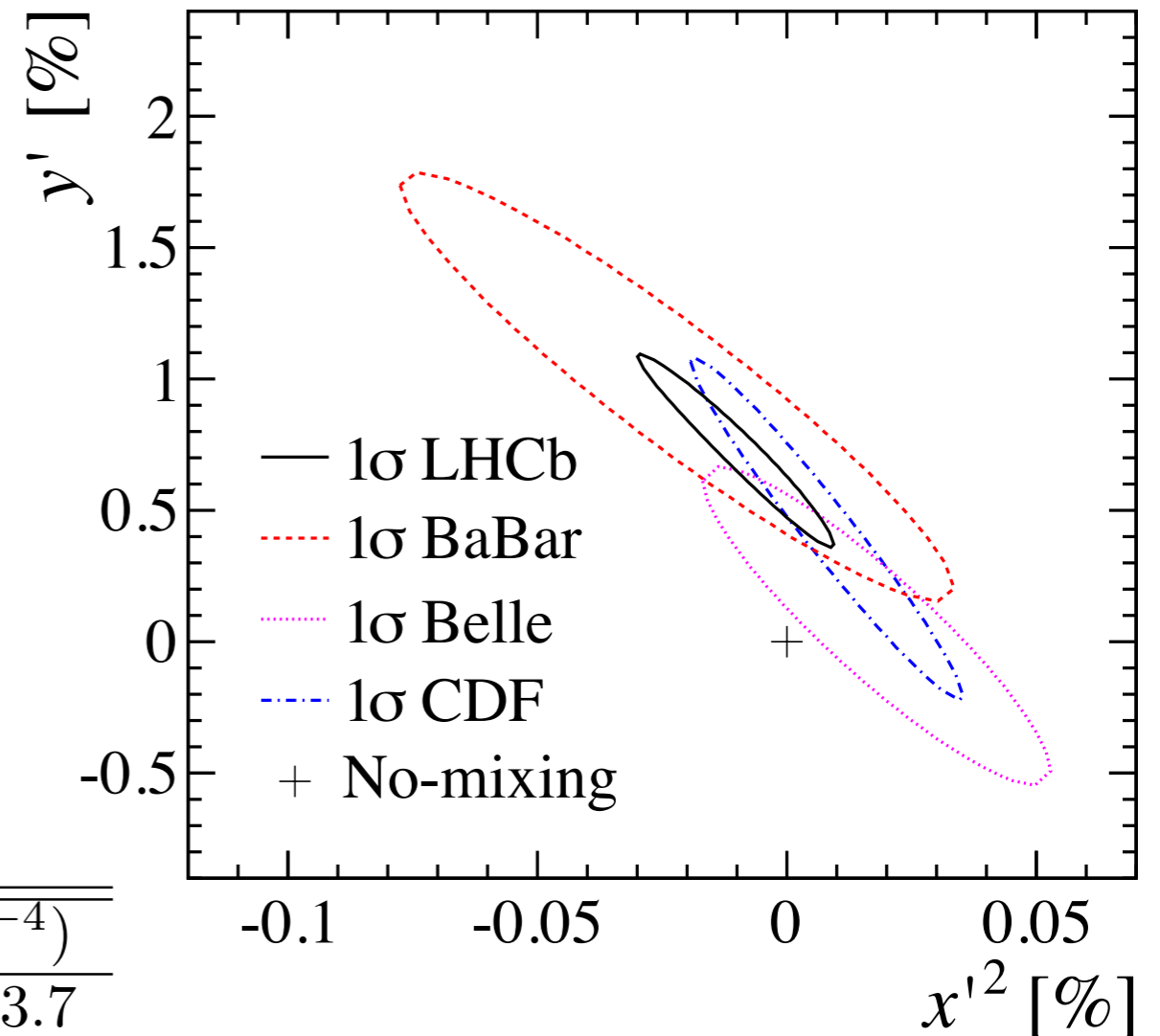
Few months after LHCb...

- ...also CDF presented an observation of charm mixing using WS $D^0 \rightarrow K^+ \pi^-$ decays
 - Full Tevatron Run II dataset
 - Signal yield comparable to LHCb, but worse signal-to-background ratio
 - No-mixing excluded at 6.1σ



Comparison between available measurements

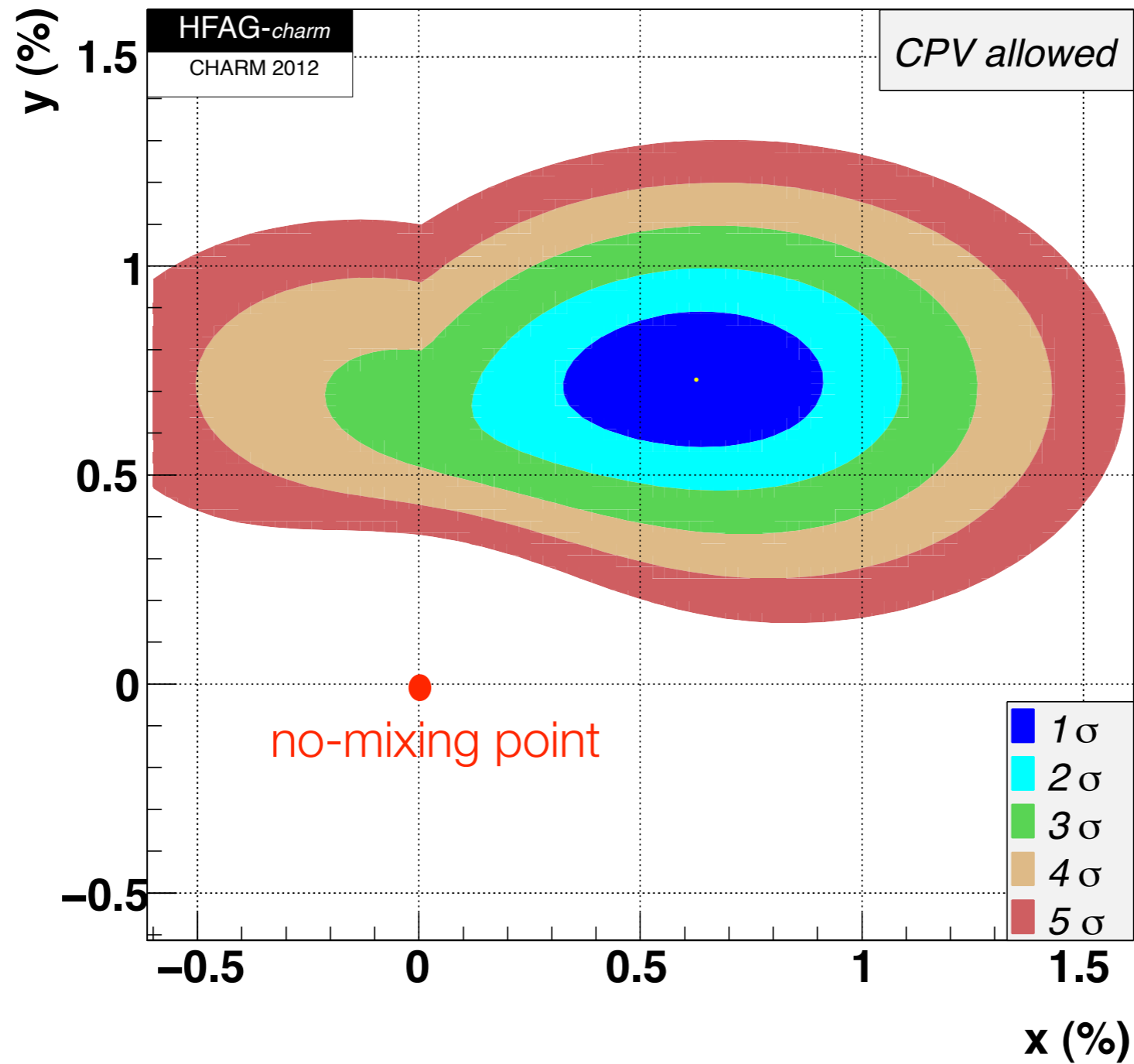
- LHCb measurement nicely agree with other experiments
- Results dominated by statistical uncertainties
 - Fit with no systematics estimates 6%, 10% and 11% smaller uncertainties on R_D , y' and x'^2 , respectively



Experiment	R_D (10^{-3})	y' (10^{-3})	x'^2 (10^{-4})
BaBar	3.03 ± 0.19	9.7 ± 5.4	-2.2 ± 3.7
Belle	3.64 ± 0.17	$0.6^{+0.4}_{-3.9}$	$1.8^{+2.1}_{-2.3}$
CDF	3.51 ± 0.35	4.3 ± 4.3	0.8 ± 1.8
LHCb	3.52 ± 0.15	7.2 ± 2.4	-0.9 ± 1.3

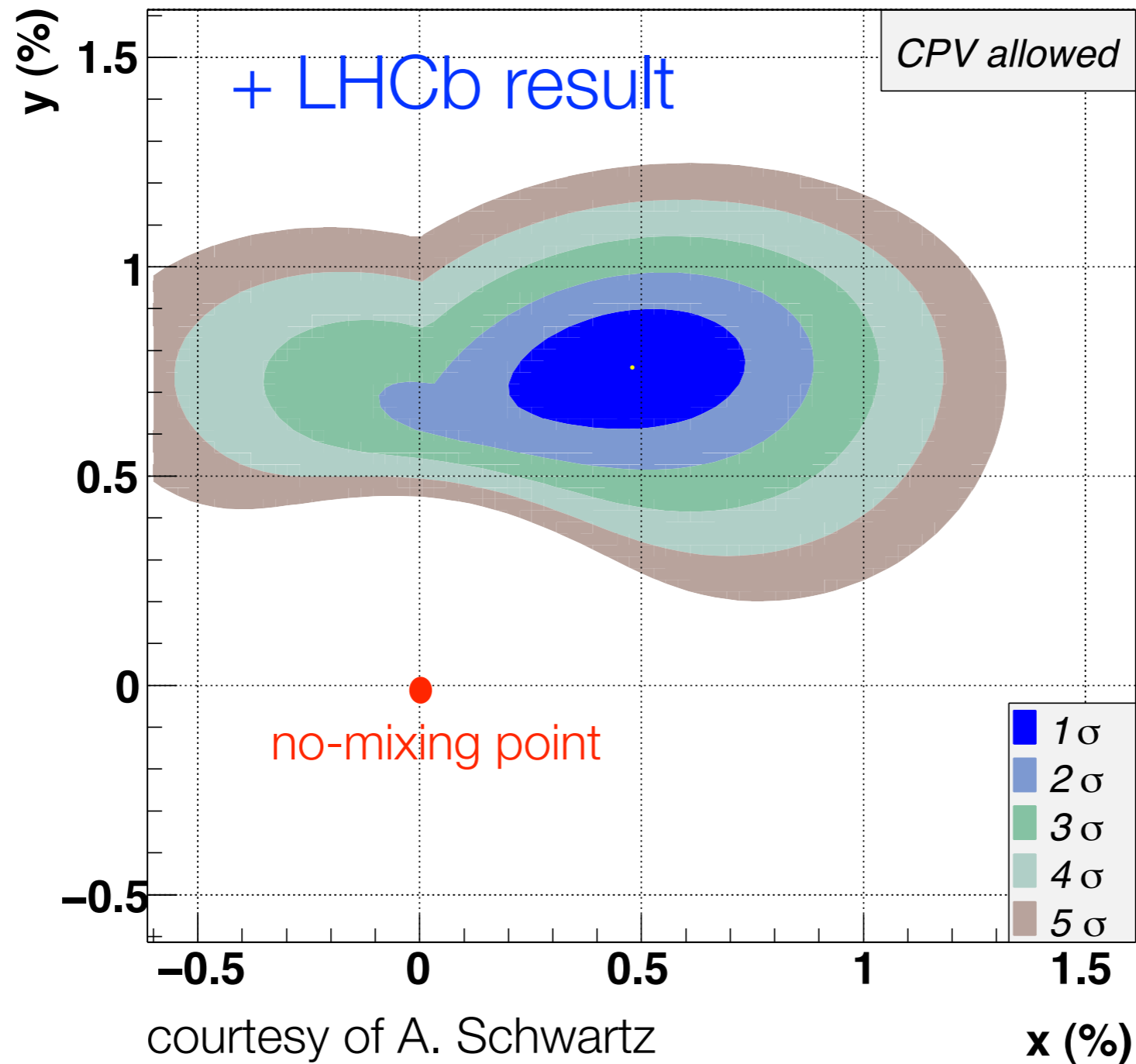
[BaBar: Phys. Rev. Lett. 98 \(2007\) 211802](#)
[Belle: Phys. Rev. Lett. 96 \(2006\) 151801](#)
[CDF: Public Note 10990](#)

Impact on world average



$$x = (0.63 \pm 0.19)\%$$
$$y = (0.73 \pm 0.11)\%$$

Impact on world average

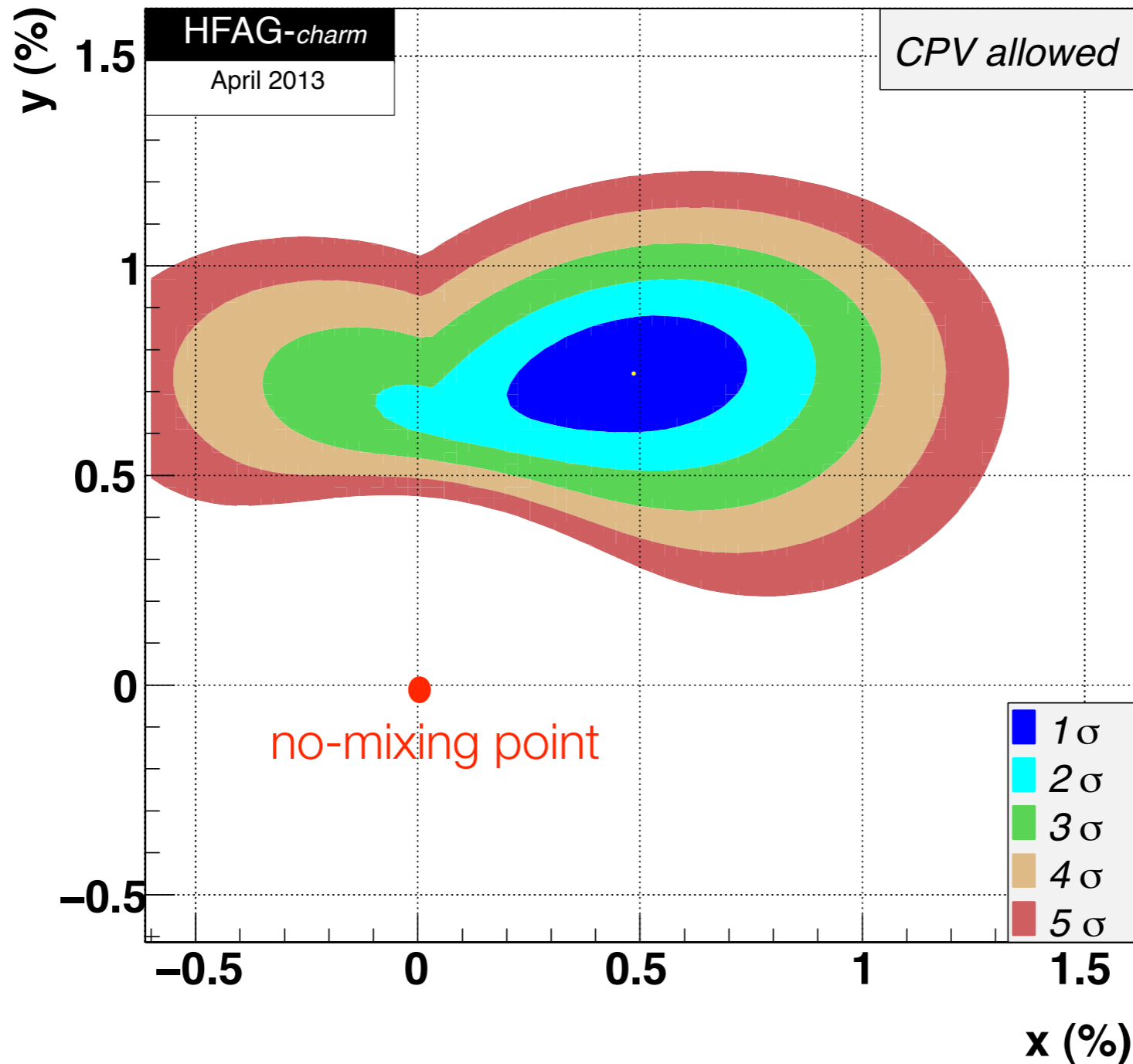


$$x = (0.63 \pm 0.19)\%$$
$$y = (0.73 \pm 0.11)\%$$

+ LHCb result

$$x = (0.48 \pm 0.18)\%$$
$$y = (0.76 \pm 0.10)\%$$

Impact on world average



$$x = (0.63 \pm 0.19)\%$$
$$y = (0.73 \pm 0.11)\%$$

+ LHCb result

$$x = (0.48 \pm 0.18)\%$$
$$y = (0.76 \pm 0.10)\%$$

Current HFAG average
(new CDF result not yet included)

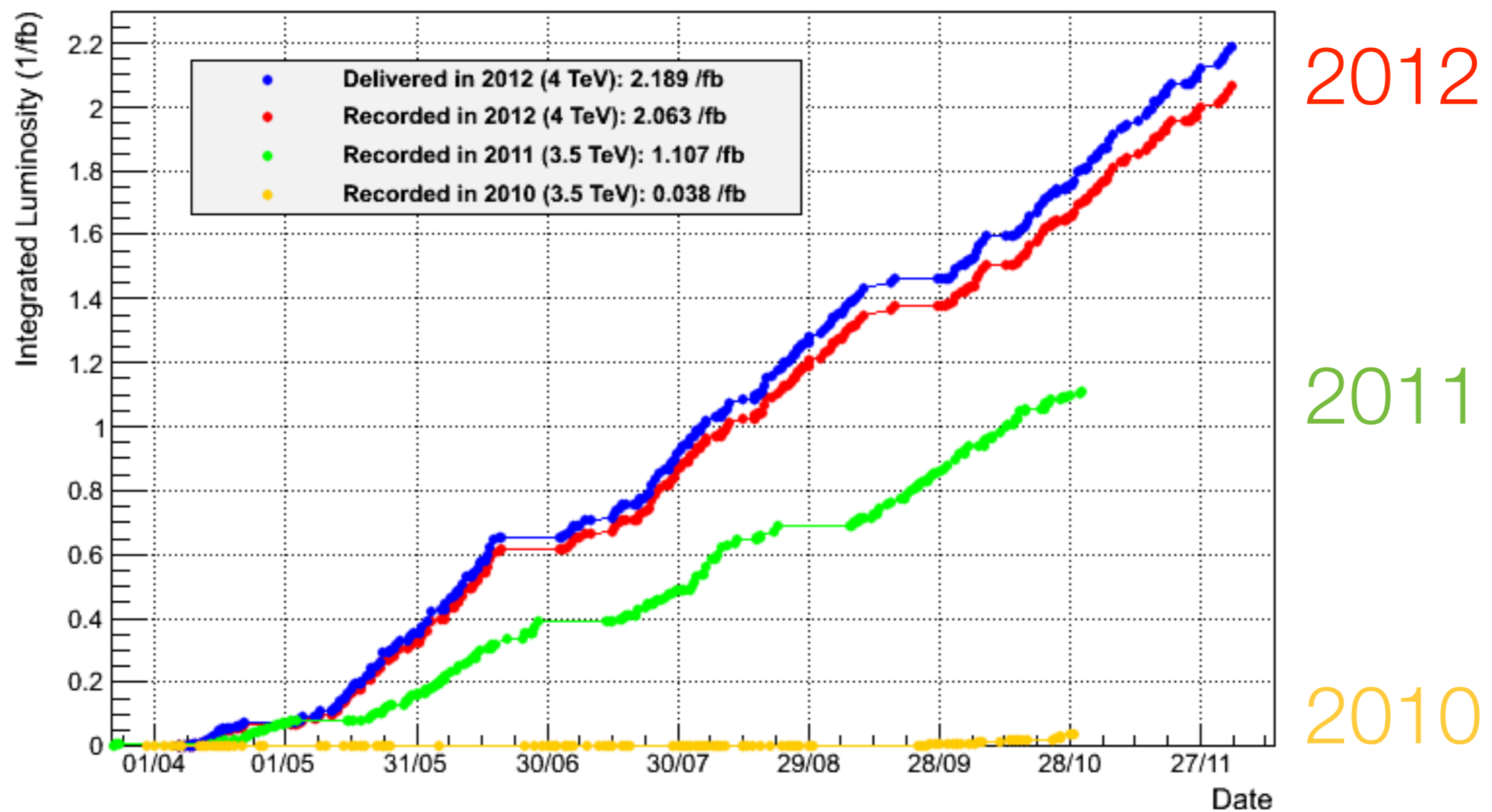
$$x = (0.49 \pm 0.18)\%$$
$$y = (0.74 \pm 0.09)\%$$

Conclusions

- Charm physics is a unique probe of beyond-standard model flavor effects, quite complementary to tests in K and B systems
 - It is quite plausible that new physics contributions affect mostly the up sector
- Presented the first observation of charm mixing from a single measurement using $D^0 \rightarrow K^+ \pi^-$ and $D^0 \rightarrow K^- \pi^+$ decays reconstructed in 1.0 fb^{-1} of LHCb data [[Phys. Rev. Lett. 110 \(2013\) 101802](#)]
 - The measured values of the mixing parameters are compatible with and have substantially better precision than those from other measurements
- LHCb will soon start challenging the standard model with many precision measurements of charm dynamics...

Expect more charm to come...

Recorded luminosity more than doubled during 2012 data-taking...
stay tuned for new results to come



Backup slides

Definition of $D^0\pi_s$ mass

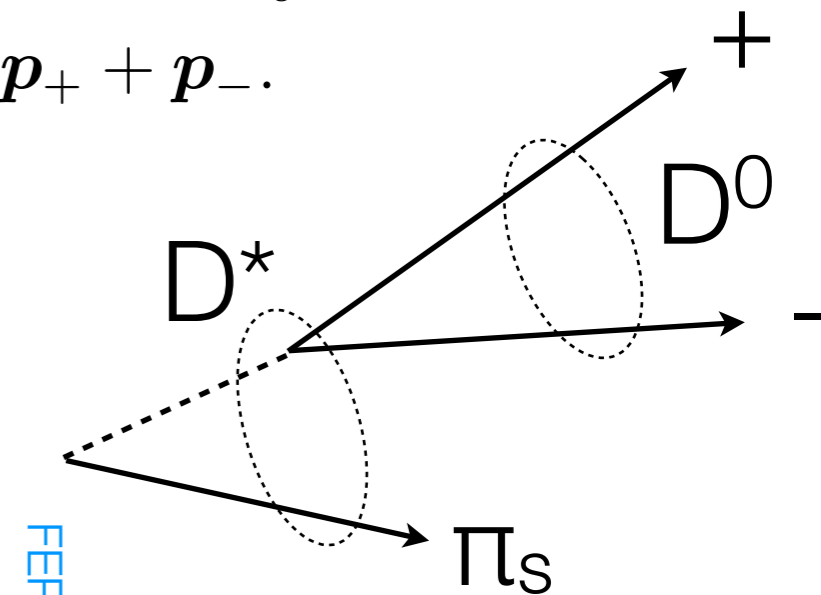
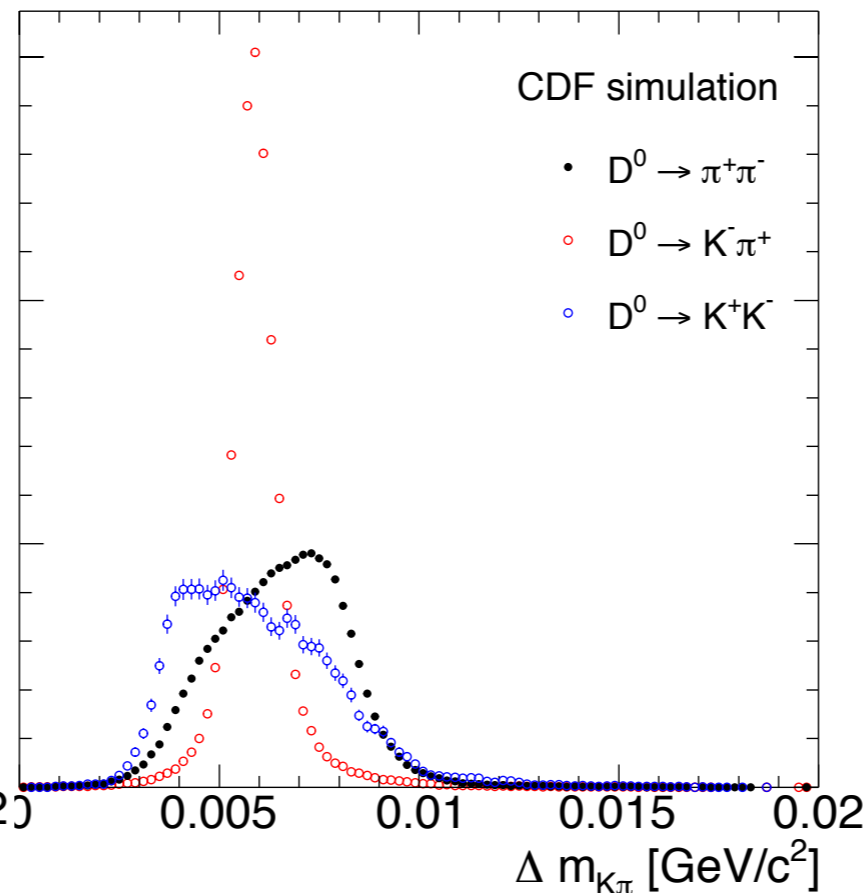
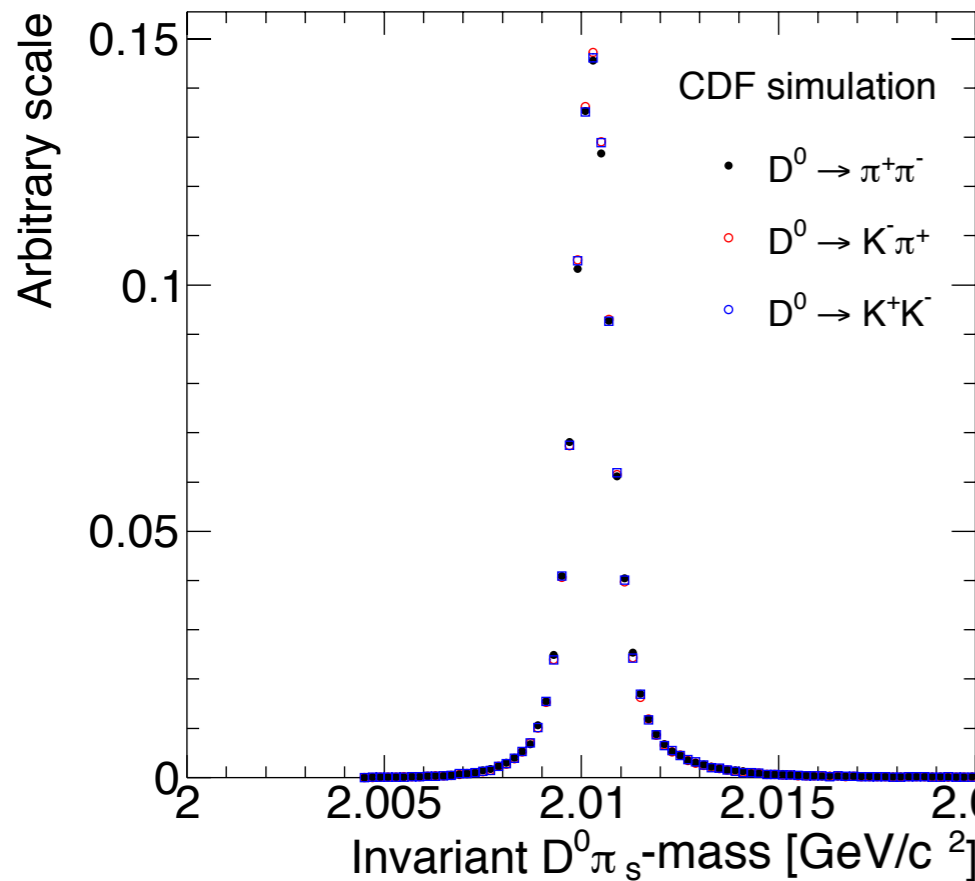
- No mass hypothesis for D^0 final state particle
- Equivalent to Δm when mass hypo is correct
- 2-body decays have same M_{D^*} but different Δm distributions

$$M(D^0\pi_s) = \sqrt{E_{D^*}^2 - p_{D^*}^2},$$

$$E_{D^*}^2 = (m_{D^0}^2 + p_{D^0}^2) + (m_{\pi}^2 + p_{\pi_s}^2),$$

$$\mathbf{p}_{D^*} = \mathbf{p}_{D^0} + \mathbf{p}_{\pi_s},$$

$$\mathbf{p}_{D^0} = \mathbf{p}_+ + \mathbf{p}_-.$$



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Other (neglected) systematics

	ΔR_D	$\Delta y'$	$\Delta x'^2$
Asymmetries in detection or production	$<0.001\sigma$	$<0.001\sigma$	$<0.001\sigma$
VELO length scale	0	0.003σ	0.001σ
Multiple candidates	0.02σ	0.06σ	0.07σ

Experimental assumption

- The acceptance/efficiency for WS events is the same as for RS
- valid up to terms that are quadratic in detection/production asymmetries:

$$\begin{aligned} R^{\text{obs}} &= \frac{N_{WS+}^{\text{obs}} + N_{WS-}^{\text{obs}}}{N_{RS+}^{\text{obs}} + N_{RS-}^{\text{obs}}} \\ &= R \frac{(1 + A_P)(1 - \delta_{K\pi})(1 + \delta_{\pi_s}) + (1 - A_P)(1 + \delta_{K\pi})(1 - \delta_{\pi_s})}{(1 + A_P)(1 + \delta_{K\pi})(1 + \delta_{\pi_s}) + (1 - A_P)(1 - \delta_{K\pi})(1 - \delta_{\pi_s})} \\ &\approx R (1 - 2A_P\delta_{K\pi} - 2\delta_{K\pi}\delta_{\pi_s} + \text{products of four asymmetries}), \end{aligned}$$

- corrections are $O(10^{-4})$ then completely negligible

New HFAG average

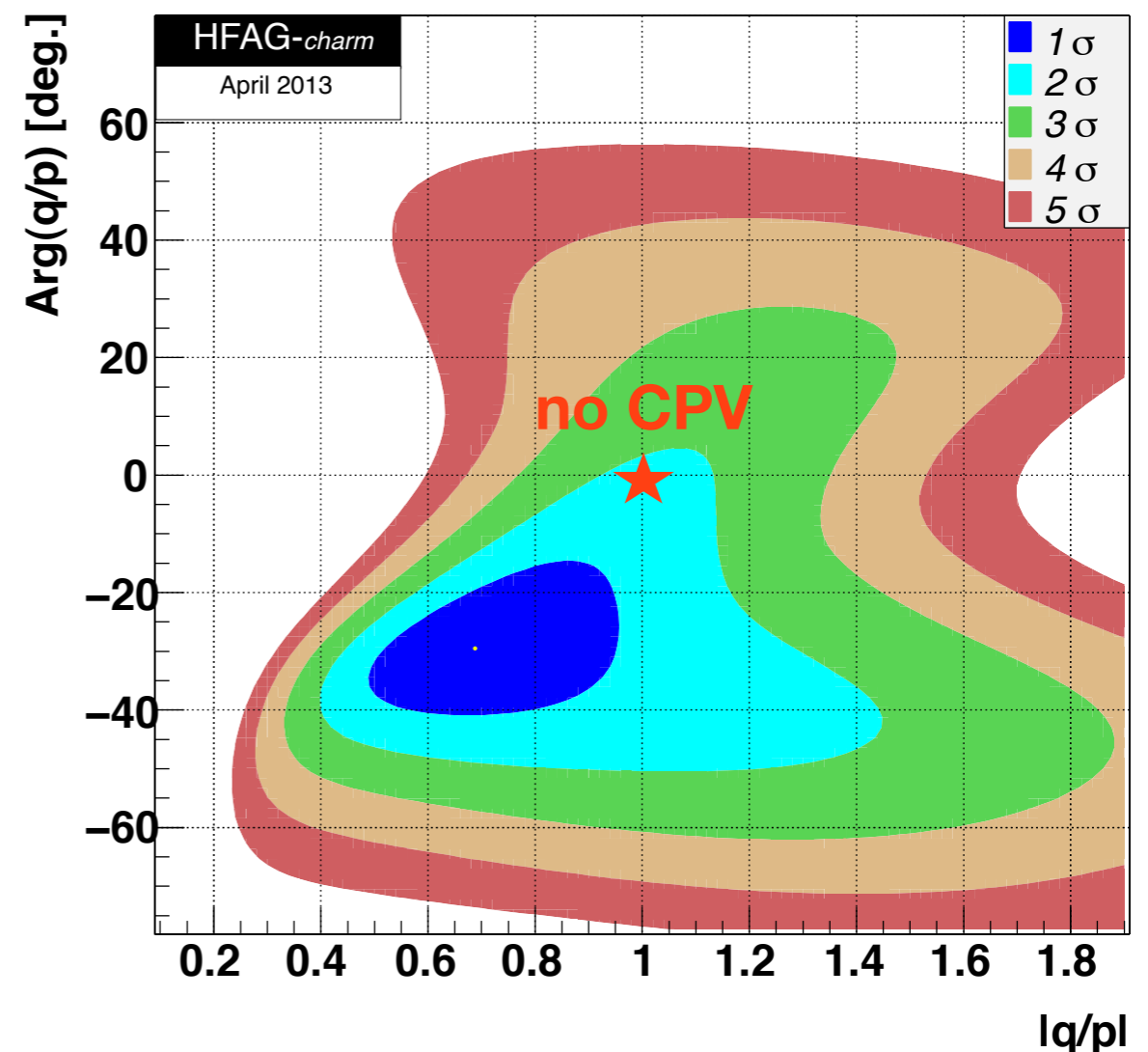
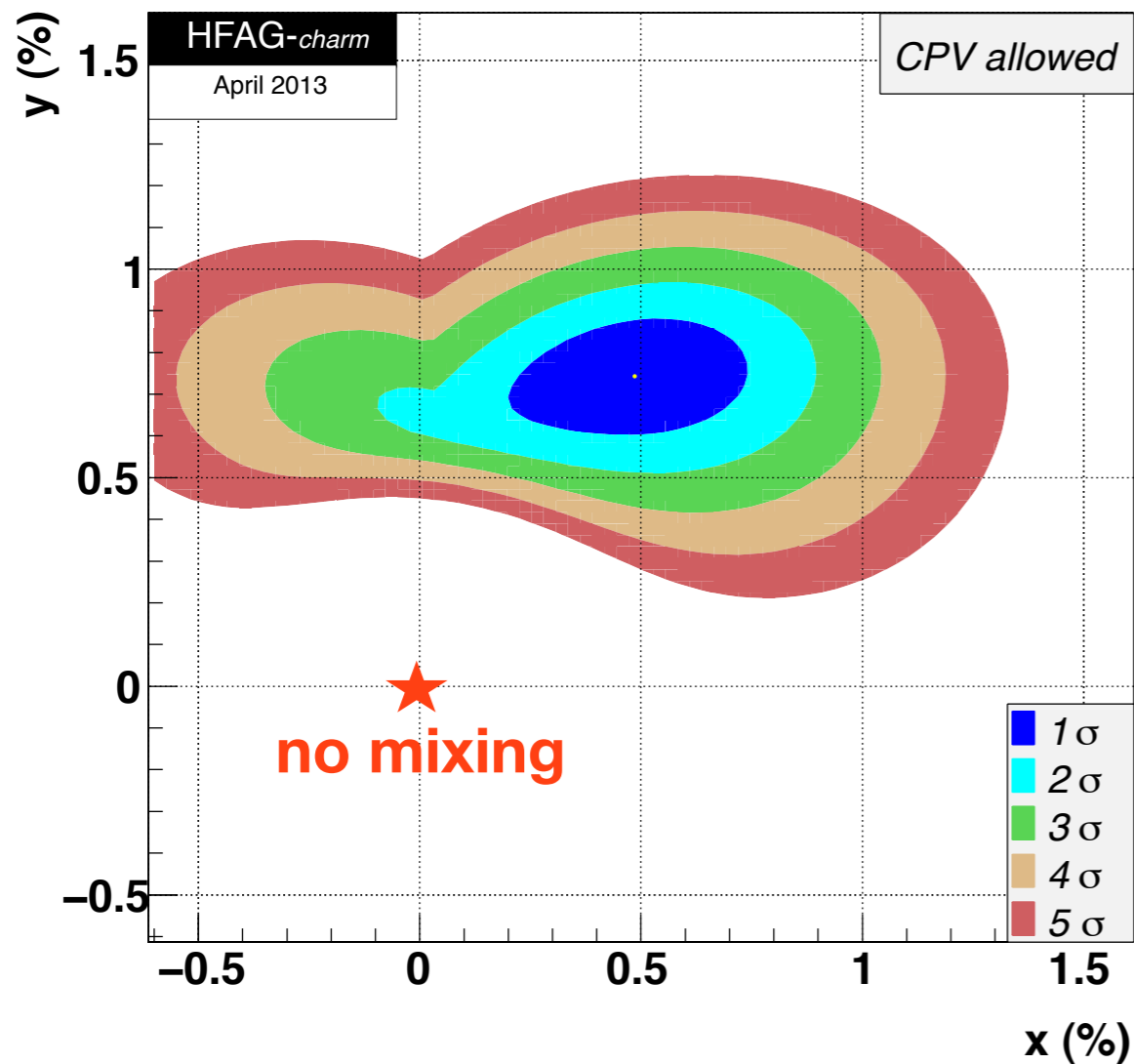
NB: new CDF result not yet included

$$x = (0.49^{+0.17}_{-0.18})\%$$

$$y = (0.74 \pm 0.09)\%$$

$$|q/p| = (0.69^{+0.17}_{-0.14})\%$$

$$\phi = (-29.6^{+8.9}_{-7.5})^\circ$$



New HFAG average

NB: new CDF result not yet included

Parameter	No CPV	No direct CPV	CPV -allowed	CPV -allowed 95% C.L.
x (%)	$0.49^{+0.17}_{-0.18}$	0.46 ± 0.18	$0.49^{+0.17}_{-0.18}$	[0.10, 0.81]
y (%)	0.66 ± 0.09	0.67 ± 0.09	0.74 ± 0.09	[0.56, 0.92]
δ ($^\circ$)	$10.8^{+10.3}_{-12.3}$	$11.4^{+10.5}_{-12.7}$	$19.5^{+8.6}_{-11.1}$	[-9.6, 35.4]
R_D (%)	0.347 ± 0.006	0.347 ± 0.006	$0.350^{+0.007}_{-0.006}$	[0.337, 0.362]
A_D (%)	—	—	-2.6 ± 2.2	[-6.9, 1.7]
$ q/p $	—	$1.04^{+0.07}_{-0.06}$	$0.69^{+0.17}_{-0.14}$	[0.44, 1.07]
ϕ ($^\circ$)	—	$-1.6^{+2.4}_{-2.5}$	$-29.6^{+8.9}_{-7.5}$	[-44.6, -7.5]
$\delta_{K\pi\pi}$ ($^\circ$)	$21.3^{+23.4}_{-23.8}$	$22.9^{+23.7}_{-24.0}$	$25.1^{+22.3}_{-23.0}$	[-20.6, 69.2]
A_π	—	—	0.16 ± 0.21	[-0.25, 0.57]
A_K	—	—	-0.16 ± 0.20	[-0.56, 0.23]
x_{12} (%)	—	0.46 ± 0.18	—	[0.10, 0.80]
y_{12} (%)	—	0.67 ± 0.09	—	[0.50, 0.85]
ϕ_{12} ($^\circ$)	—	$4.8^{+9.2}_{-7.4}$	—	[-11.7, 35.9]

Time-dependent fit configuration

$$\chi^2(r_i, t_i, \sigma_i | \boldsymbol{\theta}) = \sum_i \left(\frac{r_i - R(t_i)[1 - \Delta_B(t_i)] - R_p(t_i)}{\sigma_i} \right)^2 + \chi_B^2 + \chi_p^2,$$

$$R(t) = R_D + \sqrt{R_D} y' t + \frac{x'^2 + y'^2}{4} t^2,$$

$$\Delta_B(t) = f_B^{RS}(t) \left[1 - \frac{R_D}{R(t)} \right],$$

$$f_B^{RS}(t) = B_0 \operatorname{Erf}(B_1 t + B_2) \times 10^{-2},$$

$$\chi_B^2 = (B_0 - b_0 \quad B_1 - b_1 \quad B_2 - b_2) \mathbf{V}_B^{-1} \begin{pmatrix} B_0 - b_0 \\ B_1 - b_1 \\ B_2 - b_2 \end{pmatrix},$$

$$R_p(t) = P_0(1 + P_1 t) \times 10^{-2},$$

$$\chi_p^2 = (P_0 - p_0 \quad P_1 - p_1) \mathbf{V}_p^{-1} \begin{pmatrix} P_0 - p_0 \\ P_1 - p_1 \end{pmatrix},$$

Cross-checks

- We perform the measurement in statistically independent sub-samples of the data and find consistent results
 - different data-taking periods,
 - magnet polarities,
 - number of reconstructed primary vertices
- Also use alternative decay-time binning schemes or alternative fit methods to separate signal and background, and find no significant variations in the estimated mixing parameters