Higgs production at N3LO beyond threshold

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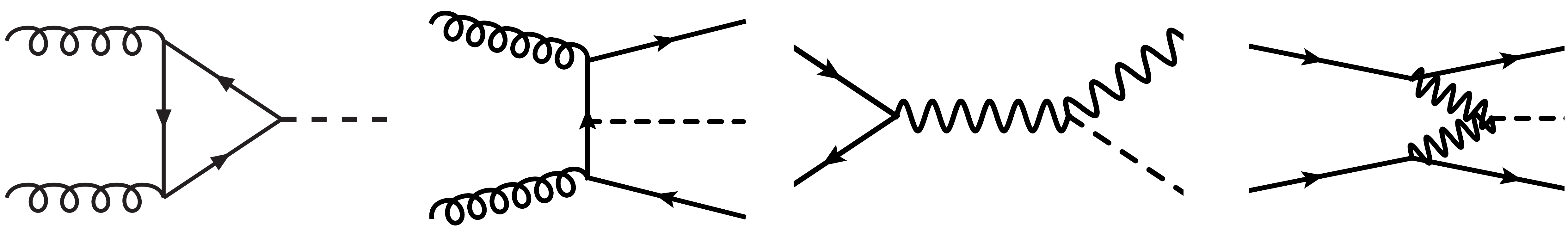
LAL, Orsay, 24/10/201

Higgs physics at LHC

- Establishing whether the BEH mechanism and its boson is SM-like will be of outmost importance for the run of the LHC.
- Higgs-boson production modes at the LHC:



Gluon fusion   TTH   Higgs strahlung   VBF

- Current status for the total cross section: [D. André @ ICHEP 2014]

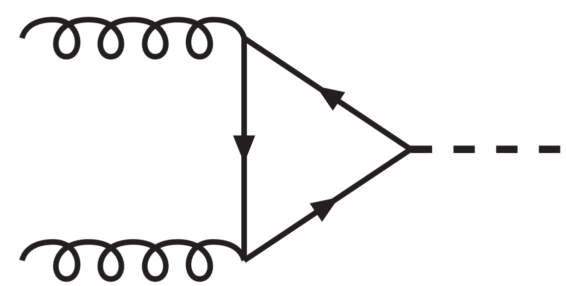
$$\sigma/\sigma_{\rm SM} = 1.00 \pm 0.13 \left[\pm 0.09(\text{stat.})^{+0.08}_{-0.07}(\text{theo.}) \pm 0.07(\text{syst.})\right]$$

➡ Theo. and exp. uncertainties are of the same order.

➡ Need to improve our theory predictions!

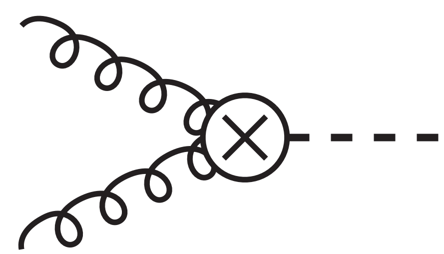
The gluon fusion cross section

- The dominant Higgs production mechanism at the LHC is gluon fusion.
  ➡ Loop-induced process.



- For a light Higgs boson, the dimension five operator describing a tree-level coupling of the gluons to the Higgs boson

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1\, H\, G^a_{\mu\nu}\, G_a^{\mu\nu}$$



- Top-mass corrections known at NNLO.

[Harlander, Ozeren; Pak, Rogal, Steinhauser; Ball, Del Duca, Marzani, Forte, Vicini; Harlander, Mantler, Marzani, Ozeren]

- In the rest of the talk, I will only concentrate on the effective theory.

The gluon fusion cross section

- The gluon fusion cross section is given in perturbation theory by

$$\sigma(p\,p \to H + X) = \tau \sum_{ij} \int_{\tau}^{1} dz\, \mathcal{L}_{ij}(z)\, \hat{\sigma}_{ij}(\tau/z)$$

- The (partonic) cross section depends up to an overall scale only on the ratio

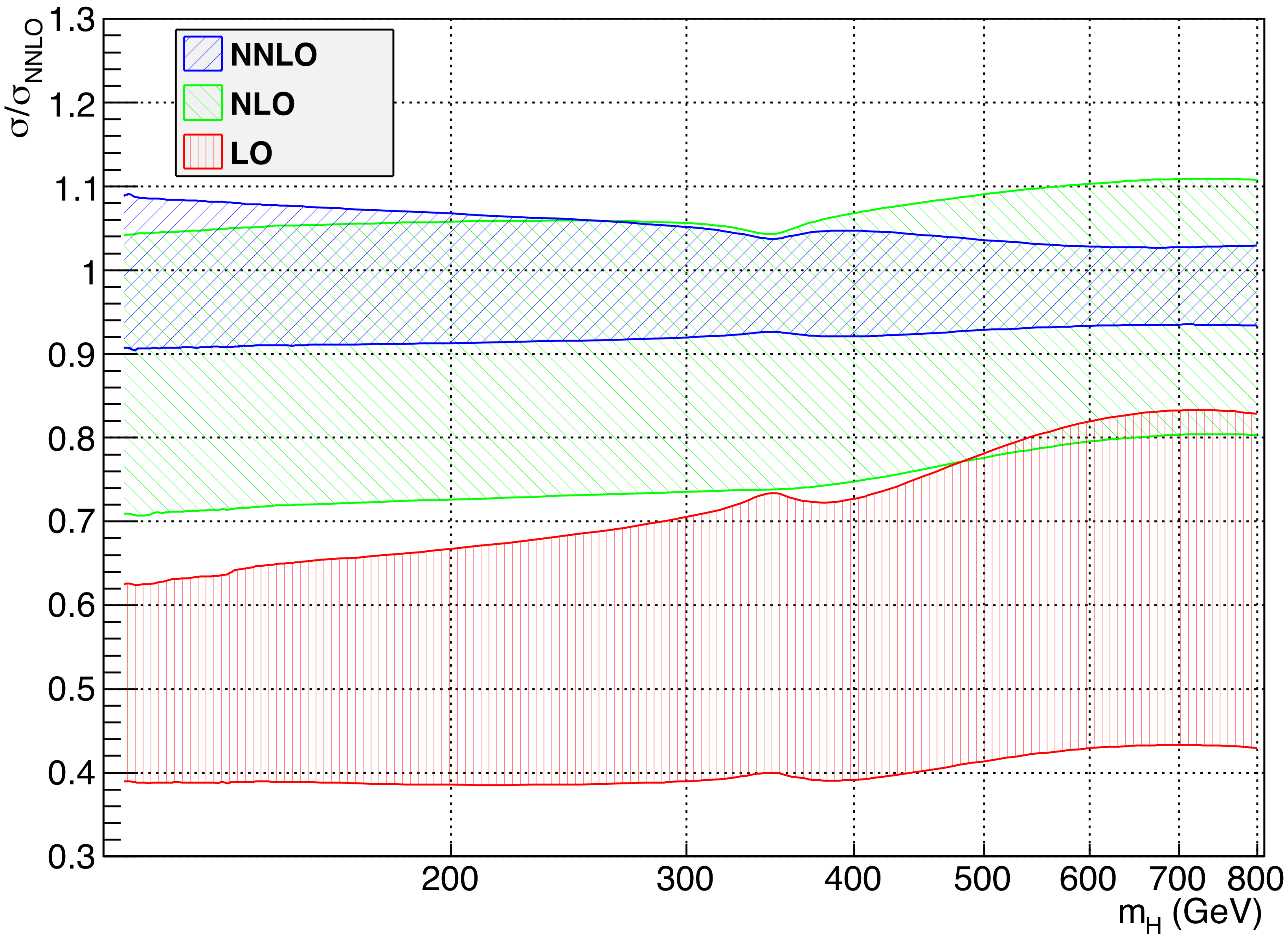
$$\tau = \frac{m^2}{s} \qquad\qquad z = \frac{m^2}{\hat{s}}$$

- The partonic cross section known at NLO and NNLO.

[Dawson; Djouadi, Spira, Zerwas; Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]

- The inclusive Higgs cross section is known to be ‘plagued’ by large perturbative corrections.

The gluon fusion cross section



[Plot from Anastasiou, Bühler, Herzog, Lazopoulos]

[Results for 8 TeV]

	σ	$\delta\sigma$
LO	9.6 pb	~ 25%
NLO	16.7 pb	~ 20%
N2LO	19.6 pb	~ 7 - 9%
N3LO	???	~ 4 - 8%

[Fixed order only]

The gluon fusion cross section

- We need one more order in the perturbative expansion, N3LO.
- So far no complete computation is available.
  - ➡ Scale variation at N3LO is known.

  [Moch, Vogt; Ball, Bonvini, Forte, Marzani, Ridolfi; Bühler, Lazopoulos]
- Several approximate N3LO results exist.

  [Ball, Bonvini, Forte, Marzani, Ridolfi; de Florian, Mazzitelli, Moch, Vogt]
  - ➡ How good are these approximations..?
  - ➡ Only full computation can tell…
- Challenge: Never has an N3LO computation been performed so far...
  - ➡ Uncharted territory!
  - ➡ New conceptual challenges.

Outline

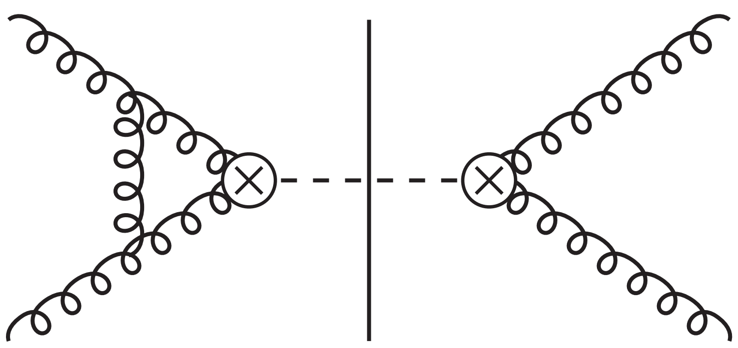
- Higgs production at N3LO
- The soft-virtual cross-section at N3LO.
- Approximate cross-sections at N3LO.
- Going beyond the soft-virtual approximation.

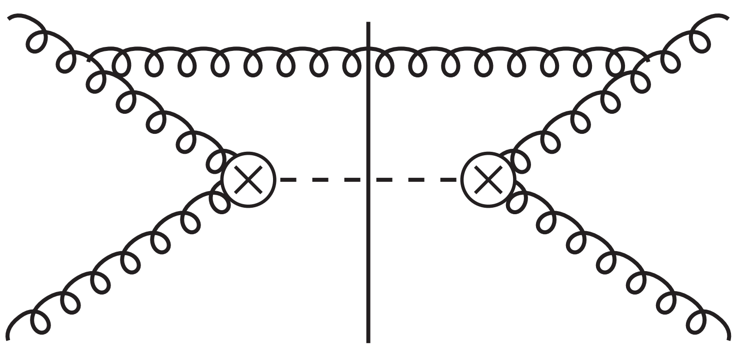
Higgs production at N3LO

The gluon fusion cross section

- At NLO, there are two contributions (~1991):

[Dawson; Djouadi, Spira, Zerwas]





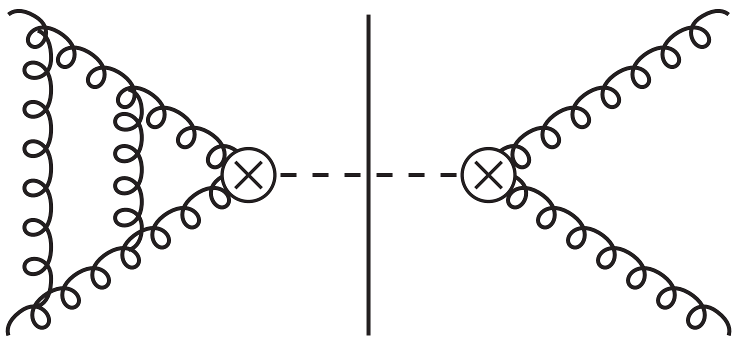
Virtual corrections ('loops')   Real emission

- Both contributions are individually divergent:
  - ➡ UV divergences are handled by renormalization.
  - ➡ IR divergences cancelled by PDF counterterms.

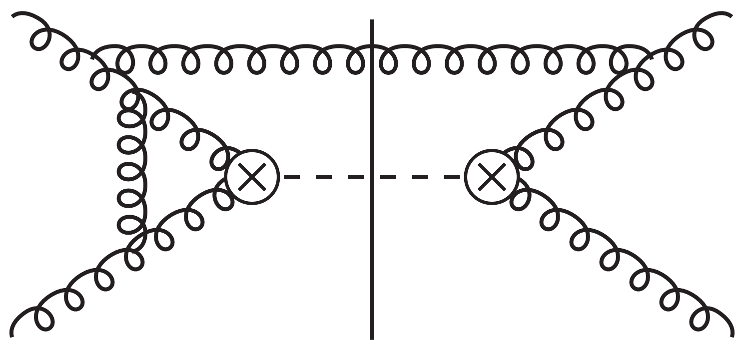
The gluon fusion cross section

- At NNLO, there are three contributions (2002):

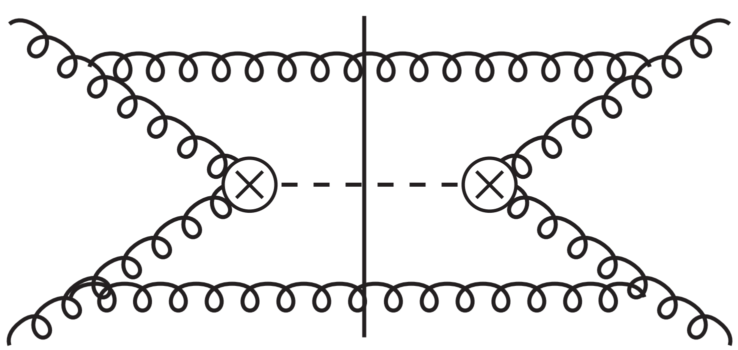
  [Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]



Double virtual



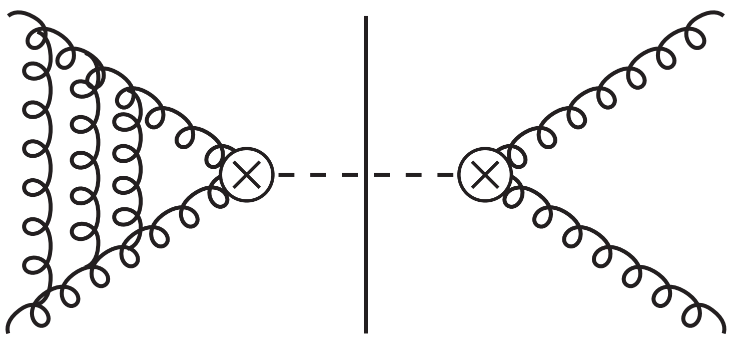
Real-virtual



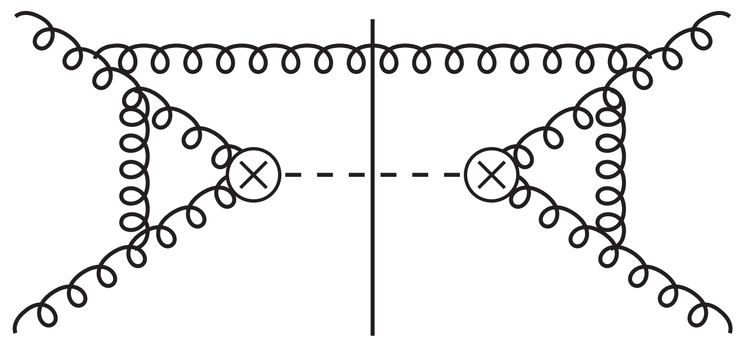
Double real

The gluon fusion cross section

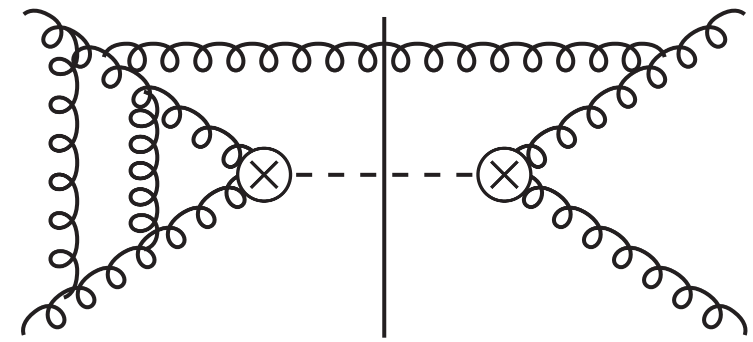
- At N3LO, there are five contributions:



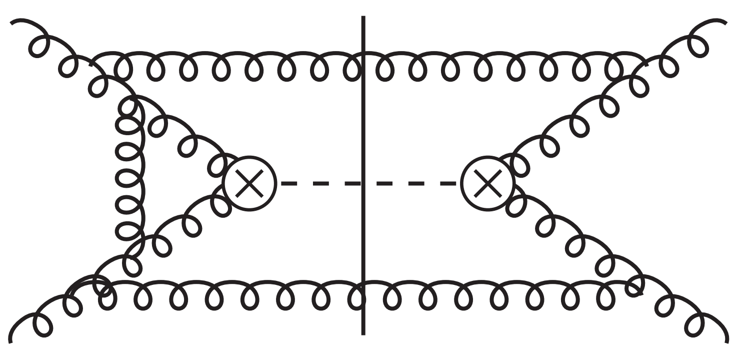
Triple virtual



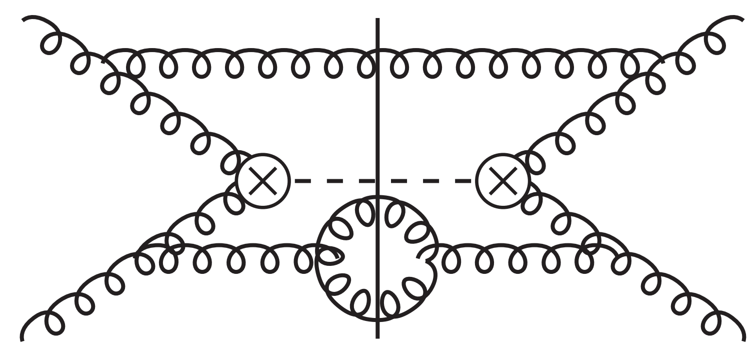
Real-virtual squared



Double virtual real



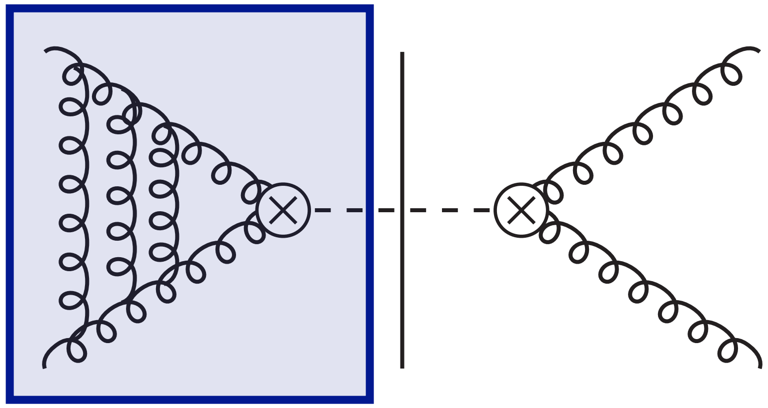
Double real virtual



Triple real

Triple virtual corrections

- The triple virtual corrections are directly related to the QCD form factor

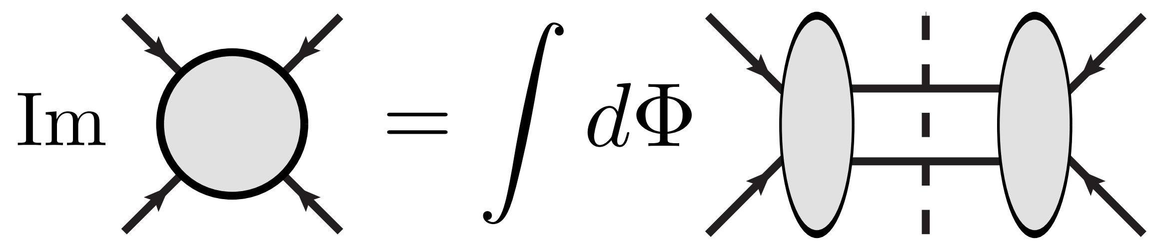


- The QCD form factor is known
  - ➡ at one loop.
  - ➡ at two loops. [Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maître]
  - ➡ at three loops. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

- It is not the loops that are the problem!

Unitarity

- Optical theorem:



➡ Discontinuities of loop amplitudes are phase space integrals.

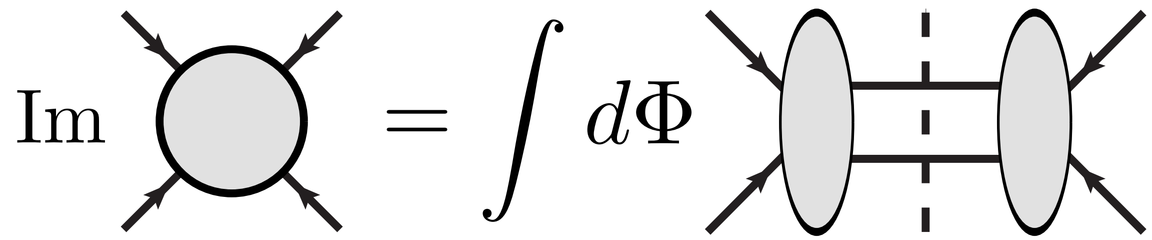
- Discontinuities of loop integrals are given by Cutkosky's rule:

$$\frac{1}{p^2 - m^2 + i\varepsilon} \to \delta_+(p^2 - m^2) = \delta(p^2 - m^2)\,\theta(p^0)$$

- These relations are at the heart of all the unitarity-based approaches to loop computations.

Reverse-unitarity

- Optical theorem:



- We can read the optical theorem ‘backwards’ and write inclusive phase space integrals as unitarity cuts of loop integrals. [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]

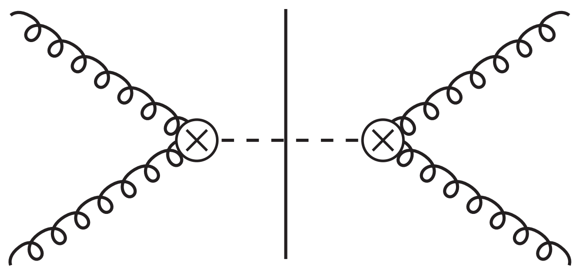
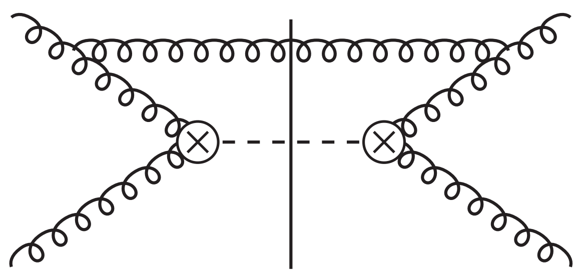
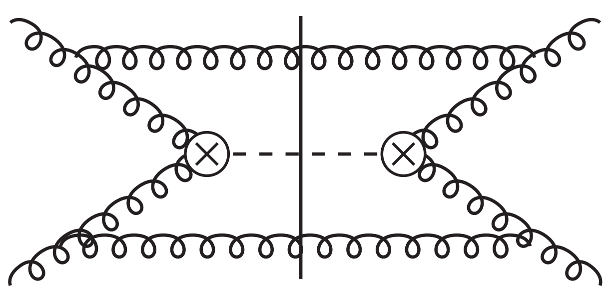
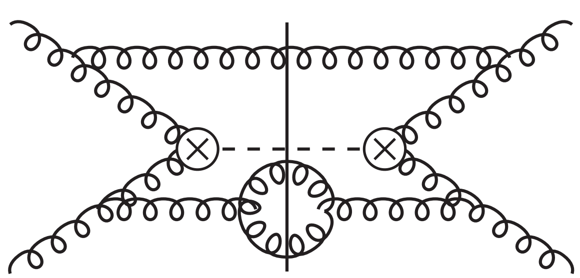
  ➡ Rather than computing phase-space integrals, we can compute loop integrals with cuts!

  ➡ Makes inclusive phase space integrals accessible to all the technology developed for multi-loop computations!

    - Integration-by-parts & differential equations.

Reverse-unitarity @ N3LO

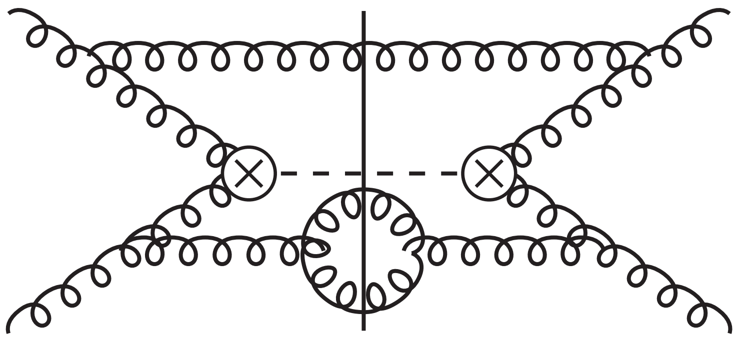
Growth in complexity for real emission

LO		1 diagram	1 integral
NLO		10 diagrams	1 integral
NNLO		381 diagrams	18 integrals
N3LO		26565 diagrams	~500 integrals

The threshold expansion

- ~ 500 master integrals only for triple real double real NNLO).

  ➡ Tough nut to crack!



- The gluon fusion cross section depends on one single parameter:

$$z = \frac{m^2}{s} \qquad \bar{z} = 1 - z$$

- Close to threshold ($z \sim 1$), we can approximate the triple real cross section by a power series:

$$\hat{\sigma}(z) = \sigma_{-1} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

- Goal: Compute cross section as a series around threshold!

The soft-virtual cross section at N3LO

The soft-virtual approximation

- The

$$\hat{\sigma}(z) = \boxed{\sigma_{-1}} + \sigma_0 + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

- The soft-virtual term receives contributions from a 'pole' at $z \sim 1$:

$$(1-z)^{-1+n\epsilon} = \frac{\delta(1-z)}{n\,\epsilon} + \left[\frac{1}{1-z}\right]_+ + n\epsilon\left[\frac{\log(1-z)}{1-z}\right]_+ + \mathcal{O}(\epsilon^2)$$

- Plus-distribution terms already known. [Moch, Vogt

- Complete three-loop corrections are contained the delta function term.

  ➡ The soft-virtual term contains the complete three-loop corrections plus the correction from the emission of up to three soft gluons.

The soft-virtual approximation

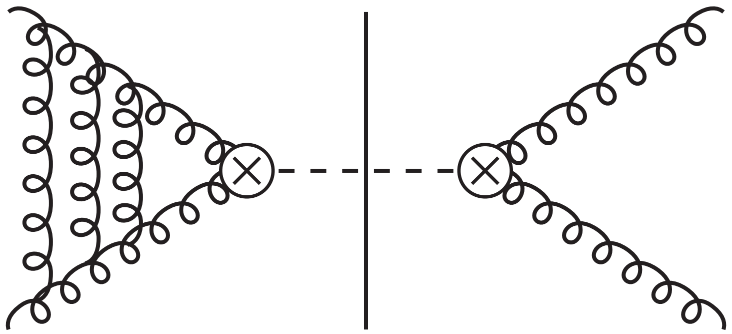
- At NLO and NNLO, the soft-virtual term reads ($\mu_R = \mu_F = m_H$)

$$\hat{\sigma}^{SV}_{gg}(z) = \frac{\pi\, C(\mu^2)^2}{v^2\,(N^2-1)^2} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k \hat{\eta}^{(k)}(z)$$

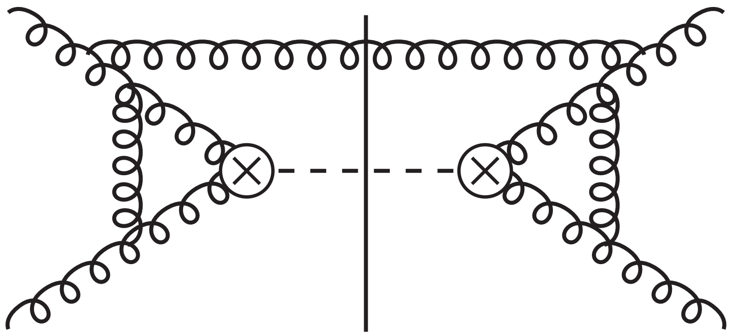
$$\hat{\eta}^{(0)}(z) = \delta(1-z) \qquad\qquad \hat{\eta}^{(1)}(z) = 2\,C_A\,\zeta_2\,\delta(1-z) + 4\,C_A \left[\frac{\log(1-z)}{1-z}\right]_+$$

$$\begin{aligned}
\hat{\eta}^{(2)}(z) &= \delta(1-z)\left\{C_A^2\left(\frac{67}{18}\,\zeta_2 - \frac{55}{12}\,\zeta_3 - \frac{1}{8}\,\zeta_4 + \frac{93}{16}\right) + N_F\left[C_F\left(\zeta_3 - \frac{67}{48}\right) - C_A\left(\frac{5}{9}\,\zeta_2 + \frac{1}{6}\,\zeta_3 + \frac{5}{3}\right)\right]\right\} \\
&+ \left[\frac{1}{1-z}\right]_+ \left[C_A^2\left(\frac{11}{3}\,\zeta_2 + \frac{39}{2}\,\zeta_3 - \frac{101}{27}\right) + N_F\,C_A\left(\frac{14}{27} - \frac{2}{3}\,\zeta_2\right)\right] \\
&+ \left[\frac{\log(1-z)}{1-z}\right]_+ \left[C_A^2\left(\frac{67}{9} - 10\,\zeta_2\right) - \frac{10}{9}\,C_A\,N_F\right] \\
&+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ \left(\frac{2}{3}\,C_A\,N_F - \frac{11}{3}\,C_A^2\right) + \left[\frac{\log^3(1-z)}{1-z}\right]_+ 8\,C_A^2\,.
\end{aligned}$$

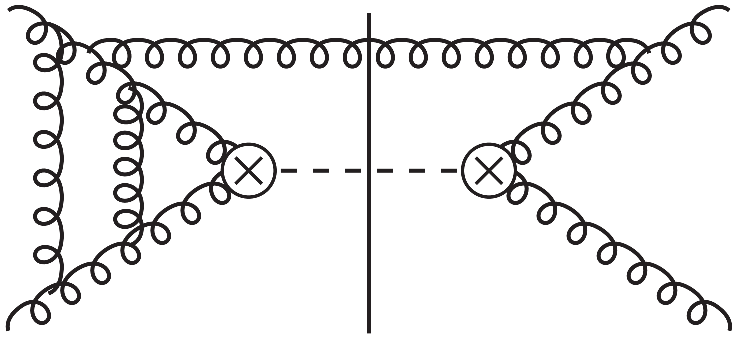
N3LO status: soft-virtual



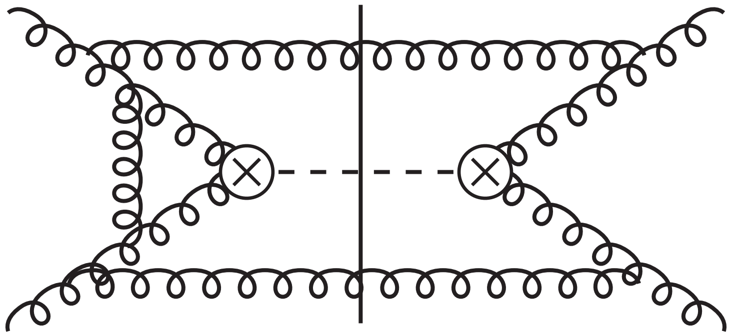
✓ Triple virtual



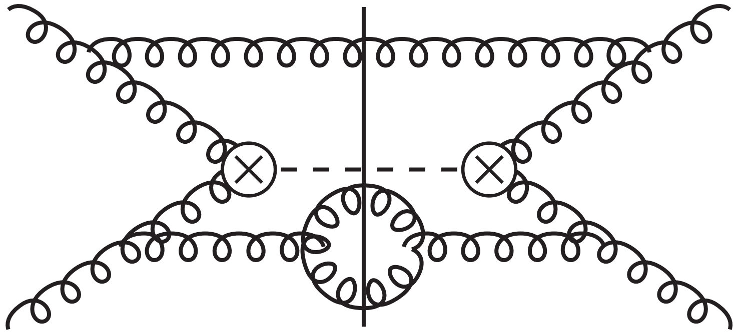
✓ Real-virtual squared



✓ Double virtual real



✓ Double real virtual



✓ Triple real

✓ +

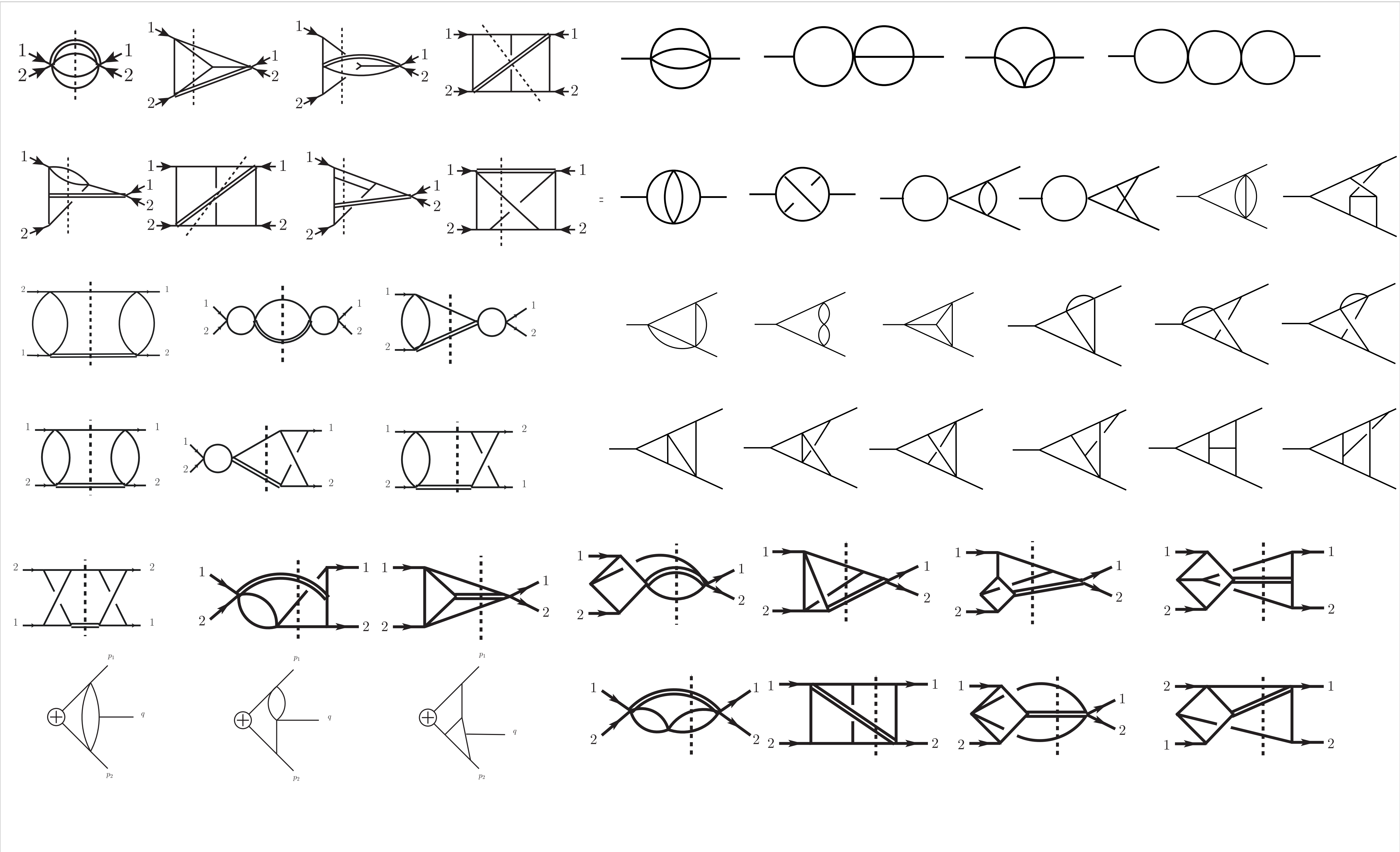
The soft-virtual approximation

- The computation of the first term has been completed!

  [Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

- Many different contributions are needed:
  - ➡ 22 three-loop. [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]
  - ➡ 3 double-virtual-real. [CD Gehrmann, Li, Zhu]
  - ➡ 7 real-virtual-squared. [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore]
  - ➡ 10 double-real-virtual. [Anastasiou, CD, Dulat, Furlan, Herzog, Mistlberger; Li, von Manteuffel, Schabinger, Zhu]
  - ➡ 8 triple real. [Anastasiou, CD, Dulat, Mistlberger]
  - ➡ three-loop splitting functions. [Moch, Vermaseren, Vogt]
  - ➡ three-loop beta function. [Tarasov, Vladimirov, Zharkov; Larin, Vermaseren]
  - ➡ three-loop Wilson coefficient. [Chetyrkin, Kniehl, Steinhauser; Schroeder, Steinhauser; Chetyrkin, Kuhn, Sturm]

The integrals



Higgs soft-virtual @ N3LO

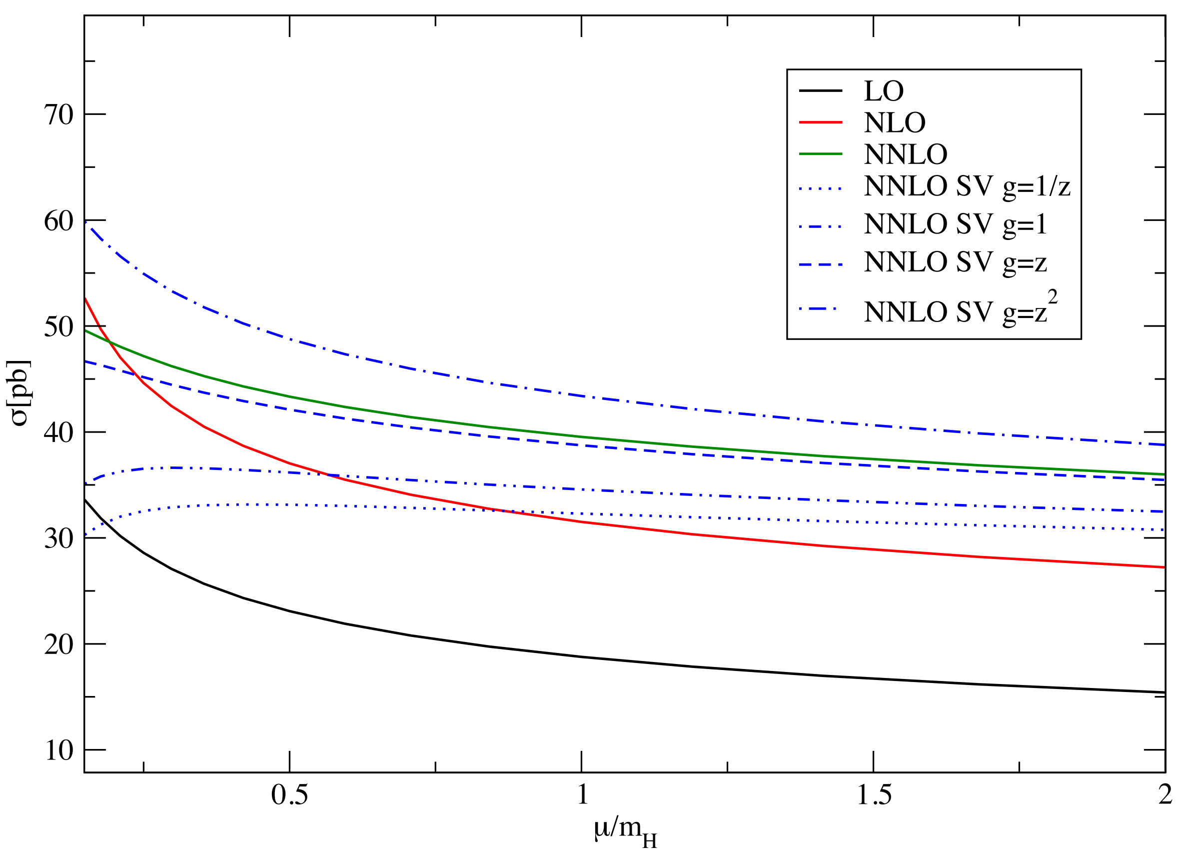
$$
\begin{aligned}
\hat\eta^{(3)}(z) = \delta(1-z)\Bigg\{ & C_A^3\left(-\frac{2003}{48}\zeta_6+\frac{413}{6}\zeta_3^2-\frac{7579}{144}\zeta_5+\frac{979}{24}\zeta_2\zeta_3-\frac{15257}{864}\zeta_4-\frac{819}{16}\zeta_3+\frac{16151}{1296}\zeta_2+\frac{215131}{5184}\right)\\
&+N_F\Bigg[C_A^2\left(\frac{869}{72}\zeta_5-\frac{125}{12}\zeta_3\zeta_2+\frac{2629}{432}\zeta_4+\frac{1231}{216}\zeta_3-\frac{70}{81}\zeta_2-\frac{98059}{5184}\right)\\
&\quad+C_A C_F\left(\frac{5}{2}\zeta_5+3\zeta_3\zeta_2+\frac{11}{72}\zeta_4+\frac{13}{2}\zeta_3-\frac{71}{36}\zeta_2-\frac{63991}{5184}\right)+C_F^2\left(-5\zeta_5+\frac{37}{12}\zeta_3+\frac{19}{18}\right)\Bigg]\\
&+N_F^2\Bigg[C_A\left(-\frac{19}{36}\zeta_4+\frac{43}{108}\zeta_3-\frac{133}{324}\zeta_2+\frac{2515}{1728}\right)+C_F\left(-\frac{1}{36}\zeta_4-\frac{7}{6}\zeta_3-\frac{23}{72}\zeta_2+\frac{4481}{2592}\right)\Bigg]\Bigg\}\\
&+\left[\frac{1}{1-z}\right]_+\Bigg\{C_A^3\left(186\zeta_5-\frac{725}{6}\zeta_3\zeta_2+\frac{253}{24}\zeta_4+\frac{8941}{108}\zeta_3+\frac{8563}{324}\zeta_2-\frac{297029}{23328}\right)+N_F^2 C_A\left(\frac{5}{27}\zeta_3+\frac{10}{27}\zeta_2-\frac{58}{729}\right)\\
&\quad+N_F\Bigg[C_A^2\left(-\frac{17}{12}\zeta_4-\frac{475}{36}\zeta_3-\frac{2173}{324}\zeta_2+\frac{31313}{11664}\right)+C_A C_F\left(-\frac{1}{2}\zeta_4-\frac{19}{18}\zeta_3-\frac{1}{2}\zeta_2+\frac{1711}{864}\right)\Bigg]\Bigg\}\\
&+\left[\frac{\log(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(-77\zeta_4-\frac{352}{3}\zeta_3-\frac{152}{3}\zeta_2+\frac{30569}{648}\right)+N_F^2 C_A\left(-\frac{4}{9}\zeta_2+\frac{25}{81}\right)\\
&\quad+N_F\Bigg[C_A^2\left(\frac{46}{3}\zeta_3+\frac{94}{9}\zeta_2-\frac{4211}{324}\right)+C_A C_F\left(6\zeta_3-\frac{63}{8}\right)\Bigg]\Bigg\}\\
&+\left[\frac{\log^2(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(181\zeta_3+\frac{187}{3}\zeta_2-\frac{1051}{27}\right)+N_F\Bigg[C_A^2\left(-\frac{34}{3}\zeta_2+\frac{457}{54}\right)+\frac{1}{2}C_A C_F\Bigg]-\frac{10}{27}N_F^2 C_A\Bigg\}\\
&+\left[\frac{\log^3(1-z)}{1-z}\right]_+\Bigg\{C_A^3\left(-56\zeta_2+\frac{925}{27}\right)-\frac{164}{27}N_F C_A^2+\frac{4}{27}N_F^2 C_A\Bigg\}\\
&+\left[\frac{\log^4(1-z)}{1-z}\right]_+\left(\frac{20}{9}N_F C_A^2-\frac{110}{9}C_A^3\right)+\left[\frac{\log^5(1-z)}{1-z}\right]_+ 8C_A^3\,.
\end{aligned}
$$

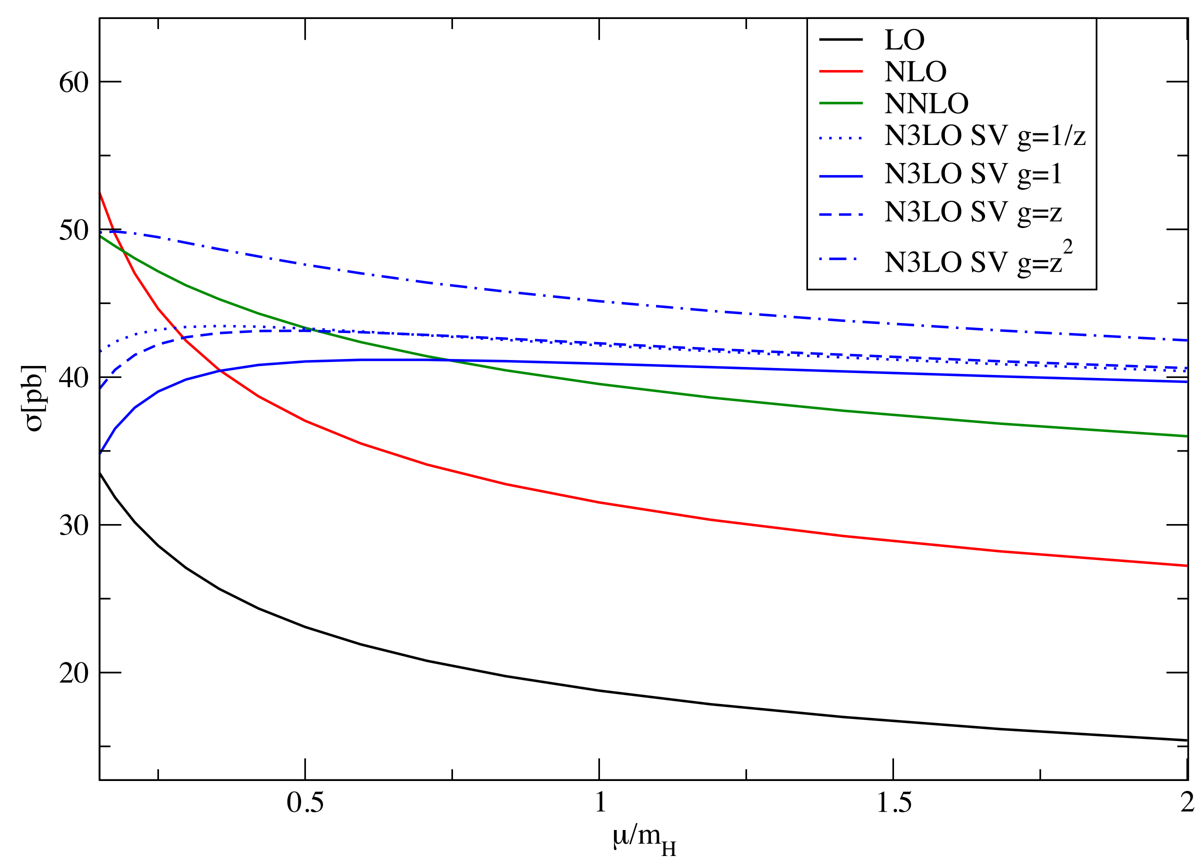
[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

Higgs soft-virtual @ N3LO

- Caveat!
- Source of ambiguity:

$$\int dx_1\, dx_2\, \left[f_i(x_1)\, f_j(x_2) z g(z)\right] \left[\frac{\hat{\sigma}_{ij}(s,z)}{z g(z)}\right]_{\text{threshold}} \qquad \lim_{z\to 1} g(z) = 1$$





[Herzog, Mistlberger]

Going beyond soft-virtual

- Can we go beyond the soft-virtual approximation..?
  - ➡ More terms in the expansion..?
  - ➡ Result in full kinematics..?

- Can we improve the soft-virtual result and do phenomenology..?
  - ➡ Recent approximate N3LO results..?

    [Ball, Bonvini, Forte, Marzani, Ridolfi; de Florian, Mazzitelli, Moch, Vogt]

  - ➡ How good are these approximations..?

Approximate cross sections at N3LO

Approximate N3LO results

- Recently, approximate results at N3LO have been presented that include terms beyond the soft-virtual approximation (gluons only).
- Ball, Bonvini, Forte, Marzani, Ridolfi:
  - ➡ Soft-virtual term at N3LO.
  - ➡ High-energy behaviour, including top-mass effects at N3LO.
  - ➡ Analyticity.
- de Florian, Mazzitelli, Moch, Vogt:
  - ➡ Soft-virtual term at N3LO.
  - ➡ First three logarithms from the next term in the expansion, + numerical guesses for the missing logarithms.

Mellin-space vs. z-space

$$\hat{\sigma}(N) = \int_0^1 dz\, z^{N-1}\, \hat{\sigma}(z) \qquad \hat{\sigma}(z) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}\, z^{-N}\, \hat{\sigma}(N)$$

- Mellin-space is the natural language for resummation.

	z-space	Mellin-space
Soft / threshold limit:	$z \to 1$	$N \to \infty$
High-energy limit:	$z \to 0$	'small' N

- Experience from lower orders: numerical convergence of soft expansion better in Mellin-space.

The high-energy limit

- The leading behaviour of the cross section at small N is known at N3LO.
  - ➡ In the infinite top-mass limit. [Hautmann]
  - ➡ Including finite top-mass effects. [Ball, Del Duca, Forte, Marzani, Vicini]
- Infinite top-mass not compatible with the high-energy limit
  - ➡ Tension between $m_t \gg 1$ and $s \gg 1$.
- If one includes the correct high-energy limit (and requires the correct analytic behaviour in z-space), we find ~16% increase compared to NNLO (8 TeV, $\mu_R = m_H$, gluons only).

  [Ball, Bonvini, Forte, Marzani, Ridolfi]

  - ➡ To be compared to ~6% from expanding resummation to N3LO.

Subleading soft terms

- Recently, the first three next-to-soft terms were published:

$$\hat{\sigma}(z) = \sigma_{-1} + \boxed{\sigma_0} + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

$$-\,512 C_A^3 \ln^5(1-z) + \left\{1728 C_A^3 + \frac{640}{3} C_A^2 \beta_0\right\} \ln^4(1-z)$$

$$+ \quad \left\{\left(-\frac{1168}{3} + 3584\zeta_2\right) C_A^3 - \left(\frac{2512}{3} + \frac{\xi_H^{(3)}}{3}\right) C_A^2 \beta_0 - \frac{64}{3} C_A \beta_0^2\right\} \ln^3(1-z)$$

$\xi_H^{(3)} \simeq 300$ [de Florian, Mazzitelli, Moch, Vogt]

- In Mellin-space:

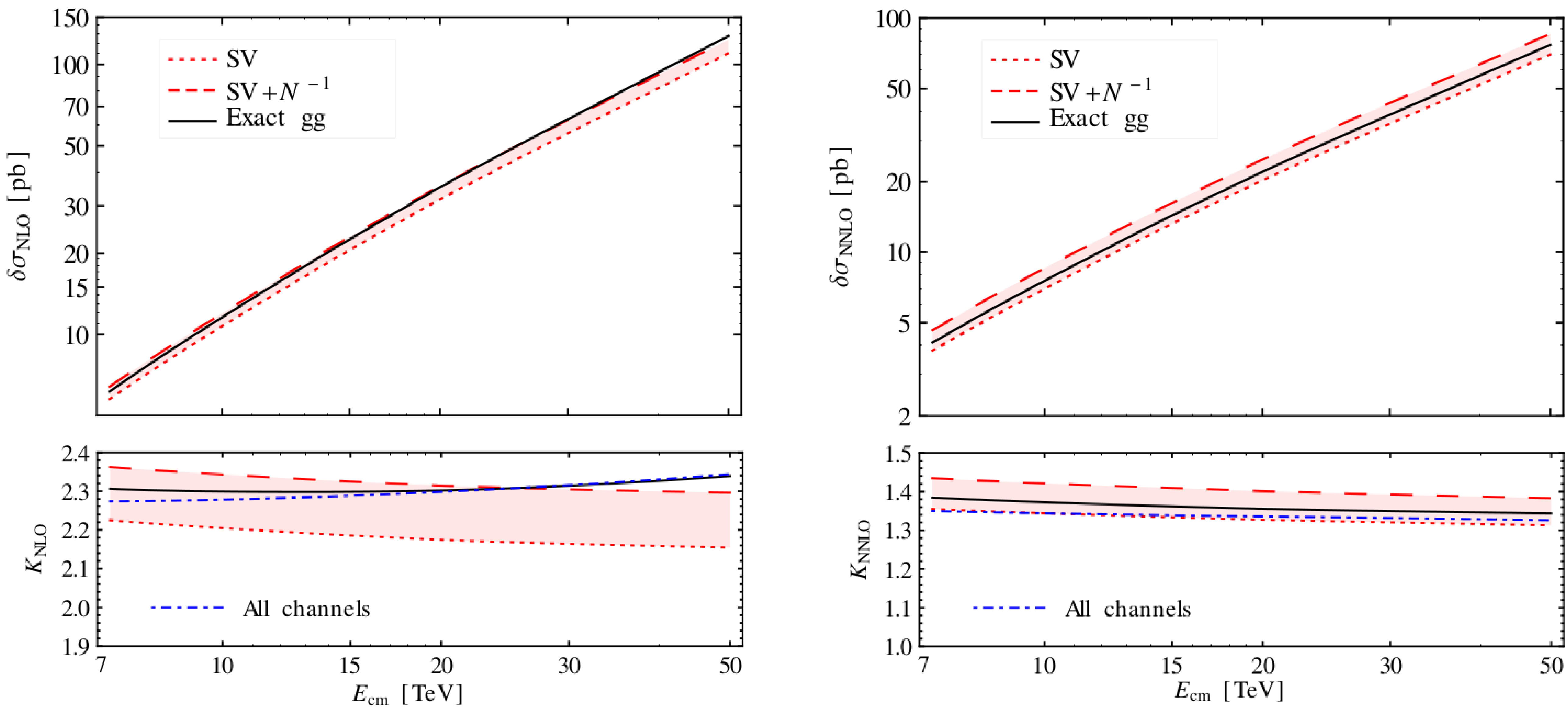
Estimated/guessed from DY

$$\ln^5 N + 5.701 \ln^4 N + 18.9 \ln^3 N + \boxed{46 \ln^2 N + 18 \ln N + 9}$$

- Leads to an increase of ~10-13% (14TeV, $\mu_R = m_H$, gluons only).

Validity of approximation

- " … approximation works well at lower orders…"



[Plots from de Florian, Mazzitelli, Moch, Vogt]

Going beyond the soft-virtual approximation

State of the art at N3LO

- gg   Soft-virtual   [Moch, Vogt; Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

  First 3 next-to-soft logs [de Florian, Mazzitelli, Moch, Vogt]

  Full next-to-soft   Full first three logs (exact)

- gq   First next-to-soft log   [Almasy, Lo Presti, Vogt]

  Full next-to-soft   Full first three logs (exact)

- qqbar   Full first three logs (exact)

- qq   Full first three logs (exact)

- qQ   Full first three logs (exact)

Towards full kinematics

- We have the full contribution from
  - ➡ Emission of one parton at one loop, all channels.

    [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore]
  - ➡ Emission of one parton at two loops, all channels.

    [Dulat, Mistlberger; CD, Gehrmann]
  - ➡ UV and PDF counterterms, all channels.

    [Höschele, Hoff, Pak, Steinhauser, Ueda; Bühler, Lazopoulos]
- We know that all the poles must cancel when we combine ALL contribution.
  - ➡ The knowledge of the previous contributions is enough to fix the first three logarithm in all channels.

    [Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

Next-To-Soft Contribution (gg)

$$
\begin{aligned}
\hat{\eta}^{(3)}_{gg}(z)\Big|_{(1-z)^0} = &-8\,N^3\,\log^5(1-z) + \left(\frac{353}{9}N^3 - \frac{20}{9}N^2N_f\right)\log^4(1-z) \\
&+\left[\left(56\,\zeta_2 - \frac{3469}{54}\right)N^3 + \frac{205}{18}N^2N_f - \frac{4}{27}NN_f^2\right]\log^3(1-z) \\
&+\left\{\left(-181\,\zeta_3 - \frac{2147}{12}\zeta_2 + \frac{2711}{27}\right)N^3 + \left[\left(\frac{545}{48}\zeta_2 - \frac{4139}{216}\right)N^2 + \frac{1}{4}\right]N_f \right. \\
&\quad\left. + \frac{59}{108}NN_f^2\right\}\log^2(1-z) \\
&+\left\{\left(77\,\zeta_4 + 362\,\zeta_3 + \frac{2375}{18}\zeta_2 - \frac{9547}{108}\right)N^3 + \left[\left(-\frac{223}{12}\zeta_3 - \frac{1813}{72}\zeta_2 + \frac{8071}{324}\right)N^2 \right.\right. \\
&\quad\left.\left. +3\,\zeta_3 + \frac{1}{24}\zeta_2 - \frac{17}{4}\right]N_f + \left(\frac{4}{9}\zeta_2 - \frac{163}{324}\right)NN_f^2\right\}\log(1-z) \\
&+\left(-186\,\zeta_5 + \frac{725}{6}\zeta_2\zeta_3 - \frac{821}{12}\zeta_4 - \frac{32849}{216}\zeta_3 - \frac{11183}{162}\zeta_2 + \frac{834419}{23328}\right)N^3 \\
&\quad+\left[\left(\frac{19}{8}\zeta_4 + \frac{1789}{72}\zeta_3 + \frac{4579}{324}\zeta_2 - \frac{527831}{46656}\right)N^2 - \frac{1}{4}\zeta_4 - \frac{149}{72}\zeta_3 - \frac{5}{24}\zeta_2 + \frac{5065}{1728}\right]N_f \\
&\quad+\left(-\frac{5}{27}\zeta_3 - \frac{19}{36}\zeta_2 + \frac{49}{729}\right)NN_f^2\,.
\end{aligned}
$$

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger]

Next-To-Soft Contribution

- We can compute the full contribution to the second term in the threshold expansion

$$\hat{\sigma}(z) = \sigma_{-1} + \boxed{\sigma_0} + (1-z)\,\sigma_1 + \mathcal{O}(1-z)^2$$

  ➡ Receives contribution from both gg and gq channels.

- Needed some rethinking of our technology for double-real emission at one loop.

  ➡ There are now contributions from collinear virtual gluons.

- We find full agreement with known results for leading logarithms. [Almasy, Lo Presti, Vogt; de Florian, Mazzitelli, Moch, Vogt]

  ➡ In particular $\xi_H^{(3)} = \frac{896}{3} \simeq 298.666\ldots$

Ambiguity in z-space

- Ambiguity:

$$\sigma = \tau^{1+\alpha} \sum_{ij} \left(f_i^{(\alpha)} \otimes f_j^{(\alpha)} \otimes \frac{\hat{\sigma}_{ij}(z)}{z^{1+\alpha}} \right)(\tau) \qquad f_i^{(\alpha)}(x) \equiv \frac{f_i(x)}{x^\alpha}$$

➡ Full hadronic cross section cross section is independent order-by-order of α.

- Truncating the soft expansion introduces a dependence on α:

$$\frac{\hat{\sigma}_{ij}(z)}{z^{1+\alpha}} \simeq \hat{\sigma}_{ij}(z)\big|_{(1-z)^{-1}} + \hat{\sigma}_{ij}(z)\big|_{(1-z)^{0}} + \alpha(1-z)\,\hat{\sigma}_{ij}(z)\big|_{(1-z)^{-1}} + \mathcal{O}(1-z)^1$$

➡ Soft-expansion introduces an ambiguity, which can have numerical impact.

- Is this ambiguity also present in Mellin-space..?

Ambiguity in Mellin-space

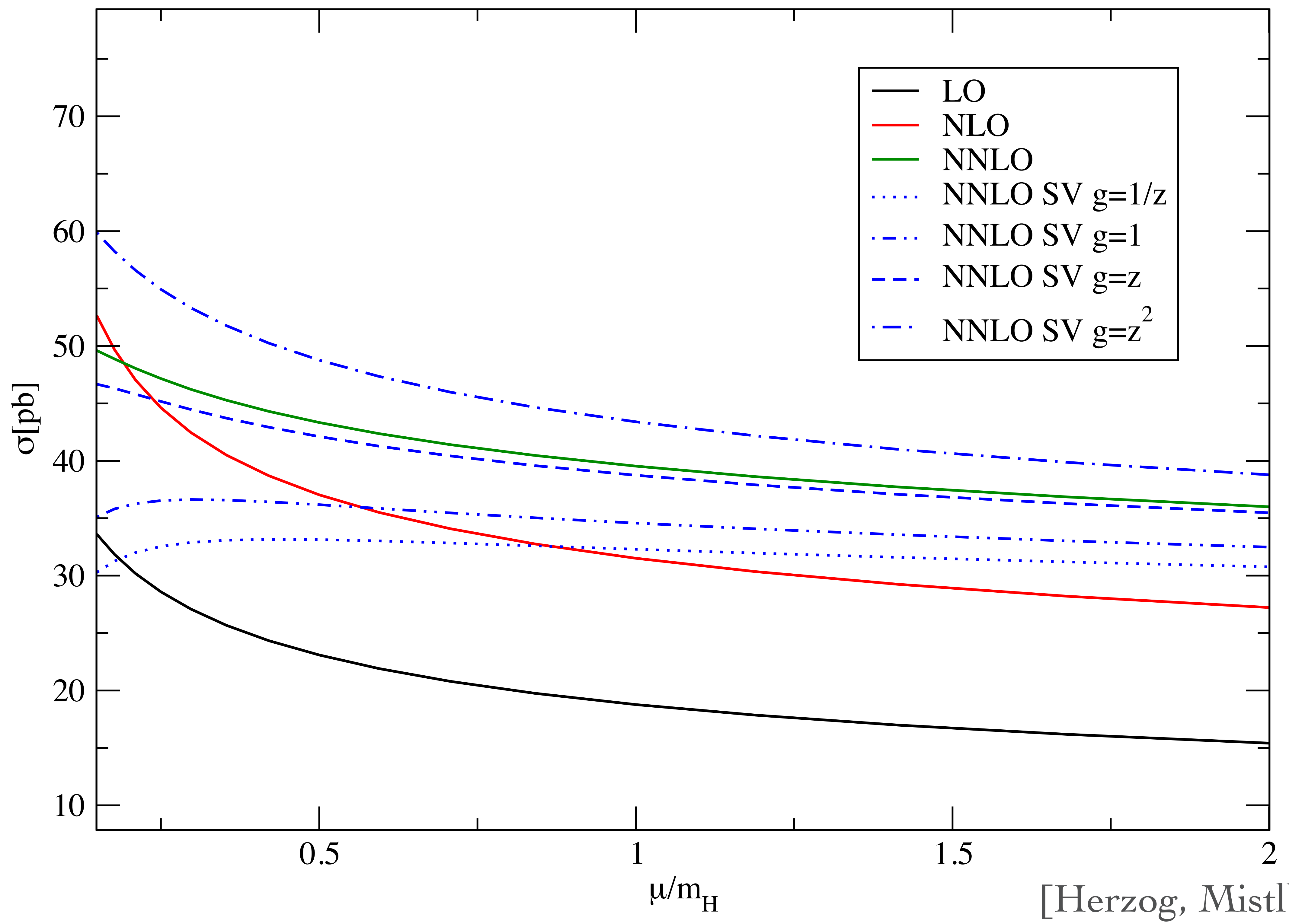
- Multiplying by z^α in z-space corresponds to shifting $N \to N + \alpha$ in Mellin-space.

$$\hat{\sigma}(N) = \int_0^1 dz\, z^{N-1}\, \hat{\sigma}(z)$$

- The threshold limit $N \to \infty$ is obviously insensitive to this!

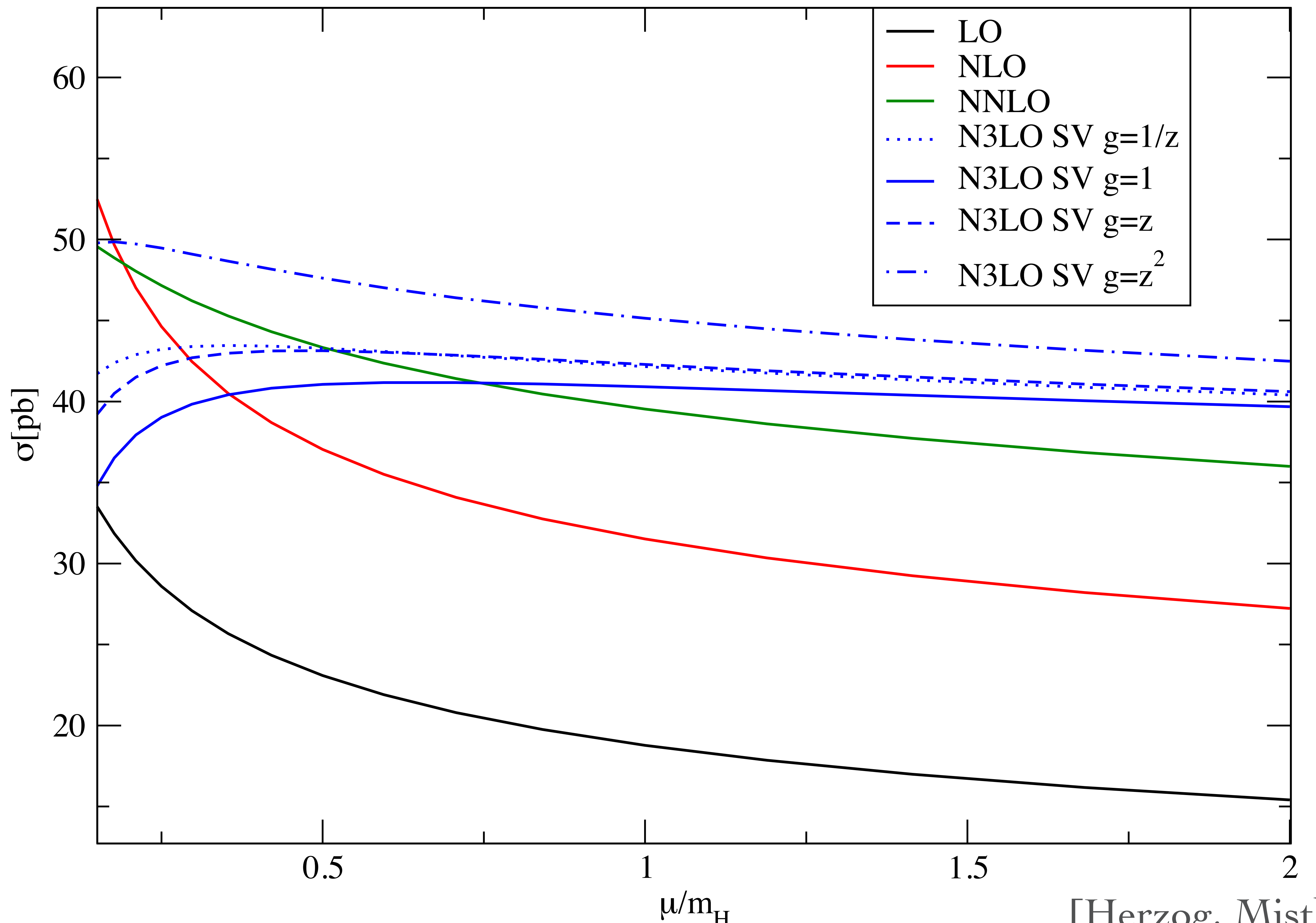
- In order to quantify the validity of approximate cross sections via threshold expansion, we study the dependence of the result on α.

Soft-virtual NNLO



[Herzog, Mistlberger]

Soft-virtual N3LO



[Herzog, Mistlberger]

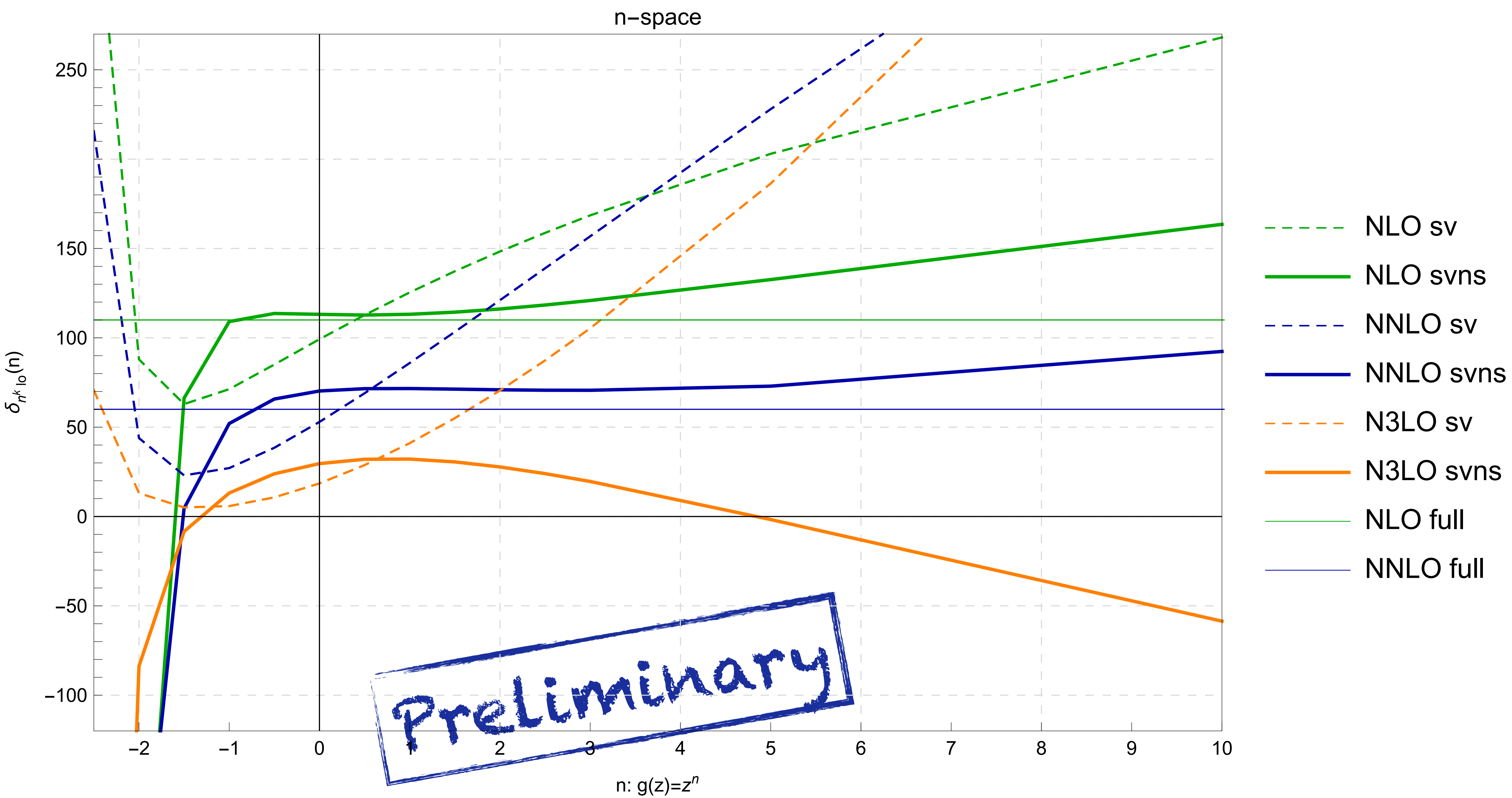
Dependence on the truncation

		Soft-virtual			Next-to-soft		
α	g(z)	NLO ~ 110%	NNLO ~ 60%	N3LO	NLO ~ 110%	NNLO ~ 60%	N3LO
-2	$\frac{1}{z^3}$	3331.71	1998.46	730.957	−54 238.2	−32 593.5	−12 229.
$-\frac{9}{8}$	$\frac{1}{z^{17/8}}$	112.646	60.7583	18.7323	−565.695	−413.316	−138.71
-1	$\frac{1}{z^2}$	87.9371	43.9049	13.0597	−278.802	−235.064	−83.8003
$-\frac{1}{2}$	$\frac{1}{z^{3/2}}$	62.9118	23.0081	5.0715	66.2443	4.58479	−8.37025
0	$\frac{1}{z}$	71.2825	27.0973	5.84748	109.079	52.0453	13.1455
$\frac{1}{2}$	$\frac{1}{\sqrt{z}}$	85.0509	38.4733	10.7073	113.637	65.7434	23.9023
1	1	99.2279	52.9352	18.5346	113.146	70.25	29.6145
$\frac{5}{4}$	$z^{1/4}$	106.092	60.8134	23.3797	112.856	71.1298	31.1678
$\frac{3}{2}$	$\sqrt{z}$	112.75	68.9784	28.7748	112.75	71.5425	32.04
2	z	125.418	85.9054	41.0442	113.177	71.6204	32.1418
$\frac{5}{2}$	$z^{3/2}$	137.253	103.339	55.0482	114.368	71.293	30.5585
3	z^2	148.331	121.057	70.5521	116.148	70.9235	27.7393

[14 TeV, $\mu = mH$, gluons only]

Preliminary

Dependence on the truncation



Conclusion

- The computation of the Higgs cross section at N3LO moves forward at a steady pace!
  - ➡ Soft-virtual contribution known.
  - ➡ Next-to-soft contribution known (noth gg & gQ).
  - ➡ First three logs known exactly for all channels.
  - ➡ Contribution form single-real emission fully known.
- Approximate results should be taken with a grain of salt!
  - ➡ Only full result for N3LO cross section will be the final judge!
- Stay tuned!