Données Manquantes : 2 exemples
Séparation de RFI sur SMOS
& Video Inpainting

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GT SPU "Traitement données spatiales"
27 septembre, 2014

SMOS images restoration from L1a data: A sparsity-based variational approach

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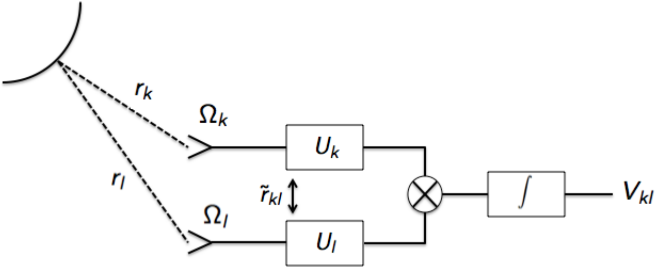
Interferometry principle

- Measure the phase difference of incident radiation
- Cross-correlation between all pairs of receivers to obtain the *Visibility Function* V_{kl}
- T_b can be obtained indirectly from V_{kl}



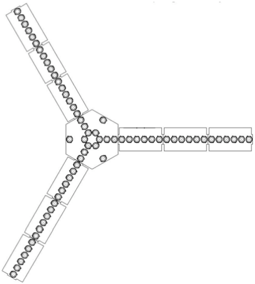
$$V_{k,l} = \frac{1}{\Omega_k \Omega_l} \iint_{||\xi|| \leq 1} \frac{U_k(\xi) U_l^*(\xi) \tilde{r}_{kl}(t)}{\sqrt{1 - ||\xi||^2}} (T_b(\xi) - T_r) e^{-i2\pi \mathbf{u}_{kl}^T \xi} d\xi$$

[Corbella et. al. 2004]

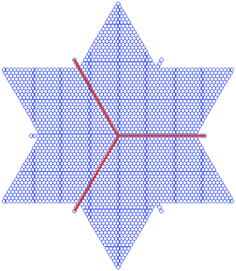
The MIRAS instrument

Antenna configuration

- Support of T_b is the unit circle
  - Optimum sampling grid on visibilities is an hexagonal grid
  - Two possible configurations: triangular or Y shaped arrays
  - Frequency coverage is larger for Y-shaped (but does not cover the entire hexagonal domain)



[Camps et al. 1998]



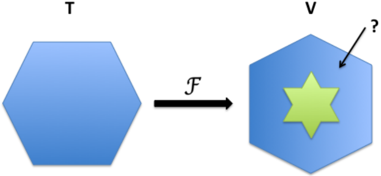
Recovering T_b from visibilities

$$V_{k,l} = \frac{1}{\Omega_k \Omega_l} \iint_{||\xi|| \le 1} \frac{U_k(\xi) U_l^*(\xi) \tilde{r}_{kl}(t)}{\sqrt{1 - ||\xi||^2}} \overbrace{(T_b(\xi) - T_r)}^{T} e^{-i2\pi \mathbf{u}_{kl}^T \xi} d\xi$$

- Consider the discrete version of this linear operator, it can be stated by means of matrix **G**:

$$\mathbf{G} T = V$$

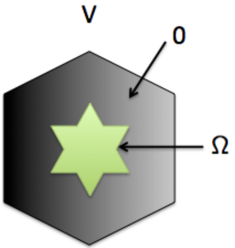
- $\dim(T) > \dim(V)$: the problem is under constrained



Generation of L1B: Zero padding regularization

Anterrieu 2004

$$\min_T \|V - \mathbf{G}T\|_2^2$$
$$\text{s.t. } (I - P_\Omega)T = 0$$
$$\text{with} P_\Omega = \mathcal{F}^{-1} Z_\Omega Z_\Omega^* \mathcal{F}$$



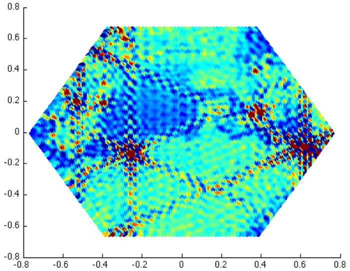
- This problem can be reformulated as:

$$\hat{T} = \arg\min_{\hat{t} \in \Omega} \|V - \overbrace{\mathbf{G}\mathcal{F}^{-1} Z_\Omega}^{\mathbf{A}} \hat{t}\|_2^2$$

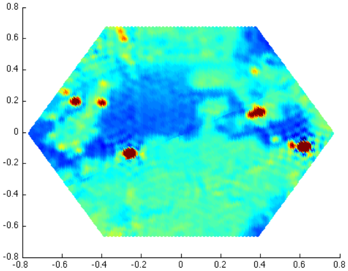
- $\hat{T}$ is SMOS L1B data product, V is SMOS L1A data product
- T can be simply recovered from $\hat{T}$ by $T = \mathcal{F}^{-1} Z_\Omega \hat{T}$

Zero padding limitations

- Strong Gibbs effects: illegal transmitters introduce outliers
- Poor spectral extrapolation: limited resolution



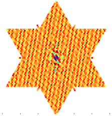
Direct Method: $T_b = \mathcal{F}^{-1}(\hat{T})$

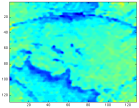


Regularized Method - Blackmann: $T_b = \mathcal{F}^{-1}(B\hat{T})$

Objetives of the present work

- Recover the brightness temperature directly from visibilities





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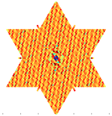
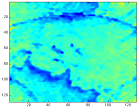
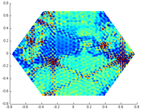
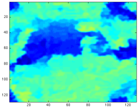




- Remove noise and signal effects generated from illegal emissions (outliers)





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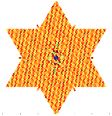

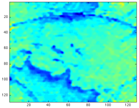

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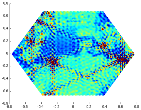
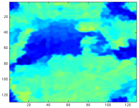
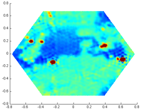
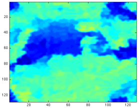




- Extrapolate the image spectrum to minimize Gibbs effects





Proposed method

Main idea : Separate 3 sources

u : Original brightness temperature image

$\Rightarrow$ TV semi-norm

o : RFI Outliers

$\Rightarrow$ Sparsity norm (ℓ^1 or ℓ^0)

n : Gaussian measurement noise

$\Rightarrow$ ℓ^2 data fidelity term

Variational Formulation

Proposed method

$$\min_{u,o} \left\{ \underbrace{\frac{1}{2}\|\mathbf{G}(o+u) - V\|_2^2}_{E_1(u,o)} + \underbrace{\lambda(\mathrm{TV}_{\mathcal{H}}(u) + \mu S(o))}_{E_2(u,o)} \right\} \qquad (1)$$

where

- μ Trade-off between sparsity and regularity
- λ is chosen to satisfy $\|\mathbf{G}(o+u) - V\|_2^2 \leq |\Omega|\sigma^2$

- $\mathrm{TV}_{\mathcal{H}}(u)$: *Total Variation* for $\mathcal{H}$-bandlimited images [Moisan 2007] $\rightarrow$ reduces staircaising effect

[Moisan 2007] *How to discretize the total variation of an image?* Proc. Appl. Math. Mech., 2007

Proposed Method: Numerical Implementation

Two stage process:

Stage one

Solve the minimization problem with sparsity term $S(o) = \|o\|_1$

- the problem is convex
- can be solved iteratively with a Forward-Backward algorithm
- converges to a global minimum

Stage two

Starting from the previous solution, we solve the same problem with $S(o) = \|o\|_0$

- the problem is non-convex due to the ℓ_0 norm
- for this functional the Forward-Backward algorithm converges to a local minimum [Blumensath and Davies 2005]

Proposed Method: Numerical Implementation

Algorithm

The k-th iteration starting from seed $x^0 = (u^0, o^0)$ is

$$\begin{cases} x^{k+1/2} &= x^k - \gamma \nabla E_1(x^k) \\ x^{k+1} &= \mathrm{prox}_{\gamma E_2}(x^{k+1/2}). \end{cases}$$

Differential operator

$$\nabla E_1(u,o) = (\mathbf{G}^*\mathbf{G}(u+o) - V, \mathbf{G}^*\mathbf{G}(u+o) - V)\,.$$

Proximal operators

- $\mathrm{prox}_{\gamma E_2}(u,o) = \left(\mathrm{prox}_{\gamma\lambda \mathrm{TV}}(u), \mathrm{prox}_{\gamma\lambda\mu S}(o)\right)$
- $\mathrm{prox}_{\gamma TV}$: modified version of [Chambolle 2004] with spectral projection
- $\mathrm{prox}_{\gamma\|\cdot\|_1}$: the *soft-threshold* or *shrinkage* operator
- $\mathrm{prox}_{\gamma\|\cdot\|_0}$: the *hard-threshold* operator

Implementation limitations

- $\mathbf{G}^*\mathbf{G}$ is a huge full matrix of size 16384x16384
- Explicit multiplication by this matrix on each iteration is impractical.
- Change of basis to Fourier domain:

$$\nabla E_1(u,o) = F^*((\mathbf{GF}^*)^*\mathbf{GF}^* F(u+o) - (\mathbf{GF}^*)^* V)$$

- $\mathbf{FG}^*\mathbf{GF}^*$ is even bigger than $\mathbf{G}^*\mathbf{G}$ (32768×32768) but highly sparse: to keep 99.99% we need 0.008 coefficients.

Final algorithm

Final algorithm

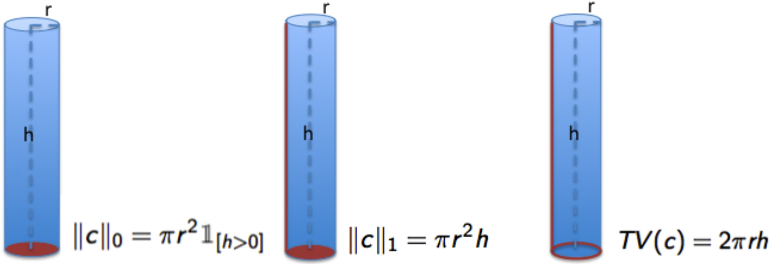
1. Set $S(\cdot) = \|\cdot\|_1$
2. Initialize λ
   a. Iterate until convergence (FB)

$$\begin{cases} u^{k+1/2} & = & u^k - \gamma F^*(\mathbf{FG}^*\mathbf{GF}^*F(u+o) - \mathbf{FG}^*V \\ o^{k+1/2} & = & o^k - \gamma F^*(\mathbf{FG}^*\mathbf{GF}^*F(u+o) - \mathbf{FG}^*V \\ u^{k+1} & = & \mathrm{prox}_{\gamma\lambda\,\mathrm{TV}_{\mathcal{H}}}(u^{k+1/2}) \\ o^{k+1} & = & s_{\gamma\lambda\mu}(o^{k+1/2}). \end{cases}$$

   b. If $\|\mathbf{GF}^*F(o+u) - V\|_2^2 - |\Omega|\sigma^2| \leq \epsilon$, update λ and go to a)
3. Set $S(\cdot) = \|\cdot\|_0$ and go to 2)

Numerical implementation: μ selection

We model an outlier as a cylinder c of radius r and height h:



μ selection

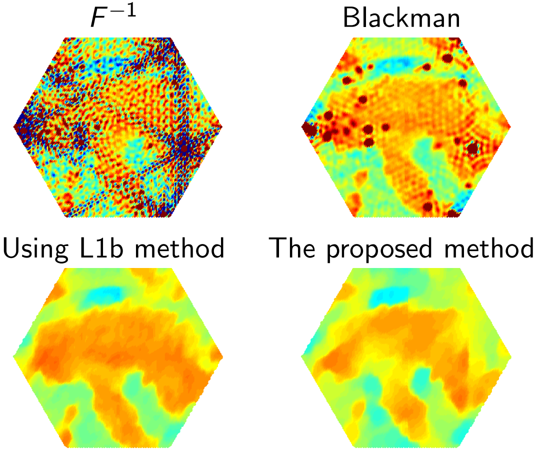
- If ℓ_1 then c is an outlier if $TV(c) \geq \mu \|c\|_1$, leading to a $\mu \leq \frac{2}{r}$
- If ℓ_0 then c is an outlier if $TV(c) \geq \mu \|c\|_0$, i.e. $\mu \leq \frac{2h}{r}$

Experiments on real data

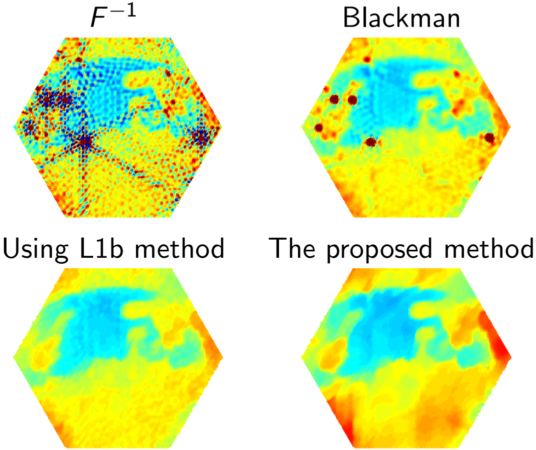
 

Google Earth view of two of the regions used on the experiments. The left one corresponds to snapshot 996 and the right one to snapshot 1050

Experiments on real data
Comparison between previous works and our method.



This snapshot corresponds to Central Europe, with Italy clearly visible. Color scale mapped between 0 and 300 K.



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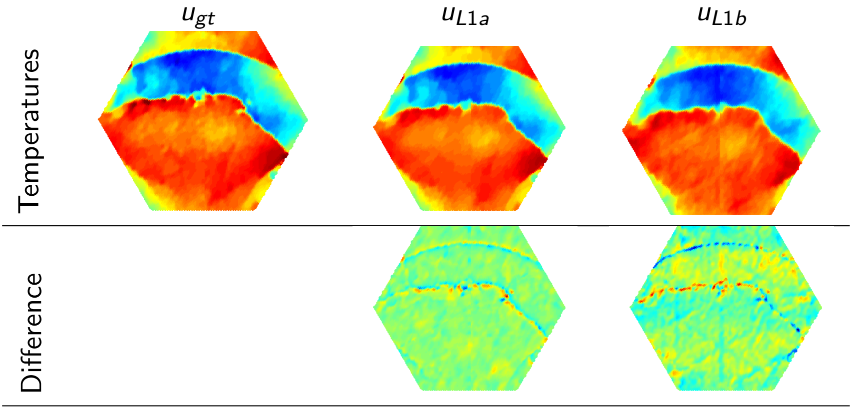

	L_1	L_2	L_∞
$u_{L1a} - u_{gt}$	3.197598	5.418558	57.679203
$u_{L1b} - u_{gt}$	9.587280	12.994680	87.467700

Results from simulated data. The error is measured over all the image, not only the free of aliasing (FOA) zone.

Conclusions

We propose a variational method to restore images from the L1A SMOS data product.

- The method models the observations as the superposition of three components on the spatial domain:
  - The target brightness temperature map u
  - The outliers image o due to the illegal emissions
  - A gaussian noise image n
- The method is numerically tractable by a change of basis from spatial to spectrum domain
- The method also extrapolates the spectral domain of u thanks to the total variation semi-norm
- The method is general enough to be used for other restoration problems

Future work

- Better separation if RFI are finely localized [Veterli 2002], [Candes 2013], [Duval 2014]
- More detailed restoration with patch-based (non-local) regularization