Forecasting of 21cm Intensity Mapping

1. Visibility and Correlation

First of all, let's assume the small-angle approximation. For a pair of receiver $\alpha = (i, j)$, the visibility $V_\alpha(\mathbf{u}_\parallel)$ could be expressed as

$$V_\alpha = V_{ij} = \int df\ e^{2\pi i f u_\parallel}\ V_{ij}(f) = \int df\ d^2\hat{\mathbf{n}}\ e^{2\pi i\ f u_\parallel} e^{2\pi i\ \hat{\mathbf{n}}\cdot\mathbf{u}_{ij}} A_i(\hat{\mathbf{n}}, f) A_j(\hat{\mathbf{n}}, f)\ T(\hat{\mathbf{n}}, f), \tag{1}$$

where $A_i(\hat{\mathbf{n}}, f)$ is the response pattern of the $i-$th receiver, and $T(\hat{\mathbf{n}}, f)$ is the temperature of the sky. If we define $A_\alpha(\hat{\mathbf{n}}, f)$

$$A_\alpha(\hat{\mathbf{n}}, f) = A_{ij}(\hat{\mathbf{n}}, f) = A_i(\hat{\mathbf{n}}, f) A_j(\hat{\mathbf{n}}, f) \tag{2}$$

Then equation (1) could be expressed as

$$V_\alpha = \int df\ d^2\hat{\mathbf{n}} e^{2\pi i [f u_\parallel + \hat{\mathbf{n}}\cdot\mathbf{u}_\alpha]} A_\alpha(\hat{\mathbf{n}}, f) T(\hat{\mathbf{n}}, f), \tag{3}$$

Therefore, for the signal part, the observed visibility is a 3D Fourier transformation of temperature convolved with window function A_α. Writing in more compact form, define 3 vector $\mathbf{\Theta} = \{n_x, n_y, f\}$, and its Fourier conjugate $\mathbf{U} = \{u_x, u_y, u_\parallel\}$, therefore above definition could be simplified as

$$\begin{aligned} V_\alpha(u_\parallel) &= \int d^3\Theta\ e^{2\pi i\ \mathbf{\Theta}\cdot\mathbf{U}}\ A_\alpha(\mathbf{\Theta})\ T(\mathbf{\Theta}) \\ &= \int d^3\mathbf{K}\ \widetilde{A}_\alpha(\mathbf{U} - \mathbf{K})\ \widetilde{T}(\mathbf{K}). \end{aligned} \tag{4}$$

Then, one is interested in the covariance matrix of visibility

$$C^s_{\alpha\beta} = \langle V_\alpha V^*_\beta \rangle = \int d^3K d^3K' \ \left[\widetilde{A}_\alpha(\mathbf{U}_\alpha - \mathbf{K}) \widetilde{A}^*_\beta(\mathbf{U}_\beta - \mathbf{K}')\right]\ \langle \widetilde{T}(\mathbf{K}) \widetilde{T}^*(\mathbf{K}')\rangle, \tag{5}$$

which further related to the temperature power spectrum

$$P_T(\mathbf{k}) = (2\pi)^3 \delta_D(\mathbf{k} - \mathbf{k}') \langle \widetilde{T}(\mathbf{k}) \widetilde{T}^*(\mathbf{k}')\rangle. \tag{6}$$

Please note the difference between $u - v - u_\parallel$ space $\mathbf{K}$ and the Fourier space $\mathbf{k}$.

One further simplify

$$C^s_{\alpha\beta} = \int d^3K\ \left[\widetilde{A}_\alpha(\mathbf{U}_\alpha - \mathbf{K}) \widetilde{A}^*_\beta(\mathbf{U}_\beta - \mathbf{K}')\right]\ \frac{1}{d_A^2(z) y(z)} P_{\Delta T}(d_A k_\perp, y k_\parallel; z) \tag{7}$$

and $y(z)$

$$y(z) = \frac{\lambda_{21}(1+z)^2}{H(z)} \tag{8}$$

convert the frequency to distance.

For the noise part, it will only has diagonal contributions

$$C^n_{\alpha\beta} = \left(\frac{2k_B T_{sys}}{\eta_A A_D}\right)^2 \frac{1}{\Delta f t_{int} n_b} \delta_{\alpha\beta}, \tag{9}$$

where T_{sys} the sysmtem temperature, η_A aperture efficiency, A_D the physical area, n_b the number of each baseline, and t_{int} the integration time.

2. Fisher Matrix

Once we have correlation function of observed data, the Fisher matrix could be simply expressed in the standard form

$$F_{ij} = \mathrm{tr}\left[\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial p_i}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial p_j}\right] \tag{10}$$

Since the correlation among different frequency bin are tiny, it could be simplified

$$F_{ij} = \sum_f F^f_{ij} = \sum_f \mathrm{tr}\left[\mathbf{C}_\mathrm{f}^{-1}\frac{\partial \mathbf{C}_\mathrm{f}}{\partial p_i}\mathbf{C}_\mathrm{f}^{-1}\frac{\partial \mathbf{C}_\mathrm{f}}{\partial p_j}\right] \tag{11}$$

3. Beyond the Small-angle Approximation

The use of Fourier transformation is only valid for small angle approximation. To extend, one could either use spherical harmonic, or as Tegmark suggested, define visibility as the Fourier transformation of modifed window function.

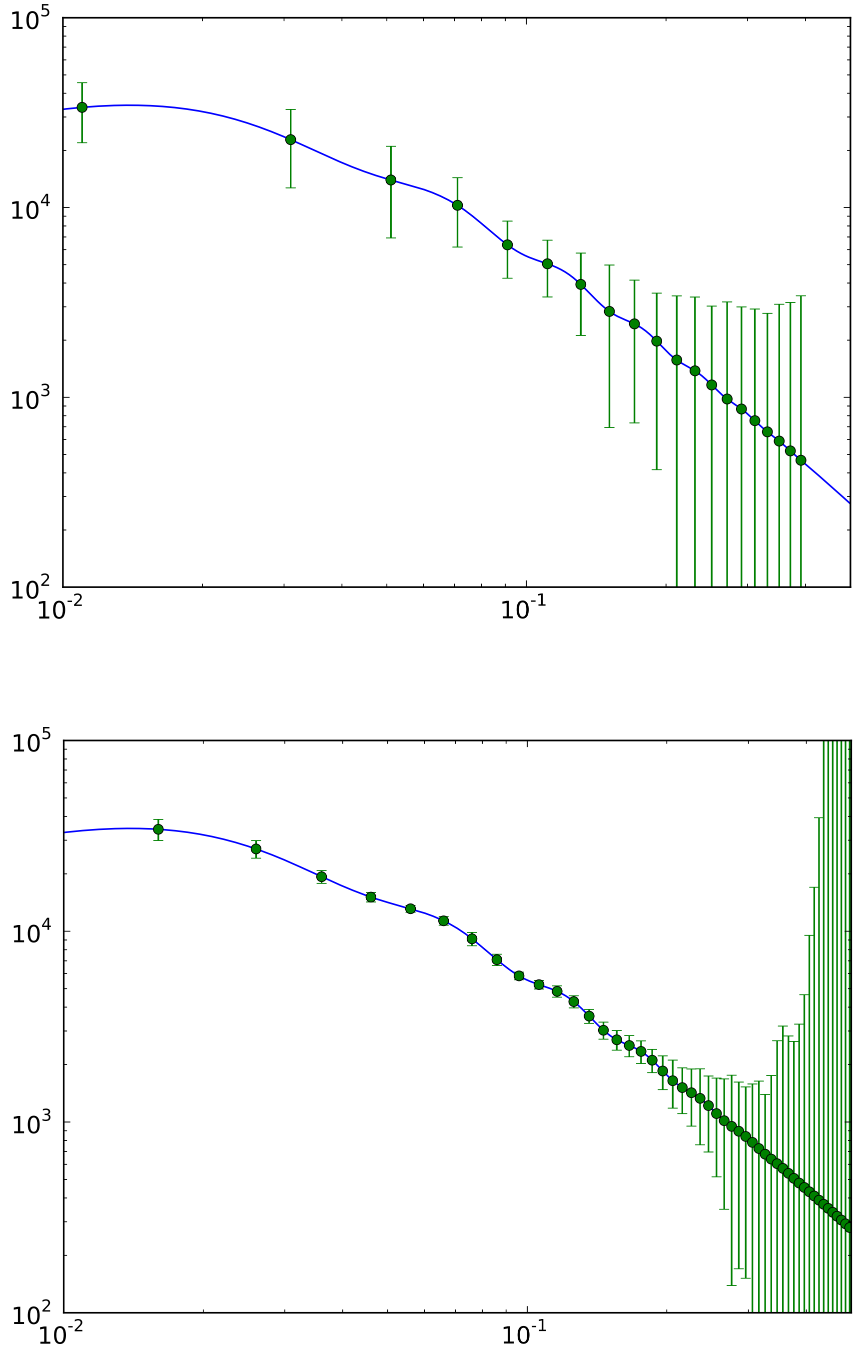

Fig. 1.— Error of band power, only diagonal elements are shown. *Upper*: Single pointing, with $\Delta k = 0.02h/Mpc$. *Lower*: Multi-pointing, with $\Delta k = 0.01h/Mpc$. Note the small scale constraint is not improved since I simply multiply the pointing number in front of the Fisher matrix with reduced integration time of each pointing to mimic the multi-pointing. 4 cylinders with 25 m width and 100 m long, only 10 feeds in each cylinder, and assuming each feeds respondes to 100/10 m space in that direction.

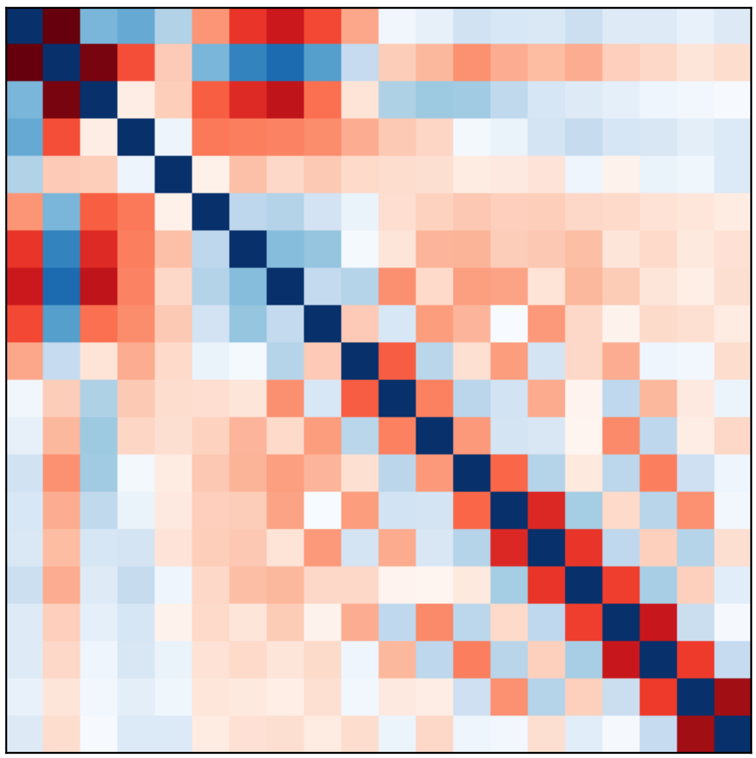

Fig. 2.— correlation matrix of band power, blue for positive correlations, and red for negative correlations, $\Delta k = 0.02h/Mpc$. See the strong correlation around diagonal. Assuming the same cylinder array as previous figure.