

Sky map reconstruction in spherical coordinates

From (u,v) plane to the (l,m) plane

Jiao Zhang

2014-11-13

method

- Sky map $I(\alpha, \delta)$ reconstruction for a transit instrument from visibilities.
- α : RA East-west; $\delta = \pi/2 - \theta$: dec North-South.
- Fourier: $(\alpha, \delta) \rightarrow (u, v); I(\alpha, \delta) \rightarrow F(u, v)$
a given pair of antenna: $(\Delta x, \Delta y) \rightarrow BeamUV(u, v)$
- θ : colatitude; φ : longitude.
- Spherical harmonics: $(\theta, \varphi) \rightarrow (m, l); I(\theta, \varphi) \rightarrow A(l, m)$
 $(\Delta x, \Delta y) + (\theta_0, \varphi_0) \rightarrow BeamLM(l, m)$

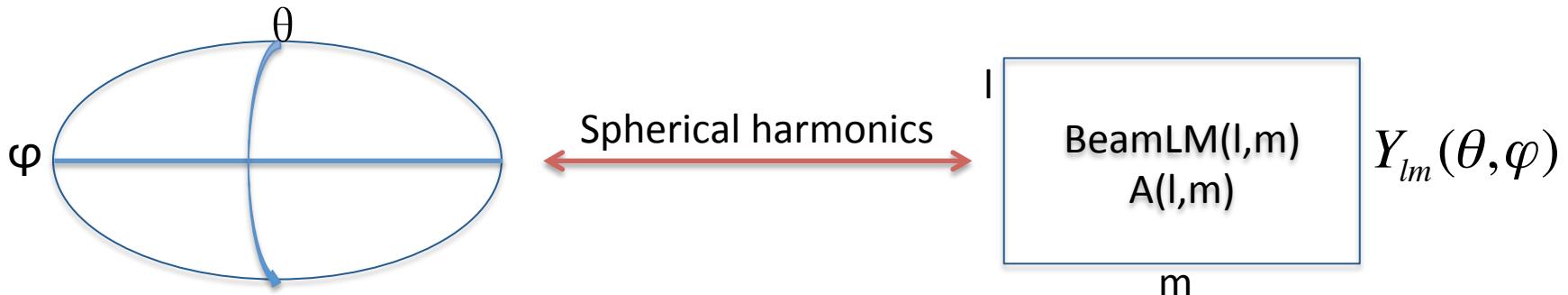
- Spherical coordinate:
coordinate transform from local tangent plane (antenna baselines) to 3D Cartesian coordinate.

The diagram illustrates the coordinate transform. On the left, a blue coordinate system shows 'Longitude, Latitude' with a diagonal vector and axes labeled 'NS' (up) and 'EW' (right). A red arrow points from this system to the right, labeled 'Δr in earth coordinate'. Below this arrow is the equation:

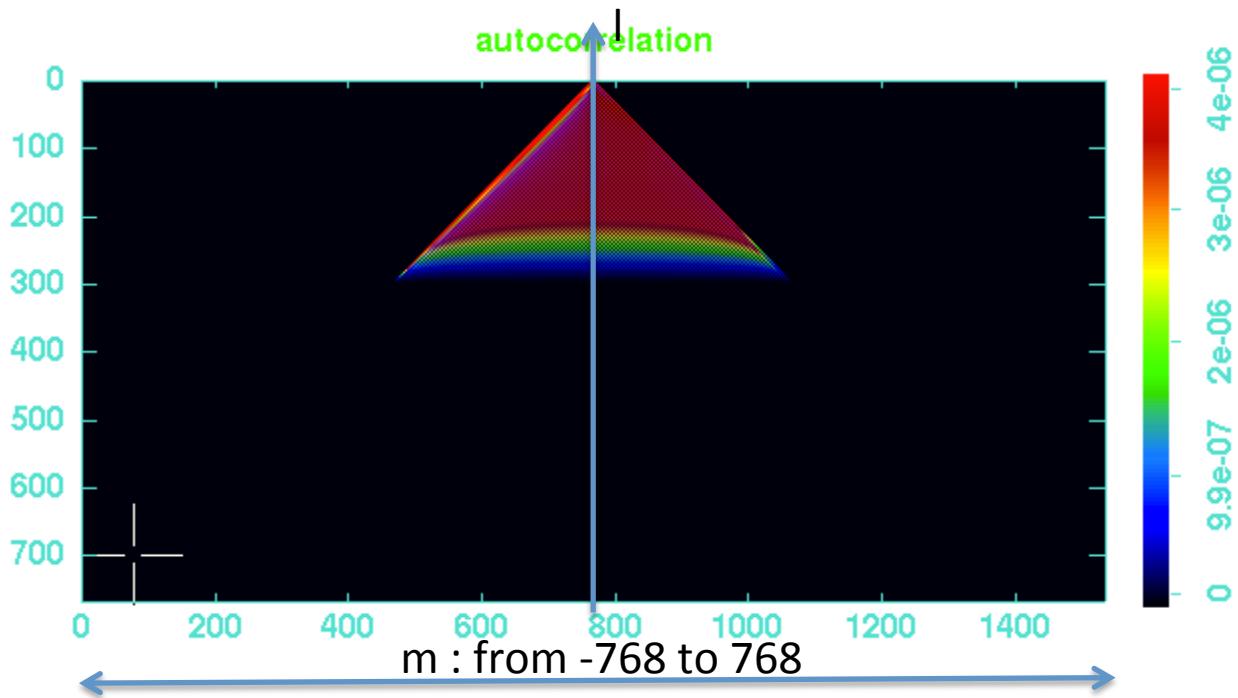
$$\vec{\Delta r} = \begin{pmatrix} -bdx * \sin \varphi - bdy * \cos \theta \cos \varphi + bdz * \sin \theta \cos \varphi \\ bdx * \cos \varphi - bdy * \cos \theta \sin \varphi + bdz * \sin \theta \sin \varphi \\ bdy * \sin \theta + bdz * \cos \theta \end{pmatrix}$$

- We can express beam as

$$Beam(\theta, \varphi) \times e^{i\vec{k} \times \vec{\Delta r}} \rightarrow BeamLM(l, m)$$

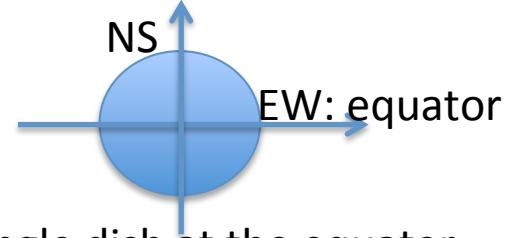


Few examples of BeamLM(l,m)

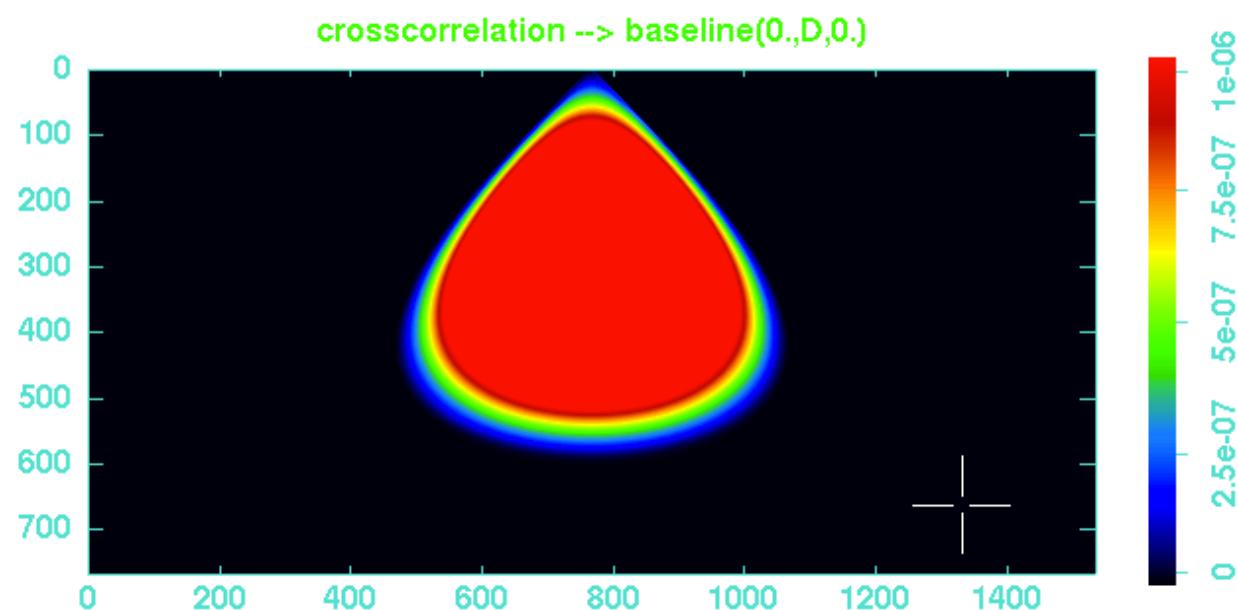


D=10m; I_{max}=768

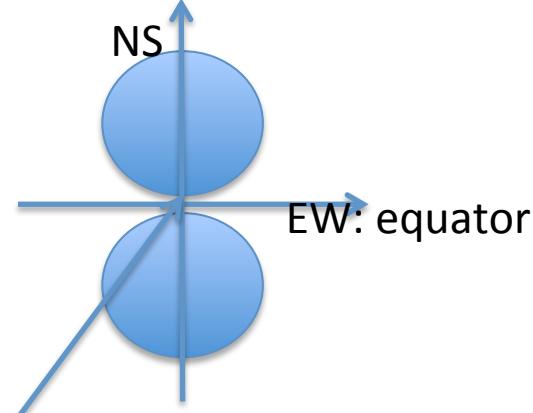
Autocorrelation Baseline (0.,0.,0.)



Single dish at the equator,
 $\theta=\pi/2$; $\varphi=0$

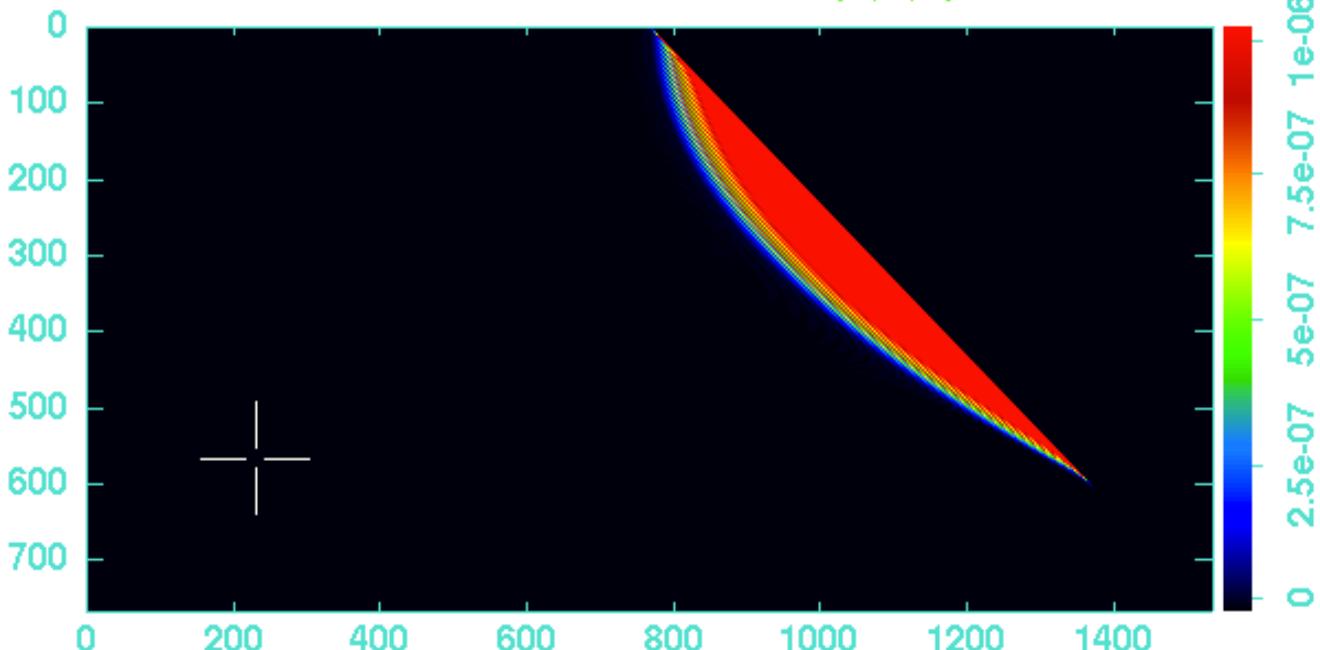


Crosscorrelation: Baseline (0.,D,0.) in tangent plane

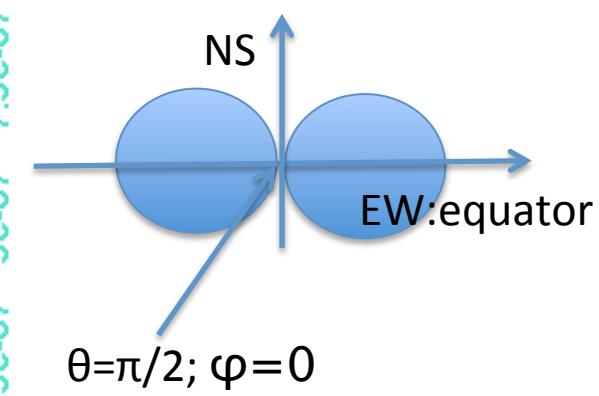


$$\theta=\pi/2; \varphi=0$$

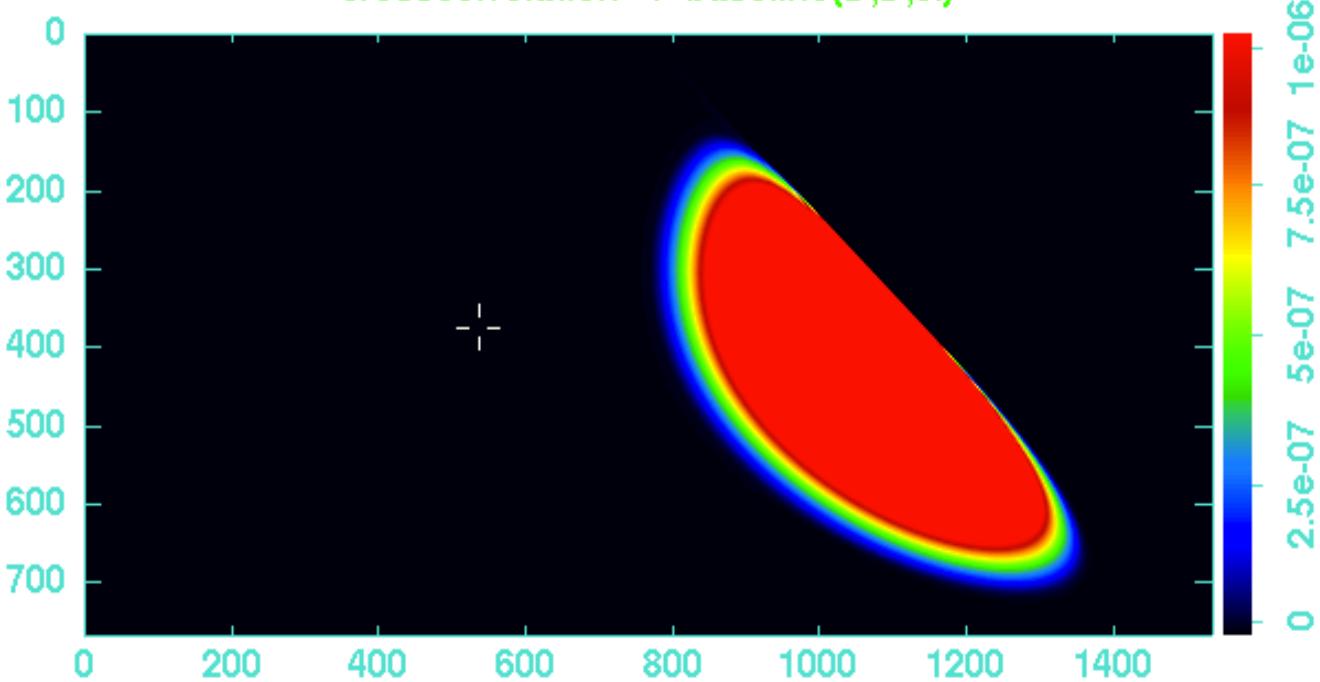
crosscorrelation --> baseline(D,0,0.)



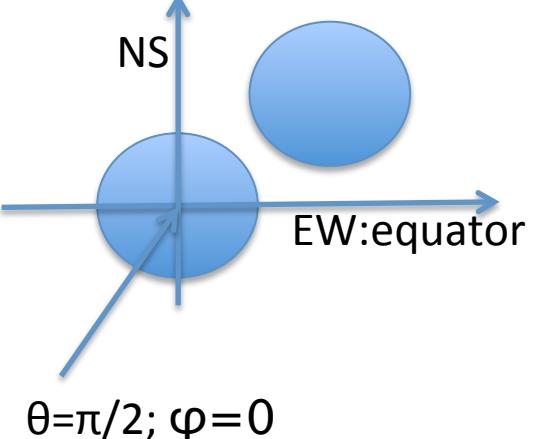
Crosscorrelation : Baseline
(D,0.,0.) in tangent plane



crosscorrelation --> baseline(D,D,0.)



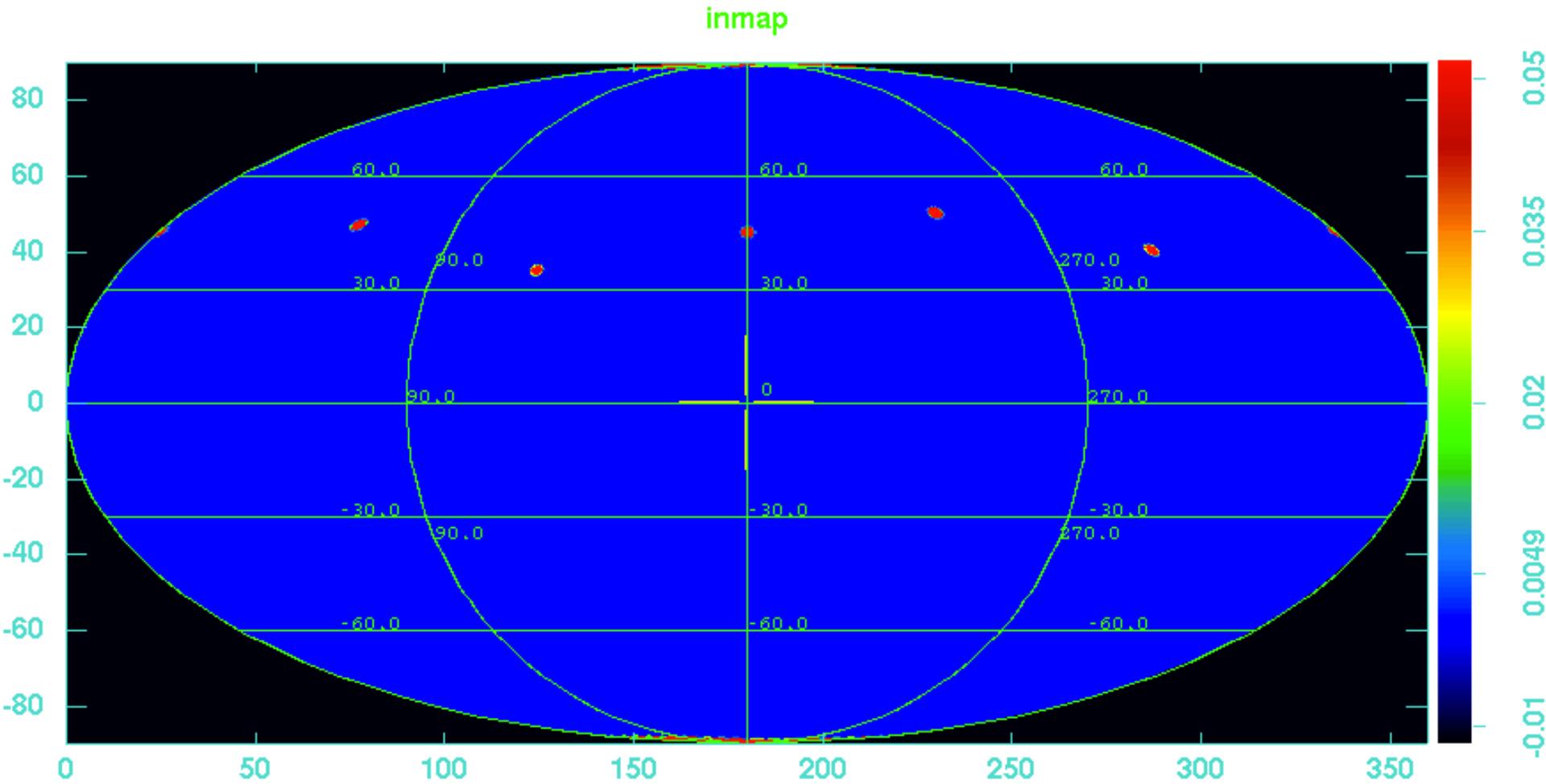
Crosscorrelation : Baseline
(D,D,0.) in tangent plane



Sky map reconstruction in spherical harmonics

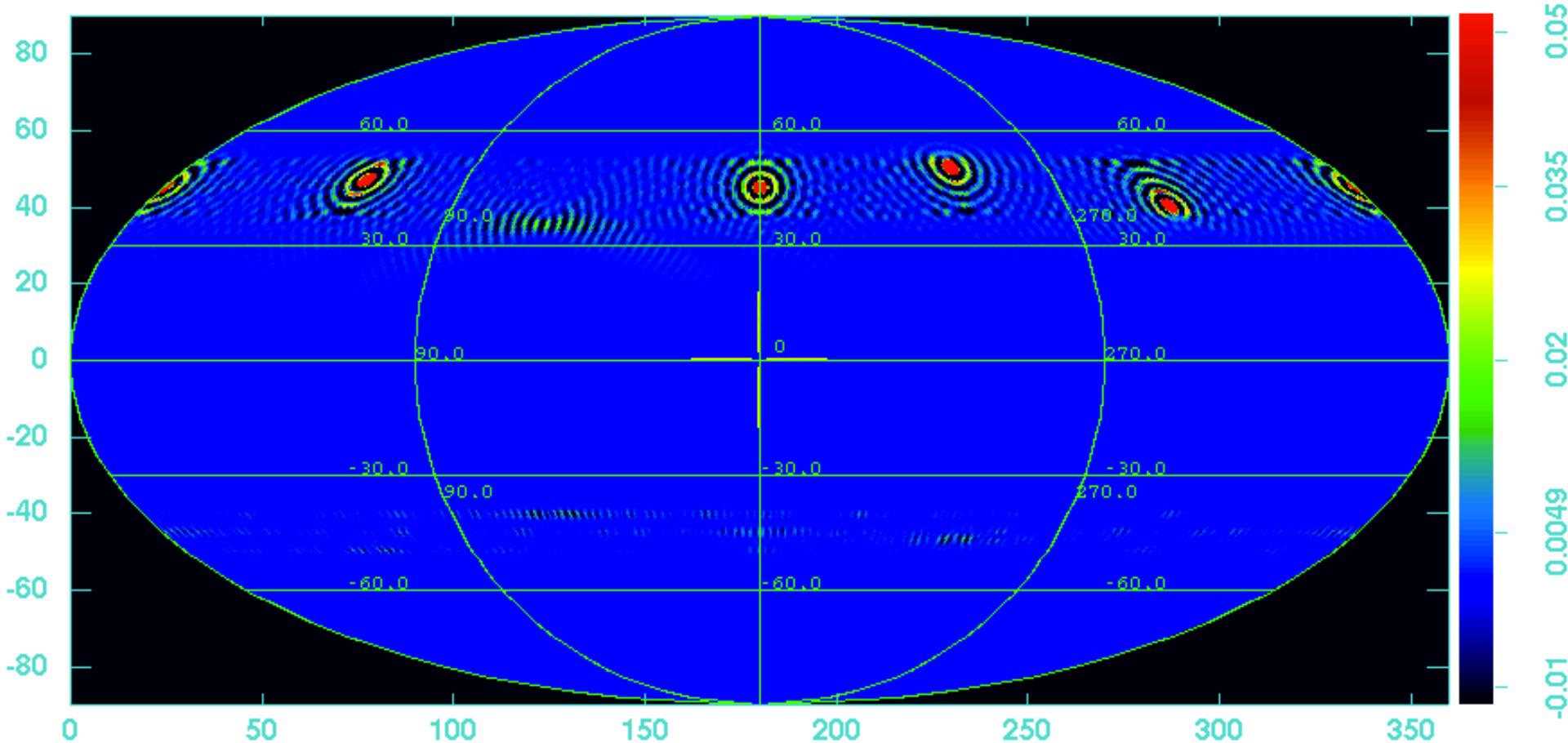
$$\tilde{V}_m^{ij} = \sum_{l,m} a_{lm} * B(l,m) + noise_m^{ij} : \text{Calculate Alm from Visibilities and BeamLM}(l,m)$$

Input 8 expended source : 1 north pole, 1 south pole, 6 around latitude 45 degrees;
Scan strategy : 11 scan per 1 degree centered at 45 degrees.



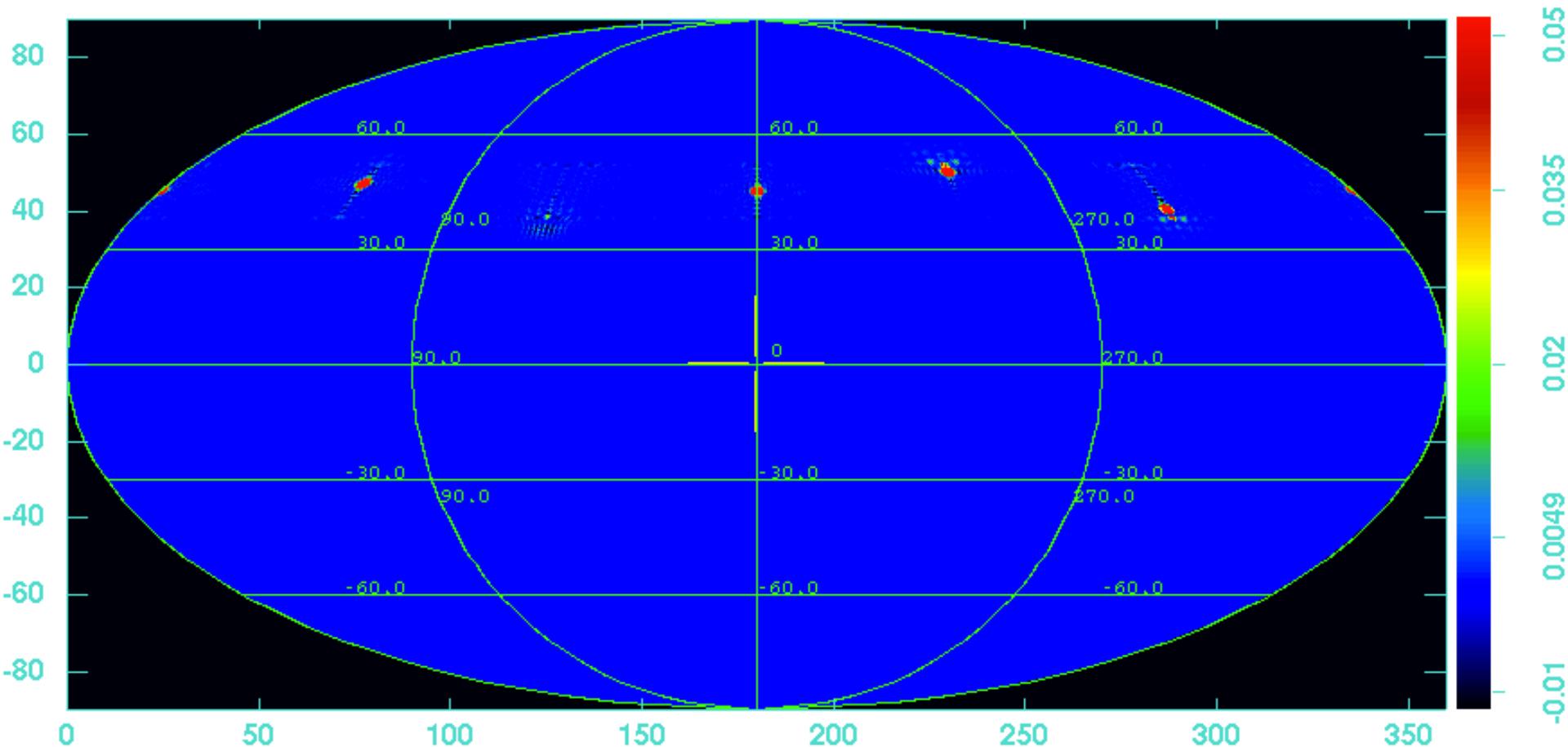
Single dish case

recmap single dish with 11 scan



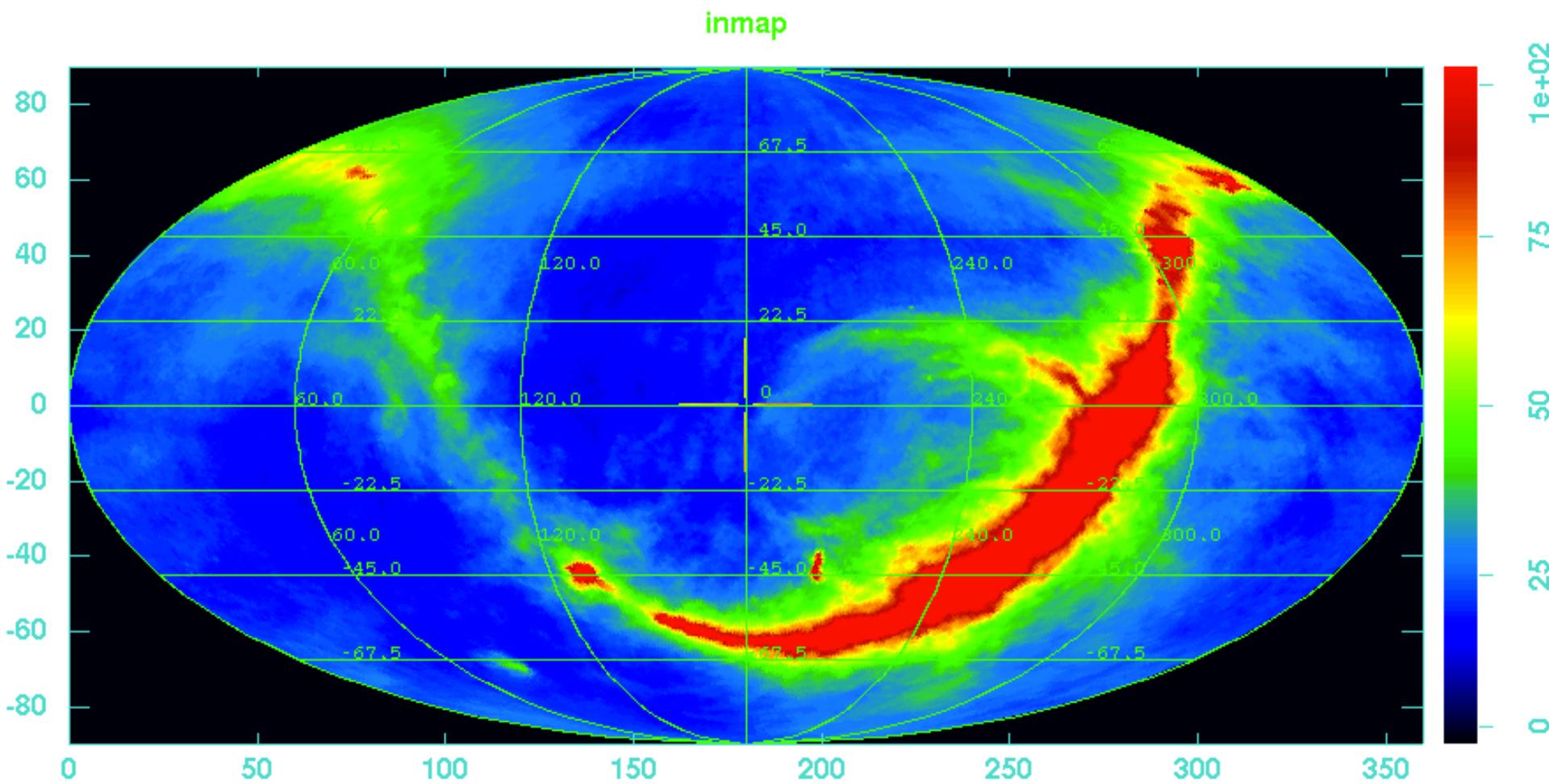
2 by 2 regular dishes case

recmap for 2 by 2 regular dishes with 11 scan



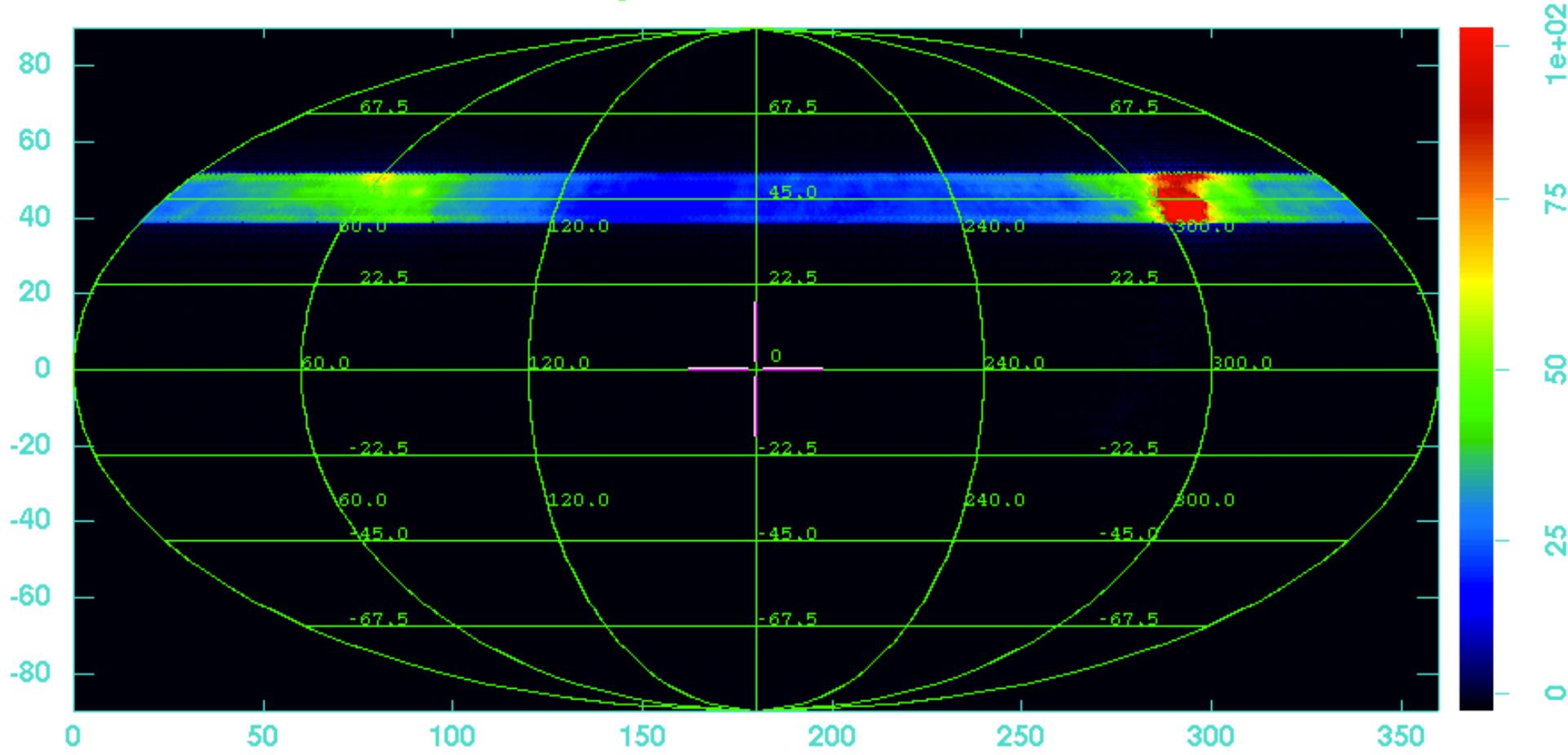
Input synchrotron map

Scan strategy : 11 scan per 1 degree centered at 45 degrees.



Interferometer : 4 by 4 regular dish array

16 regular dishes with 11 scan



Next steps

- Detailed study of reconstructed map properties
- Application to PAON-4 & Tianlai