Precision determination of the top-quark mass

Sven-Olaf Moch

Universität Hamburg

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Introduction (I)

Classical mechanics

- Mass is defined as product of density and volume of matter
  - classical concept

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PRINCIPIA MATHEMATICA.

DEFINITIONES.

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Motus totius est summa motuum in partibus singulis; ideoque in corpore duplo majore, æquali cum velocitate, duplus est, & dupla cum velocitate quadruplus.

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A

Atomic theory

- Mass is conserved Lavoisier
- Mass of body is sum of mass of its constituents
  $M(X) = N_A m_a(X)$ Avogadro

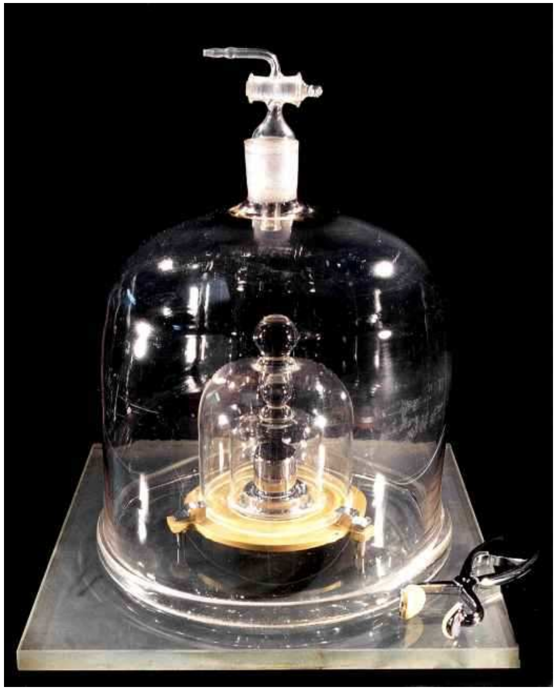
Introduction (II)

Kilogram

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

Orginal des Bureau International des Poids et Mesures

- International prototype kilogram (IPK): made in 1889, 39 mm high, alloy of platinum and iridium



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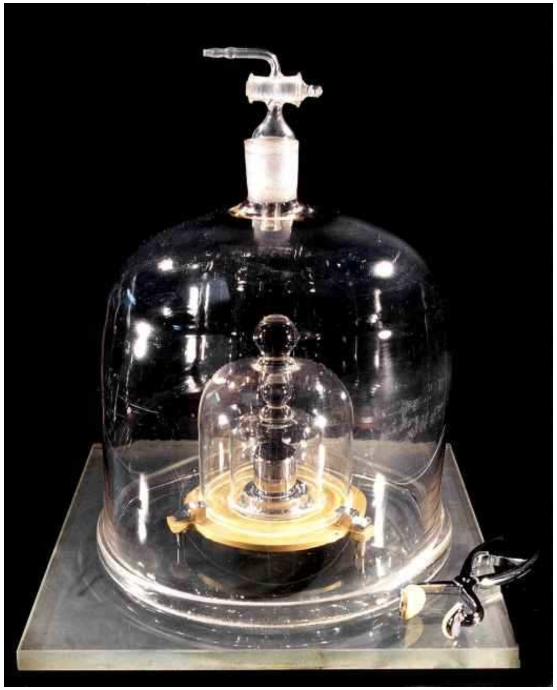

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Special relativity

- Equivalence principle
  $E = mc^2$ Einstein

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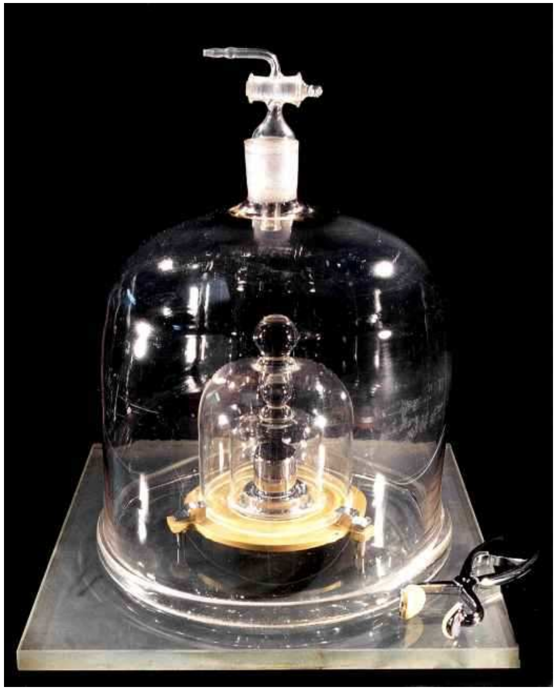

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Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top-quark mass m_t

Quantum field theory

QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} \quad = \quad -\frac{1}{4} F^a_{\mu\nu} F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i \left(\mathrm{i} \not{D} - m_q \right)_{ij} q_j$$

  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_\mu \delta_{ij} + \mathrm{i} g_s \left(t_a \right)_{ij} A^a_\mu$

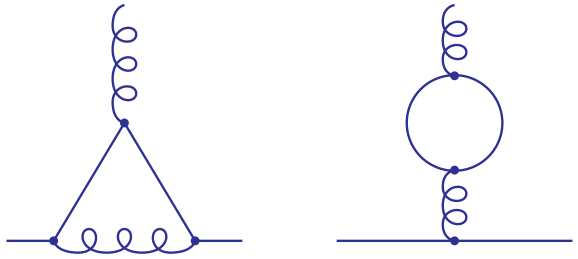
- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2/(4\pi)$
  - quark masses m_q
- Parameters of Lagrangian have no unique physical interpretation
  - radiative corrections require definition of renormalization scheme

Challenge

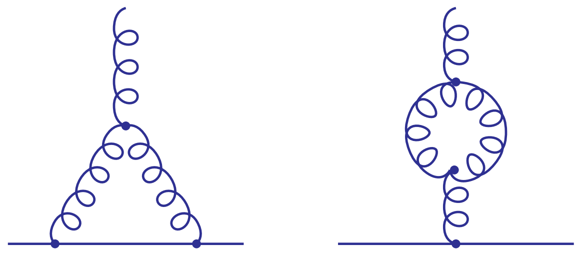
- Suitable observables for measurements of α_s, m_q, . . .
  - comparison of theory predictions and experimental data

Coupling constant renormalization

- Running coupling constant α_s from radiative corrections, e.g. one loop

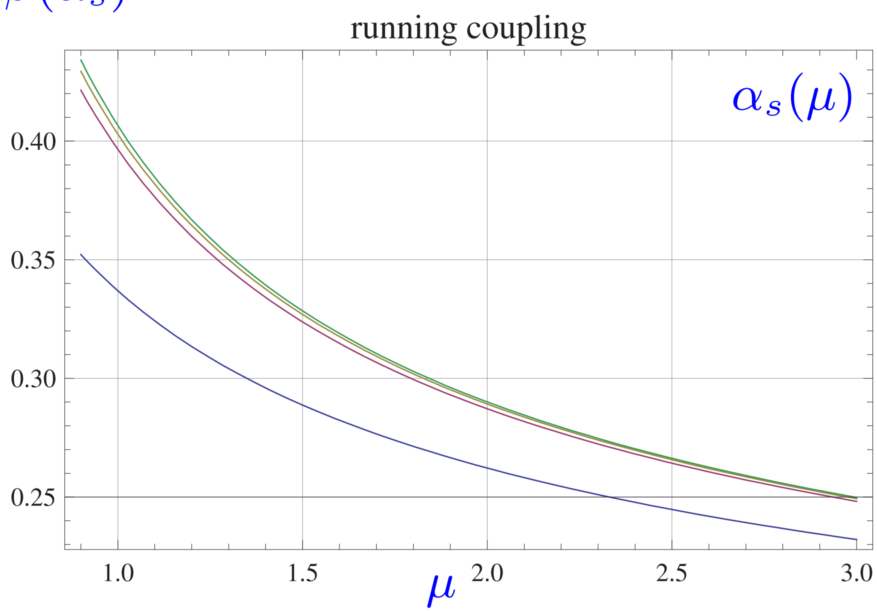


– screening (like in QED)



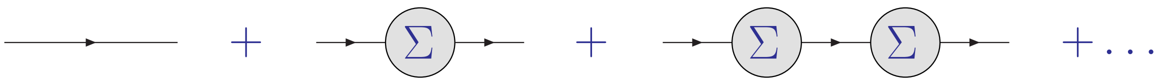
– anti-screening (color charge of g)

- QCD beta function $\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu) = \beta(\alpha_s)$
  - perturbative expansion to four loops
    van Ritbergen, Vermaseren, Larin '97
  - very good convergence of perturbative series even at low scales



Quark mass renormalization

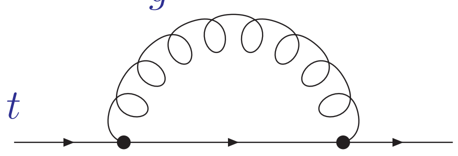
- Heavy-quark self-energy $\Sigma(p, m_q)$



$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

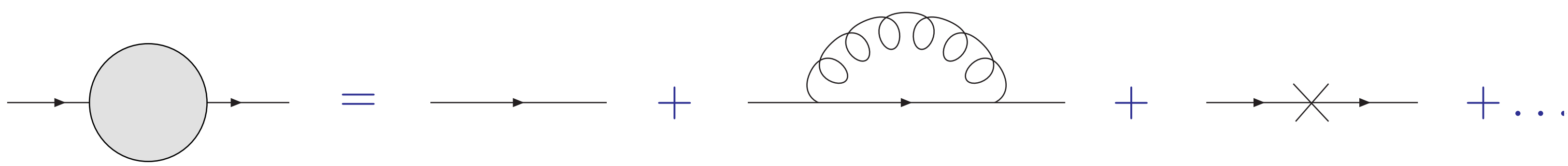
QCD

- QCD corrections to self-energy $\Sigma(p, m_q)$
  - dimensional regularization $D = 4 - 2\epsilon$
  - one-loop: UV divergence $1/\epsilon$ (Laurent expansion)



$$\Sigma^{(1),\mathrm{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_q^2}\right)^{\epsilon} \left\{ (\not{p} - m_q) \left(-C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left(3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}$$

- Relate bare and renormalized mass parameter $m_q^{\mathrm{bare}} = m_q^{\mathrm{ren}} + \delta m_q$



$\Sigma^{\mathrm{ren}}(p, m_q)$ $\qquad$ $(Z_\psi - 1)\not{p} - (Z_m - 1) m_q$

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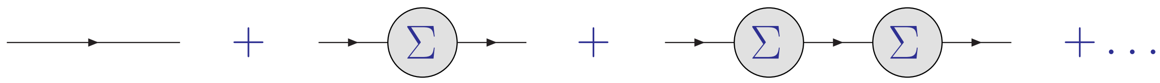
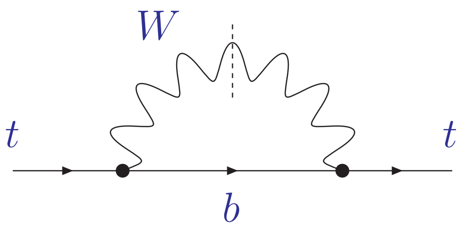


$$= \frac{\mathrm{i}}{\not{p} - m_q - \Sigma(p, m_q)}$$

EW sector



- EW corrections to top-quark self-energy
  - on-shell intermediate (virtual) W-boson
  - m_t complex parameter with imaginary part $\Gamma_t = 2.0 \pm 0.7$ GeV
  - $\Gamma_t > 1$ GeV: top-quark decays before it hadronizes

Mass renormalization scheme

Pole mass

- Based on (unphysical) concept of top-quark being a free parton
  - m_q^{ren} coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{\not{p}=m_q} \to \not{p} - m_q^{\mathrm{pole}}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$
  - bound from lattice QCD: $\Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200$ MeV
    Bauer, Bali, Pineda '11

$\overline{MS}$ *scheme*

- $\overline{MS}$ mass definition
  - one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi\right)$$

- $\overline{MS}$ scheme induces scale dependence: $m(\mu)$

Running quark mass

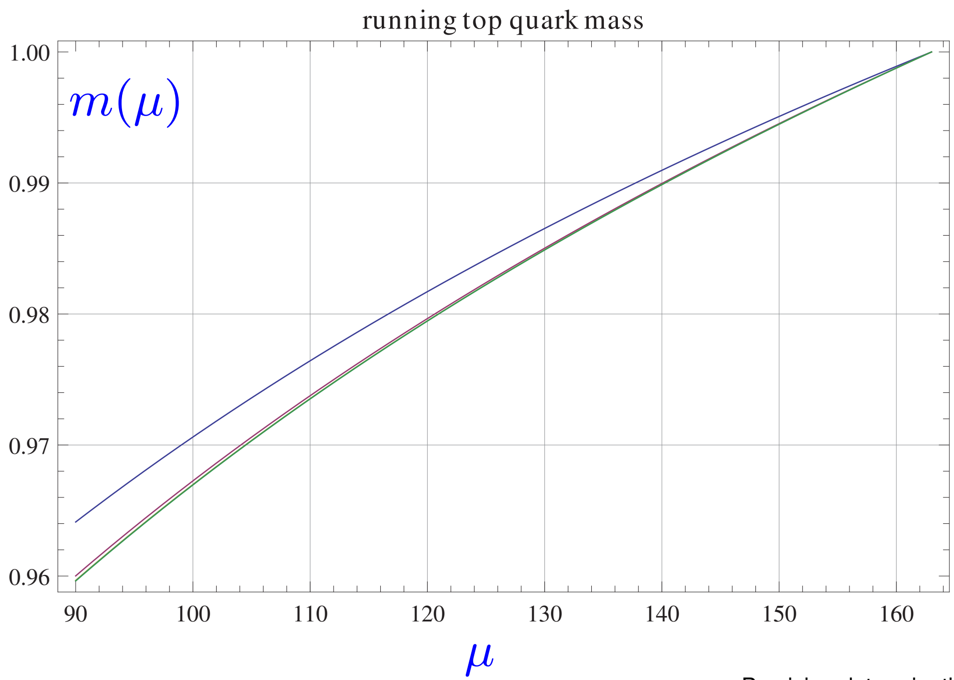
Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension γ known to four loops

    Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right) m(\mu) \; = \; \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio $m_t(163\text{GeV})/m_t(\mu)$



Scheme transformations

- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and $\overline{MS}$ mass
  - known to three loops in QCD Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99
  - EW sector known to $\mathcal{O}(\alpha_{\rm EW}\alpha_{\rm s})$
    Jegerlehner, Kalmykov '04; Eiras, Steinhauser '06
  - example: one-loop QCD

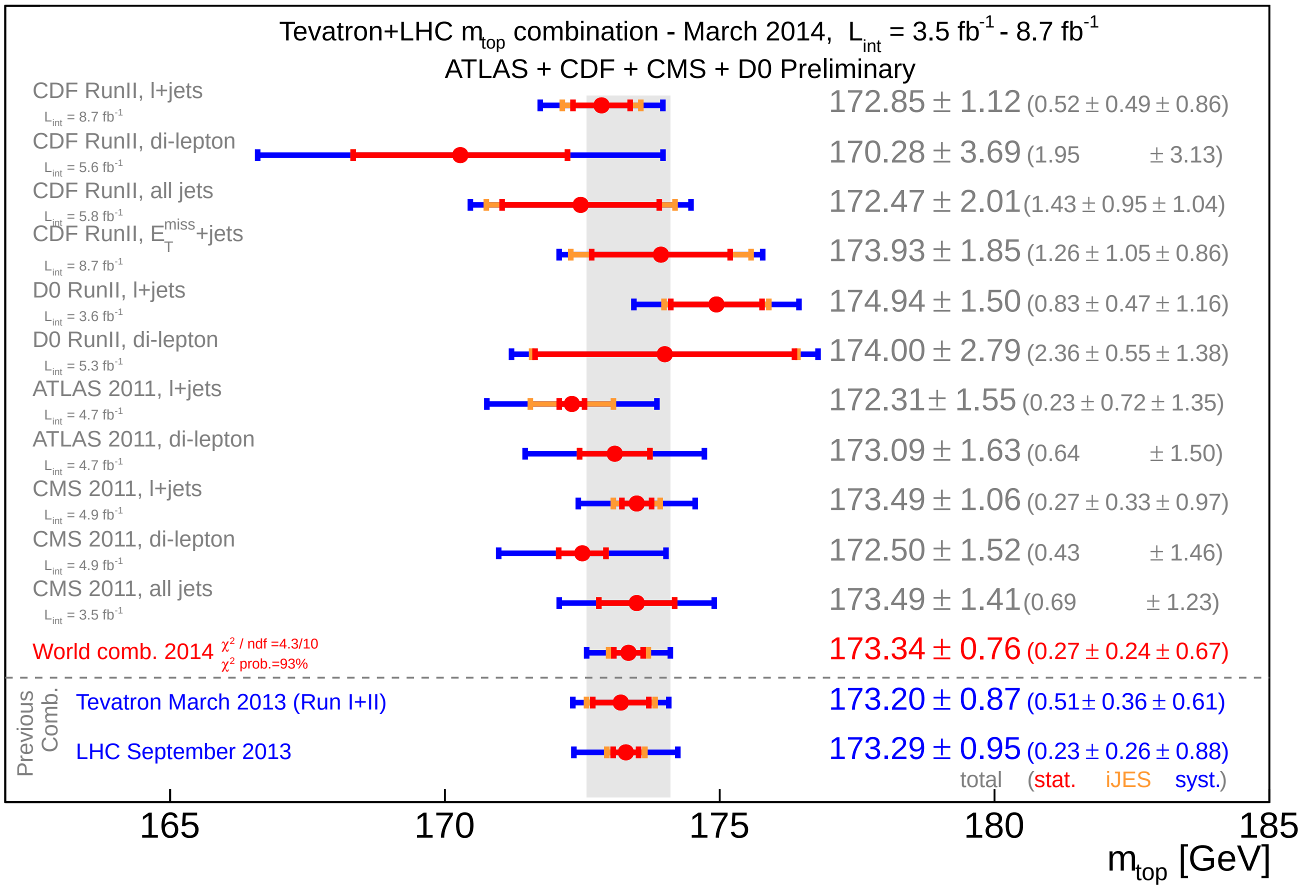
$$m^{\rm pole} = m(\mu)\left\{1 + \frac{\alpha_s(\mu)}{4\pi}\left(\frac{4}{3} + \ln\left(\frac{\mu^2}{m(\mu)^2}\right)\right) + \dots\right\}$$

Top-quark mass

What is the value of the top-quark mass ?

$$m_t = ?$$

Some Answers



World combination

Experiment: ATLAS, CDF, CMS & D0 coll. 1403.4427

$$m_t = 173.34 \pm 0.76\,\text{GeV}$$

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Theory:

That is, we can state as the final result for the likely relation between the top-quark mass measured using a given Monte Carlo event generator ("MC") and the pole mass as

$$m_{\mathrm{pole}} = m_{\mathrm{MC}} + Q_0 \left[\alpha_s(Q_0) c_1 + \ldots\right]$$

where $Q_0 \sim 1$ GeV and c_1 is unknown, but presumed to be of order 1 and, according to the argument above, presumed to be positive.

A. Buckley et al. arXiv:1101.2599

Rates, shapes and peaks

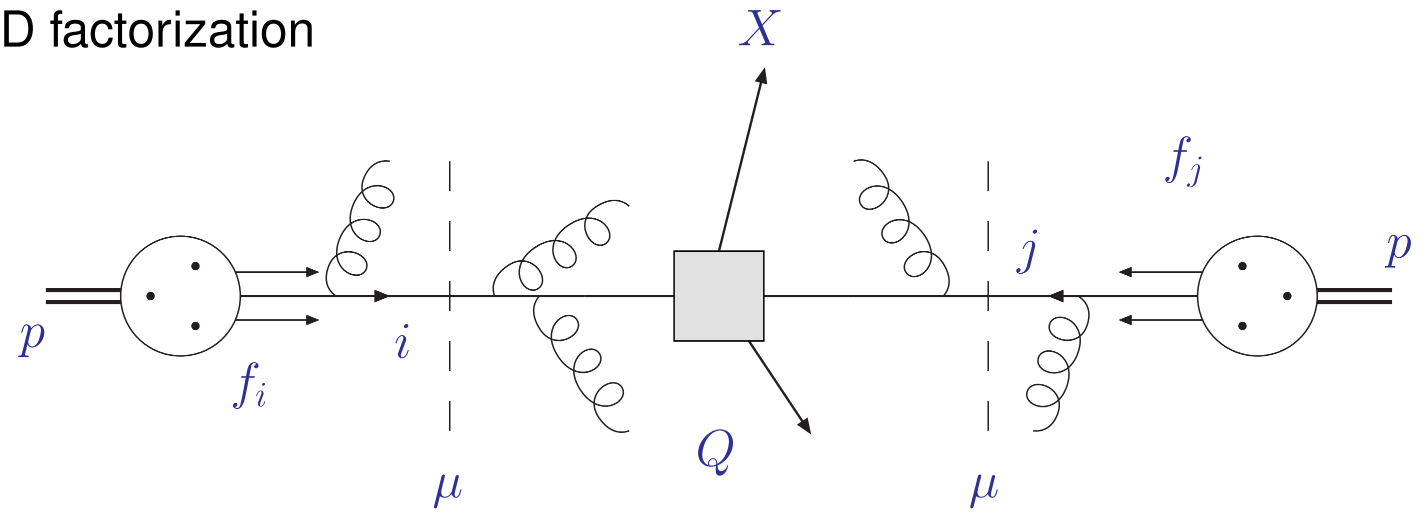
- Rates and shapes of distributions offer possibility for top mass determination with well-defined renormalization scheme
- Requirements:
  - theory predictions at least to NLO in QCD
  - sufficiently large sensitivity $\mathcal{S}$ to m_t (kinematics)

$$\left| \frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \simeq \mathcal{S} \times \left| \frac{\Delta m_t}{m_t} \right|$$

- Observables (examples):
  - inclusive cross section
  - distributions for $t\bar{t} + 1\text{jet}$ samples
  - kinematic reconstruction of top mass (Monte Carlo mass)

Top mass from total cross section

- QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

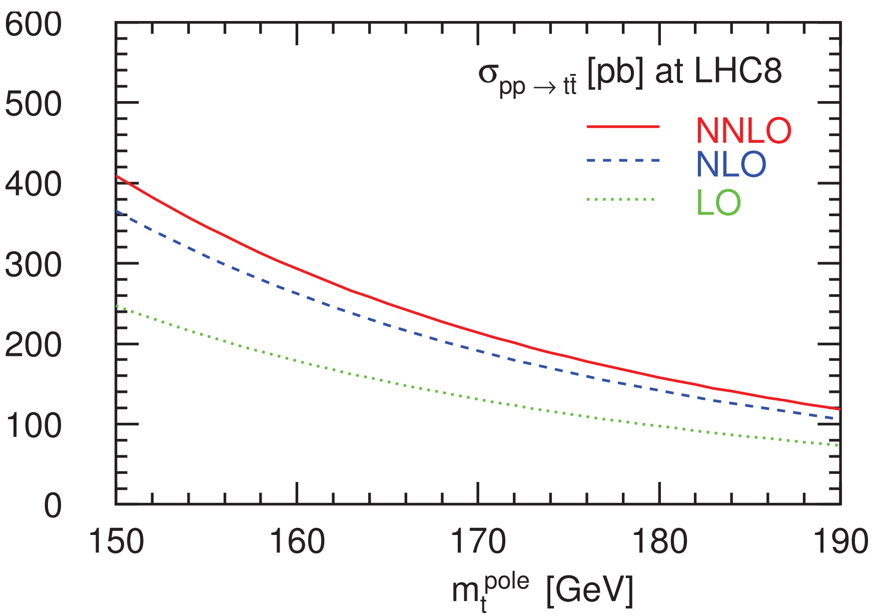
- Joint dependence on non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, masses m_X
- Intrinsic limitation in total cross section through sensitivity $\mathcal{S} \simeq \bigtriangledown$

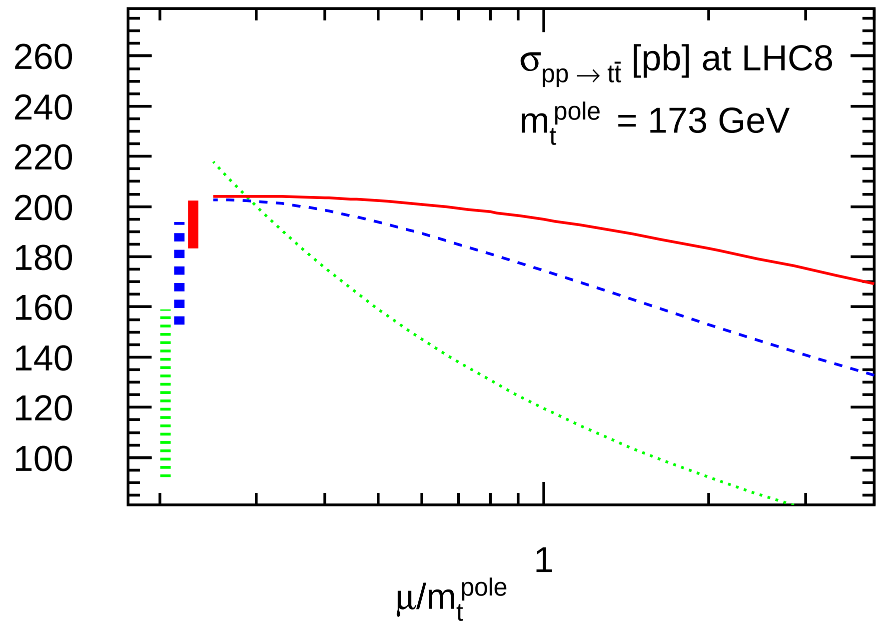
$$\left|\frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}}\right| \simeq 5 \times \left|\frac{\Delta m_t}{m_t}\right|$$

Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov '13



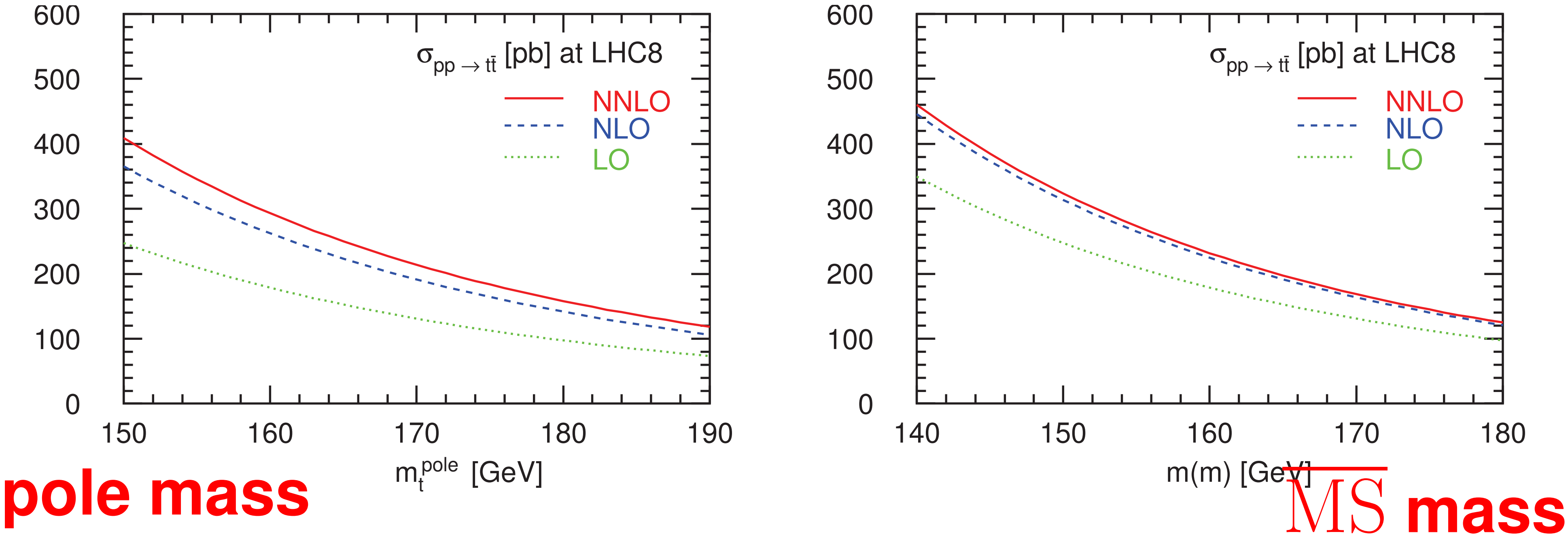


- NNLO perturbative corrections (e.g. at LHC8)
  - K-factor (NLO $\to$ NNLO) of $\mathcal{O}(10\%)$
  - scale stability at NNLO of $\mathcal{O}(\pm 5\%)$

Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass (I)

Dowling, S.M. '13



- NNLO cross section with running mass significantly improved
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity form scale variation

Total cross section with running mass

Comparison pole mass vs. $\overline{\mathrm{MS}}$ mass (II)

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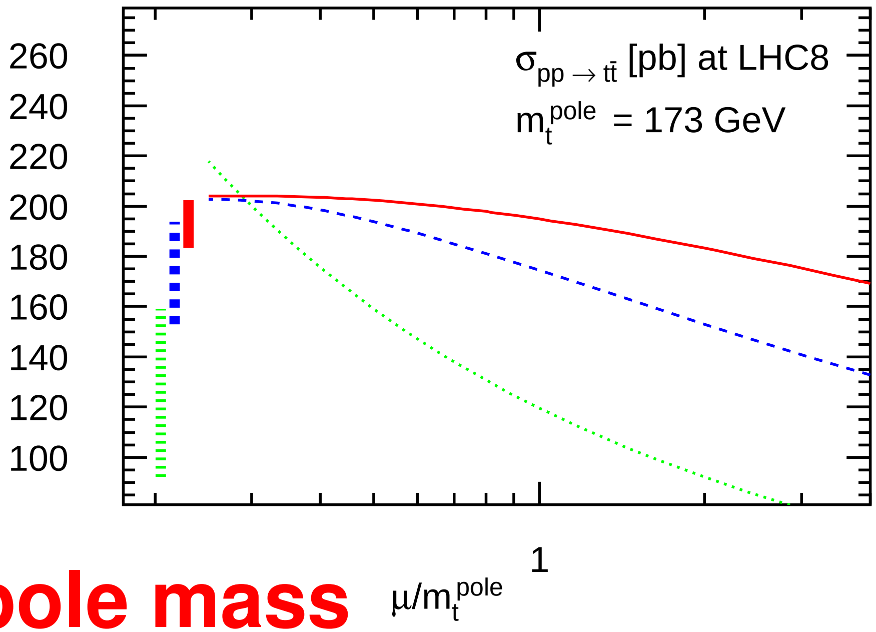
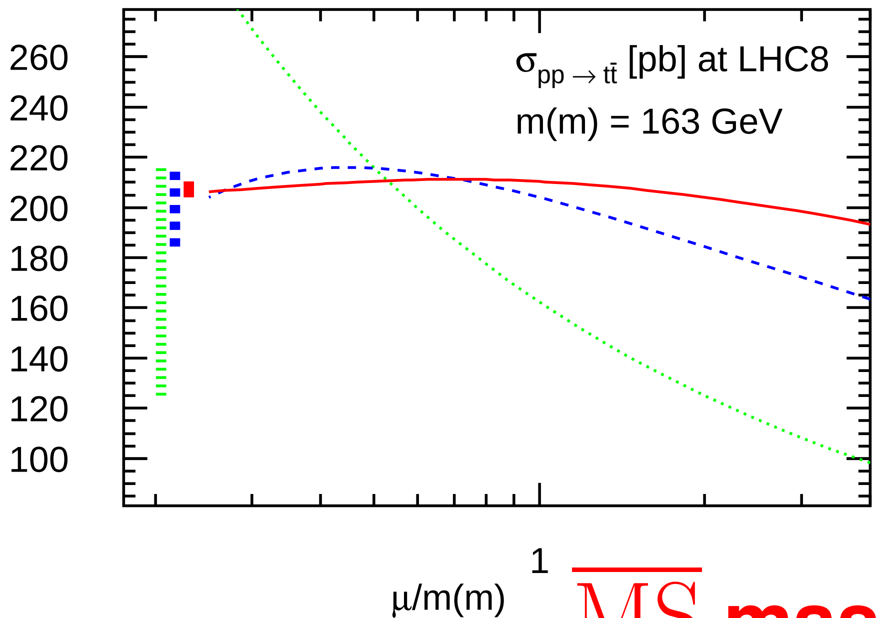


pole mass



$\overline{\mathrm{MS}}$ mass

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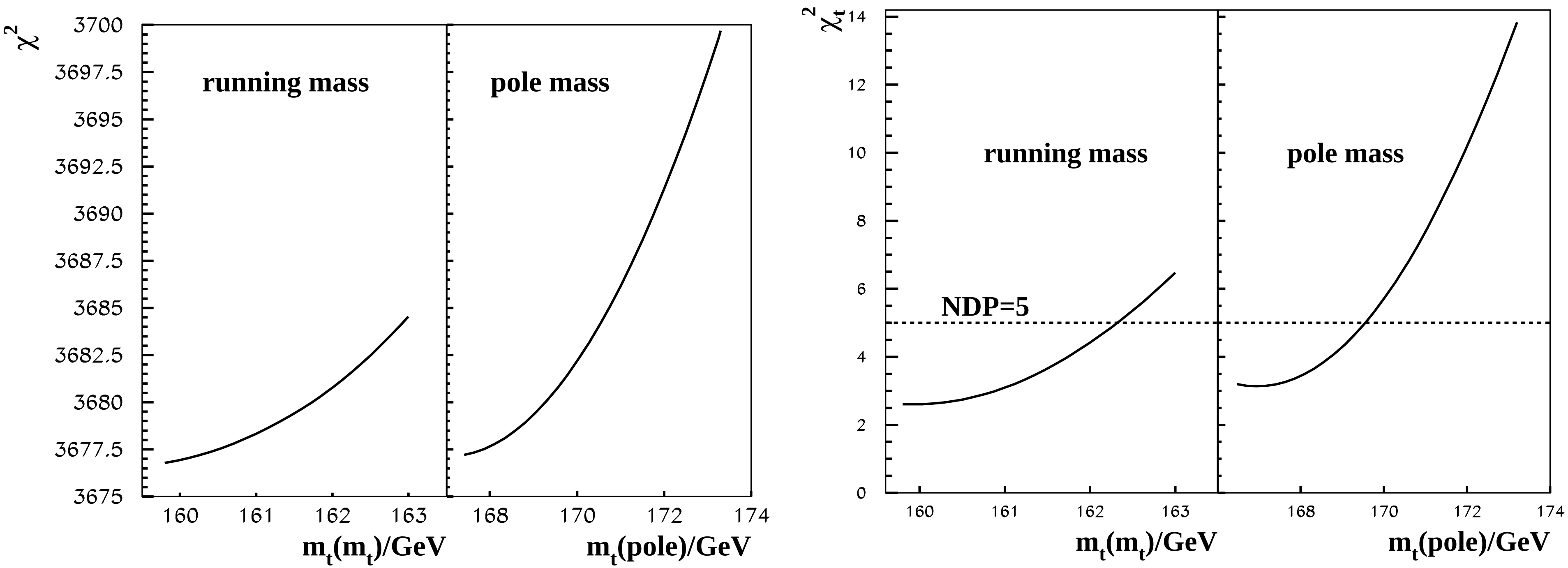
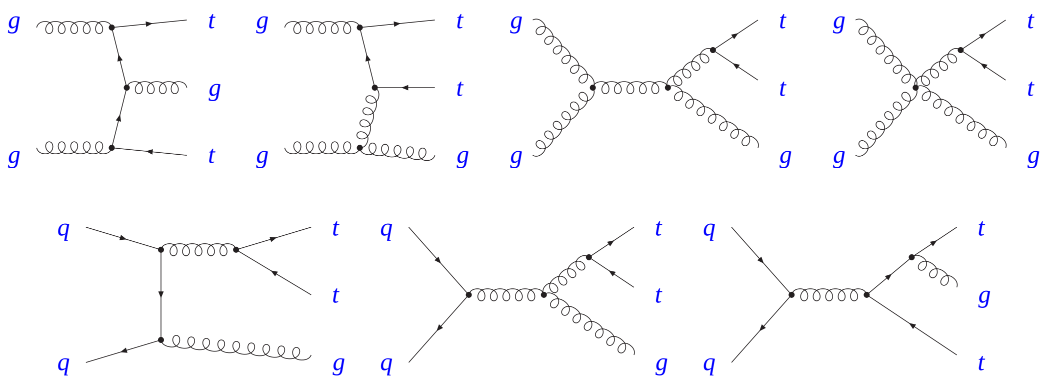
Top cross section data in ABM12 fit

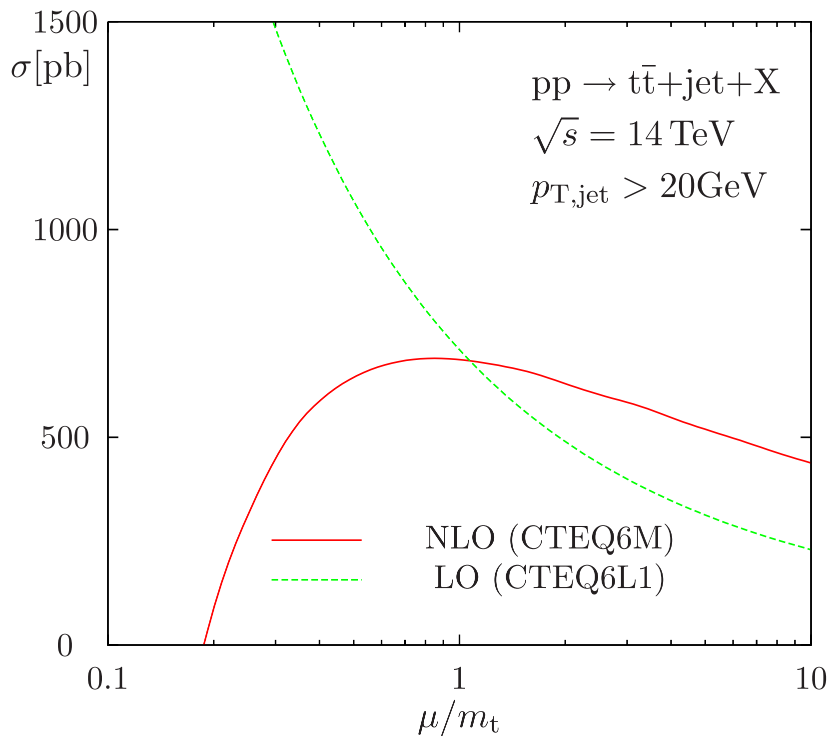
- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162.3 \pm 2.3$ GeV (equivalent to pole mass $m_t = 171.2 \pm 2.4$ GeV) Alekhin, Blümlein, S.M. '13
  - χ^2-profile steeper for pole mass (bigger impact of top-quark data and greater sensitivity to theoretical uncertainty at NNLO)



Top-quark pairs with one jet

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO QCD corrections for $t\bar{t} + 1\text{jet}$ Dittmaier, Uwer, Weinzierl '07-'08
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale $\mu_r = \mu_f = m_t$ are almost zero





- Additional jet raises kinematical threshold
  - invariant mass $\sqrt{s_{t\bar{t}+1\text{jet}}}$

Top mass with $t\bar{t}+\mathrm{jet}$*-samples*

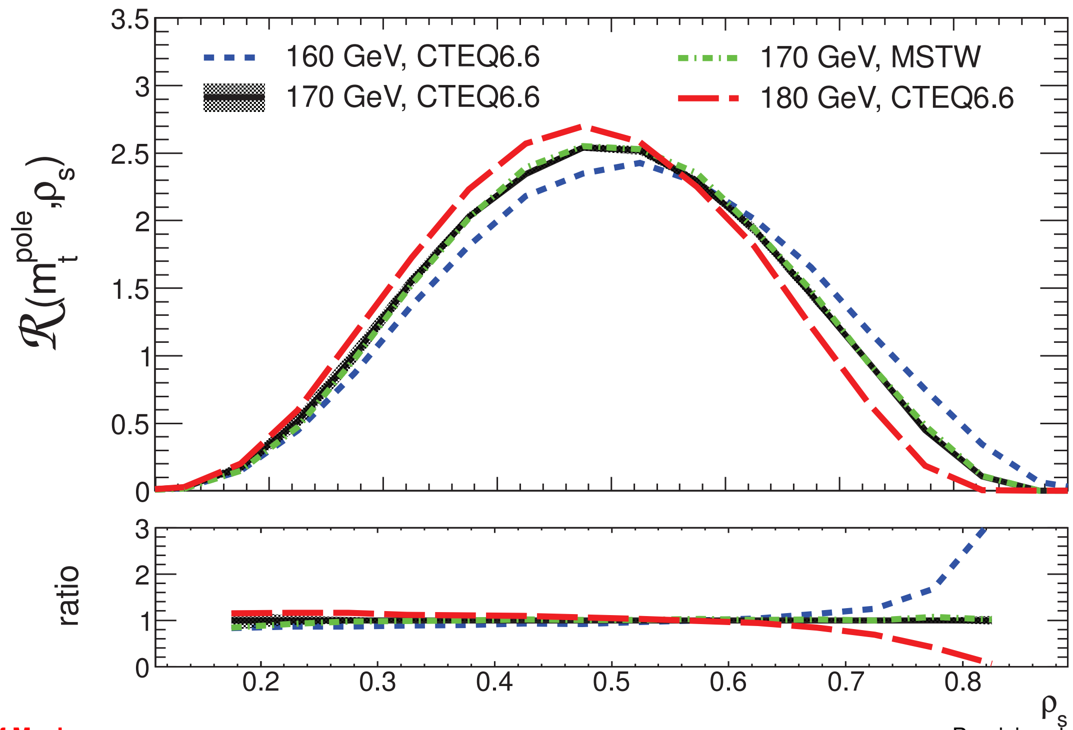
- Normalized-differential $t\bar{t}+\mathrm{jet}$ cross section

Alioli, Fernandez, Fuster, Irles, S.M., Uwer, Vos '13

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\mathrm{jet}}} \frac{d\sigma_{t\bar{t}+1\mathrm{jet}}}{d\rho_s}(m_t, \rho_s)$$

  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\mathrm{jet}}}}$ with invariant mass of $t\bar{t}+1\mathrm{jet}$ system and fixed scale $m_0 = 170\,\mathrm{GeV}$

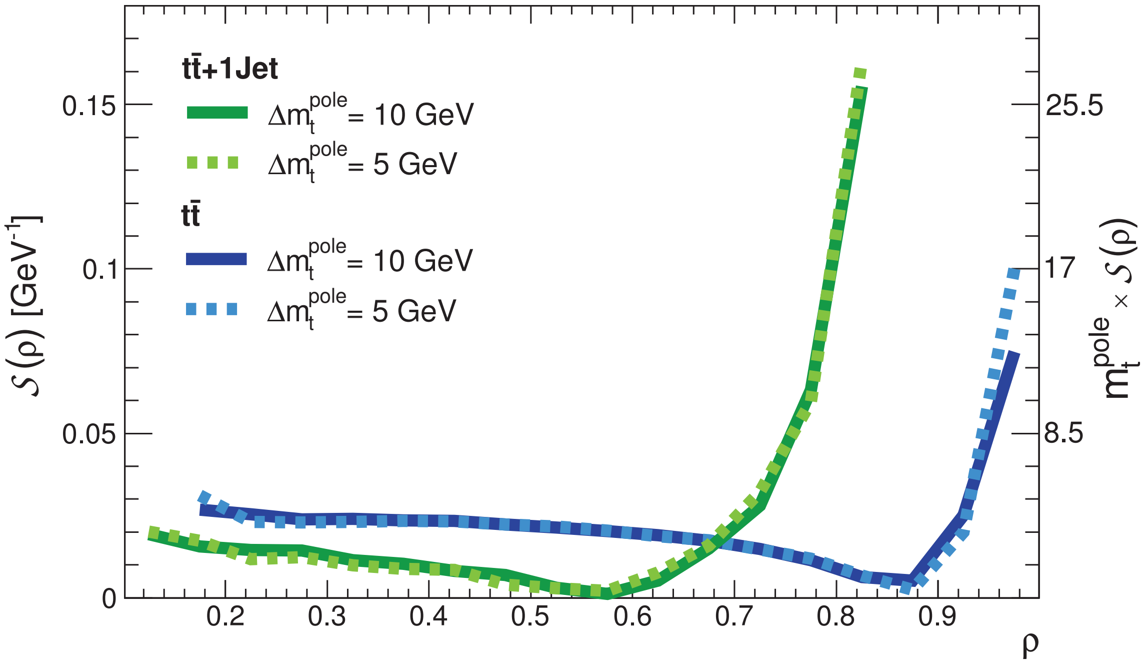
- Significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$



Mass sensitivity of $t\bar{t} + \mathrm{jet}$*-samples*

- Differential cross section $\mathcal{R}(m_t, \rho_s)$
  - good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, ...
- Increased sensitivity for system $t\bar{t} + \mathrm{jet}$ compared

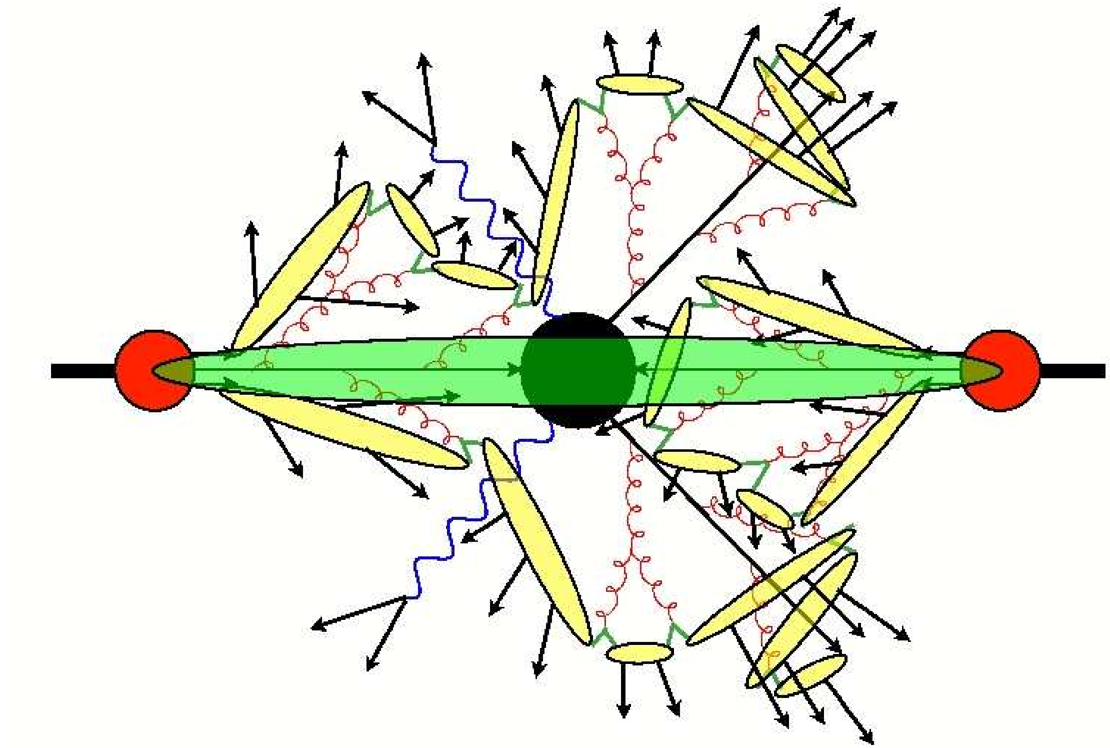
$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \simeq (m_t\mathcal{S}) \times \left|\frac{\Delta m_t}{m_t}\right|$$



- ATLAS analysis ongoing (preliminary mass reported at Top2014)

Monte Carlo mass

- Hard interaction and parton emission in QCD followed by hadronization
- Top-quark decays on shell (e.g. leptonic decay $t \to bW \to bl\bar{\nu}_l$)



[picture by B.Webber]

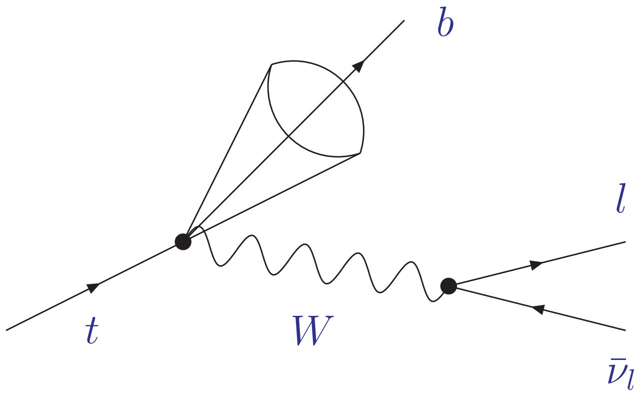
- <u>Intuition:</u> Monte Carlo mass identified with pole mass due to kinematics

$$m_q^2 \; = \; E_q^2 - p^2$$

- <u>Caveat:</u> heavy quarks in QCD interact with potential due to gluon field

Kinematic reconstruction

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W-\mathrm{boson}} + p_{b-\mathrm{jet}})^2$

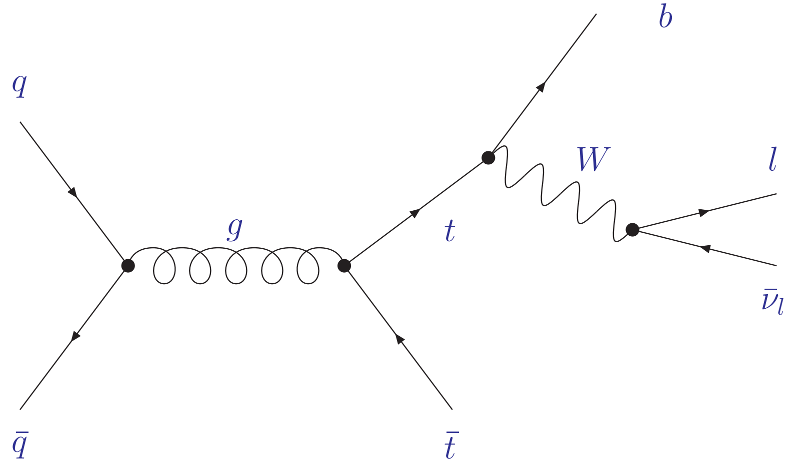


Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - uncertainties from variation of Monte Carlo parameters

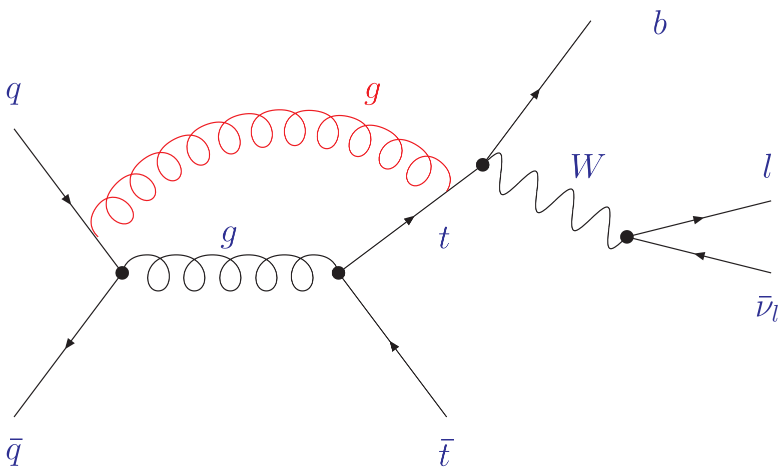
Hard scattering process

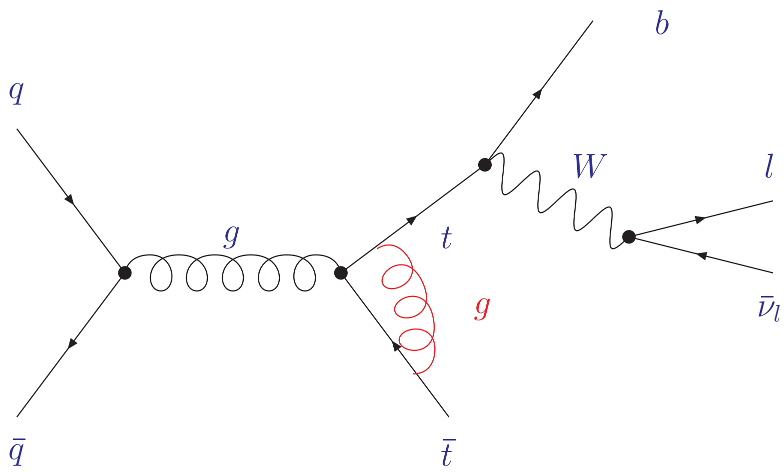
- Born process ($q\bar{q}$-channel) with leptonic decay $t \to bl\bar{\nu}_l$



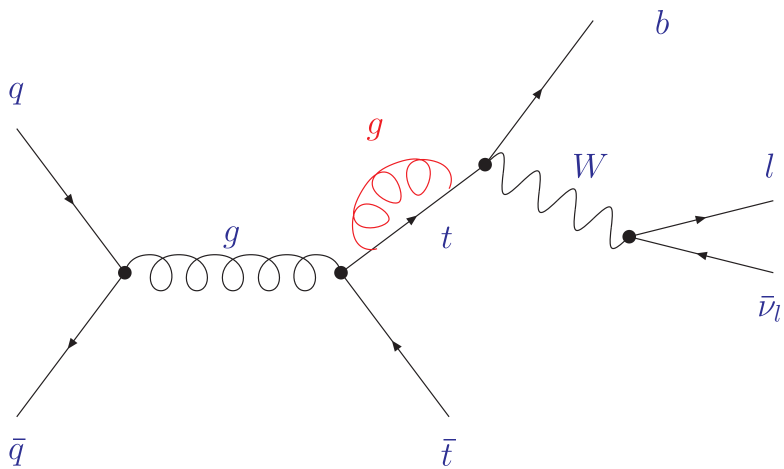
Radiative corrections

- Virtual corrections (examples): gluon exchange
  - box diagram (left) and vertex corrections (right)
  - infrared divergences cancel against real emission contributions



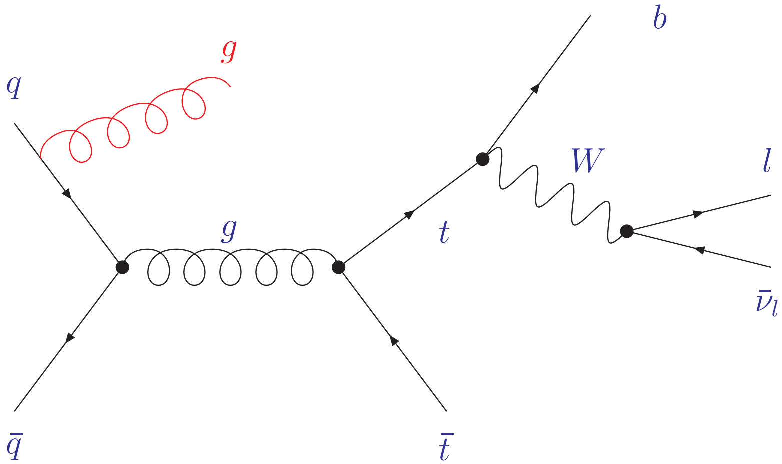


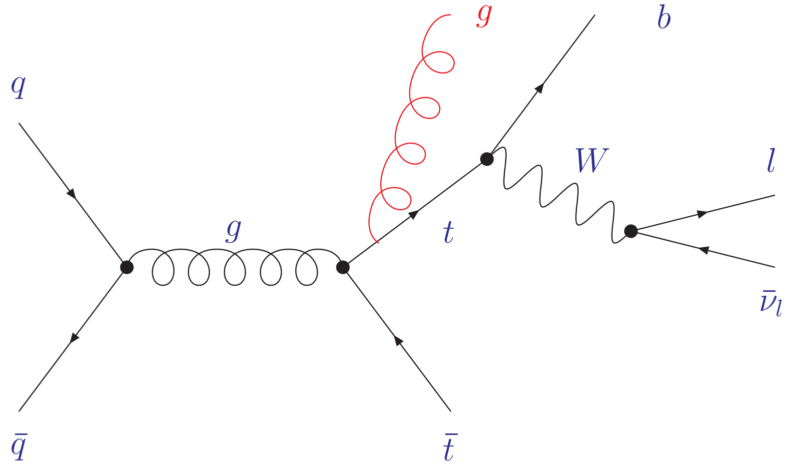
- Mass renormalization from self-energy corrections to top-quark



Radiative corrections

- Real corrections (examples): gluon emission
  - phase space integration $\rightarrow$ infrared divergences (soft/collinear singularities)





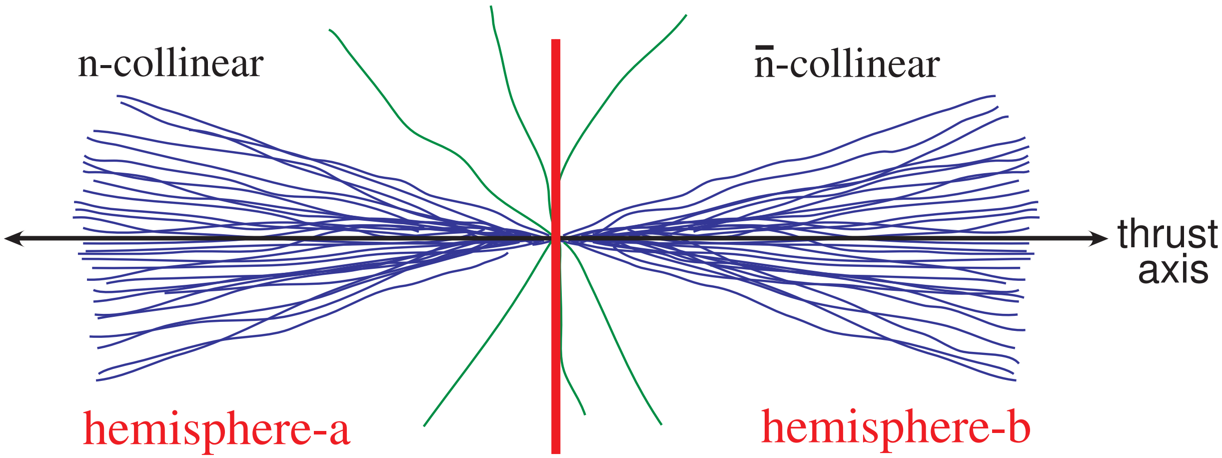
- Parton shower MC
  - emission probability modeled by Sudakov exponential with cut-off Q_0
  - leading logarithmic accuracy

$$\Delta\left(Q^2, Q_0^2\right) = \exp\left(-C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right)$$

  - subtraction of IR contributions at hadronization scale $Q_0 \simeq \mathcal{O}(1)$GeV

Mass of heavy-quark jet (I)

- Cross section for invariant mass of jet $M_{t(\bar{t})}$ in $e^+e^- \to t\bar{t}$
- Back-to-back heavy-quark jets with collinear parton emission define hemispheres Fleming, Hoang, Mantry, Stewart '07



- Cross section factorization in effective theory (SCET)

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} = \sigma_0 \underbrace{H(Q,m,\mu)}_{\text{hard fct.}} \int d\ell^+ d\ell^- \underbrace{B_+(M_t,\Gamma_t,\mu)}_{\text{jet fct.}} \underbrace{B_-(M_{\bar{t}},\Gamma_t,\mu)}_{\text{jet fct.}} \underbrace{S(\ell^+,\ell^-,\mu)}_{\text{soft fct.}}$$

- hierarchy of scales $Q \gg m_t \gg \Gamma_t \gg \Lambda_{QCD}$ and $|M_t - m_t| \simeq \Gamma_t$

Mass of heavy-quark jet (II)

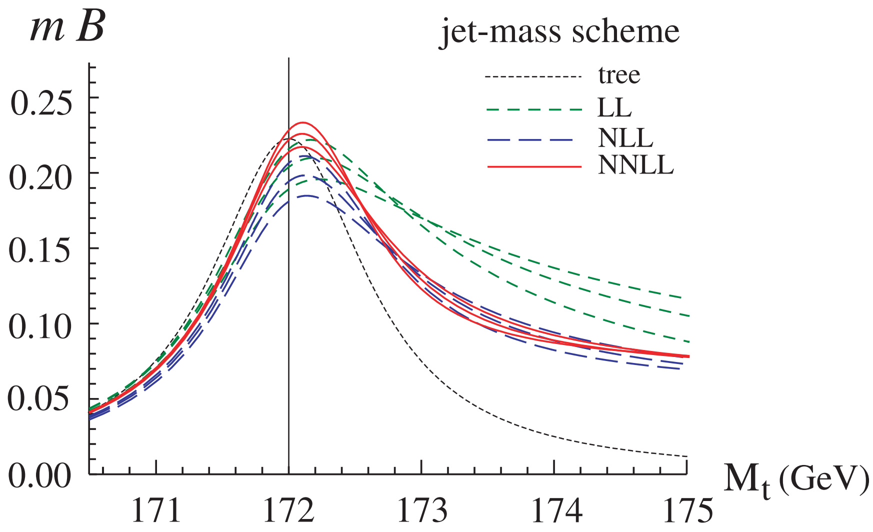
- Computation of heavy-quark jet function from discontinuity of heavy-quark propagator connected by light-like Wilson lines $W_n(0)W_n^\dagger(x)$

$$B_+(v_+ \cdot k, \Gamma_t) = \frac{-1}{8\pi N_c m} \text{Disc} \int d^4x\, e^{ik\cdot x} \langle 0 | \mathrm{T}\{\bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x)\} | 0 \rangle$$

- Computation of $B_\pm$ in perturbation theory with well-defined mass scheme
- Breit-Wigner resonance at tree level

$$B_\pm(\hat{s}, \Gamma_t) = \frac{1}{4\pi m} \text{Disc.} \left(\frac{i}{v_\pm \cdot k + i\Gamma_t/2} \right) = \frac{1}{\pi m} \frac{\Gamma_t}{\hat{s}^2 + \Gamma_t^2}$$

- Stable peak position at higher orders
  Hoang, Stewart '08
- Mass renormalization with short-distance mass
  $m_{\text{pole}} = m_{\text{short distance}} + \delta m$
- short-distance mass $m^{\text{MSR}}(R)$ probes scale of hard interaction: $R \simeq \Gamma_t$



Conversion Monte Carlo mass to pole mass (I)

Assumption

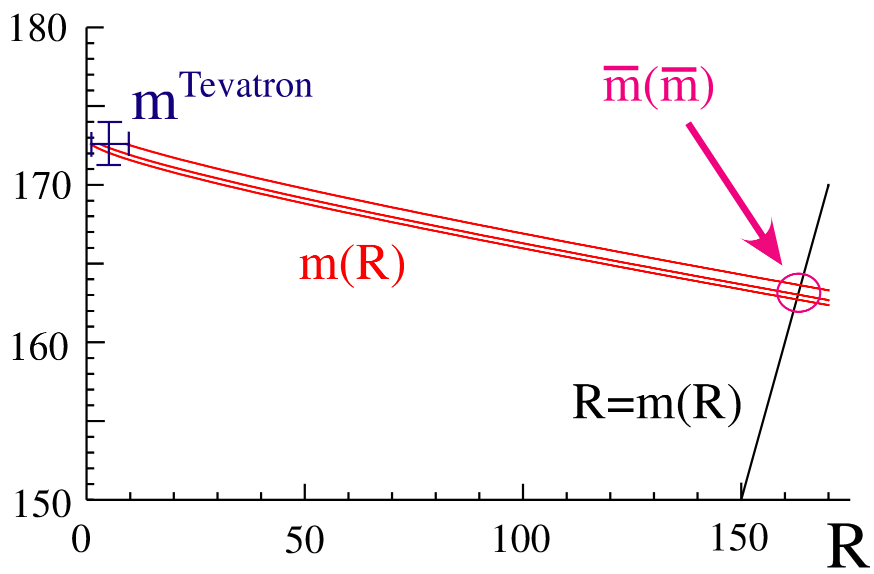
- Identify Monte Carlo mass m^{MC} with short distance mass $m^{\mathrm{MSR}}(R)$ at low scale $\mathcal{O}(1)\ \mathrm{GeV}$
  - choice for range of scale $R \simeq 1 \dots 9\mathrm{GeV}$

$$m^{\mathrm{MC}} \quad = \quad m^{\mathrm{MSR}}(R = 3^{+6}_{-2}\text{GeV})$$

Conversion Monte Carlo mass to pole mass (II)

Strategy

- Use perturbation theory to convert $m^{\rm MSR}(R)$ to $m^{\rm pole}$
- Running of $m^{\rm MSR}(R)$ mass
  Hoang, Stewart '08



- Choice 1: run $m^{\rm MSR}(R)$ from low scale to $R = m_t$: $m^{\rm MSR}(R) \to m(m)$ and convert from $m(m)$ to pole mass [arXiv:1405.4781]

$m^{\rm MSR}(1)$	$m^{\rm MSR}(3)$	$m^{\rm MSR}(9)$	$\overline{m}(\overline{m})$	$m^{\rm pl}_{\rm 1lp}$	$m^{\rm pl}_{\rm 2lp}$	$m^{\rm pl}_{\rm 3lp}$
173.72	173.40	172.78	163.76	171.33	172.95	173.45

- Choice 2: convert from $m^{\rm MSR}(R)$ at low scale directly to pole mass

$m^{\rm MSR}(1)$	$m^{\rm MSR}(3)$	$m^{\rm MSR}(9)$	$m^{\rm pl}_{\rm 1lp}$	$m^{\rm pl}_{\rm 2lp}$	$m^{\rm pl}_{\rm 3lp}$
173.72	173.40	172.78	173.72	173.87	173.98

Conversion Monte Carlo mass to pole mass (III)

Summary

$$m_{\rm pole} = 173.34 \pm 0.76\,\text{GeV (exp)} + \Delta m\text{(th)}$$

with

$$\Delta m\text{(th)} \quad = \quad {}^{+0.32}_{-0.62}\,\text{GeV}\,(m^{\rm MC} \to m^{\rm MSR}(3\text{GeV})) + 0.50\,\text{GeV}\,(m(m) \to m_{\rm pole})$$

and combined

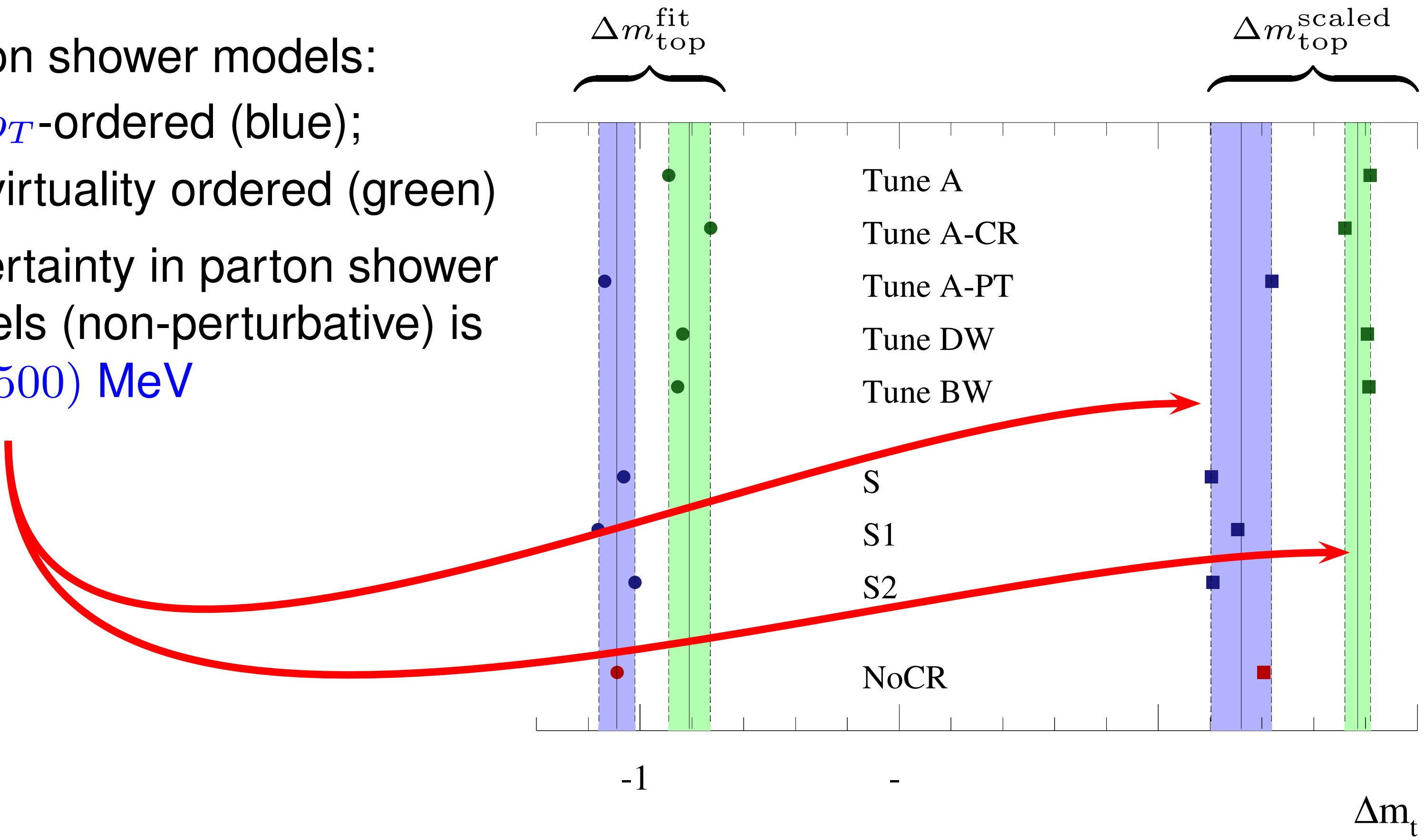
$$\Delta m\text{(th)} \quad = \quad {}^{+0.82}_{-0.62}\text{GeV}$$

In addition, unknown systematic mass shift $\mathcal{O}(1)$ GeV due to non-perturbative effects on peak position of invariant jet-mass distribution $M^{\rm peak}$ with decaying top-quark for short distance mass m_t

$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \dots) + \frac{Q\Lambda_{\rm QCD}}{m_t}$$

Non-perturbative corrections

- Simulation of top mass measurement Skands, Wicke '07
  - test of different Monte Carlo tunes for non-perturbative physics / colour reconnection
  - calibration offsets before/after scaling with jet energy scale corrections
- Parton shower models:
  - p_T-ordered (blue);
  - virtuality ordered (green)
- Uncertainty in parton shower models (non-perturbative) is $\mathcal{O}(\pm 500)$ MeV



Higgs boson mass

Experimental result

Atlas arXiv:1307.1427; CMS coll. arXiv:1312.5353; (average arXiv:1303.3570)

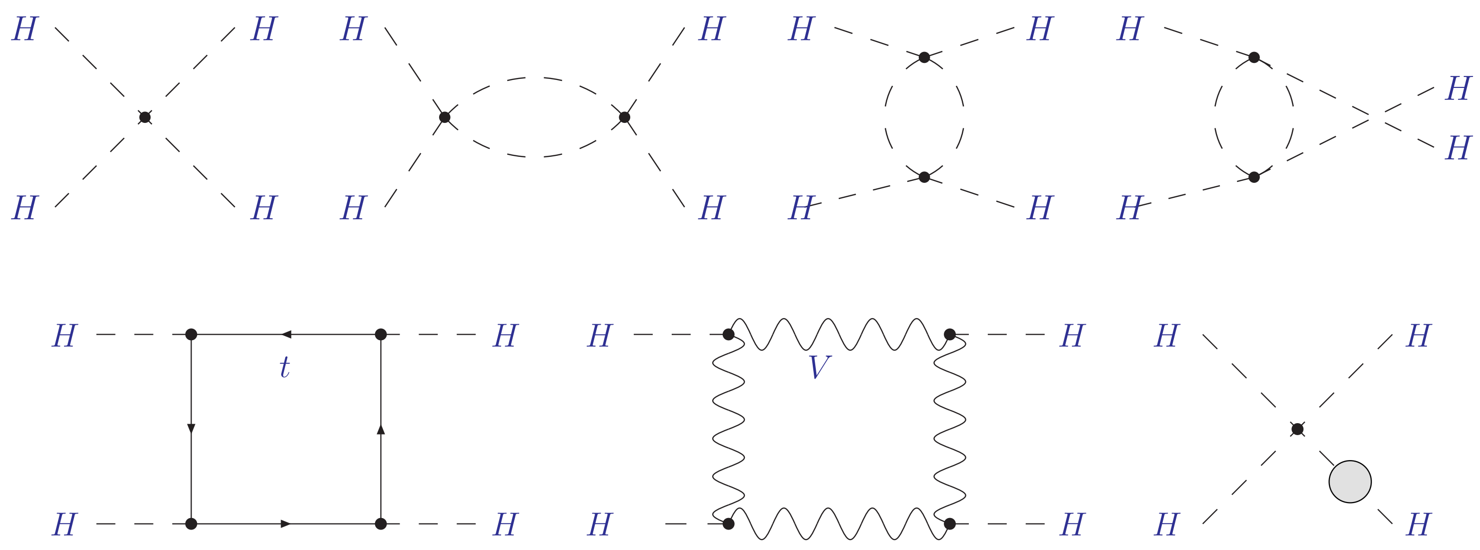
$$m_H = 125.15 \pm 0.24 \text{GeV}$$

Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left|\Phi^\dagger \Phi - \frac{v}{2}\right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \ldots$$



Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

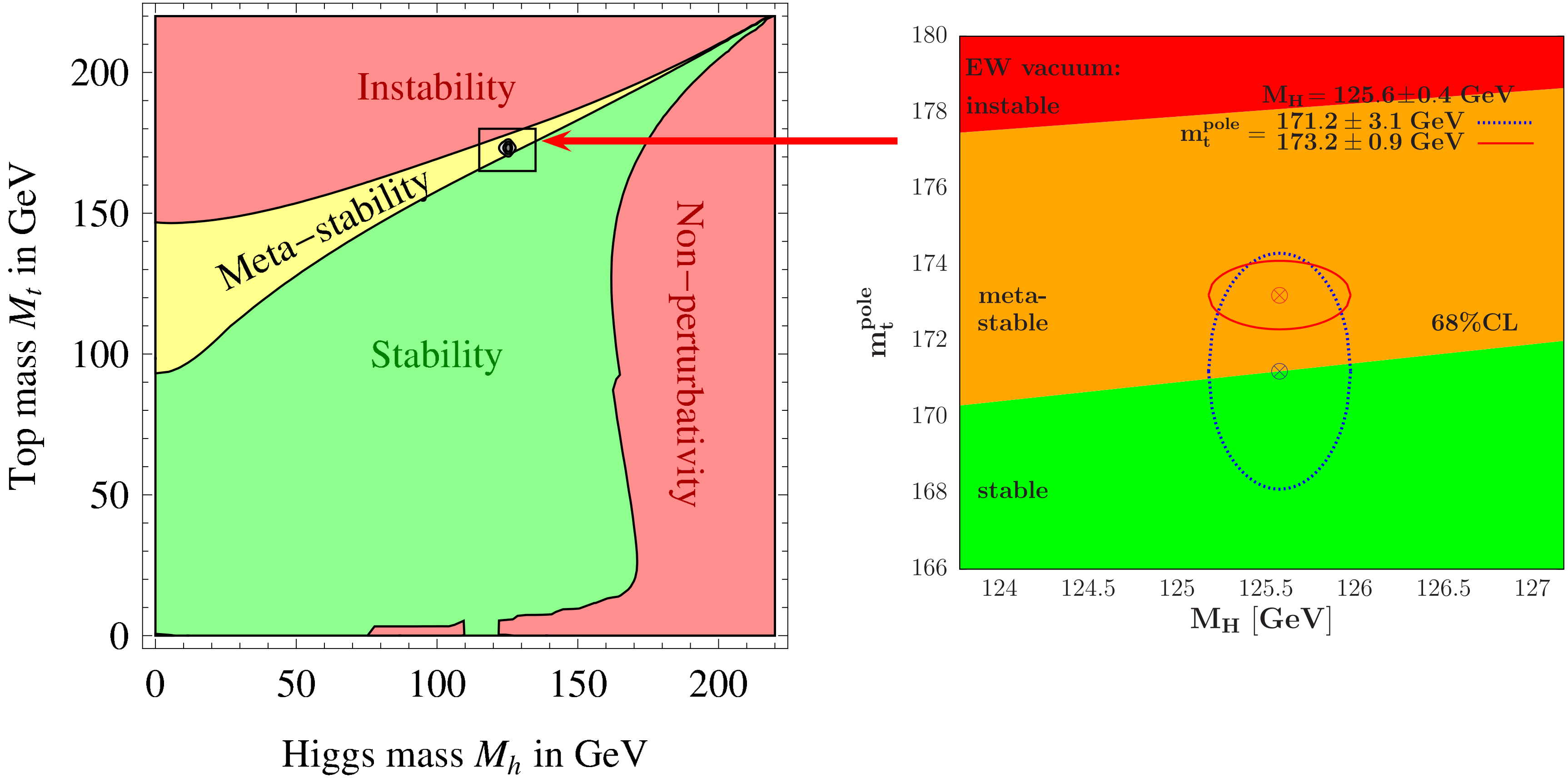
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top-quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \;\geq\; 129.6 + 2.0 \times \left(m_t^{\rm pole} - 173.34\ {\rm GeV} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.3\ \ {\rm GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov ‘12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. ‘12; Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia ‘13
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top-quark mass from total cross section (well-defined scheme)
  - $m_t^{\overline{\rm MS}}(m_t) \;=\; 162.3 \pm 2.3 \;\pm 0.7\ {\rm GeV}$ implies in pole mass scheme $m_t^{\rm pole} \;=\; 171.2 \pm 2.4 \pm 0.7\ {\rm GeV}$
  - mass determination accounts for correlation with gluon PDF and $\alpha_S(M_Z)$

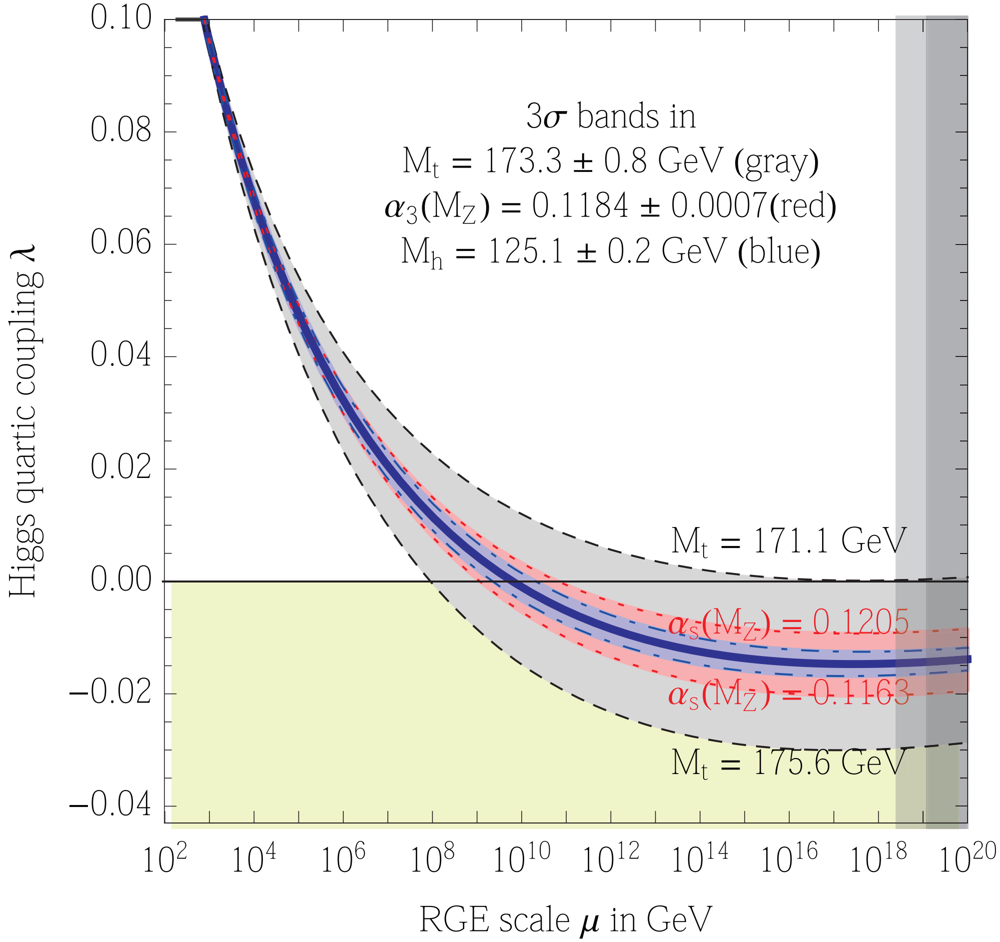
Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 125.3 \pm 6.2$ GeV
  - "fate of universe" still undecided

Higgs self-coupling



Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia '13

- Renormalization group evolution of λ with uncertainties in m_H, m_t and α_s
  - top-quark mass least precise parameter
- Vacuum stability bound at M_P in terms of m_t

$$m_t \;\leq\; (171.53 \pm 0.15 \pm 0.23_{\alpha_3} \pm 0.15_{m_h})\ \text{GeV} \;=\; (171.53 \pm 0.42)\ \text{GeV}$$

Summary

Top-quark mass

- Running mass ($\overline{\text{MS}}$ scheme) at NNLO in QCD

$$m_t(m_t) = 162.3 \pm 2.3 \pm 0.7\text{GeV}$$

Higgs mass

- Known to very high precision (pole mass)

$$m_H = 125.15 \pm 0.24\text{GeV}$$

Fate of the universe

- Still undecided . . .

Summary

Physics at the Terascale

- Discovery of Higgs boson opens new avenue for studies of Standard Model physics and beyond
- QCD and electroweak corrections at higher orders are crucial
- Precision tests of SM at LHC depend on non-perturbative parameters
  - masses m_t, M_W, m_H, ...
  - coupling constant $\alpha_s(M_Z)$
  - parton content of proton (PDFs)

Top-quark mass

- Top-quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Quality of perturbative expansion depends on scheme for top-quark mass
- Relation of Monte Carlo mass m^{MC} to pole mass with additional theory uncertainty Δm_t(th)

Future tasks

- Joint effort theory and experiment