Sparse Inpainting in Astrophysics

Jean-Luc Starck
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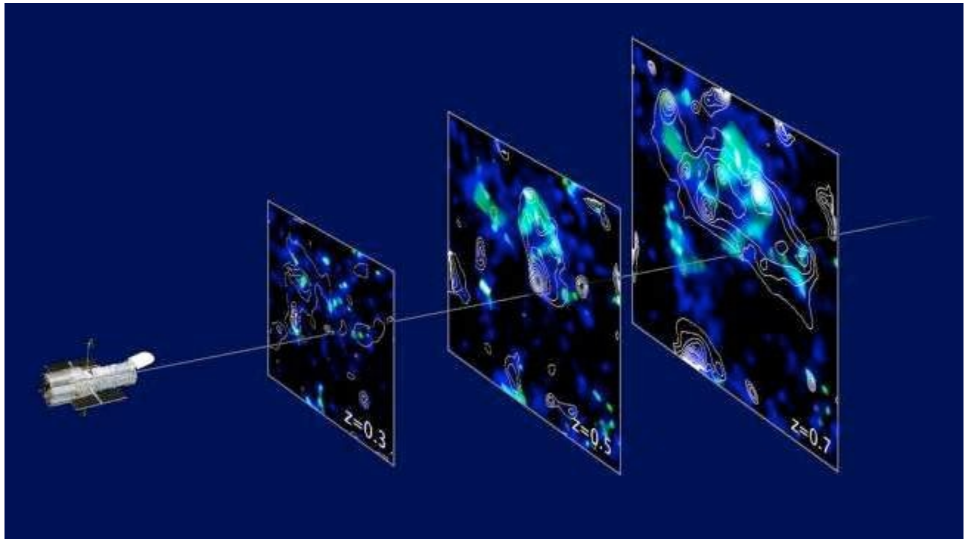
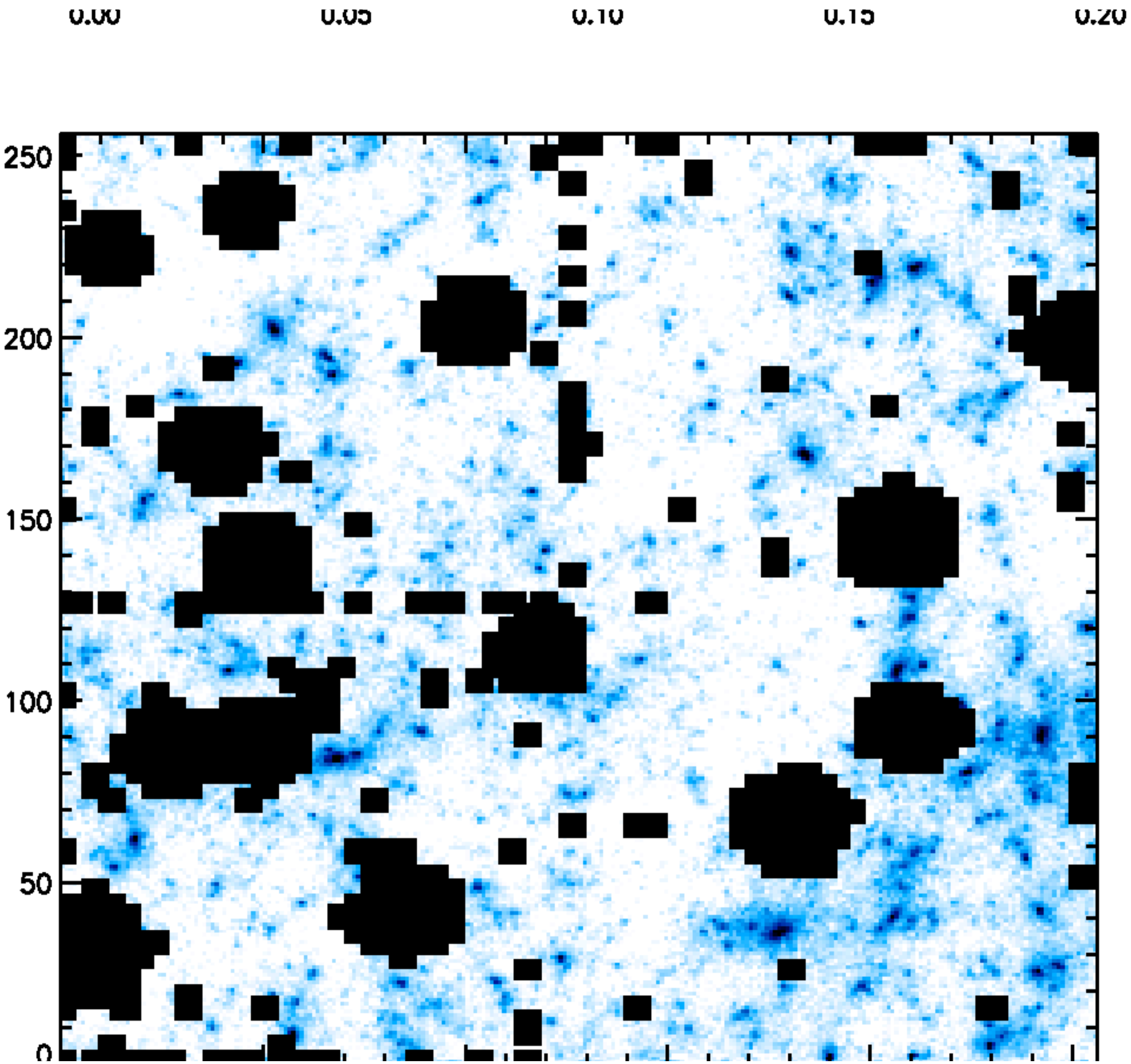


http://jstarck.free.fr

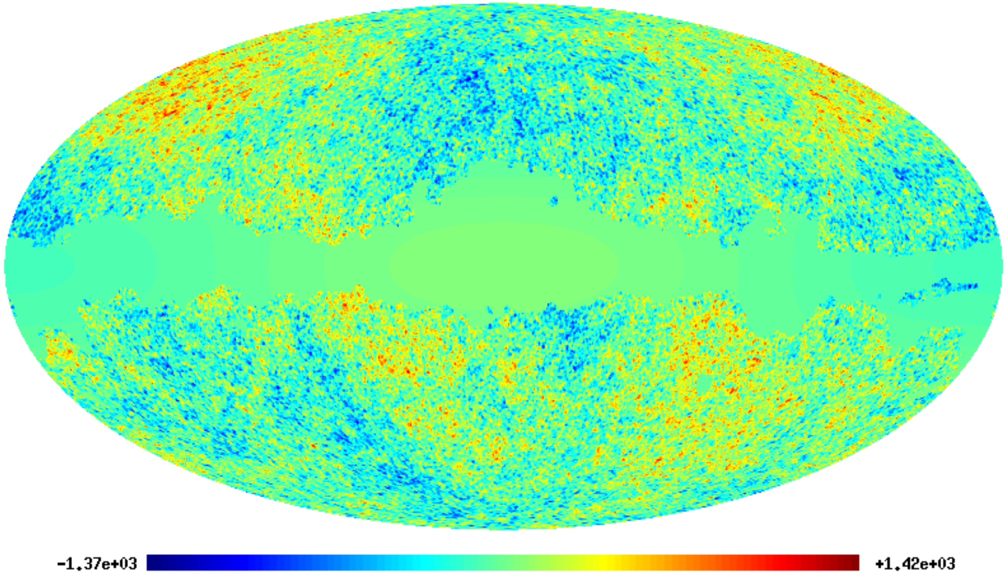
http://www.cosmostat.org

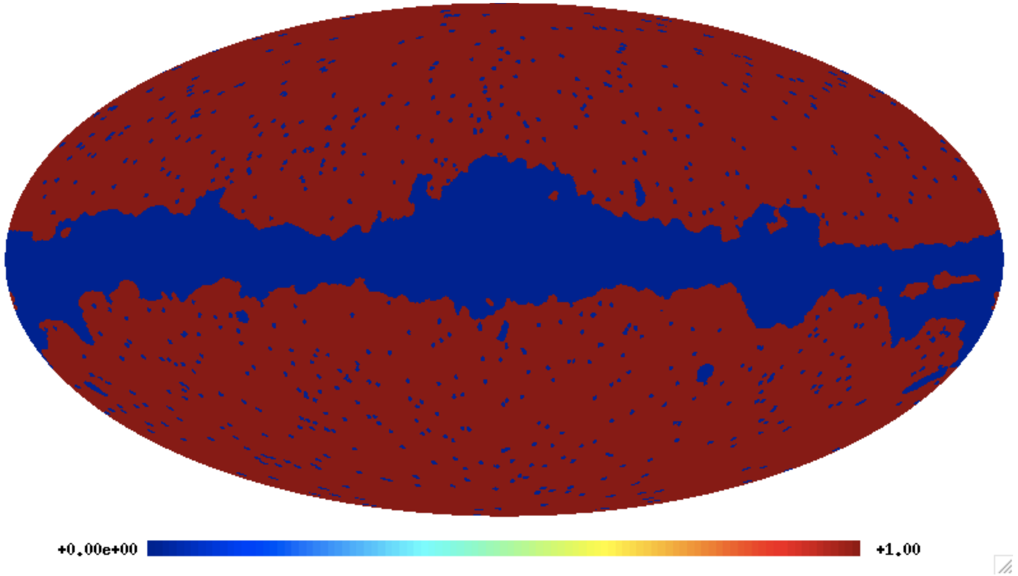
Missing Data

Bad pixels, cosmic rays, point sources in 2D images, ...



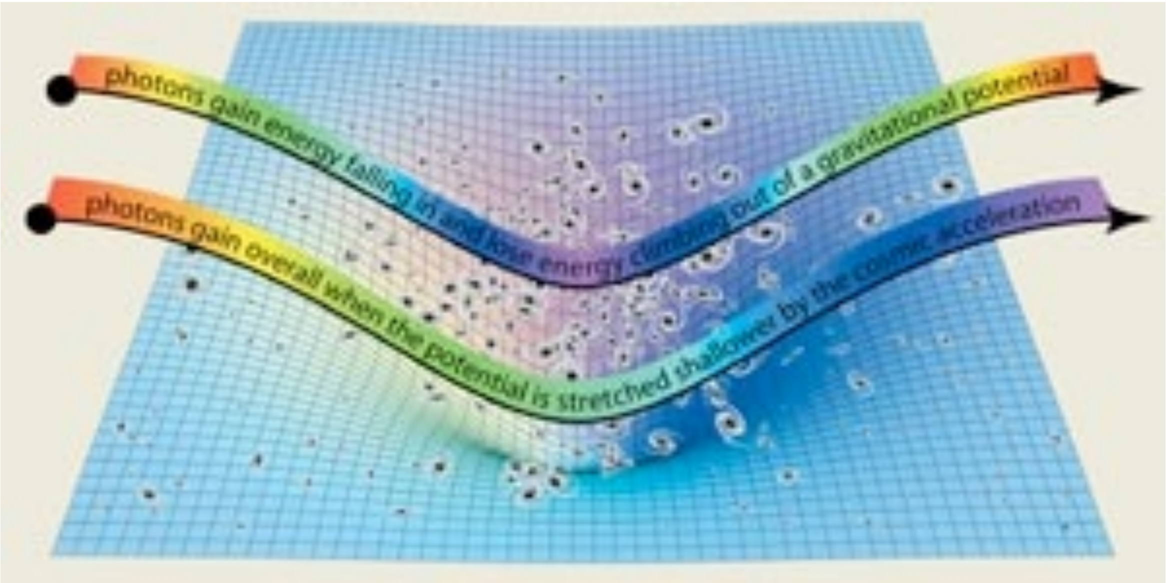
Cosmic Microwave Background





- Point sources problem
- Gaussianity/isotropy test, especially in the spherical harmonic domain (bispectrum analysis, lensing estimator, large scale studies, etc).
- Any analysis where the mask is a problem.

Integrated Sachs-Wolfe Effect (ISW)



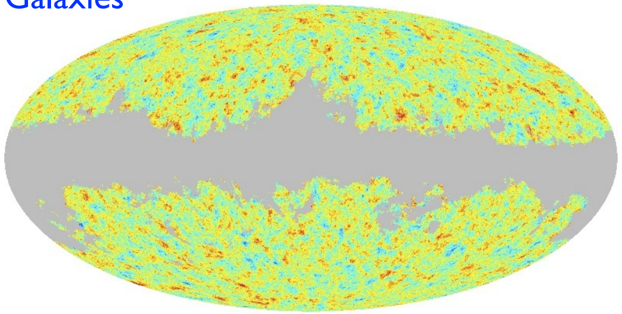
Measure of Time Variation in the Gravitational Potential on **large scales (linear)**

$$\left(\frac{\Delta T}{T}\right)_{ISW} = -2\int \frac{d\Phi}{d\eta}\, d\eta$$

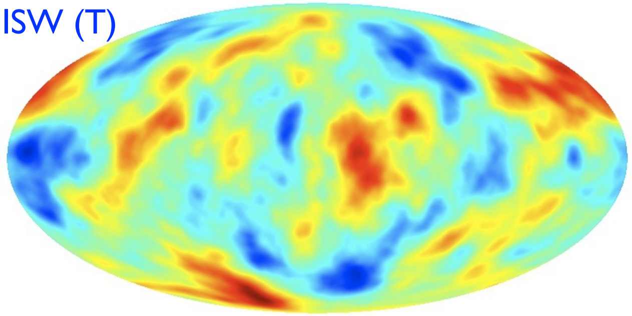
Detect by cross-correlating with local tracers of mass

Can ISW explain some of the CMB anomalies (Francis & Peacock, 2010) ?

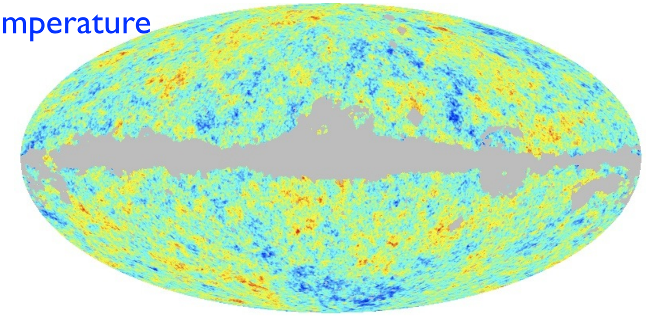
Galaxies



ISW (T)



Temperature

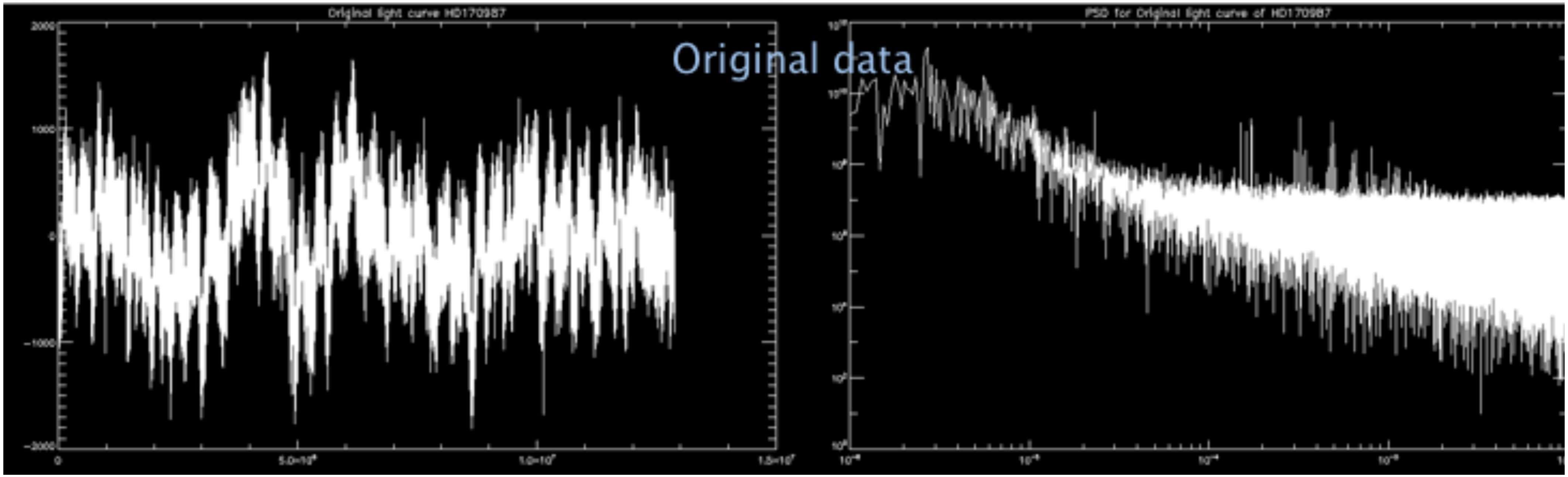


Even if you don’t believe in these, you should still remove secondary anisotropies, ..., if you can.

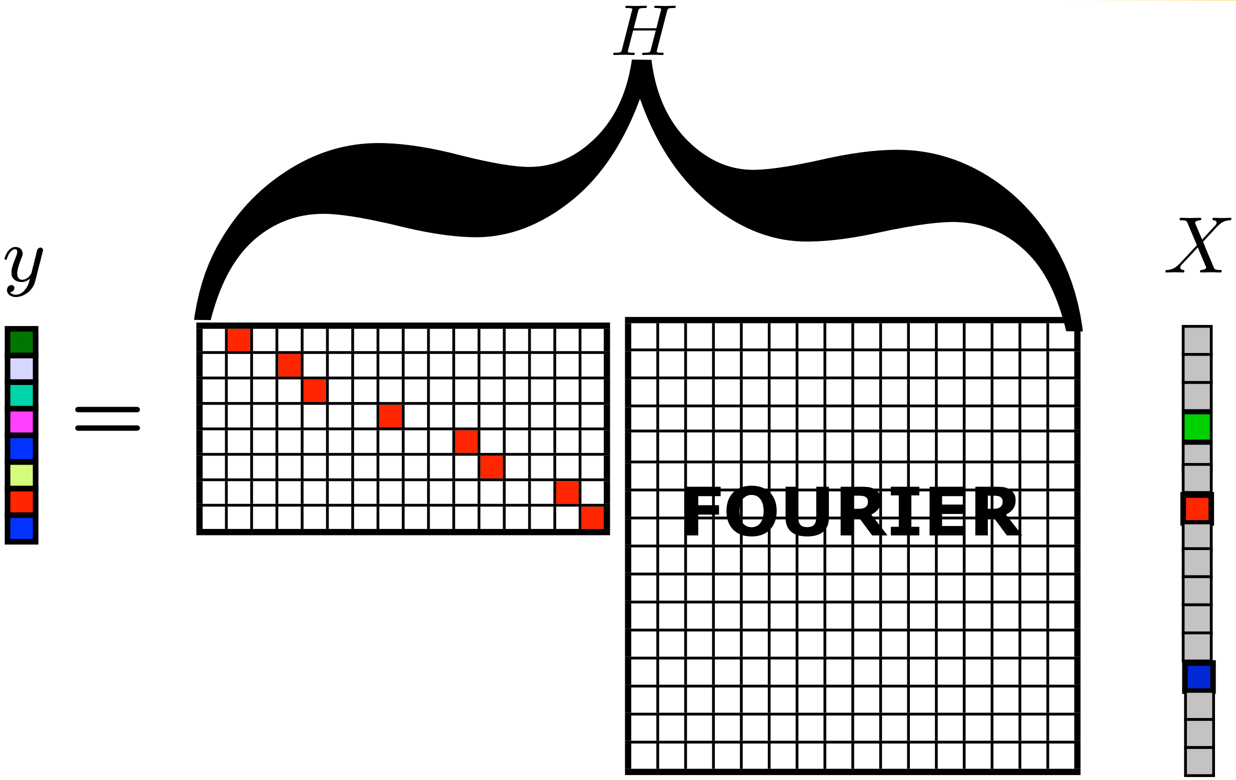
==> Galactic Mask problem when analyzing the largest scales.

Period detection in temporal series

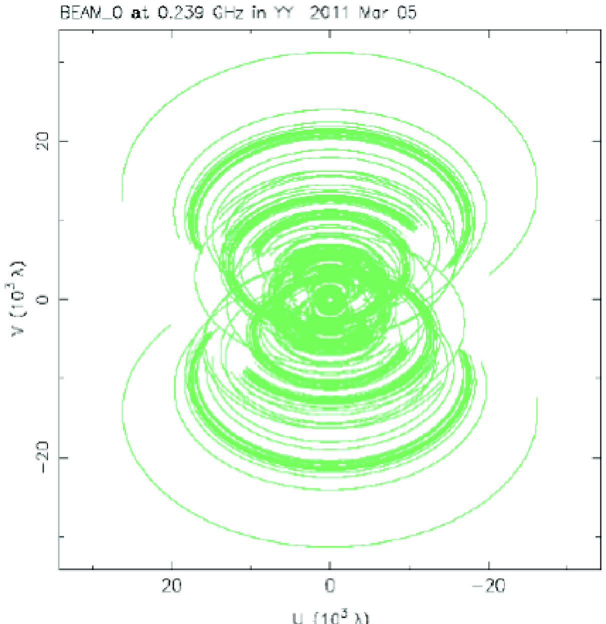
COROT: HD170987



Radio-Interferometry Image Reconstruction

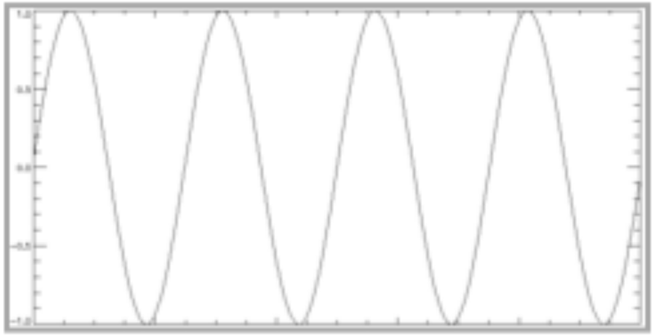


Measurement System

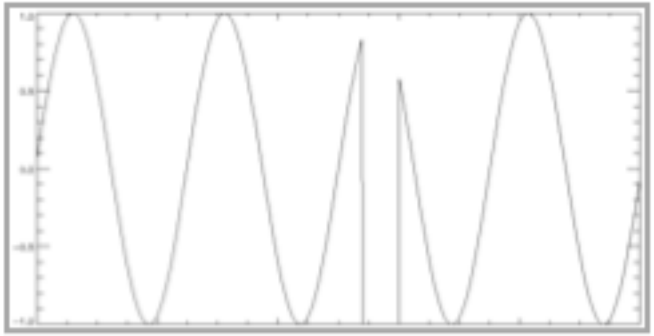


$$Y = HX + N$$

Sparsity Constraint

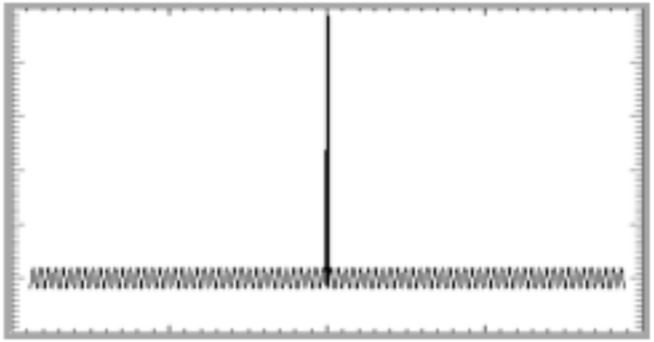


Sine curve



Truncated sine curve

TF of a sine curve



TF of a truncated sine curve

Weak Sparsity or Compressible Signals

A signal *s* (*n* samples) can be represented as sum of weighted elements of a given dictionary

Dictionary (basis, frame)

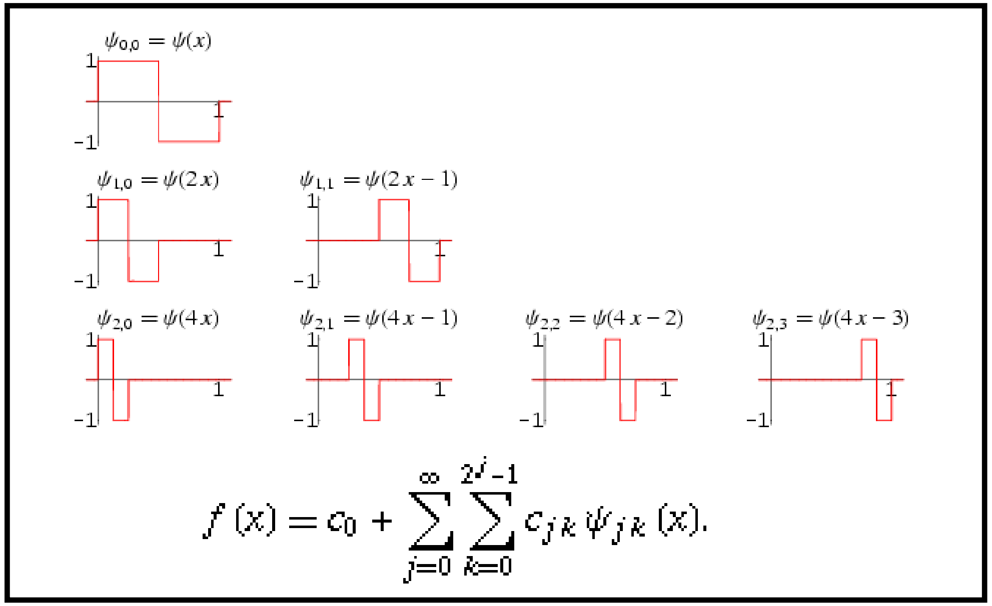
$$\Phi = \{\phi_1, \ldots, \phi_K\}$$

Atoms

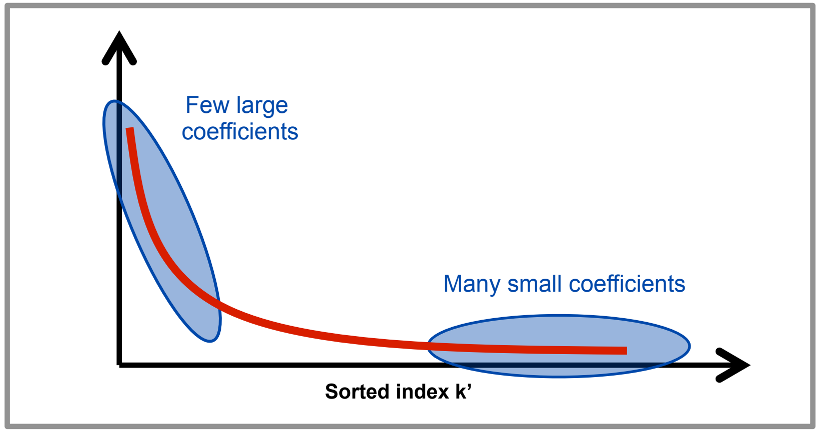
$$s = \sum_{k=1}^{K} \alpha_k \phi_k = \Phi\alpha$$

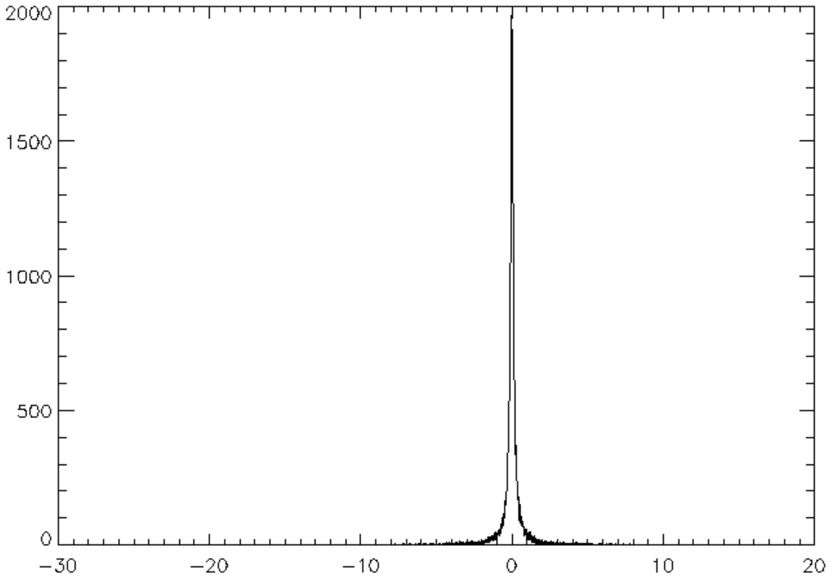
coefficients

Ex: Haar wavelet



$$f(x) = c_0 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} c_{jk} \psi_{jk}(x).$$





- Fast calculation of the coefficients
- Analyze the signal through the statistical properties of the coefficients
- Approximation theory uses the sparsity of the coefficients

How to measure sparsity ?

with $0^0 = 0,$

$$\| \alpha \|_0 = \sum_k \alpha_k^0 = \# \{\alpha_k \neq 0\}$$

Formally, the sparsest coefficients are obtained by solving the optimization problem:

(P0) Minimize $\|\alpha\|_0$ subject to $s = \phi\alpha$

It has been proposed (*to relax and*) to replace the l_0 norm by the l_1 norm (Chen, 1995):

(P1) Minimize $\|\alpha\|_1$ subject to $s = \phi\alpha$

It can be seen as a kind of convexification of (P0).

It has been shown (Donoho and Huo, 1999) that for certain dictionary, if there exists a highly sparse solution to (P0), then it is identical to the solution of (P1).

INVERSE PROBLEMS AND SPARSE RECOVERY

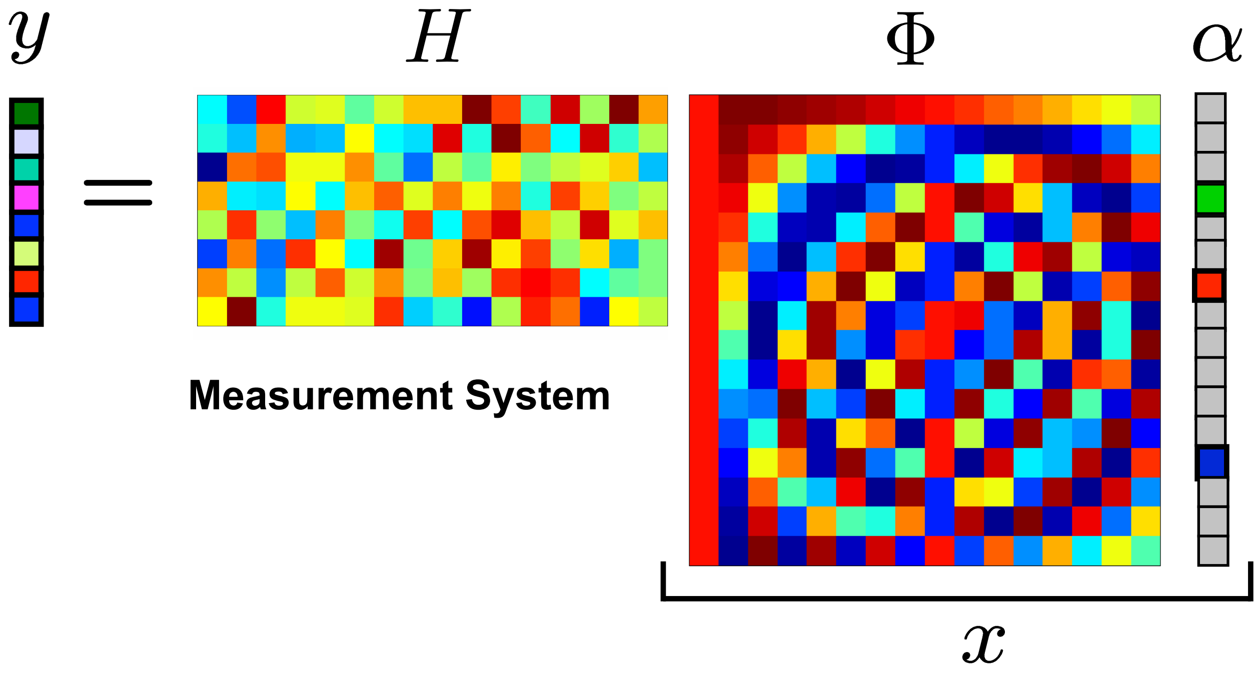
$$Y = HX + N$$

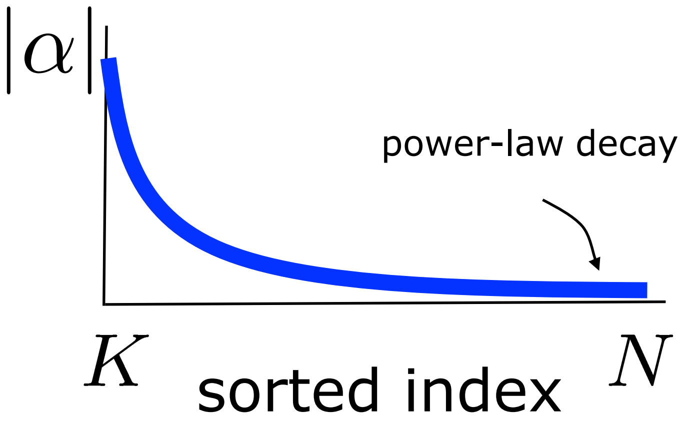
$X = \Phi\alpha$, and α is sparse

- **Denoising**
- **Deconvolution**
- **Component Separation**
- **Inpainting**
- **Blind Source Separation**
- **Minimization algorithms**
- **Compressed Sensing**

$$\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - A\Phi\alpha\|^2 \leq \epsilon$$

Very efficient recent methods now exist to solve it (proximal theory)





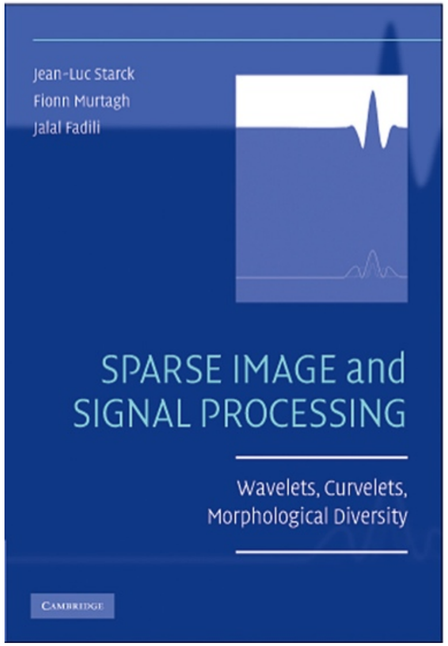
Interpolation of Missing Data

Inp  inting

$$x = \Phi\alpha$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data

$$\boxed{min_\alpha \parallel \alpha \parallel_p \quad s.t. \quad y = Mx}$$



Jean-Luc Starck
Fionn Murtagh
Jalal Fadili

SPARSE IMAGE and SIGNAL PROCESSING

Wavelets, Curvelets, Morphological Diversity

CAMBRIDGE

Proximal optimizatio: (Forward-Backward algorithm, etc)

- M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, “Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", ACHA, Vol. 19, pp. 340-358, 2005.
- M.J. Fadili, J.-L. Starck and F. Murtagh, "Inpainting and Zooming using Sparse Representations", The Computer Journal, 52, 1, pp 64-79, 2009.
- J.L. Starck, F. Murtagh, and J. Fadili, Sparse Image and Signal Processing: Wavelets, Curvelets, Morphological Diversity , Cambridge University Press, Cambridge (GB), 2010.

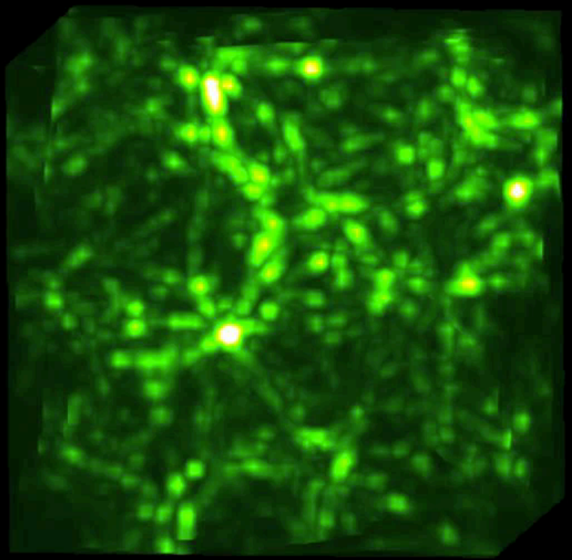
20%

50%

80%

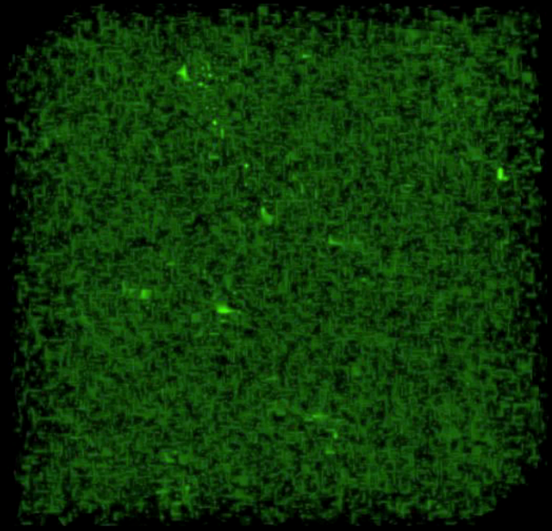
Inpainted with the curvelet dictionary (80% data missing)

MCAlab available at: http://www.greyc.ensicaen.fr/~jfadili

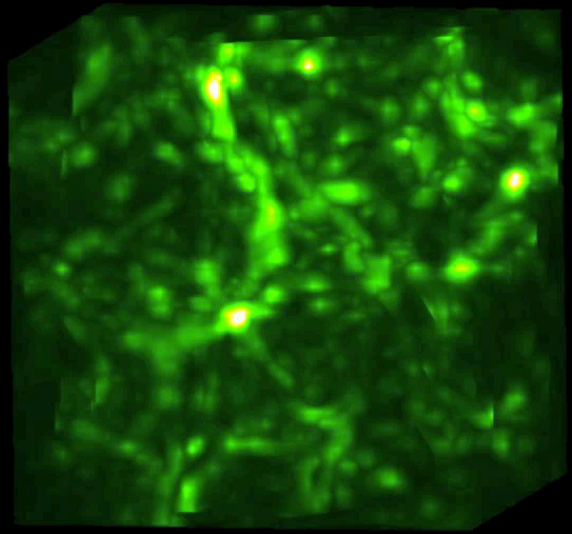


Original

Dictionary
BeamCurvelets

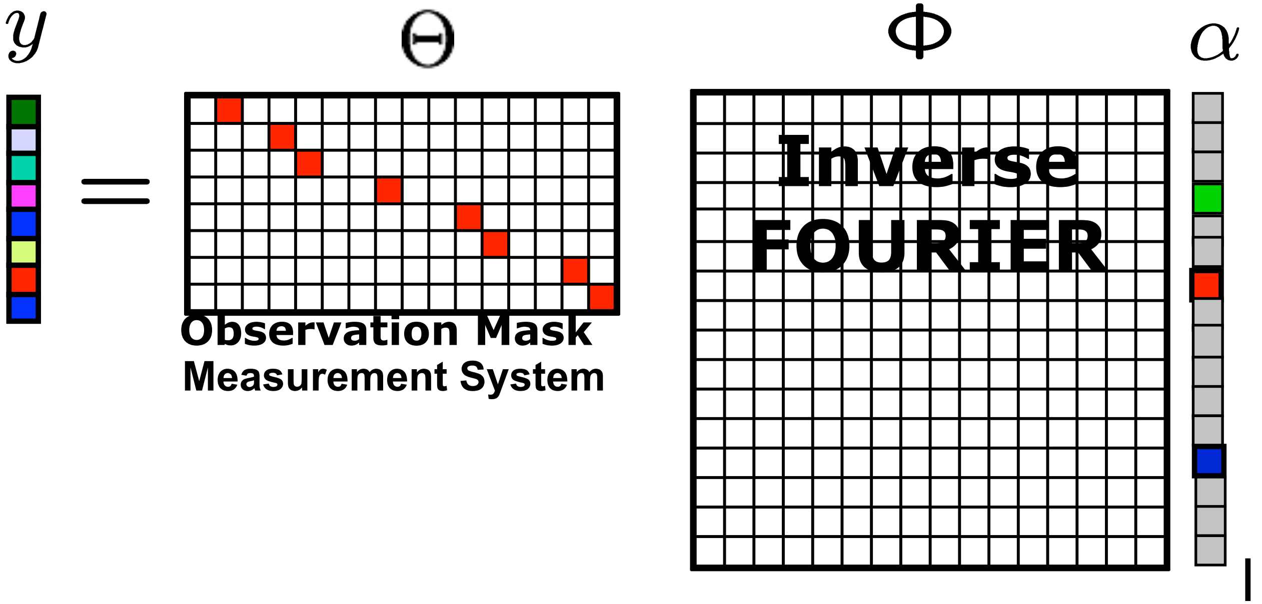


Mask



Inpainted

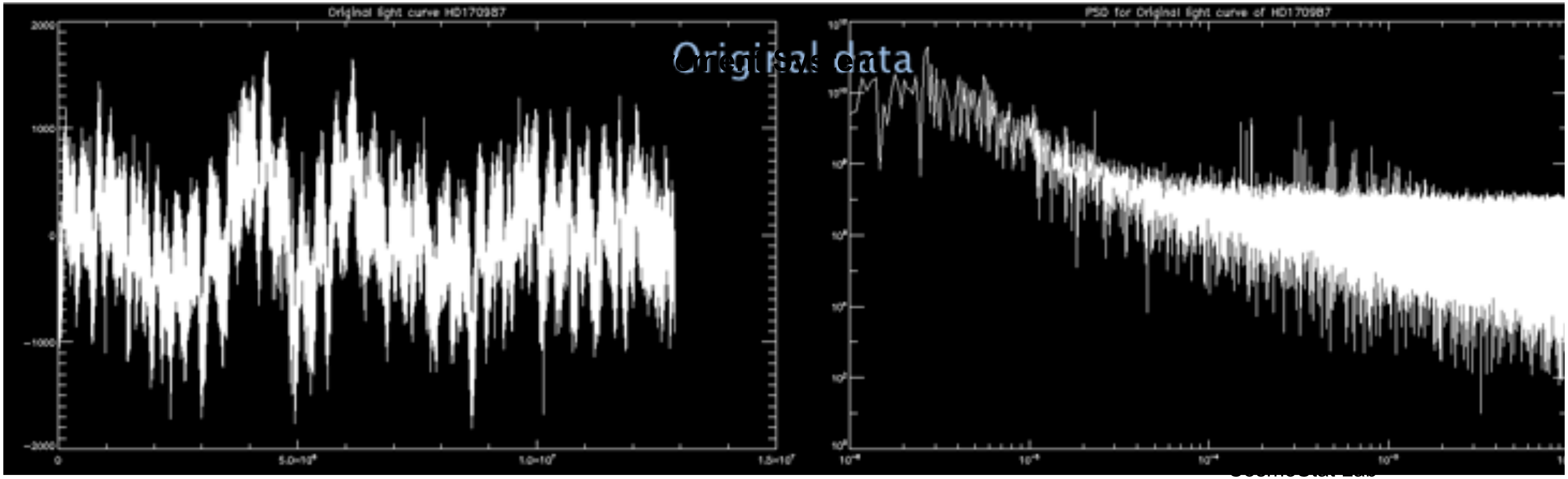
Period detection in temporal series



Inverse FOURIER

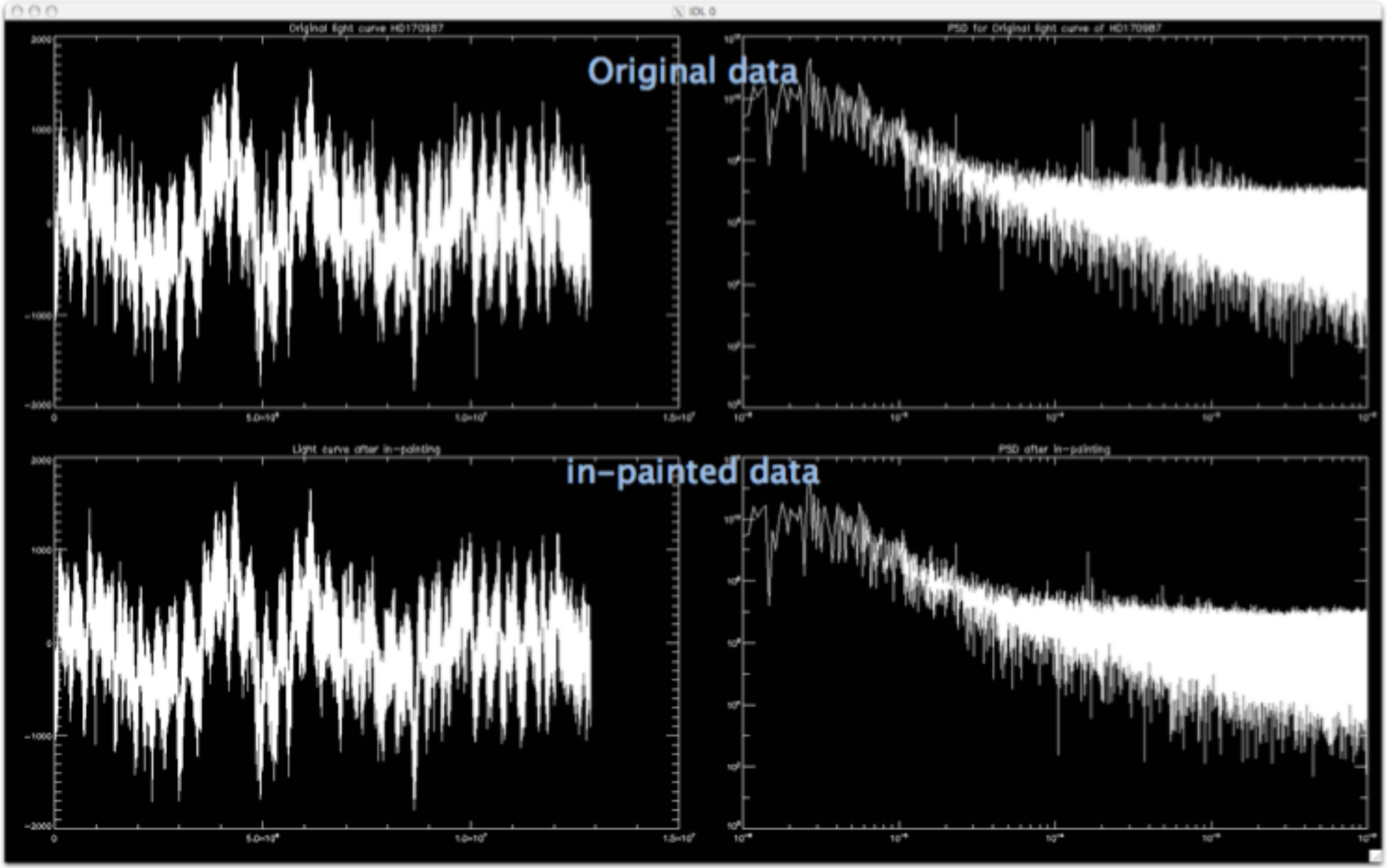
Observation Mask
Measurement System

COROT: HD170987



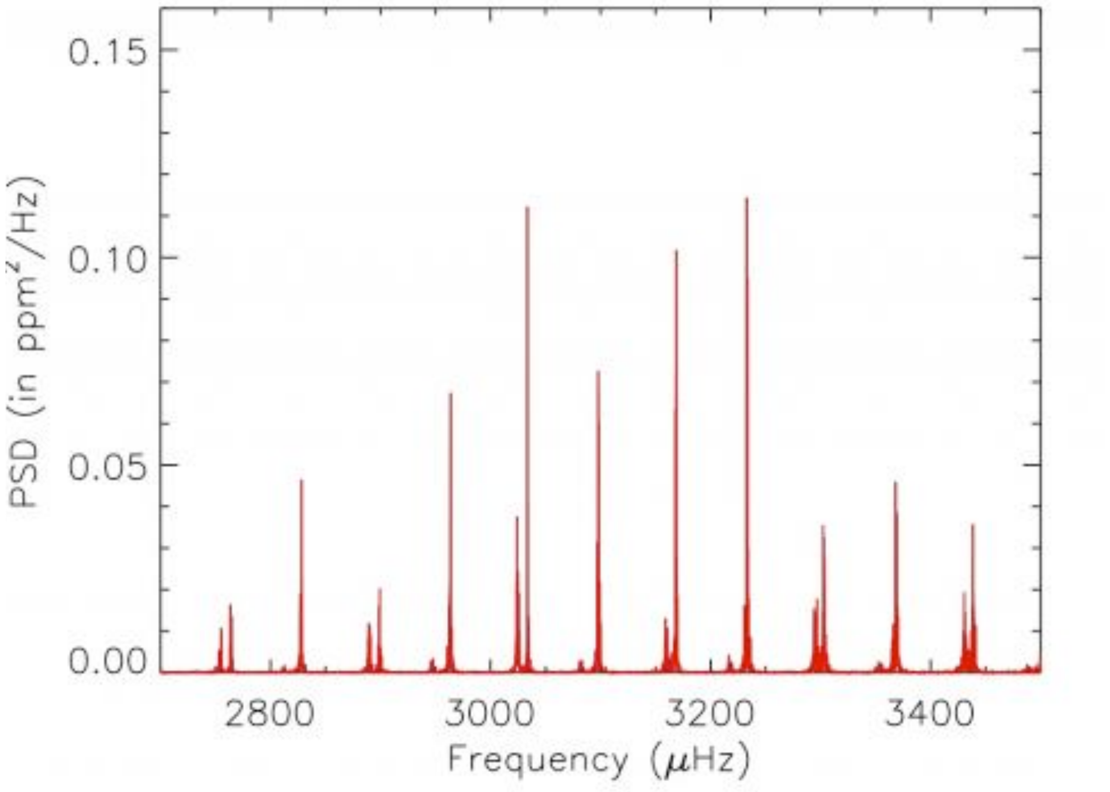
COROT: HD170987 with in-painting

arXiv:1003.5178



Sparse inpainting & asteroseismology

Gap interpolation by Inpainting methods: Application to Ground and Space-based data, S. Pires, S. Mathur, R.A. Garcia, J. Ballot, D. Stello and K. Sato, Astronomy and Astrophysics, submitted.



CoRo: sparse inpainting is in the official pipeline.
Kepler: 18.000 stars have been processed.
GOLF. ongoing tests

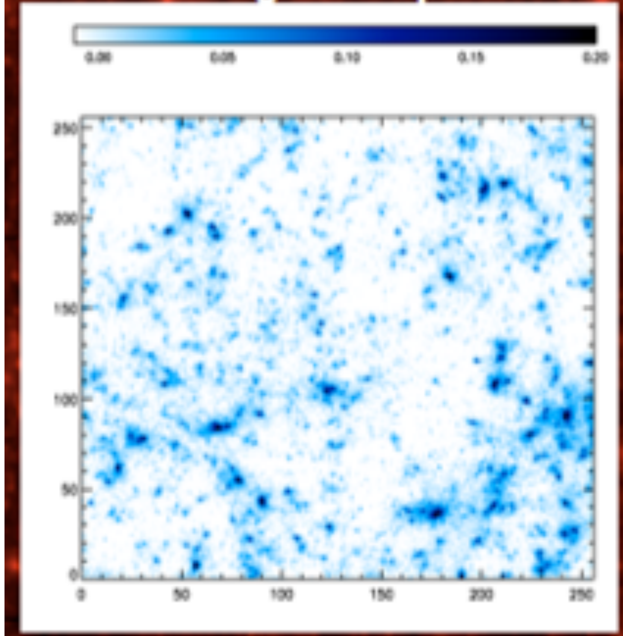
SOFTWARE K-INPAINTING : INPAINTING FOR KEPLER

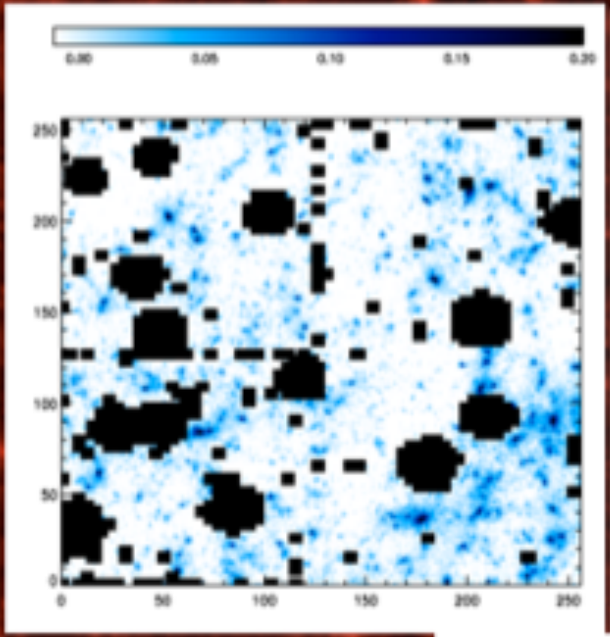
S. Pires, R. A. Garcia, S. Mathur, J. Ballot www.cosmostat.org/software.html

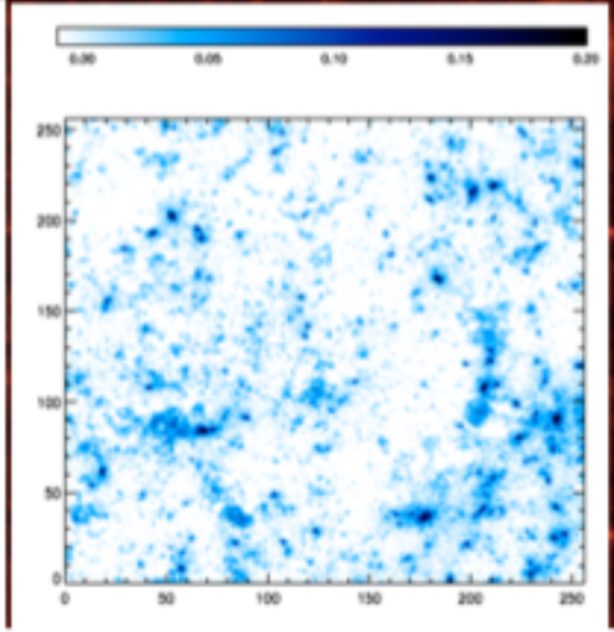
http://irfu.cea.fr/Sap/en/Phocea/Vie_des_labos/Ast/ast_visu.php?id_ast=3346

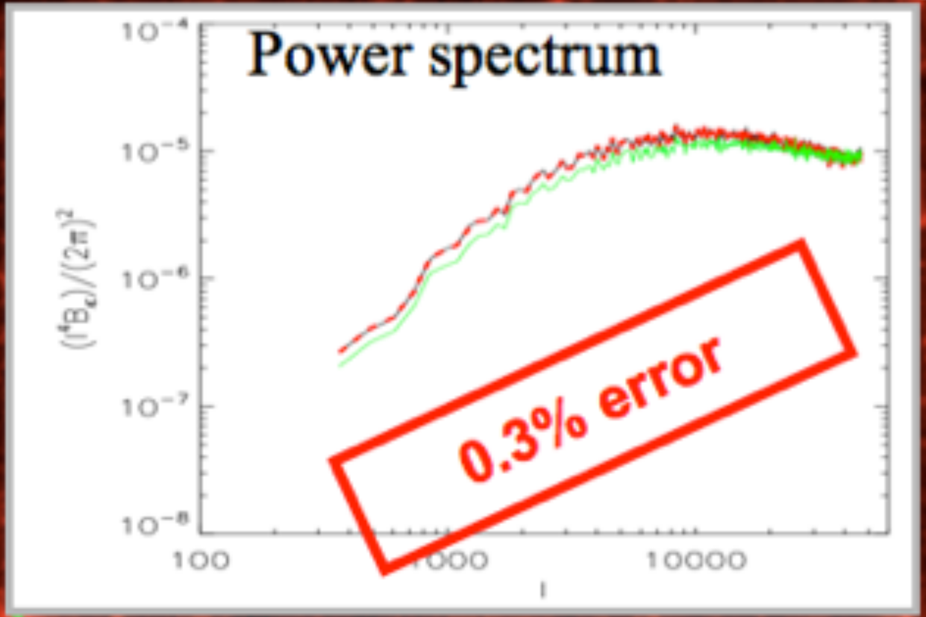
Inpainting :

S. Pires, J.-L. Starck, A. Amara, R. Teyssier, A. Refregier and J. Fadili, "FASTLens (FAst STatistics for weak Lensing) : Fast method for Weak Lensing Statistics and map making", MNRAS, 395, 3, pp. 1265-1279, 2009.

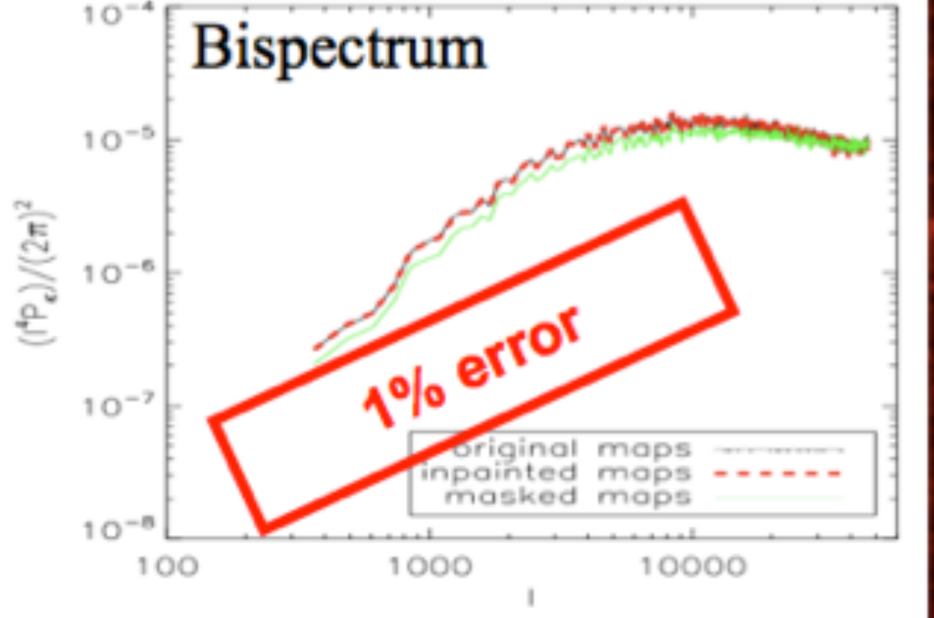

Original map


Masked map


Inpainted map

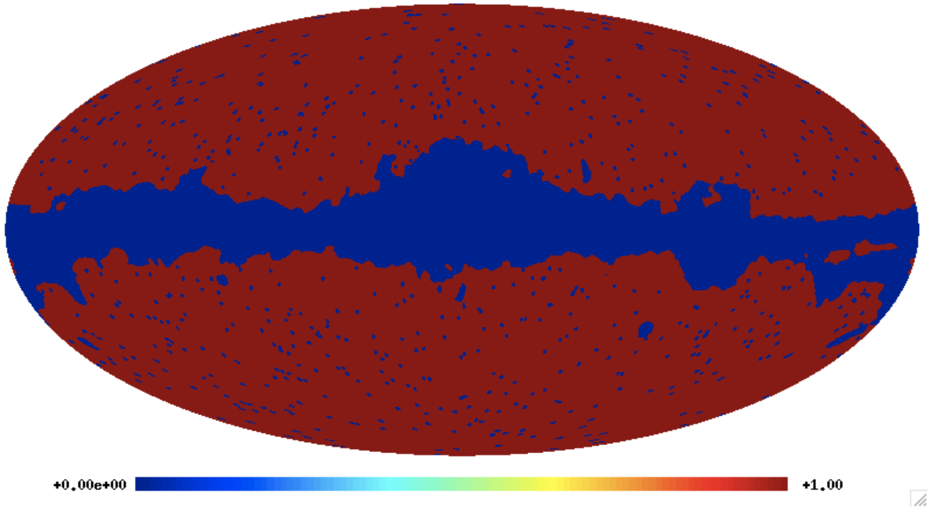

Power spectrum

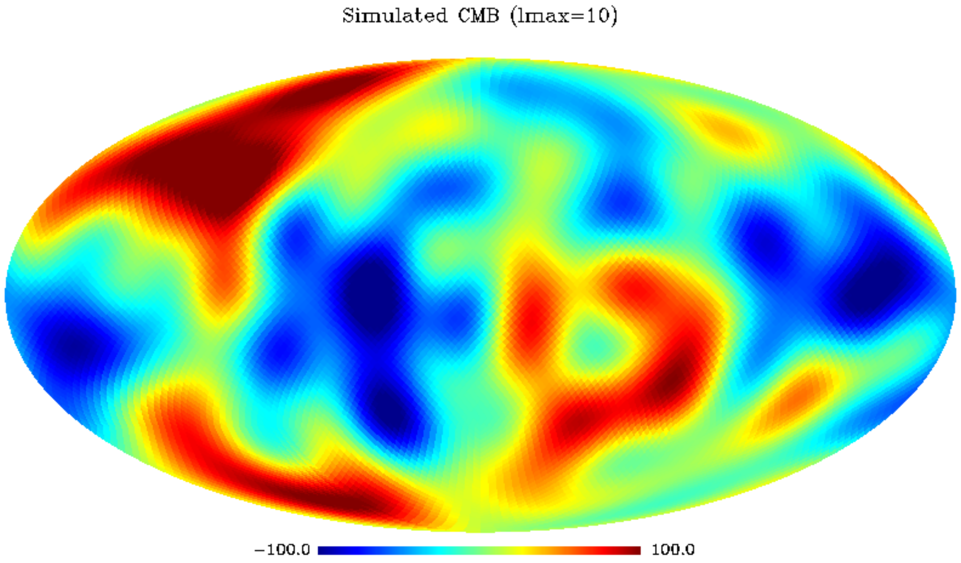
0.3% error


Bispectrum

1% error

Large Scale: Douglas-Rachford Sparse Inpainting





$$\min_{\alpha} \|\alpha\|_1 \quad \text{subject to} \quad Y = M\Phi\alpha$$

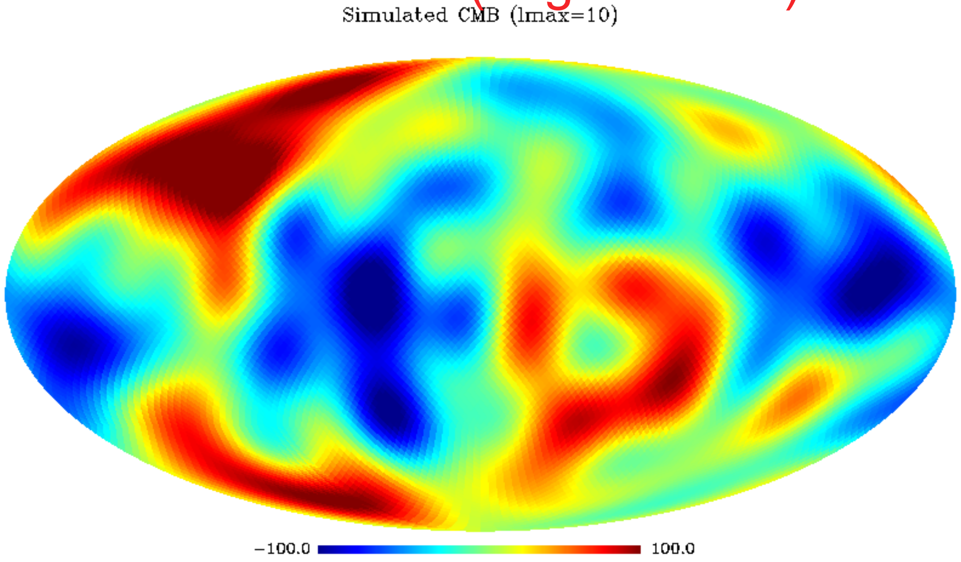
$$X = \Phi\alpha \qquad \Phi = \text{Spherical Harmonics}$$

$$\|\alpha\|_1 \quad = \sum_k \mid \alpha_k \mid$$

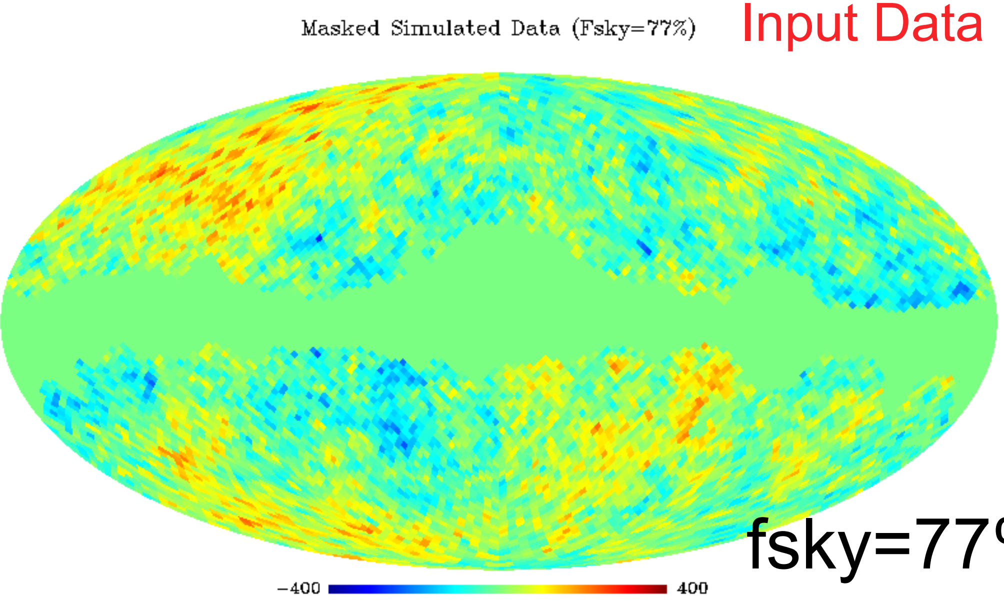
J.-L. Starck, A. Rassat, and M.J. Fadili, "Low-l CMB Analysis and Inpainting”, **Astronomy and Astrophysics** , 550, A15, 2013.

Large CMB Scale Analysis

Simulated CMB (largest scale)

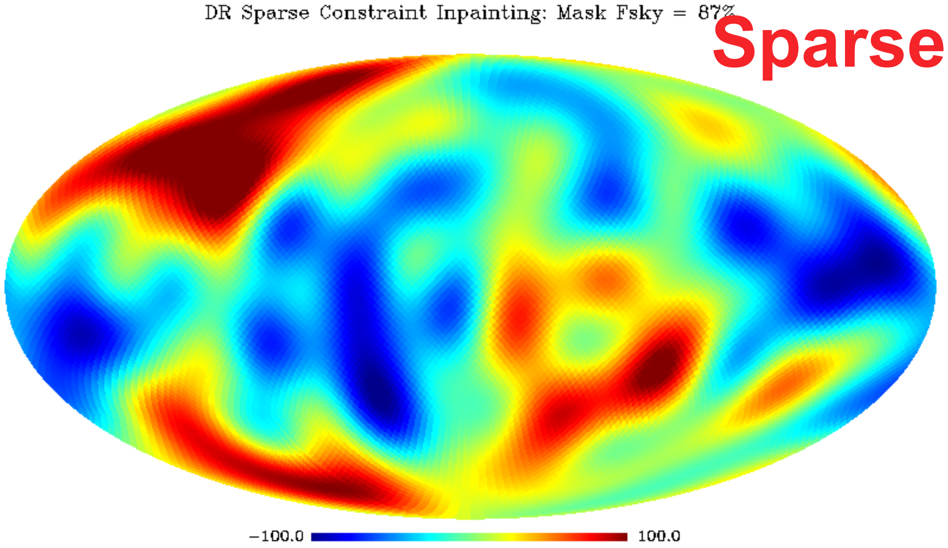

Simulated CMB (lmax=10)

Input Data

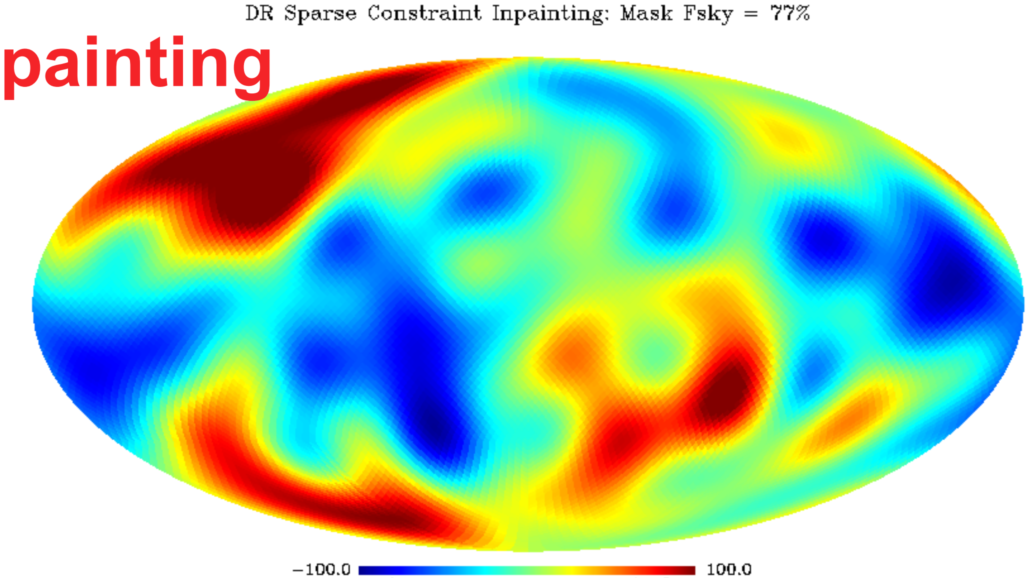

Masked Simulated Data (Fsky=77%)

fsky=77%

Sparse Inpainting


DR Sparse Constraint Inpainting: Mask Fsky = 87%

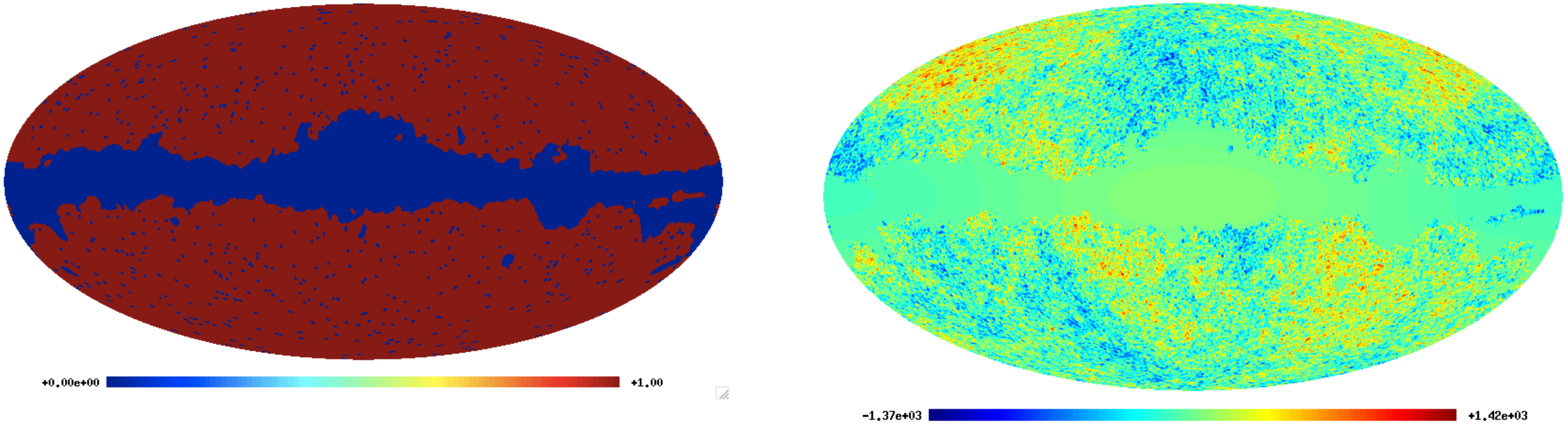
fsky=87%


DR Sparse Constraint Inpainting: Mask Fsky = 77%

fsky=77%

J.-L. Starck, A. Rassat, and M.J. Fadili, "Low-l CMB Analysis and Inpainting", **Astronomy and Astrophysics**, 550, A15, 2013.

CMB & Sparse Inpainting



- Sparse-Inpainting preserves the weak lensing signal.

- L. Perotto, J. Bobin, S. Plaszczynski, J.-L. Starck, and A. Lavabre, "Reconstruction of the CMB lensing for Planck", *Astronomy and Astrophysics*, 2010.
- S. Plaszczynski, A. Lavabre, L. Perotto, J-L Starck, "An hybrid approach to CMB lensing reconstruction on all-sky intensity maps", arxiv.org/abs/1201.5779, *Astronomy and Astrophysics*, 544, A27, 2012.

- Sparse-Inpainting preserves the ISW

- F.-X. Dupe, A. Rassat, J.-L. Starck, M. J. Fadili , “An Optimal Approach for Measuring the Integrated Sachs-Wolfe Effect”, arXiv:1010.2192, *Astronomy and Astrophysics*, 534, A51+, 2011.

- Sparse-Inpainting preserves the large scales anomalies

- A. Rassat and J-L. Starck, "On Preferred Axes in WMAP Cosmic Microwave Background Data after Subtraction of the Integrated Sachs-Wolfe Effect", Astronomy and Astrophysics , 557, id.L1, pp 7, 2013.
- A. Rassat, J-L. Starck, and F.X. Dupe, "Removal of two large scale Cosmic Microwave Background anomalies after subtraction of the Integrated Sachs Wolfe effect", Astronomy and Astrophysics , 557, id.A32, pp 15, 2013.

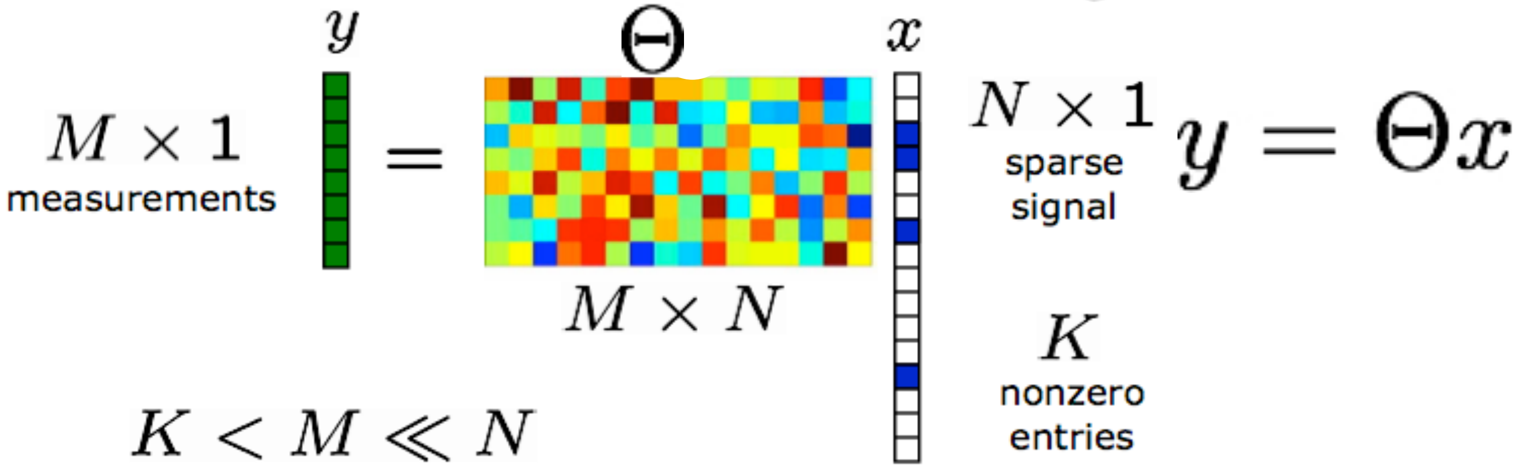
Compressed Sensing: a sampling theorem

* E. Candès and T. Tao, “Near Optimal Signal Recovery From Random Projections: Universal Encoding Strategies? “, IEEE Trans. on Information Theory, 52, pp 5406-5425, 2006.
* D. Donoho, “Compressed Sensing”, IEEE Trans. on Information Theory, 52(4), pp. 1289-1306, April 2006.
* E. Candès, J. Romberg and T. Tao, “Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information”, IEEE Trans. on Information Theory, 52(2) pp. 489 - 509, Feb. 2006.

A non linear sampling theorem

“Signals with exactly K components different from zero can be recovered perfectly from ~ K log N incoherent measurements”

Replace samples with *few linear projections* $y = \Theta x$



$$K < M \ll N$$

Sparse recovery:

Reconstruction via non linear processing:

$$\boxed{\min_{x} \|x\|_1 \quad \text{s.t.} \quad y = \Theta x}$$

Compressed Sensing Reconstruction

Measurements: $y_k = \left\langle x, \theta_k \right\rangle$

Reconstruction via non linear processing: $\min_x \|x\|_1 \quad \text{s.t.} \quad y = \Theta_\Lambda x$

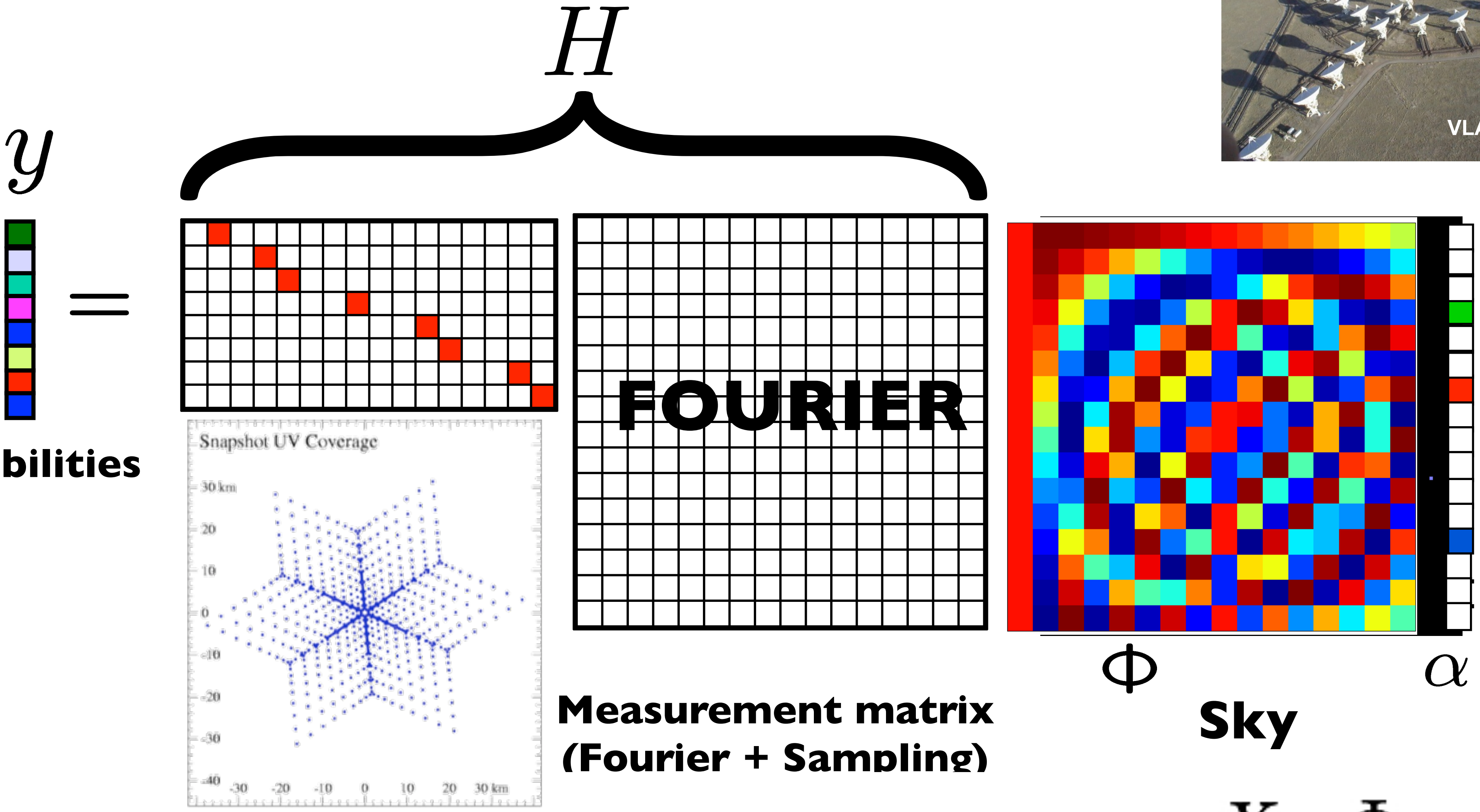
In practice, x is sparse in a given **dictionary**: $x = \Phi \alpha$

and we need to solve: $\min_\alpha \|\alpha\|_1 \quad \text{s.t.} \quad y = \Theta_\Lambda \Phi \alpha$

The mutual incoherence is defined as $\mu_{\Theta,\Phi} = \sqrt{N} \max_{i,k} \left| \left\langle \phi_i, \theta_k \right\rangle \right|$

the number of required measurements is : $m \geq C \mu^2_{\Theta,\Phi} K \log n$

Radio-Interferometry Sparse Recovery



Visibilities

Measurement matrix (Fourier + Sampling)

Sky

$$X = \Phi\alpha$$

e.g. Wavelets Tr.

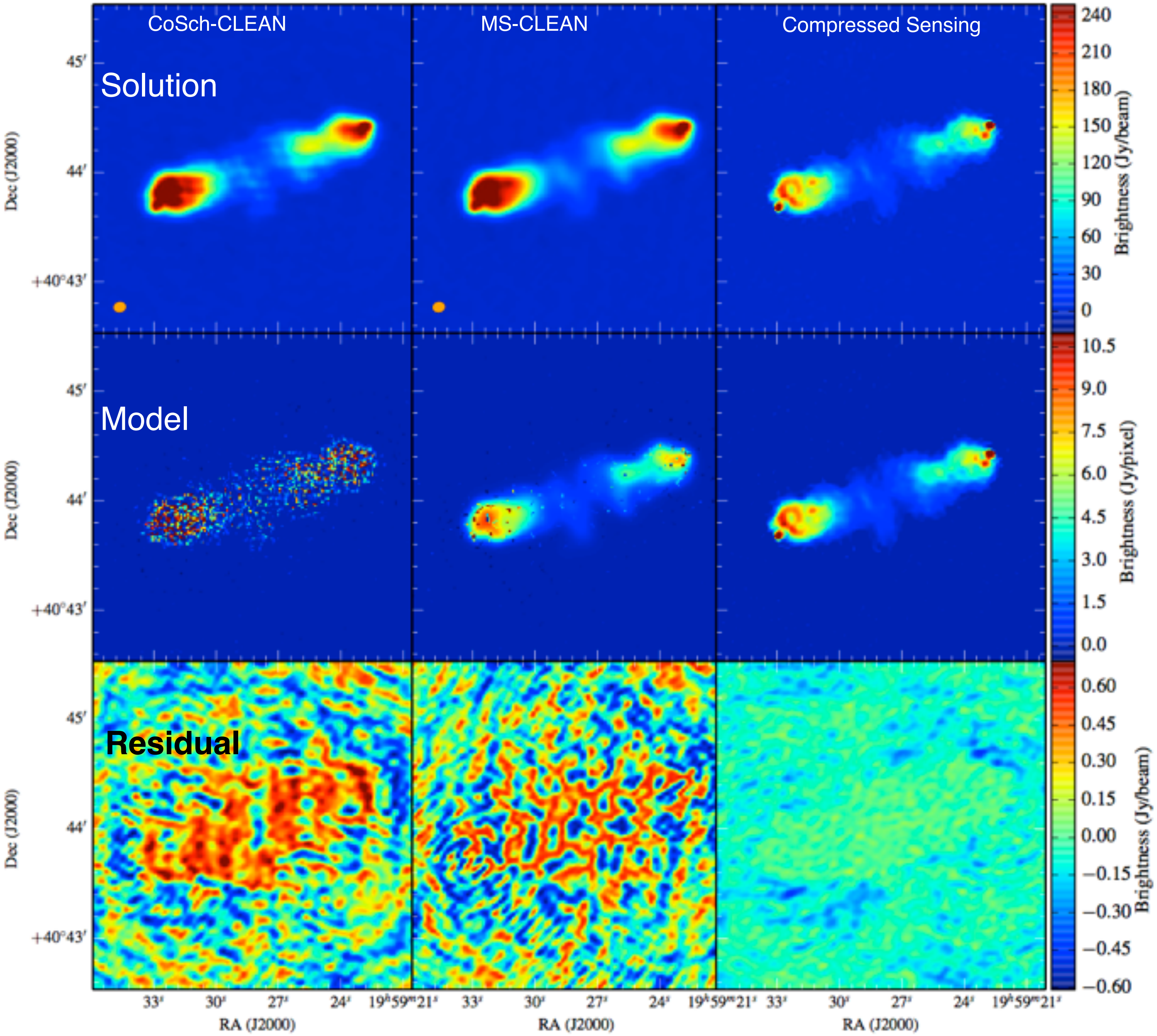
Sparse

$$Y = HX + N$$

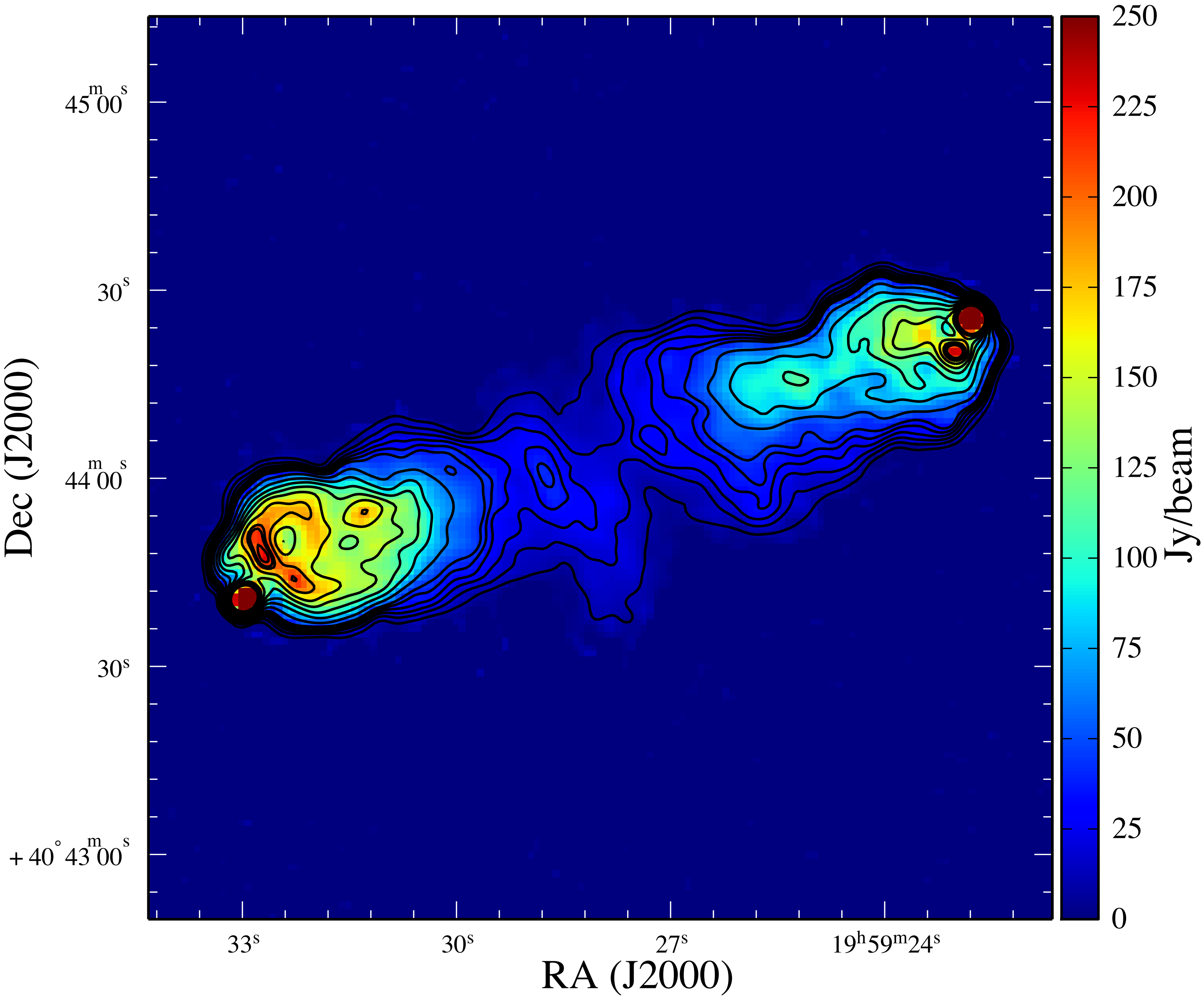
$$\min_{\alpha} \|\alpha\|_p^p \quad \text{subject to} \quad \|Y - H\Phi\alpha\|^2 \leq \epsilon$$

- Garsden et al, “LOFAR Image Sparse Reconstruction”, A&A, submitted. http://arxiv.org/abs/1406.7242

Reconstructed images of Cygnus A from the real LOFAR observations



Residual std-dev = 2,65 Jy/beam, 0,26 Jy/beam, 0,05 Jy/beam



Colorscale: reconstructed 512x512 image of Cygnus A at 151 MHz (with resolution 2.8” and a pixel size of 1”). Contours levels are [1,2,3,4,5,6,9,13,17,21,25,30,35,37,40] Jy/Beam from a 327.5 MHz Cyg A VLA image (Project AK570) at 2.5” angular resolution and a pixel size of 0.5”. Most of the recovered features in the CS image correspond to real structures observed at higher frequencies.

Conclusions on Sparse Inpainting

- ✓ **Weak lensing: Second and third order stastistics of weak lensing images.**

- ✓ **CMB: Large scale data analysis on the sphère**

- ✓ **Asteroseismology:**
  CoRo: sparse inpainting is in the official pipeline.
  Kepler: 18.000 stars have been processed.
  GOLF. ongoing tests

S. Pires et al, **Gap interpolation by Inpainting methods: Application to Ground and Space-based data**, A&A, submitted.

- ✓ **CMB Large Scale Analysis**

- ✓ **Radio-Astronomy:**
  - Photometry: similar to CLEAN on point sources.
  - Resolution: improved by a factor larger than 2 for SNR > 10.
  - Extended objects reconstruction much better than CLEAN and Multiscale CLEAN.

Garsden et al, “**LOFAR Image Sparse Reconstruction**”, A&A, submitted. http://arxiv.org/abs/1406.7242