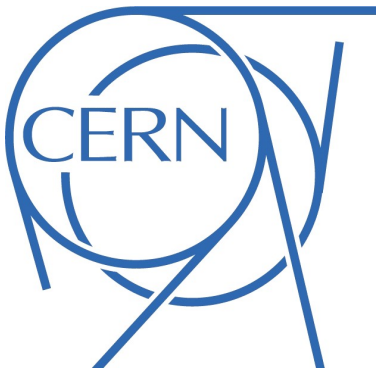


Precision flavor physics:

**Recent measurements of the  
CKM angle  $\gamma$  at LHCb**

Moritz Karbach  
[moritz.karbach@cern.ch](mailto:moritz.karbach@cern.ch)

Particle Physics Seminar LAL  
17.02.2015



# LHC, CERN, Geneva

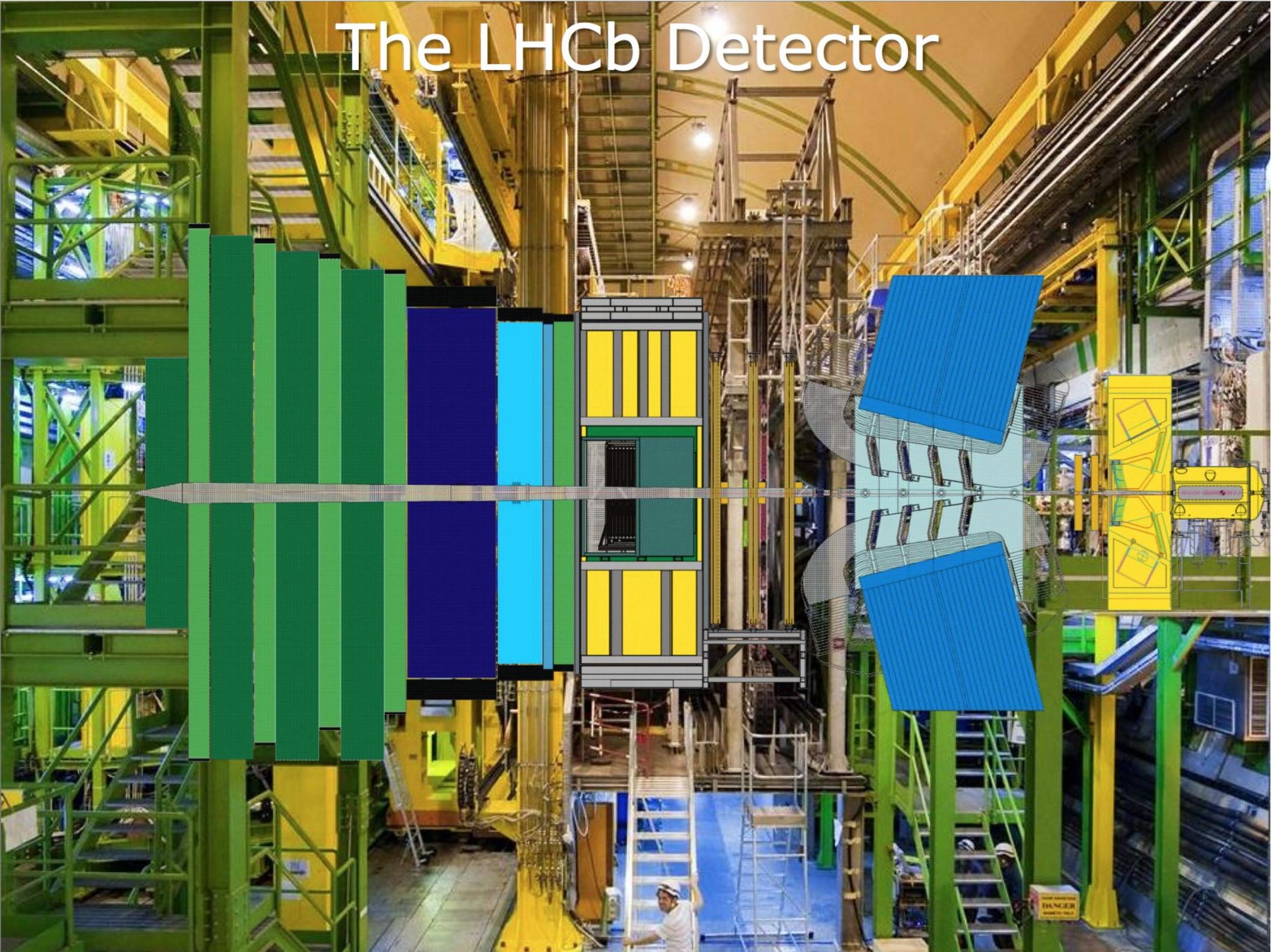


pp collisions at 7-8 TeV  
Long Shutdown 1: 2013-2015  
then: 13-14 TeV

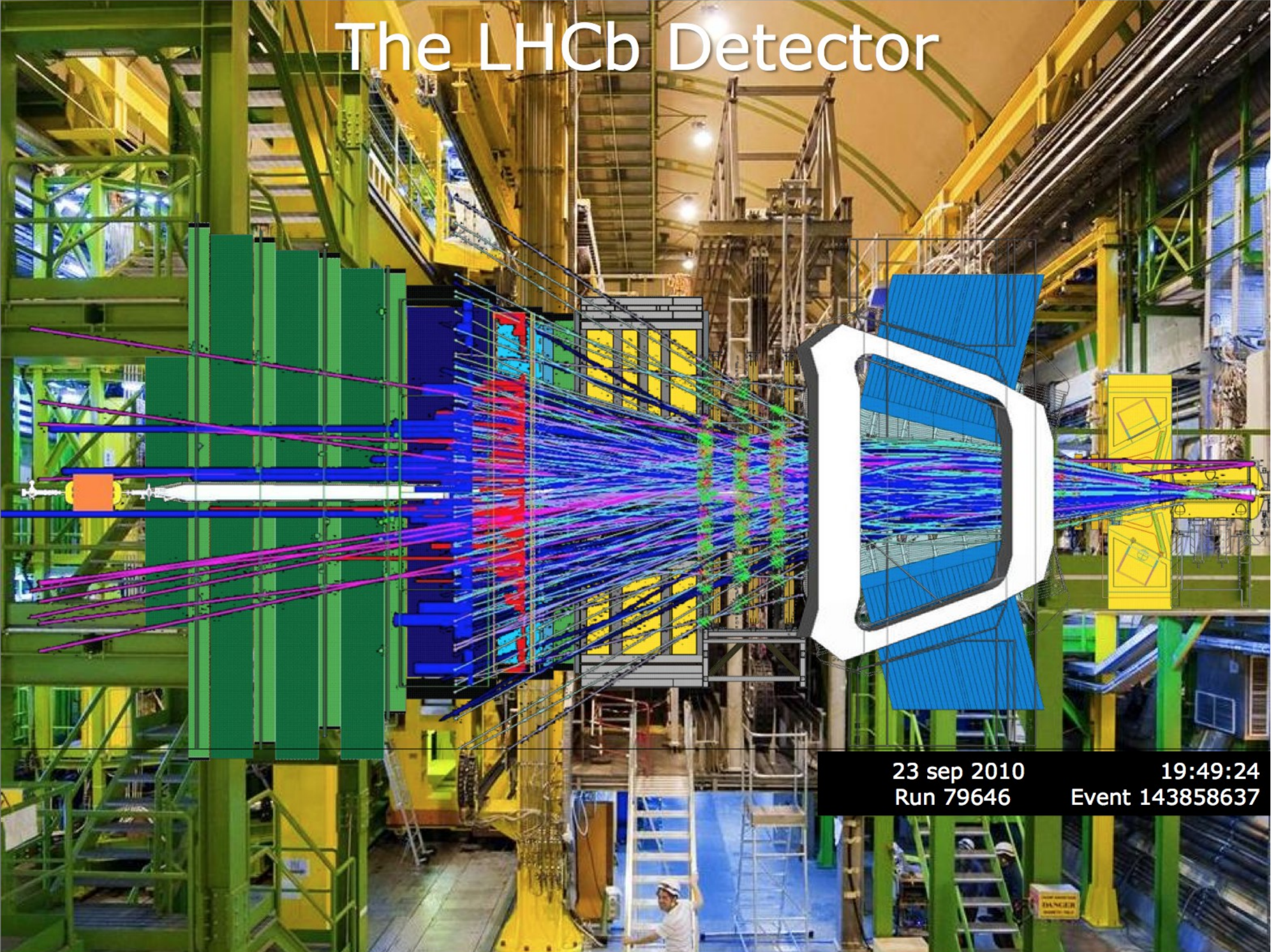
# The LHCb Detector



# The LHCb Detector



# The LHCb Detector



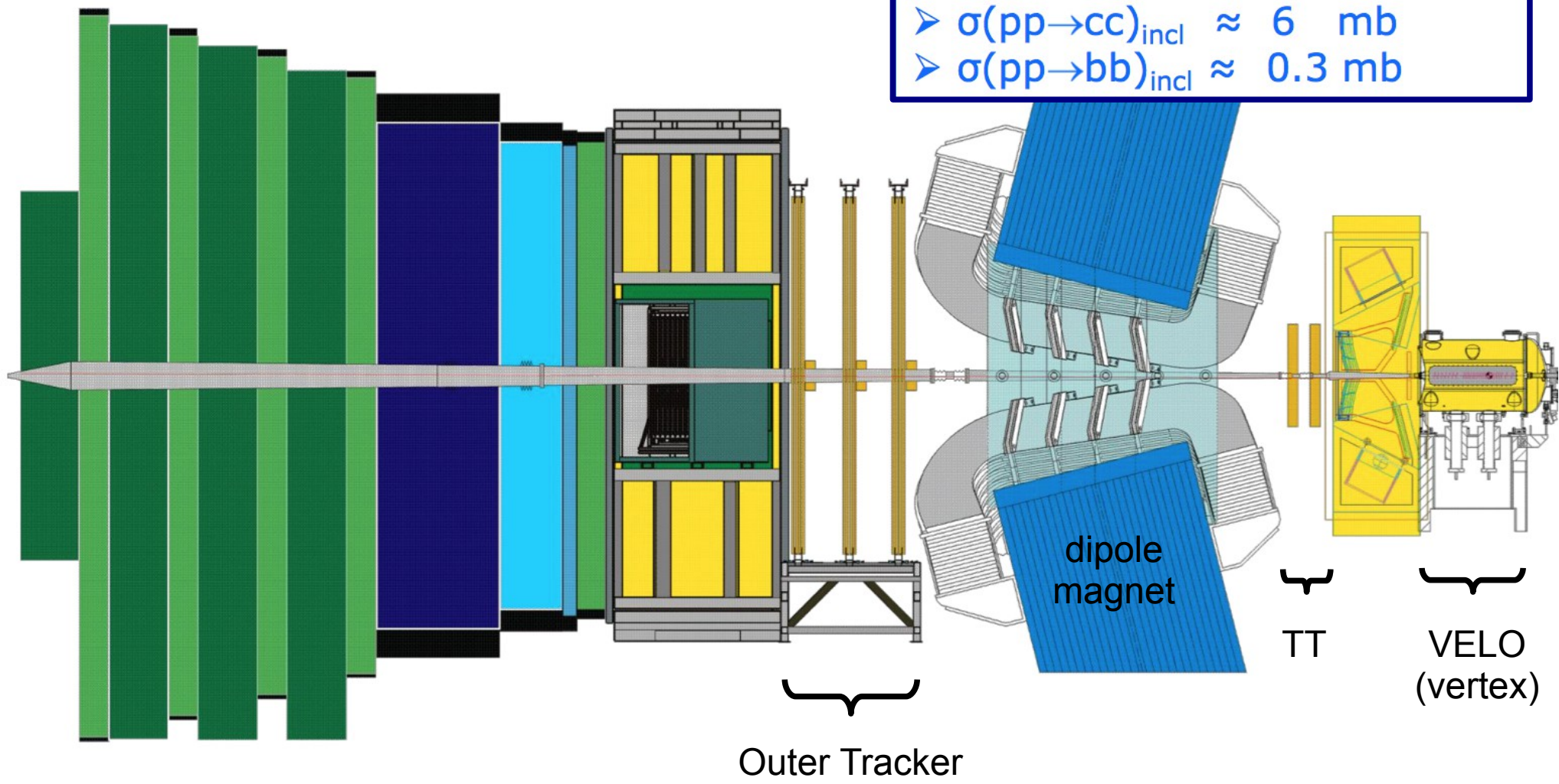
23 sep 2010  
Run 79646

19:49:24  
Event 143858637

BEWARE  
DANGER

Forward arm spectrometer

- $2 < \eta < 5$
- $\sigma(pp \rightarrow X)_{\text{inel}} \approx 60 \text{ mb}$
- $\sigma(pp \rightarrow cc)_{\text{incl}} \approx 6 \text{ mb}$
- $\sigma(pp \rightarrow bb)_{\text{incl}} \approx 0.3 \text{ mb}$



# Outer Tracker

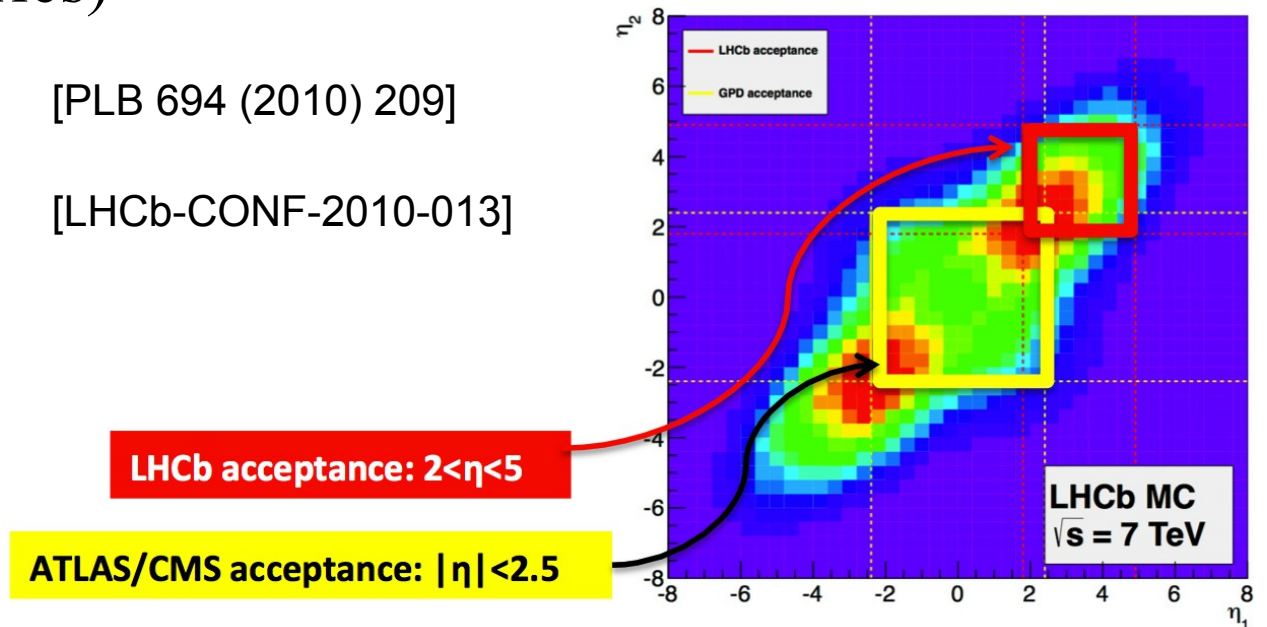
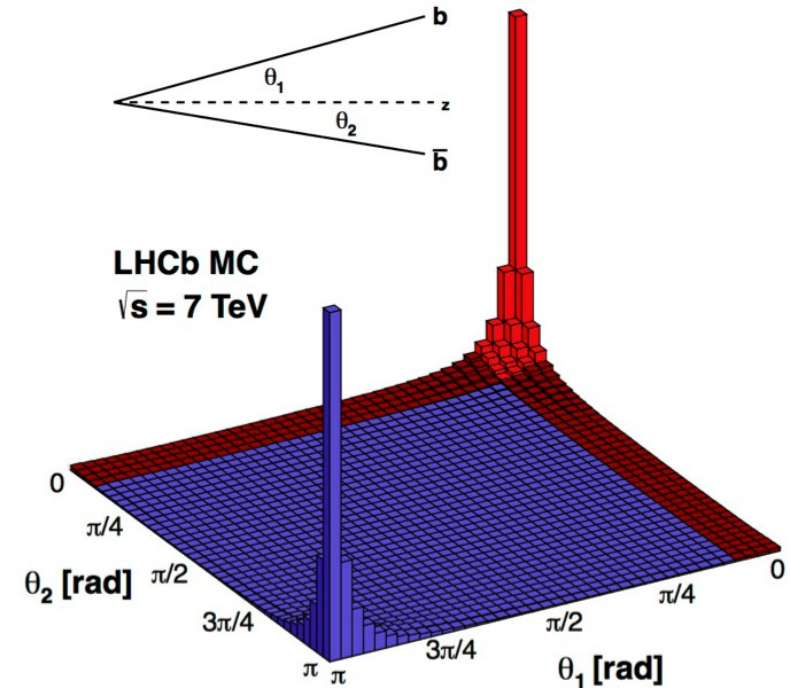


# LHCb

- one arm forward spectrometer
- $b$  pair production angles strongly correlated
- covers  $1.9 < \eta < 4.9$
- 100'000  $b\bar{b}$  pairs produced per second ( $10^4 \times B$  factories)

$$\sigma(b\bar{b}) = 284 \pm 53 \mu\text{b} \quad [\text{PLB 694 (2010) 209}]$$

$$\sigma(c\bar{c}) \approx 20 \times \sigma(b\bar{b}) \quad [\text{LHCb-CONF-2010-013}]$$



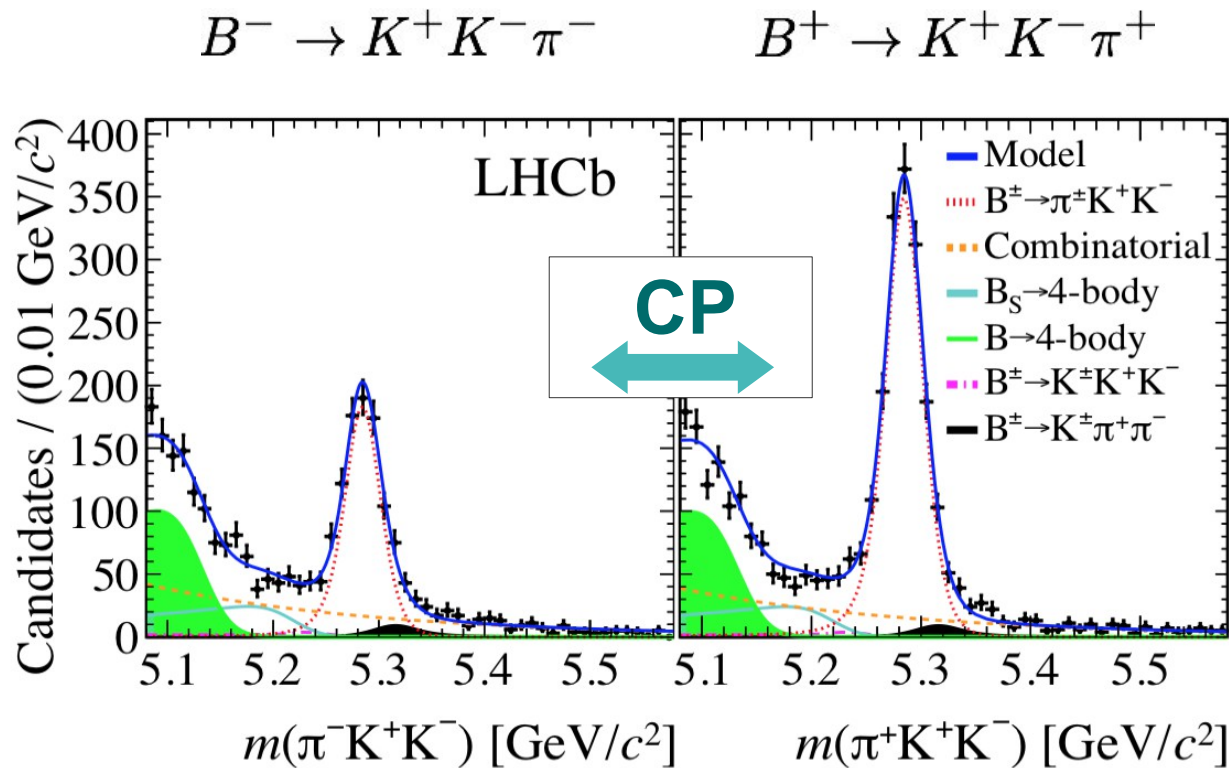


# CP violation

- Matter/Antimatter, baryon genesis
- CP violation is one crucial ingredient (Sacharov)
- The CKM matrix is the one place in the SM with CP violation.

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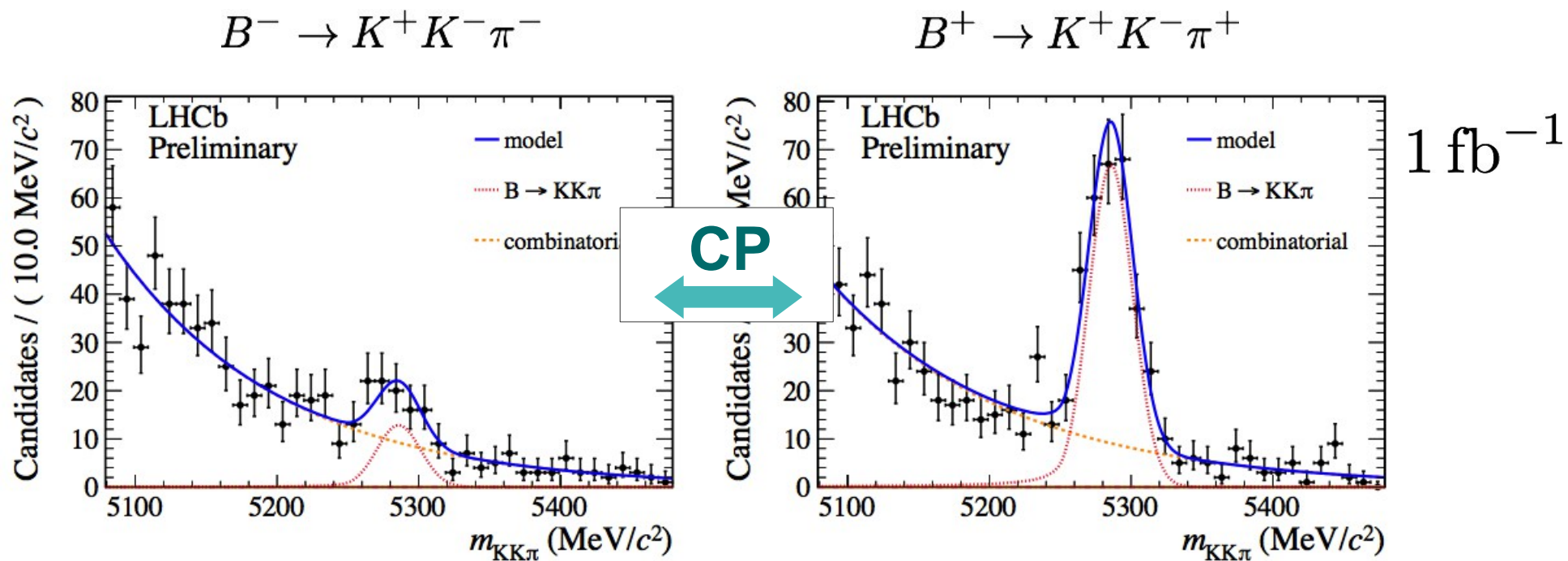


$3 \text{ fb}^{-1}$

PRDD90 (2014) 112004  
arXiv:1408.5373

# CP violation

- Matter/Antimatter, baryon genesis
- CP violation is one crucial ingredient (Sacharov)
- The CKM matrix is the one place in the SM with CP violation.

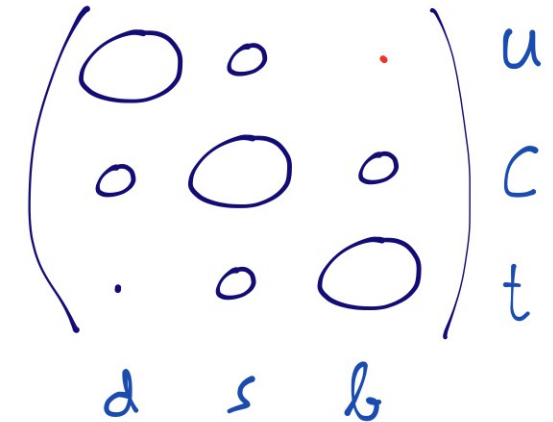


CP asymmetry in  $B \rightarrow KK\pi$  in **selected kinematic** range [LHCb-CONF-2012-028]

# CP Violation in the SM: CKM matrix

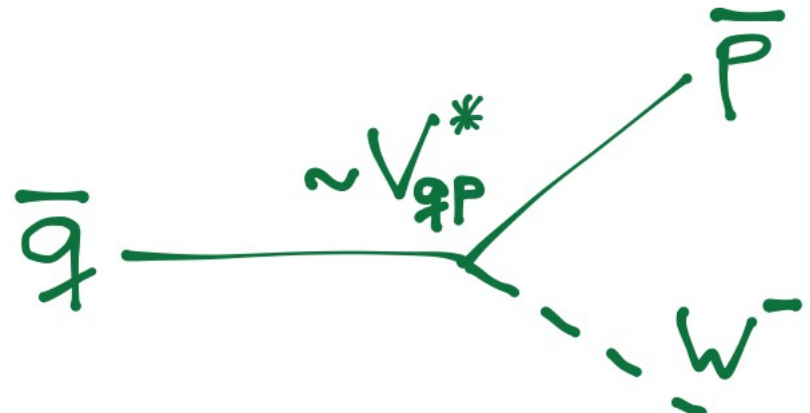
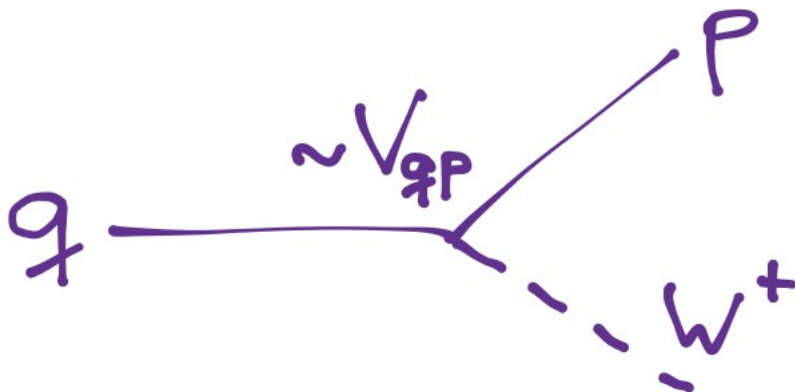
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

flavor eigenstates mass eigenstates



Cabibbo  
Kobayashi  
Maskawa

matrix elements determine transition probabilities:



# CP Violation in the SM: CKM matrix

Unitarity condition

$$V^\dagger V = 1$$

implies

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

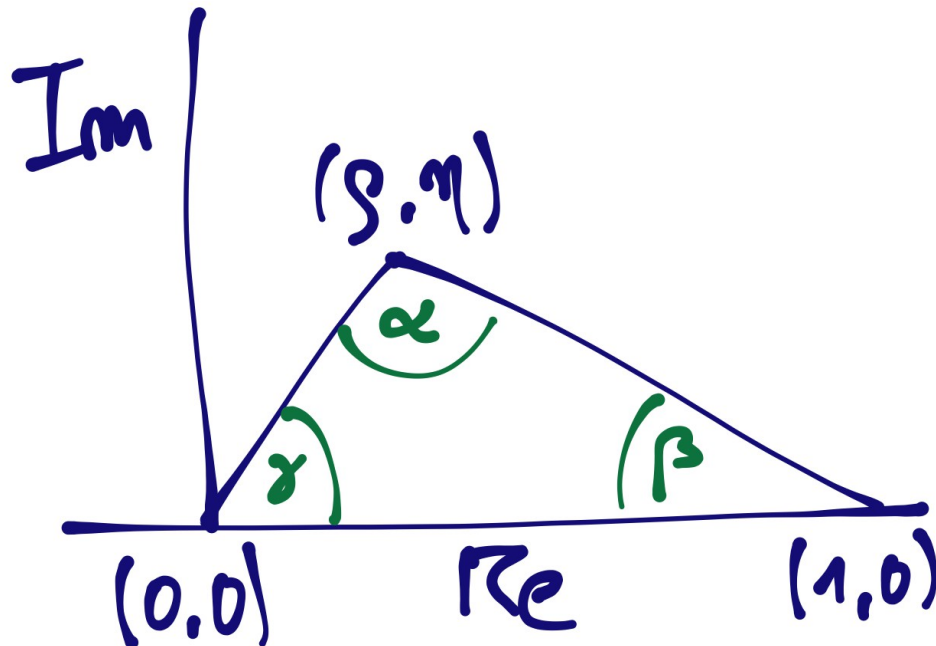
normalize it:

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

# CP Violation in the SM: CKM matrix

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

**Triangle** in the complex plane.

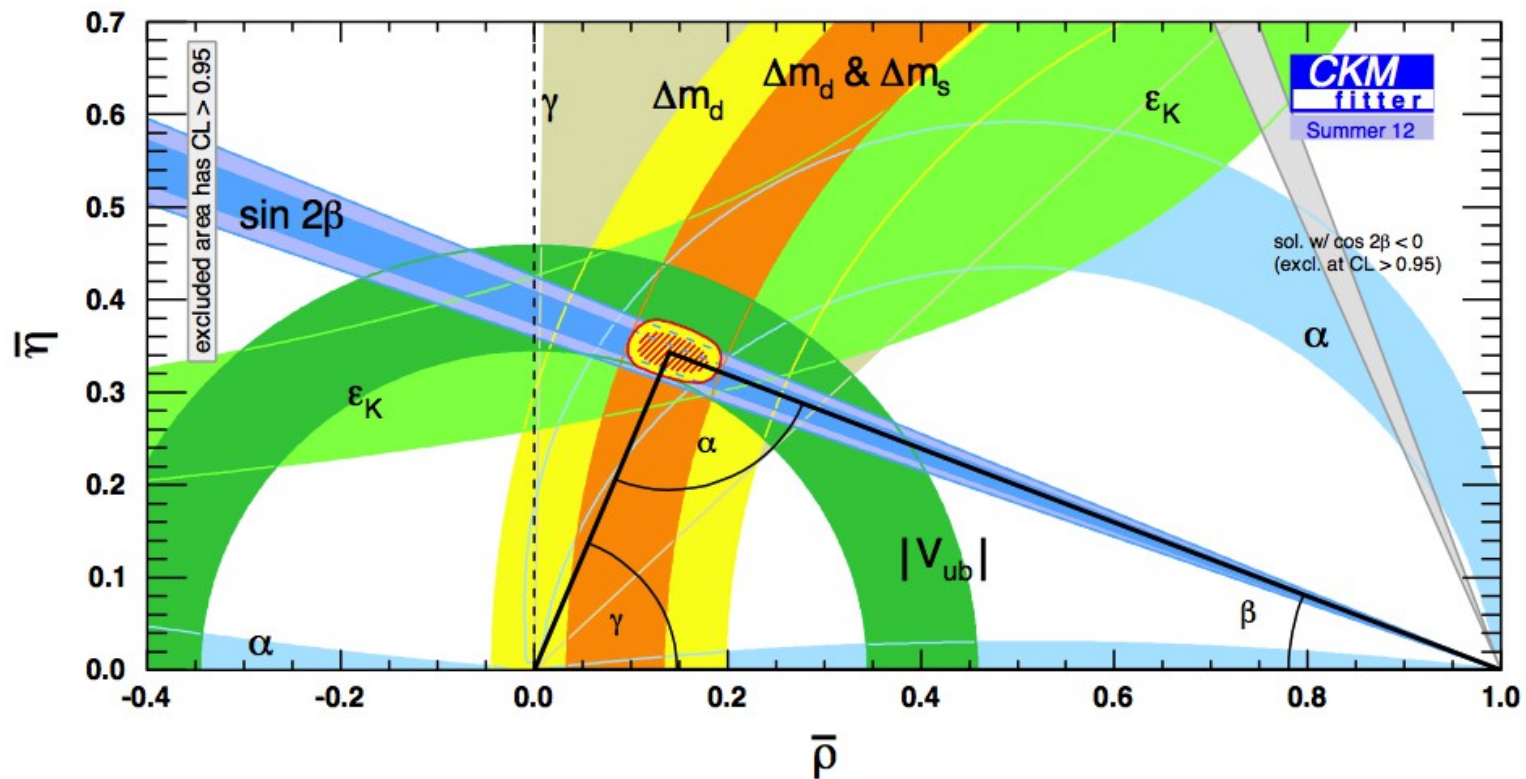


$$\gamma = \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right)$$

**Area** corresponds to the **total CP violation** in the Standard Model.

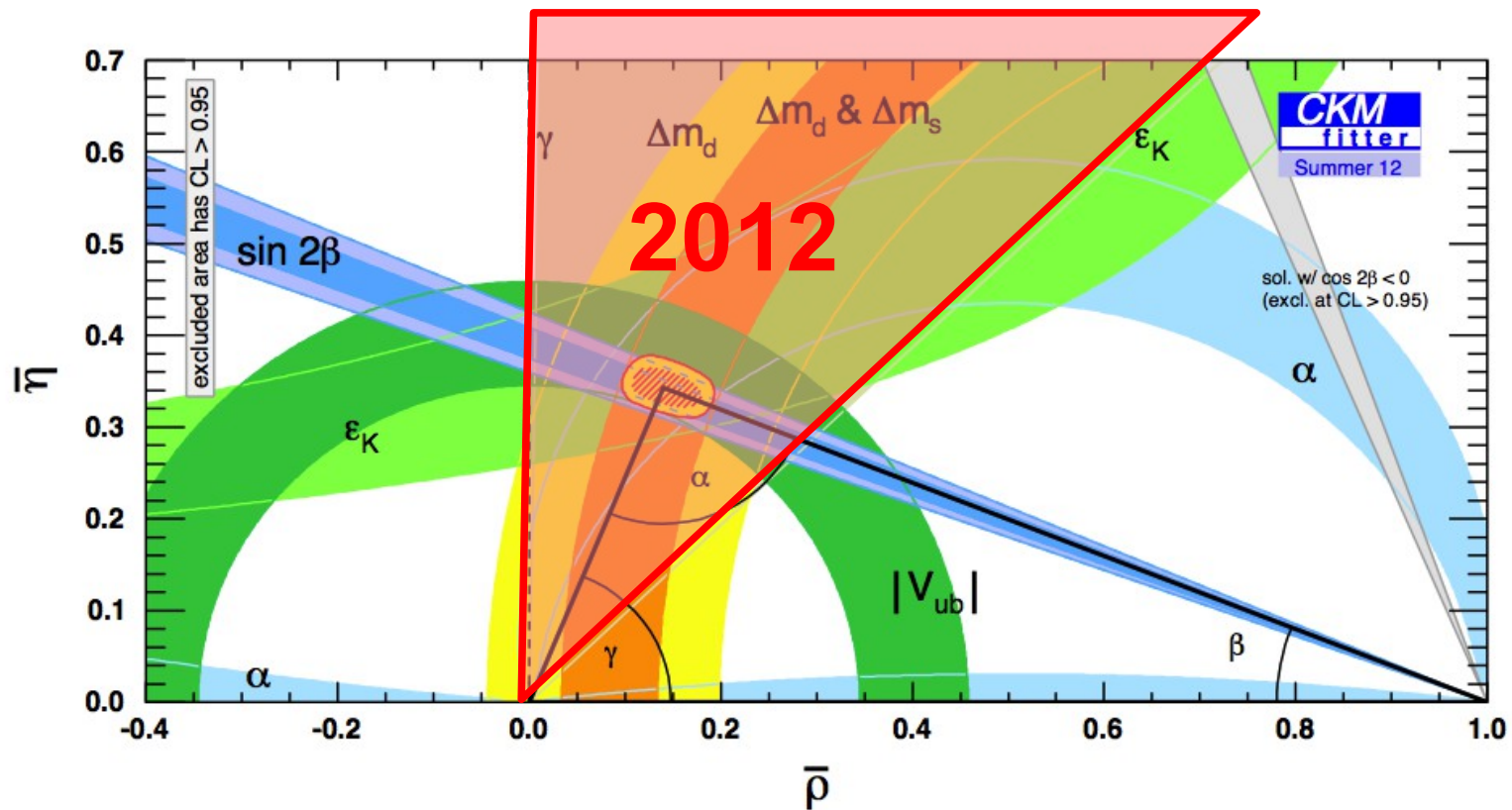
# CKM angle $\gamma$

This is the *least well known* angle of the unitarity triangle.



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# CKM angle $\gamma$

This is the *least well known* angle of the unitarity triangle.

“combined  $\gamma$  measurements”

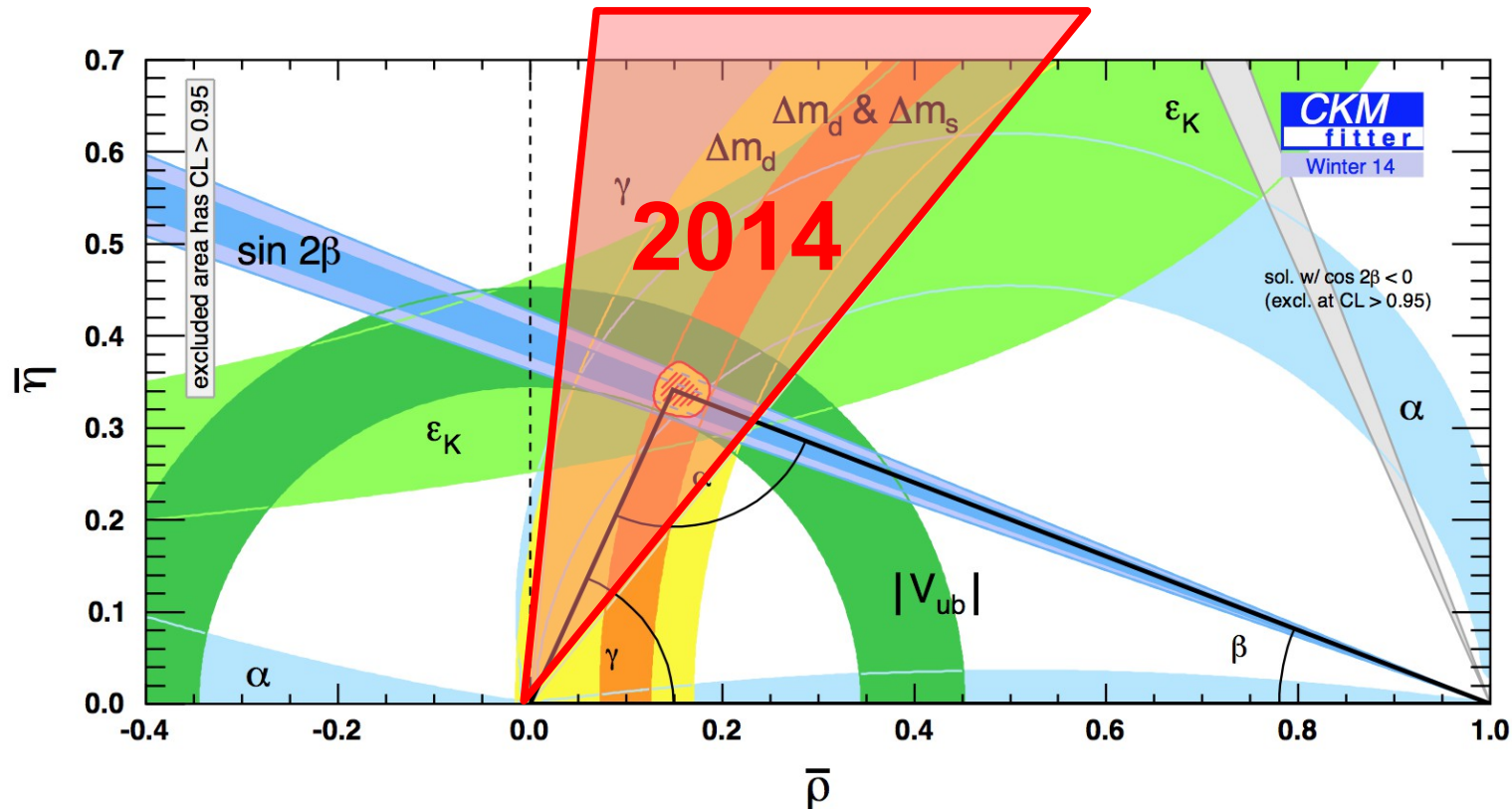
$$\gamma = (73.2^{+6.3}_{-7.0})^\circ$$

CKMfitter CKM2014

“ $\gamma$  meas. not in triangle fit”

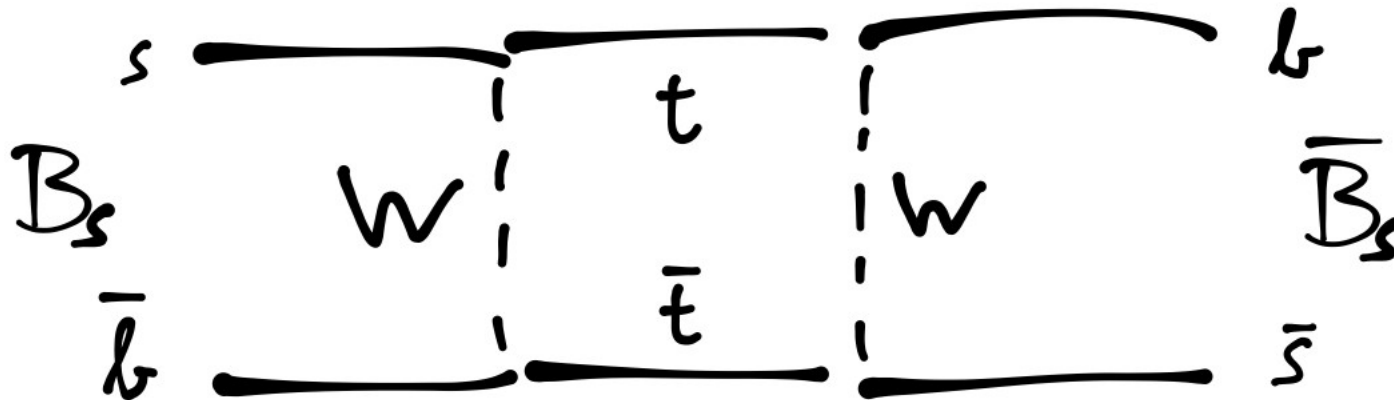
$$\gamma = (66.4^{+1.3}_{-3.3})^\circ$$

CKMfitter Moriond 2014



# New Physics in (anti)particle oscillation?

Some neutral particles can transition into their own anti-particle (mixing):

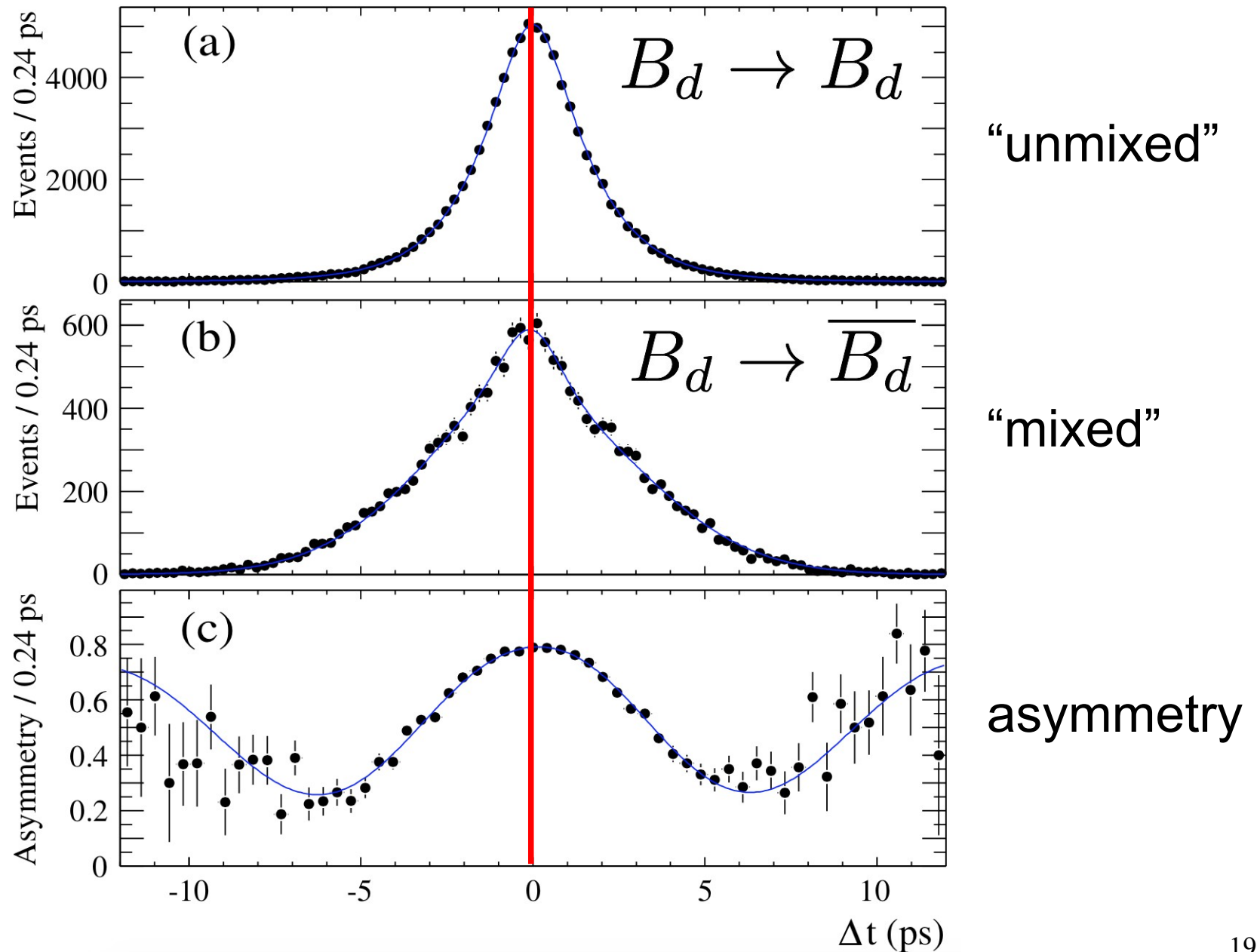


There are only 4 particles that can oscillate:

- $D^0$  mesons: *very, very* slowly
- $K^0$  mesons: very slowly
- $B_d$  mesons: slowly
- $B_s$  mesons: fast!

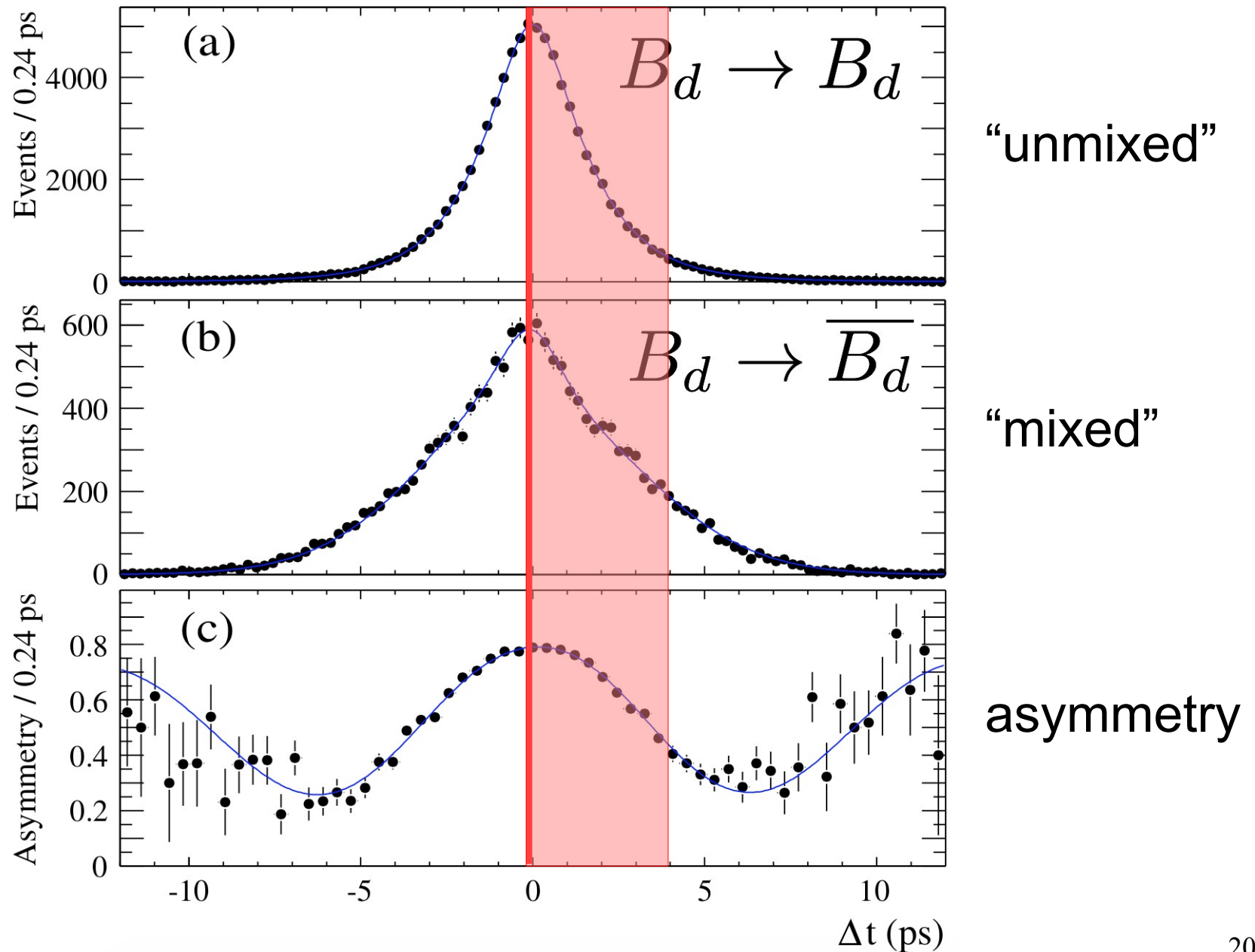
# New Physics in (anti)particle oscillation?

BaBar 2002; Physics of the B factories, arXiv:1406.6311, 2014

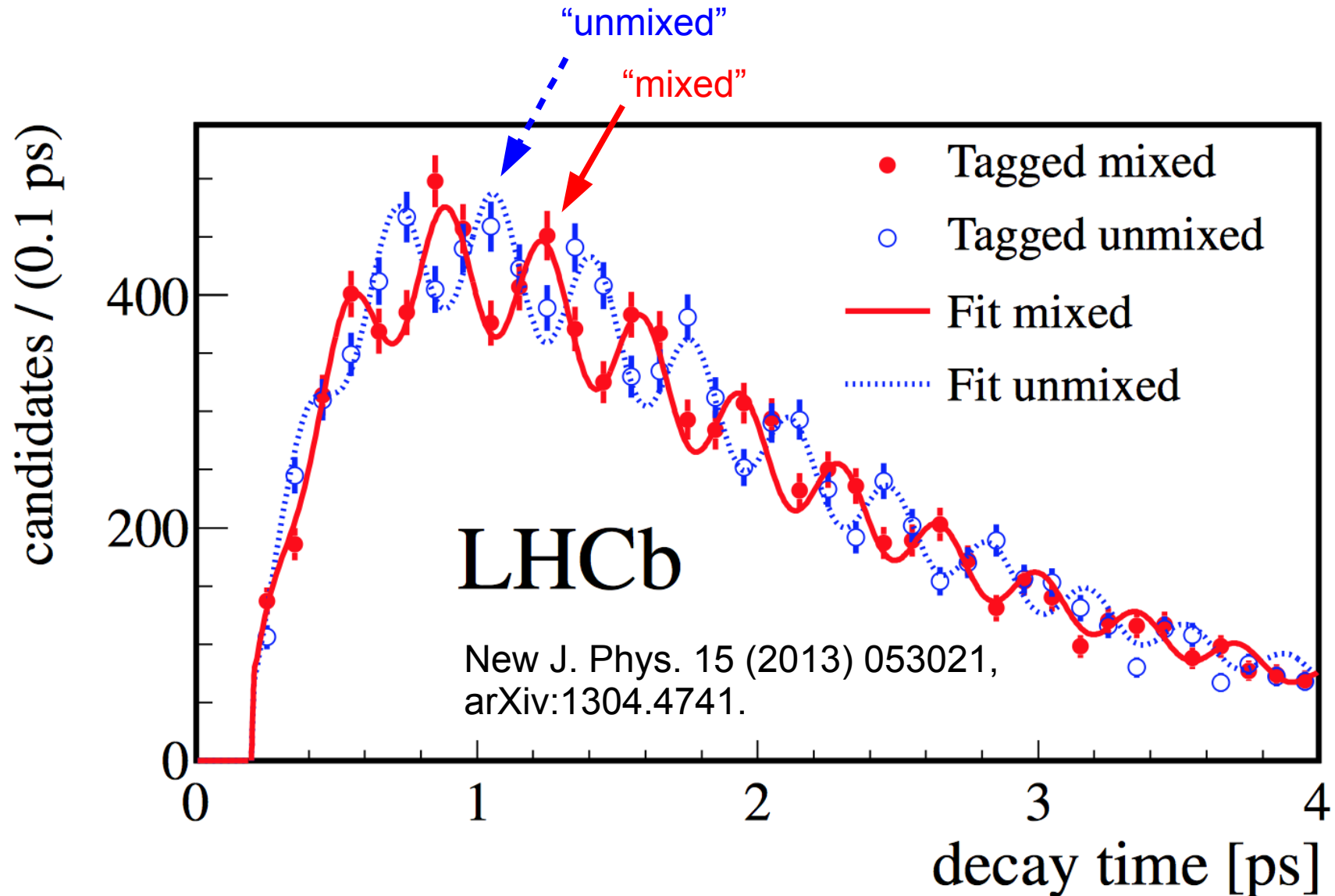


# New Physics in (anti)particle oscillation?

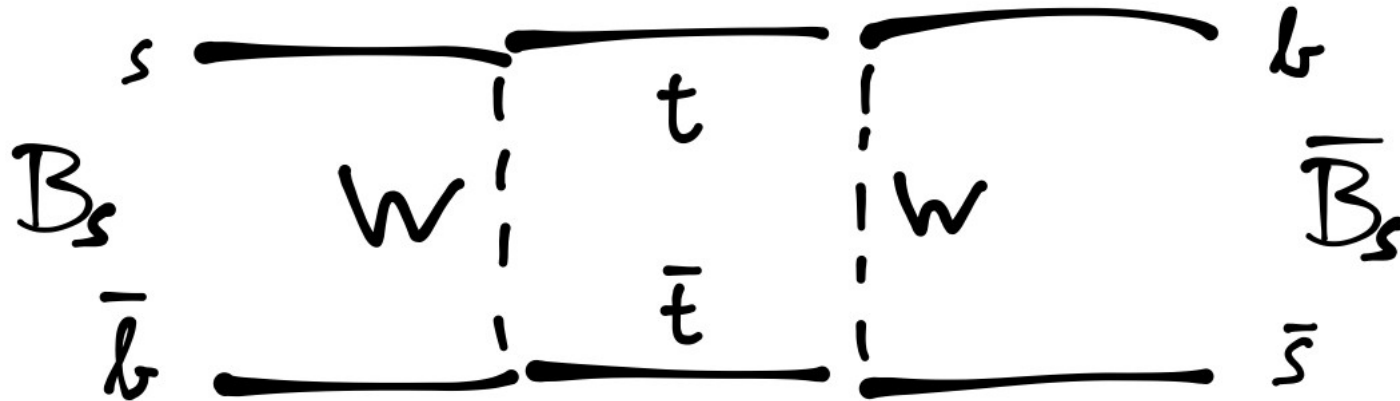
BaBar 2002; Physics of the B factories, arXiv:1406.6311, 2014



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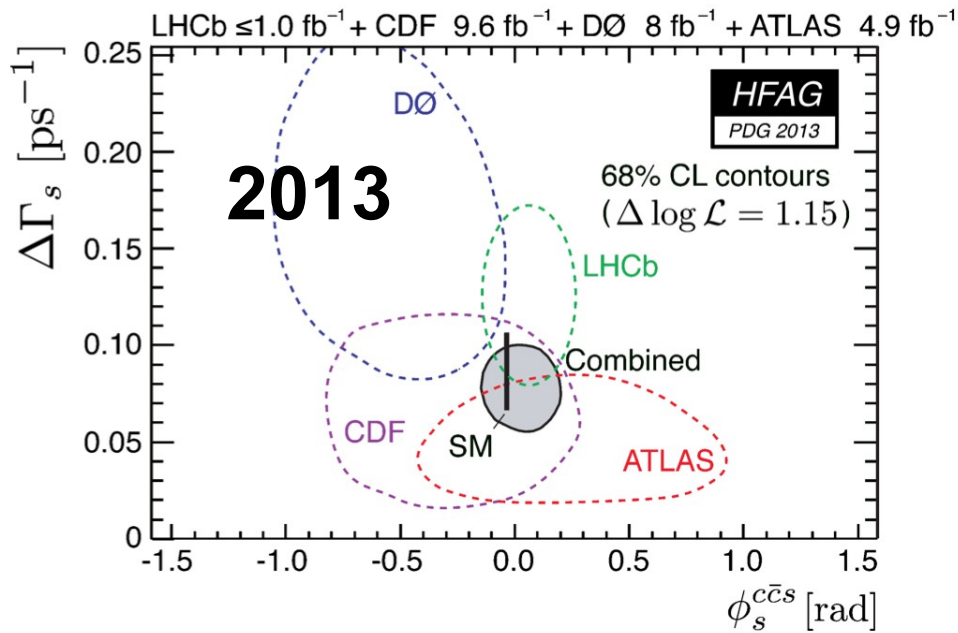


# New Physics in (anti)particle oscillation?



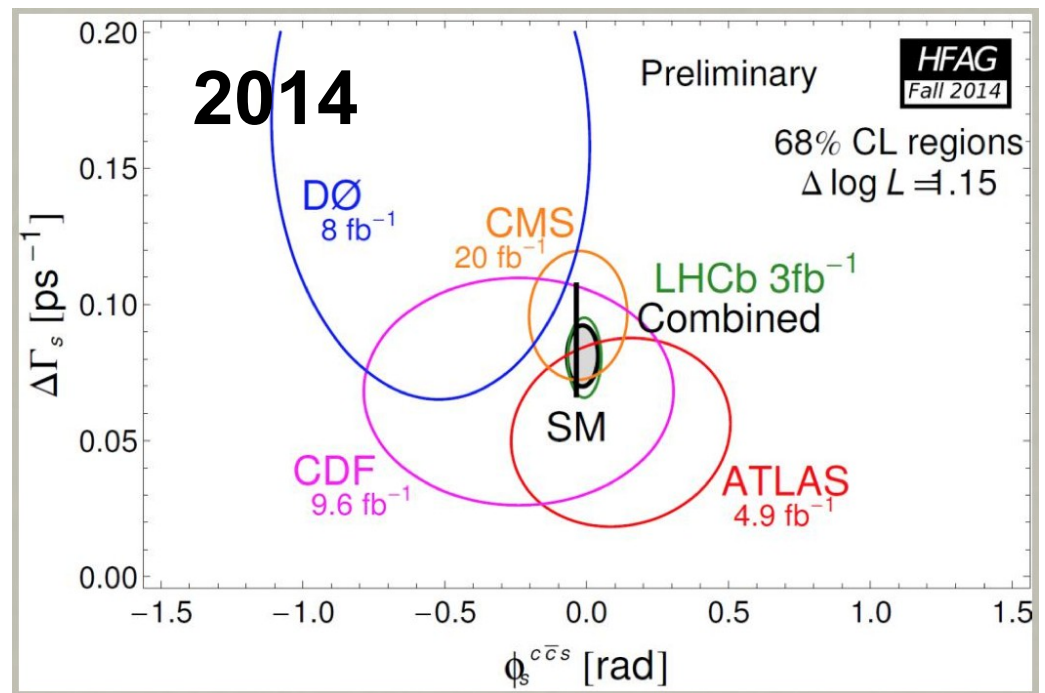
- mixing introduces a weak mixing phase  $2\beta_s$
- the mixing phase could (have been...) easily affected by new physics!

# New Physics in (anti)particle oscillation?

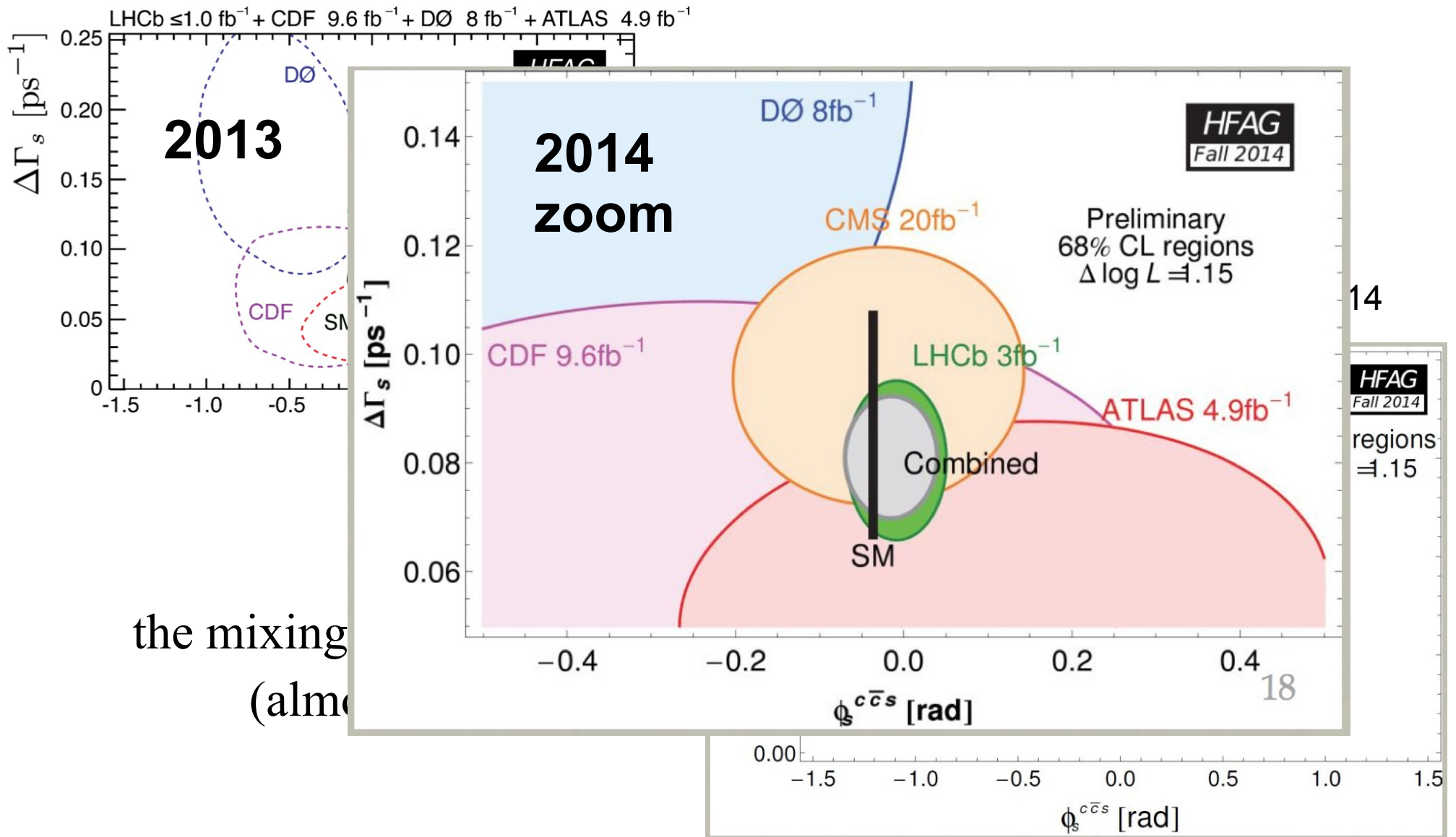


the mixing phase  $2\beta_s$   
 (almost!)

LHCb Implications Workshop Oct 2014



# New Physics in (anti)particle oscillation?





# New Physics in (anti)particle oscillation?

- Example: model independent analysis of the room for new physics in meson mixing (Ligeti et al. 2013):

$$M_{12}^{d,s} = (M_{12}^{d,s})_{\text{SM}} \times (1 + h_{d,s} e^{2i\sigma_{d,s}})$$

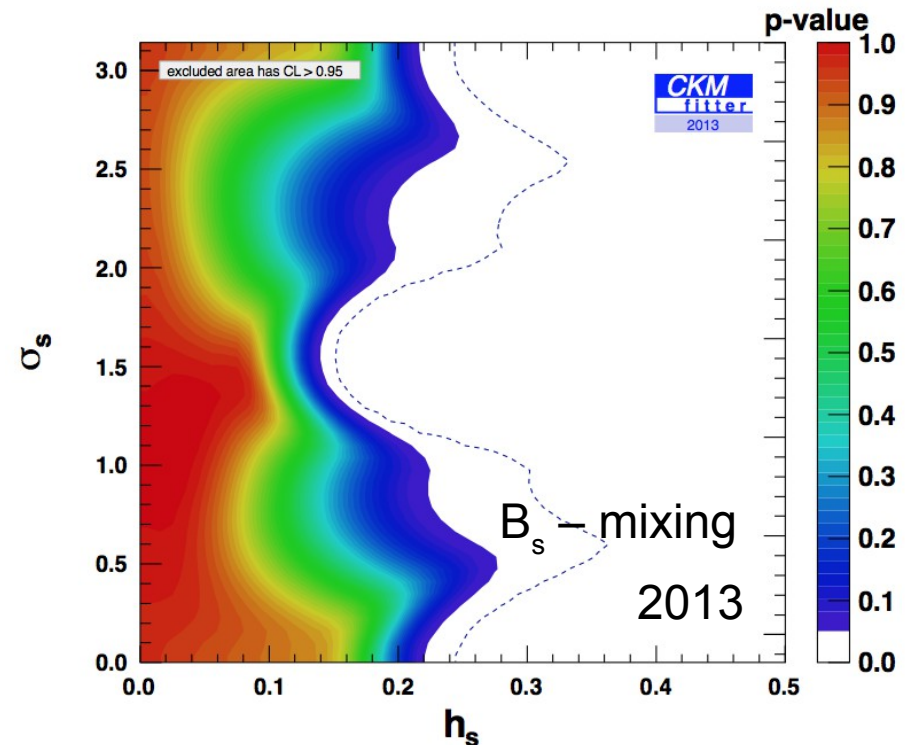
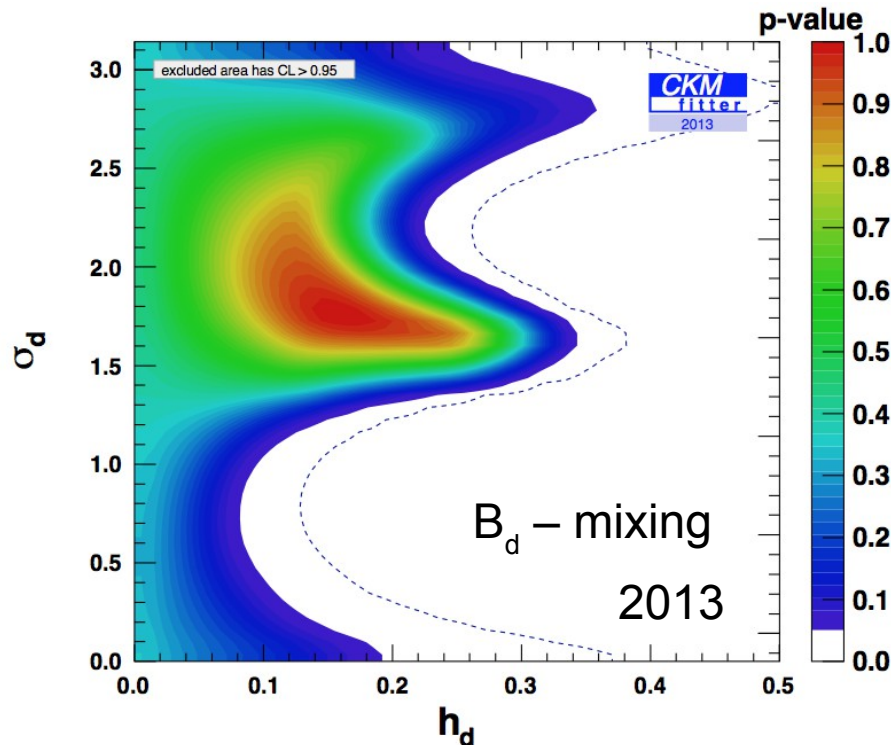
mixing operator

NP magnitude

NP phase

plots assume this value:

$$\gamma = (68^{+8.0}_{-8.5})^\circ$$



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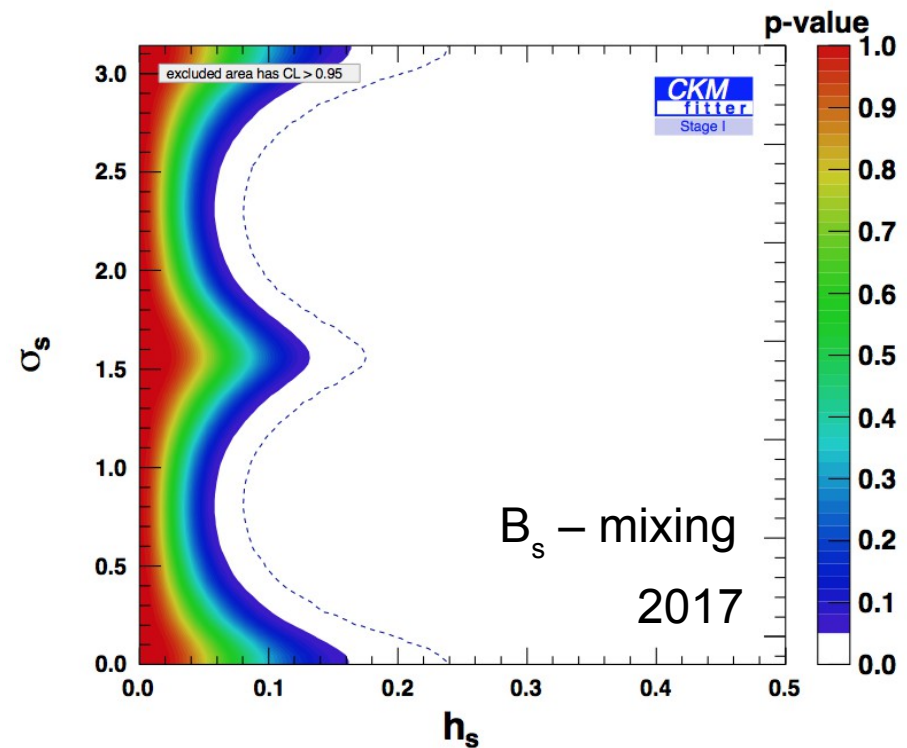
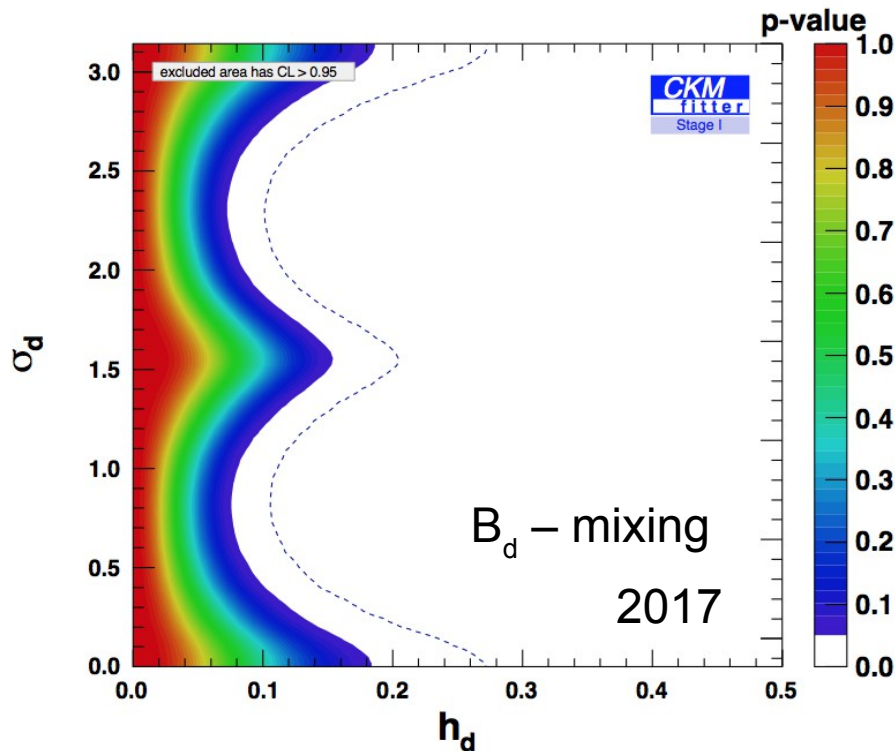
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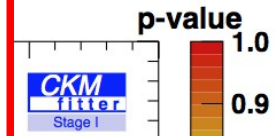
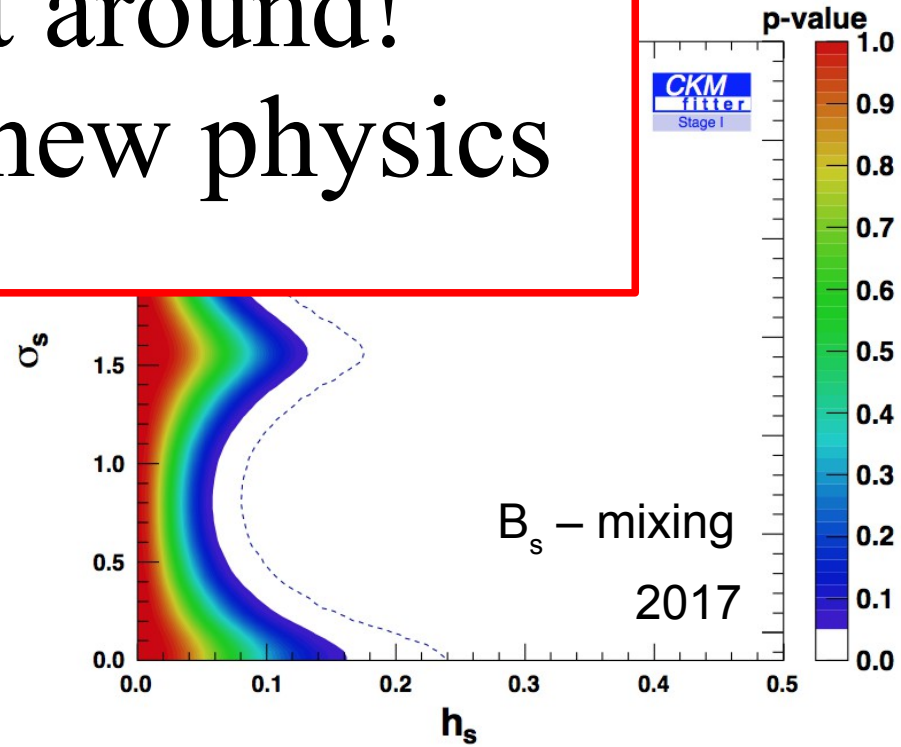
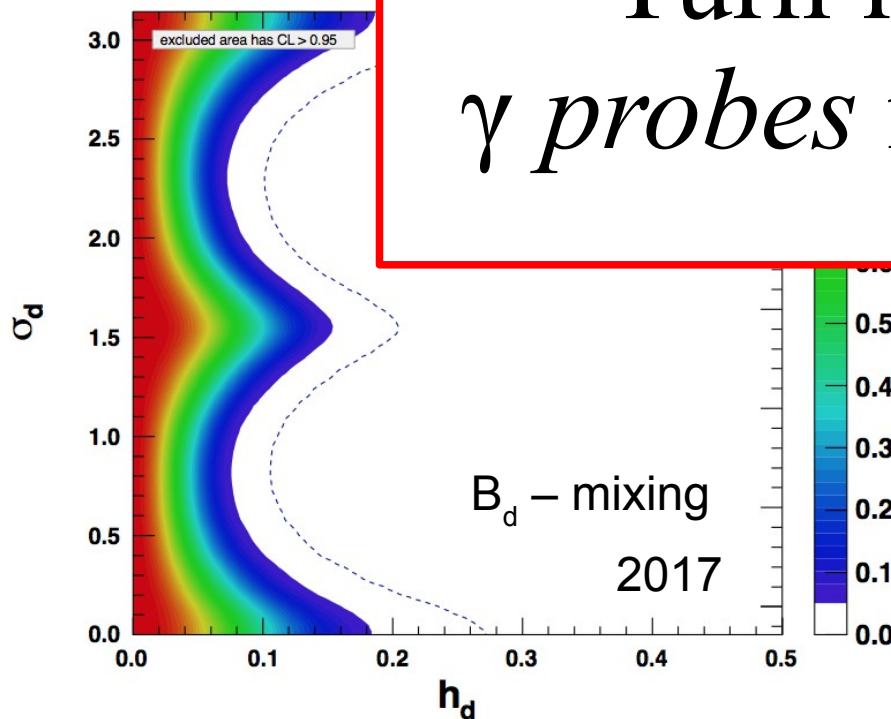
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plots assume this value:

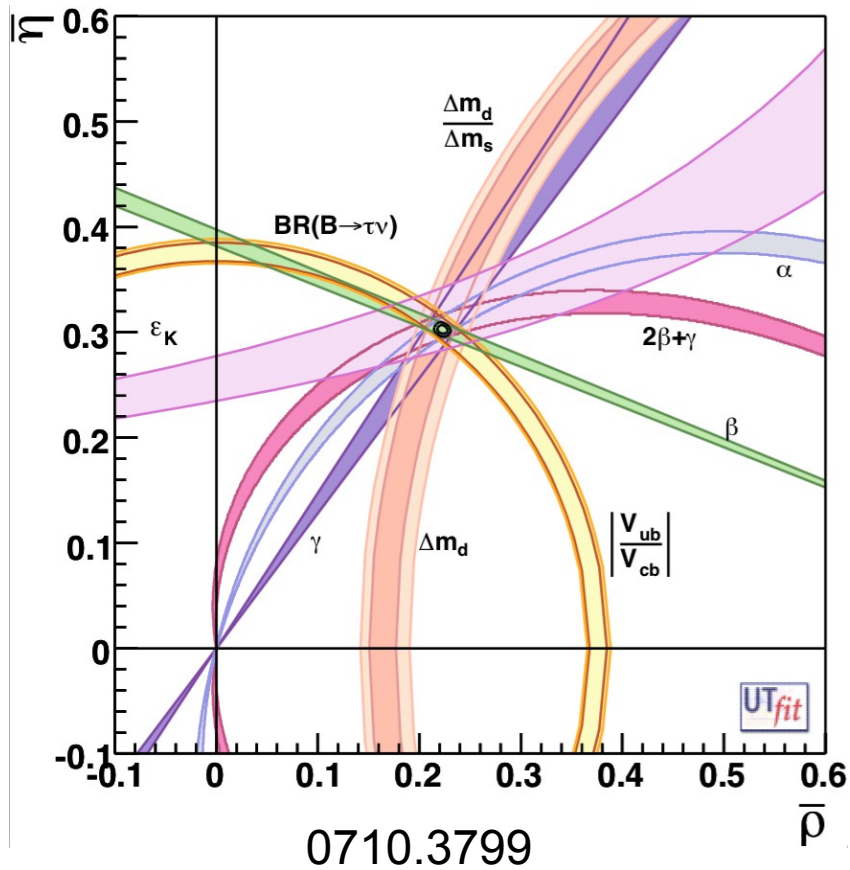
mixing operator

$$\left(68^{+4.0}_{-4.0}\right)^\circ$$

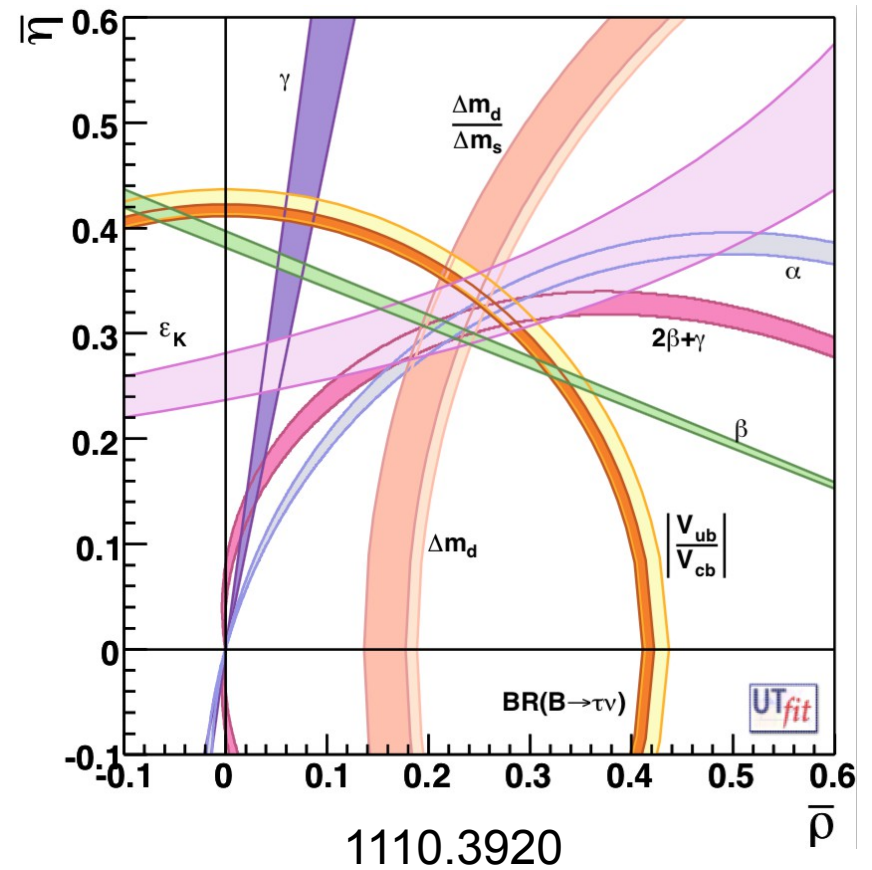
Turn it around!  
 *$\gamma$  probes new physics*



# the ultimate test

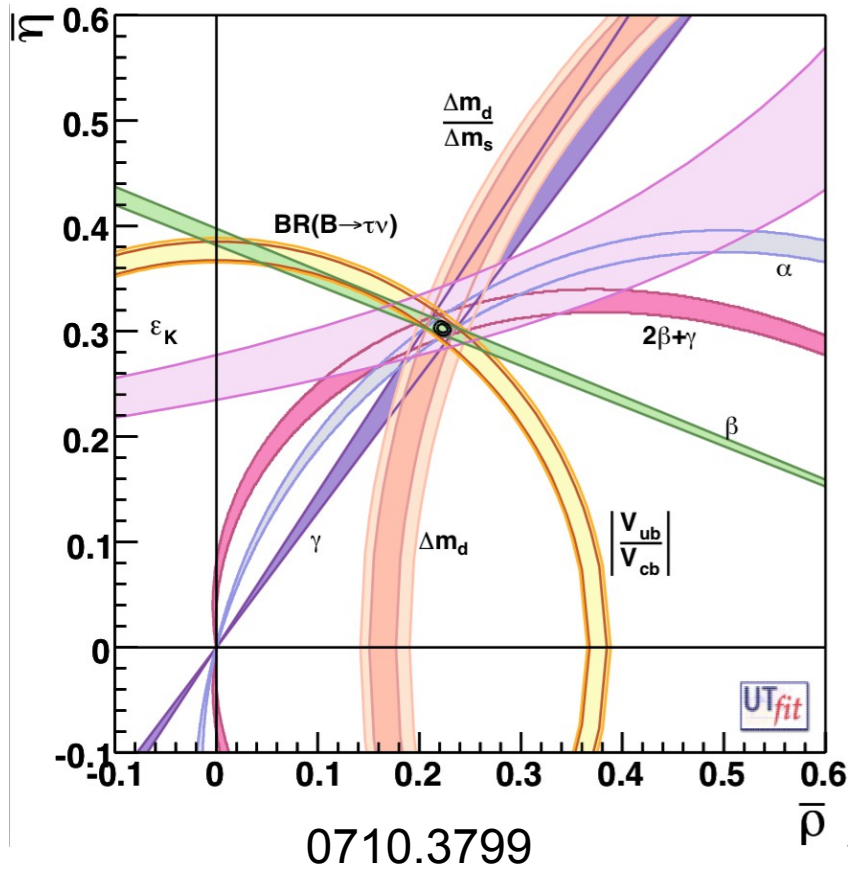


*“the nightmare”*

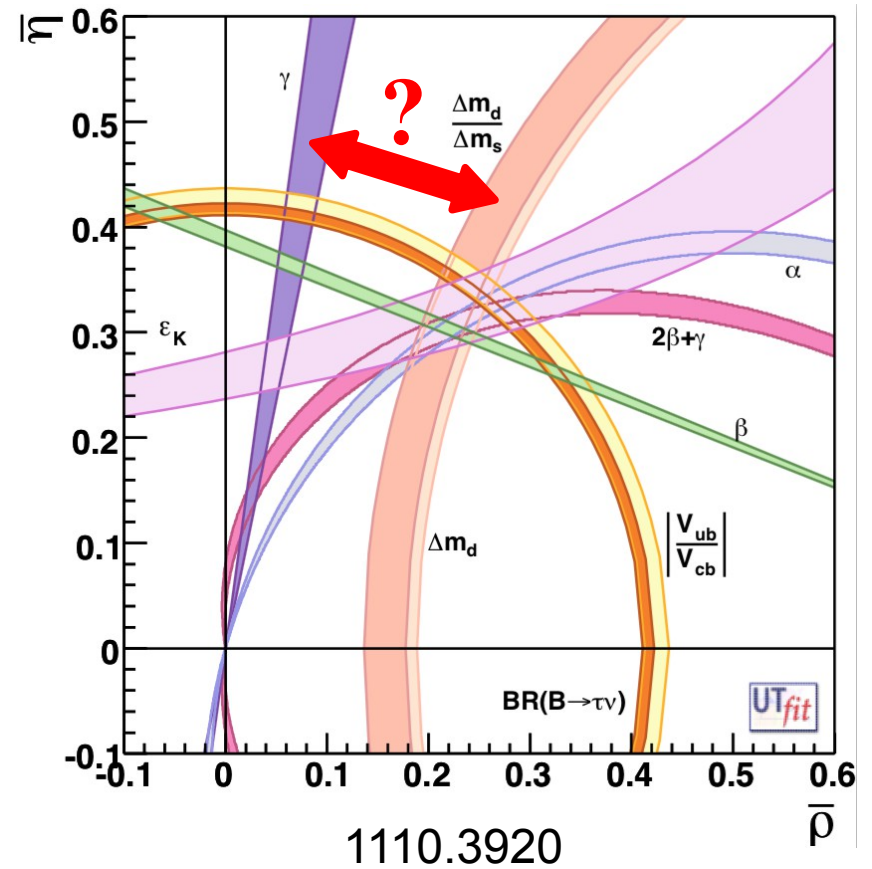


*“the dream”*

# the ultimate test

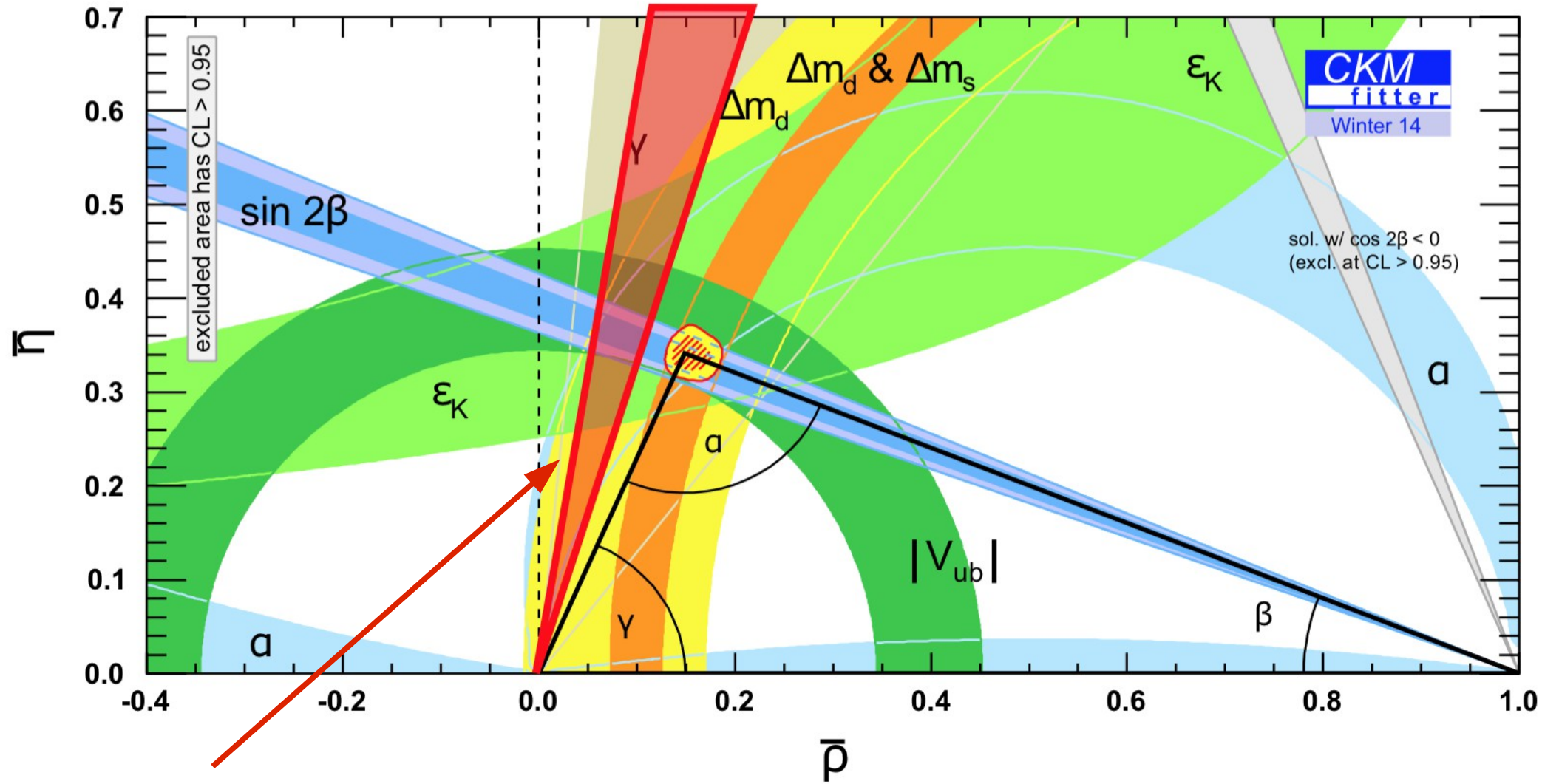


*“the nightmare”*



*“the dream”*

# the ultimate test



LHCb precision in 2020

# $\gamma$ is known very well

theory

- $\gamma$  can be determined entirely from **tree decays**.
  - this is a **unique** property among all CP violation parameters
  - hadronic parameters can all be **determined from the data**
  - **negligible** theoretical uncertainty (Zupan and Brod 2013):

$$\delta\gamma/\gamma \approx \mathcal{O}(10^{-7})$$

JHEP 1401 (2014) 051,  
arXiv:1308.5663.

- $\gamma$  can probe for new physics at extremely **high energy scales** (Zupan)
  - (N)MFV new physics scenarios:  $\sim\mathcal{O}(10^2 \text{ TeV})$
  - gen. FV new physics scenarios:  $\sim\mathcal{O}(10^3 \text{ TeV})$

$\gamma$  is **not** known very well

experiment

it is quite challenging to measure!

- The decay rates are small.

$$\text{BR}(B^- \rightarrow DK^-, D \rightarrow \pi K) \approx 2 \times 10^{-7}$$

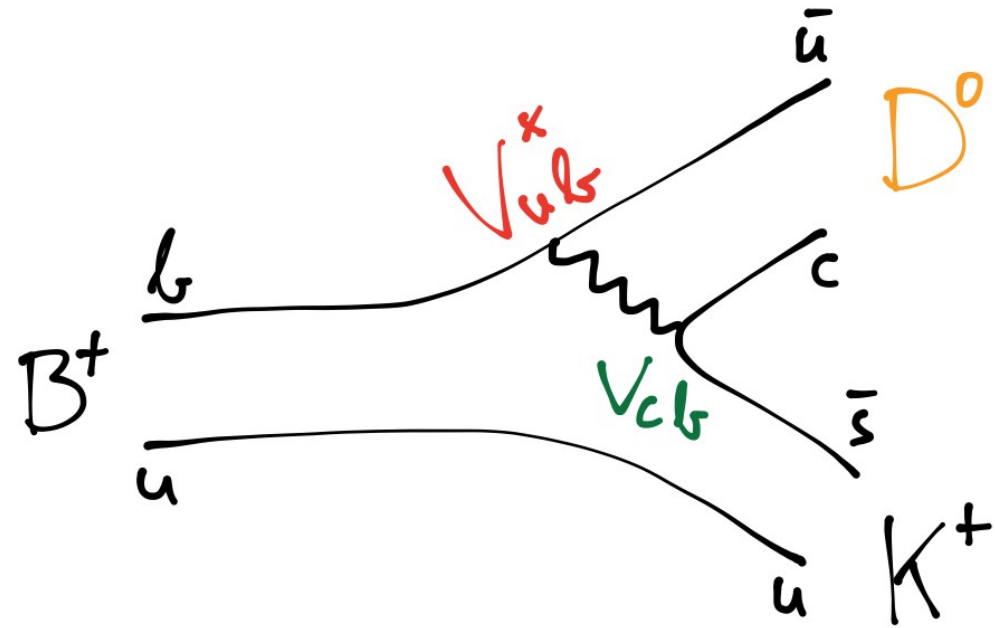
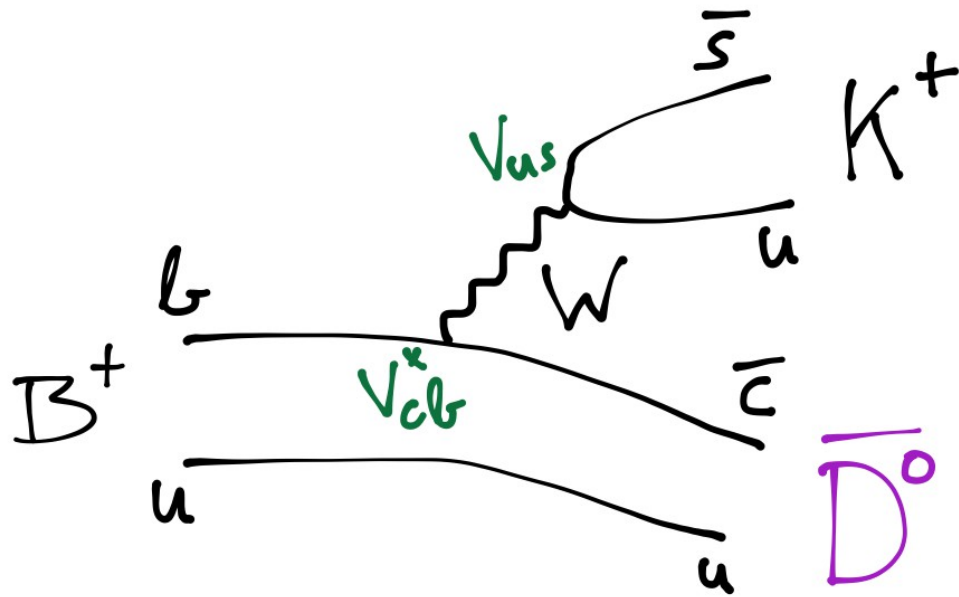
- Low interference effects of typically 10%.
- Fully hadronic decays – hard to trigger on.
- Many channels contain a  $K_S$  in the final state – low efficiency.
- Many channels contain a  $\pi^0$  in the final state – very challenging at LHCb.
- Many decay channels involved.
- Many observables – statistically challenging.



First<sup>(\*)</sup> method to measure  $\gamma$

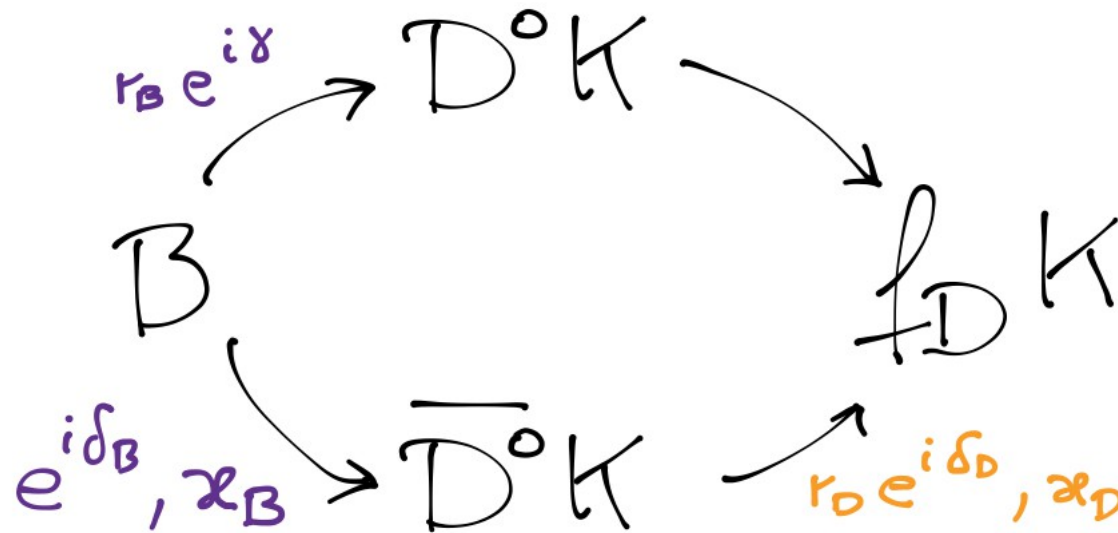
(\*) of this talk

# first method to measure $\gamma$



We need to reconstruct the  $D/\bar{D}$  meson in a final state accessible to both to achieve interference.

# first method to measure $\gamma$



Depending on the final state  $f_D$  the method is called:

**“GLW”**

Gronau, London, Wyler (1991)

Phys. Lett. B253 (1991) 483

Phys. Lett. B265 (1991) 172

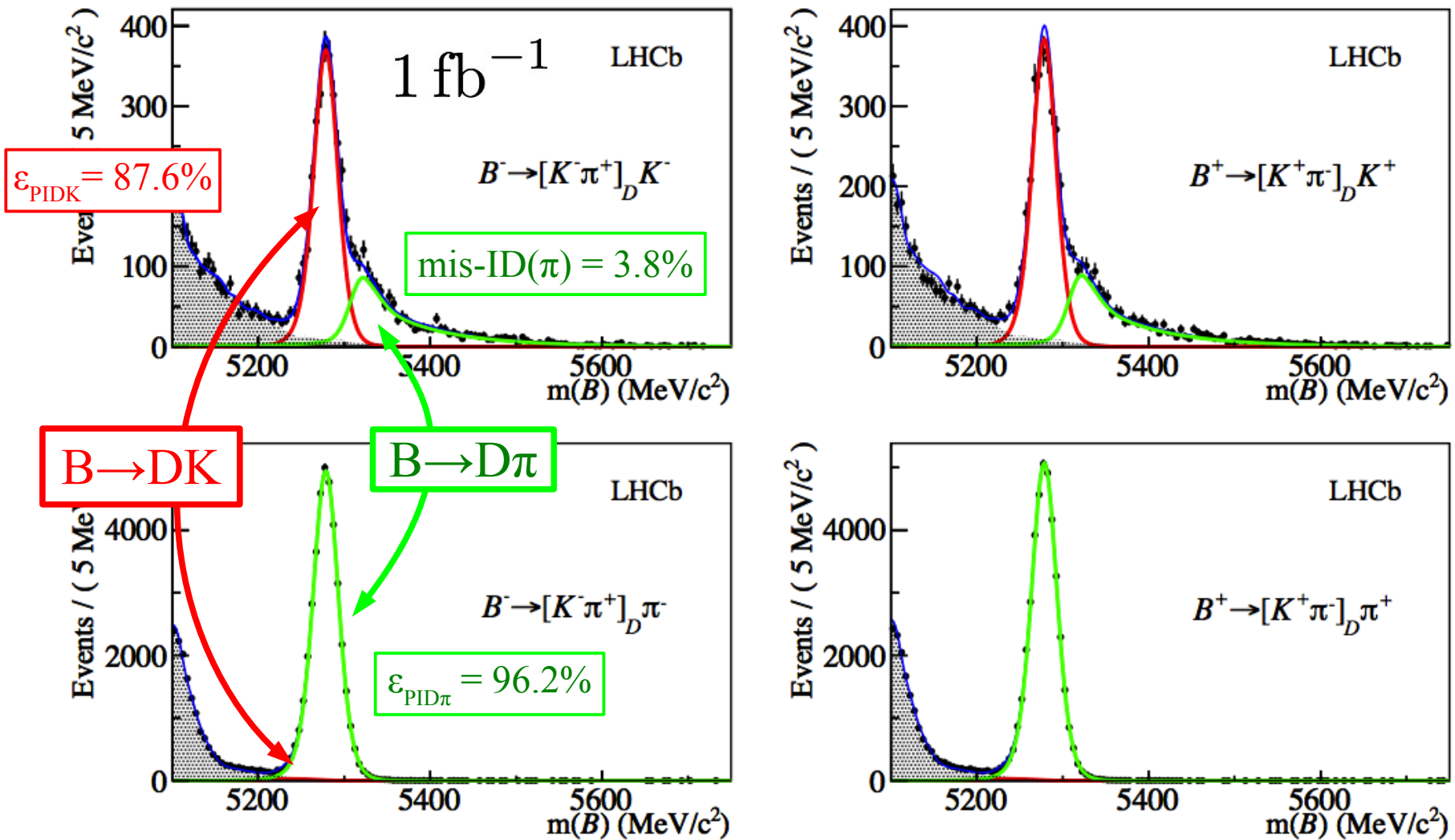
**“ADS”**

Atwood, Dunietz, Soni (1997, 2001)

Phys. Rev. D63 (2001) 036005

Phys. Rev. Lett. 78 (1997) 3257

# $B \rightarrow D(K\pi)h$ : ADS favored mode

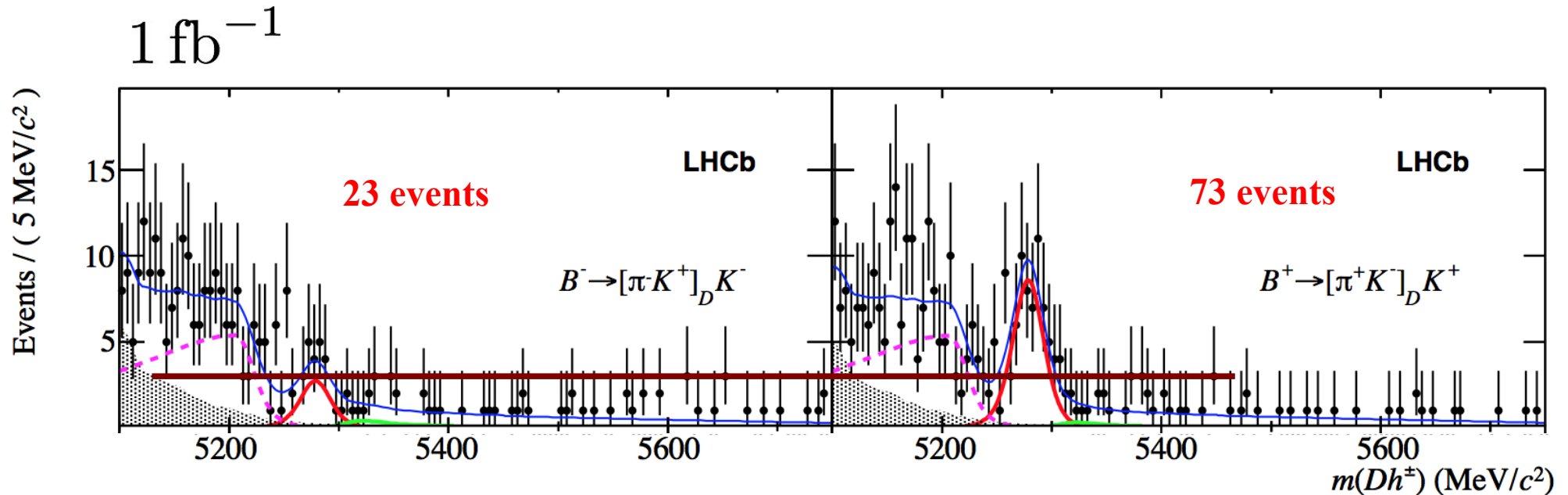


Phys. Lett. B712 (2012) 203, arXiv:1203.3662.

# $B \rightarrow D(\pi K)K$ : ADS **suppressed** mode

$$\mathcal{B}(B^\pm \rightarrow D_{ADS} K^\pm) \approx 2 \cdot 10^{-7} \quad (!!)$$

$$A_{CP} = -0.520 \pm 0.150 \pm 0.021$$



Phys. Lett. B712 (2012) 203, arXiv:1203.3662.

# first method to measure $\gamma$

- Define observables as **yield ratios** (many systematics cancel).
- Charge **asymmetries**:

$$A_h^f = \frac{\Gamma(B^- \rightarrow [f]_D h^-) - \Gamma(B^+ \rightarrow [f]_D h^+)}{\Gamma(B^- \rightarrow [f]_D h^-) + \Gamma(B^+ \rightarrow [f]_D h^+)}$$

- **Kaon/pion** ratio:

$$R_{K/\pi}^f = \frac{\Gamma(B^\pm \rightarrow [f]_D K^\pm)}{\Gamma(B^\pm \rightarrow [f]_D \pi^\pm)}$$

Form a system of equations.  
Need more observables than  
parameters!

→ many different D decays

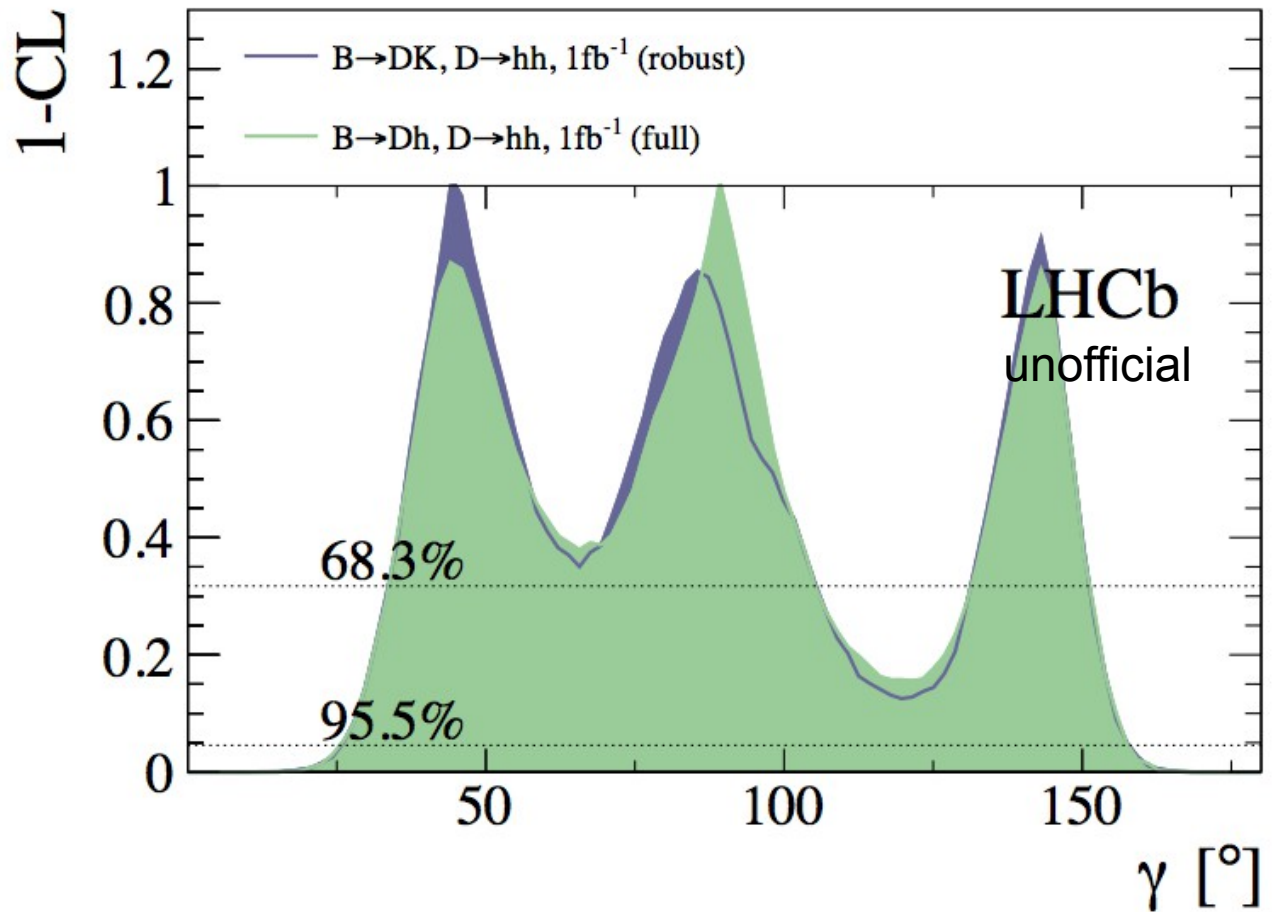
- **Suppressed/favored** decay ratio (2-body example):

$$R_h^\pm = \frac{\Gamma(B^\pm \rightarrow [\pi^\pm K^\mp]_D h^\pm)}{\Gamma(B^\pm \rightarrow [K^\pm \pi^\mp]_D h^\pm)}$$

$$= r_B^2 + r_D^2 + 2r_B r_D \cos(\underbrace{\pm\gamma + \delta_B + \delta_D}_{\text{strong phase diff.}})$$

strong phase diff.

# first method to measure $\gamma$



# Second method to measure $\gamma$



# second method: “GGSZ”

- **Idea:** perform an GLW/ADS type analysis in every bin of the  $D$  decay phase space
- GGSZ uses  $B \rightarrow DK$  followed by self-conjugate three-body final states

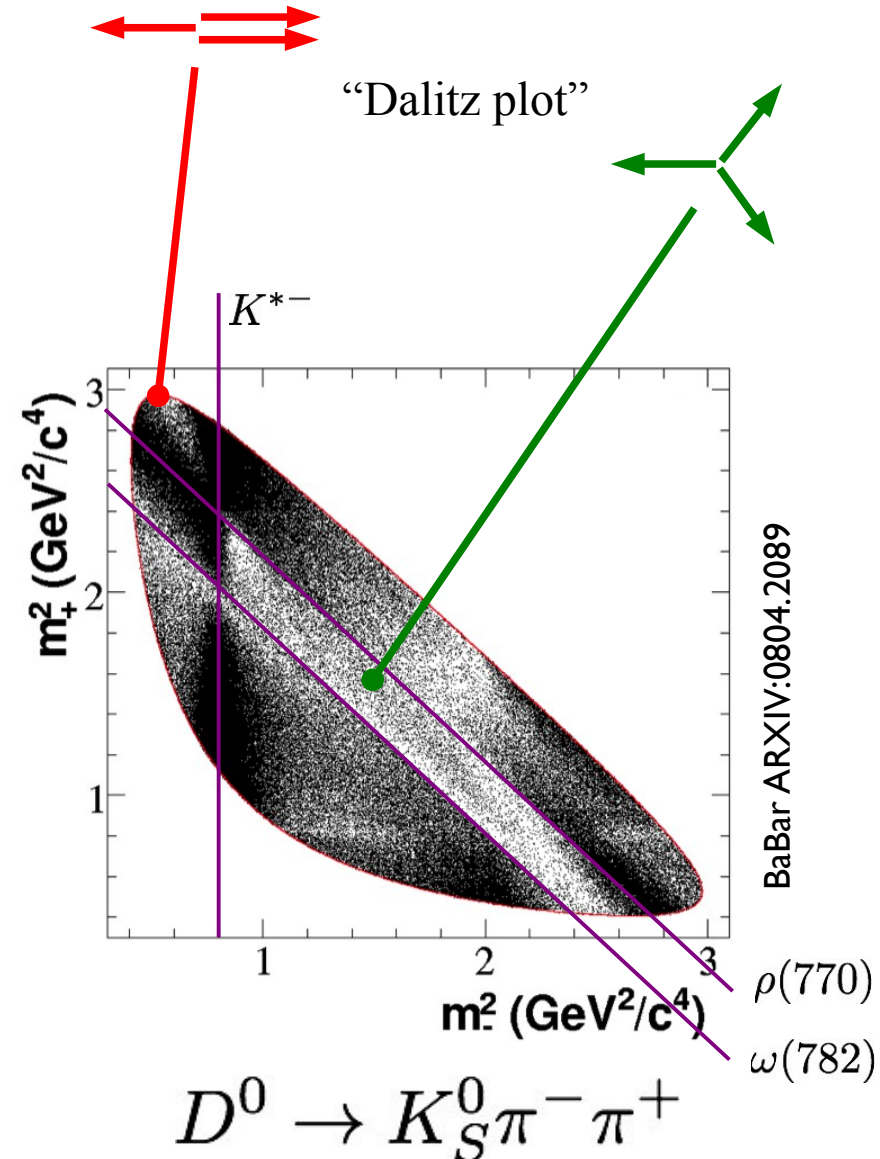
$$D^0 \rightarrow K_S^0 \pi^- \pi^+$$

$$D^0 \rightarrow K_S^0 K^- K^+$$

- Most precise at B-factories.
- Observables: the “cartesian coordinates”

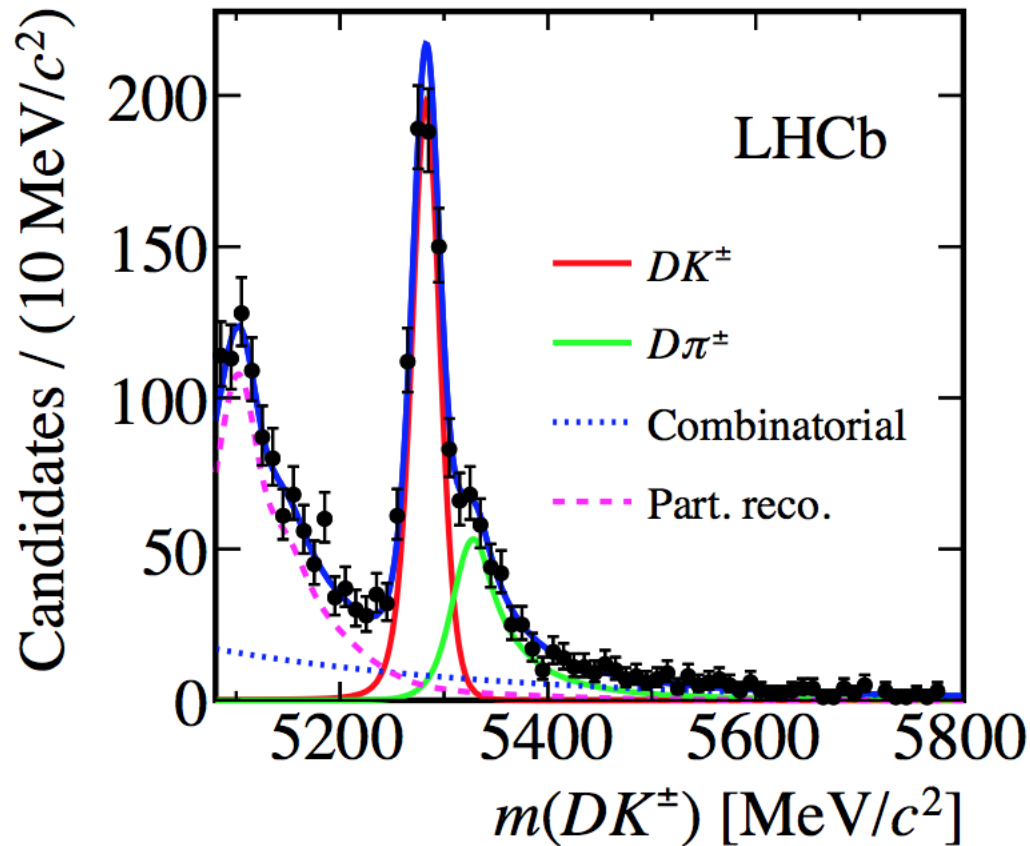
$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

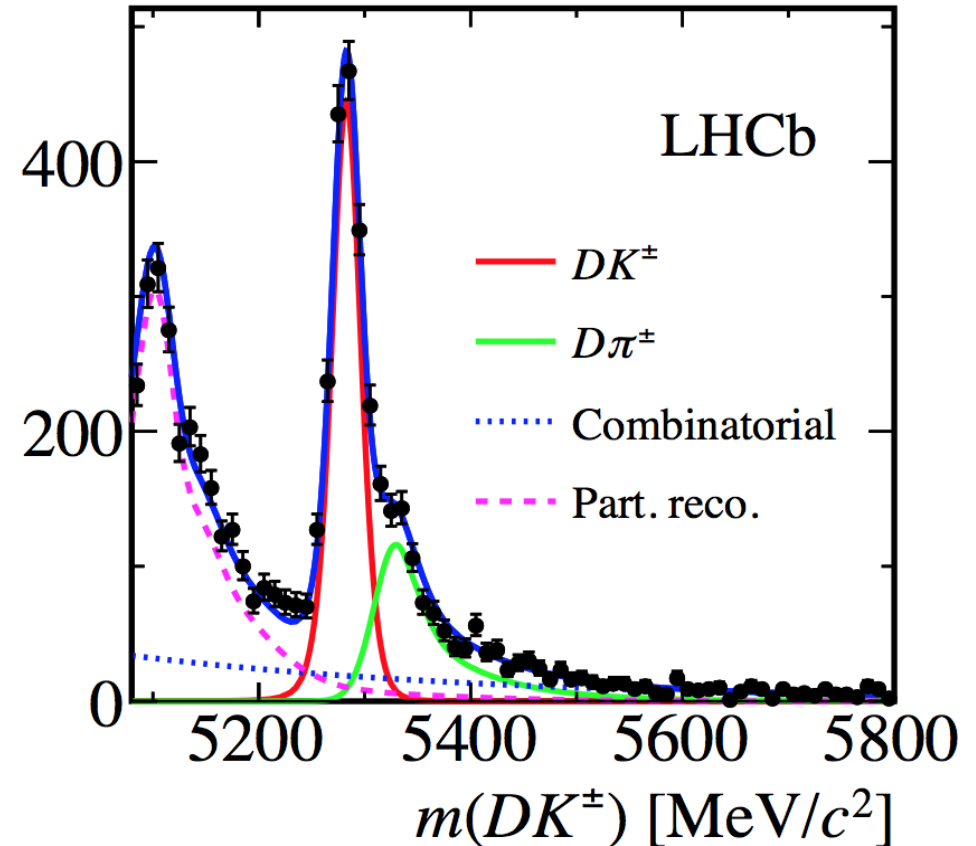


# second method: “GGSZ”

“long  $K_S$ ” (N~700)

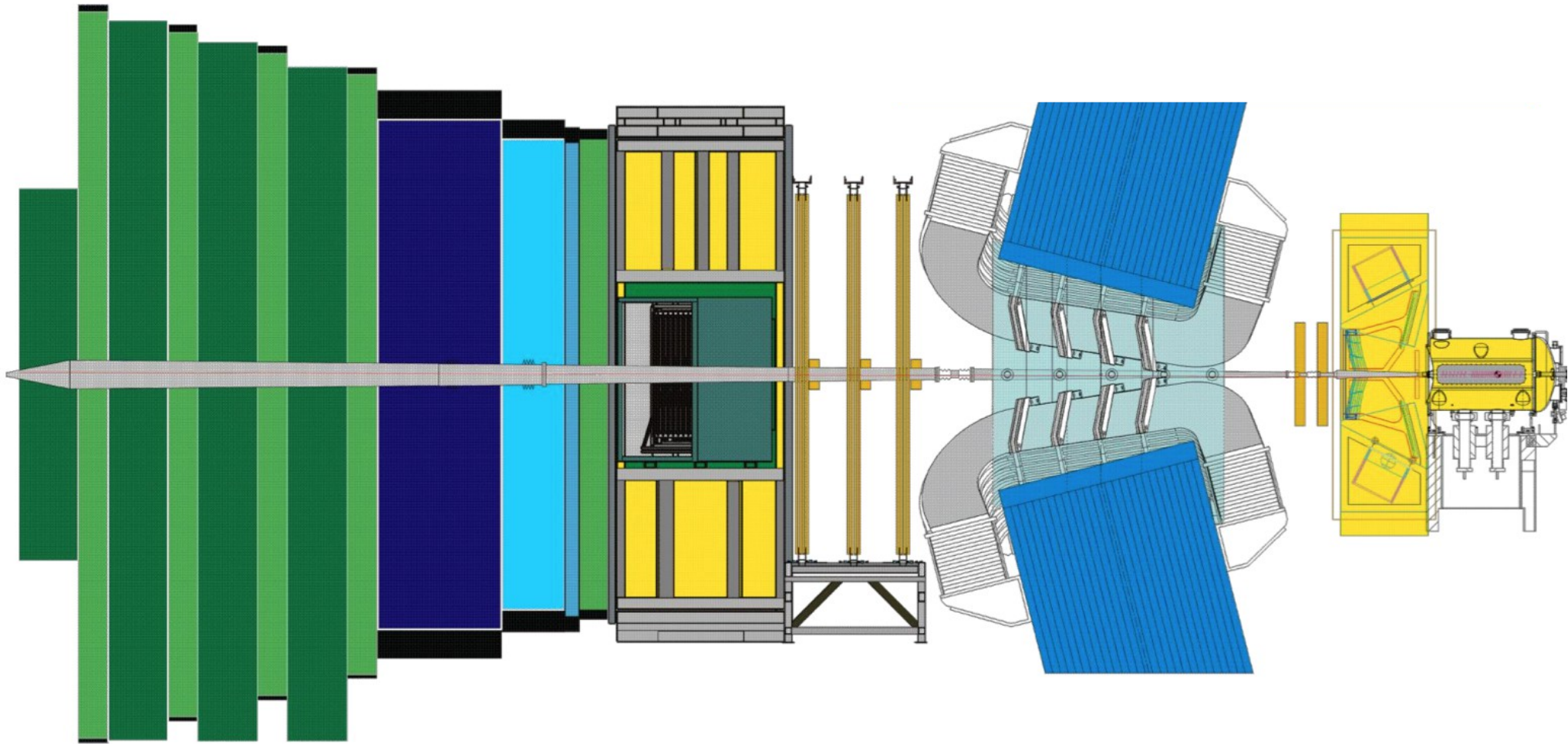


“downstream  $K_S$ ” (N~1600)

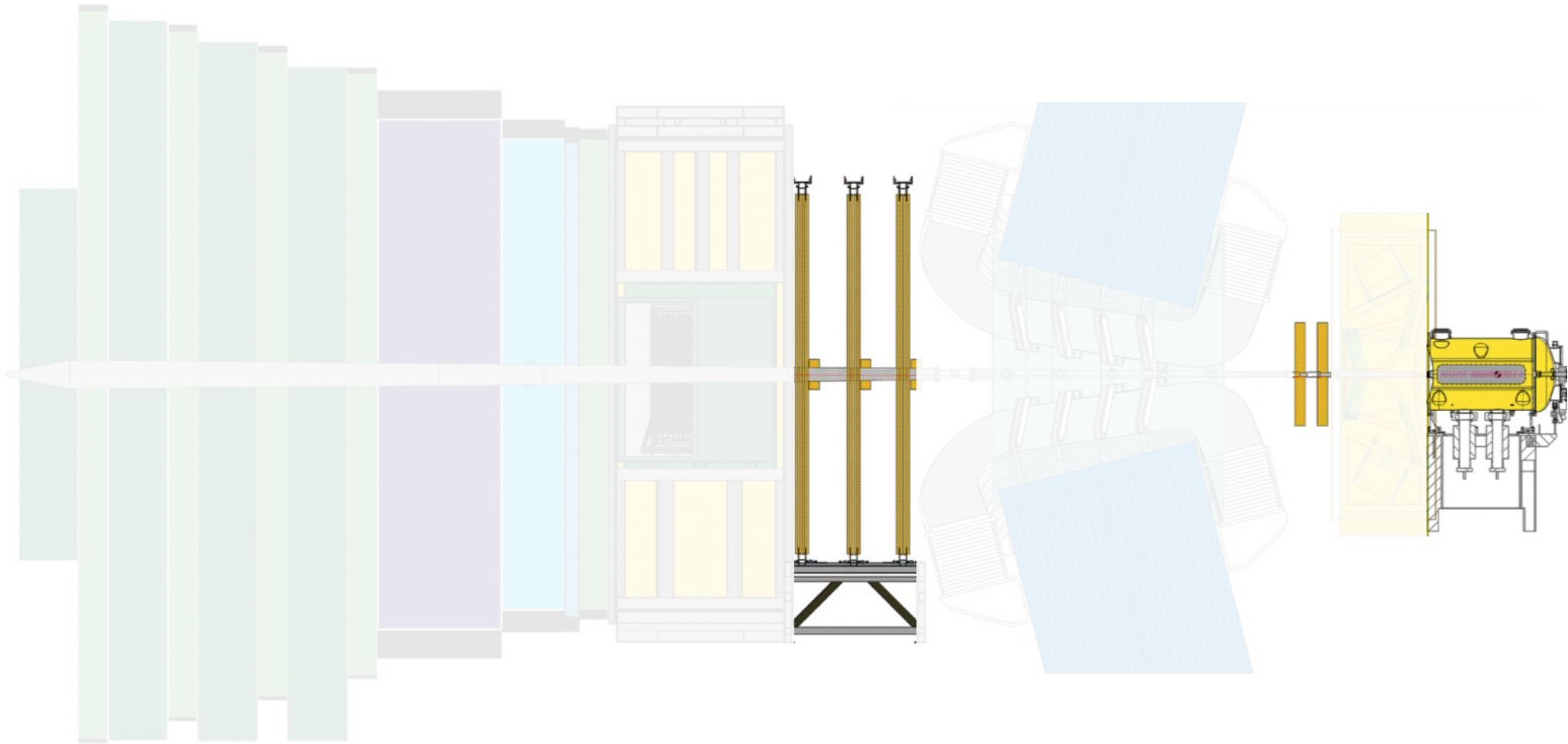


JHEP 1410 (2014) 97, arXiv:1408.2748.

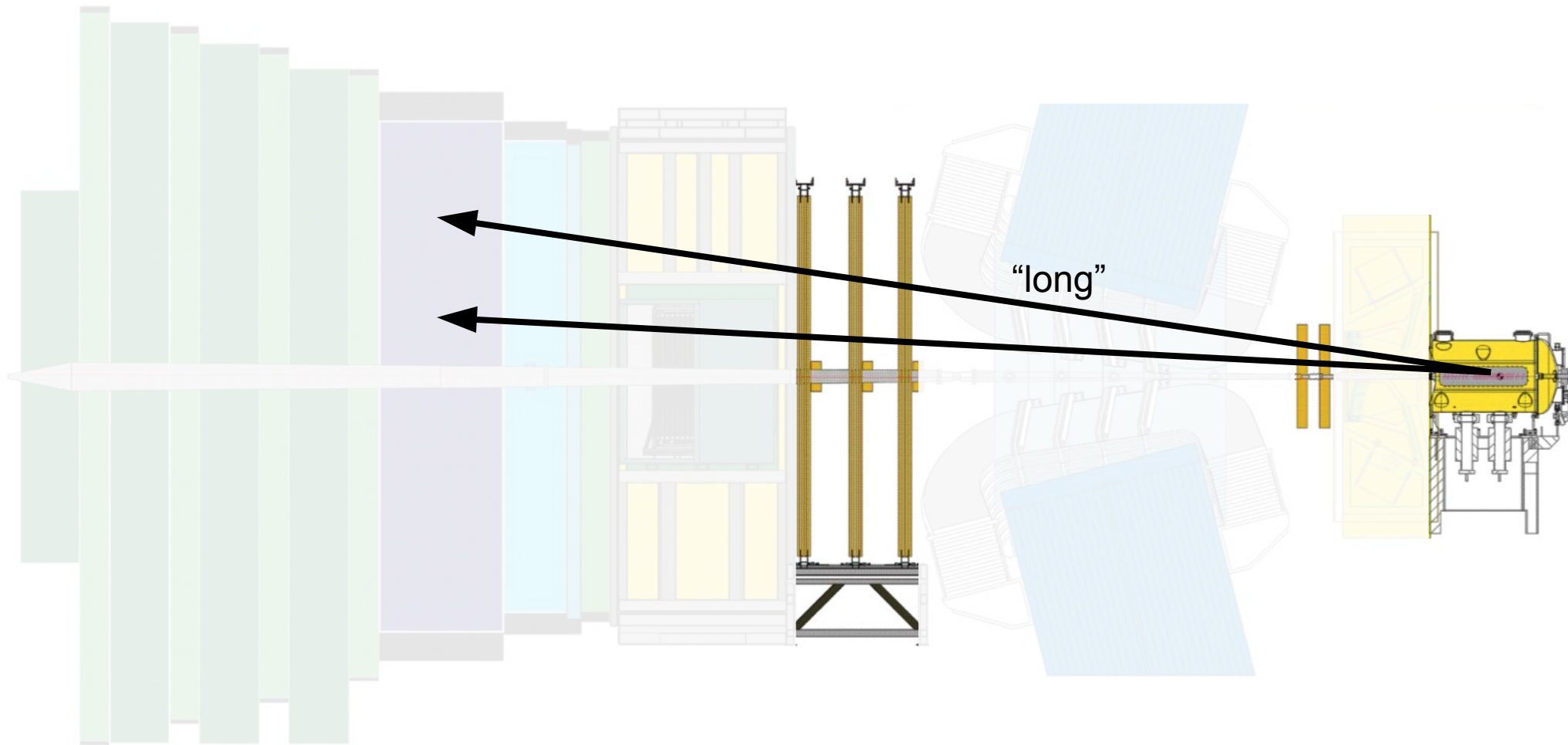
# $K_S$ reconstruction



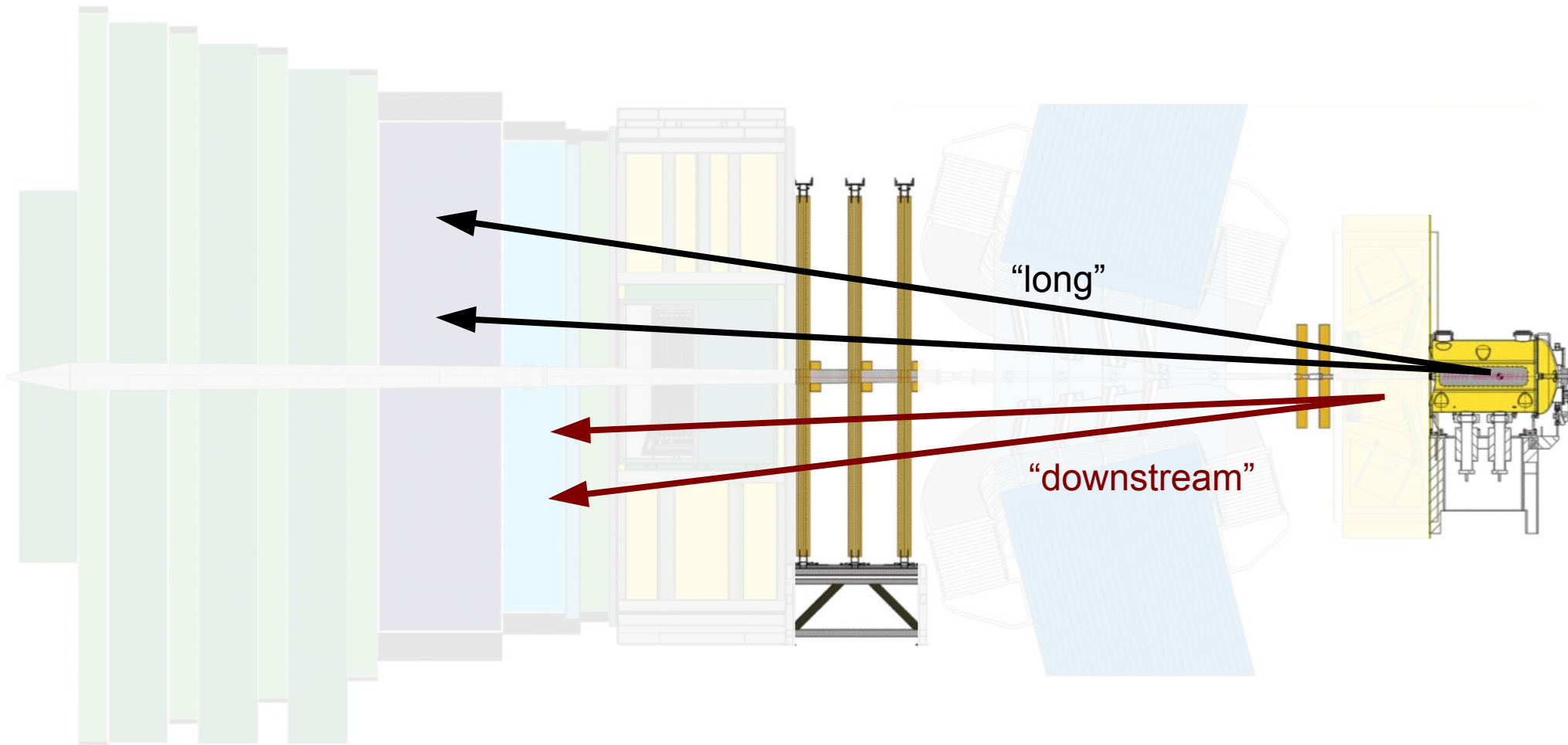
# $K_S$ reconstruction



# $K_S$ reconstruction

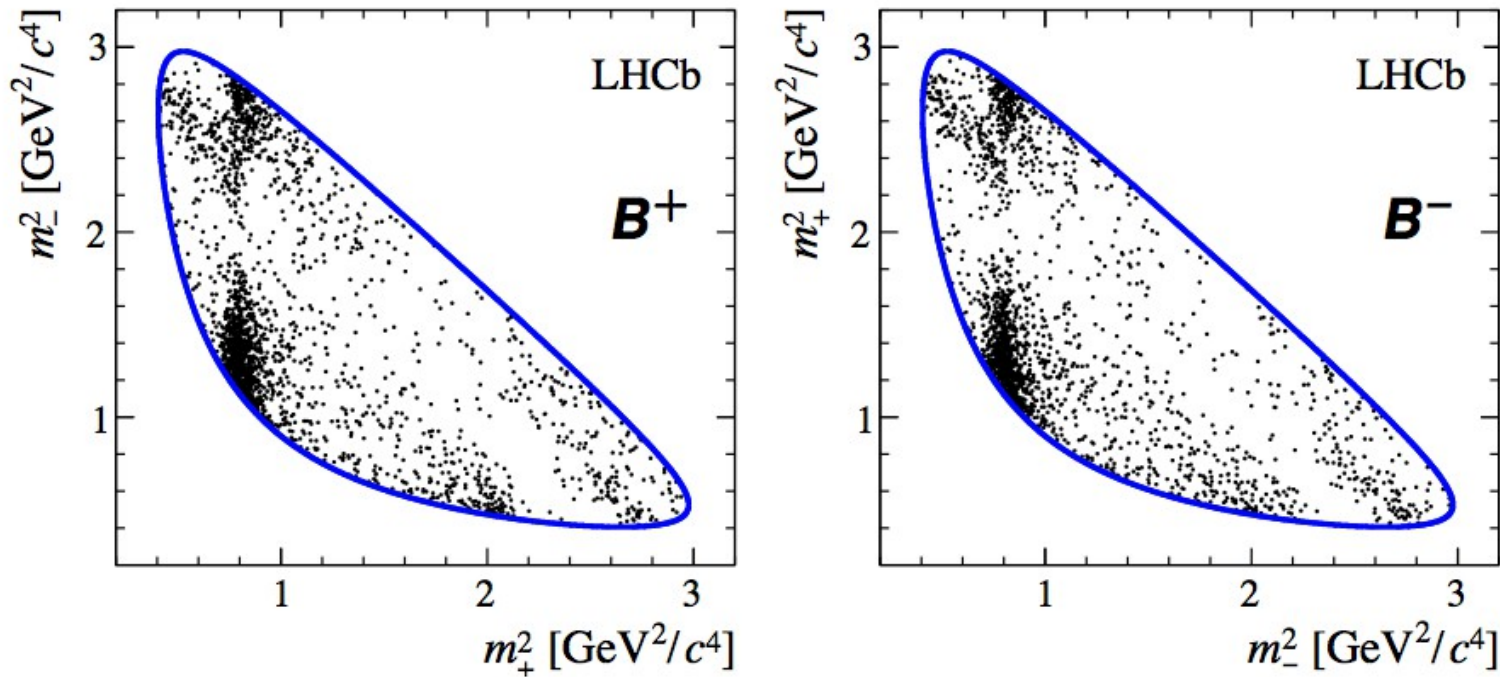


# $K_S$ reconstruction



# second method: “GGSZ”

$K_S^0\pi^+\pi^-$  data ( $\sim 2600$  candidates):

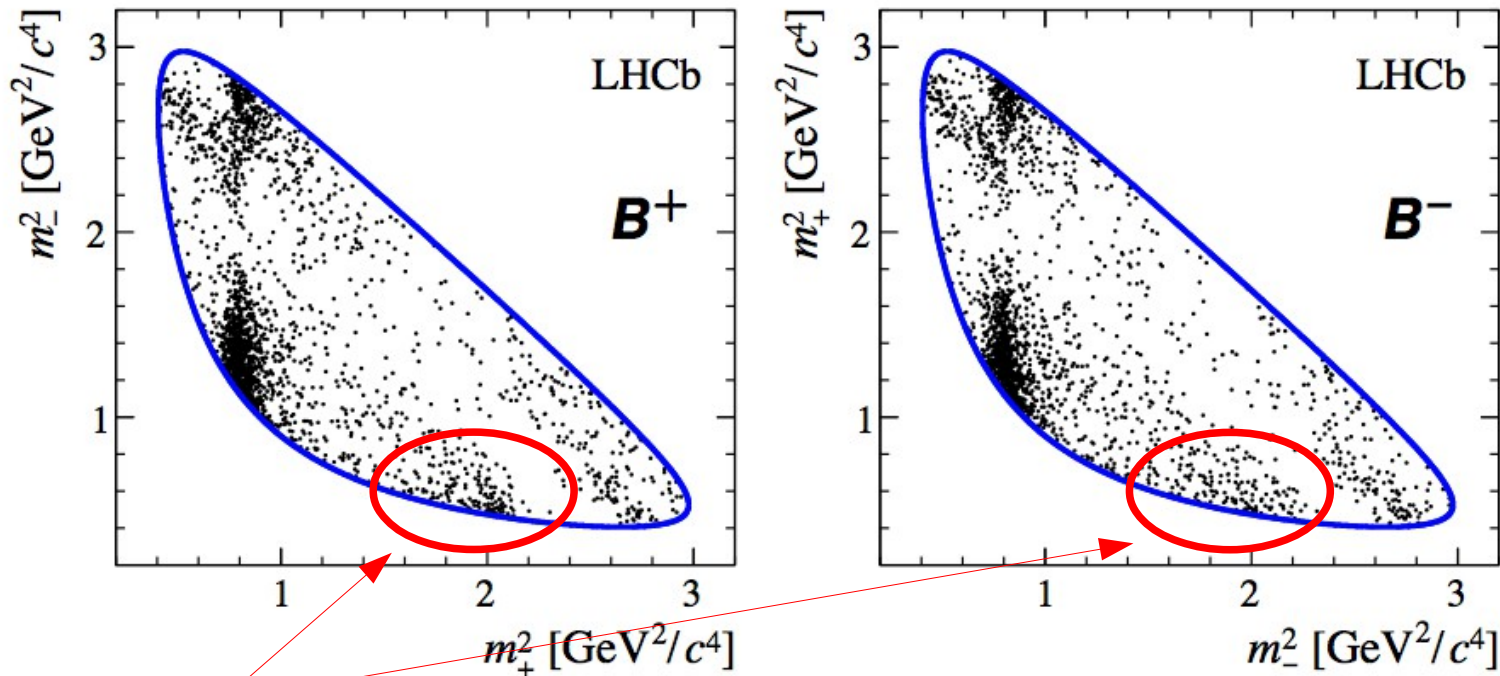


$$m_{\pm}^2 \equiv m^2(K_S^0\pi^{\pm})$$

JHEP 1410 (2014) 97, arXiv:1408.2748.

# second method: “GGSZ”

$K_S^0\pi^+\pi^-$  data ( $\sim 2600$  candidates):



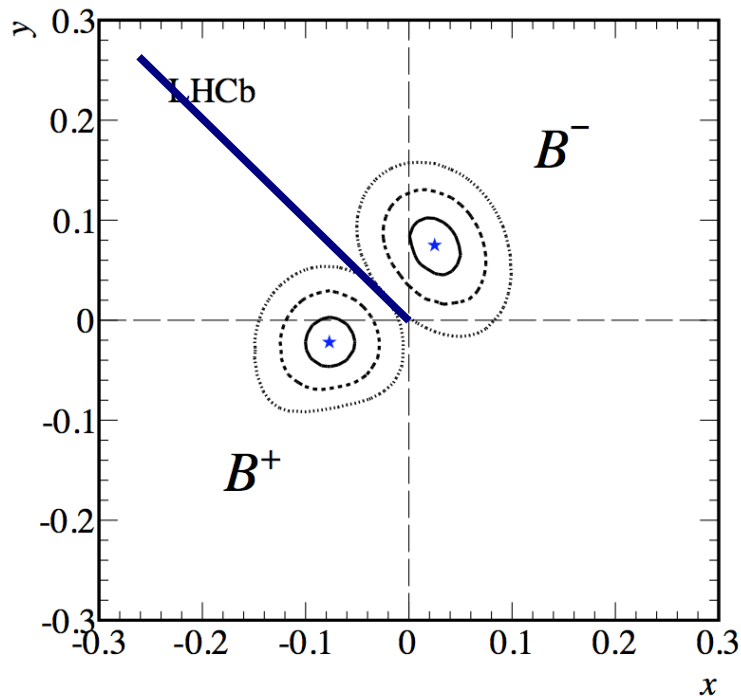
CP violation (?)

$$m_{\pm}^2 \equiv m^2(K_S^0\pi^{\pm})$$

JHEP 1410 (2014) 97, arXiv:1408.2748.



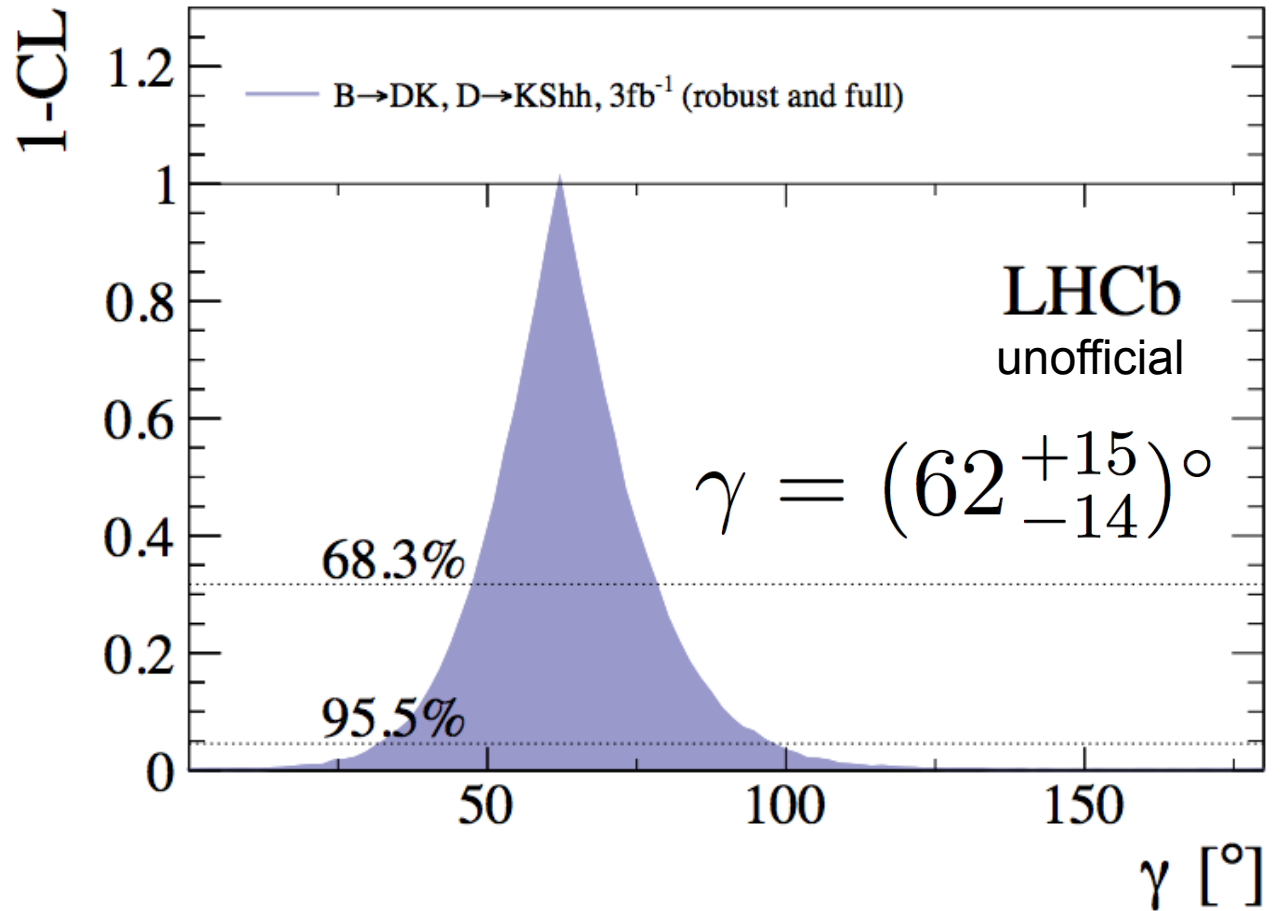
# second method: “GGSZ”



$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

JHEP 1410 (2014) 97,  
arXiv:1408.2748.



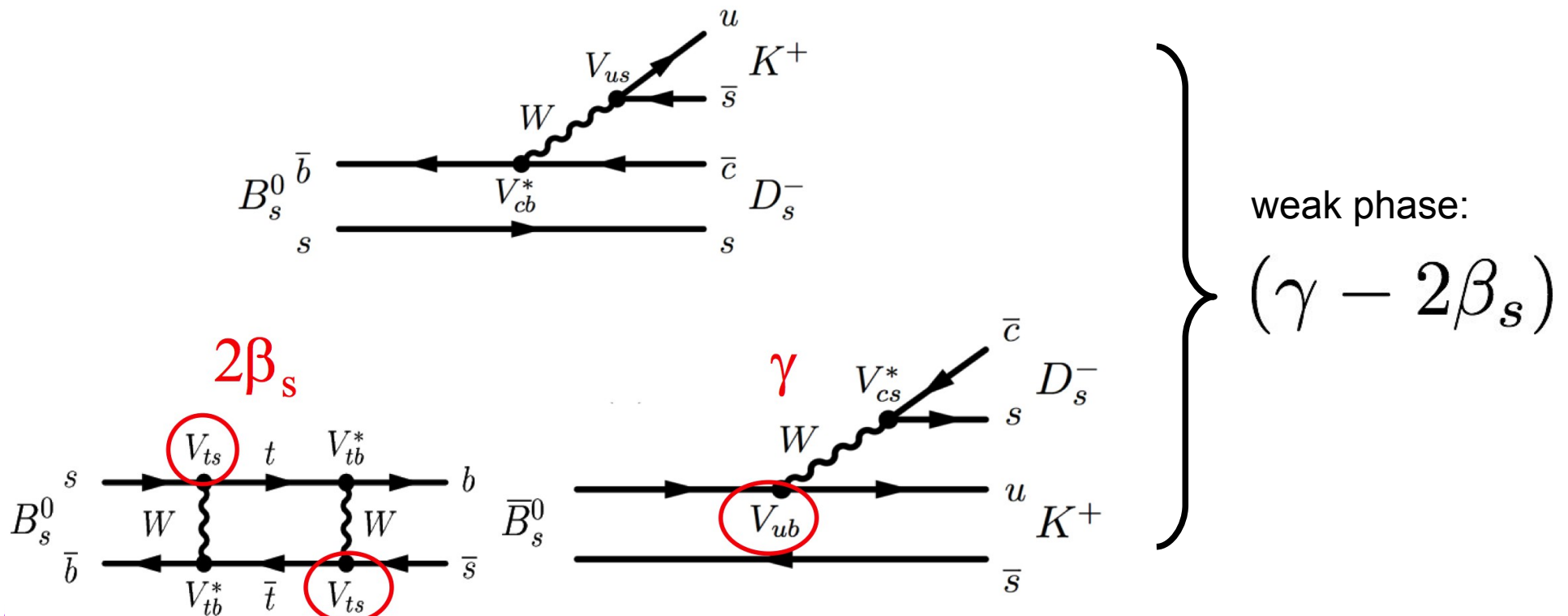
World's most precise *single* measurement!

# Third method to measure $\gamma$

# third method: time-dependent

Using charged-particle final states, interference is achieved through **mixing**.

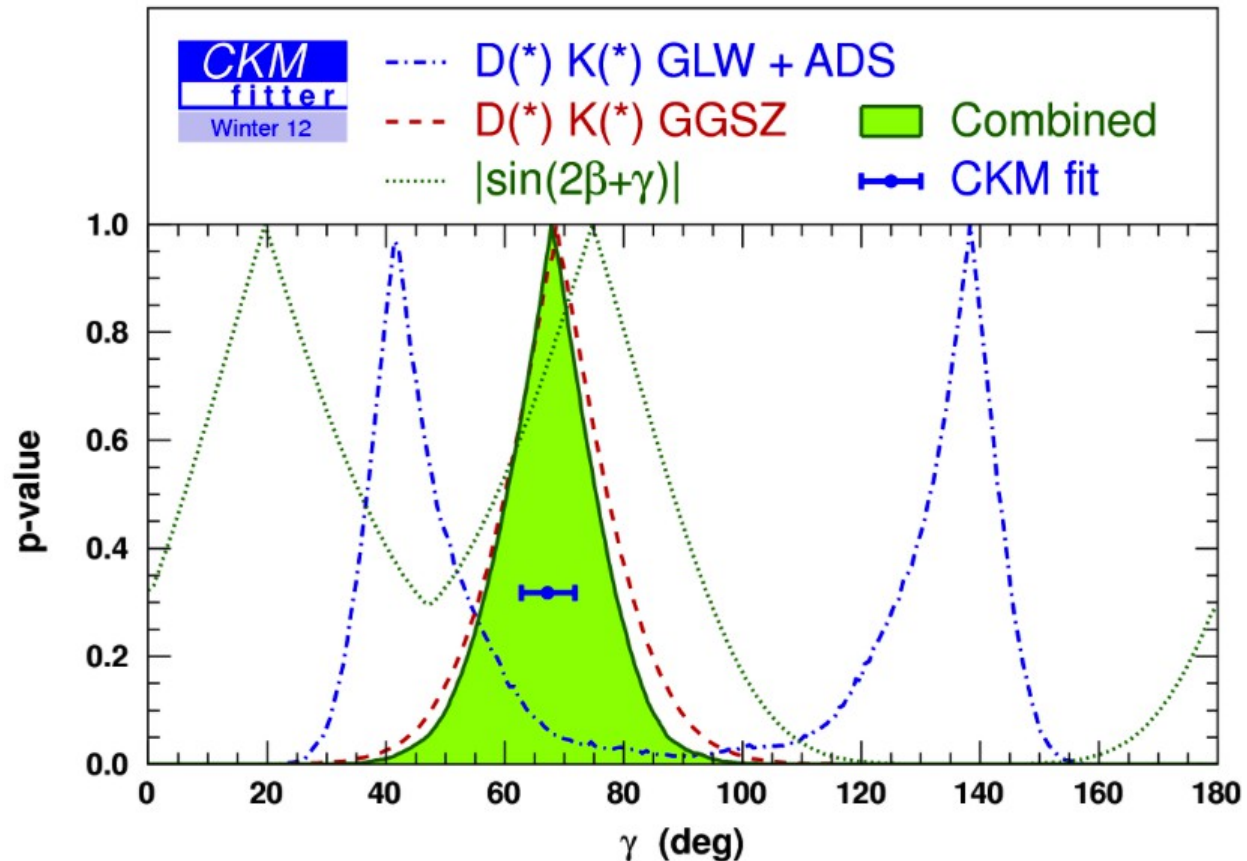
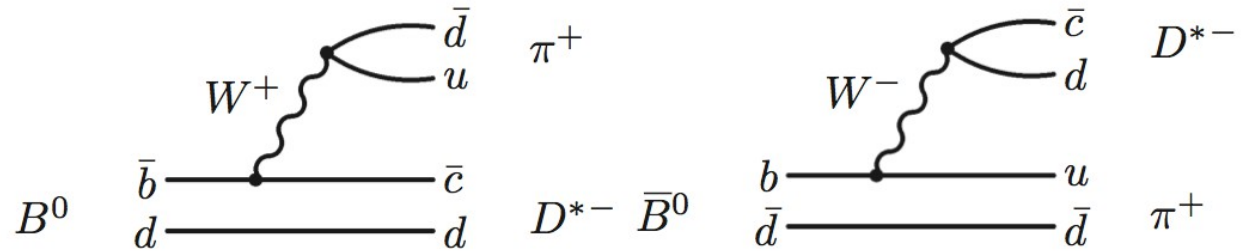
$$B_s \rightarrow D_s^\pm K^\mp$$



Phys. Lett. B387 (1996) 361, arXiv:hep-ph/9605221.

# third method: time-dependent

B-factories performed such measurements with  $B^0 \rightarrow D^+ \pi^-$ , constraining  $\sin(2\beta+\gamma)$



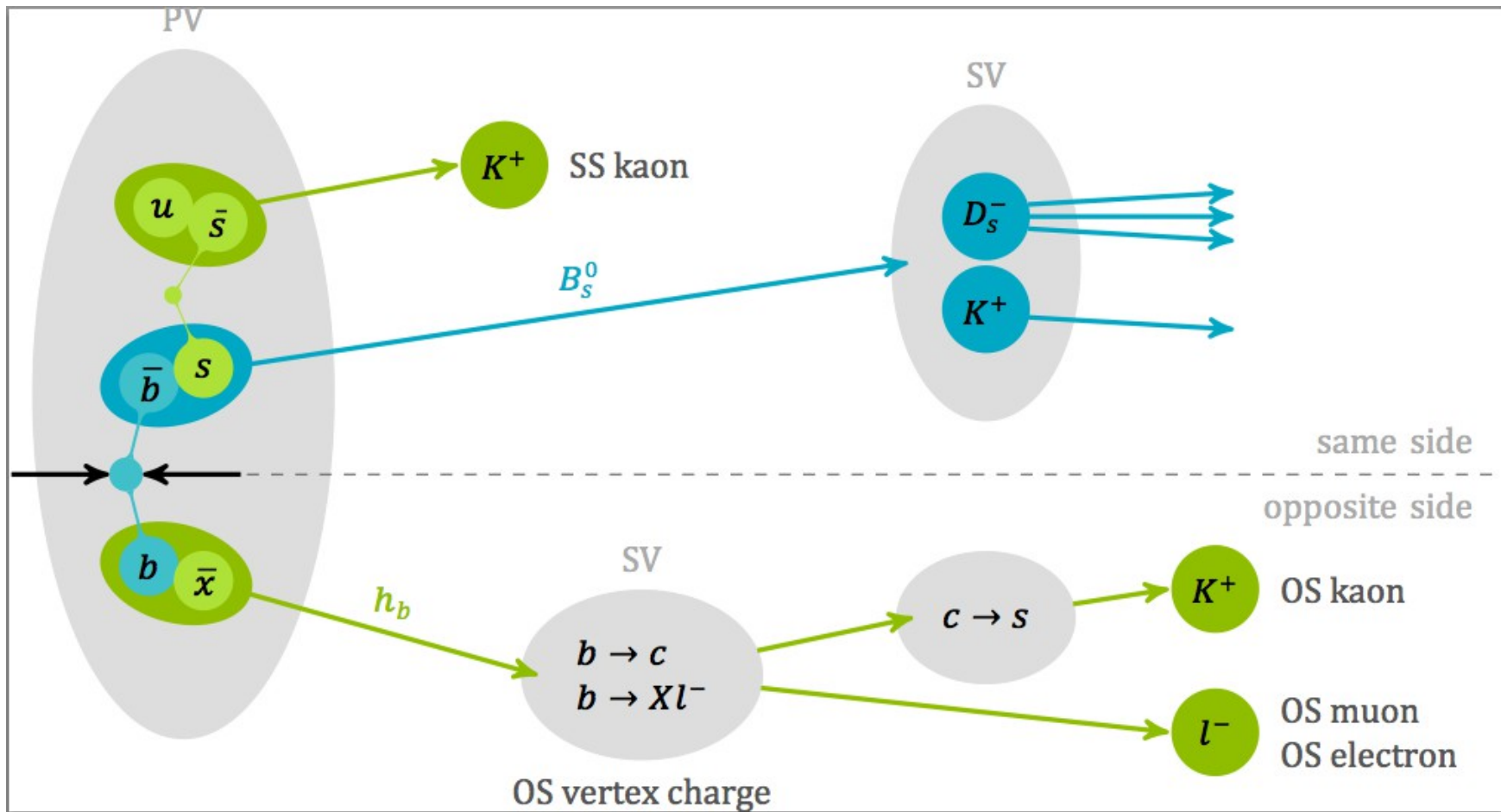
BaBar 2005,  
hep-ex/0504035

Belle 2011,  
arXiv:1102.0888

# third method: time-dependent

- $B_s$  is much better suited than  $B_d$  !
- expected **large interference** effects of  $\sim 40\%$
- finite decay width difference adds sensitivity:  
 $\Delta\Gamma_s = 0.091 \pm 0.011 \text{ ps}^{-1}$  (HFAG fall 2012)
  
- It is still a pure, **clean tree decay**.
  
- **Only possible at LHCb:**
  - $B_s$  statistics: large b-quark production cross section
  - fully hadronic: full real-time reconstruction on trigger level
  - time resolution:  $\sigma(t) \sim 50\text{fs}$
  - flavor tagging: distinguish  $B_s$  from anti- $B_s$  (tagging power  $\sim 5\%$ )

# flavor tagging

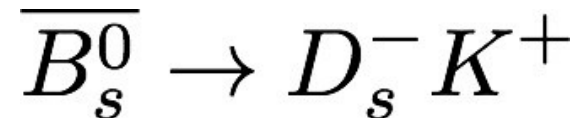
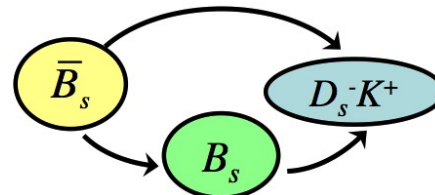
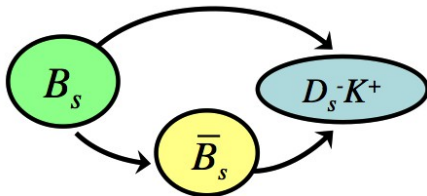
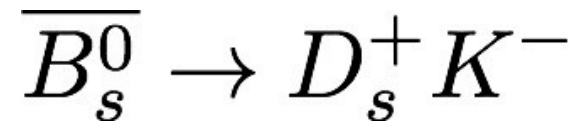
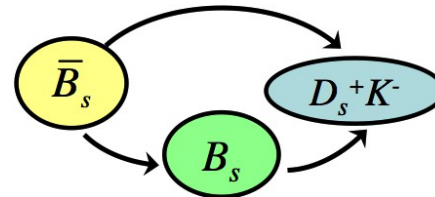
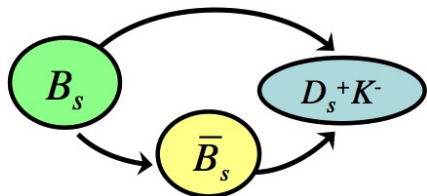


$$\epsilon_{\text{eff}} = \epsilon(1 - 2\omega)^2$$

$$\sigma \propto 1/\sqrt{\epsilon_{\text{eff}}}$$

$$\epsilon_{\text{eff}} = 5.07\% \text{ (for } B_s \rightarrow D_s K \text{)}$$

# third method: time-dependent



each has their own time dependence ...

# third method: time-dependent

$$\frac{d\Gamma_{B_s^0 \rightarrow f}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2}|A_f|^2(1 + |\lambda_f|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t) \right] \quad (1)$$

$$\frac{d\Gamma_{\bar{B}_s^0 \rightarrow f}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2}|A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f \cos(\Delta m_s t) + S_f \sin(\Delta m_s t) \right] \quad (2)$$

$$\frac{d\Gamma_{\bar{B}_s^0 \rightarrow \bar{f}}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2}|\bar{A}_{\bar{f}}|^2(1 + |\bar{\lambda}_{\bar{f}}|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_{\bar{f}} \cos(\Delta m_s t) - S_{\bar{f}} \sin(\Delta m_s t) \right] \quad (3)$$

$$\frac{d\Gamma_{B_s^0 \rightarrow \bar{f}}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2}|\bar{A}_{\bar{f}}|^2 \left| \frac{q}{p} \right|^2 (1 + |\bar{\lambda}_{\bar{f}}|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_{\bar{f}} \cos(\Delta m_s t) + S_{\bar{f}} \sin(\Delta m_s t) \right] \quad (4)$$



# third method: time-dependent

$$\frac{d\Gamma_{B_s^0 \rightarrow f}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2}|A_f|^2(1 + |\lambda_f|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t) \right] \quad (1)$$

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$$\frac{d\Gamma_{\bar{B}_s^0 \rightarrow \bar{f}}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2}|\bar{A}_{\bar{f}}|^2(1 + |\bar{\lambda}_{\bar{f}}|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_{\bar{f}} \cos(\Delta m_s t) - S_{\bar{f}} \sin(\Delta m_s t) \right] \quad (3)$$

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# third method: time-dependent

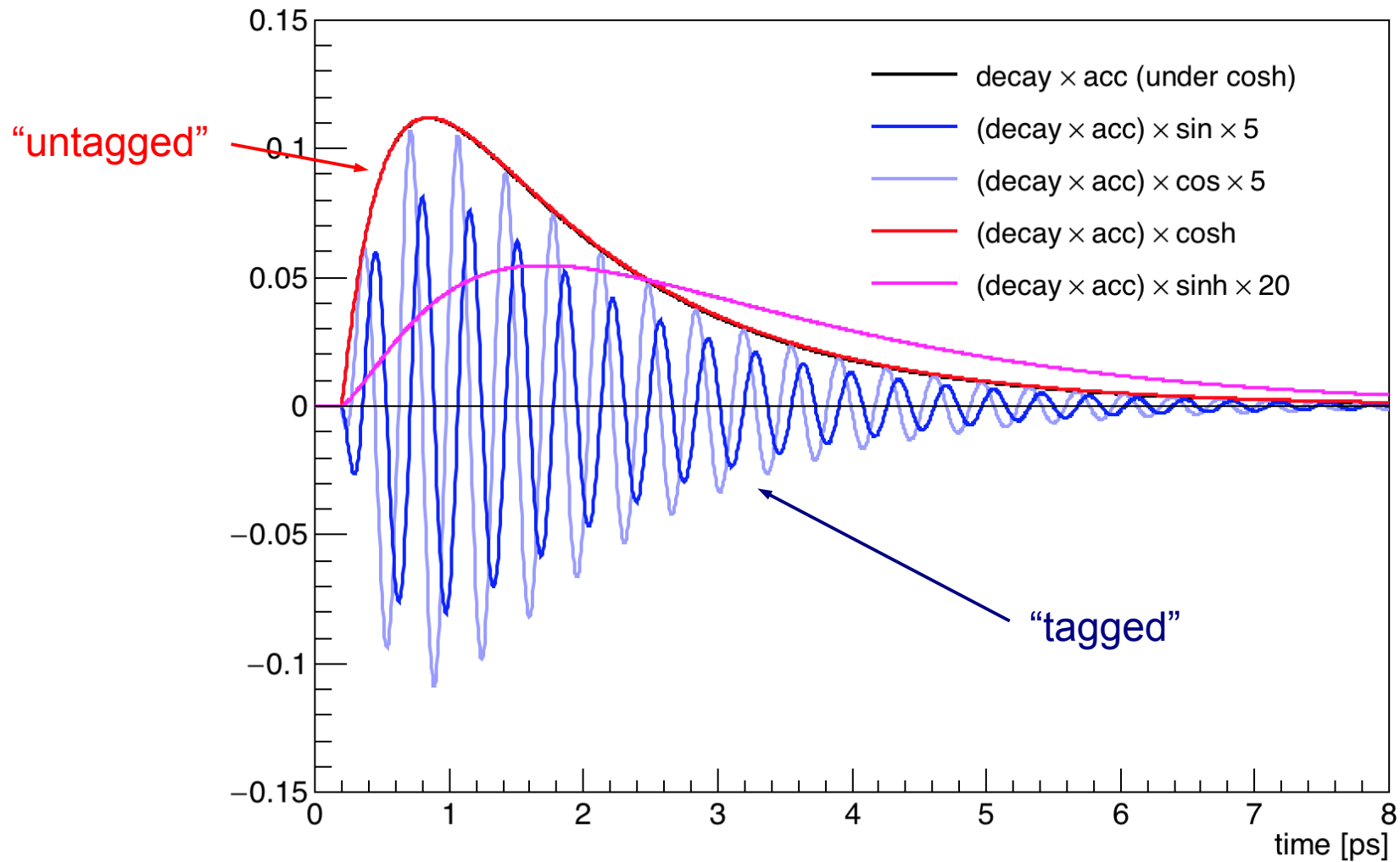
$$\frac{d\Gamma_{B_s^0 \rightarrow f}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2} |A_f|^2 (1 + |\lambda_f|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_f \cos(\Delta m_s t) - S_f \sin(\Delta m_s t) \right] \quad (1)$$

$$\frac{d\Gamma_{\bar{B}_s^0 \rightarrow f}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2} |A_f|^2 \left| \frac{p}{q} \right|^2 (1 + |\lambda_f|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_f \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) - C_f \cos(\Delta m_s t) + S_f \sin(\Delta m_s t) \right] \quad (2)$$

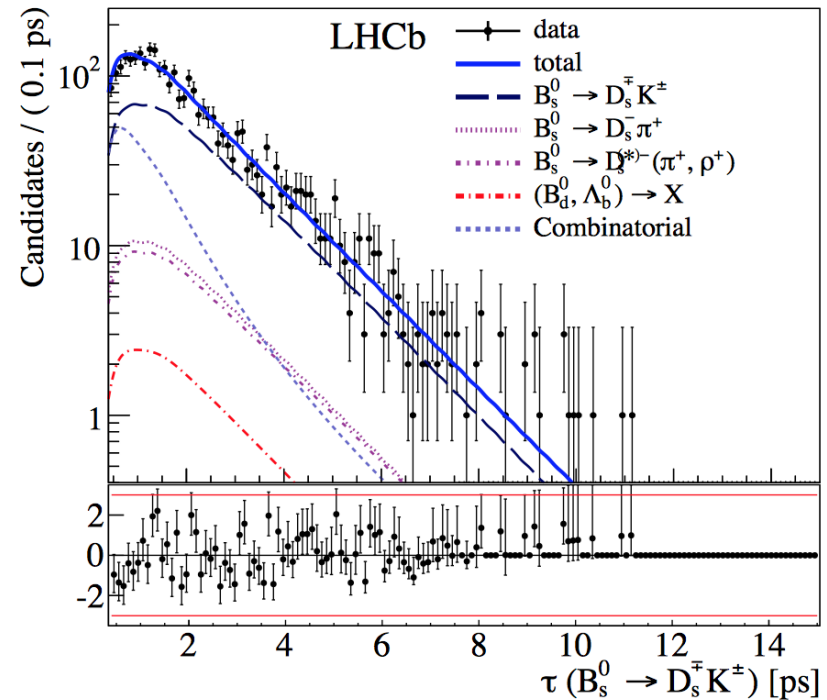
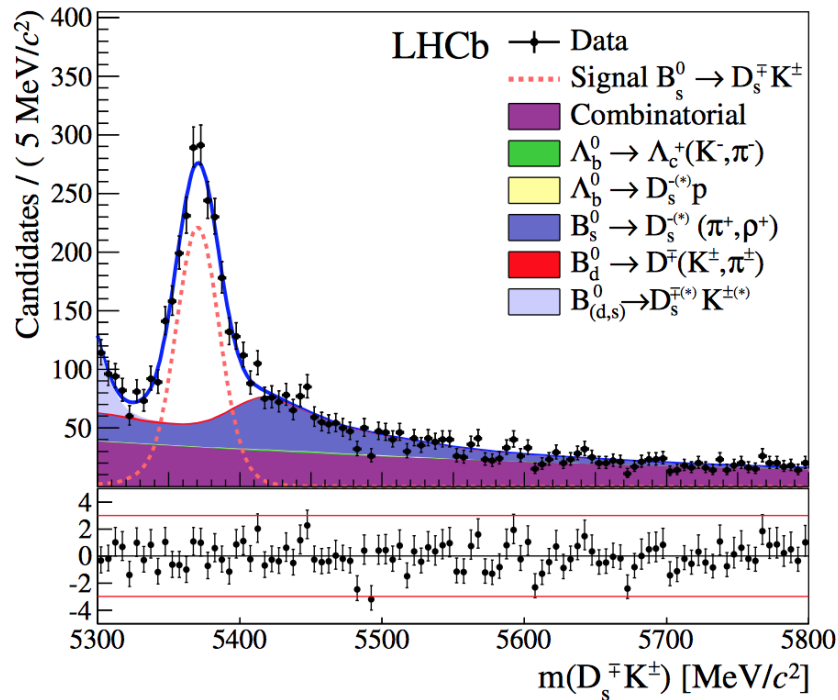
$$\frac{d\Gamma_{\bar{B}_s^0 \rightarrow \bar{f}}(t)}{dt e^{-\Gamma_s t}} = \frac{1}{2} |\bar{A}_{\bar{f}}|^2 (1 + |\bar{\lambda}_{\bar{f}}|^2) \left[ \cosh\left(\frac{\Delta\Gamma_s t}{2}\right) + D_{\bar{f}} \sinh\left(\frac{\Delta\Gamma_s t}{2}\right) + C_{\bar{f}} \cos(\Delta m_s t) - S_{\bar{f}} \sin(\Delta m_s t) \right] \quad (3)$$

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# third method: time-dependent



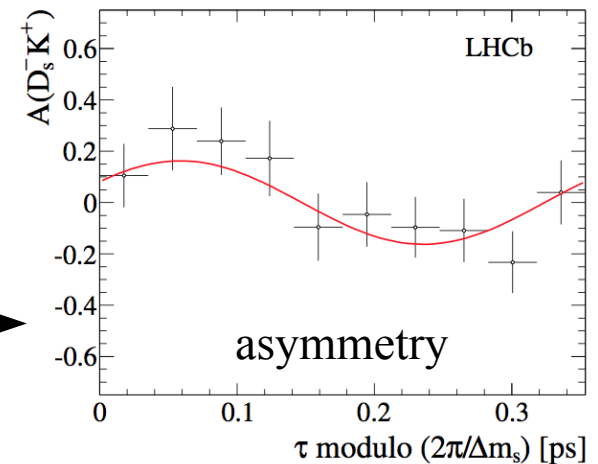
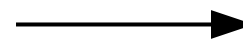
# third method: time-dependent



$$N_{\text{sig}} \sim 1770 (1 \text{ fb}^{-1})$$

JHEP 1411 (2014) 060,  
arXiv:1407.6127.

hint of an oscillation!



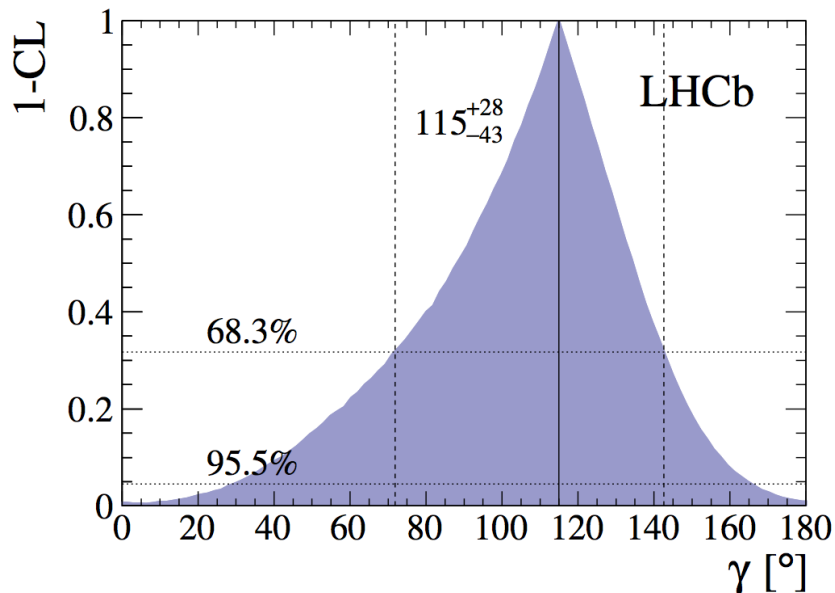
# third method: time-dependent

Assuming the Bs mixing phase to be (LHCb, arXiv:1304.2600)

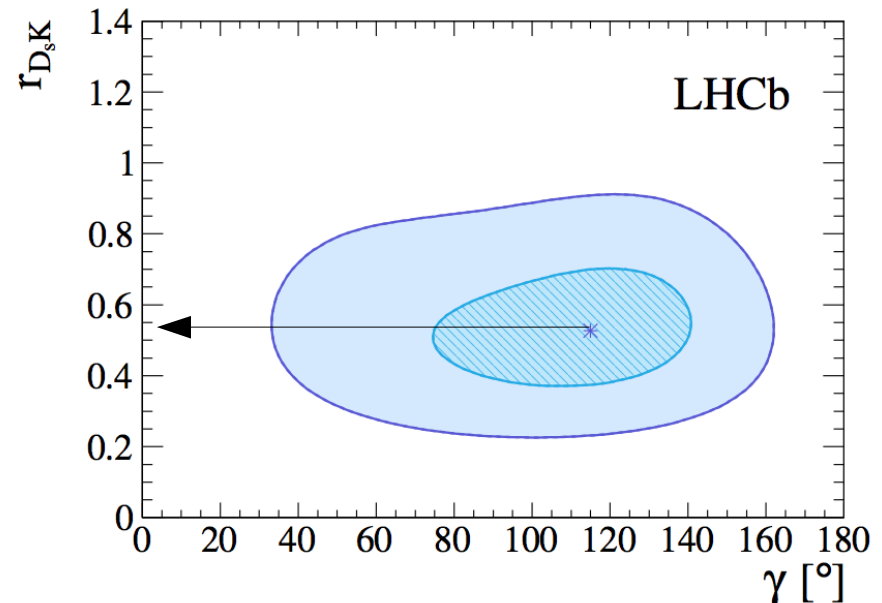
$$\phi_s = 0.01 \pm 0.07 \text{ (stat)} \pm 0.01 \text{ (syst) rad}$$

we constrain  $\gamma$  (arXiv:1407.6127):

$$\gamma = (115^{+28}_{-43})^\circ$$



interference parameter is large!



# Combining all LHCb tree-level $\gamma$ measurements

# $\gamma$ combination

Two combinations:

---

<b>robust</b>	$B \rightarrow DK\text{-like}$
full	$B \rightarrow DK\text{-like}$ and $B \rightarrow D\pi$

---

Inputs:

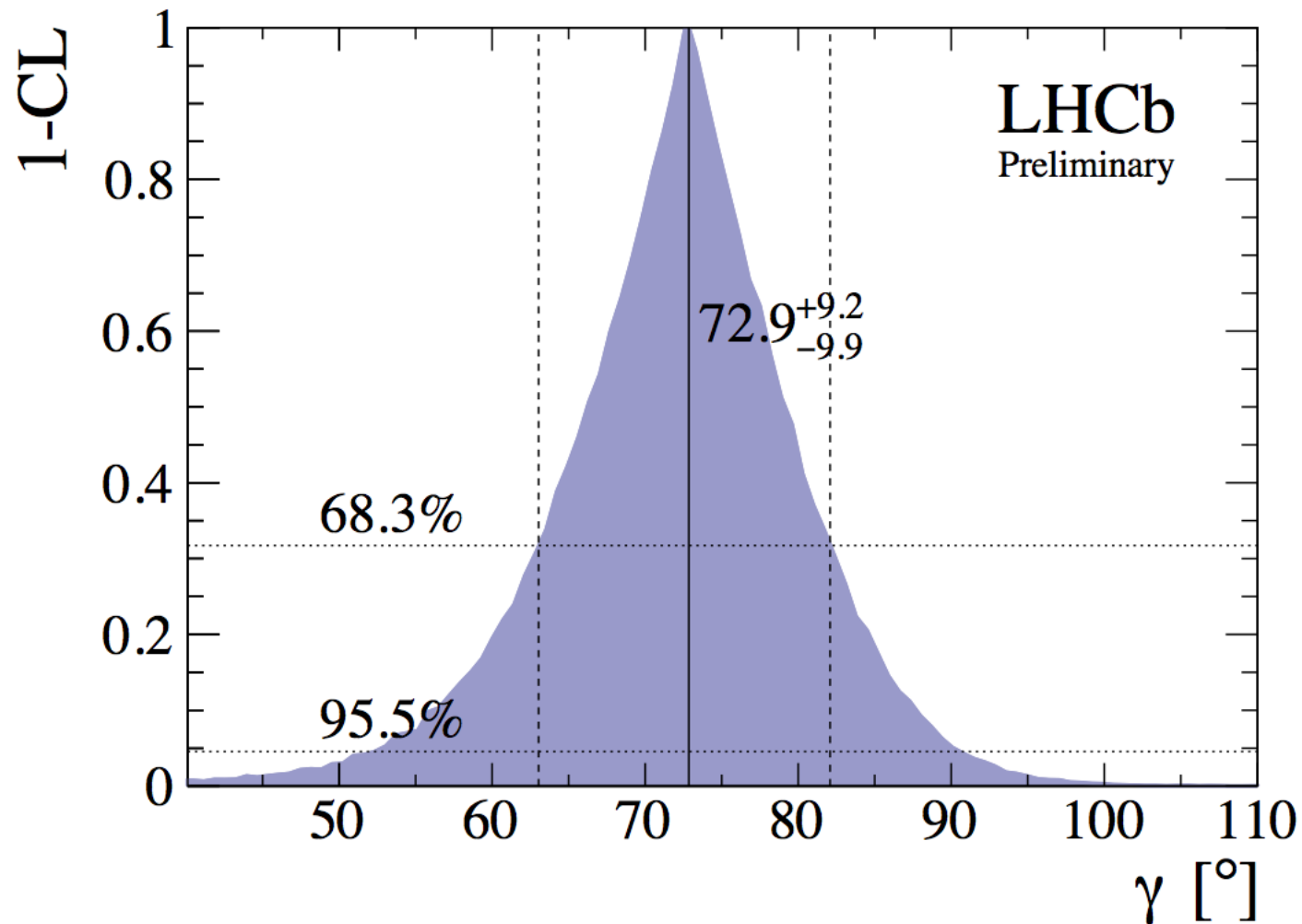
- ▶  $B^+ \rightarrow Dh^+, D \rightarrow hh$ , GLW/ADS,  $1 \text{ fb}^{-1}$  1203.3662
- ▶  $B^+ \rightarrow Dh^+, D \rightarrow K\pi\pi\pi$ , ADS,  $1 \text{ fb}^{-1}$  1303.4646
- ▶ **updated:**  $B^+ \rightarrow DK^+, D \rightarrow K_S^0 hh$ , model-ind. GGSZ,  $3 \text{ fb}^{-1}$  1408.2748
- ▶ **new:**  $B^+ \rightarrow DK^+, D \rightarrow K_S^0 K\pi$ , GLS,  $3 \text{ fb}^{-1}$  1402.2982
- ▶ **new:**  $B^0 \rightarrow D^0 K^{*0}, D \rightarrow hh$ , GLW/ADS,  $3 \text{ fb}^{-1}$  1407.8136
- ▶ **new:**  $B_s^0 \rightarrow D_s^\mp K^\pm$ ,  $1 \text{ fb}^{-1}$  1407.6127



# $\gamma$ combination

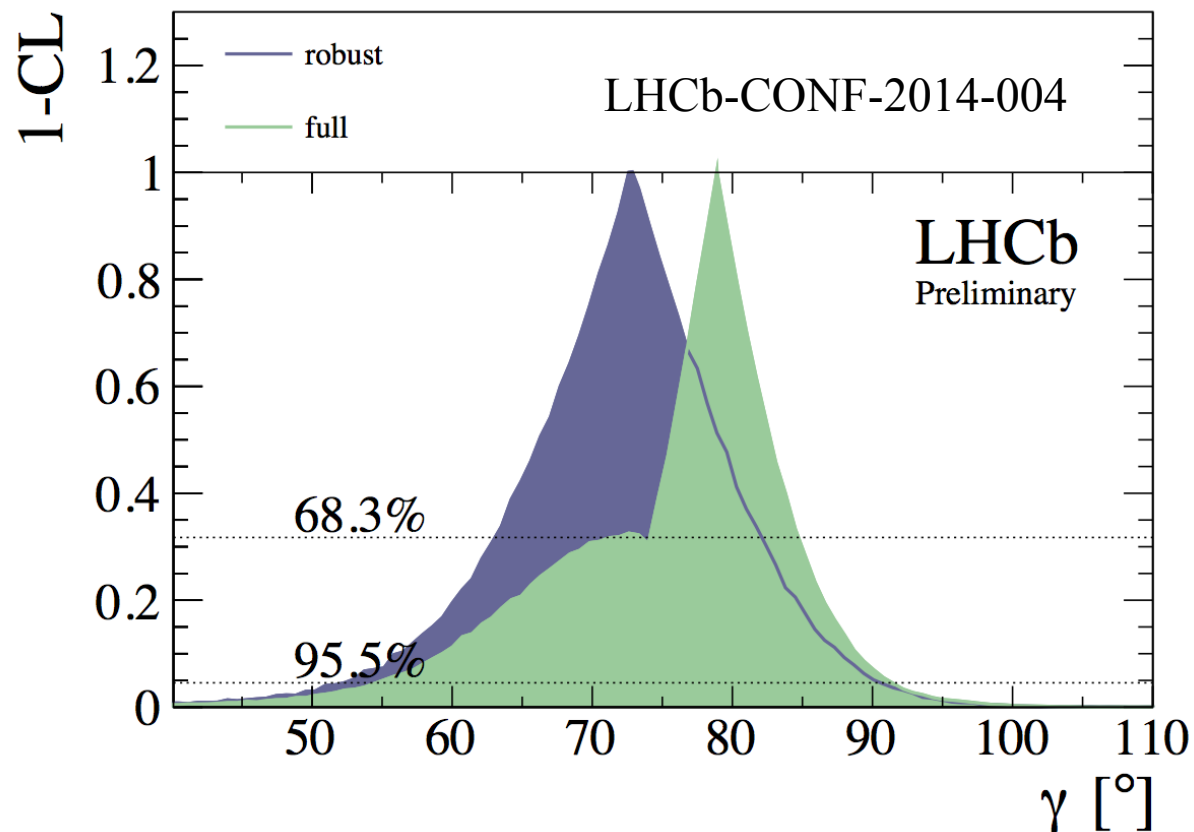
LHCb-CONF-2014-004

Frequentist (plugin-method)



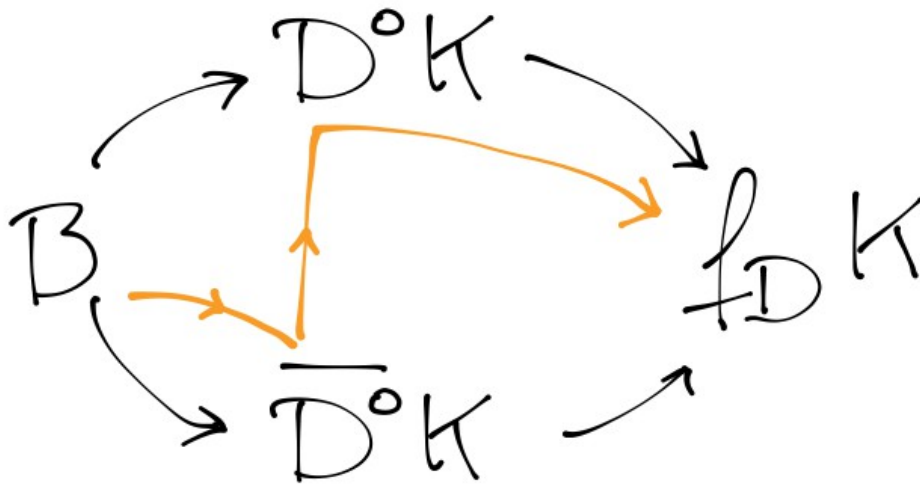
# on the way to the degree precision

- We add another channel:  $B \rightarrow D^0\pi$ , “full combination”
- Less sensitivity to  $\gamma$ , but larger statistics
- A fluctuation causes much increased apparent sensitivity, and highly non Gaussian behavior – to be interpreted with care!



# on the way to the degree precision

- The effect of  $D^0$  mixing does affect the determination of  $\gamma$ .
- Already accounted for in the LHCb combination!
- Next up: also  $K^0$  mix ...



$$\Delta \approx \sqrt{x_D^2 + y_D^2} / r_B$$

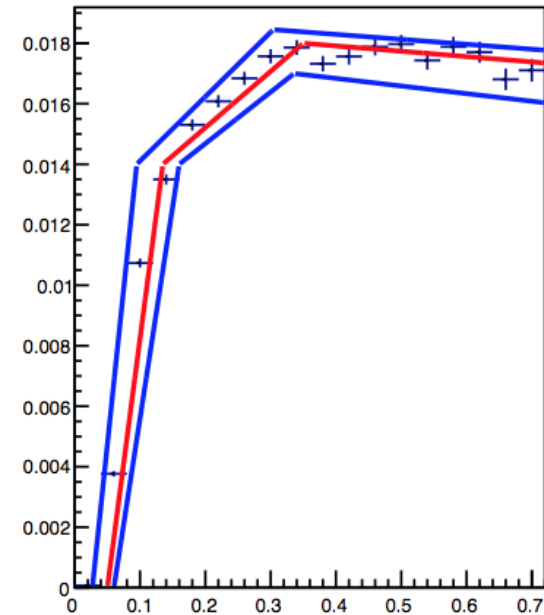


Figure: LHCb  $D^0$  decay time acceptance for  $B^+ \rightarrow DK^+$ ,  $D \rightarrow hh$ .

# $\gamma$ combination

**Table:** Summary of results for  $\gamma$  from the  $B$  factories BaBar and Belle, and from LHCb, and combiners. Errors correspond to 68% confidence or credibility.

experiment	result	date
BaBar	$(69^{+17}_{-16})^\circ$	Jan 2013
Belle	$(68^{+15}_{-14})^\circ$	Jan 2013
LHCb 1–3 fb <sup>-1</sup> prelim.	$(67 \pm 12)^\circ$	Apr 2013
LHCb 1 fb <sup>-1</sup>	$(72.6^{+9.7}_{-17.2})^\circ$	Aug 2013
LHCb 1–3 fb <sup>-1</sup> prelim.	$(72.9^{+9.2}_{-9.9})^\circ$	Sep 2014
UTfit	$(68.3 \pm 7.5)^\circ$	post Moriond 2014
CKMfitter	$(70.0^{+7.7}_{-9.0})^\circ$	Moriond / Jun 2014
CKMfitter	$(73.2^{+6.3}_{-7.0})^\circ$	Sep 2014

# Outlook

# Updates of the existing

Many inputs yet to be updated to 3 fb<sup>-1</sup>:

- ▶  $B^+ \rightarrow Dh^+, D \rightarrow hh$ , GLW/ADS, 1 fb<sup>-1</sup> 1203.3662
- ▶  $B^+ \rightarrow Dh^+, D \rightarrow K\pi\pi\pi$ , ADS, 1 fb<sup>-1</sup> 1303.4646
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- ▶ **new:**  $B^0 \rightarrow D^0 K^{*0}, D \rightarrow hh$ , GLW/ADS, 3 fb<sup>-1</sup> 1407.8136
- ▶ **new:**  $B_s^0 \rightarrow D_s^\mp K^\pm$ , 1 fb<sup>-1</sup> 1407.6127

# More channels to be added

There are many more possibilities:

$$B^+ \rightarrow Dh^+, D \rightarrow K\pi\pi^0 \quad \text{ADS}$$

$$B^+ \rightarrow Dh^+, D \rightarrow \pi\pi\pi^0 \quad \text{GLW}$$

$$B^+ \rightarrow Dh^+, D \rightarrow KK\pi\pi \quad \text{GGSZ}$$

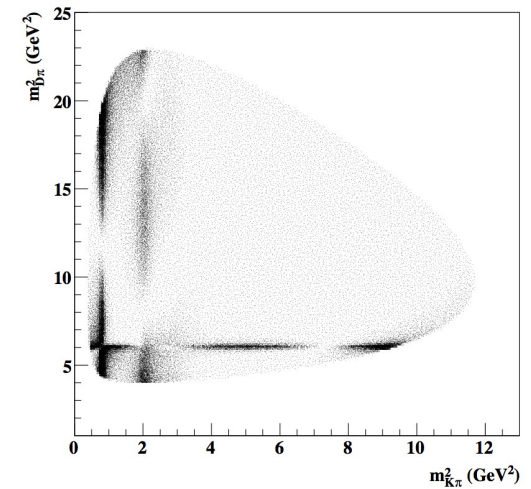
$$B^0 \rightarrow DK^{*0}, D \rightarrow Khh \quad \text{GGSZ}$$

$$B^+ \rightarrow DK\pi\pi, D \rightarrow hh, Khh \quad \text{GLW/ADS/GGSZ}$$

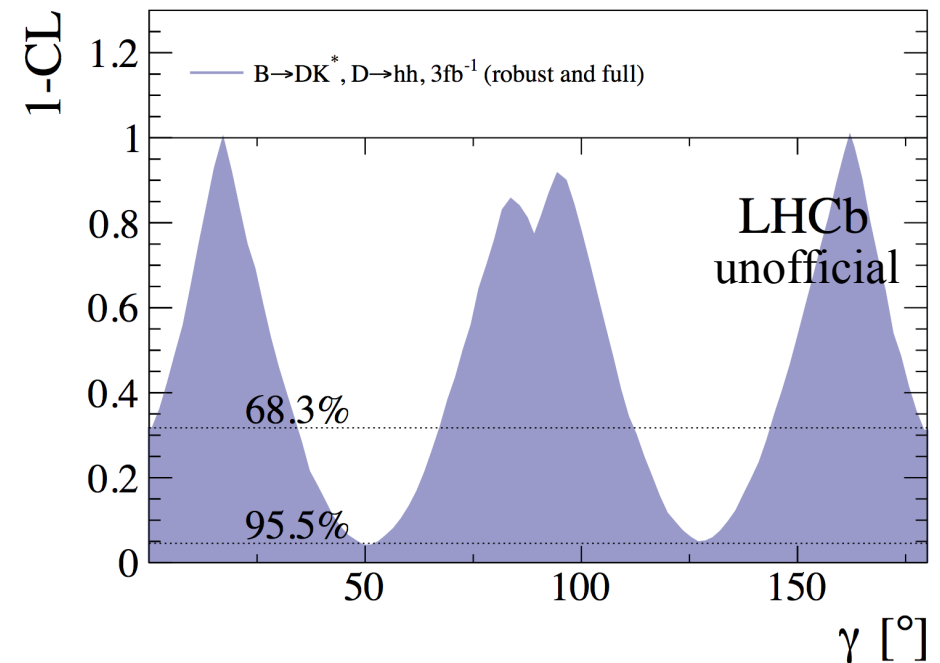
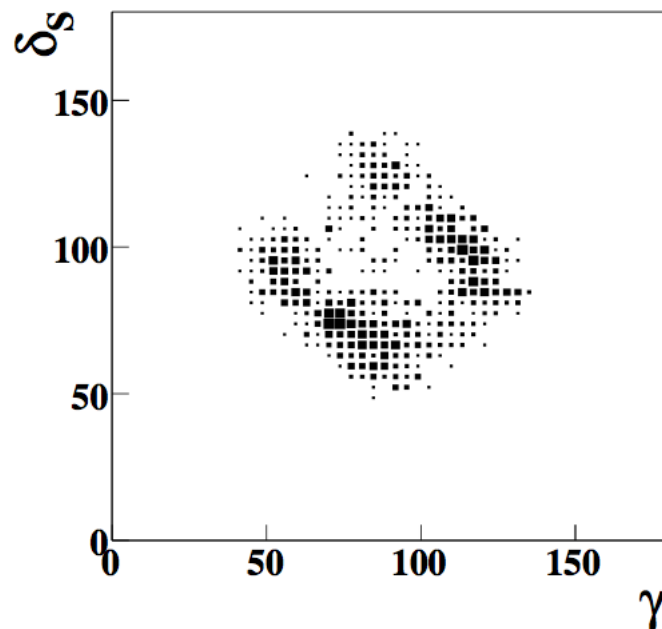
...

# A new method

- Idea: analyze the  $B^0$  Dalitz plot in  $B^0 \rightarrow D^0 K \pi$
- Gershon, Williams [arXiv:0909.1495]
- This resolves ambiguities!
- A 10deg error on  $\gamma$  seems not unreasonable! (Dalitz model unknown).



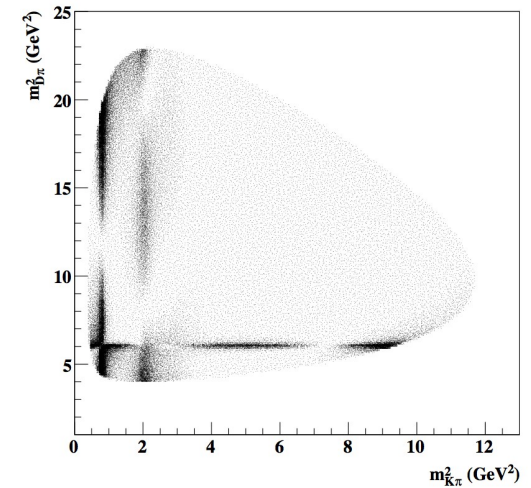
$B^0 \rightarrow D^0 K^{*0} \rightarrow D^0 K \pi$   
already contributes now!



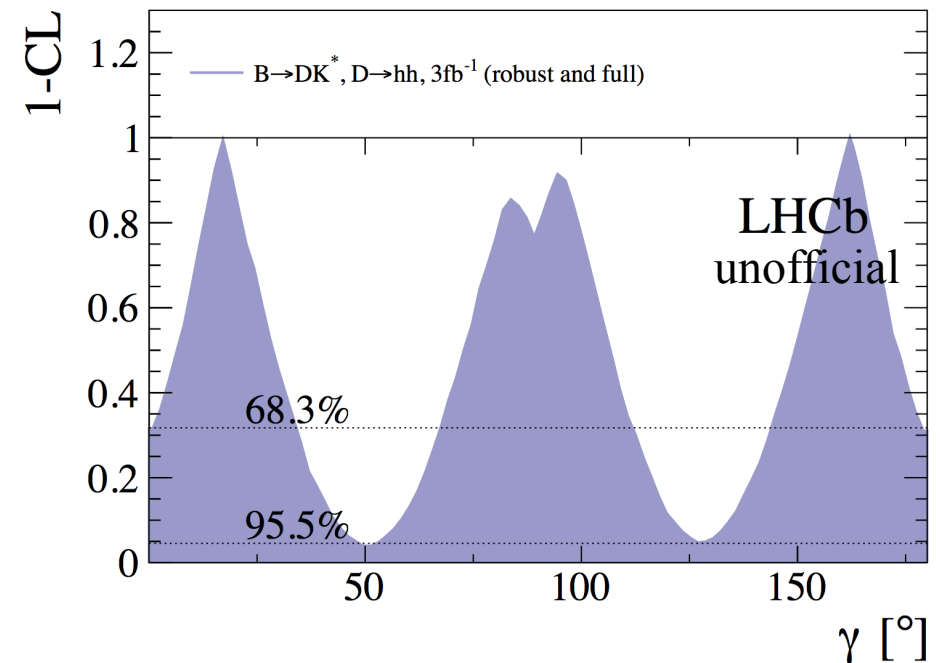
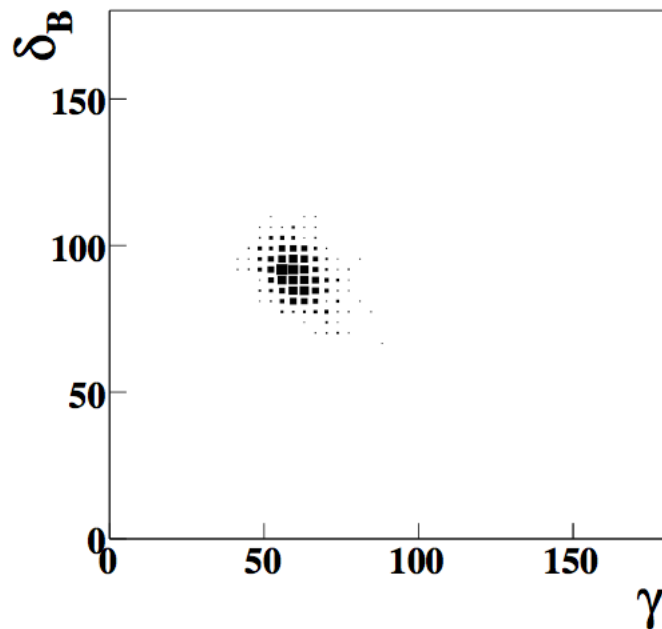


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$B^0 \rightarrow D^0 K^{*0} \rightarrow D^0 K \pi$   
already contributes now!



# LHCb Run2 expectations

Table 28: Statistical sensitivities of the LHCb upgrade to key observables. For each observable the expected sensitivity is given for the integrated luminosity accumulated by the end of LHC Run 1, by 2018 (assuming  $5 \text{ fb}^{-1}$  recorded during Run 2) and for the LHCb Upgrade ( $50 \text{ fb}^{-1}$ ). An estimate of the theoretical uncertainty is also given – this and the potential sources of systematic uncertainty are discussed in the text.

Type	Observable	LHC Run 1	LHCb 2018	LHCb upgrade	Theory
☺ ☺ (☺)	$B_s^0$ mixing				
	$\phi_s(B_s^0 \rightarrow J/\psi \phi)$ (rad)	0.050	0.025	<b>0.009</b>	$\sim 0.003$
	$\phi_s(B_s^0 \rightarrow J/\psi f_0(980))$ (rad)	0.068	0.035	<b>0.012</b>	$\sim 0.01$
	$A_{\text{sl}}(B_s^0)$ ( $10^{-3}$ )	2.8	1.4	<b>0.5</b>	0.03
☺ ☺ ☺	Gluonic penguin				
	$\phi_s^{\text{eff}}(B_s^0 \rightarrow \phi \phi)$ (rad)	0.15	0.10	<b>0.023</b>	0.02
	$\phi_s^{\text{eff}}(B_s^0 \rightarrow K^{*0} \bar{K}^{*0})$ (rad)	0.19	0.13	<b>0.029</b>	$< 0.02$
	$2\beta^{\text{eff}}(B^0 \rightarrow \phi K_S^0)$ (rad)	0.30	0.20	<b>0.04</b>	0.02
Right-handed currents	$\phi_s^{\text{eff}}(B_s^0 \rightarrow \phi \gamma)$	0.20	0.13	<b>0.030</b>	$< 0.01$
	$\tau^{\text{eff}}(B_s^0 \rightarrow \phi \gamma)/\tau_{B_s^0}$	5%	3.2%	<b>0.8%</b>	0.2%
Electroweak penguin	$S_3(B^0 \rightarrow K^{*0} \mu^+ \mu^-; 1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.04	0.020	<b>0.007</b>	0.02
	$q_0^2 A_{\text{FB}}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)$	10%	5%	<b>1.9%</b>	$\sim 7\%$
	$A_1(K \mu^+ \mu^-; 1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.09	0.05	<b>0.017</b>	$\sim 0.02$
	$\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)/\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)$	14%	7%	<b>2.4%</b>	$\sim 10\%$
Higgs penguin	$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$ ( $10^{-9}$ )	1.0	0.5	<b>0.19</b>	0.3
	$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-)/\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$	220%	110%	<b>40%</b>	$\sim 5\%$
☺ (☺ ☺)	Unitarity triangle				
	$\gamma(B \rightarrow D^{(*)} K^{(*)})$	$7^\circ$	$4^\circ$	<b><math>1.1^\circ</math></b>	negligible
☺ (☺ ☺)	angles				
	$\gamma(B_s^0 \rightarrow D_s^\mp K^\pm)$	$17^\circ$	$11^\circ$	<b><math>2.4^\circ</math></b>	negligible
	$\beta(B^0 \rightarrow J/\psi K_S^0)$	$1.7^\circ$	$0.8^\circ$	<b><math>0.31^\circ</math></b>	negligible
Charm	$A_\Gamma(D^0 \rightarrow K^+ K^-)$ ( $10^{-4}$ )	3.4	2.2	<b>0.5</b>	–
CP violation	$\Delta A_{\text{CP}}$ ( $10^{-3}$ )	0.8	0.5	<b>0.12</b>	–

Smileys indicate “on trackness”, added by Tim Gershon (LHCb Implications Workshop Oct 2014)

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Type	Observable	LHC Run 1	LHCb 2018	LHCb upgrade	Theory
☺ ☺ (☺)	$B_s^0$ mixing				
	$\phi_s(B_s^0 \rightarrow J/\psi \phi)$ (rad)	0.050	0.025	<b>0.009</b>	$\sim 0.003$
	$\phi_s(B_s^0 \rightarrow J/\psi f_0(980))$ (rad)	0.068	0.035	<b>0.012</b>	$\sim 0.01$
	$A_{\text{sl}}(B_s^0)$ ( $10^{-3}$ )	2.8	1.4	<b>0.5</b>	0.03
☺ ☺	Gluonic penguin				
	$\phi_s^{\text{eff}}(B_s^0 \rightarrow \phi \phi)$ (rad)	0.15	0.10	<b>0.023</b>	0.02
	$\phi_s^{\text{eff}}(B_s^0 \rightarrow K^{*0} \bar{K}^{*0})$ (rad)	0.19	0.13	<b>0.029</b>	$< 0.02$
	$2\beta^{\text{eff}}(B^0 \rightarrow \phi K_S^0)$ (rad)	0.30	0.20	<b>0.04</b>	0.02
Right-handed currents	$\phi_s^{\text{eff}}(B_s^0 \rightarrow \phi \gamma)$	0.20	0.13	<b>0.030</b>	$< 0.01$
	$\tau^{\text{eff}}(B_s^0 \rightarrow \phi \gamma)/\tau_{B_s^0}$	5%	3.2%	<b>0.8%</b>	0.2%
Electroweak penguin	$S_3(B^0 \rightarrow K^{*0} \mu^+ \mu^-; 1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.04	0.020	<b>0.007</b>	0.02
	$q_0^2 A_{\text{FB}}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)$	10%	5%	<b>1.9%</b>	$\sim 7\%$
	$A_1(K \mu^+ \mu^-; 1 < q^2 < 6 \text{ GeV}^2/c^4)$	0.09	0.05	<b>0.017</b>	$\sim 0.02$
	$\mathcal{B}(B^+ \rightarrow \pi^+ \mu^+ \mu^-)/\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)$	14%	7%	<b>2.4%</b>	$\sim 10\%$
Higgs penguin	$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$ ( $10^{-9}$ )	1.0	0.5	<b>0.19</b>	0.3
	$\mathcal{B}(B^0 \rightarrow \mu^+ \mu^-)/\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)$	220%	110%	<b>40%</b>	$\sim 5\%$
☺ (☺ ☺)	Unitarity triangle				
	$\gamma(B \rightarrow D^{(*)} K^{(*)})$	7°	4°	<b>σ(γ) ≈ 4°</b>	
☺ (☺ ☺)	angles				
	$\gamma(B_s^0 \rightarrow D_s^\mp K^\pm)$	17°	11°		
	$\beta(B^0 \rightarrow J/\psi K_S^0)$	1.7°	0.8°		
Charm	$A_\Gamma(D^0 \rightarrow K^+ K^-)$ ( $10^{-4}$ )	3.4	2.2	<b>0.5</b>	–
CP violation	$\Delta A_{\text{CP}}$ ( $10^{-3}$ )	0.8	0.5	<b>0.12</b>	–

Smileys indicate “on trackness”, added by Tim Gershon (LHCb Implications Workshop Oct 2014)

# current systematic effects

Tree-level measurements of  $\gamma$  will **not be limited** by systematics for a long time (not at **100 times** the current dataset).

going well beyond  
LHCb upgrade!

- **first method** ( $B \rightarrow DK$  GLW/ADS)
  - instrumental charge asymmetries (known to the per-mille level,  $B \rightarrow J/\psi K$  asymmetry needed as input, magnet polarity flip)
  - calibration of particle identification

**example result:**

$$A_{CP} = 0.0849 \pm 0.0201(\text{stat.}) \pm 0.0010(\text{syst.})$$

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Tree-level measurements of  $\gamma$  will **not be limited** by systematics for a long time (not at **100 times** the current dataset).

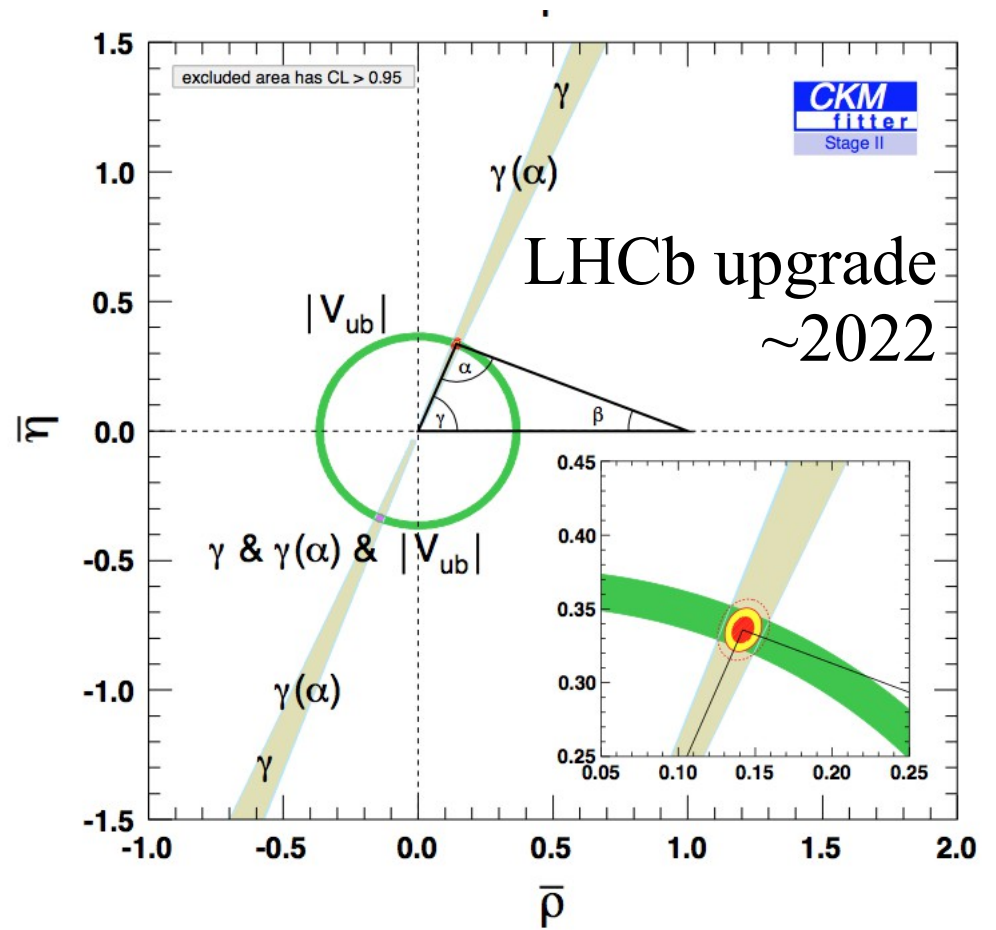
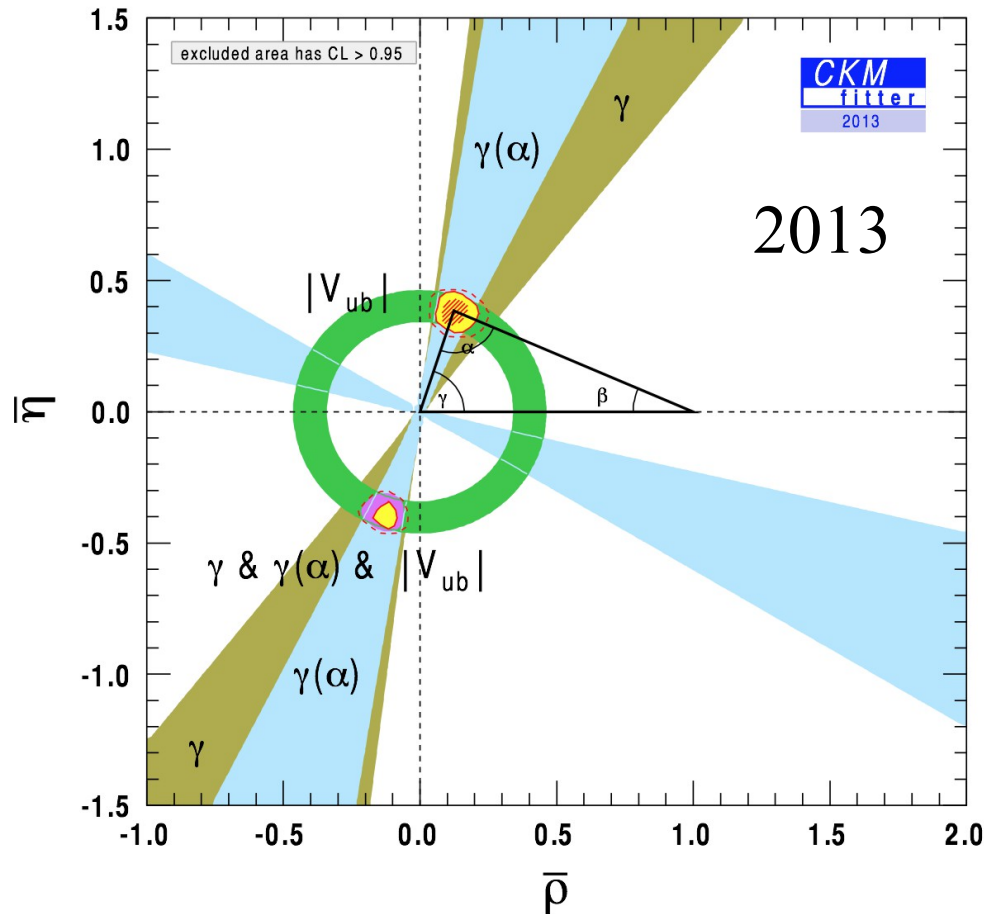
going well beyond  
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- **first method** ( $B \rightarrow DK$  GLW/ADS)
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    - calibration of particle identification
  - **second method** ( $B \rightarrow DK$  GGSZ)
    - efficiency corrections over the Dalitz plot
  - **third method** ( $B_s \rightarrow D_s K$  time dependent)
    - decay time resolution
    - decay time acceptance
    - knowledge of  $\Delta m_s$ ,  $\Delta \Gamma_s$ ,  $\Gamma_s$
- } completely different sources!

# Conclusion

# Conclusion

LHCb is getting closer to a tree-level precision measurement of the CKM triangle!  
 (Might need a little help with  $|V_{ub}|$  though!)

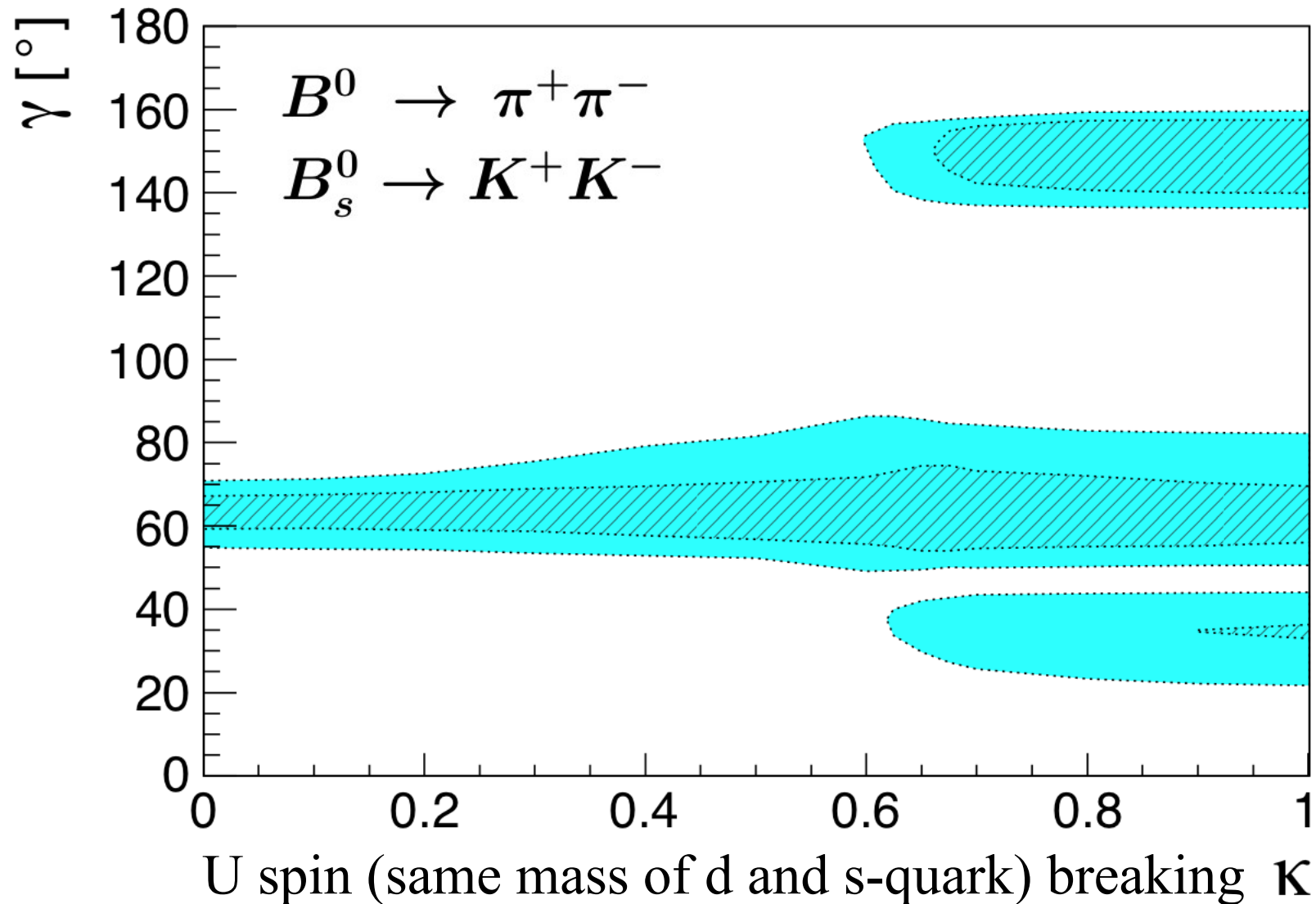


Ligeti et al., 1309.2293

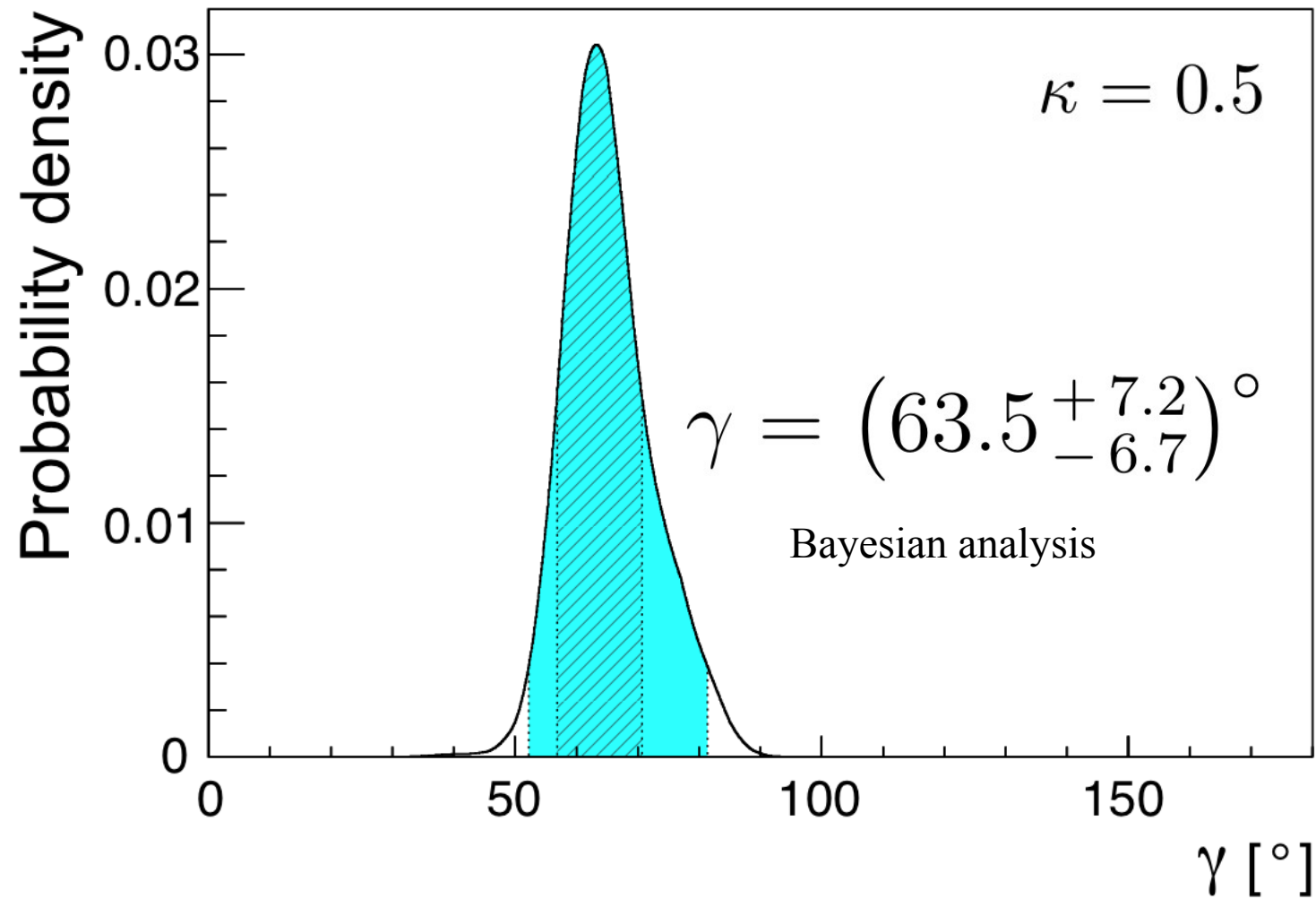
# Backup



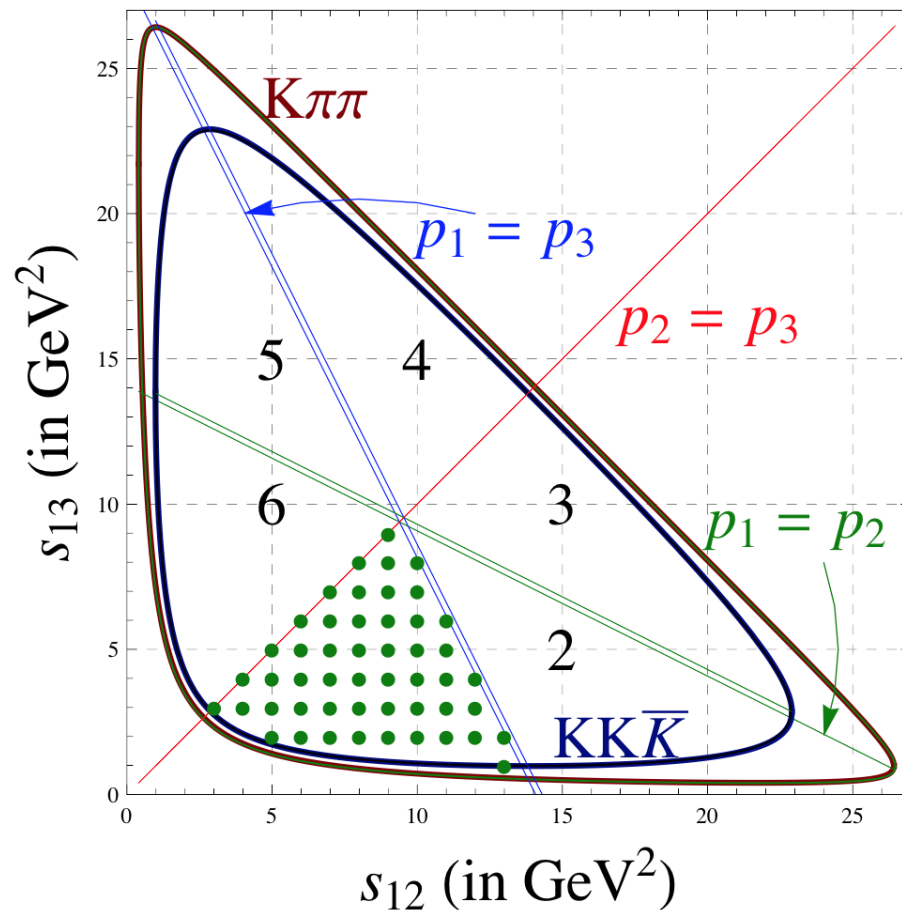
# “ $\gamma$ from loops”



# “ $\gamma$ from loops”



# “ $\gamma$ from loops”



**New method by**  
London, Bhattacharya,  
Imbeault, Rey-Le Lurier:

$$B \rightarrow hhh$$

$$h = K, \pi$$

$$\gamma = (77 \pm 3)^\circ$$

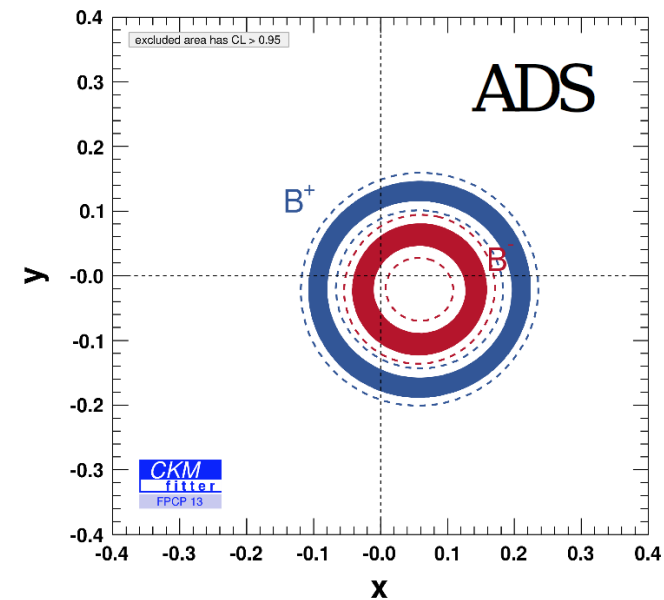
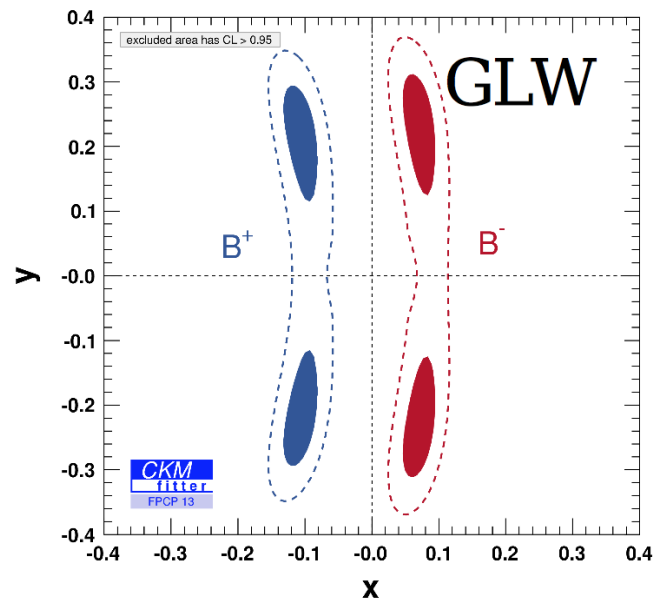
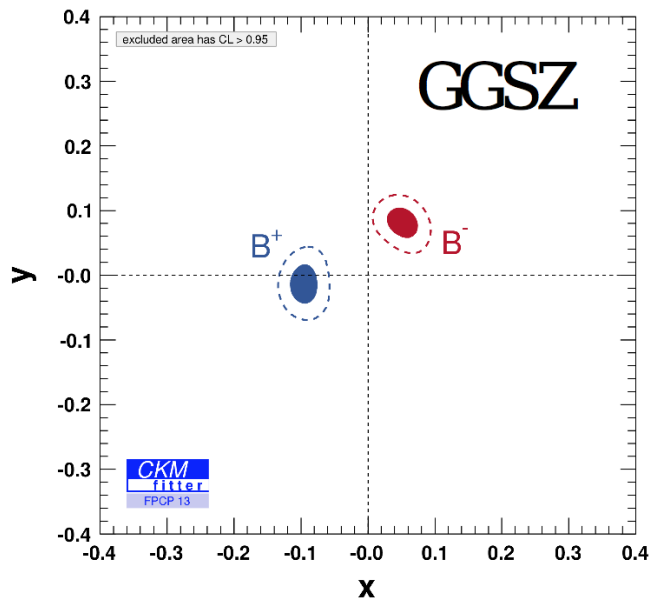
(take with a grain of salt)

FIG. 1: Kinematic boundaries and symmetry axes of  $B \rightarrow K\pi\pi$  and  $B \rightarrow KK\bar{K}$  Dalitz plots. The symmetry axes divide each plot into six zones, five of which are marked 2-6. The fifty dots in the region of overlap of the first of six zones from all Dalitz plots are used for the  $\gamma$  measurement.

**Well suited  
for LHCb!**

# GGSZ or the “Dalitz” method

illustration:



Karim Trabelsi, CKM2014

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma) \quad y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

# Combination

Table 2: Observables used in the robust combination.

LHCb Analysis	Observables
$B^+ \rightarrow DK^+, D \rightarrow hh, \text{GLW/ADS}$	$A_{CP}^{DK, KK}, A_{CP}^{DK, \pi\pi}, R_{K/\pi}^{KK}, R_{K/\pi}^{\pi\pi}, R_{K/\pi}^{K\pi}, A_{\text{fav}}^{DK, K\pi}, R_+^{DK, K\pi}, R_-^{DK, K\pi}$
$B^+ \rightarrow DK^+, D \rightarrow K\pi\pi\pi, \text{ADS}$	$R_+^{DK, K3\pi}, R_-^{DK, K3\pi}, A_{\text{fav}}^{DK, K3\pi}$
$B^+ \rightarrow DK^+, D \rightarrow K_s^0 hh, \text{model-independent GGSZ}$	$x_-, x_+, y_-, y_+$
$B^+ \rightarrow DK^+, D \rightarrow K_s^0 K\pi, \text{GLS}$	$R_{DK, \text{fav/sup}}^{K_S K\pi}, A_{\text{fav}}^{DK, K_S K\pi}, A_{\text{sup}}^{DK, K_S K\pi}$
$B^0 \rightarrow DK^{*0} \text{GLW/ADS}$	$A_{CP}^{DK^{*0}, KK}, A_{\text{fav}}^{DK^{*0}, K\pi}, R_{CP}^{DK^{*0}, KK}, A_{CP}^{DK^{*0}, \pi\pi}, R_{CP}^{DK^{*0}, \pi\pi}, R_+^{DK^{*0}, K\pi}, R_-^{DK^{*0}, K\pi}$
$B_s^0 \rightarrow D_s^\mp K^\pm$	$C_f, A_f^{\Delta\Gamma}, A_{\bar{f}}^{\Delta\Gamma}, S_f, S_{\bar{f}}$
Auxiliary Input	Observables
CLEO-c	$\kappa_D^{K3\pi}, \delta_D^{K3\pi}$
Belle, CLEO	$R_{WS}(D \rightarrow K\pi\pi\pi)$
CLEO	$R_D^{K_S K\pi}, \kappa_D^{K_S K\pi}, \delta_D^{K_S K\pi}$
LHCb toy	$\kappa_B^{DK^{*0}}$
LHCb	$\phi_s$
HFAG	$x_D, y_D, \delta_D^{K\pi}, R_D^{K\pi}, A_{CP}^{\text{dir}}(KK), A_{CP}^{\text{dir}}(\pi\pi)$

# Combination

Table 3: Confidence intervals and central values for the robust combination.

quantity	robust combination
$\gamma$ ( $^\circ$ )	72.9
68% CL ( $^\circ$ )	[63.0, 82.1]
95% CL ( $^\circ$ )	[52.0, 90.5]
$r_B^{DK}$	0.0914
68% CL	[0.0826, 0.0997]
95% CL	[0.0728, 0.1078]
$\delta_B^{DK}$ ( $^\circ$ )	126.8
68% CL ( $^\circ$ )	[115.3, 136.7]
95% CL ( $^\circ$ )	[101.6, 145.2]

# Combination

Table 4: Observables used in the full combination in addition to those of the robust combination given in Table 2.

$B^+ \rightarrow DK^+, D \rightarrow hh, \text{GLW/ADS}$	$A_{CP}^{D\pi, KK}, A_{CP}^{D\pi, \pi\pi}, A_{\text{fav}}^{D\pi, K\pi}, R_+^{D\pi, K\pi}, R_-^{D\pi, K\pi}$
$B^+ \rightarrow DK^+, D \rightarrow K\pi\pi\pi, \text{ADS}$	$R_+^{D\pi, K3\pi}, R_-^{D\pi, K3\pi}, A_{\text{fav}}^{D\pi, K3\pi}, R_{K/\pi}^{K3\pi}$

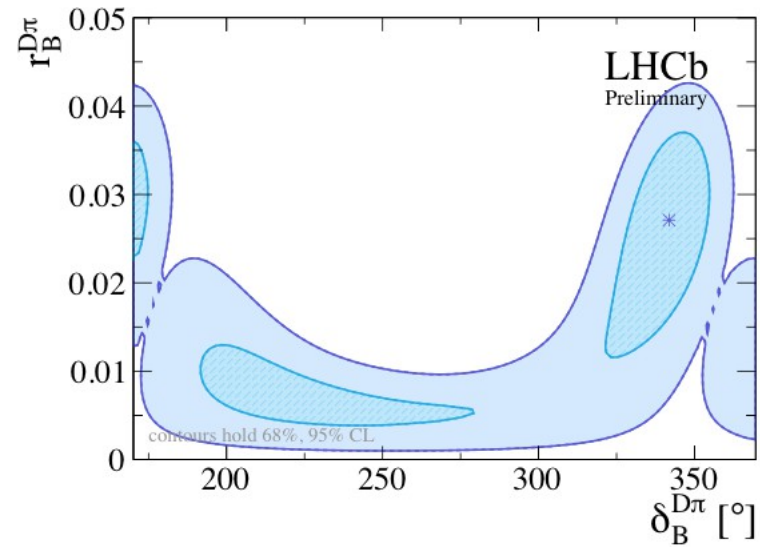
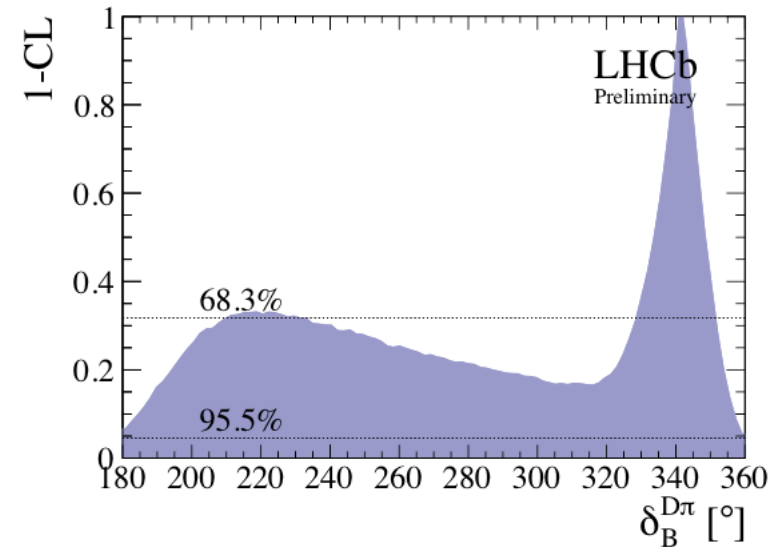
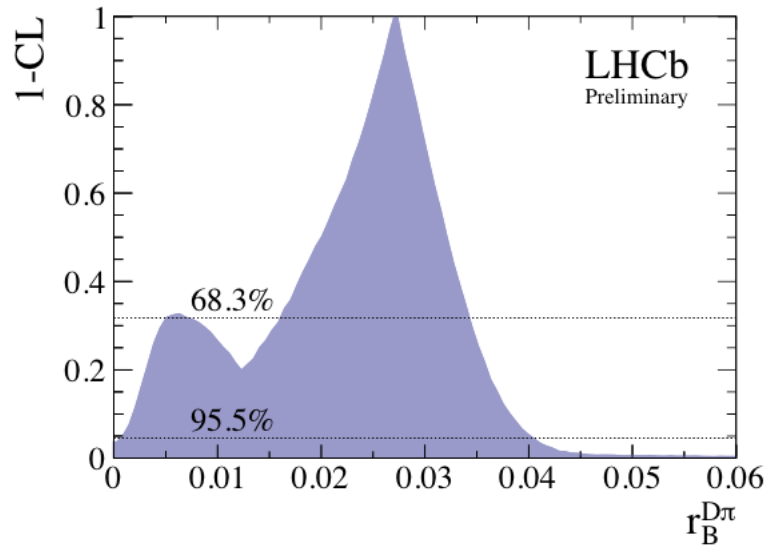
# Combination

Table 5: Confidence intervals and central values for the full combination. The two columns correspond to the two minima found by the fit. The most probable value is given in the left column, corresponding to a large value of  $r_B^{D\pi}$ .

quantity	full	
$\gamma$ ( $^\circ$ )	78.9	72.8
68% CL ( $^\circ$ )	[71.5, 84.7]	
95% CL ( $^\circ$ )	[54.6, 91.4]	
$r_B^{DK}$	0.0928	
68% CL	[0.0845, 0.1008]	
95% CL	[0.0732, 0.1085]	
$\delta_B^{DK}$ ( $^\circ$ )	128.9	
68% CL ( $^\circ$ )	[118.9, 137.9]	
95% CL ( $^\circ$ )	[102.0, 145.9]	
$r_B^{D\pi}$	0.027	0.006
68% CL	[0.016, 0.034]	[0.005, 0.007]
95% CL	[0.001, 0.040]	
$\delta_B^{D\pi}$ ( $^\circ$ )	341.8	215.6
68% CL ( $^\circ$ )	[328.7, 351.4]	[210.2, 231.5]
95% CL ( $^\circ$ )	no constraint	



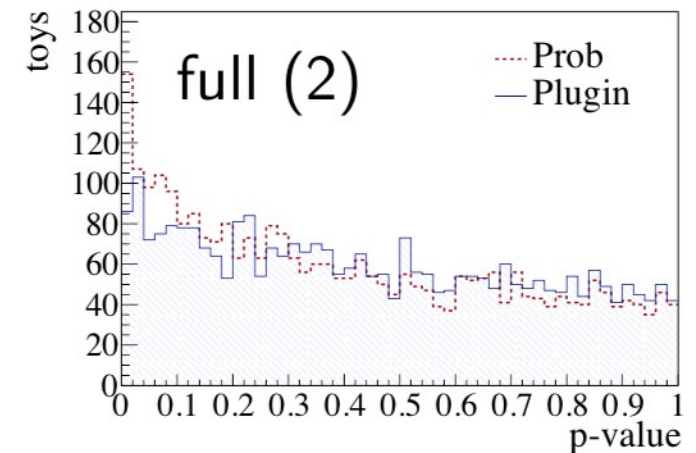
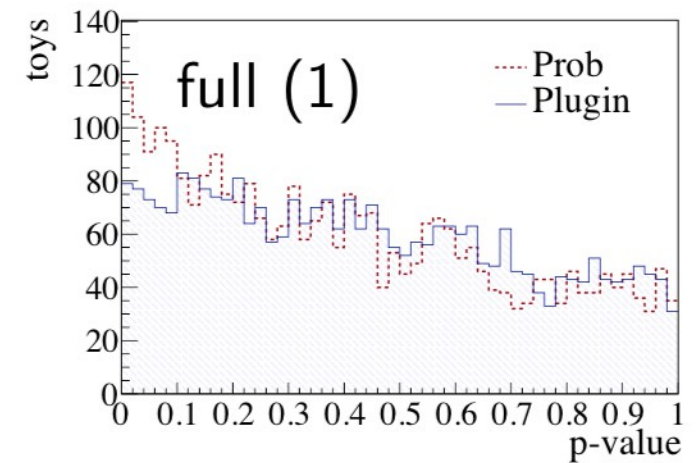
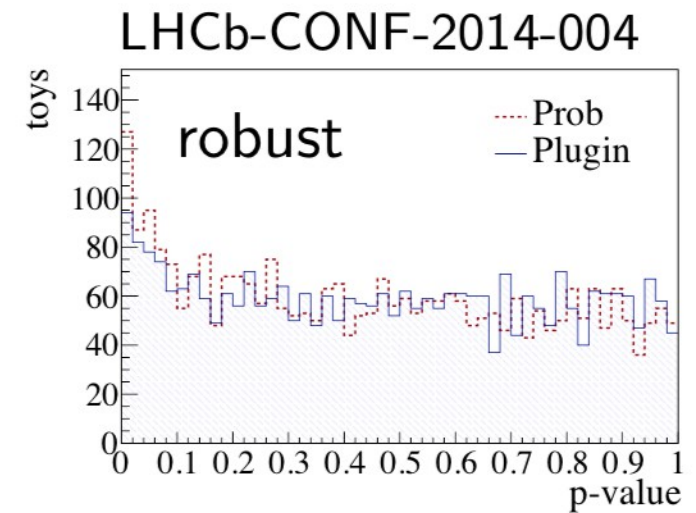
# Combination



# Coverage test

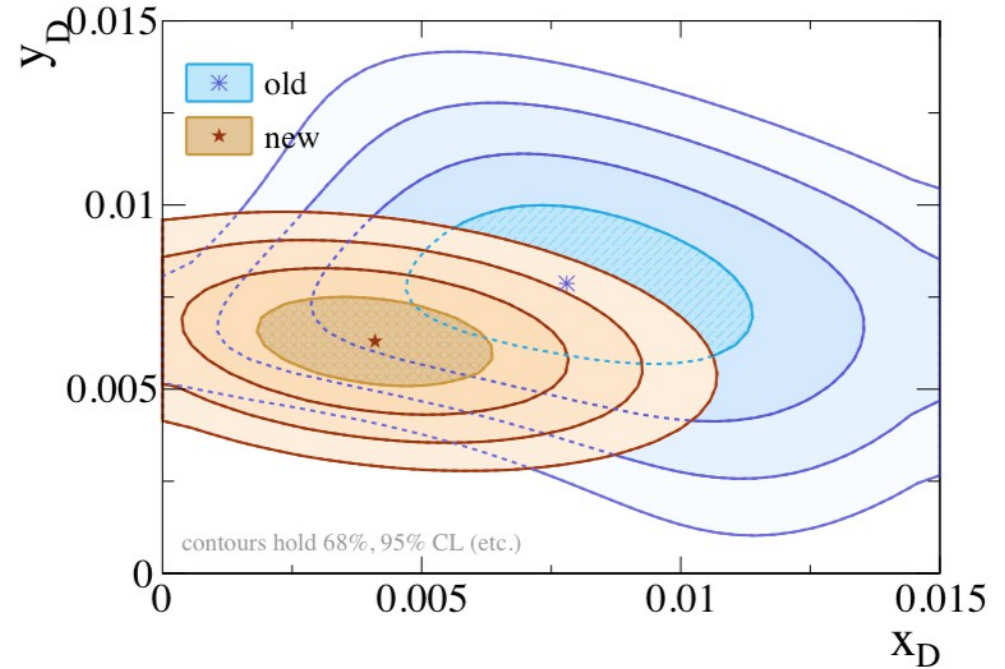
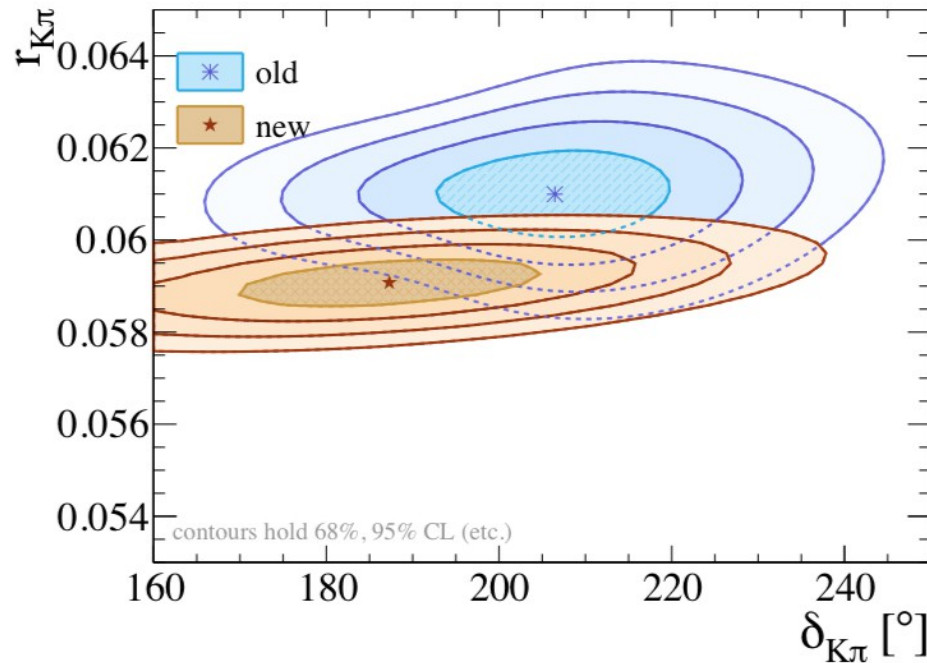
- ▶ We test the frequentist coverage at the minima of the combinations.
- ▶ We find that the profile likelihood construction undercovers quite a bit.
- ▶ The robust plugin method has good coverage.
- ▶ The coverage of the full combination is worse than of the robust. Expected due to the low value of  $r_B^{D\pi}$ .

$\eta = 0.683$	$\alpha$ (prof. LH.)	$\alpha$ (plugin)
robust	0.6158	0.6494
full (1), $r_B^{D\pi} = 0.027$	0.5593	0.6154
full (2), $r_B^{D\pi} = 0.006$	0.5454	0.6120



# Auxiliary input from HFAG

comparing old and new



**Figure:** Profile likelihood contours: The “old” contour corresponds to what was used in the previous (2013) combination (the 2009 CLEO input [25] together with the 2013 LHCb charm mixing measurement [26]). The “new” contour is what is used in this combination (HFAG 2014). The contours are two-dimensional 1–4 $\sigma$  contours.

# Auxiliary input from HFAG

The parameter  $R_D^{K\pi}$  is the squared ratio of the doubly-Cabibbo-suppressed amplitude  $D^0 \rightarrow \pi^- K^+$  to the favored one  $D^0 \rightarrow K^- \pi^+$ . It is not the ratio of branching ratios. It gets often measured in time-dependent wrong-sign  $D^0$  mixing measurements:

$$R_{WS} = R_D^{K\pi} + \sqrt{R_D^{K\pi}} \left( x \cos(\delta_D^{K\pi}) \pm y \sin(\delta_D^{K\pi}) \right) \frac{t}{\tau} + \frac{x_D^2 + y_D^2}{4} \left( \frac{t}{\tau} \right)^2$$

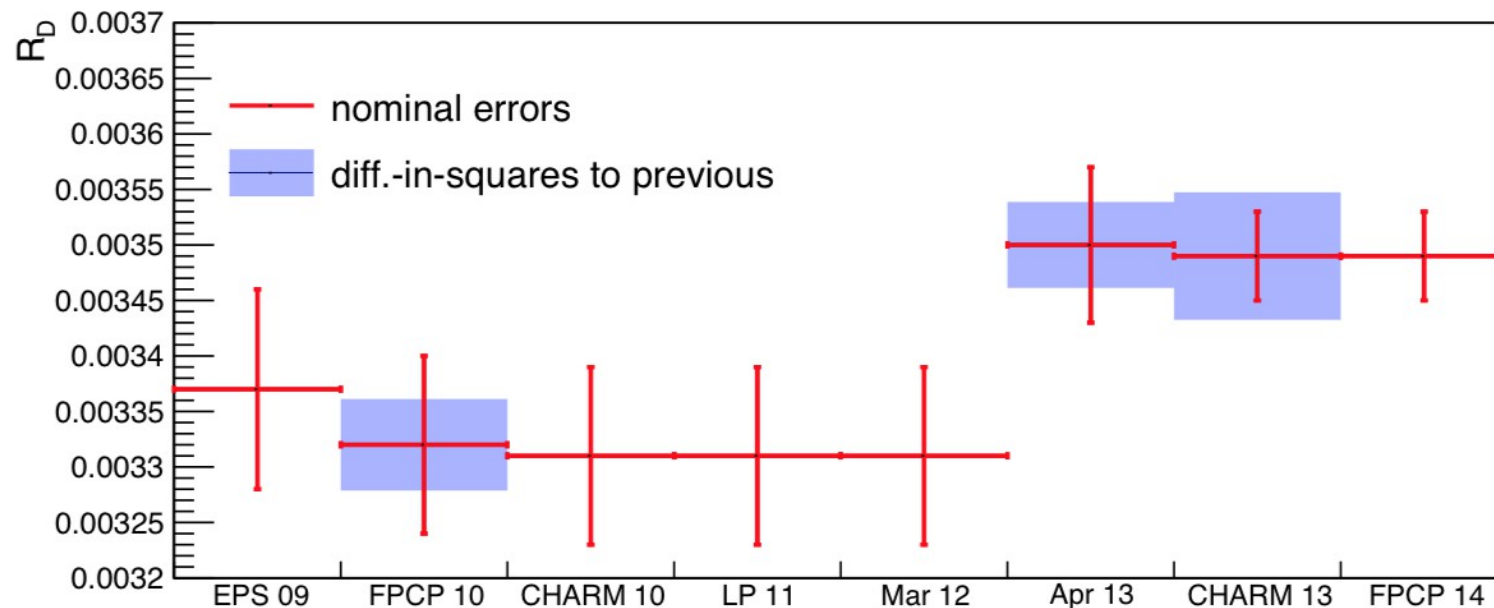


Figure: Evolution of HFAG results on  $R_D^{K\pi}$ .