Sky map reconstruction

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Map making method

Sky map reconstruction from visibilities for a transit telescope

Full α scan means we could perform Fourier transform (FFT)

$$V_{ij}(\alpha, \beta_0) \quad (0 < \alpha < 2\pi) \longrightarrow \widetilde{V_{ij}}(u, \beta_0) \quad for\ all\ u$$

$$\widetilde{V_{ij}}(u, \beta_0) = \mathscr{F}(V_{ij}(\alpha, \beta_0)) = \sum_v F_{sky}(u, v) F_{beam}(u - \frac{\Delta x}{\lambda}, v - \frac{\Delta y}{\lambda}) \exp(i 2\pi v \beta_0)$$

unknown: $F_{sky}(u, v)$

Known: A matrix

We could express our function as matrix, (we keep only one Vij for a set of antenna with the same baseline, to simplify numerical handling)

The full problem of all $V_{ij}(u)$ could be separated into a set of independent problems for each u which is independent with each other.

$$\begin{pmatrix} \cdots \\ \widetilde{V_{ij}}(u, \beta_0) \\ \cdots \end{pmatrix} = A \times \begin{pmatrix} \cdots \\ F_{sky}(u, v) \\ \cdots \end{pmatrix} + noise \longrightarrow \begin{pmatrix} \cdots \\ \widetilde{V_{ij}}(\beta_0) \\ \cdots \end{pmatrix} = A_{u_0} \times \begin{pmatrix} \cdots \\ F_{sky}(u_0, v) \\ \cdots \end{pmatrix} + noise$$

Fixed u: u --> u_0

A_{u0}

$$\begin{pmatrix} \widetilde{V_{11}}(\beta_0) \\ \widetilde{V_{11}}(\beta_1) \\ \cdots \\ \widetilde{V_{11}}(\beta_n) \\ \widetilde{V_{ij}}(\beta_0) \\ \widetilde{V_{ij}}(\beta_1) \\ \cdots \\ \widetilde{V_{ij}}(\beta_n) \end{pmatrix} = \begin{pmatrix} F_b(v_1) \exp 2i\pi\beta_0 & F_b(v_2) \exp 2i\pi\beta_0 & \cdots & F_b(v_m) \exp 2i\pi\beta_0 \\ & \cdots & & \\ F_b(v_1) \exp 2i\pi\beta_n & F_b(v_2) \exp 2i\pi\beta_n & \cdots & F_b(v_m) \exp 2i\pi\beta_n \\ F_{bij}(v_1) \exp 2i\pi y_0 & F_{bij}(v_2) \exp 2i\pi\beta_0 & \cdots & F_{bij}(v_m) \exp 2i\pi\beta_0 \\ & \cdots & & \\ F_{bij}(v_1) \exp 2i\pi\beta_n & F_{bij}(v_2) \exp 2i\pi\beta_n & \cdots & F_{bij}(v_m) \exp 2i\pi\beta_n \end{pmatrix} \times \begin{pmatrix} F_s(v_1) \\ \cdots \\ F_s(v_m) \end{pmatrix} + noise$$

$$noise = \begin{pmatrix} n_{11}(\beta_0) \\ n_{11}(\beta_1) \\ \cdots \\ n_{11}(\beta_n) \\ n_{ij}(\beta_0) \\ n_{ij}(\beta_1) \\ \cdots \\ n_{ij}(\beta_n) \end{pmatrix}$$

- Now, we use pseudo-inverse (singular value decomposition) considering noise covariance matrix N.
- $N = < n^t n >$:noise covariance matrix on $\widetilde{V_{ij}}(u, \beta_0)$
- We assume that matrix N is positive and symmetric

$$B = (A^t N^{-1} A)^{-1} A^t N^{-1} = (N^{-\frac{1}{2}} A)^{-1} N^{-\frac{1}{2}}$$

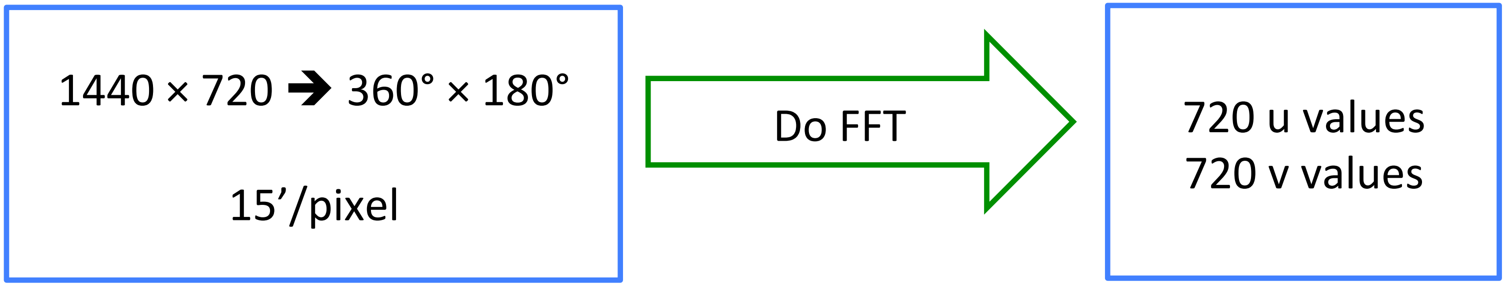
Pseudo-inverse with N

$$\begin{pmatrix} \cdots \\ \widehat{F}_{sky}(u_0, v) \\ \cdots \end{pmatrix} = \left((N^{-\frac{1}{2}} A_{u_0})^{-1} N^{-\frac{1}{2}} \right) \times \begin{pmatrix} \cdots \\ \widetilde{V_{ij}}(\beta_0) \\ \cdots \end{pmatrix}$$

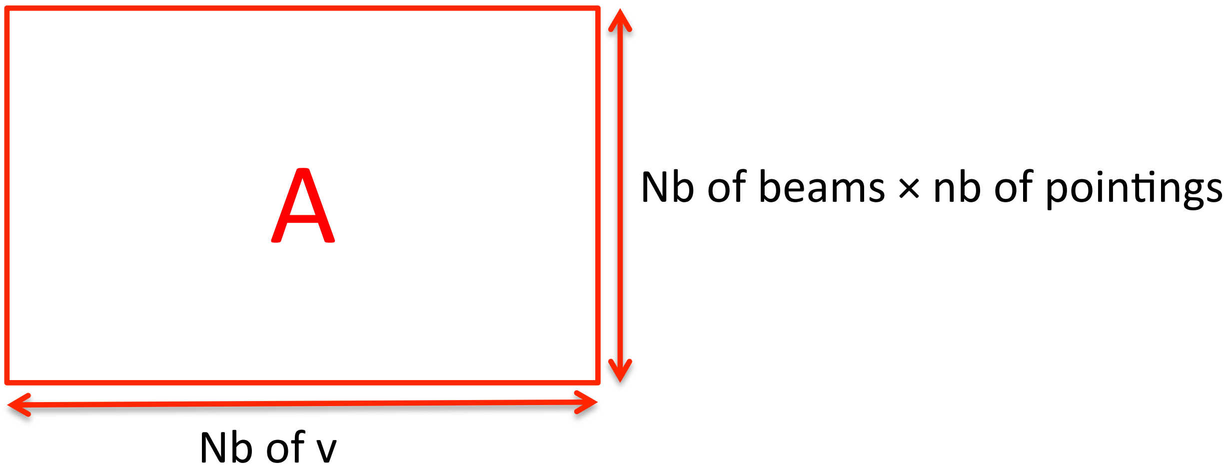
B

- We could compute F_{sky}^hat(u_0,v) error covariance matrix for each u_0

$$\sigma^2_{\widehat{F}} = BNB^t$$

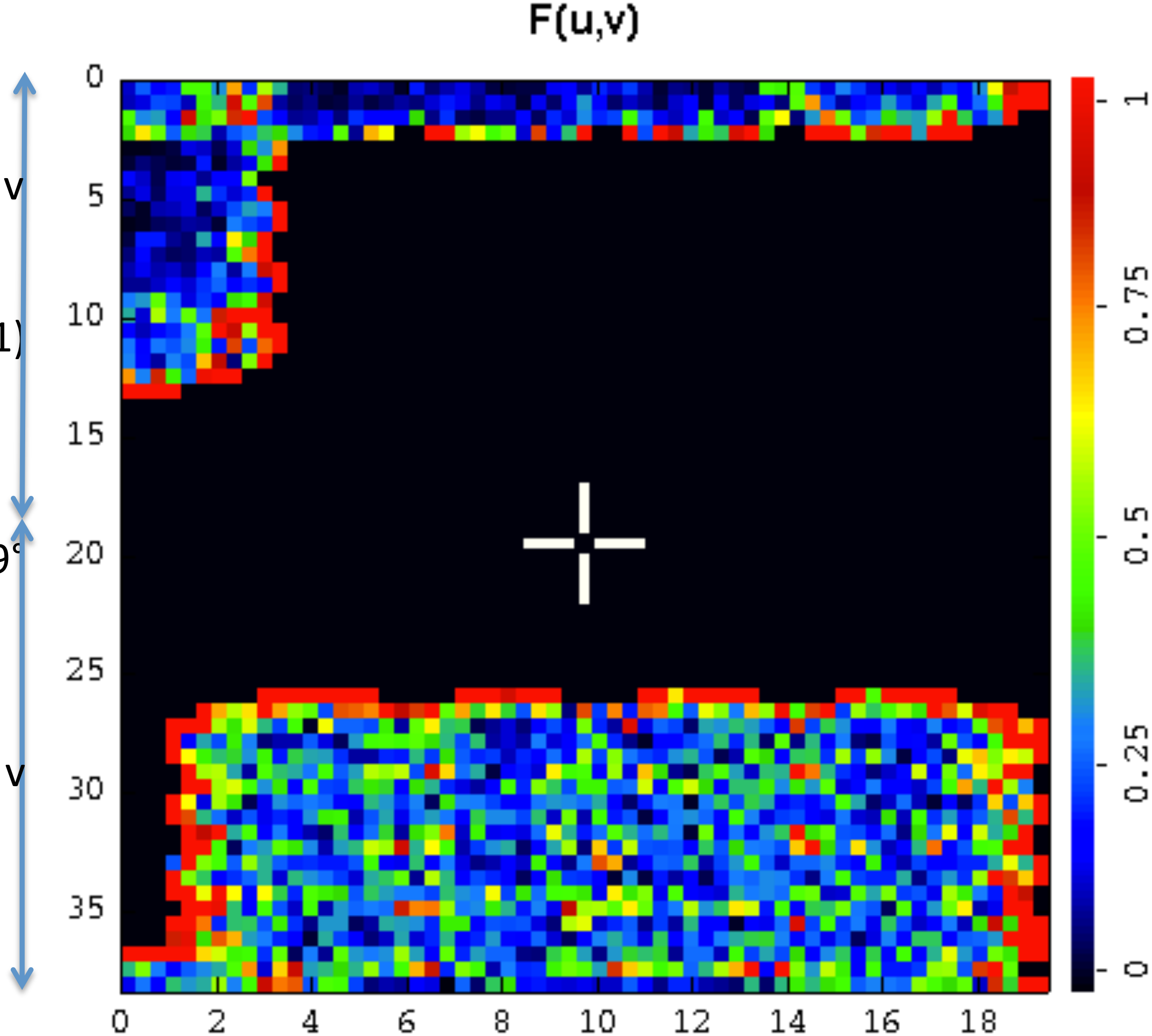


- For each frequency and for each u, we solve $B=A^{-1}$



- PAON4: 7 beams; resolution ≈ 50’/pixel
- 16 dishes with T shape: 66 beams;

L-shape: $\widehat{F}(u,v)$ of white noise



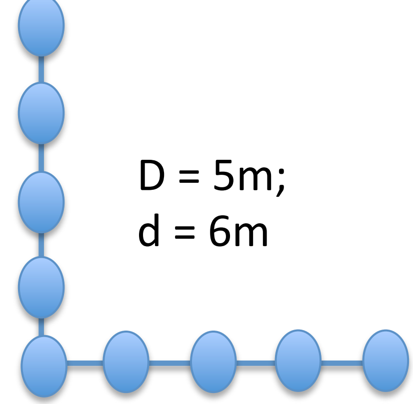
Positive v

v(rad^-1)

V: 20 -> 2.9°

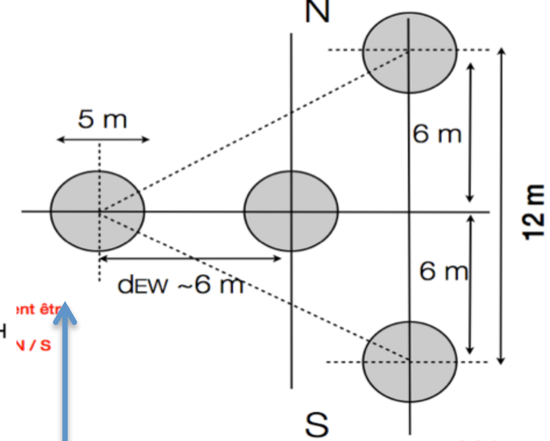
Negative v

u(rad^-1): 10 -> 5.7°; 18 -> 3.1°



D = 5m;
d = 6m

PAON4: $\widehat{F}(u,v)$ of white noise

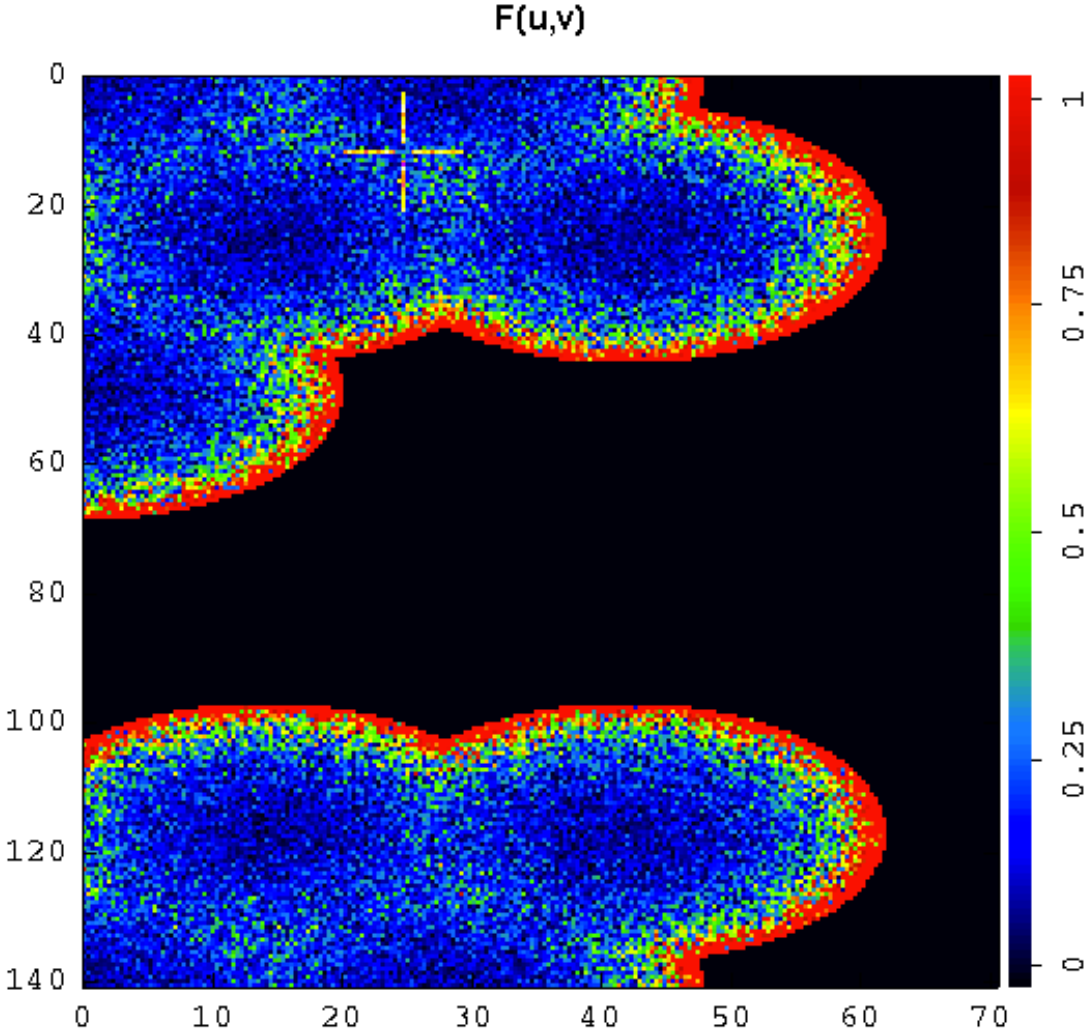


v(rad^-1)

V: 20 -> 2.9°

V: 60 -> 57'

V: 120 -> 28'

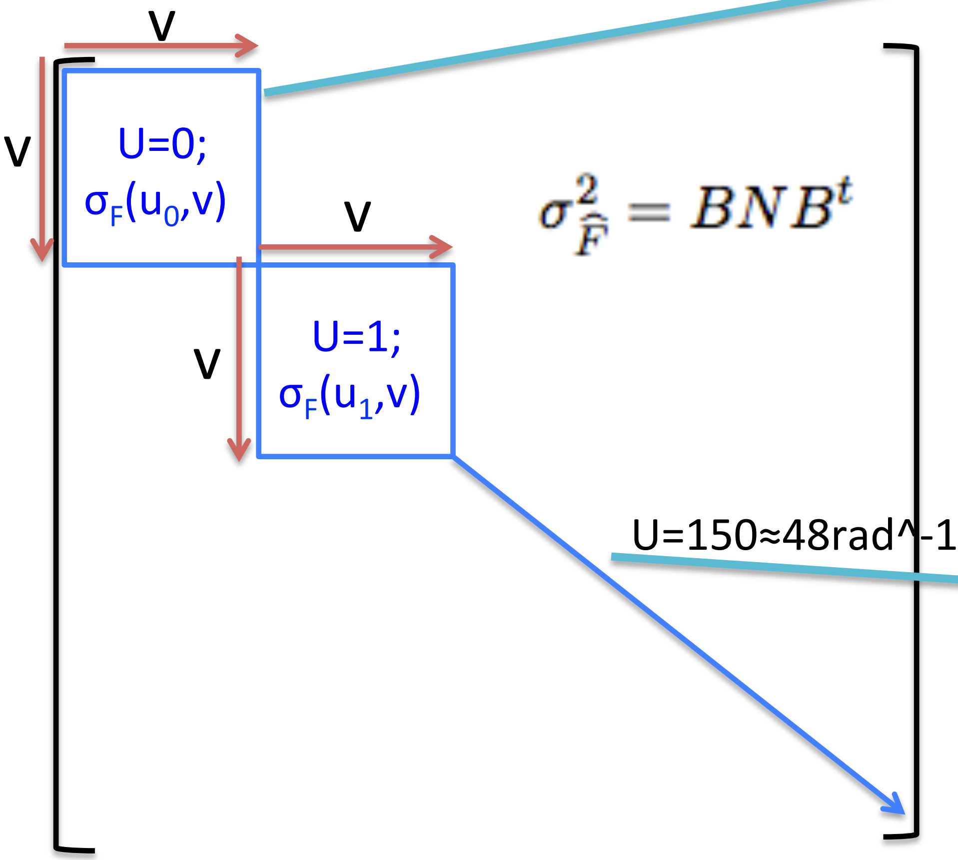


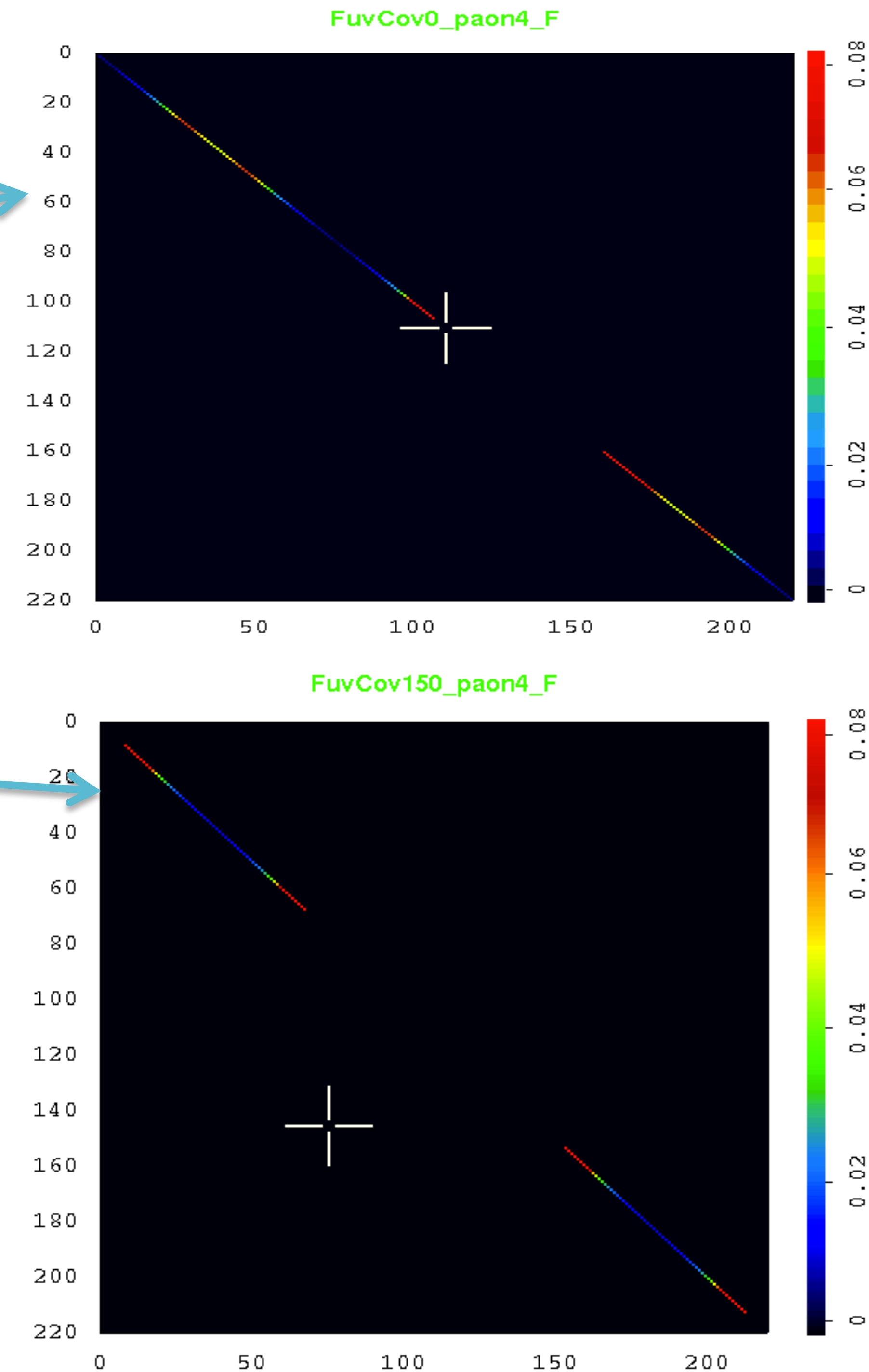
Positive v

Negative v

u(rad^-1): 10 -> 5.7°; 30 -> 1.9°; 60 -> 57'

Reconstruct F(u,v) error covariance matrix

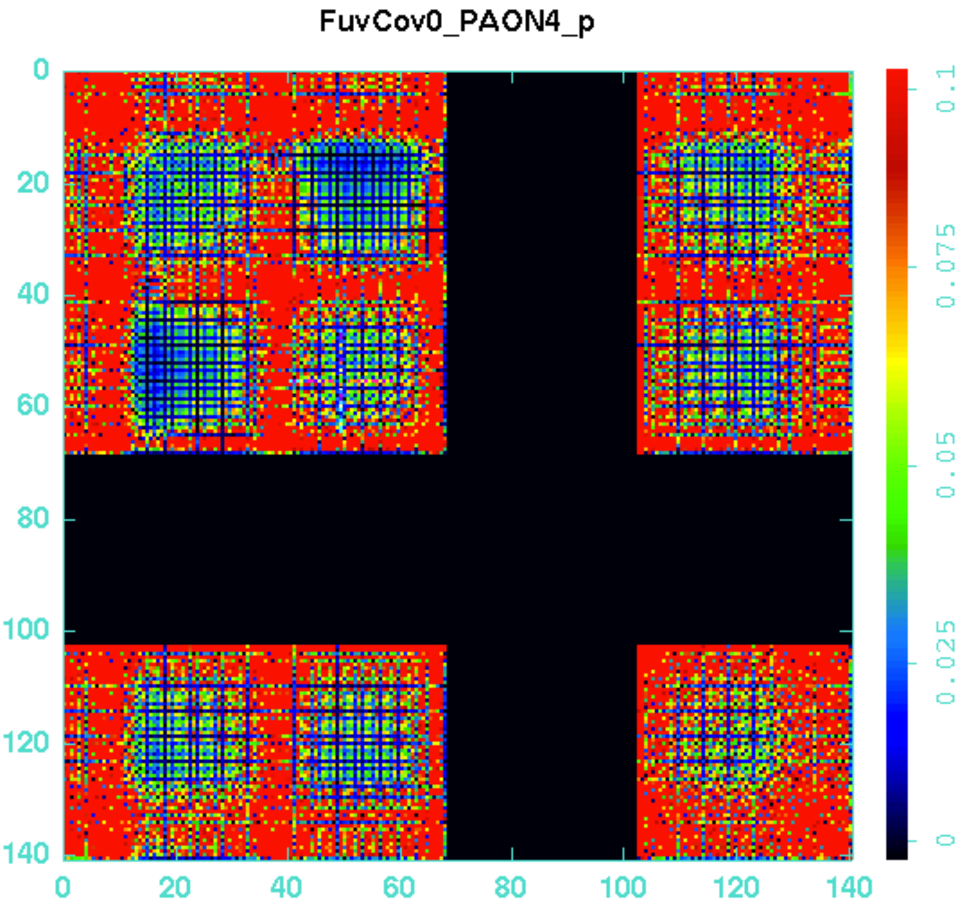




Full NS scan

Reconstruct F(u,v) error covariance matrix for partial NS scan

U=0



U=150≈48rad^-1