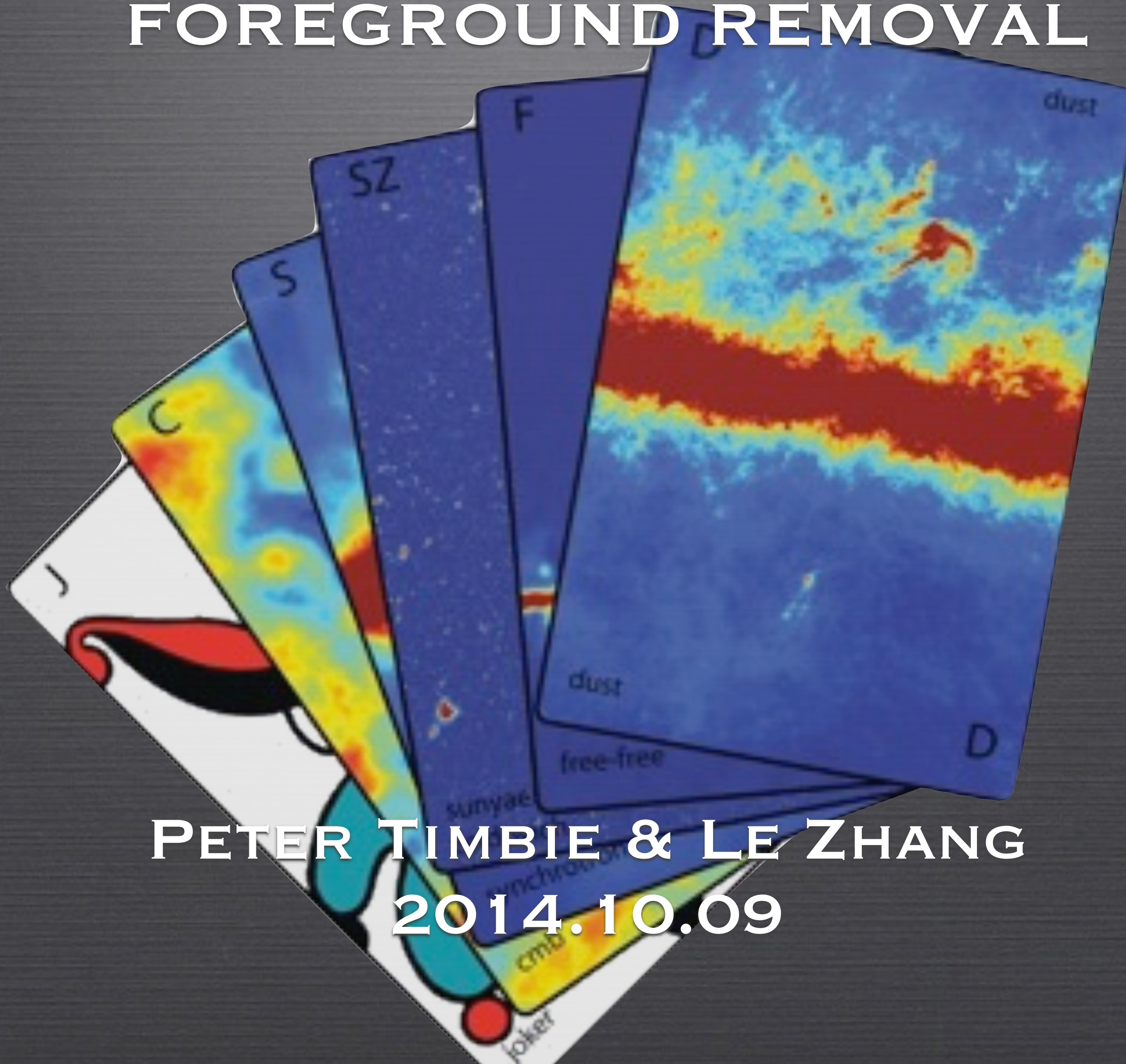
BAYESIAN SEMI-BLIND COMPONENT SEPARATION FOR FOREGROUND REMOVAL



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CMB reconstruction: ICA Approach

★***POL**arized **E**xpectation-**M**aximization **I**ndependent **C**omponent **A**nalysis*
★*adaptation to polarization of the Spectral Matching ICA (SMICA) algorithm* [Delabrouille *et al.* 2003]

$$R_Y = A R_S A^T + R_N$$

★*sets of parameters to extract, several levels of a priori*

Blind $\theta_{\mathrm{blind}}(b) = \left\{ A, R_s(b), \mathrm{diag}\big(R_n(b)\big) \right\}$

CMB-fixed $\theta_{\mathrm{CMB-fixed}}(b) = \left\{ A_{i,j \neq \mathrm{CMB}}, R_s(b), \mathrm{diag}\big(R_n(b)\big) \right\}$

A-fixed $\theta_{\mathrm{A-fixed}}(b) = \left\{ R_s(b), \mathrm{diag}\big(R_n(b)\big) \right\}$

★*Expectation-Maximization (EM) algorithm* [Dempster *et al.* 1977]

E-step: *computation of the conditional statistics from θ_i (gaussian a priori)*

M-step: *maximization of the likelihood and update of the parameters to compute θ_{i+1}*

Maximization of the likelihood analytically guaranteed

iterations

Aumont & Macías-Pérez 2007

Application to 21cm data cube

- Data model: $V(u,v,\nu) = s(u,v,\nu) + \sum M_{\nu c}.f_c(u,v) + n(u,v,\nu)$
- Power spectrum: $<ss^*> = \mathbf{F}_{\|}^{\dagger}\mathbf{P}_{3D}(k)\mathbf{F}_{\|};$ $<f_c f_c^*> = C_c(l)$
- Semi-Blind search: find the best-fitted $\mathbf{P}_{3D}(k)$, $C(l)$ and M to data without any assumption about foregrounds and signal and only fix $N=<nn^*>$
- Instrument effects: beam pattern, bandpass and frequency-dependent uv-coverage could mix the angular Fourier modes and radial Fourier modes. We will consider these effects in future

THE EM ALGORITHM

Basic steps in the EM algorithm

The implementation of the EM algorithm starts by choosing an initial guess for the unknown parameters, $\boldsymbol{\theta}^0$, which is used in the first iteration of the algorithm. Then, at each iteration $j+1$ the following basic steps are performed

- **i)** $\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}^j$.
- **ii)** E-step: Compute $Q(\boldsymbol{\theta}', \boldsymbol{\theta})$.
- **iii)** M-step: Find $\boldsymbol{\theta}^{j+1}$ such that $Q(\boldsymbol{\theta}^{j+1}, \boldsymbol{\theta}) \geq Q(\boldsymbol{\theta}', \boldsymbol{\theta})$ for all $\boldsymbol{\theta}' \in \boldsymbol{\Theta}$.

The iterative procedure is stopped when a fixed point is reached so that $\boldsymbol{\theta}^{j+1} = \boldsymbol{\theta}^{j+1}$.

The functional $Q(\boldsymbol{\theta}, \boldsymbol{\theta}^k)$ is given by

$$
\begin{aligned}
Q\left(\boldsymbol{\theta} \,|\, \boldsymbol{\theta}^j\right) &= \mathrm{E}_{\boldsymbol{s} \,|\, \boldsymbol{\theta}^j} \left[\log p(\boldsymbol{y}_{1..K}, \boldsymbol{s}_{1..K} \,|\, \boldsymbol{\theta}) \,|\, \boldsymbol{y}_{1..K}, \boldsymbol{\theta}^j\right] \\
&= \int_{\boldsymbol{s}_{1..K}} \log p(\boldsymbol{y}_{1..K}, \boldsymbol{s}_{1..K} \,|\, \boldsymbol{\theta})\, p(\boldsymbol{s}_{1..K} \,|\, \boldsymbol{y}_{1..K}, \boldsymbol{\theta}^j)\, d\,\boldsymbol{s}_{1..K}
\end{aligned}
$$

y=As+n Consider a probability model p(y, s|θ) for a pair (y, s) of random variables with θ a parameter set. The maximization of the log-likelihood l(θ) can be made easier by considering the EM functional:

$$l(\theta, \theta') = \int \log(p(y, s|\theta))\ p(s|y, \theta')ds.$$

Both the E step and the M step turn out to be straight- forward; EM step amounts to solving

$$0 = \int \frac{\partial \log(p(y, s|\theta^{(n+1)}))}{\partial \theta}\ p(s|y, \theta^{(n)})\, ds.$$

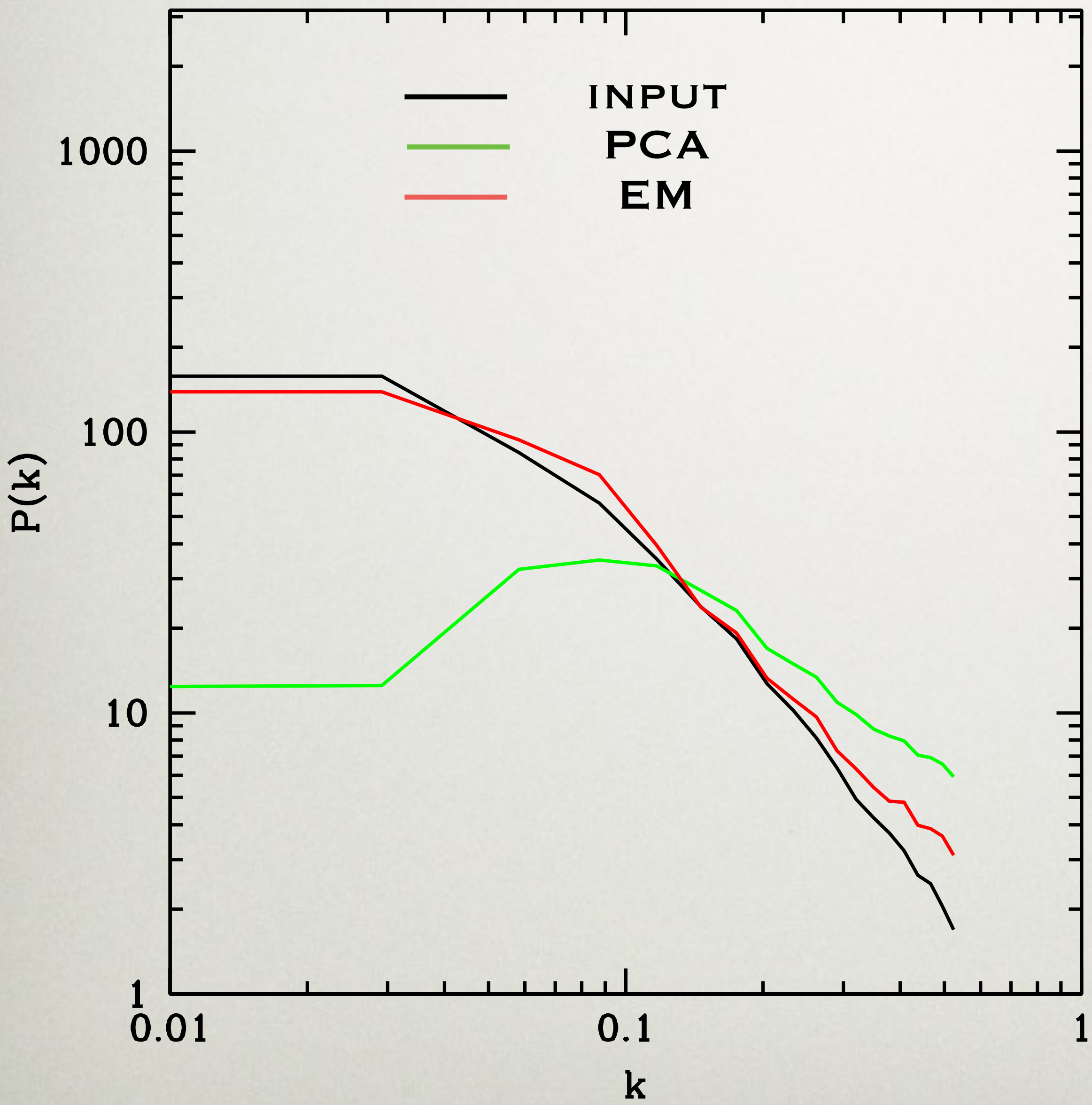
update scheme:

$$\begin{aligned} A &= \widetilde{R}_{ys}(\theta')\widetilde{R}_{ss}(\theta')^{-1} \\ \sigma_i^2 &= \left[\widetilde{R}_{yy}(\theta') - \widetilde{R}_{ys}(\theta')\widetilde{R}_{ss}(\theta')^{-1}\widetilde{R}_{sy}(\theta')\right]_{ii} \\ P_i(q) &= [\widetilde{R}_{ss}(\theta', q)]_{ii} \end{aligned}$$

where

$$\begin{aligned} C(q) &= (A^\dagger R_n^{-1} A + R_s(q)^{-1})^{-1} \\ W(q) &= (A^\dagger R_n^{-1} A + R_s(q)^{-1})^{-1} A^\dagger R_n^{-1} \\ \widetilde{R}_{ss}(q) &= W(q)\hat{R}_y(q)W(q)^\dagger + C(q) \\ \widetilde{R}_{sy}(q) &= W(q)\hat{R}_y(q) \end{aligned}$$

HI Power spectrum recovery



- **Data cube(32^3 grids):** 20°x20° sky patch 800-830 MHZ(z~0.8)
- **Foregrounds:** synchrotron, free-free, points source
- **PCA**: remove the first 2 eigen-foregrounds with largest eigenvalues
- **EM**: assume 3 independent foreground components
- Consider noise (S/N=~3) in uv-plane, but not include beam pattern and incomplete uv-coverage