Modification of Gravity

LPT Orsay, CNRS

Paris 21cm Intensity Mapping workshop

General Relativity

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- Mach's principle
- Equivalence principle

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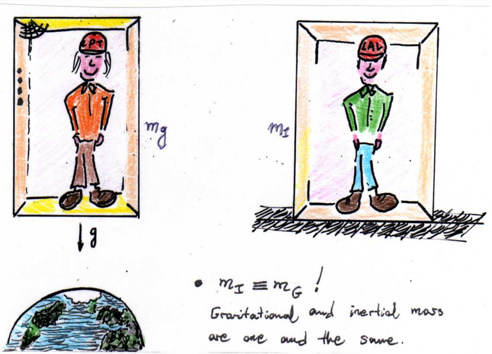


General Relativity

GR is based on two important principles:

- Mach's principle The presence of matter curves the geometry of spacetime
- Equivalence principle Locally a free-falling observer and an inertial observer are indistinguishable
- This means:
  -Gravity is a local condition of spacetime
  -Gravity sees all (including vacuum energy!)
  -In Newtonnian gravity m_I and m_G happen to be the same, in GR it is a founding principle

GR is a unique theory

- Theoretical consistency: In 4 dimensions, consider $\mathcal{L} = \mathcal{L}(\mathcal{M}, g, \nabla g, \nabla\nabla g)$. Then Lovelock's theorem in $D = 4$ states that GR with cosmological constant is the unique metric theory emerging from,

$$S_{(4)} = \int_{\mathcal{M}} d^4x \sqrt{-g^{(4)}}\,[R - 2\Lambda]$$

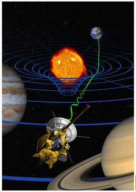
giving,

  - Equations of motion of 2^{nd}-order (Ostrogradski no-go theorem 1850!)
  - given by a symmetric two-tensor, $G_{\mu\nu} + \Lambda g_{\mu\nu}$
  - and admitting Bianchi identities.

Under these hypotheses GR is the unique massless-tensorial 4 dimensional theory of gravity!

Experimental and observational data in weak gravity

- Experimental consistency:
  -Excellent agreement with solar system tests
  -Strong gravity tests on binary pulsars
  -Laboratory tests of Newton's law (tests on extra dimensions)

Time delay of light  Planetary tajectories 

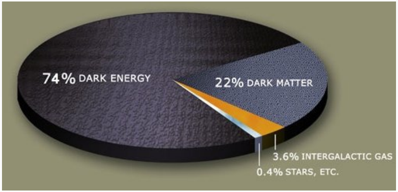
...

Q: What is the matter content of the Universe today?

Assuming homogeinity-isotropy and GR

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

cosmological and astrophysical observations dictate the matter content of the Universe today:



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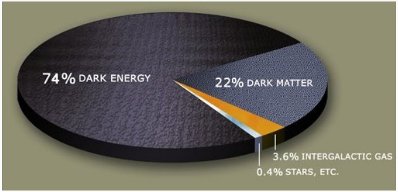

A: -Only a 4% of matter has been discovered in the laboratory. We hope to see more at LHC. But even then...

If we assume only ordinary sources of matter (DM included) there is disagreement between local, astrophysical and cosmological data.

Universe is accelerating → Enter the cosmological constant

Easiest way out: Assume a tiny cosmological constant $\rho_\Lambda = \frac{\Lambda_{obs}}{8\pi G} = (10^{-3} eV)^4$, ie modify Einstein's equation by,

$$G_{\mu\nu} + \Lambda_{obs} g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Cosmological constant introduces $\sqrt{\Lambda}$ and generates a cosmological horizon
- $\sqrt{\Lambda}$ is as tiny as the inverse size of the Universe today, $r_0 = H_0^{-1}$
- Note that $\frac{\text{Solar system scales}}{\text{Cosmological Scales}} \sim \frac{10 \text{ A.U.}}{H_0^{-1}} = 10^{-14}$
- Theoretically the cosmological constant should be huge [S Weinberg Rev. Mod. Phys. 1989]

$\Lambda_{obs} \sim \Lambda_{vac} + \Lambda_{pot} + \Lambda_{bare}$

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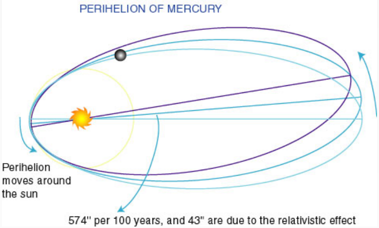
Maybe Λ_{obs} is not a cosmological constant.

What if the need for exotic matter or cosmological constant is the sign for novel gravitational physics at very low energy scales or large distances.



-Same situation at the advent of GR.
-A next order correction with one additional parameter was enough to save Newton's laws (at the experimental precision of the time..)
-Success of GR is not the advance of Mercury's perihelion, modification of gravity cannot only be "an explanation" of the cosmological constant.

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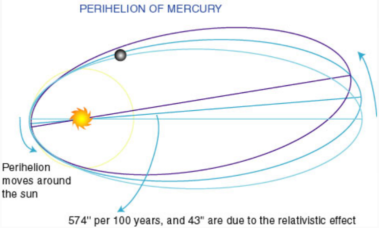

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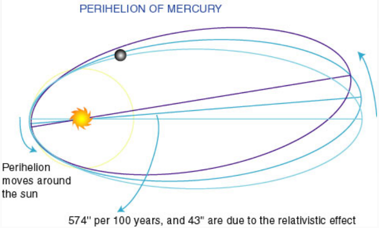

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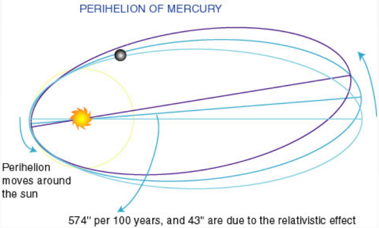

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"Black holes have no hair" ([Wheeler])

During gravitational collapse...

Black holes eat or expel surrounding matter

their stationary phase is characterized by a limited number of charges and no details

black holes are bald...

GR satisfies stringent black hole theorems.

No hair arguments dictate that adding degrees of freedom lead to singular solutions...

Birkhoff theorem

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Birkhoff theorem in GR

Relates strong gravity to weak gravity regimes in GR

- A spherically symmetric source in vacuum has the geometry of a Schwarzschild black hole!
- Fundamental length scale of a source $r_s = 2GM$
- Sun $r_{Sch} \sim 3km$ hence exterior gravitational field is the Schwarzschild solution and linear regime of GR is an excellent approximation.
- Typical strength $\epsilon = \frac{r_{Sch}}{r_d}$ and $\epsilon \sim 10^{-5}$ for solar system.
- ... Eddington parameters associated to GR agree to local gravity observations, $ds^2 \sim -(1 - 2\Phi + 2\beta\Phi^2)dt^2 + (1 + 2\gamma\Phi + ...)dx^i dx^j$
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General issues to deal with

- Since GR is unique we need to introduce new and genuine gravitational degrees of freedom!
- They must not lead to higher derivative equations of motion. For then additional degrees of freedom are ghosts and vacuum is unstable (Ostrogradski theorem 1850 [Woodard 2006, Rubakov 2014])
- Matter must not directly couple to novel gravity degrees of freedom. Matter sees only the metric and evolves in metric geodesics. As such EEP is preserved and space-time can be put locally in an inertial frame.
- Novel degrees of freedom need to be screened from local gravity experiments. Need a well defined GR local limit (Chameleon [Khoury 2013], Vainshtein [Babichev and Deffayet 2013]).
- Exact solutions essential in modified gravity in order to understand strong gravity regimes and novel characteristics. Need to deal with no hair paradigm.
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Possible modified gravity theories

- Assume extra dimensions : Extension of GR to Lovelock theory with modified yet second order field equations. Braneworlds DGP model RS models, Kaluza-Klein compactfication
- Graviton is not massless but massive! dRGT theory and bigravity theory. Theories are unique.
- 4-dimensional modification of GR: Scalar-tensor theories, $f(R)$, Galileon/Hornedski theories.
- Lorentz breaking theories: Horava gravity, Einstein Aether theories [Audren, Blas, Lesgourgues and Sibiryakov]
- Theories modifying geometry: inclusion of torsion, choice of geometric connection [Olmo 2012]

Scalar-tensor theories

- are the simplest modification of gravity with one additional degree of freedom
- Admit a uniqueness theorem due to Horndeski 1973
- contain or are limits of other modified gravity theories. $F(R)$ is a scalar tensor theory in disguise
- (Can) have insightful screening mechanisms (Chameleon, Vainshtein)
- Include terms that can screen classically a big cosmological constant (Fab 4 [CC, Copeland, Padilla and Saffin 2012])

Jordan-Brans-Dicke theory [Sotiriou 2014]

Simplest scalar tensor theory

$$S_{\rm BD} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\varphi R - \frac{\omega_0}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - m^2(\varphi-\varphi_0)^2\right) + S_m(g_{\mu\nu},\psi)$$

- ω_0 Brans Dicke coupling parameter fixing scalar strength
- $\phi = \phi_0$ constant gives GR solutions (with a cosmological constant) but spherically symmetric solutions are not unique!
- For spherical symmetry we find,

$$\gamma \equiv \frac{h_{ij}|_{i=j}}{h_{00}} = \frac{2\omega_0 + 3 - exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}{2\omega_0 + 3 + exp\left[-\sqrt{\frac{2\varphi_0}{2\omega_0+3}}mr\right]}$$

- where $\gamma = 1 + (2.1 \pm 2.3)\times 10^{-5}$
- $\omega_0 > 40000$
- Need a more complex version in order to screen the scalar mode locally

Galileons/Horndeski [Horndeski 1973], [Deffayet et.al.]

What is the most general scalar-tensor theory

with second order field equations?

Horndeski has shown that the most general action with this property is

$$S_H = \int d^4x \sqrt{-g}\,(L_2 + L_3 + L_4 + L_5)\ , \qquad (1)$$

where

$$L_2 = K(\phi, X), \qquad (2)$$

$$L_3 = -G_3(\phi, X)\Box\phi, \qquad (3)$$

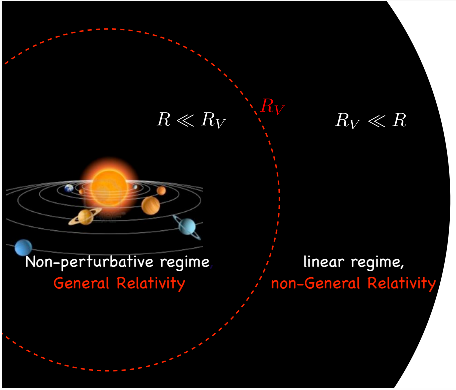
$$L_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \qquad (4)$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right], \qquad (5)$$

the G_i are unspecified functions of ϕ and $X \equiv -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$ and $G_{iX} \equiv \partial G_i/\partial X$.

- In fact same action as covariant Galileons [Deffayet, Esposito-Farese, Vikman]
- Theory screens generically scalar mode locally by the Vainshtein mechanism.

Implementing the Vainshtein mechanism



- MG theories with higher order kinetic terms sensitive to non-linear effects at small distances for $R_{NL} >> R_{Solar} >> R_{Sch}$.
- Where GR had a linear approximation MG does not!
- Vainshtein scale $R_V >> R_S$. A kind of Schw. scale for MG theories $R_V = R_V(m, r_{Sch})$
- Vainshtein mechanism gives GR as classical limit due to non-linear self-interaction.
- Restoring GR in massive gravity, in scalar-tensor models etc.

Review on the Vainshtein mechanism
EB & C.Deffayet 2013

Horndeski theory and black holes

Consider the action [Babichev and CC],

$$S = \int d^4x \sqrt{-g} \left[\zeta R - 2\Lambda - \eta \left(\partial\phi\right)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right],$$

- Important property: Scalar field has translational invariance : $\phi \to \phi$+const.,
- Scalar field equation :

$$\nabla_\mu J^\mu = 0, \quad J^\mu = \left(\eta g^{\mu\nu} - \beta G^{\mu\nu}\right) \partial_\nu \phi.$$

- Key assumption: $\beta G^{rr} - \eta g^{rr} = 0$ and $\phi(t,r) = q\,t + \psi(r)$
  Spherical symmetry: $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
- Solution: $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2$ with $\Lambda_{\text{eff}} = -\zeta\eta/\beta$
- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- $q^2\eta = \Lambda - \Lambda_{eff} > 0$
- Hence for arbitrary $\Lambda > \Lambda_{eff}$ fixes q, integration constant.
- Λ_{eff} is a geometric acceleration screening the vacuum cosmological constant

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  Spherical symmetry: $ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$
- Solution: $f = h = 1 - \frac{\mu}{r} + \frac{\eta}{3\beta} r^2$ with $\Lambda_{\text{eff}} = -\zeta\eta/\beta$
- The effective cosmological constant is not the vacuum cosmological constant. In fact,
- $q^2 \eta = \Lambda - \Lambda_{eff} > 0$
- Hence for arbitrary $\Lambda > \Lambda_{eff}$ fixes q, integration constant.
- Λ_{eff} is a geometric acceleration screening the vacuum cosmological constant

Horndeski theory and black holes

Consider the action [Babichev and CC],

$$S = \int d^4x\sqrt{-g}\left[\zeta R - 2\Lambda - \eta\left(\partial\phi\right)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right],$$

- Important property: Scalar field has translational invariance : $\phi \to \phi$+const.,
- Scalar field equation :

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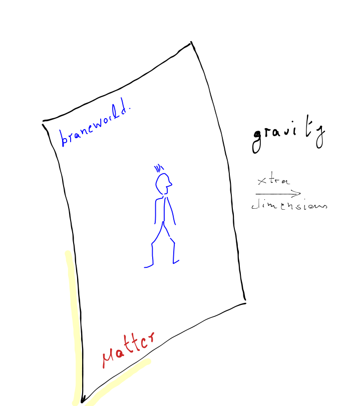
Gravity in higher dimensions

- Inspired from string theory
- Assume a metric theory in higher dimensions. Is it a unique theory? Lovelock theorem [CC 2014]
- small compact extra dimensions, the Kaluza-Klein paradigm relates higher dimensional metric theories to 4 dimensional modified gravity theories.
- Braneworld idea and how strong curvature can accommodate large extra dimensions (RS and DGP models)

Braneworld

Central idea

Matter lives on a distributional brane
gravity lives in a higher dimensional space-time



Braneworld

Interesting phenomenology

DGP IR modification, RS UV modification

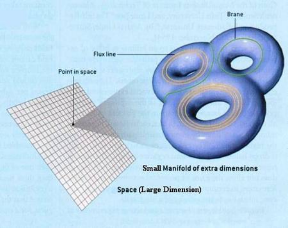
- Introduction of self-acceleration idea [Deffayet]
- Understanding of FRW braneworld cosmology
- Difficulty in understanding cosmological perturbations and localised black holes
- DGP and decoupling limit makes connection with Galileons in 4 dimensions

Kaluza-Klein reduction

Lovelock $\rightarrow$ Scalar Tensor

- Start with 5 or 6 dimensional Lovelock theory.

$$\int d^4x \, d^N X \sqrt{-g} \left[R + \alpha \hat{\mathcal{G}} \right]$$



- Consider small compact extra dimensions...
- Question: What does Kaluza-Klein reduction of Lovelock theory give?
- Higher order scalar tensor Galileon terms (2nd order EOM!)

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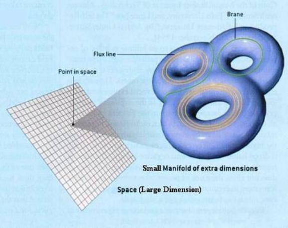

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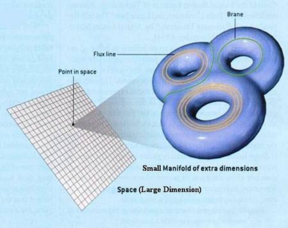

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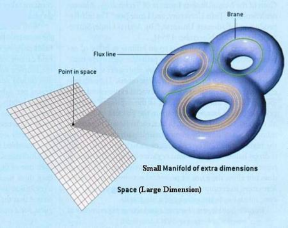

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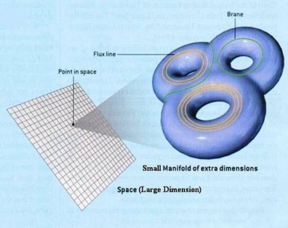

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$$S_{KK} = M\int d^4x\sqrt{-g}e^{-2\phi}\left(R^{(4)} - 4\omega(\nabla\phi)^2\right) + \ldots$$

$$+ M\int d^4x\sqrt{-g}e^{-2\phi}\left(\hat{\mathcal{G}}^{(4)} + G^{(4)}_{ab}\nabla^a\phi\nabla^b\phi + (\nabla\phi)^4 + \nabla^2\phi(\nabla\phi)^2\right)$$

Bigravity and massive gravity

- If $m_G = 0$ then gravity interaction is infinite. But, if $m_G \sim O(\epsilon)$ then priori dark energy could be due to a finite graviton mass.
- Central idea: Introduce graviton potential. We need an extra metric field: $g_{\mu\nu}, f_{\mu\nu}$ so that $U = U(\gamma)$ and $\gamma^{\mu}_{\ \nu} = \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$
- $f_{\mu\nu}$ is dynamical: bigravity theory. Matter only couples to $g_{\mu\nu}$ which is thus the physical metric

$$
\begin{aligned}
S[g,f] &= \frac{1}{2\kappa_g^2} \int d^4x \sqrt{-g}\, R(g) + \frac{1}{2\kappa_f^2} \int d^4x \sqrt{-f}\, \mathcal{R}(f) \\
&- \frac{{}^{2}_{G}}{\kappa^2} \int d^4x \sqrt{-g}\, \mathcal{U}[g,f] \,, \qquad (6)
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$\kappa^2 = \kappa_g^2 + \kappa_f^2$ and m_G is the graviton mass.

- Two parameter unique graviton potential in order to avoid Boulware Deser ghost
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Conclusions

- Modifying GR is a difficult task but with countable possibilities.
- Intuition from GR can be misleading. Perturbation theory problematic due to additional scales
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