# Sky map reconstruction from visibilities for a transit telescope

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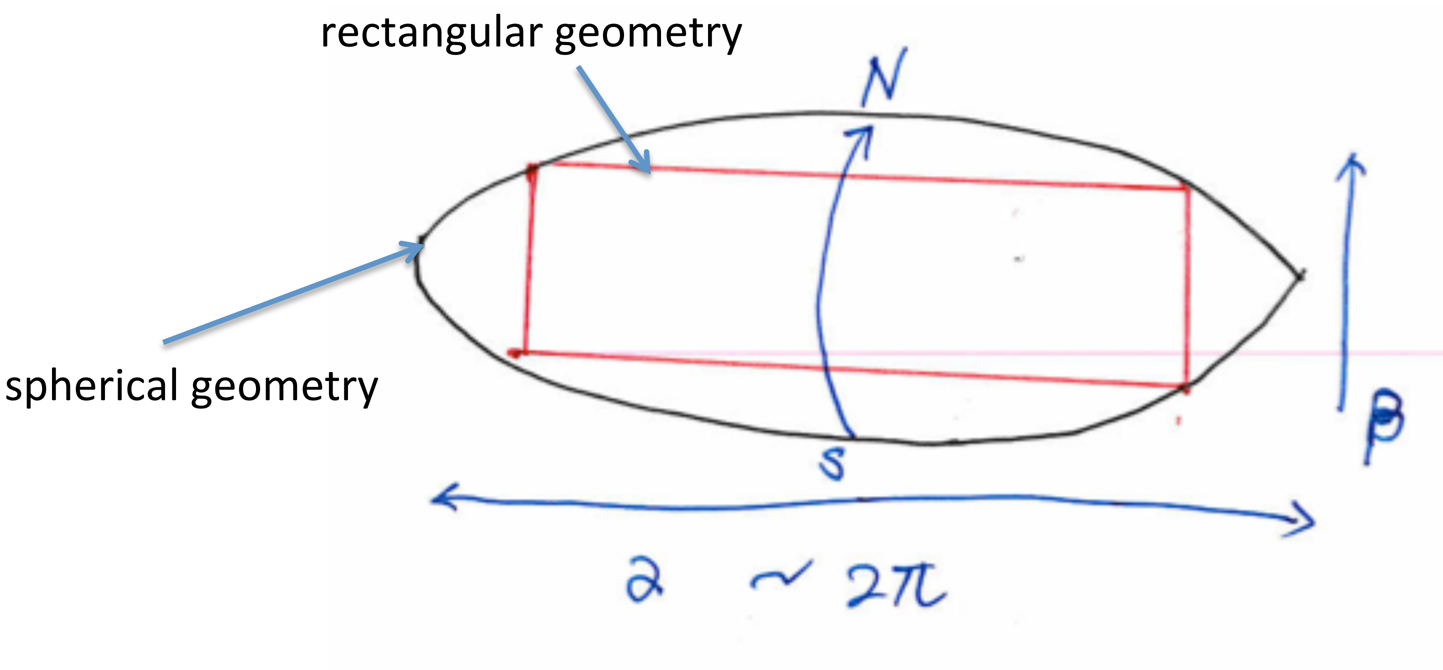
# aim

- Evaluate sky reconstruction performance for different interferometer configurations and survey strategy.
- We consider rectangular geometry at present (rectangular alpha,delta maps , Fourier transform to the UV plane)
- The method could be applied to spherical geometry later


rectangular geometry
N
spherical geometry
S
β
α ~ 2π

# Methods

- One pointing means $\alpha_0$, $\beta_0$
- If we could scan all of $\alpha$,$\beta$ pointings, our measurement is given by

$$\begin{aligned} mes(\alpha,\beta) &= \mathscr{F}(F_{sky}(u,v) \times F_{beam}(u,v)) + noise \\ &= sky(\alpha,\beta) \circledast beam(\alpha,\beta) + noise \end{aligned}$$

- Actually, we use transit telescope.

→ Full EW ($\alpha$) scan at fixed $\beta$.

- We work for partial NS scan and combine them.

Time (earth rotation)

Pointing in NS direction

$$mes(\alpha_0, \beta_0) = \iint d\alpha d\beta\, sky(\alpha, \beta) beam(\alpha - \alpha_0, \beta - \beta_0) \exp[i2\pi(\alpha \frac{\Delta x}{\lambda} + \beta \frac{\Delta y}{\lambda})]$$

$$= \iint du dv F_{sky}(u, v) F_{beam}(u - \frac{\Delta x}{\lambda}, v - \frac{\Delta y}{\lambda}) \exp(i2\pi u \alpha_0) \exp(i2\pi v \beta_0)$$

$$= \sum_u \sum_v F_{sky}(u, v) F_{beam}(u - \frac{\Delta x}{\lambda}, v - \frac{\Delta y}{\lambda}) \exp(i2\pi u \alpha_0) \exp(i2\pi v \beta_0)$$

Visibility

Sky is finite, so we could use sum instead of integral

# Full α scan means we could perform Fourier transform (FFT)

$$V_{ij}(\alpha, \beta_0) \quad (0 < \alpha < 2\pi) \longrightarrow \widetilde{V_{ij}}(u, \beta_0) \quad for\ all\ u$$

$$\widetilde{V_{ij}}(u, \beta_0) = \mathscr{F}(mes(\alpha, \beta_0)) = \sum_v F_{sky}(u, v) F_{beam}(u - \frac{\Delta x}{\lambda}, v - \frac{\Delta y}{\lambda}) \exp(i2\pi v \beta_0)$$

We could express our function as matrix, (we keep only one Vij for a set of antenna with the same baseline, to simplify numerical handling)

The full problem of all $V_{ij}(u)$ could be separated into a set of independent problems for each u.

$$\begin{pmatrix} \cdots \\ \widetilde{V_{ij}}(u,\beta_0) \\ \cdots \end{pmatrix} = A \times \begin{pmatrix} \cdots \\ F_{sky}(u,v) \\ \cdots \end{pmatrix} + noise \longrightarrow \begin{pmatrix} \cdots \\ \widetilde{V_{ij}}(\beta_0) \\ \cdots \end{pmatrix} = A_{u0} \times \begin{pmatrix} \cdots \\ F_{sky}(u_0,v) \\ \cdots \end{pmatrix} + noise$$

**Fixed u**

$$\begin{pmatrix} V_{11}(\beta_0) \\ V_{11}(\beta_1) \\ \cdots \\ V_{11}(\beta_n) \\ V_{ij}(\beta_0) \\ V_{ij}(\beta_1) \\ \cdots \\ V_{ij}(\beta_n) \end{pmatrix} = \begin{pmatrix} F_b(v_1)\exp 2i\pi\beta_0 & F_b(v_2)\exp 2i\pi\beta_0 & \cdots & F_b(v_m)\exp 2i\pi\beta_0 \\ & \cdots & & \\ F_b(v_1)\exp 2i\pi\beta_n & F_b(v_2)\exp 2i\pi\beta_n & \cdots & F_b(v_m)\exp 2i\pi\beta_n \\ F_{bij}(v_1)\exp 2i\pi y_0 & F_{bij}(v_2)\exp 2i\pi\beta_0 & \cdots & F_{bij}(v_m)\exp 2i\pi\beta_0 \\ & \cdots & & \\ F_{bij}(v_1)\exp 2i\pi\beta_n & F_{bij}(v_2)\exp 2i\pi\beta_n & \cdots & F_{bij}(v_m)\exp 2i\pi\beta_n \end{pmatrix} \times \begin{pmatrix} F_s(v_1) \\ \cdots \\ F_s(v_m) \end{pmatrix} + noise$$

- To calculate this matrix, we use pseudo-inverse (singular value decomposition)

Pseudo-inverse

$$\begin{pmatrix} \cdots \\ \widehat{F}_{sky}(u_0, v) \\ \cdots \end{pmatrix} = A_{u_0}^{-1} \times \begin{pmatrix} \cdots \\ \widetilde{V_{ij}}(\beta_0) \\ \cdots \end{pmatrix}$$

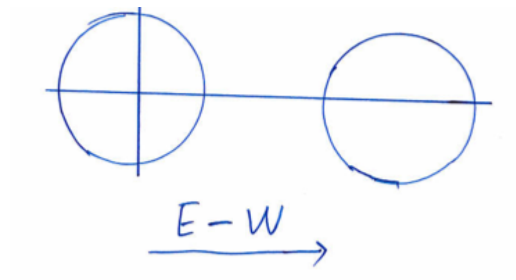
- By computing $F_{sky}(u_0,v)$ for all $u_0$ and putting them together, we could get $F_{sky}(u,v)$, then we do inverse Fourier transform to get sky(α,β)

$$\begin{pmatrix} \cdots \\ \widehat{F}_{sky}(u, v) \\ \cdots \end{pmatrix} \longrightarrow \begin{pmatrix} \cdots \\ \widehat{sky}(\alpha, \beta) \\ \cdots \end{pmatrix}$$

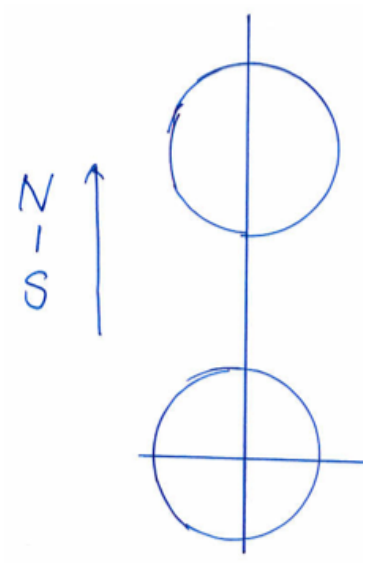
Inverse Fourier transform

# Instrument configurations

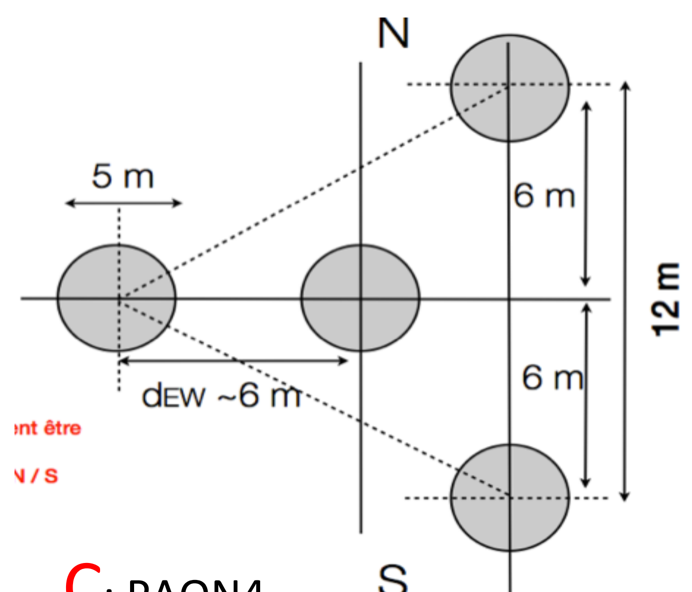
Dish size = 5 m
Distance = 2D



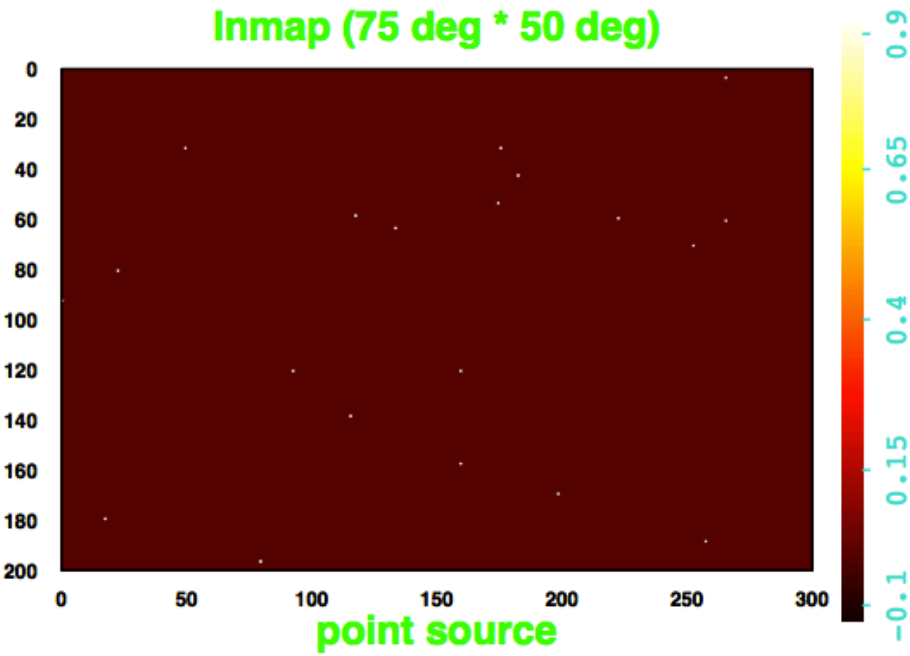

**A** : two dishes in EW

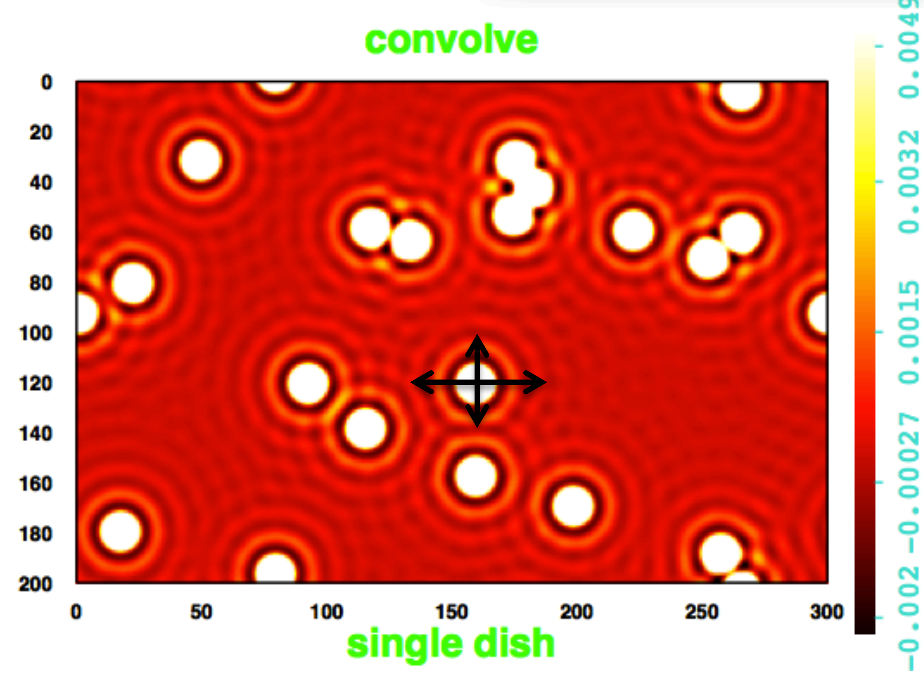


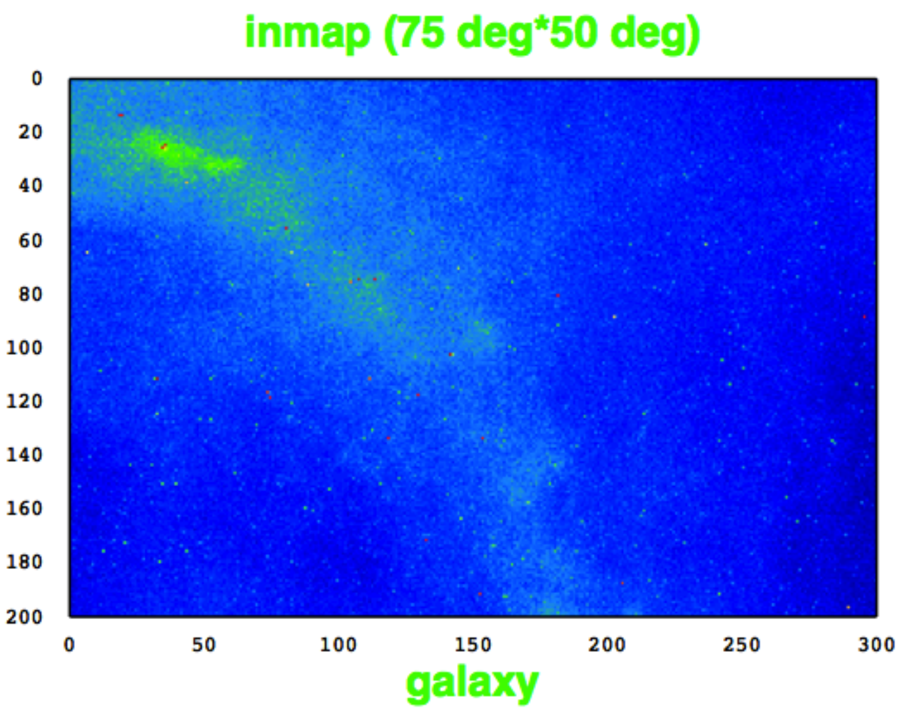

**B**: two dishes in NS

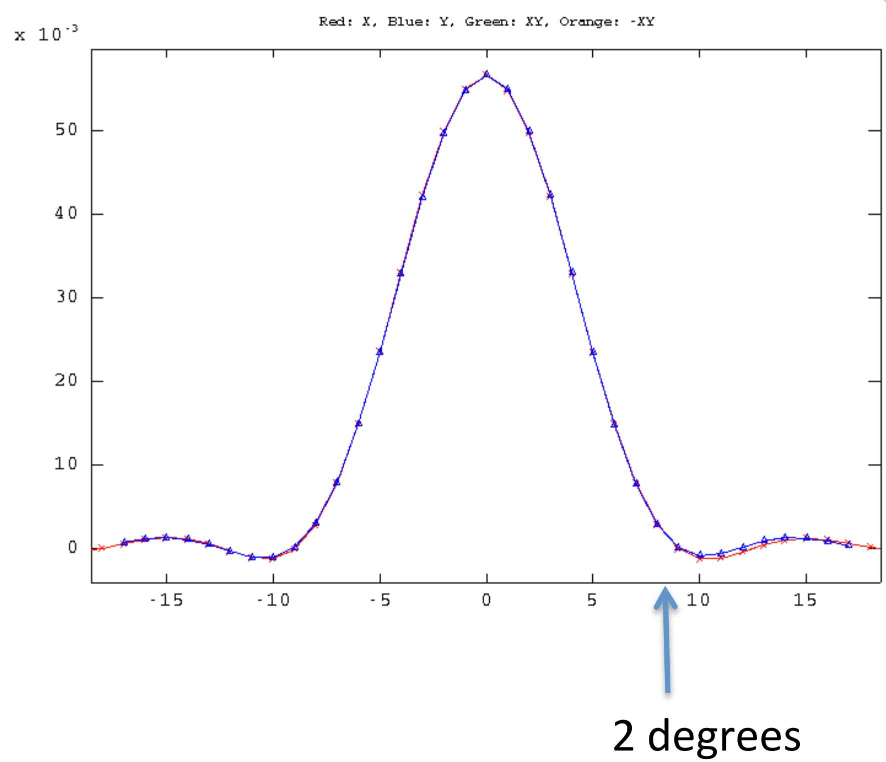


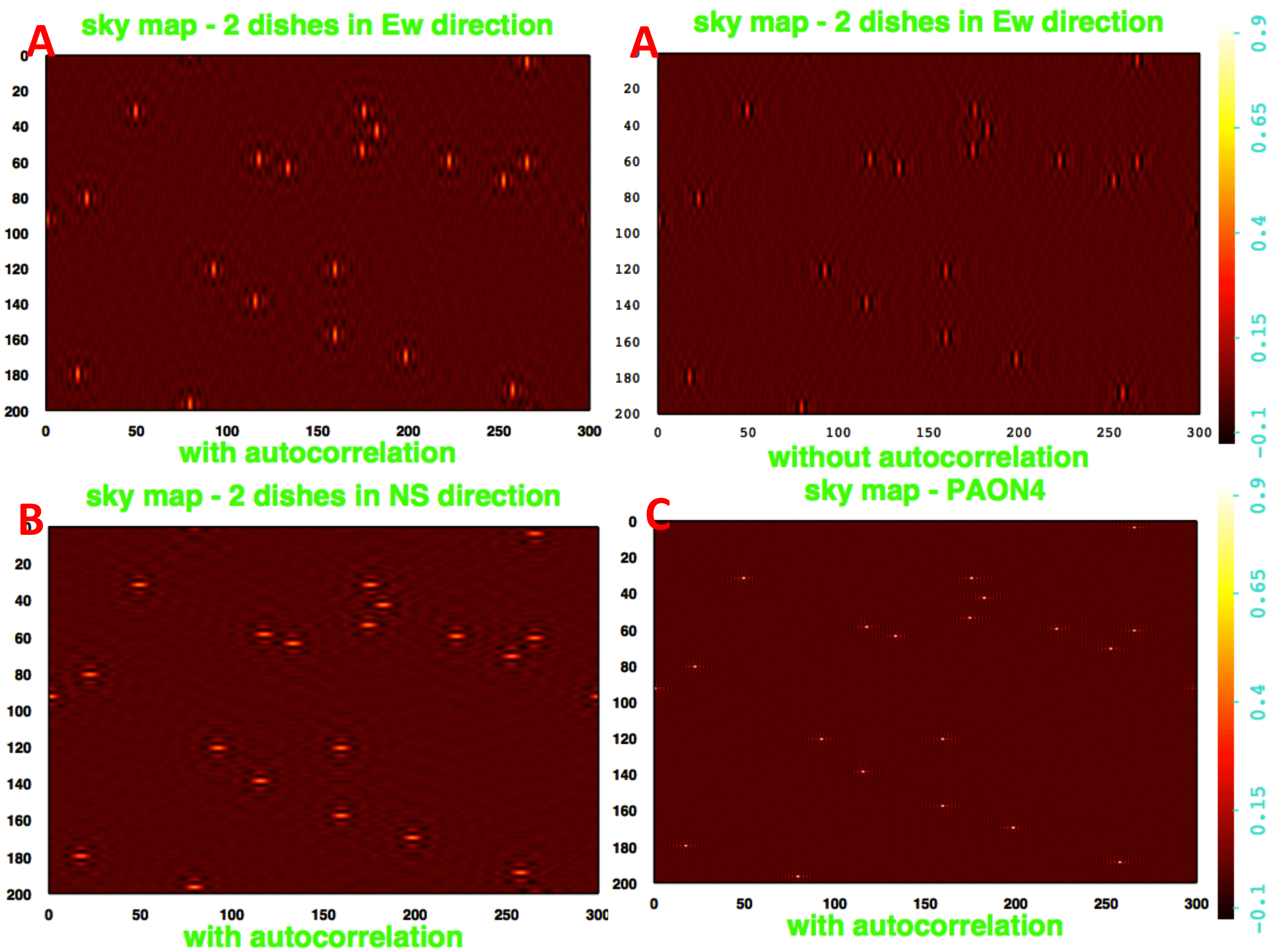

**C**: PAON4


Inmap (75 deg * 50 deg)
point source


convolve
single dish

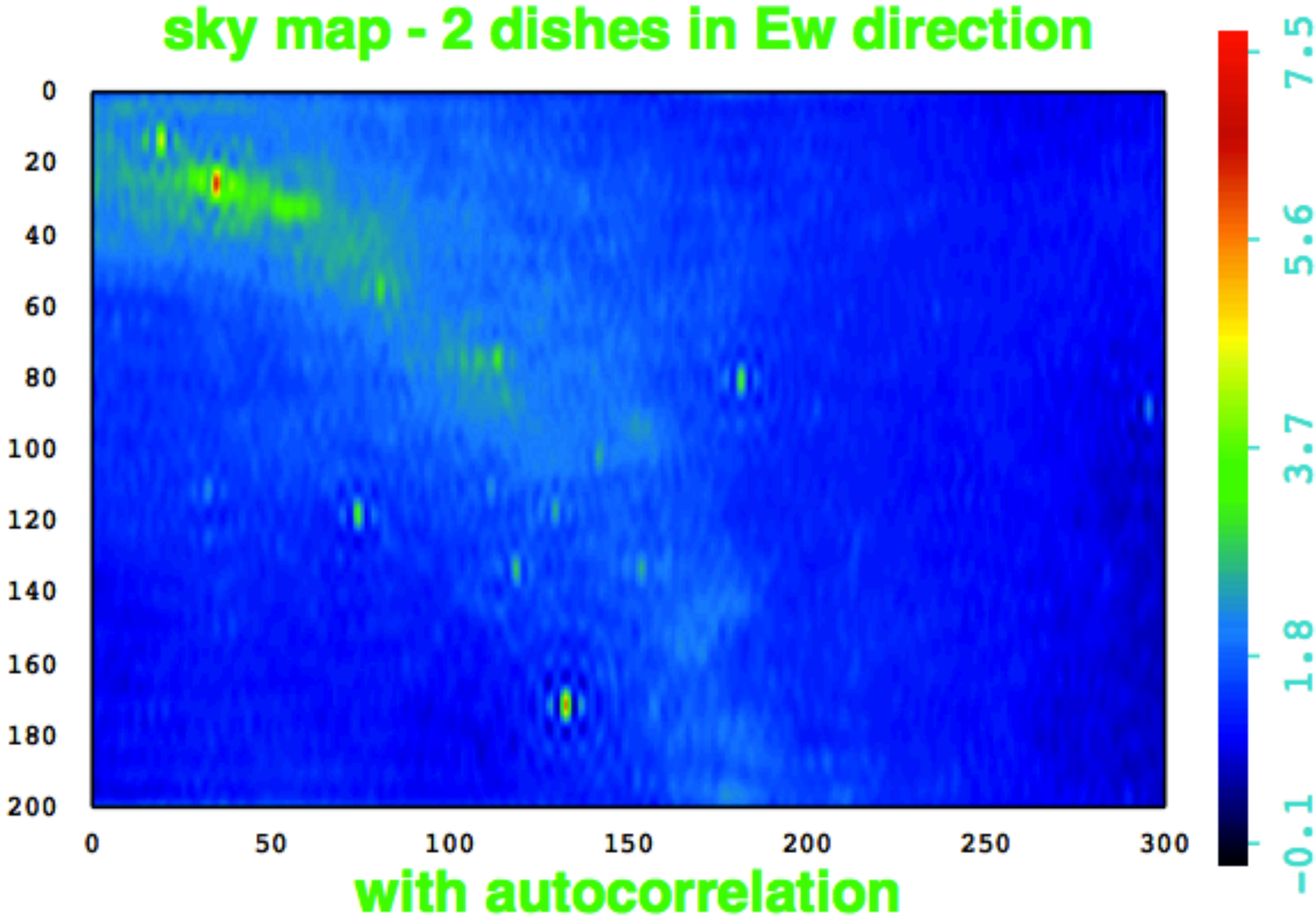

inmap (75 deg*50 deg)
galaxy


Red: X, Blue: Y, Green: XY, Orange: -XY
x 10^-3
2 degrees



A
sky map - 2 dishes in Ew direction
with autocorrelation
A
sky map - 2 dishes in Ew direction
without autocorrelation
B
sky map - 2 dishes in NS direction
with autocorrelation
C
sky map - PAON4
with autocorrelation

# A: full NS scan



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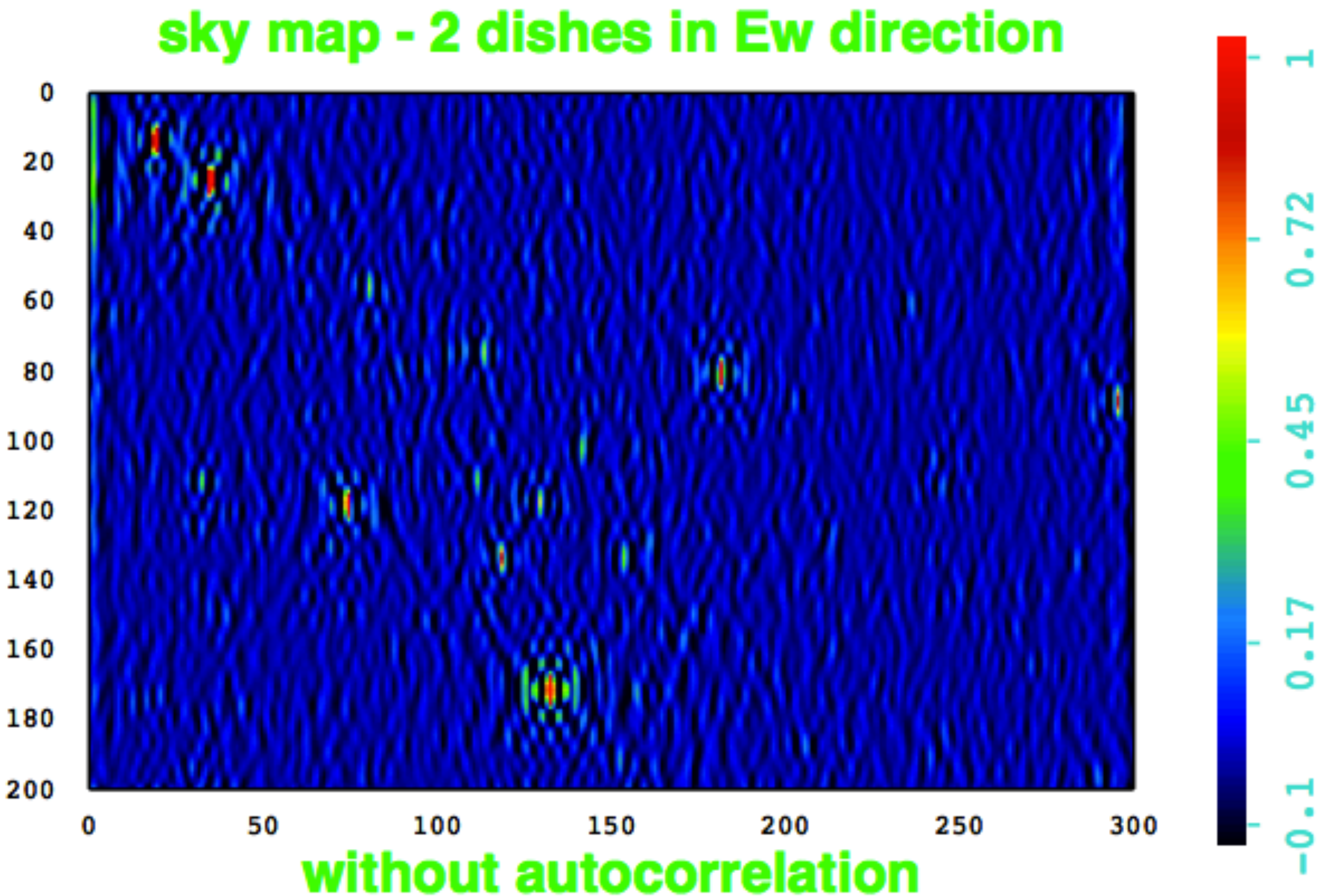

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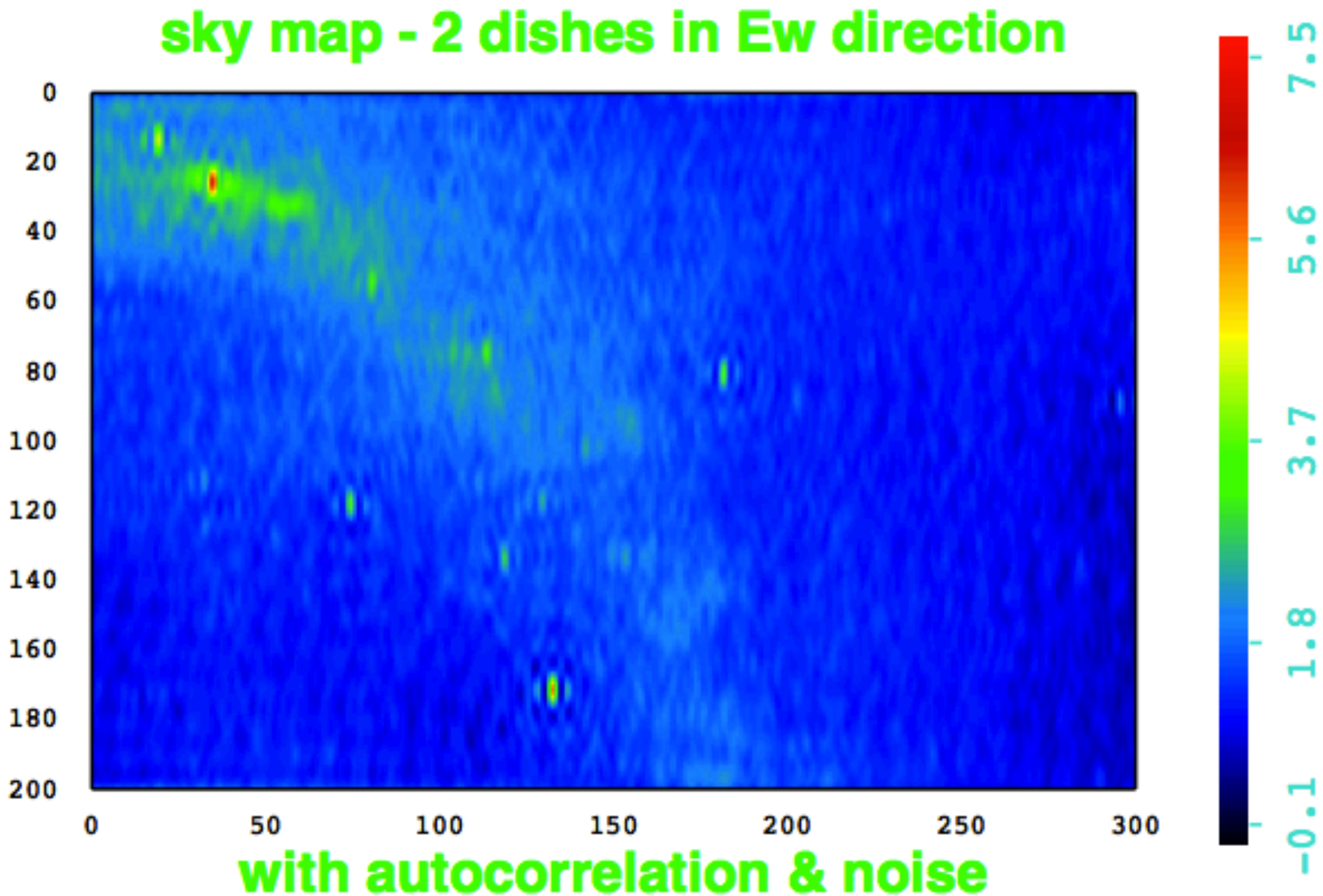
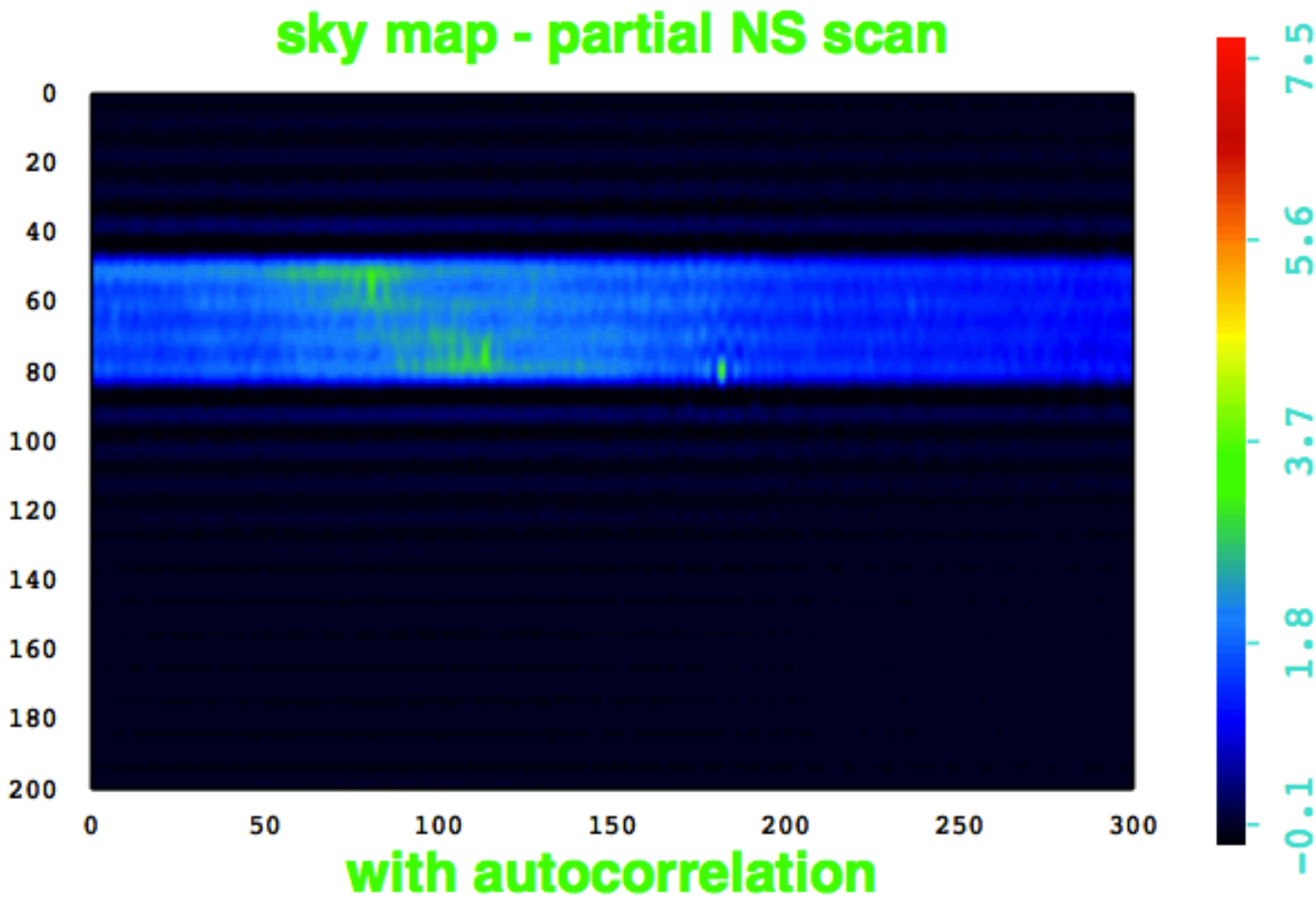
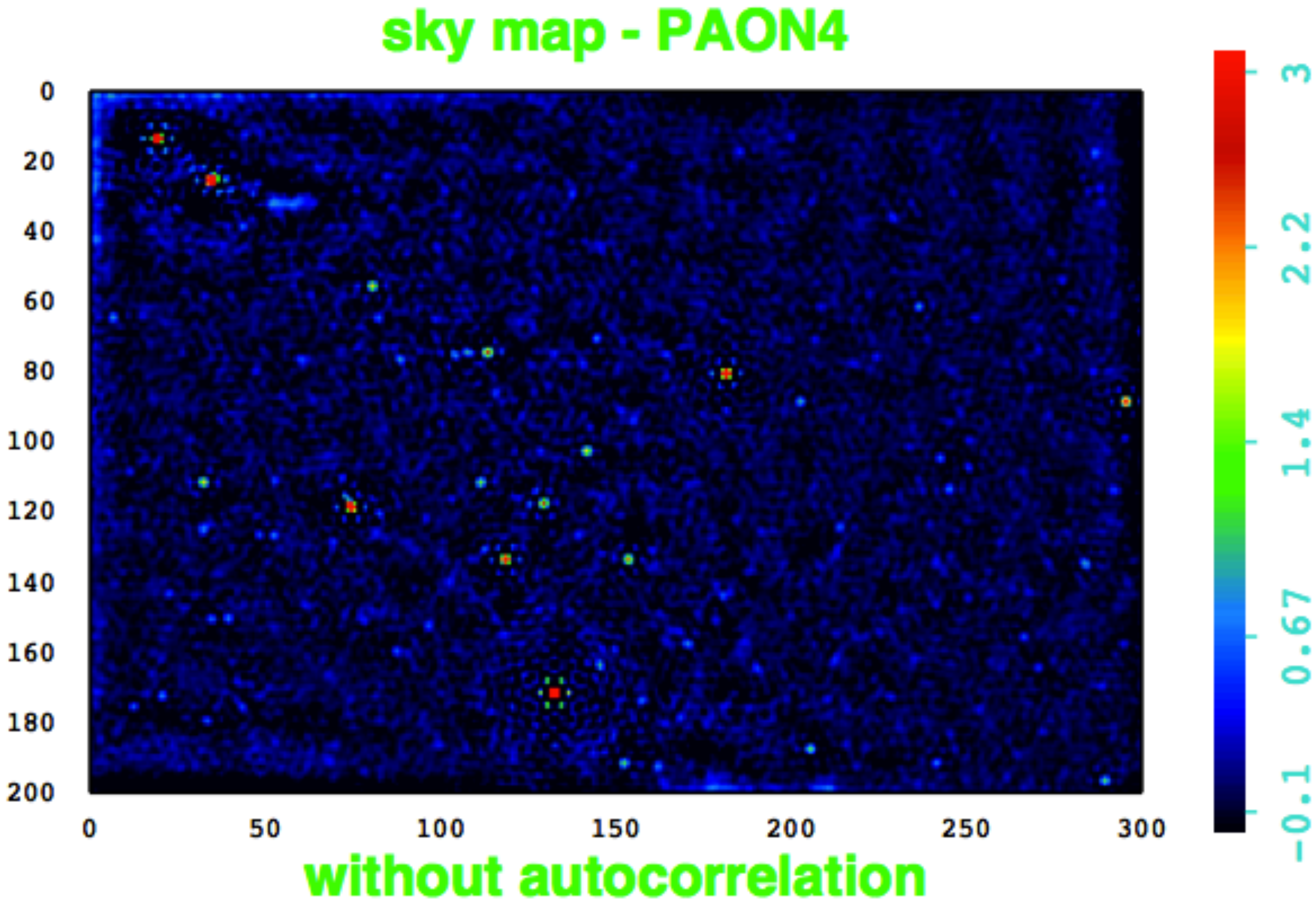


# A: paritial NS scan



# D: full NS scan

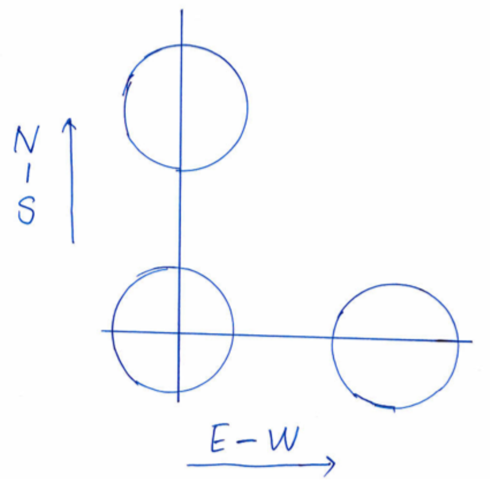


If we use this method in a true picture, what we will get?

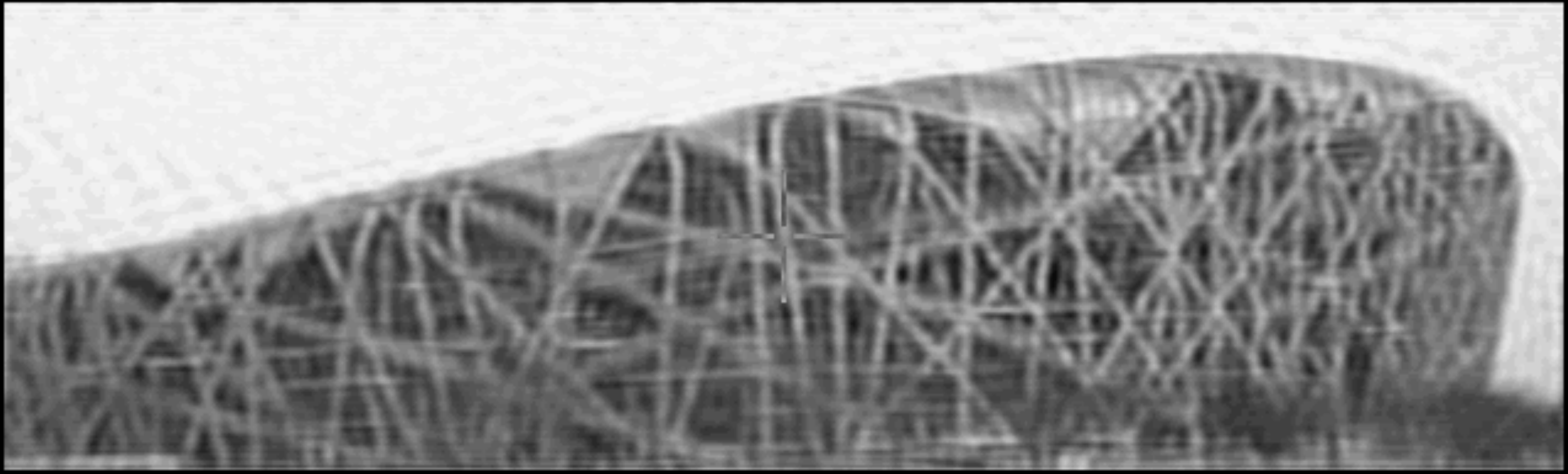
We have taken a picture of Birdsnest in Beijing.

Using 3 Dishes

Dish size = 5 m
Distance = 2D



sky map - full y, 3 dishes



with autocorrelation