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The growth index of cosmological perturbations

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Outline

- The problem: how to parametrize the growth index?
- Test of our formalism
- Prospects with Euclid-like surveys

The universe is currently accelerating [Perlmutter et al.,Planck13]

- The universe is dominated today by a hidden form of energy, dubbed Dark Energy. $w = p/\rho < -1/3$
- **OR . . .** Einsteins fields equations break down at very large cosmological scales modified gravity
- Measurements of background not enough to determine the source of acceleration → **perturbations**

Simulation of Dark Matter clustering

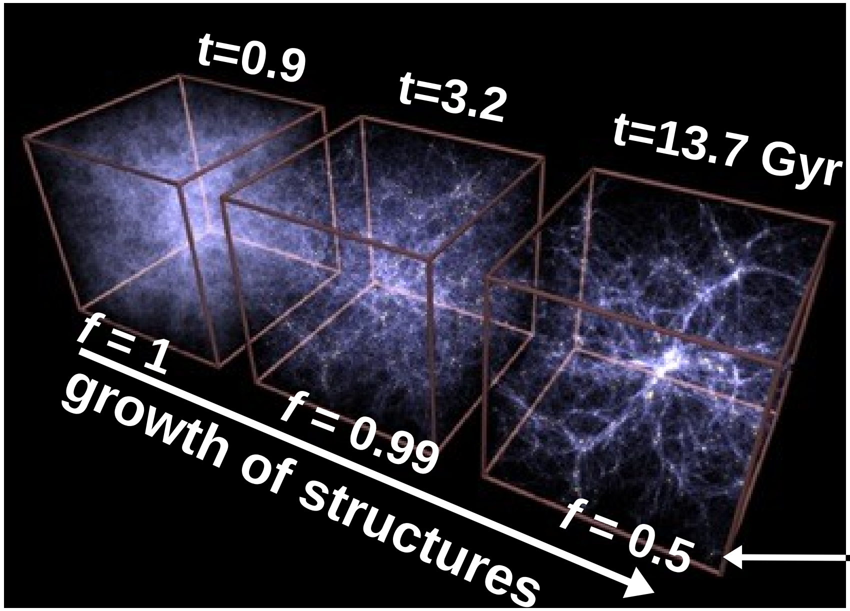


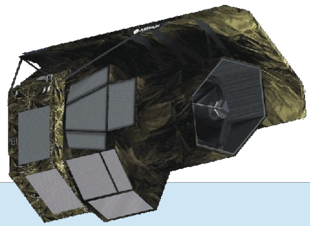
Image: MPE/V.Springel.

Can the linear growth of structures disentangle these hypotheses?

$$f' + f^2 + \left(2 + \frac{H'}{H} + \mu\right) f - \frac{3}{2}\frac{G_{\rm eff}}{G}\Omega_m = 0$$

growth rate

The problem: how to parametrize the growth rate?



$$f' + f^2 + \left(2 + \frac{H'}{H} + \mu\right) f - \frac{3}{2}\frac{G_{\rm eff}}{G}\Omega_m = 0$$

$$\boldsymbol{f}_{\rm (observed)} = \boldsymbol{f}_{\rm (predicted)}\left(\boldsymbol{P}_{\rm (background)}, \boldsymbol{P}_{\rm (perturbations)}\right)$$

- For ΛCDM $f \sim \Omega^{\gamma=0.6}$ [Peebles 1980]

growth rate

growth index

- We need a parametrization for the growth rate satisfying
  - high precision (<1% of error)
  - observer friendly (few parameters to fit)
  - theorist friendly (parameters must be easily related to theories)
  - flexible (covering a maximum of models!)

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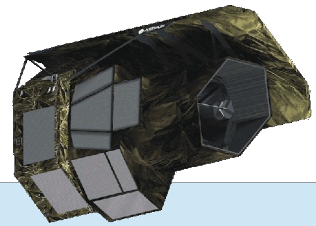

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A solution? Growth index as Taylor developpement

Ansatz : $$\gamma = \sum_{n=0}^{N} \gamma_n \frac{(\ln \Omega)^n}{n!}$$

Taylor developpement in the logarithm of Ω_m

$$\gamma_0 = \frac{3\mathfrak{G}_0 + 3\mathfrak{G}_1 - 2\mathfrak{H}_1}{2 + 2\mathfrak{X}_1 + 3\mathfrak{G}_0}$$

$$\mathfrak{H}_n = \left[\frac{d^n}{d(\ln\Omega)^n}\left(\frac{d\ln H}{d\ln a}\right)\right]_{\ln\Omega=0}$$

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model dependend

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Exemple: wCDM

$$\mathfrak{G}_0 = 1, \ \mathfrak{G}_1 = 0, \ \mathfrak{H}_1 = \tfrac{3}{2} w, \ \mathfrak{X}_1 = -3w \longrightarrow \gamma_0 = \frac{3(1-w)}{5-6w}$$

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A general recursion formula for higher order coefficients (n>0):

$$\gamma_n = 3\frac{\mathfrak{G}_{n+1} + \sum_{k=1}^{n+1}\binom{n+1}{k}\mathfrak{G}_{n+1-k}B_k(1-y_1,-y_2,-y_3,\ldots,-y_k)}{(n+1)(2+2(n+1)\mathfrak{X}_1+3\mathfrak{G}_0)} - \frac{B_{n+1}(y_1,y_2,\ldots,y_{n+1}) + \sum_{k=2}^{n+1}\binom{n+1}{k}\mathfrak{X}_k(n+2-k)\gamma_{n+1-k} + \mathfrak{H}_{n+1}}{(n+1)(1+(n+1)\mathfrak{X}_1+\frac{3}{2}\mathfrak{G}_0)}$$

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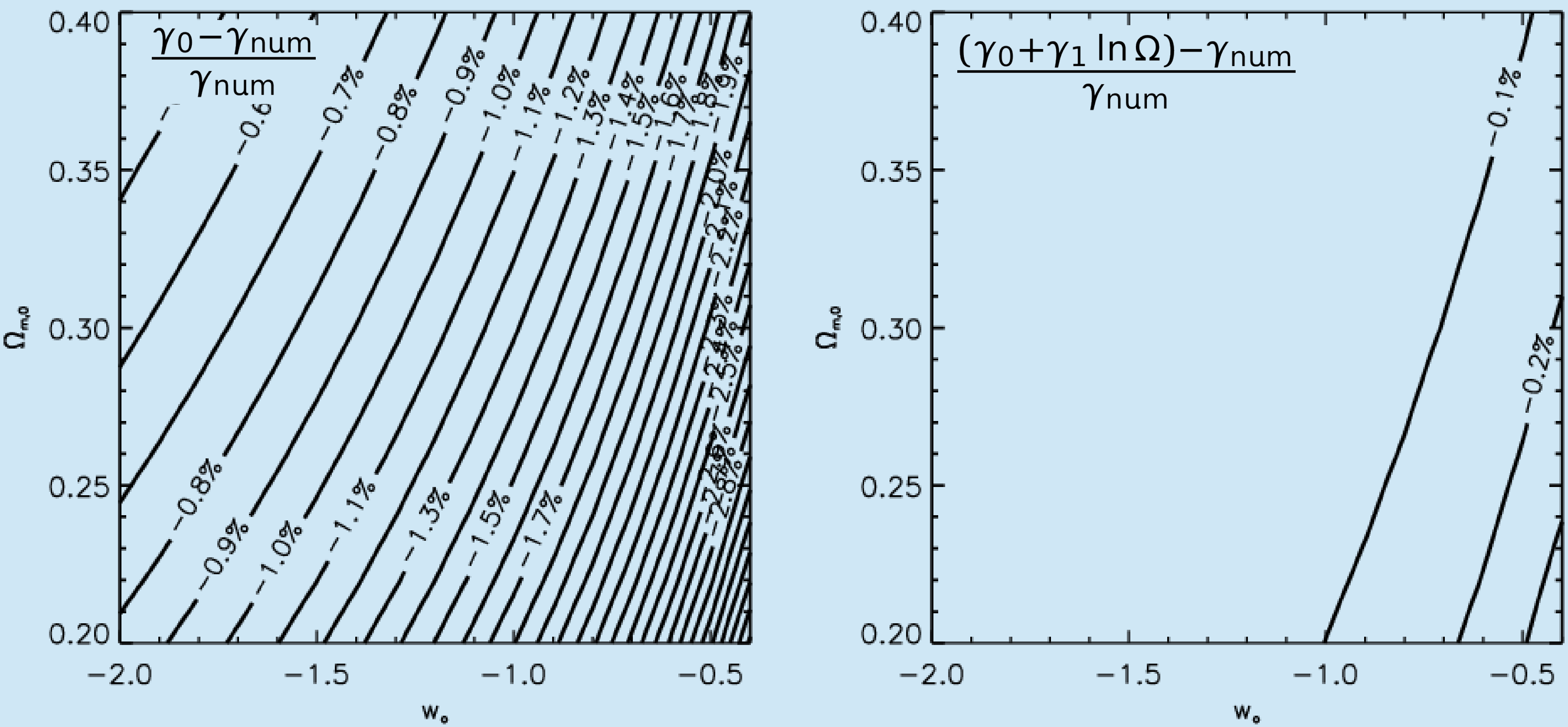
Precision

- What is the precision of the approximation?
- How many orders do we need?

wCDM | w(a)CDM | Clust. Quint.



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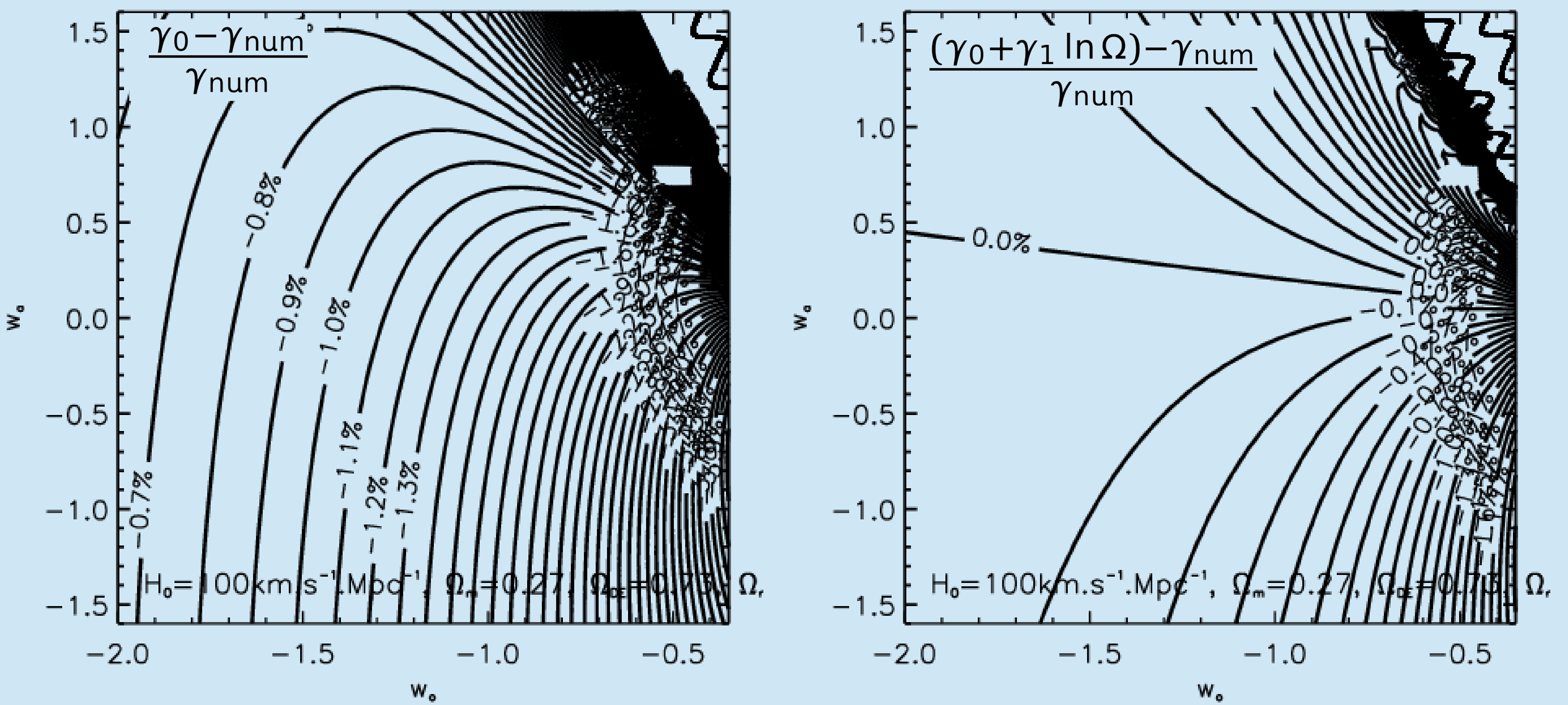

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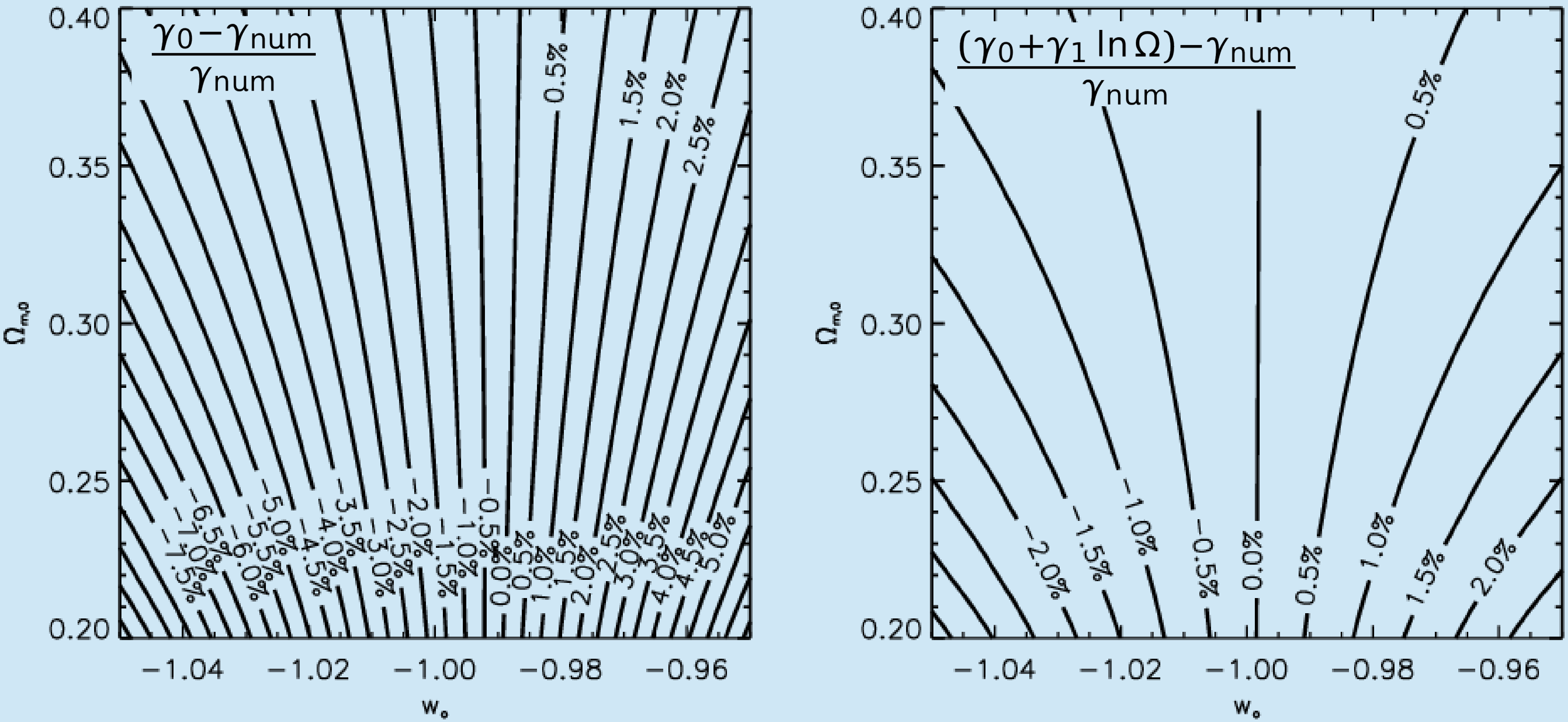
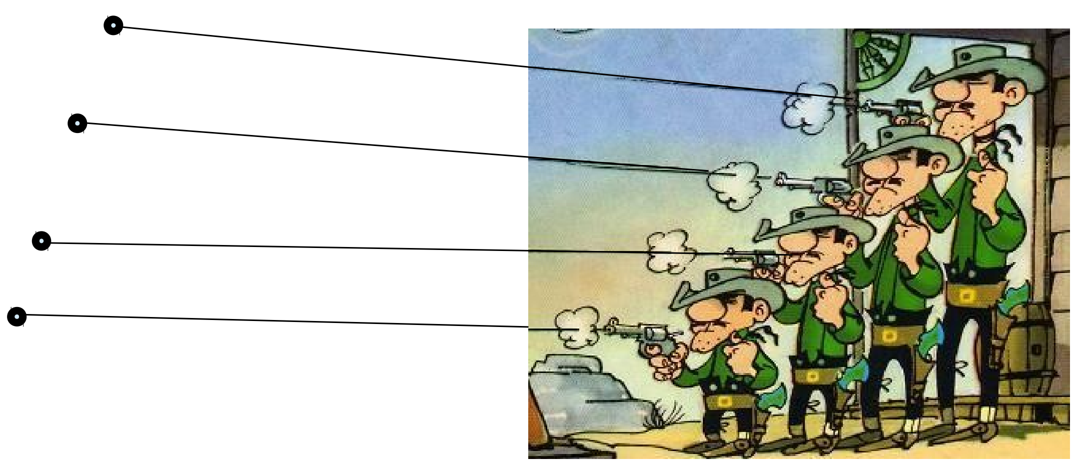


Connection theory-observation

Model	γ_0	γ_1	γ_2	...
ΛCDM	$\frac{6}{11}$	$-\frac{15}{2057}$	$\frac{410}{520\,421}$	...
wCDM ($w=-0.9$)	0.5481	−0.0078	0.0009	...
wCDM ($w=-1.1$)	0.5431	−0.0068	0.0007	...
w(a)CDM ($w_o=-0.9, w_a=0.5$)	0.5538	−0.0072	−0.0010	...
w(a)CDM ($w_o=-1.1, w_a=0.5$)	0.5467	−0.0064	0.0004	...
Clust. Quint. ($w=-0.9$)	$\frac{243}{520}$	$\frac{1\,356\,669}{42\,723\,200}$	$\frac{6\,260\,762\,727}{294\,362\,848\,000}$	...
Clust. Quint. ($w=-1.1$)	$\frac{363}{580}$	$-\frac{3\,467\,739}{61\,224\,800}$	$\frac{733\,481\,309}{17\,200\,342\,250}$	...
flat DGP	$\frac{11}{16}$	$\frac{7}{5632}$	−0.0466	...

All coefficients are analytically predicted!



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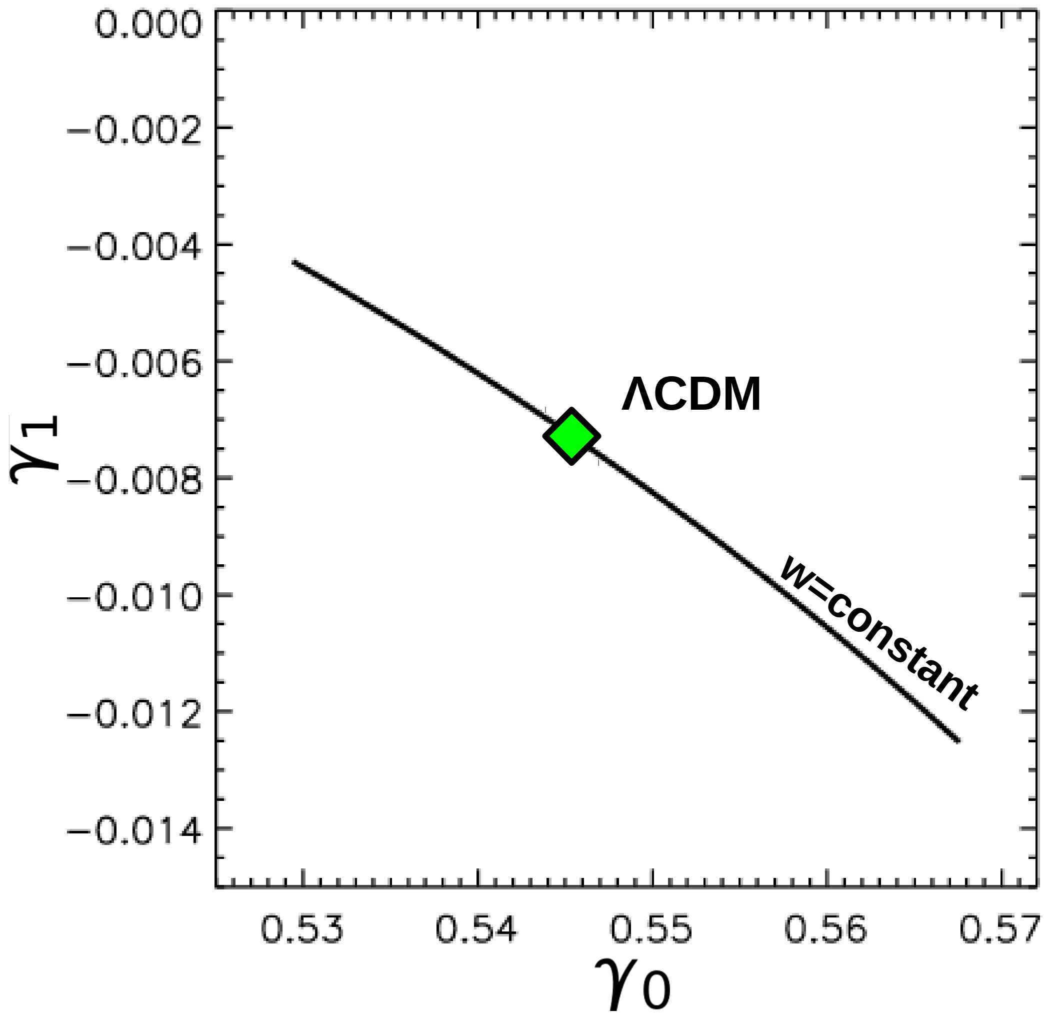

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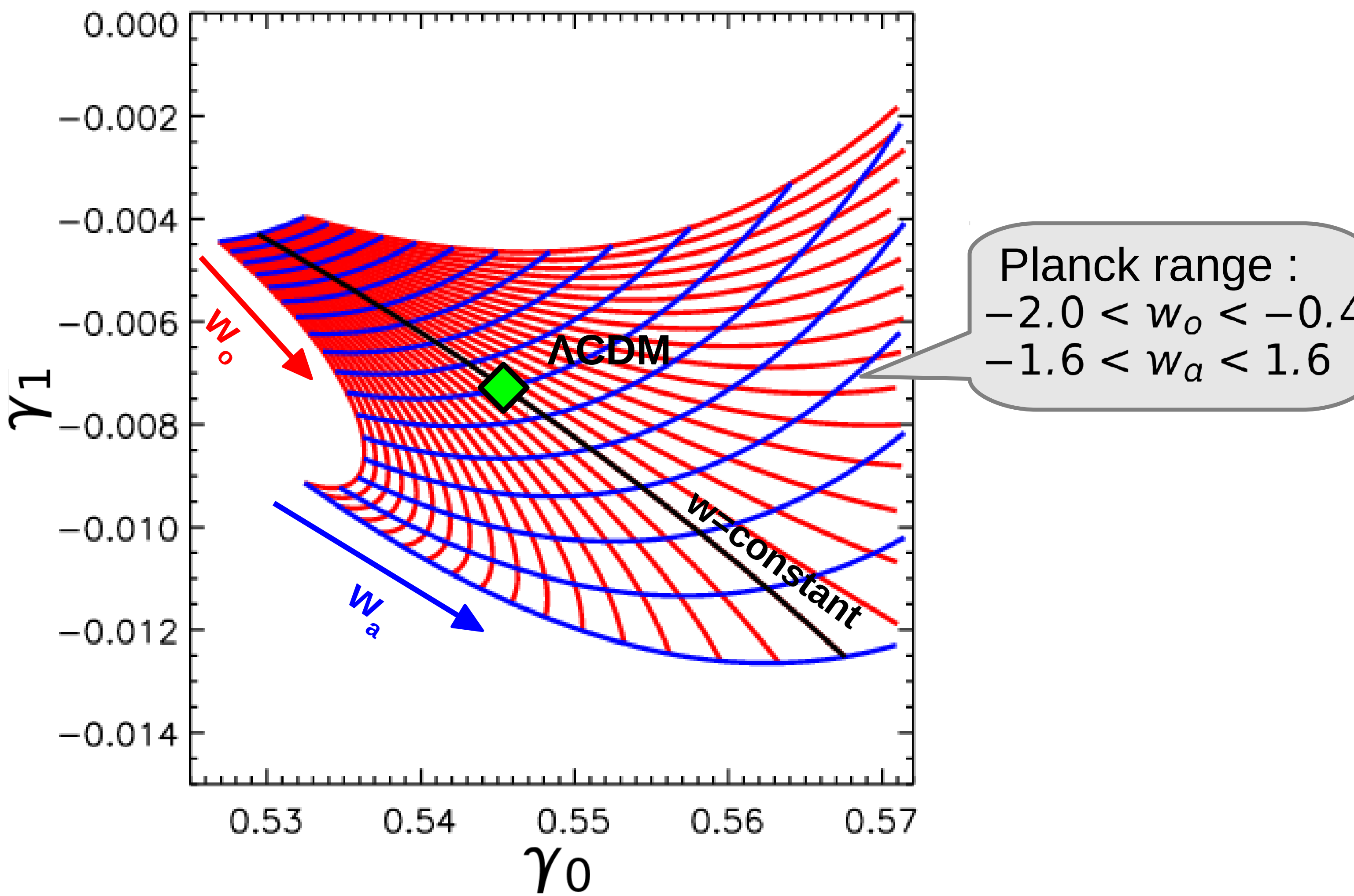

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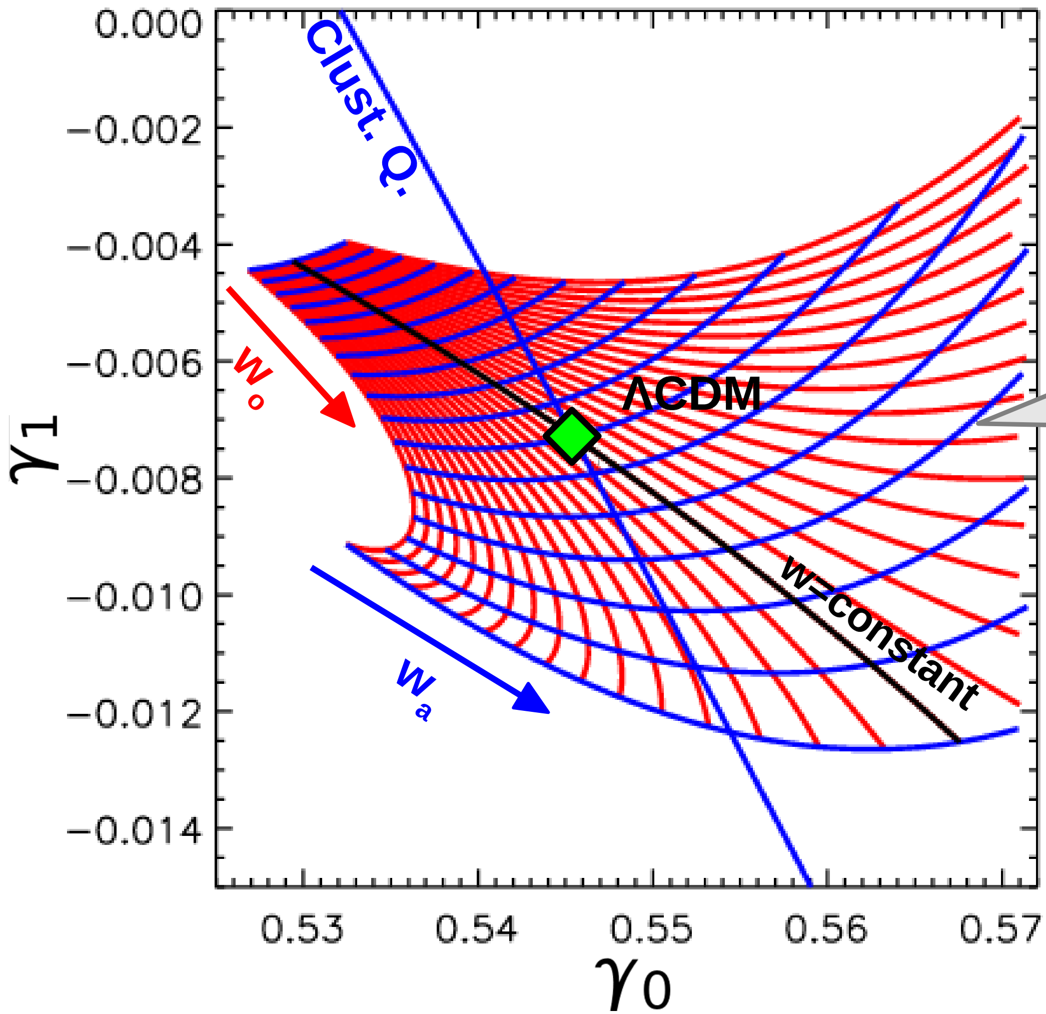

Planck range :
$-2.0 < w_o < -0.4$
$-1.6 < w_a < 1.6$

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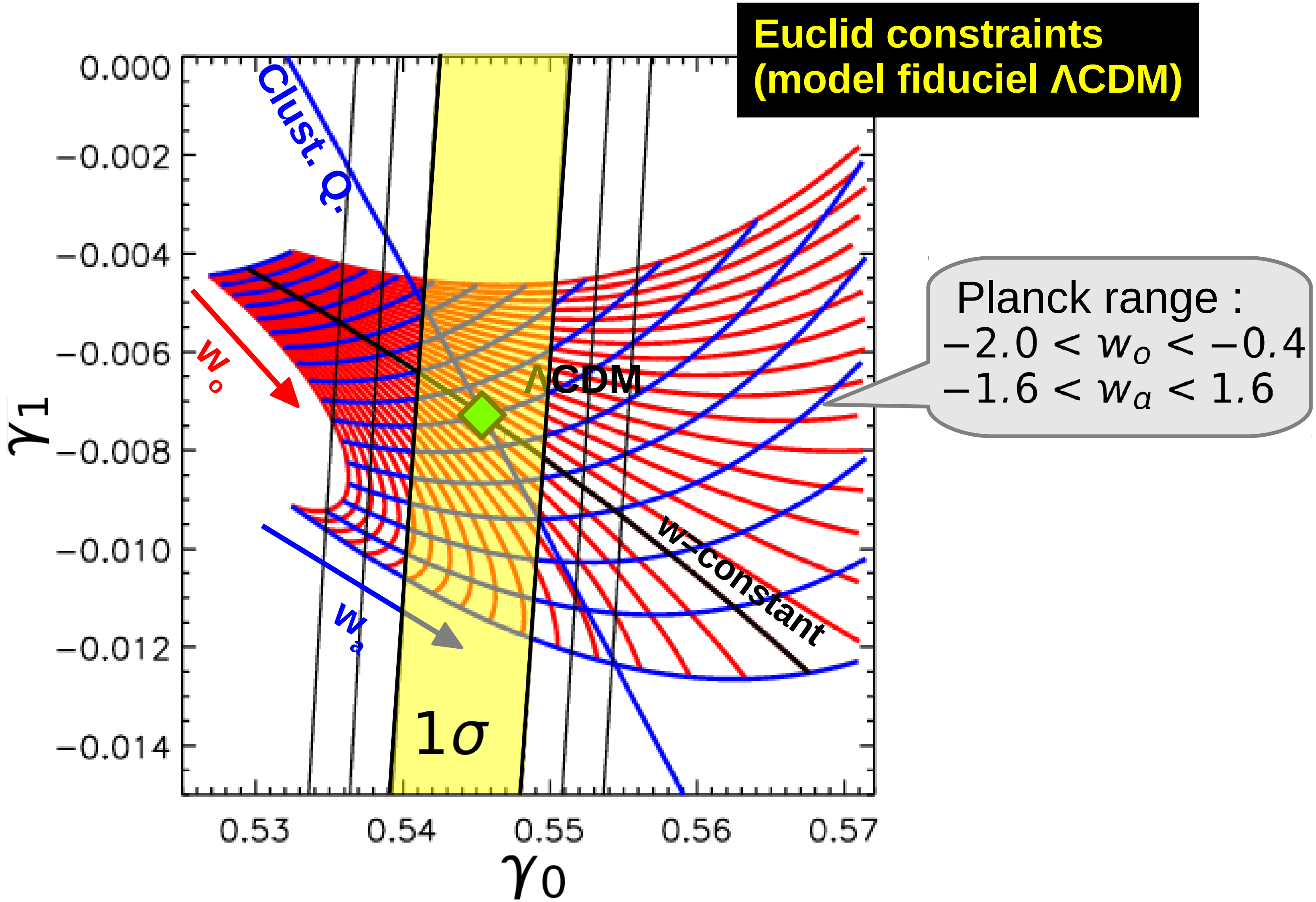

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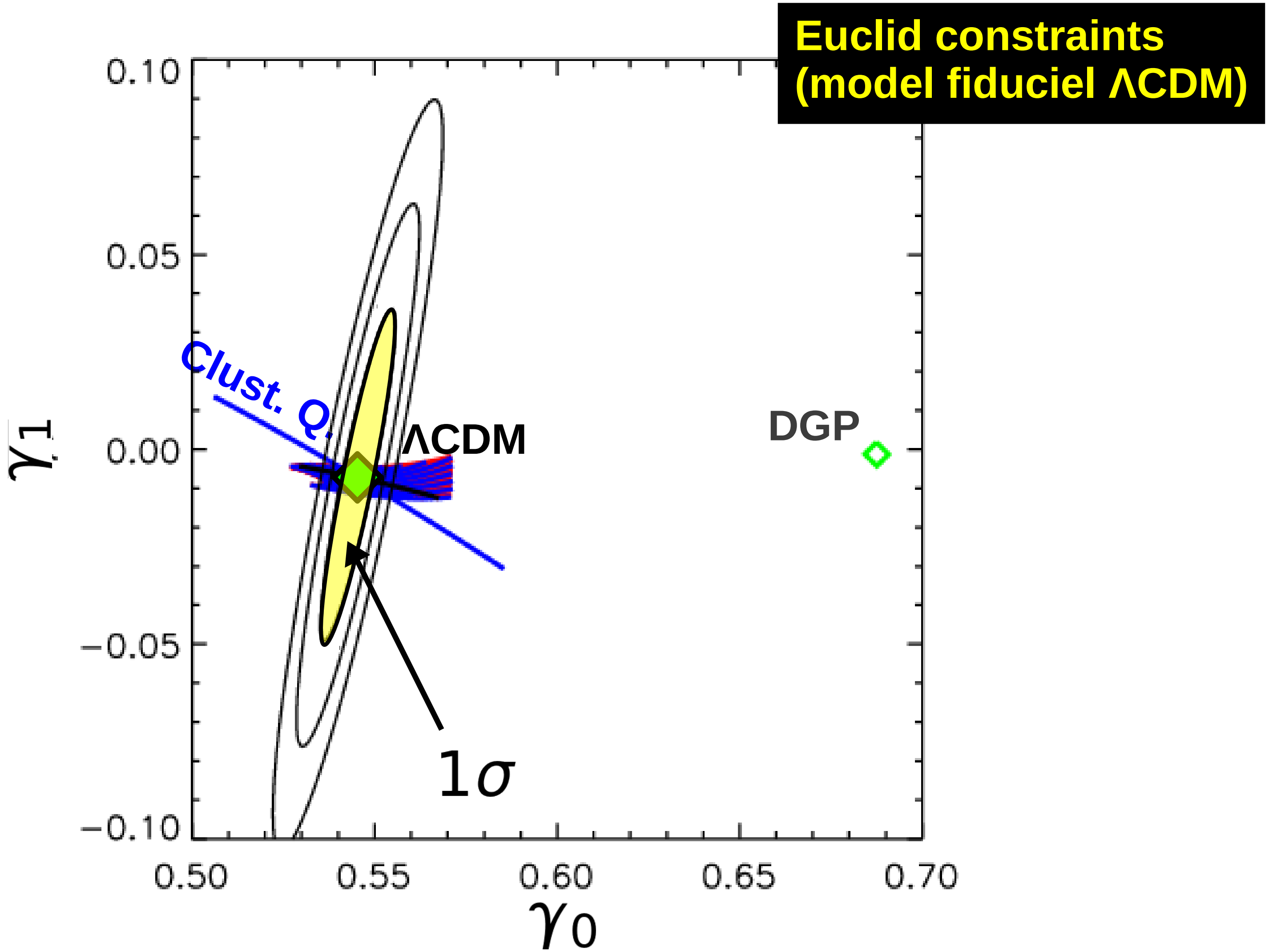


Conclusions

- We present an analytical parametrization that is

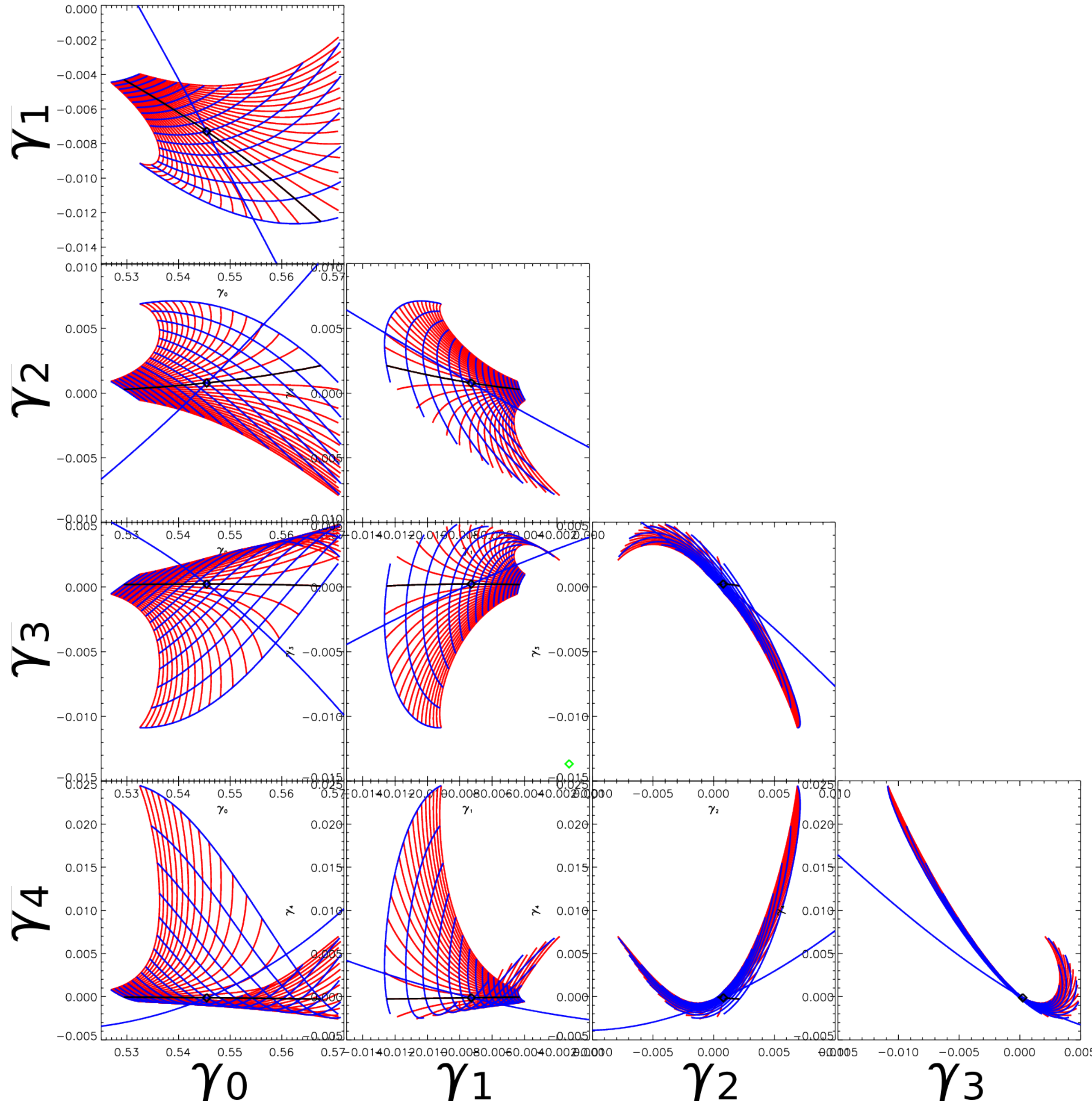
  a) observer friendly (few coefficients needed)

  b) theorist friendly (links observational results to specific models)

  c) precise

- What's next: extending the parametrization to include f(R) and more in general EFT

Analysis of an Euclid-like surveys



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Clust. Quint. ($w = -0.95$)	0.5061	0.0135	−0.01246	...
Clust. Quint. ($w = -1.05$)	0.5854	−0.0307	0.0190	...
flat DGP	0.6875	−0.0012	−0.0466	...

All coefficients are analytically predicted!

What observations can measure ...

Label	z	$f\sigma_8$	γ	Ref
THF	0.02	0.398 ± 0.065	$0.56^{+0.11}_{-0.09}$	1
DNM	0.02	0.314 ± 0.048	$0.71^{+0.10}_{-0.09}$	2
6dF	0.07	0.423 ± 0.055	$0.54^{+0.09}_{-0.08}$	3
2dF	0.17	0.510 ± 0.060	$0.43^{+0.10}_{-0.08}$	4
LRG1	0.25	0.351 ± 0.058	$0.77^{+0.16}_{-0.14}$	5
LRG2	0.37	0.460 ± 0.038	$0.55^{+0.09}_{-0.08}$	5
WZ1	0.22	0.390 ± 0.078	$0.67^{+0.19}_{-0.15}$	6
WZ2	0.41	0.428 ± 0.044	$0.64^{+0.12}_{-0.11}$	6
WZ3	0.6	0.403 ± 0.036	$0.76^{+0.14}_{-0.13}$	6
WZ4	0.78	0.493 ± 0.065	$0.38^{+0.28}_{-0.24}$	6
BOSS	0.57	0.415 ± 0.034	$0.71^{+0.12}_{-0.11}$	7
VVDS	0.77	0.490 ± 0.180	$0.40^{+0.89}_{-0.59}$	8

Euclid $0.7 - 2.0$ $\delta\gamma/\gamma \sim 0.01$ 2018