Solar models, solar neutrinos and helioseismology

F. L. Villante – Università dell' Aquila and LNGS-INFN

<u>Outline</u>

- The present situation
- The solar composition problem
- The role of CNO neutrinos
- Summary and conclusions

Hydrogen Burning: PP chain and CNO cycle

The Sun is powered by nuclear reactions that transform H into ⁴He:



The pp chain is responsible for about 99% of the total energy (and neutrino) production.

C, N and O nuclei are used as catalysts for hydrogen fusion.

CNO cycle is responsible for about 1% of the total neutrino (and energy) budget. Important for more advanced evolutionary stages

The solar neutrino spectrum



The present situation

Experimental results agree with Standard Solar Models (SSM) + v flavor oscillations:

ν flux	AGSS09	GS98	Solar
$\Phi_{\rm pp}$	$6.03(1\pm0.006)$	$5.98(1\pm 0.006)$	$6.05(1^{+0.003}_{-0.011})$
$\Phi_{ m pep}$	$1.47(1\pm 0.012)$	$1.44(1\pm 0.012)$	$1.46(1^{+0.010}_{-0.014})$
Φ_{Be}	$4.56(1\pm 0.07)$	$5.00(1\pm 0.07)$	$4.82(1^{+0.05}_{-0.04})$
$\Phi_{ m B}$	$4.59(1\pm 0.14)$	$5.58(1\pm 0.14)$	$5.00(1 \pm 0.03)$
$\Phi_{ m hep}$	$8.31(1\pm 0.30)$	$8.04(1\pm 0.30)$	$18(1^{+0.4}_{-0.5})$
$\Phi_{ m N}$	$2.17(1\pm 0.14)$	$2.96(1\pm 0.14)$	≤ 6.7
Φ_{O}	$1.56(1\pm 0.15)$	$2.23(1\pm 0.15)$	≤ 3.2
$\Phi_{ m F}$	$3.40(1\pm 0.16)$	$5.52(1\pm 0.16)$	≤ 59

Serenelli, Haxton, Pena-Garay, ApJ 2011

Units:

pp: 10¹⁰ cm ² s⁻¹; Be: 10⁹ cm ² s⁻¹; pep, N, O: 10⁸ cm ² s⁻¹; B, F: 10⁶ cm ² s⁻¹; hep: 10³ cm ² s⁻¹

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$\Phi_{\rm B}$	$4.59(1\pm0.14)$	$5.58(1\pm0.14)$	$5.00(1 \pm 0.03)$	(SK,SNO,Borexino)
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Solar thermometer!

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Units: pp: 10 ¹⁰ cr Be: 10 ⁹ cn pep, N, O: B. E: 10 ⁶ c	$m^{2} s^{-1};$ $n^{2} s^{-1};$ $10^{8} cm^{2} s^{-1};$ $m^{2} s^{-1};$			Solar thermometer!
$b, r \cdot 10^{\circ} c$	$m^2 s^{-1}$ CNO pc	outrinoc		
пер. 10-е				
	• No (direct detection		
	• Loos	se upper bounds o	btained by com	bining the
	une	rent expt results		

Serenelli, Haxton, Pena-Garay, ApJ 2011

Future goals

• Final confirmation of MSW transition (or looking for new physics):

The v_e survival probability at $E_v \sim 1$ -3MeV probes transition between vacuum and matter dominated regimes

Sensitive to new physics effects (mass varying neutrinos, sterile neutrinos, NSI, etc.)

Combined analysis of SK I-IV PRL 112 (2014) 091805 Provide a 2.7 σ observation of daynight effect;



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Seeing all the solar v branches
 → solving the solar composition problem

Helioseismology

The Sun is a non radial oscillator. The observed oscillation frequencies can be used to determine the properties of the Sun. Linearizing around a known solar model:



Impressive agreement with SSM predictions ...

Basu et al, 2009 Sound speed in the sun $c^2 = \partial P / \partial \rho |_{\rm ad} = \gamma \, u$ 0.010 $(c_{sun}(\mathbf{r})-\overline{c}(\mathbf{r}))/\overline{c}(\mathbf{r})$ (GS98) 0.00 -0.005 0.2 0.4 0.6 0.8 r/R_o

Surface helium abundance $Y_{\rm b} = 0.2485 \pm 0.0035$ $Y_{\rm b} = 0.243$ (GS98)

Inner radius of the solar convective envelope

 $R_{\rm b}/R_{\odot} = 0.713 \pm 0.001$ $R_{\rm b}/R_{\odot} = 0.712$ (GS98)

... till few years ago

Asplund et al. 05 (AGS05); Asplund et al. 09 (AGSS09)

Re-determination of the photospheric abundances of nearly all available elements (inputs for SSM calculations)

Improvements with respect to previous analysis^(*):

- 3D model instead of the classical 1D model of the lower solar atmosphere
- Careful and very demanding selection of the spectral lines... AVOID blends!!! NOT TRIVIAL!!!
- Careful choice of the atomic and molecular data NOT TRIVIAL!!!!
- NLTE instead of the classical LTE hypothesis... WHEN POSSIBLE !!!
- Use of ALL indicators (atoms as well as molecules, CNO)





The solar composition problem

AGS05 and AGSS09

Downward revision of heavy elements photospheric abundances ...

Element	GS98	AGSS09	δz_i	1
С	8.52 ± 0.06	8.43 ± 0.05	0.23	
Ν	7.92 ± 0.06	7.83 ± 0.05	0.23	
Ο	8.83 ± 0.06	8.69 ± 0.05	0.38	
Ne	8.08 ± 0.06	7.93 ± 0.10	0.41	-
Mg	7.58 ± 0.01	7.53 ± 0.01	0.12	-
Si	7.56 ± 0.01	7.51 ± 0.01	0.12	
\mathbf{S}	7.20 ± 0.06	7.15 ± 0.02	0.12	
Fe	7.50 ± 0.01	7.45 ± 0.01	0.12	1
Z/X	0.0229	0.0178	0.29	1
$[I/H] \equiv 1$	$\log\left(N_I/N_H ight)$	+ 12		

The solar composition problem



... leads to SSMs which do not correctly reproduce helioseismic observables

	AGSS09	GS98	Obs.	
$Y_{\rm b}$	$0.2319(1\pm0.013)$	$0.2429(1\pm0.013)$	0.2485 ± 0.0035	/~ Aσ discremencies)
$R_{ m b}/R_{\odot}$	$0.7231 (1 \pm 0.0033)$	$0.7124(1 \pm 0.0033)$	0.713 ± 0.001	(~ 40 discrepancies)
$\Phi_{ m pp}$	$6.03(1\pm0.005)$	$5.98(1\pm 0.005)$	$6.05(1^{+0.003}_{-0.011})$	
$\Phi_{ m Be}$	$4.56(1\pm 0.06)$	$5.00(1\pm 0.06)$	$4.82(1^{+0.05}_{-0.04})$	
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$\Phi_{ m O}$	$1.56(1\pm0.10)$	$2.23(1\pm0.10)$	≤ 3.2	

So what ...

Is there something **wrong** or **unaccounted** in solar models?

Is the **chemical evolution** not understood (extra mixing?) or peculiar (accretion?) with respect to other stars?

Are properties of the solar matter (e.g. **opacity**) correctly described?

Is this discrepancy pointing at **new physics** (e.g. WIMPs in the solar core?)

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Note that:

It is not just the problem of deciding between AGSS09 (new) and GS98 (old and presumably wrong) abundances

 \rightarrow Theory of stellar interiors confronts with stellar atmosphere models

The Sun provide the **benchmark** for stellar evolution. If there is something wrong in solar models, then this is wrong for all the stars ...

A quantitative analysis of the solar composition problem

To combine observational infos, we need an estimator that is **non-biased** and that can be used as a **figure-of-merit** for solar models with different composition:

$$\chi^{2} = \min_{\{\xi_{I}\}} \left[\sum_{Q} \left(\frac{\delta Q - \sum_{I} \xi_{I} C_{Q,I}}{U_{Q}} \right)^{2} + \sum_{I} \xi_{I}^{2} \right] .$$
Fogli et al. 2002
$$\delta Q = \frac{Q_{obs} - Q}{Q}$$
where:
$$\{\delta Q\} = \{\delta \Phi_{B}, \delta \Phi_{Be} \ \delta Y_{b}, \delta R_{b}; \delta c_{1}, \delta c_{2}, \dots, \delta c_{30}\}$$
⁷Be and ⁸B neutrino
fluxes
$$Surface helium and
convective radius
$$Sound \text{ speed data points}
(from Basu et al, 2009)$$
and:
$$U_{Q}$$
Uncorrelated (observational) errors
and:
$$U_{Q,I}$$
Correlated (systematical) uncertainties$$

We consider 18 input parameters:

$$\{I\} = \{ \text{opa, age, diffu, lum,} \\ S_{11}, S_{33}, S_{34}, S_{17}, S_{e7}, S_{1,14}, S_{\text{hep}}, \\ C, N, O, Ne, Mg, Si, S, Fe \}$$
 Nuclear Composition

The status of the AGSS09 standard solar model

The SSM implementing the AGSS09 composition provides a poor fit of the observational data (χ^2 / d.o.f. = 72.5/34; χ^2_{obs} = 42.9 ; χ^2_{syst} = 29.6)

$$\chi^2 \equiv \chi^2_{\rm obs} + \chi^2_{\rm syst} = \sum_Q \tilde{X}_Q^2 + \sum_I \tilde{\xi}_I^2$$

 $\overline{\xi}_{I} \equiv Pulls \text{ of systematic}$ $\tilde{X}_{Q} \equiv rac{\delta Q_{
m obs} - \sum_{I} \tilde{\xi}_{I} C_{Q,I}}{U_{Q}}$

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The distribution of the pulls of systematics highlight tensions in the model:



Obs. data requires an increase of the metal abundance of the sun, in particular for light elements (O, Ne).

Inferring the solar composition ...

We take the **surface abundances** (with respect to hydrogen) as free parameters:

$$z_{\rm j} \equiv Z_{\rm j,b}/X_{\rm b}$$

We group metals according to the method by which they are determined

$$1 + \delta z_{\text{CNO}} \equiv \frac{z_{\text{C}}}{\overline{z}_{\text{C}}} \equiv \frac{z_{\text{N}}}{\overline{z}_{\text{N}}} \equiv \frac{z_{\text{O}}}{\overline{z}_{\text{O}}} \qquad (photosphere)$$

$$1 + \delta z_{\text{Ne}} \equiv \frac{z_{\text{Ne}}}{\overline{z}_{\text{Ne}}} \qquad (chromosphere and corona)$$

$$1 + \delta z_{\text{Heavy}} \equiv \frac{z_{\text{Mg}}}{\overline{z}_{\text{Mg}}} \equiv \frac{z_{\text{Si}}}{\overline{z}_{\text{Si}}} \equiv \frac{z_{\text{S}}}{\overline{z}_{\text{S}}} \equiv \frac{z_{\text{Fe}}}{\overline{z}_{\text{Fe}}} \qquad (meteorites)$$

We infer the **best-fit composition** by minimizing the χ^2



- Results are presented by using the astronomical scale for logarithmic abundances ε_j in order to facilitate comparison with obs. data.
- The coloured lines are obtained by cutting at 1, 2, 3 σ confidence levels.
- The data points show the obs. values (and 1σ errors) for oxygen and iron abundances in the AGSS09, GS98 and CO5BOLD compilations.



Note that: the error budget for ⁸B and ⁷Be neutrinos is dominated by systematical uncertainties

	Age	Diffu	Lum	S_{11}	S_{33}	S_{34}	S_{17}	S_{e7}	$S_{1,14}$	Opa
$Y_{\rm b}$	-0.001	-0.012	0.002	0.001	0	0.001	0	0	0.	0.0036
$R_{ m b}$	-0.0004	-0.0029	-0.0001	-0.0006	0.0001	-0.0002	0	0	0	0.0014
$\Phi_{\rm pp}$	0	-0.002	0.003	0.001	0.002	-0.003	0	0	0	-0.0008
$\Phi_{\rm Be}$	0.003	0.022	0.014	-0.010	-0.023	0.047	0	0	0	0.009
$\Phi_{\rm B}$	0.006	0.044	0.029	-0.025	-0.022	0.046	0.075	-0.02	0	0.020
$\Phi_{\rm N}$	0.004	0.054	0.018	-0.019	0.001	-0.003	0	0	0.051	0.013
$\Phi_{\rm O}$	0.006	0.062	0.024	-0.027	0.001	-0.002	0	0	0.072	0.018

Table 1: The contributions $C_{Q,I}$ to uncertainties in theoretical predictions for helioseismic observables and solar neutrino fluxes.





♦ The AGSS09 composition is **excluded** at an high confidence level being χ^2 / d.o.f. = 176.7/32.

♦ Substantial agreement between the infos provided by the various observational constraints. The quality of the fit is quite good being χ^2 / d.o.f. = 39.6/32.

♦ The best-fit abundances are consistent at 1 sigma with GS98.

♦ The errors on the inferred abundances are smaller than what is obtained by observational determinations.

The CNO neutrino fluxes are expected to be ~50% larger than predicted by AGSS09 (this result, however, depends on the assumed heavy element grouping).

Prior: Neon-to-oxygen ratio forced at the AGSS09 value with 30% accuracy



GS98 still favored by observational data but:

- errors in the inferred abundances larger than before;
- degeneracies appear among the various δZ_i ;
- obs.data do not effectively constrain the Ne/O ratio (we recover the prior).

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What data are really saying us? ...

Metals in the Sun

• Metals give a **substantial** contribution to **opacity**:

Energy producing region ($R < 0.3 R_{o}$)

 $\kappa_{Z} \approx \frac{1}{2} \kappa_{tot}$

Fe gives the largest contribution.

Outer radiative region $(0.3 < R < 0.73 R_{o})$

 $\kappa_{Z} \sim 0.8 \kappa_{tot}$

Relevant contributions from several diff. elements (O,Fe,Si,Ne,...)

F.L. Villante and B. Ricci - Astrophys.J.714:944-959,2010 F.L. Villante – Astrophys.J.724:98-110,2010



Composition opacity change:

$$\delta \kappa_{\rm Z}(r) \simeq \sum_{j} \frac{\partial \ln \kappa(r)}{\partial \ln Z_j} \, \delta z_{\rm j}$$



What we know about the opacity profile of the present sun?

Composition opacity change

$$\delta \kappa_{\rm Z}(r) \simeq \sum_j \frac{\partial \ln \kappa(r)}{\partial \ln Z_j} \, \delta z_{\rm j}$$





Are there other effects that can provide the required opacity change?

Wrong **opacity** calculations? \rightarrow the required variations seems large wrt uncertainties

Different **distribution of metals** in the Sun? \rightarrow According to the standard assumption, metals are nearly omogeneous in the sun (elemental diffusion is responsible for a slight increase at the solar center). Is this an oversimplified picture of chemical evolution?

Is this discrepancy pointing at **new physics** (e.g. WIMPs in the solar core?)



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... Not just a problem of AGSS09.vs.GS98

CNO neutrinos

CNO neutrinos break the degeneracy between opacity and metals:



$$1 + \delta \Phi_{\nu} = \underbrace{\left(1 + \delta X_{\rm CN}\right)}_{X_{\rm CN}} \begin{bmatrix} 1 + \int dr \ K_{\nu}(r) \ \delta \kappa(r) \end{bmatrix}$$
$$X_{\rm CN} \equiv X_{\rm C}/12 + X_{\rm N}/14$$

Determines the central temperature

Total number of catalysts for CN-cycle

At present, we only have a loose upper limit on CNO neutrino fluxes:

ν flux	GS98	AGSS09	Solar
$^{-13}$ N (10 ⁸ cm ⁻² s ⁻¹)	$2.96(1 \pm 0.14)$	$2.17(1 \pm 0.14)$	≤ 6.7
$^{15}O~(10^8\mathrm{cm}^{-2}\mathrm{s}^{-1})$	$2.23(1 \pm 0.15)$	$1.56(1 \pm 0.15)$	≤ 3.3
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Will it be possible to detect CNO neutrino?

Very difficult, in practice. Not impossible, in principle

Is it possible to observe CNO neutrinos in LS?

The detection of CNO neutrinos is very difficult:

- Low energy neutrinos
- \rightarrow endpoint at about 1.5 MeV
- Continuos spectra \rightarrow do not produce recognizable features in the data.
- Limited by the background produced by beta decay of ²¹⁰Bi.



Event spectrum in ultrapure liquid scintillators (Borexino-like)

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Event spectrum in ultrapure liquid scintillators (Borexino-like)

Summary and conclusions

+ Solar neutrino physics in still interesting

+The solar composition problem is open and is potentially pointing at inadequacy in standard solar model paradigm.

+Hopefully, all the components of the solar neutrino flux measurements (pp, CNO) will be directly determined the future.

+CNO neutrino detection requires careful bkgd evaluation in existing or next future LS detectors and/or new experimental approaches.

(see Orebi-Gann@Neutrino2014 for a discussion of future expt. tecniques)

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• A step toward CNO neutrino detection

F.L. Villante et al. - Phys.Lett. B701 (2011) 336-341

Determining ²¹⁰Bi with the help of ²¹⁰Po?

$$^{210}\text{Bi} \rightarrow^{210}\text{Po} + e^- + \overline{\nu}_e$$
 $t_{Bi} = 7.232 \text{ d}$
 $^{210}\text{Po} \rightarrow^{206}\text{Pb} + \alpha$ $t_{Po} = 199.634 \text{ d}$

Event spectrum in ultrapure liquid scintillators



Deviations from the exponential decay law of Po210 can be used to determine Bi210:

$$n_{\rm Po}(t) = [n_{\rm Po,0} - n_{\rm Bi}] \exp(-t/\tau_{\rm Po}) + n_{\rm Bi}$$

Expected accuracy:

