CLOSE COLLISIONS DURING STOCHASTIC DEFLECTION OF HIGH-ENERGY CHARGED PARTICLES BY A BENT CRYSTAL

I.V. Kirillin

Akhiezer Institute for Theoretical Physics of National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

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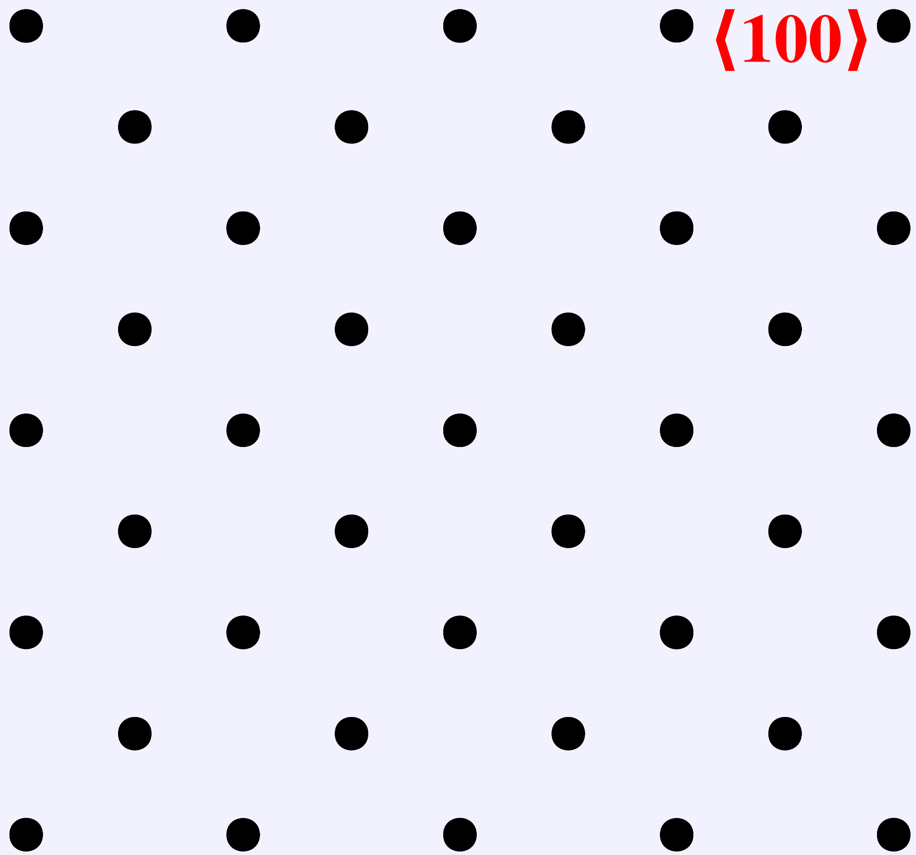
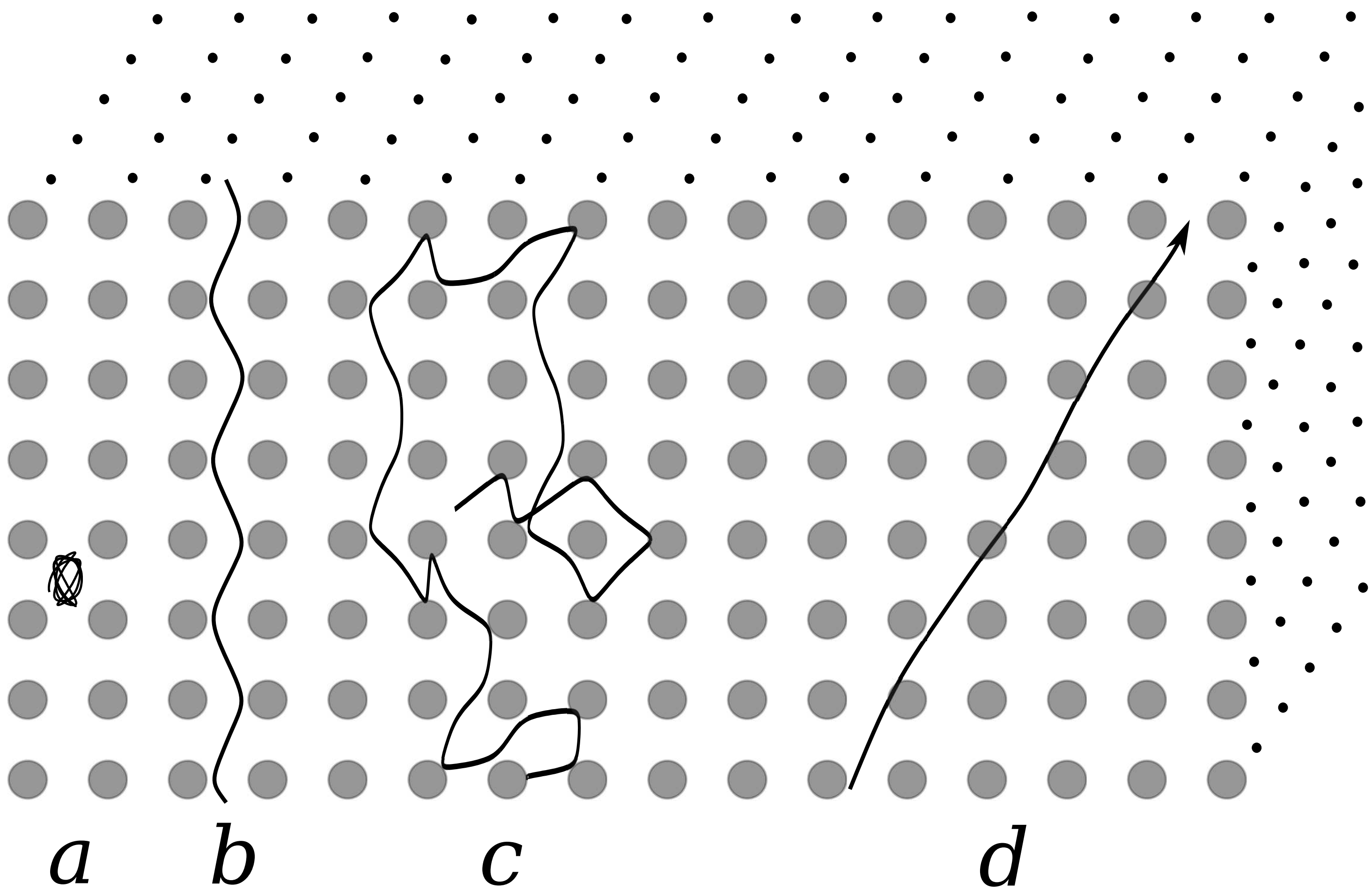
The crystal lattice of Si is a face-centered cubic diamond-type lattice



Diamond, silicon, germanium, and gray tin have this type of crystal lattice.

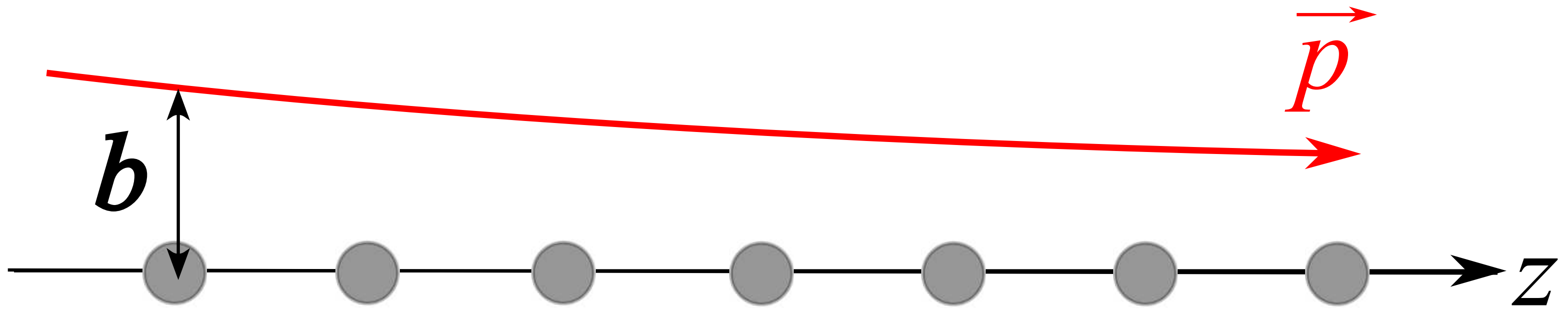


a) axial channeling;

b) planar channeling;

c) stochastic scattering;

d) strongly above-barrier motion.



$$U(\vec{r}) = \sum_n u(\vec{r} - \vec{r}_n)$$

$$U_R(\vec{\rho}) = \frac{1}{L} \int_{-\infty}^{\infty} dz \sum_n u(\vec{r} - \vec{r}_n)$$

$$\ddot{\vec{\rho}} = -\frac{c^2 q}{E} \frac{\partial}{\partial \vec{\rho}} U_R(\vec{\rho})$$

Crystal potential

if $\Phi_a(r) = \frac{Z|e|}{r}\exp(-r/R)$ then $\Phi_s(\rho) = \frac{2Z|e|}{d}K_0(\rho/R)$,

so summation $\sum_n U_R(\vec{\rho}-\vec{\rho}_n)$ couldn't be done analytically.

However, if we use Doyle-Terner approximation

$$\Phi_a(r) = \frac{2\pi\hbar^2}{|e|m_e}\sum_{i=1}^{4}\alpha_i\left(\frac{4\pi}{\beta_i+B}\right)^{3/2}\exp\left(-\frac{4\pi^2 r^2}{\beta_i+B}\right) \quad \text{then}$$

$$\Phi_s(\rho) = \frac{1}{d_a}\int_{-\infty}^{\infty} dz\Phi_a(\rho,z) = \frac{8\pi^2\hbar^2}{|e|m_e d_a}\sum_{i=1}^{4}\frac{\alpha_i}{\beta_i+B}\exp\left(-\frac{4\pi^2\rho^2}{\beta_i+B}\right)$$

and summation could be done analytically

Crystal potential

for Si $\langle 100 \rangle$ crystal axis

$$\langle \Phi_{\langle 100 \rangle}(\vec{\rho}) \rangle = \sum_{n=-\infty}^{\infty} \Phi_s(\vec{\rho} - \vec{\rho}_n) =$$

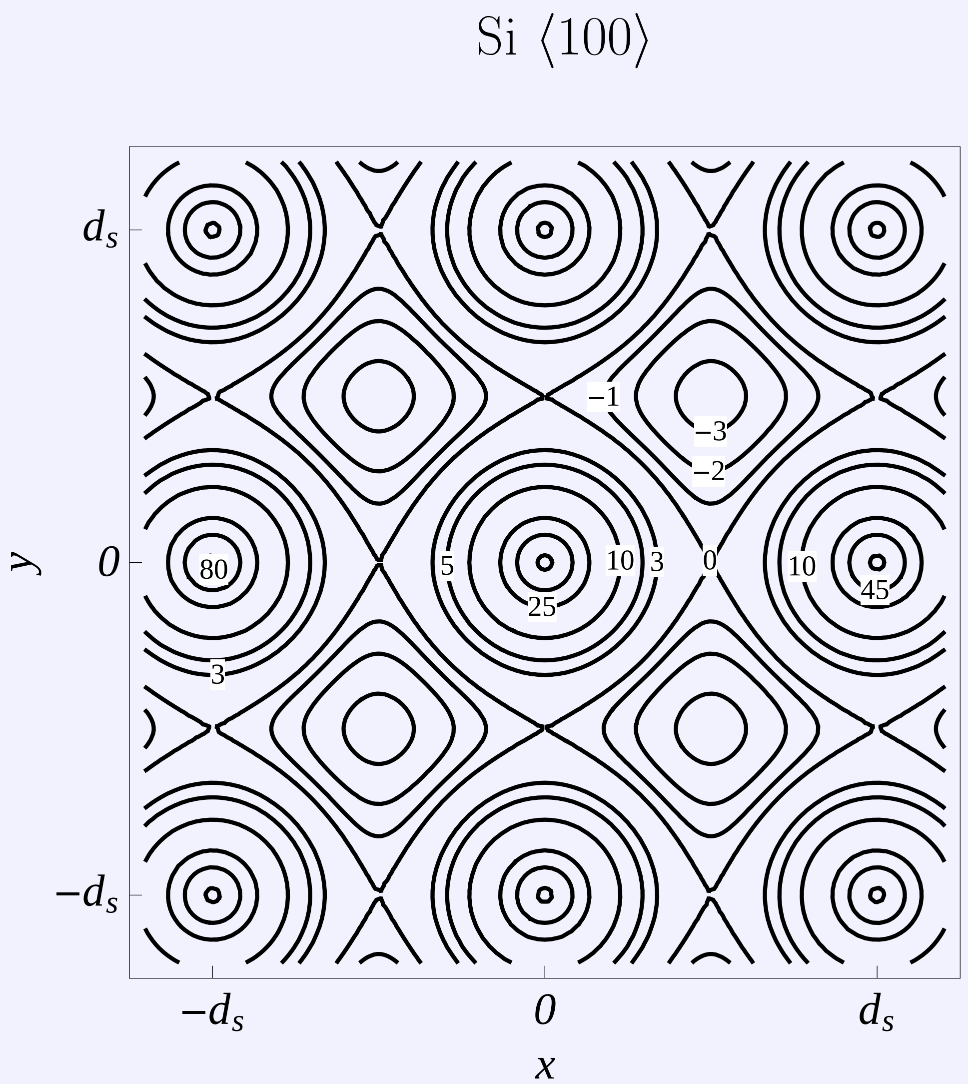
$$= \frac{2\pi\hbar^2}{|e| m_e d_a d_s^2} \sum_{i=1}^{4} \alpha_i \theta_3 \left(-\frac{x}{d_s} \middle| \frac{i(\beta_i + B)}{4\pi d_s^2} \right) \theta_3 \left(-\frac{y}{d_s} \middle| \frac{i(\beta_i + B)}{4\pi d_s^2} \right),$$

where

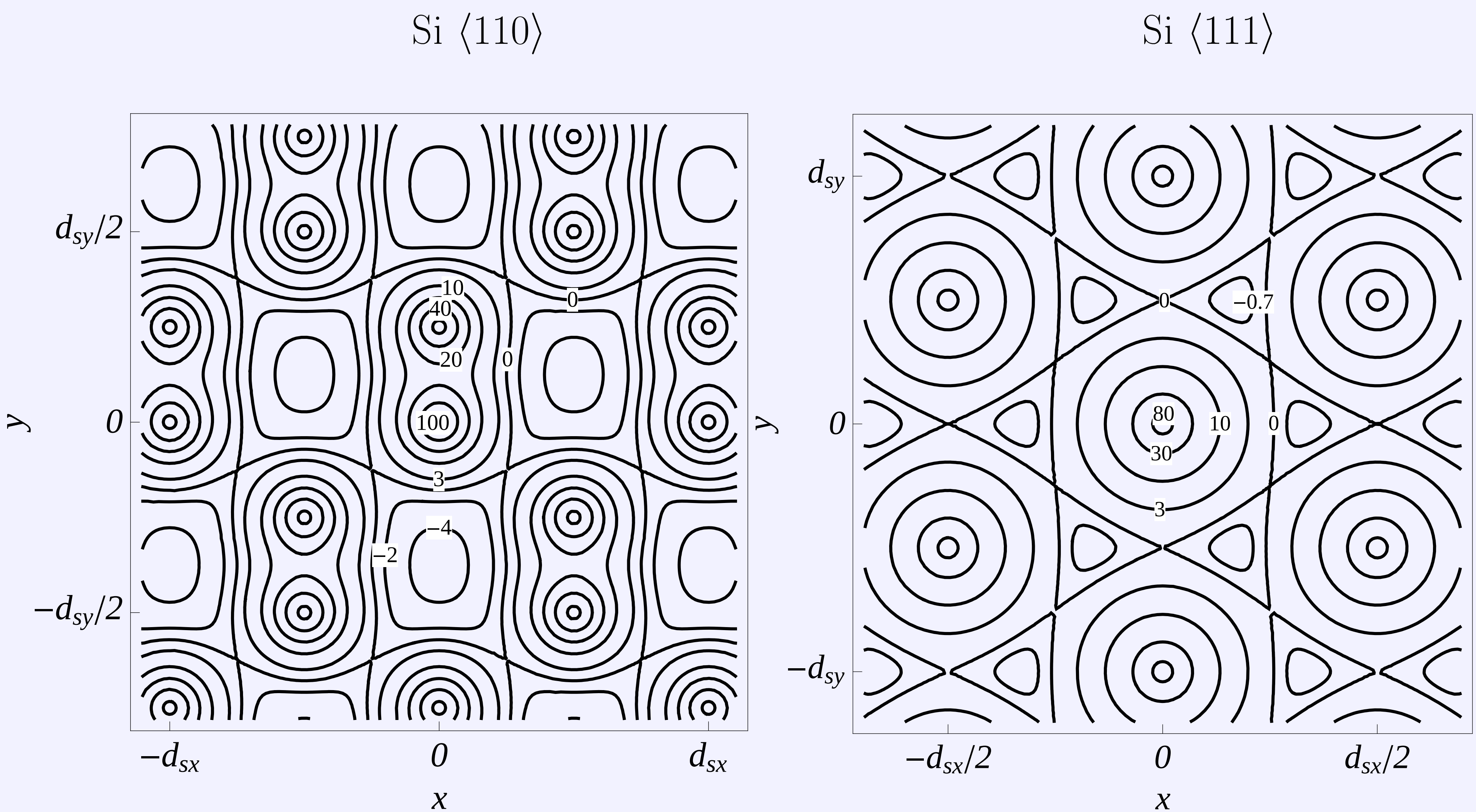
d_s — distance between neighboring crystal atomic strings,

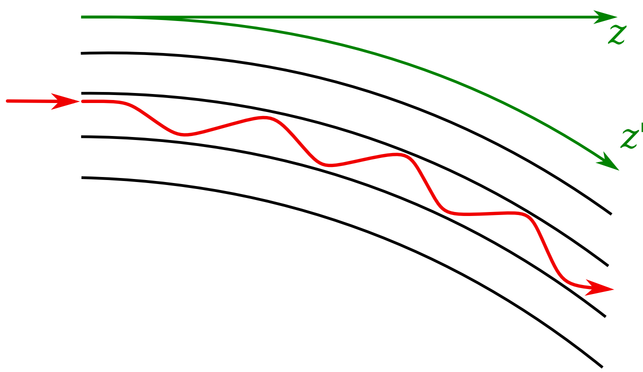
$\theta_3\,(v|w) = \sum_{n=-\infty}^{\infty} \exp(\pi i w n^2) \exp(2\pi i v n)$ — Jacobi theta function of the third type

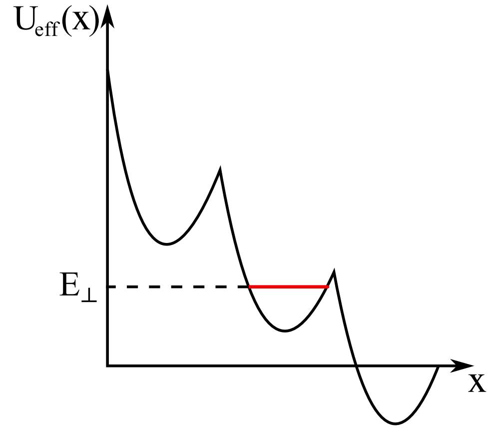
Crystal potential



Crystal potential







If the distance between particle and the center of crystal curvature is $\rho(t) = R + x(t)$, where R is the radius of crystal curvature, $x(t) \ll R$, then

$$\frac{d^2x}{dt^2} = -\frac{c^2}{E}\frac{\partial}{\partial x}U_{eff}(x),$$

where E is particle energy,

$$U_{eff}(x) = U(x) - x\frac{E}{R}.$$

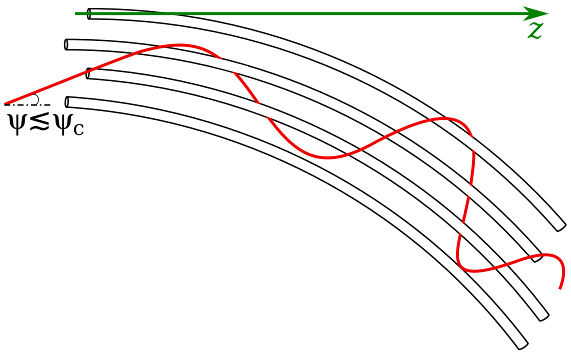
Radius of curvature R_c, at which local minima of function $U_{eff}(x)$ disappear is called the critical. If inter-planar potential is

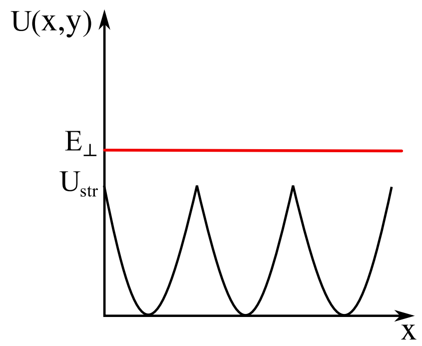
$$U_p(x) = U_{max}\frac{x^2}{(d/2)^2},$$

where d is the distance between neighboring crystal atomic planes, then

$$R_c = d\frac{E}{4U_{max}}.$$

Stochastic deflection mechanism





Greenenko-Shul'ga criterion: $\frac{l_\perp}{R\psi_c}\frac{L}{R\psi_c} < 1$

R is crystal curvature radius;

$\psi_c = \sqrt{4Z\,|qe|\,/(pvd)}$ is critical angle of axial channeling;

$Z\,|e|$ – charge of the nucleus of an atom of the crystal;

q – particle charge;

v и p – particle velocity and momenta;

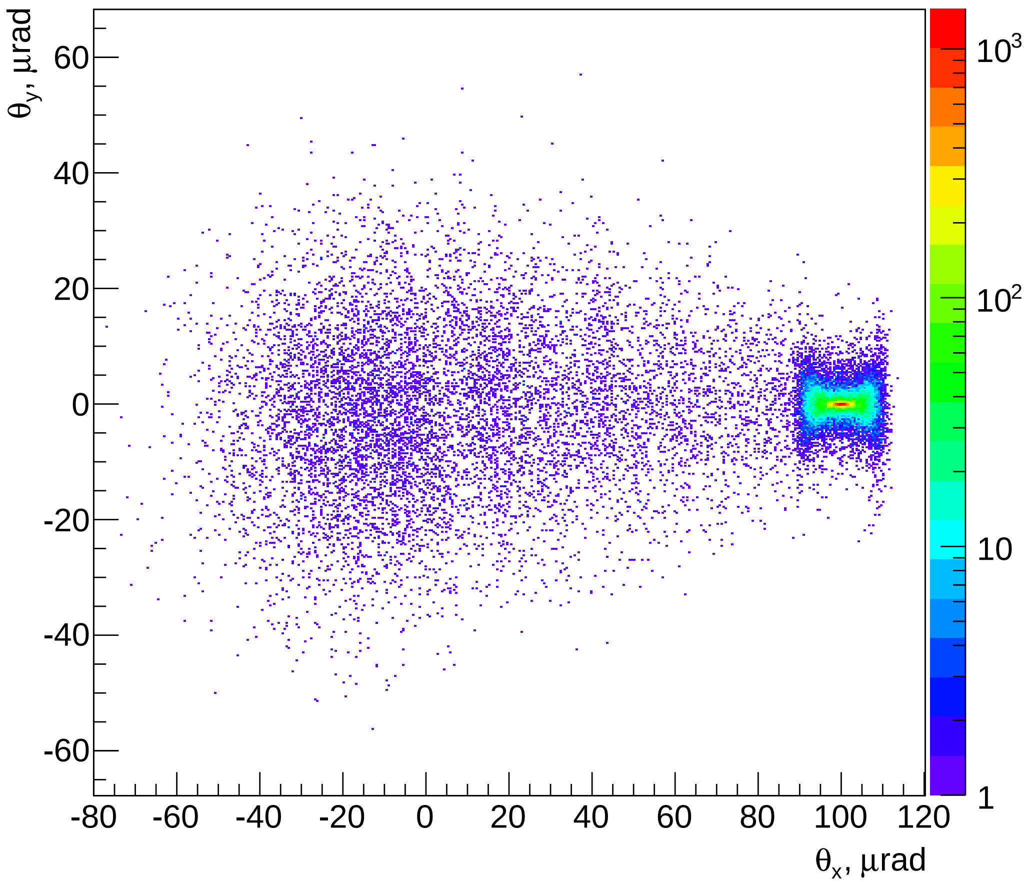
d – distance between neighboring atoms in the string;

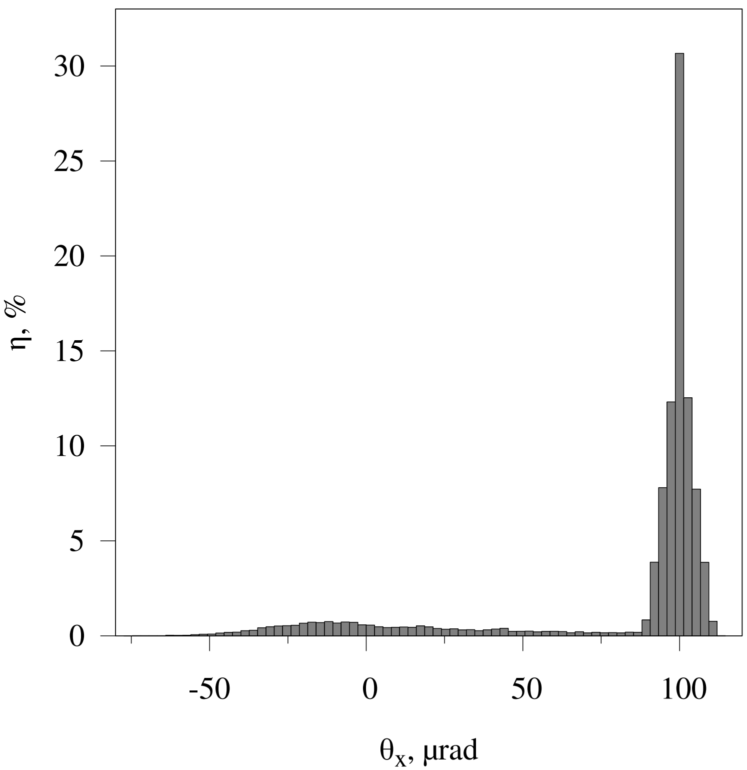
$l_\perp$ – the mean free path of the particle between successive collisions with strings of atoms in a crystal;

L – the thickness of the crystal.

Angular distribution of 270 GeV/c protons after passing through 5 mm Si crystal with radius of curvature 50 m.

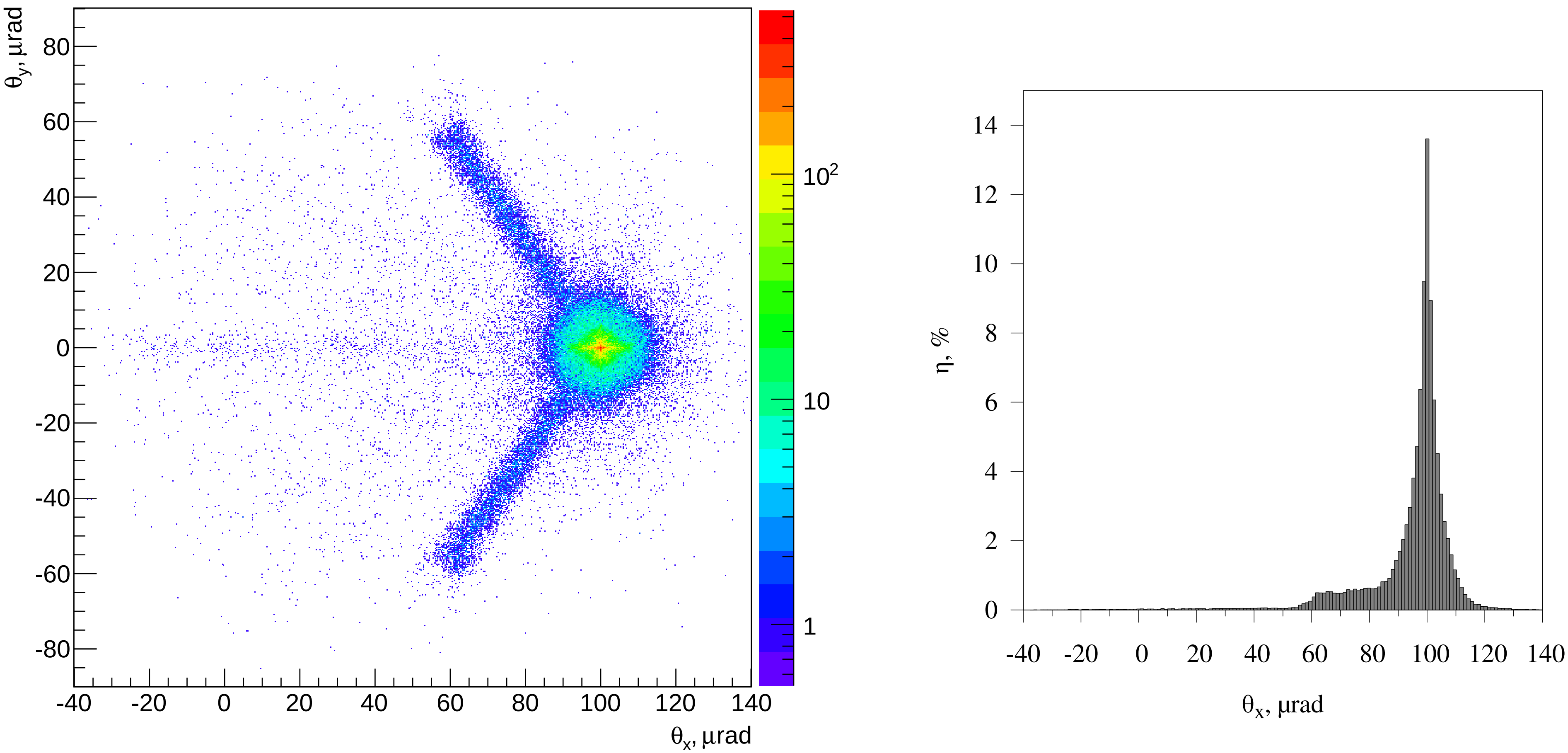
$$\theta_y^{in} = 400\ \mu\text{rad}$$





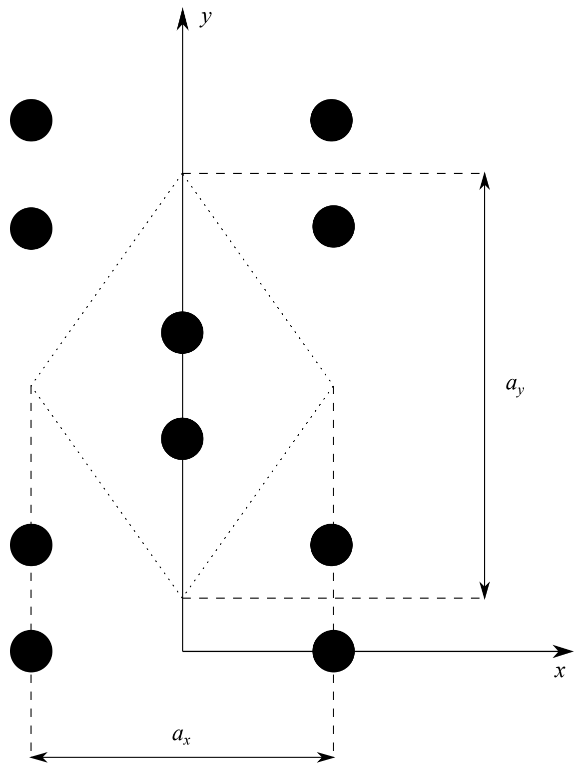
Angular distribution of 270 GeV/c protons after passing through 5 mm Si crystal with radius of curvature 50 m.

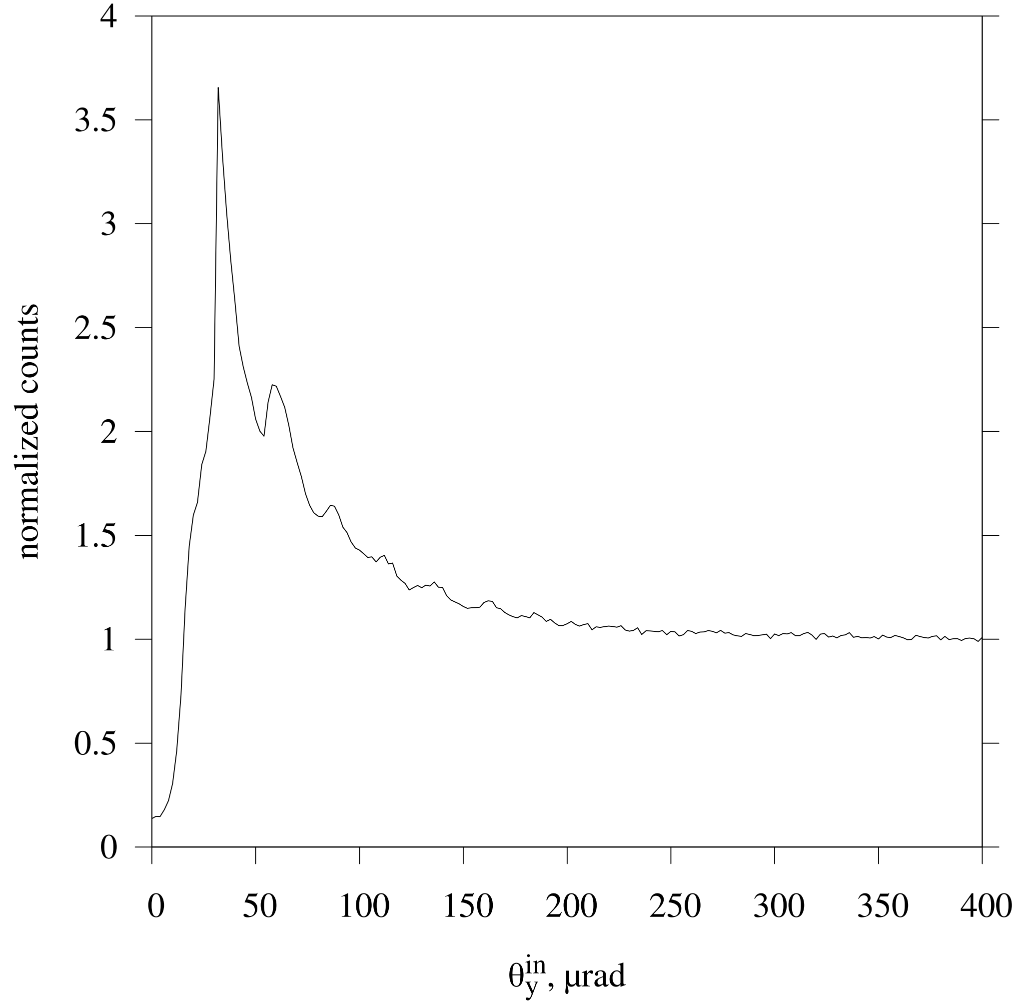
$$\theta_y^{in} = 0$$



Yu.A. Chesnokov, I.V. Kirillin, W. Scandale, N.F. Shul'ga, V.I. Truten'. Physics Letters B 731 (2014) 118–121

⟨110⟩ axis of Si and normalized probability of close collisions of protons in a bent crystal.



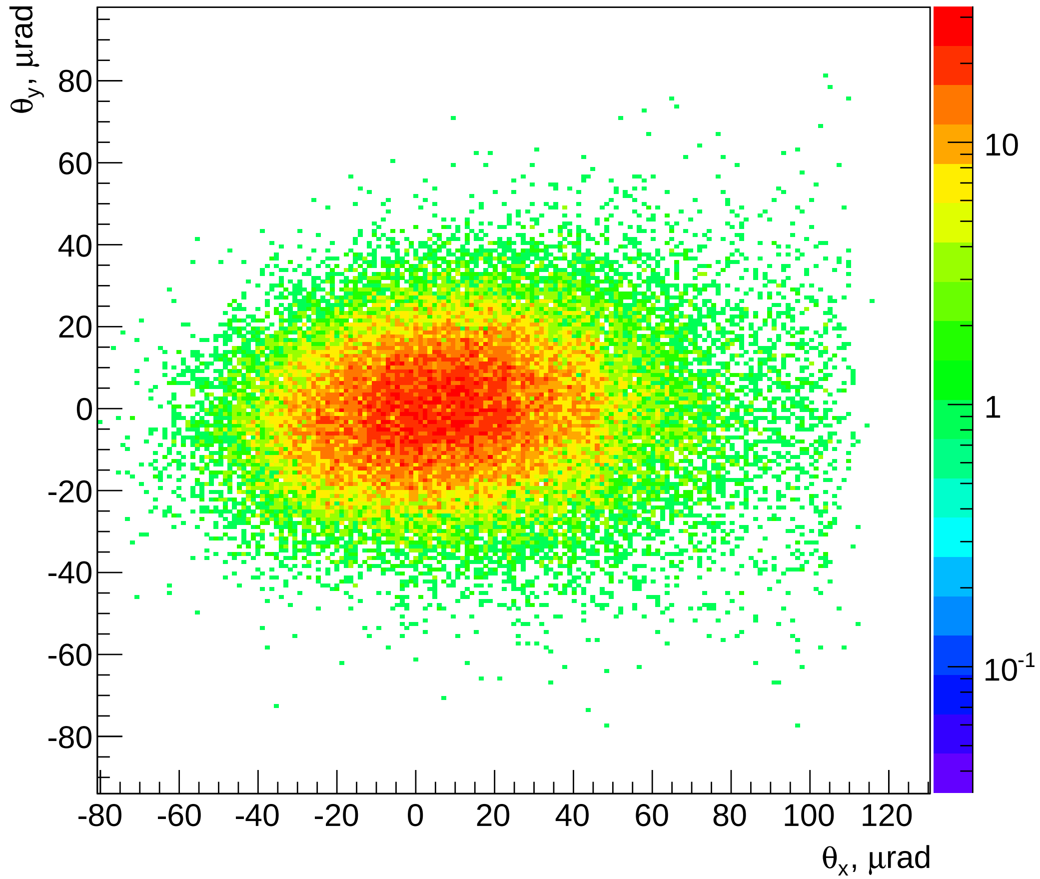


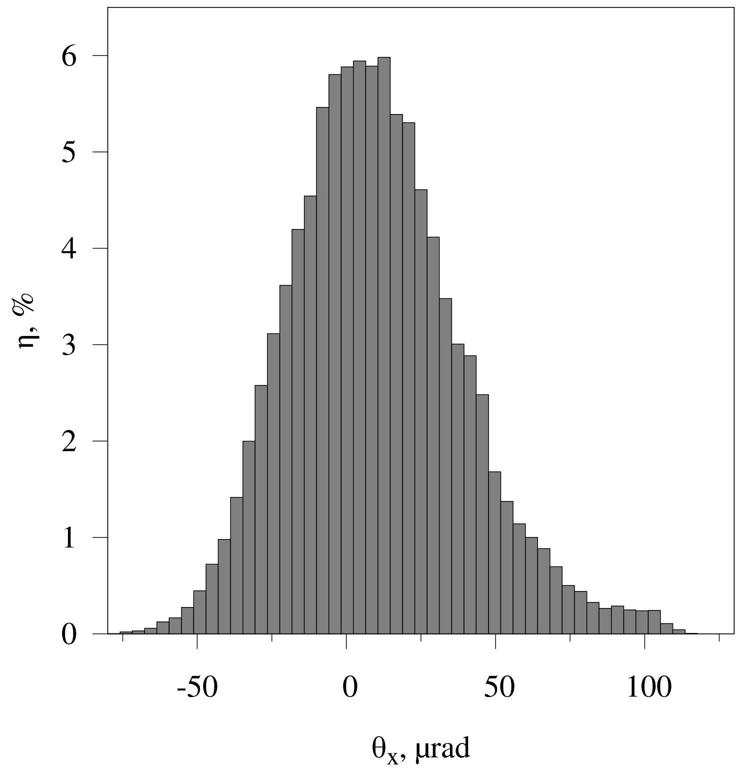
$$w_a = \frac{4\pi r_T^2}{a_x a_y} = 4\sqrt{2}\pi r_T^2/a^2 \approx 3.39 * 10^{-3}$$

$$w_p = \frac{4r_T}{a_x} = 4\sqrt{2}r_T/a \approx 78.12 * 10^{-3}$$

Angular distribution of 270 GeV/c π^--mesons after passing through 5 mm Si crystal with radius of curvature 50 m.

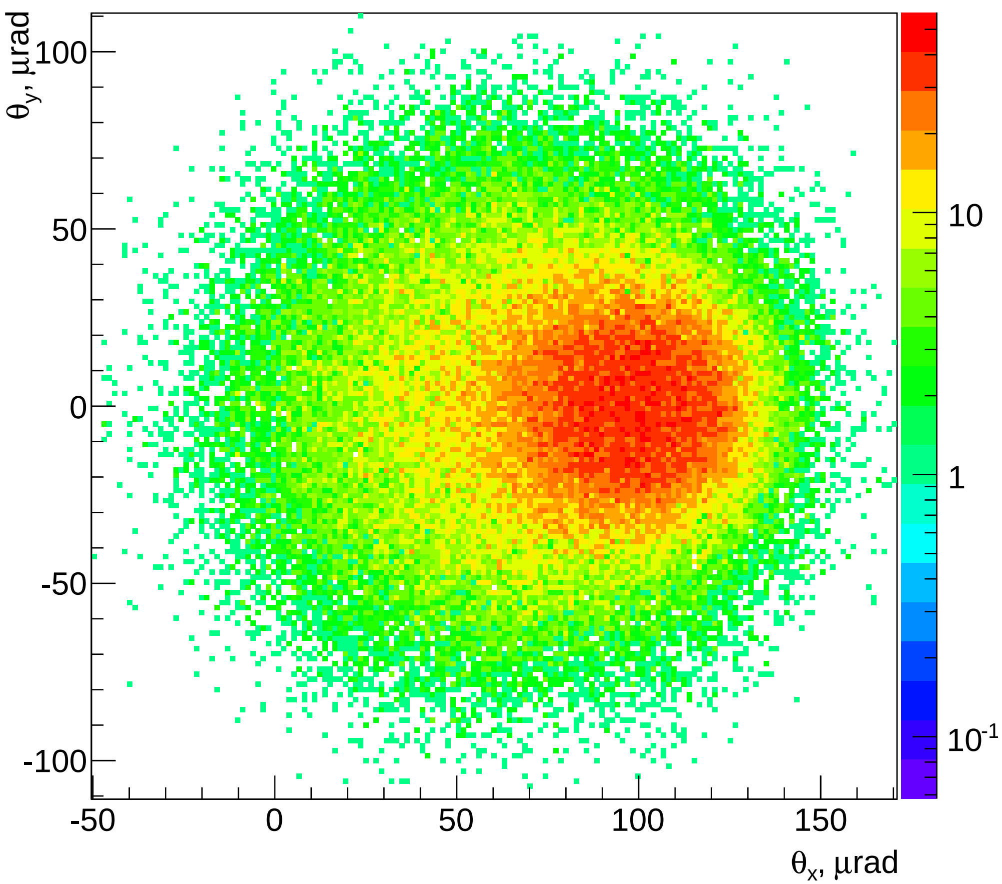
$$\theta_y^{in} = 400\ \mu\text{rad}$$

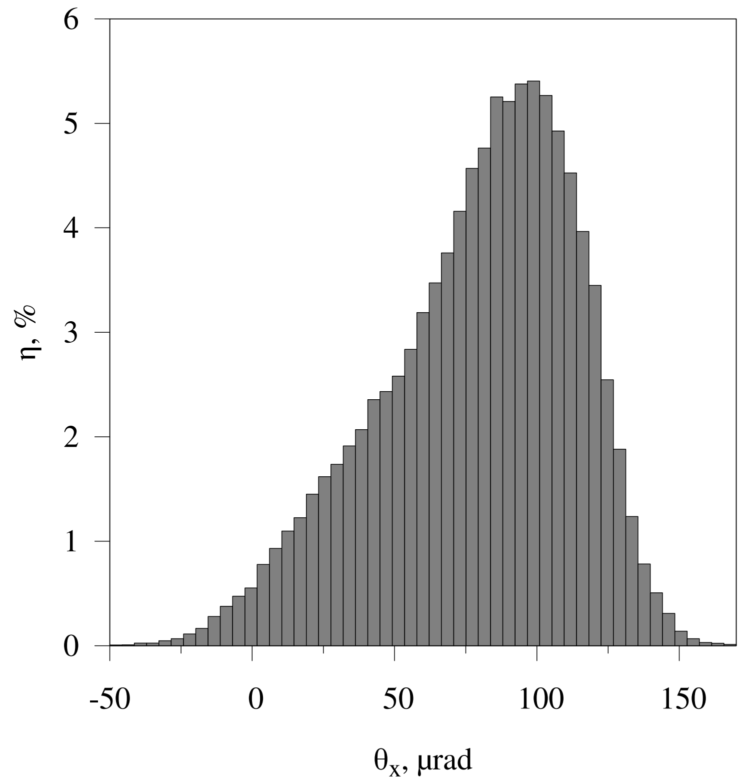




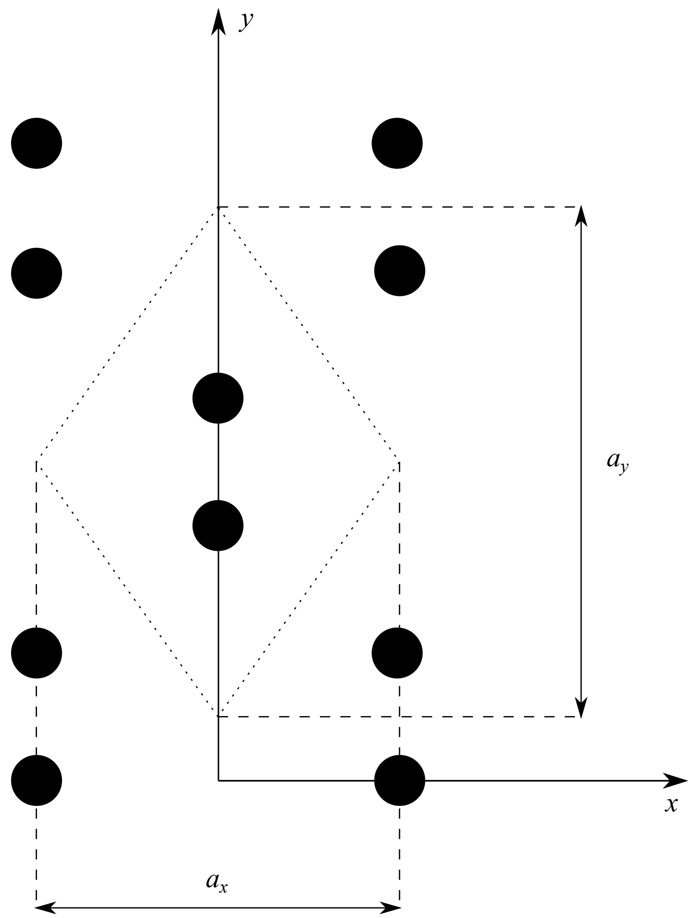
Angular distribution of 270 GeV/c π^--mesons after passing through 5 mm Si crystal with radius of curvature 50 m.

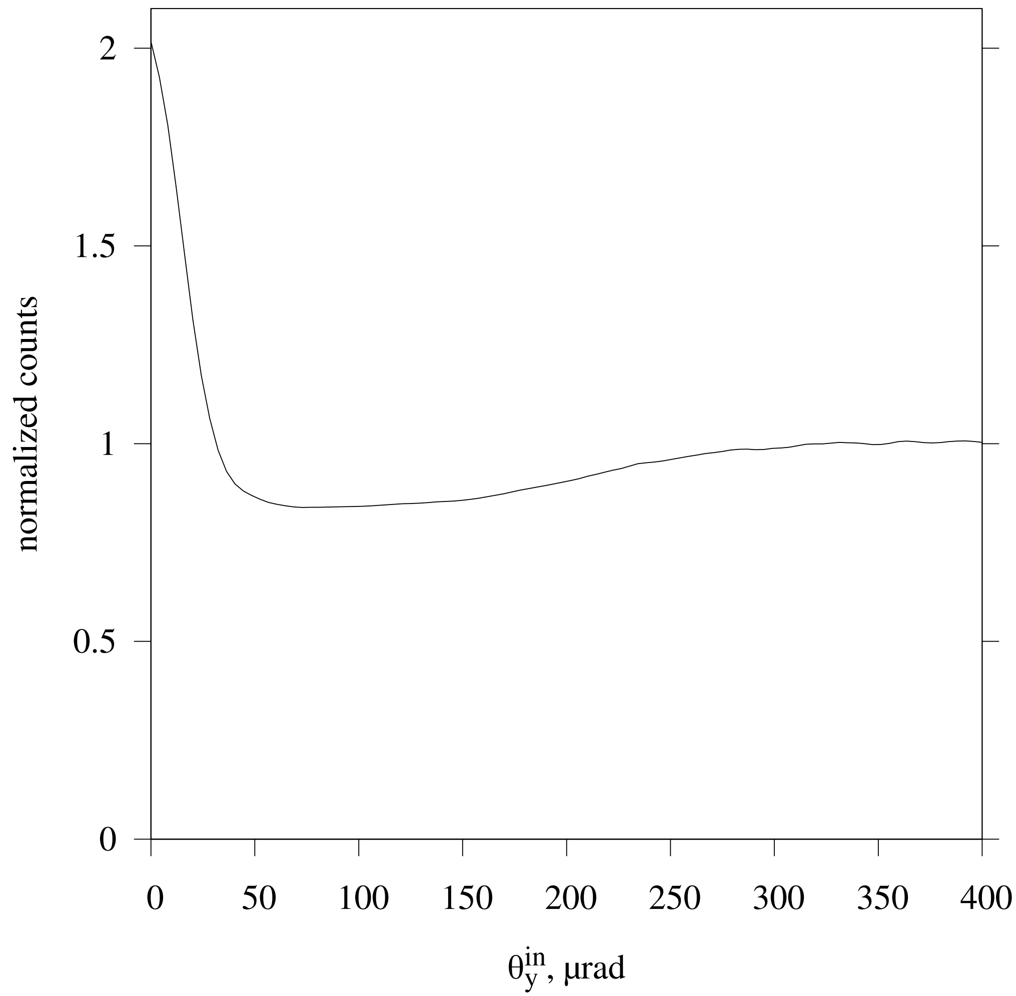
$$\theta_y^{in} = 0$$





$\langle 110 \rangle$ axis of Si and normalized probability of close collisions of π^--mesons in a bent crystal.





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