F^2-Ray: simulating the cosmic reionization *more efficiently*, and *more robustly*

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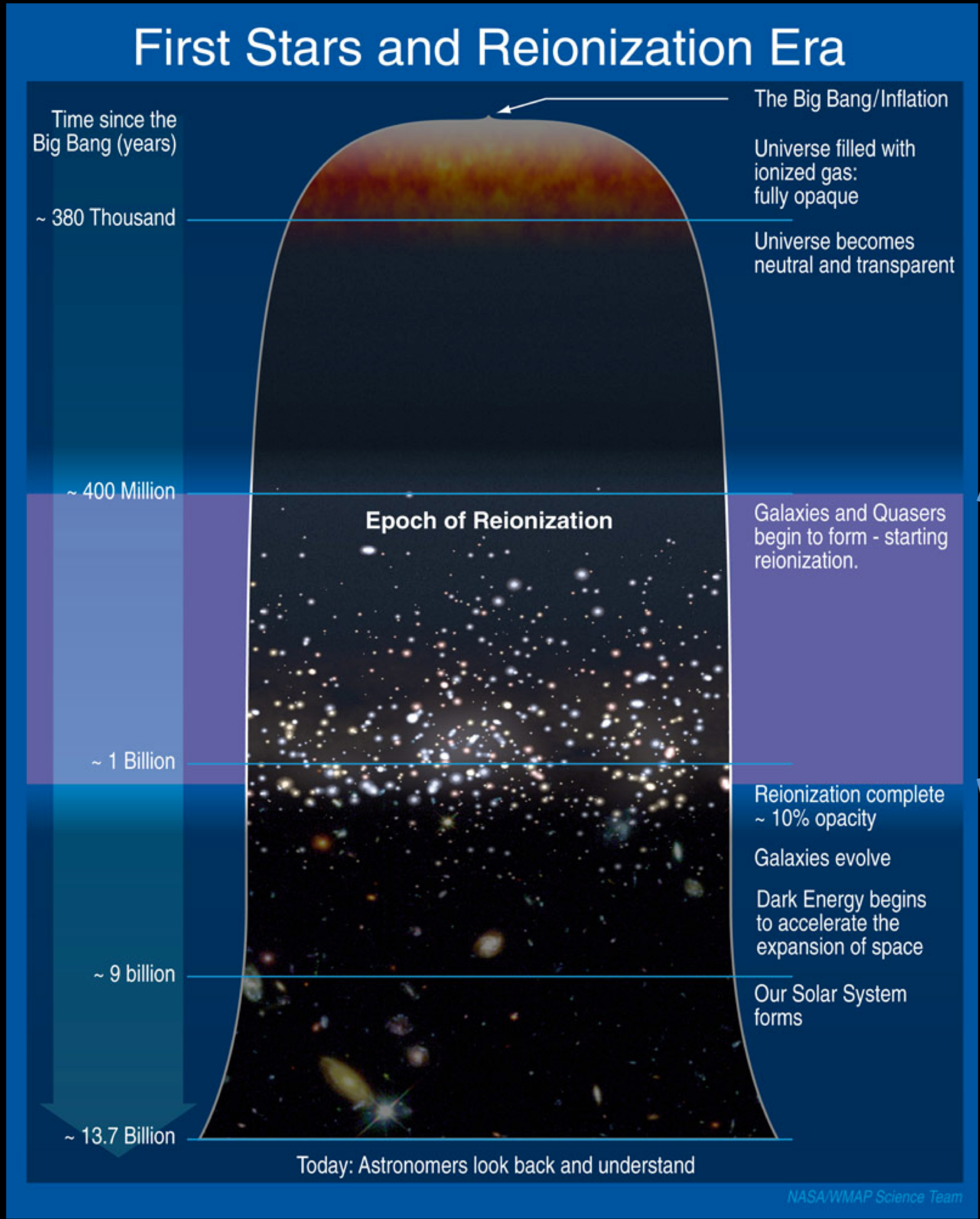
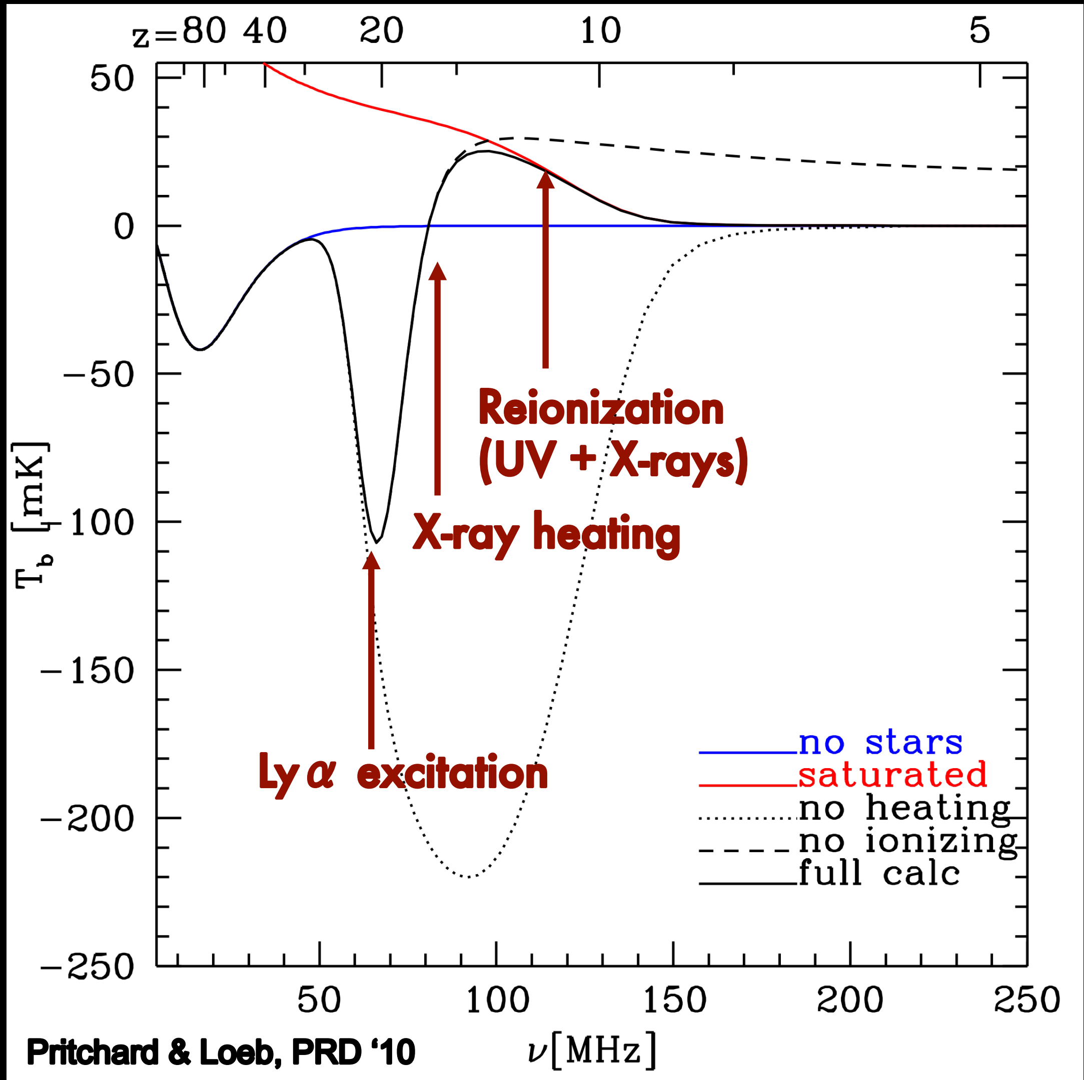


Credit: NASA/WMAP Science Team

Credit: A. Loeb '06

Observable: the 21-cm emission line



LOFAR, MWA, PAPER, 21CMA, GMRT, EDGES

SKA

LEDA, DARE

Central task of numerical RT simulation

solving for (comoving) specific intensity $I_\nu(t, \mathbf{x}, \nu, \hat{n})$

✧ Lagrangian scheme: follow the individual rays

- Ray-tracing methods

  Abel+ '99, Abel & Wandelt '02
  FLASH (Fryxell+ '00)
  Sokasian+ '01
  Razoumov+ '02, Razoumov & Cardall '05
  C^2Ray (Mellema+ '06)
  Susa '06
  McQuinn+ '07
  Trac & Cen '07
  TRAPHIC (Pawlik & Schaye '08, '10)
  START (Hasegawa & Umemura '10)
  SimpleX, SimpleX2 (Kruip+ '10, Paardekooper+ '10)
  AREPO (Petkova & Springel '08, '10, '11)

- Monte Carlo methods

  CRASH, CRASH2 (Ciardi+ '01, Maselli+ '03, '09)
  LICORICE (Semelin+ '07, Baek+ '09, '10)
  SPHRAY (Altay+ '08)

Memory loads may increase dramatically for long mean free path photon packets.

Computational cost may be increasing with the number of sources. Accuracy may be reduced for long mean free path rays.

Central task of numerical RT simulation

solving for (comoving) specific intensity $I_\nu(t, \mathbf{x}, \nu, \hat{n})$

✧ Eulerian scheme: stay on the grid and solve for RT equation

➢ Moments methods

Gnedin & Abel '01

Whalen & Norman '06

ATON/CUDATON (Aubert & Teyssier '08, '10) – *see Pierre Ocvirk's talk*

Finlator+ '09

Pros: – independent of source number

– less memory requirement [only photon density (1), flux density (3), tensor density (6)]

Cons: the solution of a moment equation at a given order requires the input of a higher moment. To close it, prescriptions instead of principles are applied.

Central task of numerical RT simulation

solving for (comoving) specific intensity $I_\nu(t, \mathbf{x}, \nu, \hat{n})$

✧ **Eulerian scheme: stay on the grid and solve for RT equation**

➢ Introducing the new algorithm for numerical RT simulation

F²-Ray

Pros: – independent of source number

– less memory requirement

– based on a new exact solution to RT equation in Eulerian scheme.

F^2-Ray Idea 1: radiative transfer equation in Eulerian scheme

$$\frac{a\partial I_\nu}{c\partial t} + \hat{n}\cdot\nabla I_\nu - \frac{aH}{c}\frac{\partial I_\nu}{\partial \ln\nu} = S_\nu - I_\nu\Gamma_\nu$$

Advection term

Non-local in x-space
But, local in k-space

$$\mathrm{i}\,\hat{n}\cdot\vec{k}\,\widetilde{I_\nu}(t,\mathbf{k},\nu,\hat{n})$$

Cosmological *redshifting*

Source emissivity

Photoionization

$$\Gamma_\nu = \sum_{\mathrm{HI,HeI,HeII}} \frac{n_i(t,\mathbf{x})}{a^2}\sigma_{\gamma i}(\nu)$$

Local in x-space
But, convolution (mixing modes) in k-space

F^2-Ray Idea 1: radiative transfer equation in Eulerian scheme

First, compute the mean intensity

$$\bar{I}_\nu(t,\nu) = \int_{t_0}^{t} \frac{c dt_s}{a} \bar{S}_\nu(t_s,\nu_s) \exp[-\bar{\tau}_{\rm I}(t_s,\nu_s,t)]$$

Then, Fourier transform the fields, and write them in vectors of N elements

$$\widetilde{\mathbf{\Delta I}_\nu}(t,\nu,\hat{n}) = \begin{bmatrix} \widetilde{\Delta I_\nu}(\mathbf{k_1},t,\nu,\hat{n}) \\ \vdots \\ \widetilde{\Delta I_\nu}(\mathbf{k_{N-1}},t,\nu,\hat{n}) \end{bmatrix}$$

$$\widetilde{\mathbf{\Delta \Gamma}_\nu}(t,\nu) = \begin{bmatrix} \widetilde{\Delta \Gamma_\nu}(\mathbf{k_1},t,\nu) \\ \vdots \\ \widetilde{\Delta \Gamma_\nu}(\mathbf{k_{N-1}},t,\nu) \end{bmatrix}$$

$$\widetilde{\mathbf{\Delta S}}(t,\nu) = \begin{bmatrix} \widetilde{\Delta S}(\mathbf{k_1},t,\nu) \\ \vdots \\ \widetilde{\Delta S}(\mathbf{k_{N-1}},t,\nu) \end{bmatrix}$$

F^2-Ray Idea 1: RT equation in Fourier space becomes a *linear* differential *vector* equation

$$\frac{\partial}{\partial v}\widetilde{\Delta \mathbf{I}_\nu} = \mathbf{q} - \mathbf{p}\,\widetilde{\Delta \mathbf{I}_\nu}$$

$\mathbf{p}(t, \nu, \hat{n})$ **is a set of non-commutative matrices, i.e.**

$$[\mathbf{p}(t_1, \nu, \hat{n}), \mathbf{p}(t_2, \nu, \hat{n})] \neq 0$$

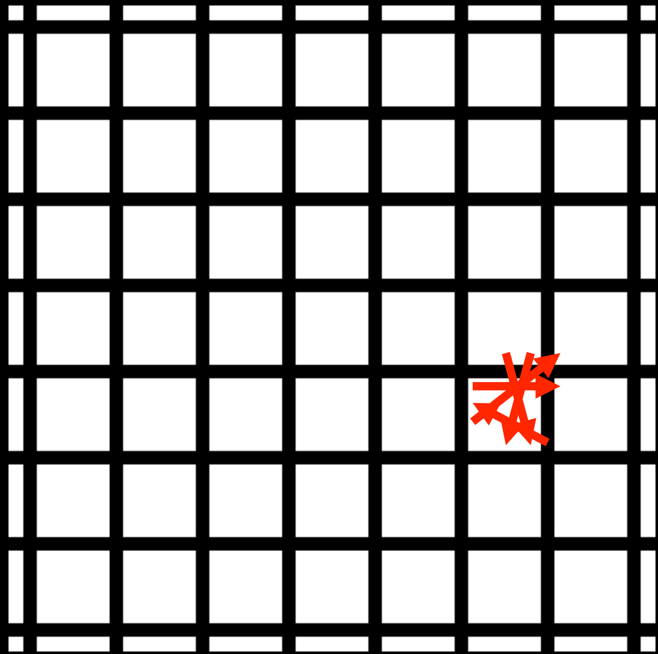
***v* is some time variable**

$\mathbf{q}(t, \nu) \sim$ effective source term.

The matrix P contains the advection (diagonal) and photoionization (off-diagonal).

We find a formal exact solution to this equation, fortunately, using the time-ordering technique (developed in solving the time-varying Schrödinger equation or in writing the Green function in quantum field theory).

F²-Ray Idea 2: (direction-summed) photon density



The gas doesn't care which direction the photons come from in terms of photoionization. Only the total intensity (integrated over directions) matters!

$$\Gamma_{\gamma\,\mathrm{HI}} = c\int_{\nu_{\mathrm{th}}}^{\infty}\frac{d\nu}{\nu}\left[\int d^2\hat{n}\, I_\nu(t,\mathbf{x},\nu,\hat{n})\right]\frac{\sigma_{\gamma\,\mathrm{HI}}(\nu)}{a^2}\kappa(\nu, x_{\mathrm{HII}})$$

F^2-Ray: Solution to RT equation in Fourier space

We prove an exact solution to the direction-summed intensity:

$$\widetilde{\Delta \mathbf{I}_\nu}(t) = \int_{t_0}^{t} \frac{c\,dt_s}{a} \mathbf{R}_{\mathbf{I}}(t, t_s) \widetilde{\Delta \mathbf{S}}_{\mathrm{eff}}(t_s)$$

Direction-summed intensity

Sum over past lightcone

"Radiative transfer matrix" from t_S to t Containing advection and photoionization

Effective source at t_S with ν_S

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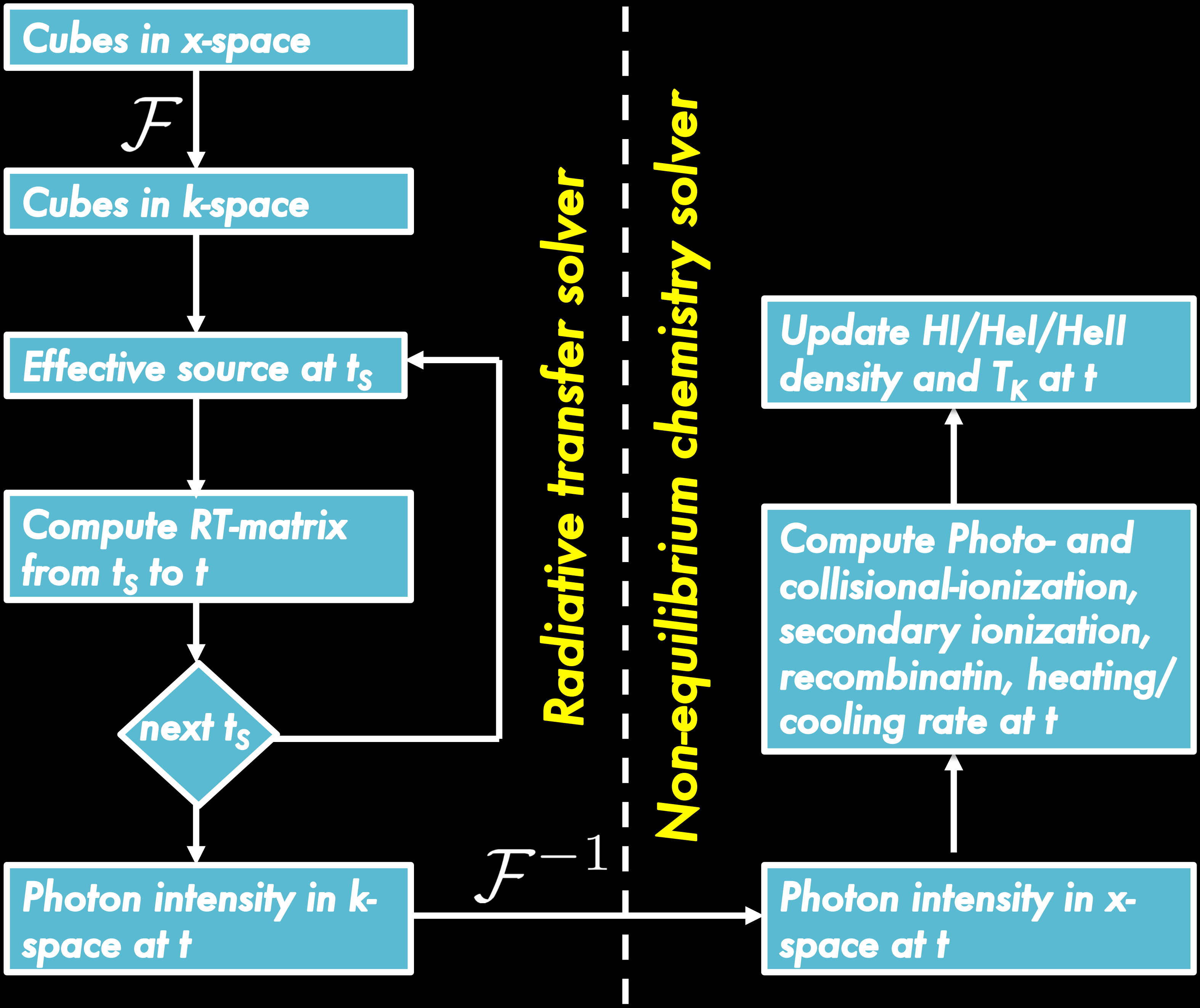
F²-Ray = Fast Fourier Ray-tracing method

F^2-Ray: work flow



F^2-Ray: pros

Robustness

✓ The F^2-Ray algorithm is based on analytic solution of RT equation in Fourier space.

✓ Photon conserving.

✓ Always use correct speed of light *c*.

✓ Automatically observe the periodic boundary condition.

F^2-Ray: pros

Computational Efficiency

✓ Computational cost is independent of the number of ionizing sources.

✓ Good scaling law, O(N), where N = No. grid.

✓ No computation to trace individual rays/packets.

Memory economy

✓ No memory for saving individual rays/packets.

F^2-Ray: pros

Applicability

✓ Source type: both point and diffuse. Can include self-regulation. Can include recombination radiation.

✓ Photon type: both UV and X-rays (short- and long- m.f.p.)

✓ Ionization: H I, He I, He II.

✓ Solve for IGM gas temperature $T_{K.}$

✓ Ionizing source emissivity can be anisotropic.

F^2-Ray: cons

<u>Applicability</u>

- Boundary condition must be periodic. Not for open b.c.

<u>Computational Efficiency</u>

✓ scale with N_t , $O(N_t)$, where N_t = No. time slices

<u>Memory economy</u>

✓ Need to save cubes (in k-space) at all past time slices. Distributed memory parallelism is necessary.