Theory Higgs production

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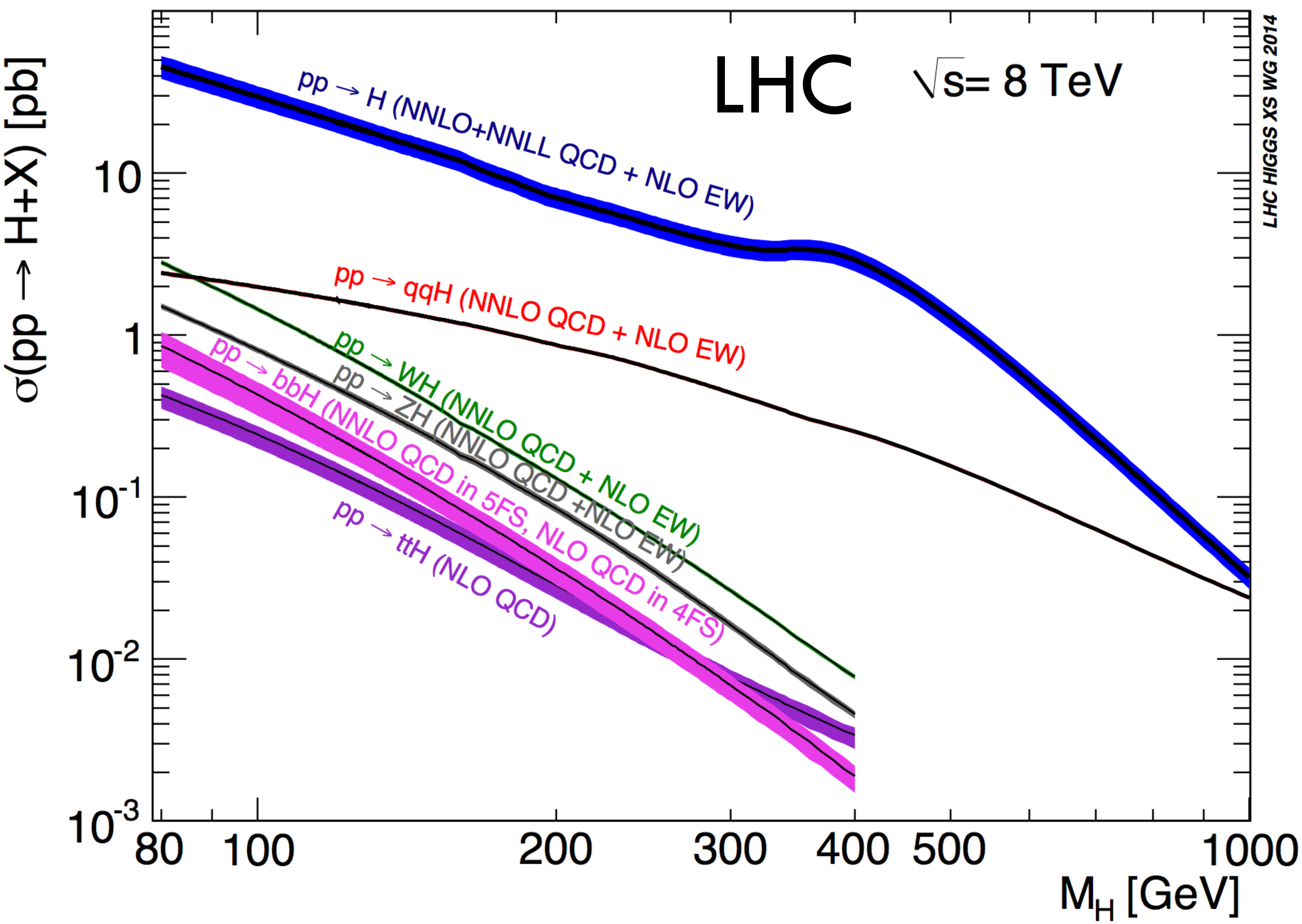
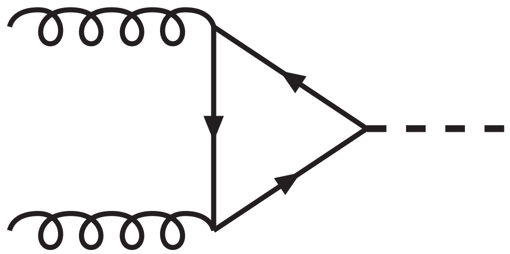
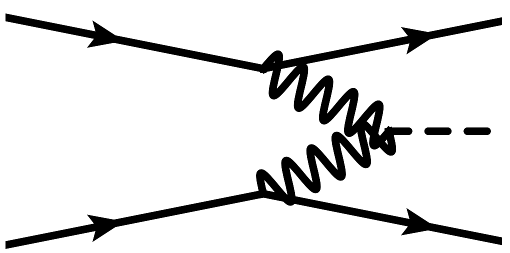
Higgs Hunting 2015
LAL Orsay, 30/07/2015

Higgs physics at the LHC

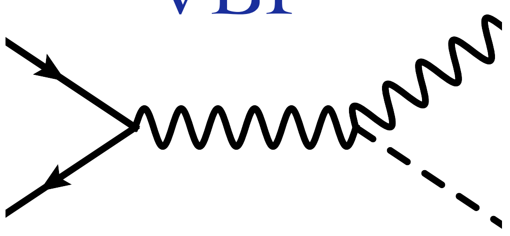




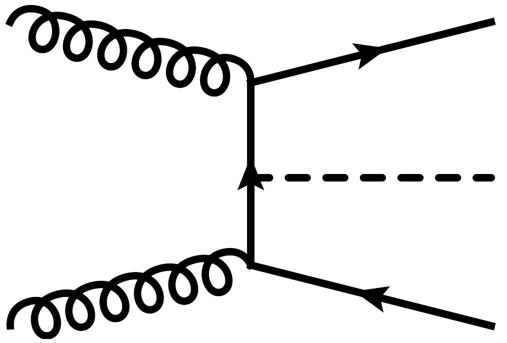
Gluon fusion



VBF

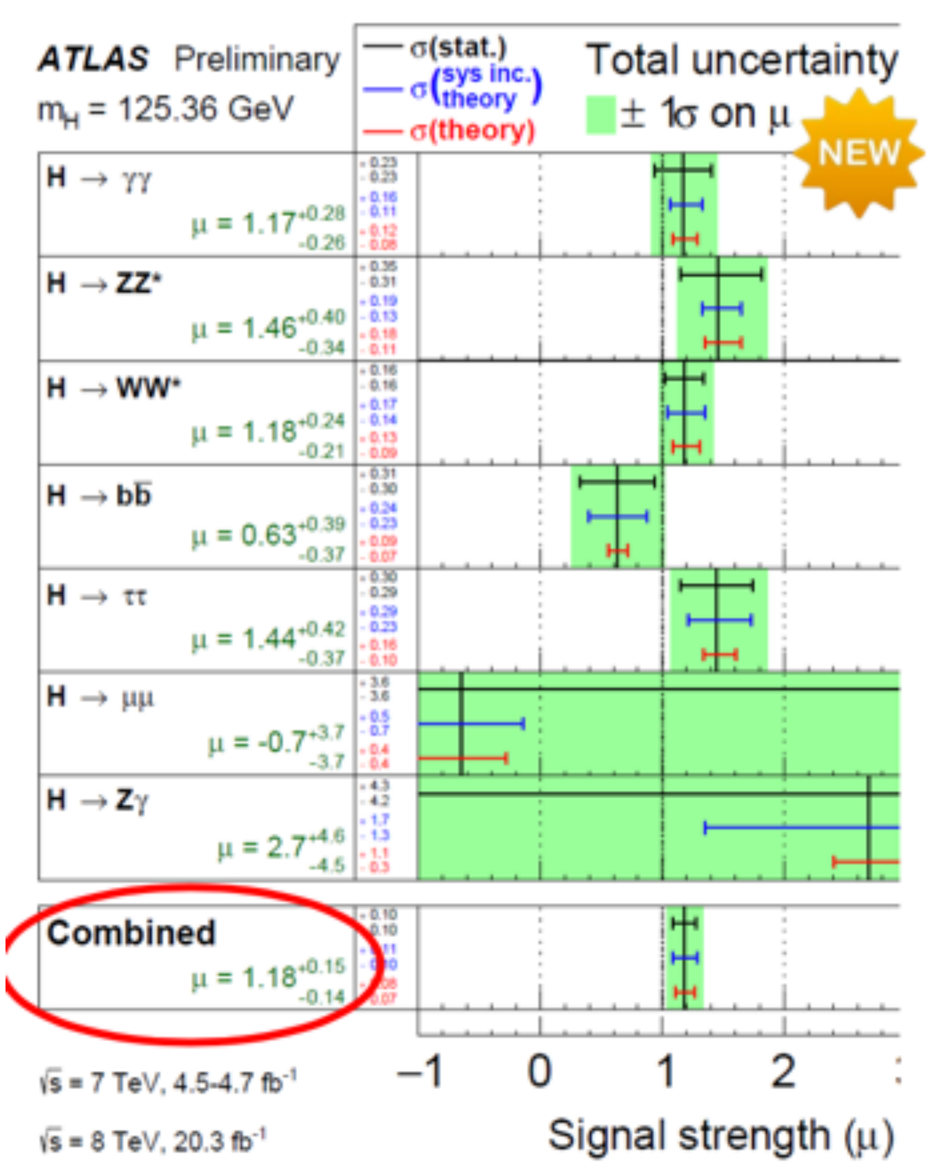


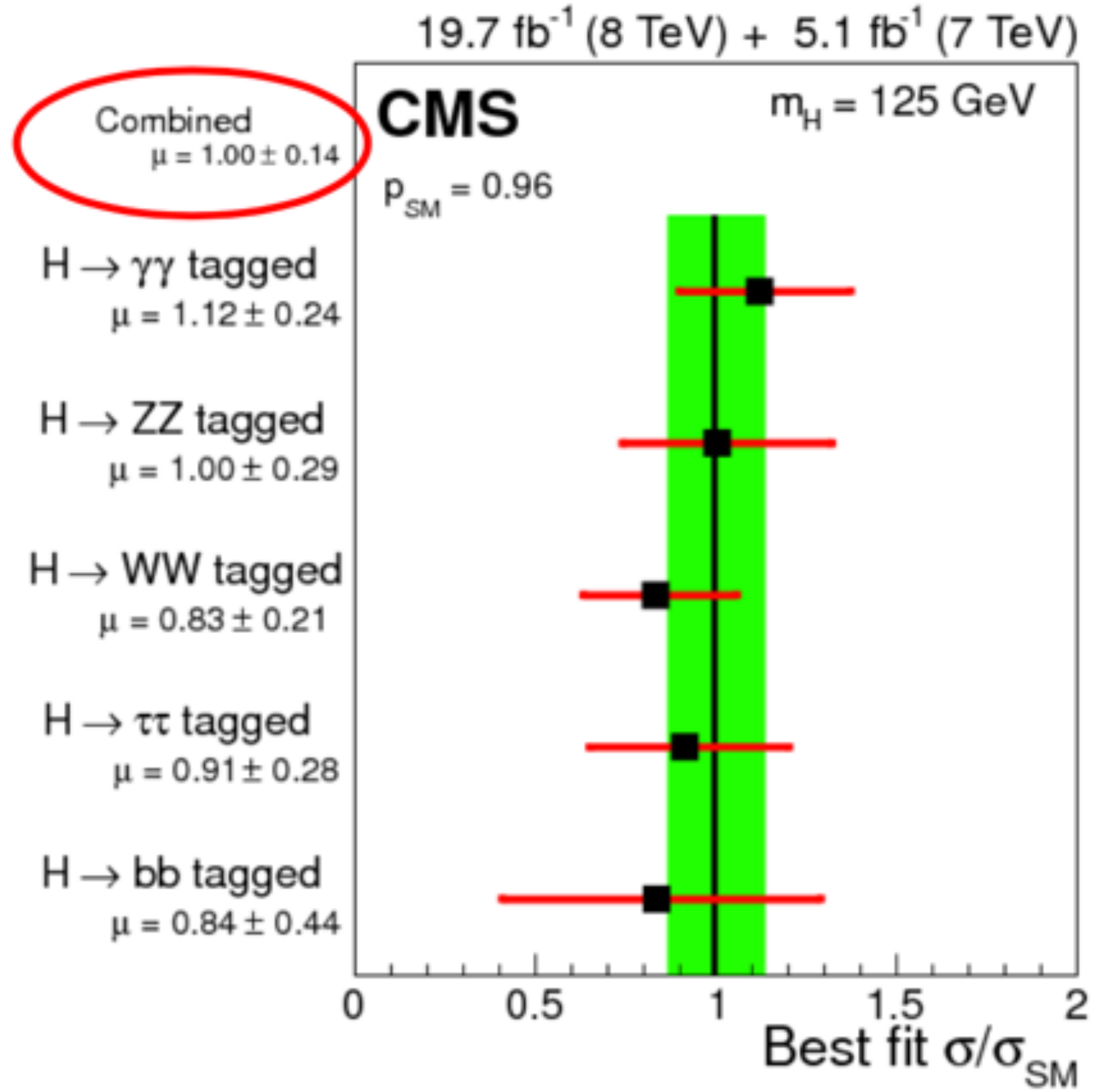
VH



TTH/bbH

Higgs physics at the LHC





$$\mu_{ATLAS} = 1.18^{+0.15}_{-0.14}$$

$$\mu_{CMS} = 1.00 \pm 0.14$$

$$\text{stat.} = {}^{+0.10}_{-0.10}$$

$$\text{sys. (inc. theo.)} = {}^{+0.11}_{-0.10}$$

$$\text{theory} = {}^{+0.08}_{-0.07}$$

[M. Dührssen @ Moriond EW 2015]

Higgs physics at the LHC

- Outline: important theoretical advances in predictions for Higgs production in the last few months:

  ➡ Fully differential NNLO predictions for VBF.

  ➡ Fully differential NNLO predictions for H + jet in gluon fusion.

  ➡ Inclusive gluon fusion cross section at N3LO.

  ➡ New PDF sets, with reduced uncertainty.

- These advances show the maturity of our tools to make precision computations in QCD!

VBF @ NNLO

Vector boson fusion

- VBF is the 2nd largest production channel at the LHC.
  - ➡ Direct access to HVV coupling.
  - ➡ Non-zero H-pT at leading order.
  - ➡ Radiation pattern allows one to disentangle ggH from VBF (VBF cuts).

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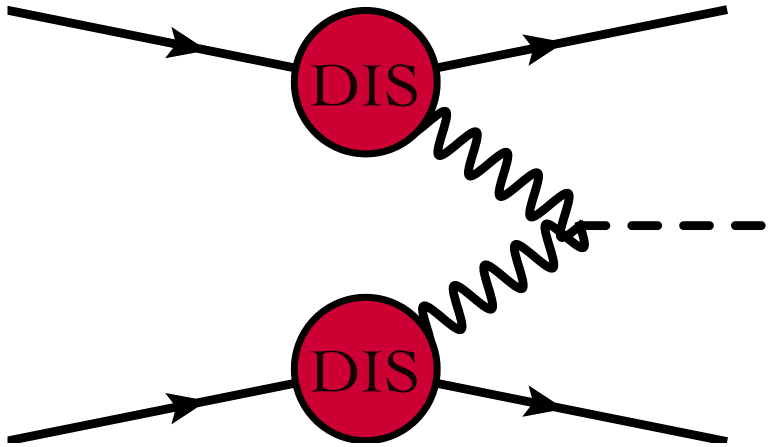
  - ➡ Radiation pattern allows one to disentangle ggH from VBF (VBF cuts).
- LO process is 2-to-3: Two-loop corrections unknown!
- Form factor approach: no colour exchange between the two quark lines.
  - ➡ Exact at NLO.
  - ➡ VBF = $(\mathrm{DIS})^2$.
  - ➡ Used to compute inclusive VBF cross section at NNLO.



[Bolzoni, Maltoni, Moch, Zaro]

Vector boson fusion

- BMMZ: NNLO effects in inclusive cross section small (~1%).
- Inclusive cross section is not quite what we want:
  - ➡ No differential information.
  - ➡ Cannot impose VBF cuts.

Vector boson fusion

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- Inclusive cross section is not quite what we want:
  - ➡ No differential information.
  - ➡ Cannot impose VBF cuts.
- Recently: first fully differential computation of VBF at NNLO in form factor approach. [Cacciari, Dreyer, Karlberg, Salam, Zanderighi]
  - ➡ Rather large NNLO corrections after VBF cuts (~5-6%)!

$$p_{T,j_1}, p_{T,j_2} > 25\,\mathrm{GeV}$$

$$|y_{j_1}|, |y_{j_2}| < 4.5$$

$$\Delta y_{j_1,j_2} > 4.5 \qquad M^2_{j_1 j_2} > (600\mathrm{GeV})^2$$

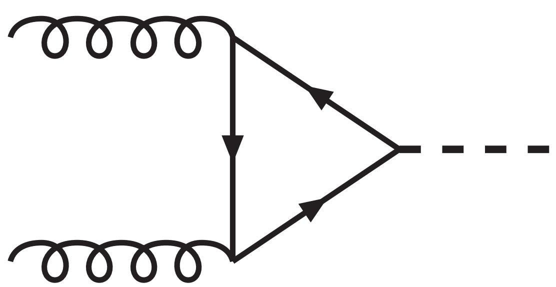
	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032\,^{+0.057}_{-0.069}$	$0.957\,^{+0.066}_{-0.059}$
NLO	$3.929\,^{+0.024}_{-0.023}$	$0.876\,^{+0.008}_{-0.018}$
NNLO	$3.888\,^{+0.016}_{-0.012}$	$0.826\,^{+0.013}_{-0.014}$
	~ 1	~5-6

➡ See Dreyer's talk this afternoon!

Gluon-fusion

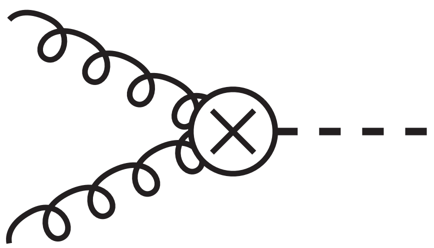
The gluon fusion cross section

- The dominant Higgs production mechanism at the LHC is gluon fusion.
  ➡ Loop-induced process.



- For a light Higgs boson, the dimension five operator describing a tree-level coupling of the gluons to the Higgs boson

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 \, H \, G^a_{\mu\nu} \, G_a^{\mu\nu}$$



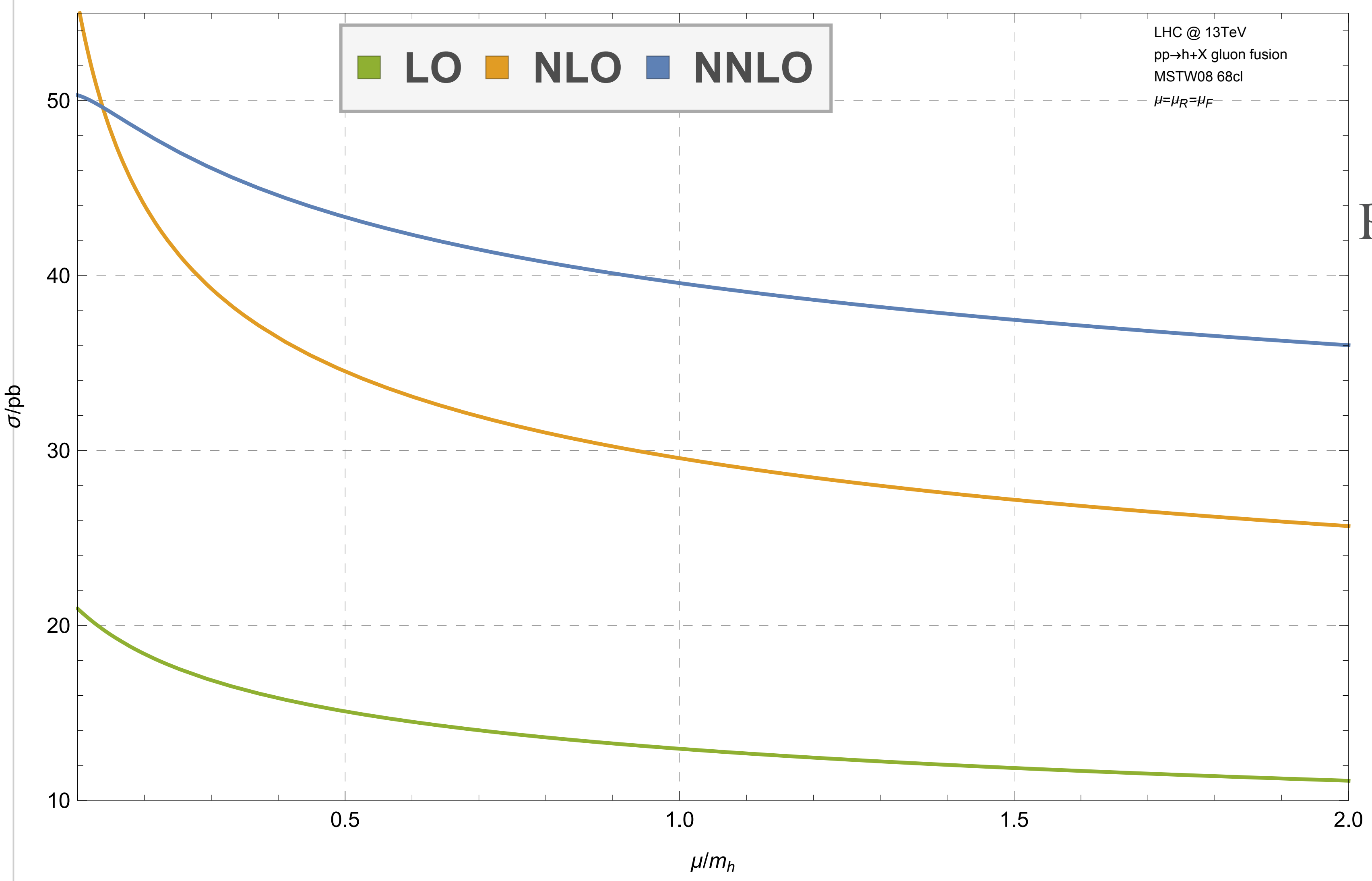
- Top-mass corrections known at NNLO.

[Harlander, Ozeren; Pak, Rogal, Steinhauser; Ball, Del Duca, Marzani, Forte, Vicini; Harlander, Mantler, Marzani, Ozeren]

- In the rest of the talk, I will only concentrate on the effective theory.

The gluon fusion cross section

- Known inclusively at NLO and NNLO, but plagued by large perturbative uncertainties.



[Dawson; Djouadi, Spira, Zerwas; Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]

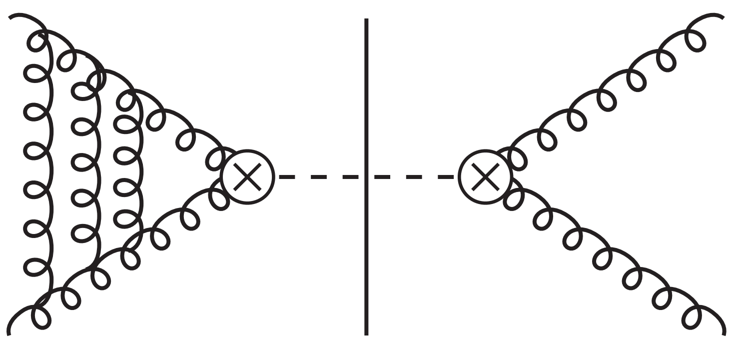
- Enormous progress over the last few months!
  - ➡ Both inclusively and differentially.

Gluon-fusion

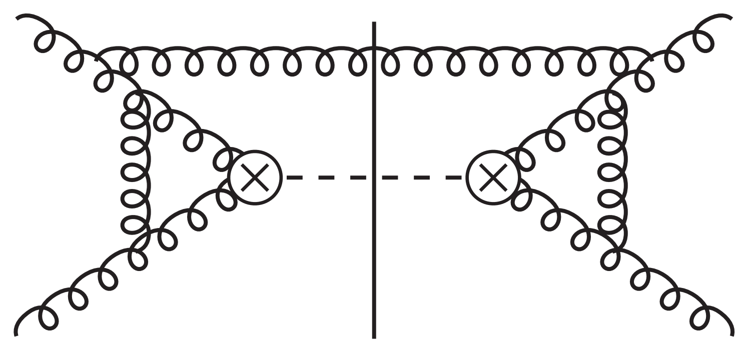
H @ N3LO

The gluon fusion cross section

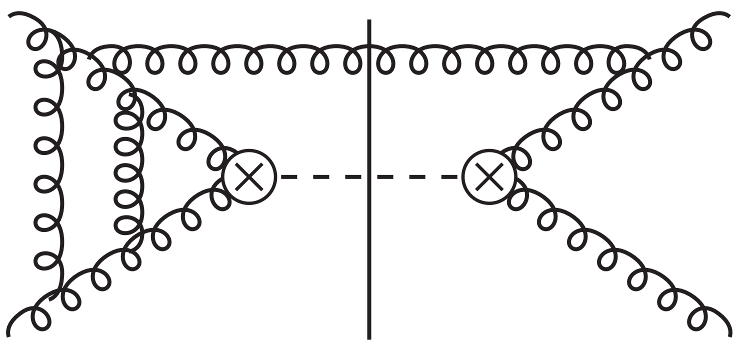
- At N3LO, there are five contributions:



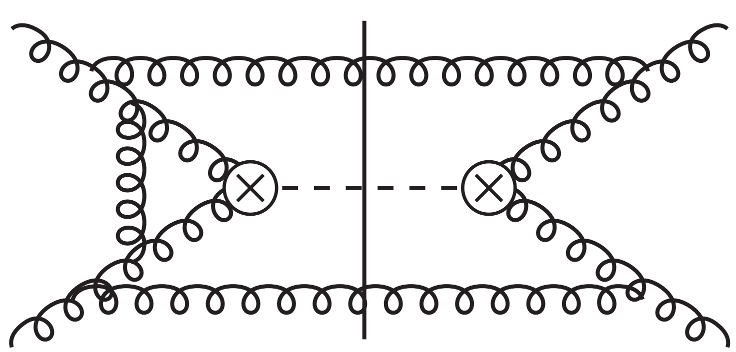
Triple virtual



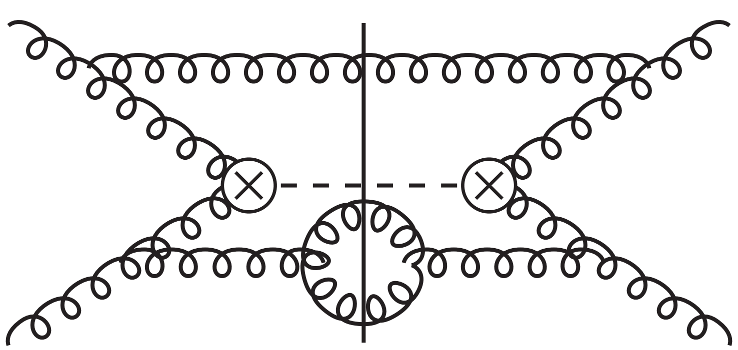
Real-virtual squared



Double virtual real



Double real virtual



Triple real

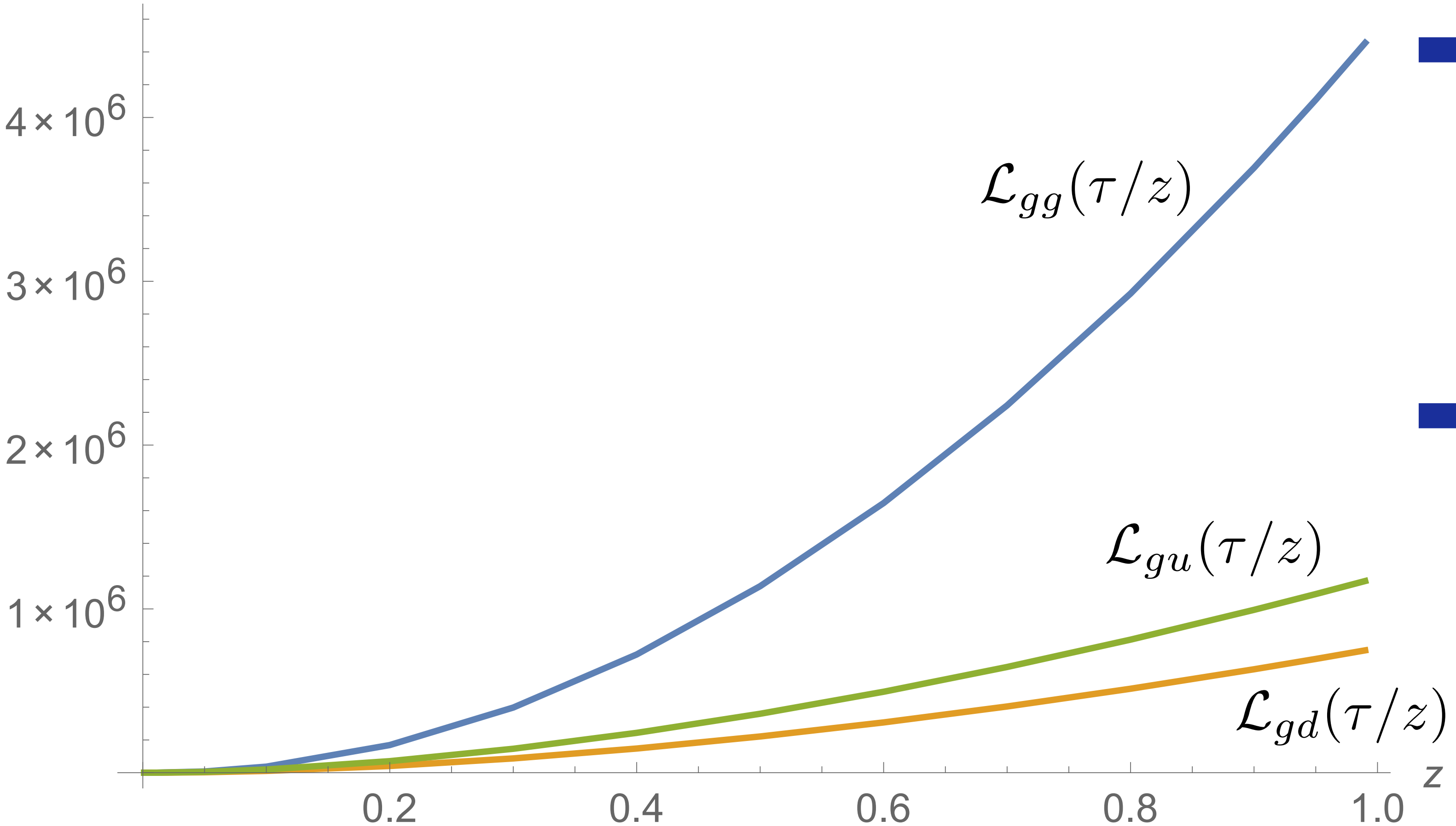
The gluon fusion cross section

- The gluon fusion cross section is given in perturbation theory by

$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z}\, \mathcal{L}_{ij}(\tau/z)\, \frac{\hat{\sigma}_{ij}(z)}{z}$$

$$z = \frac{m_H^2}{\hat{s}}$$

$$\tau = \frac{m_H^2}{S} \simeq 10^{-4}$$



➡ Main contribution from region where $z \simeq 1$.

➡ Physically: production at threshold + emission of soft partons.

Systematics of the expansion

$$\frac{\hat{\sigma}_{ij}(z)}{z} = \hat{\sigma}^{SV}\,\delta_{ig}\,\delta_{jg} + \sum_{N=0}^{\infty} \hat{\sigma}_{ij}^{(N)}\,(1-z)^N$$

- **Goal:** Compute enough terms to establish convergence.

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- The coefficients in the expansion are not constants, but they are polynomials in $\log(1-z)$.
  - ➡ At N3LO: $\hat{\sigma}_{ij}^{(N)} = \sum_{k=0}^{5} c_{ijk}^{(N)} \log^k(1-z)$
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- The first term is called the soft-virtual term and is distribution-valued:

  ➡ At N3LO: $\hat{\sigma}^{SV} = a\,\delta(1-z) + \sum_{k=0}^{5} b_k\,\left[\frac{\log^k(1-z)}{1-z}\right]_+$

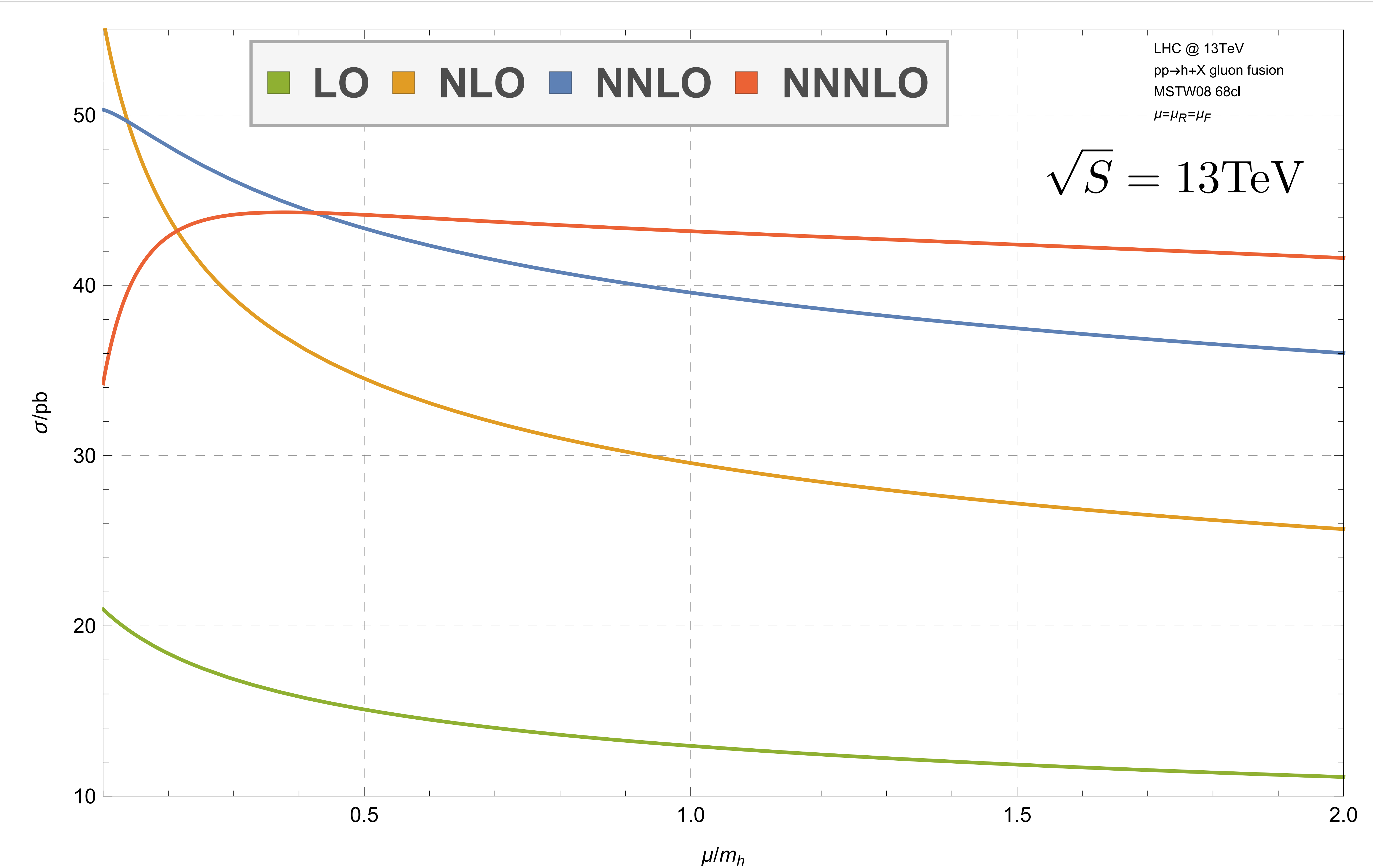
Some numbers

	NNLO	N3LO
# diagrams	~ 1.000	~ 100.000
# integrals	~50.000	517.531.178
# masters	27	1.028
# boundary conditions	5	78

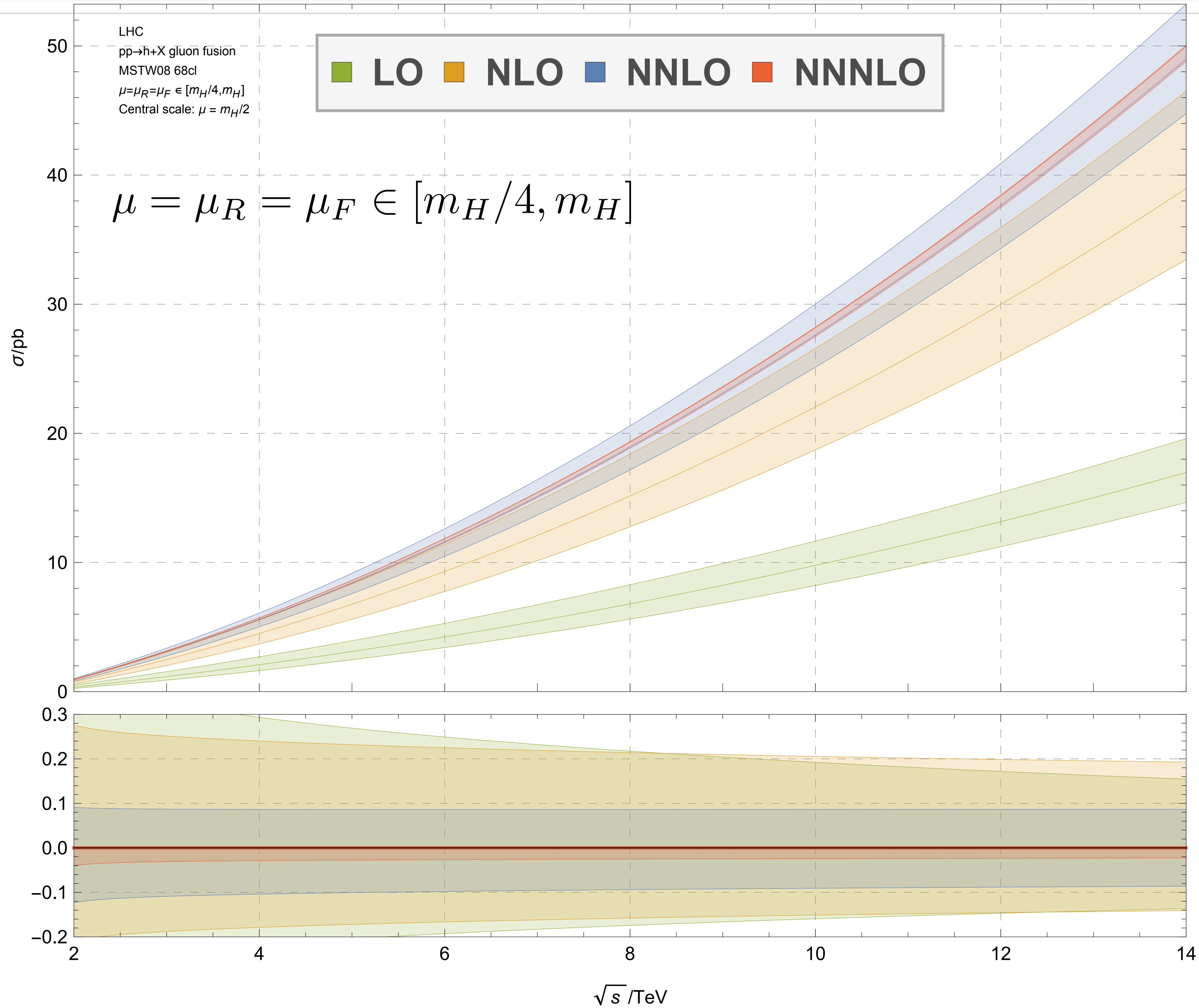
- This brings you to the edge of what is technically possible at the moment.

➡ A lot of cross talk with formal amplitudes community!

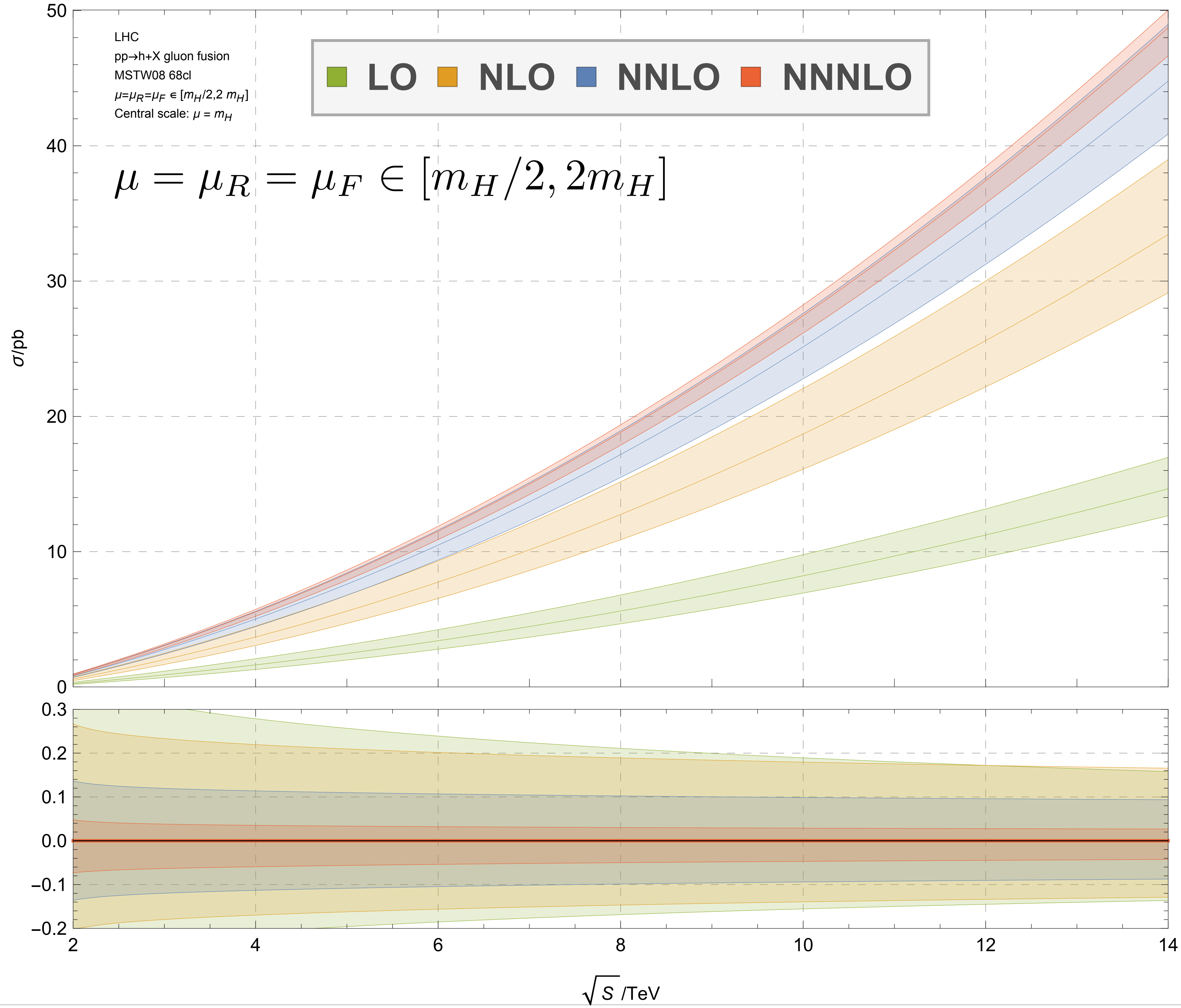
Scale variation



Energy variation



Energy variation



Uncertainties

- Remaining scale uncertainty at N3LO
  - ➡ We should think very carefully which other effects could be of the same size!
- Other sources of uncertainty:

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[Harlander, Mantler, Marzani, Ozeren]

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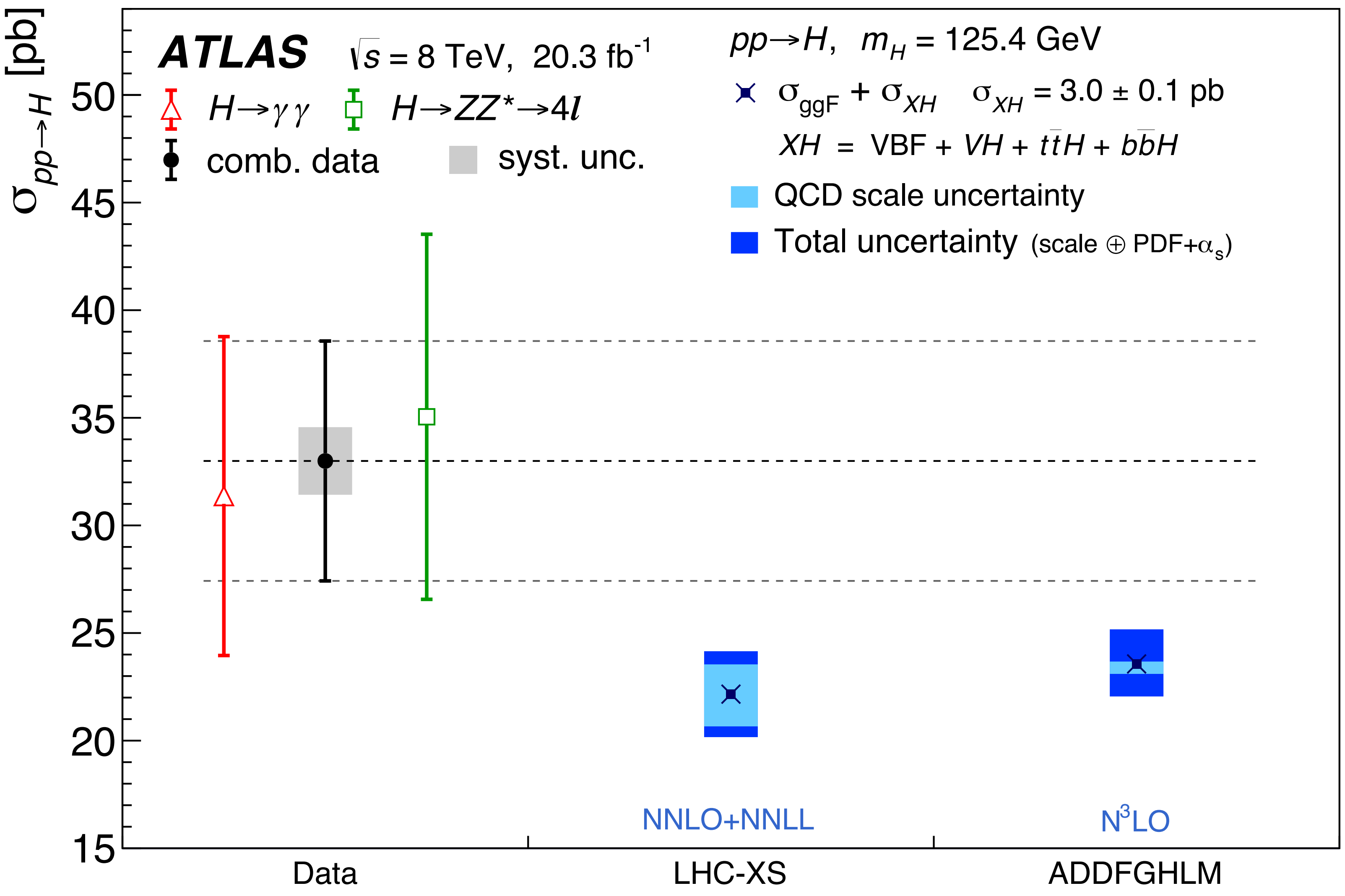
  - ➡ PDF + aS:

Scale vs. PDF uncertainty



	CT14	MMHT2014	NNPDF3.0	CT10
8 TeV	$18.66^{+2.1\%}_{-2.3\%}$	$18.65^{+1.4\%}_{-1.9\%}$	$18.77^{+1.8\%}_{-1.8\%}$	$18.37^{+1.7\%}_{-2.1\%}$
13 TeV	$42.68^{+2.0\%}_{-2.4\%}$	$42.70^{+1.3\%}_{-1.8\%}$	$42.97^{+1.9\%}_{-1.9\%}$	$42.20^{+1.9\%}_{-2.5\%}$

[CTEQ collaboration]

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[Djouadi, Gambino, Kniehl; Aglietti, Bonciani, Degrassi; Degrassi, Maltoni; Anastasiou, Boughezal, Petriello; Actis, Passarino, Sturm, Uccirati]

$$\sigma_0\left(1+\delta_{\mathrm{QCD}}+\delta_{\mathrm{EW}}\right) \quad \text{vs.} \quad \sigma_0\left(1+\delta_{\mathrm{QCD}}\right)\left(1+\delta_{\mathrm{EW}}\right)$$

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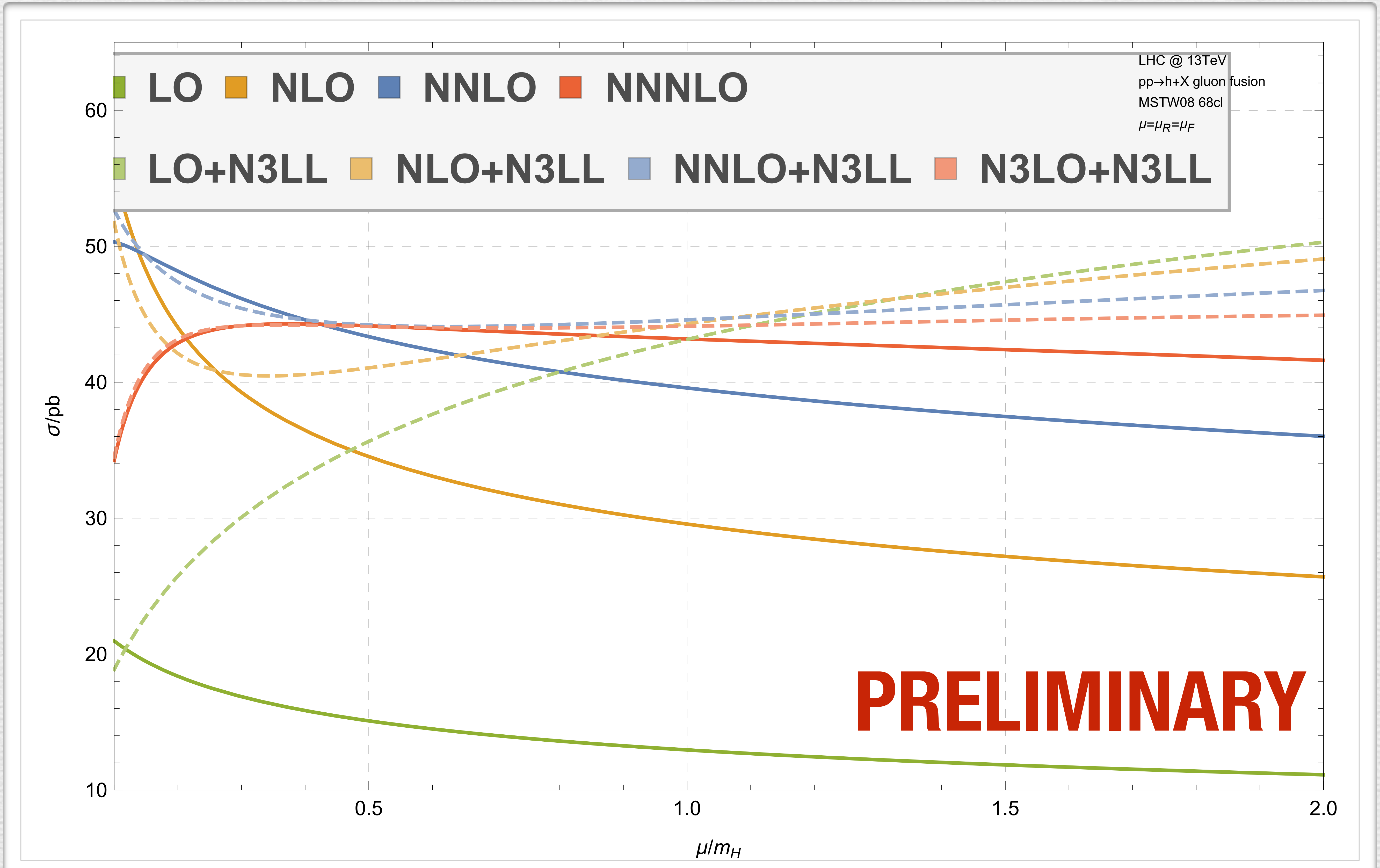
  - ➡ Missing higher orders / threshold resummation

N3LL threshold resummation



Summary for H @ N3LO

- We are currently putting together all these effects (including different 'flavours' or threshold resummation).

- $\mu = m_H/2$ seems to be a good central scale choice.
  - ➡ Reduced scale uncertainty compared to $\mu = m_H$.
  - ➡ Series seems to converge.
  - ➡ Negligible impact of soft-gluon resummation.
  - ➡ Current recommendation of HXSWG: $\mu = m_H$.

- We are reaching the point where we should critically assess our method of estimating the uncertainty by scale variation!
  - ➡ A negligible scale variation does not mean that there are no more higher-order corrections!

Gluon-fusion

H+j @ NNLO

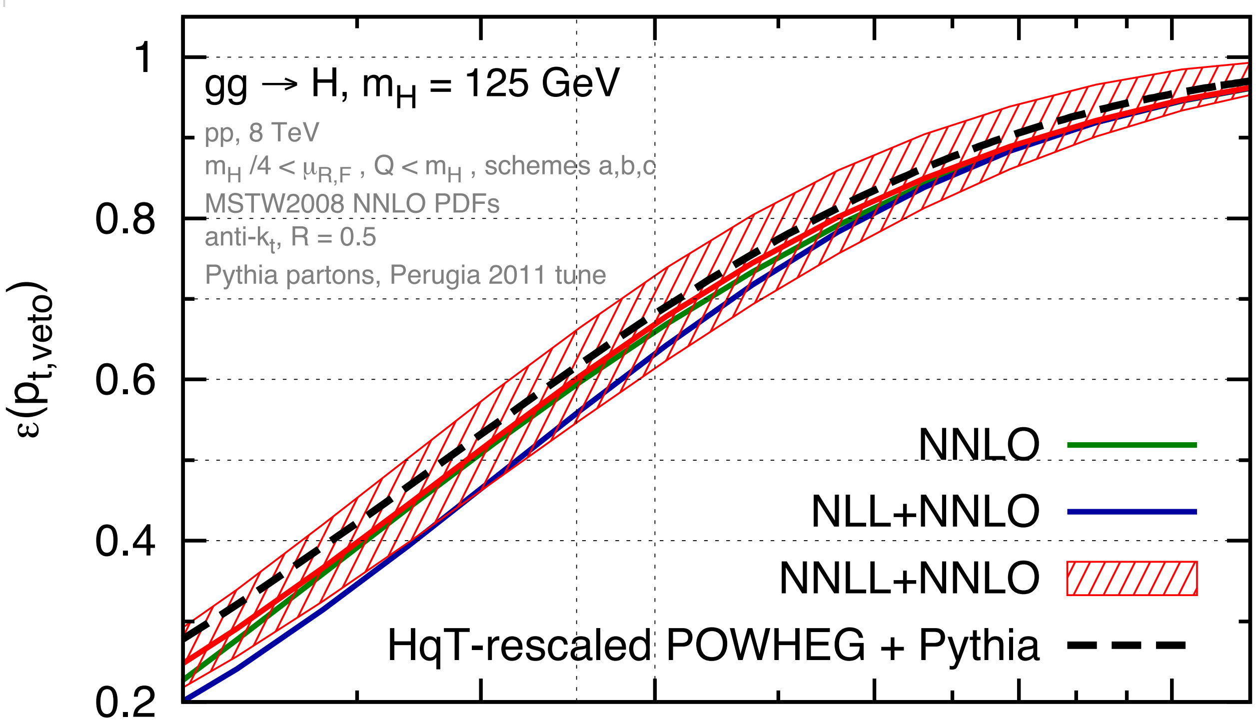
H+j @ NNLO

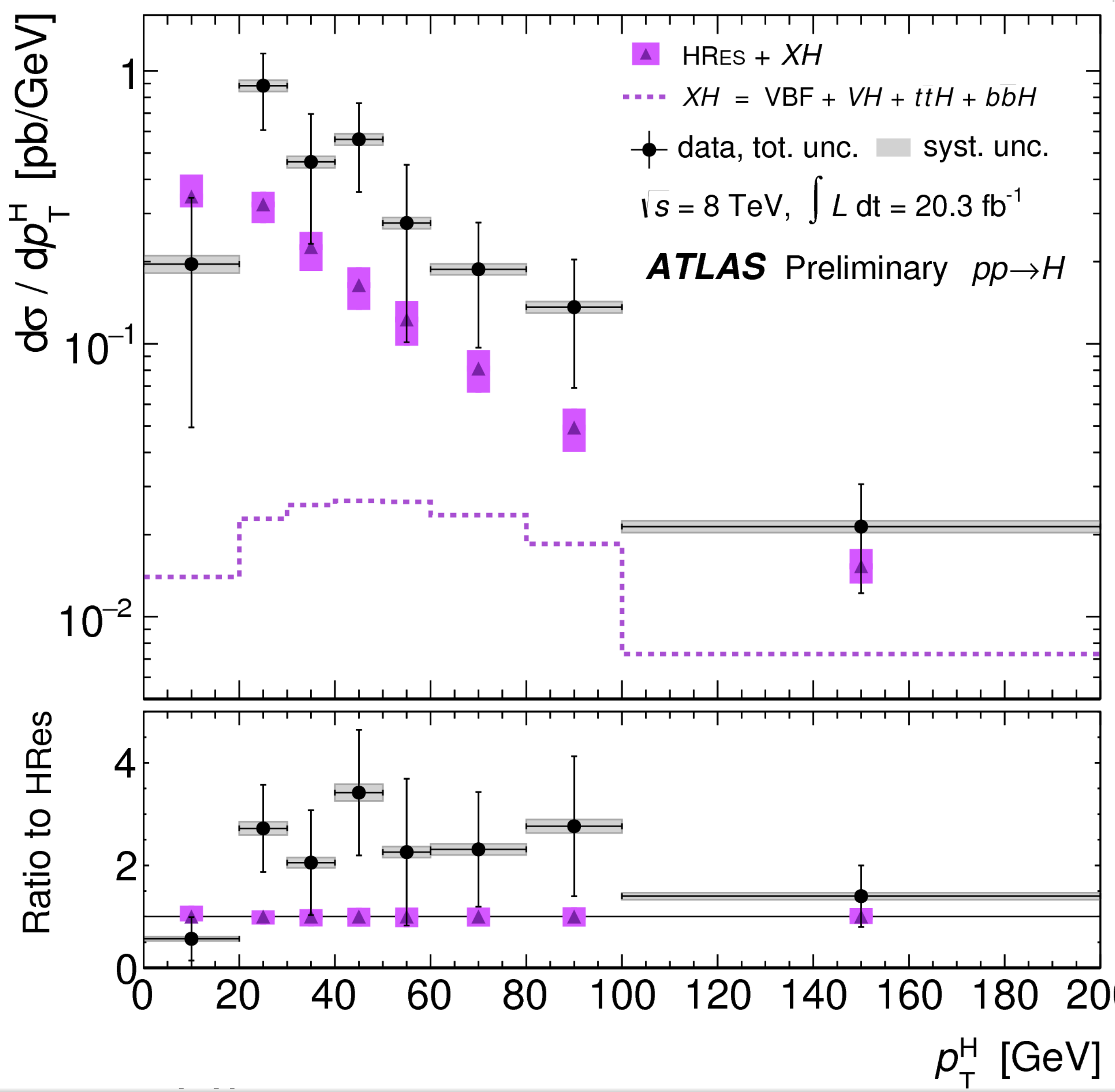
- Recently, H+j @ NNLO became available.

  [Boughezal, Caola, Melnikov, Petriello, Schulze; Boughezal, Focke, Giele, Liu, Petriello; Chen, Gehrmann, Glover, Jaquier]

  ➡ Higgs-pT beyond NLO.

  ➡ Reduced uncertainties for jet-veto efficiencies.





H+j @ NNLO

- **Now:** Predictions at NNLO accuracy for
  - ➡ (arbitrary) differential distributions
  - ➡ with (arbitrary) cuts on the final state (fiducial volume!)

H+j @ NNLO

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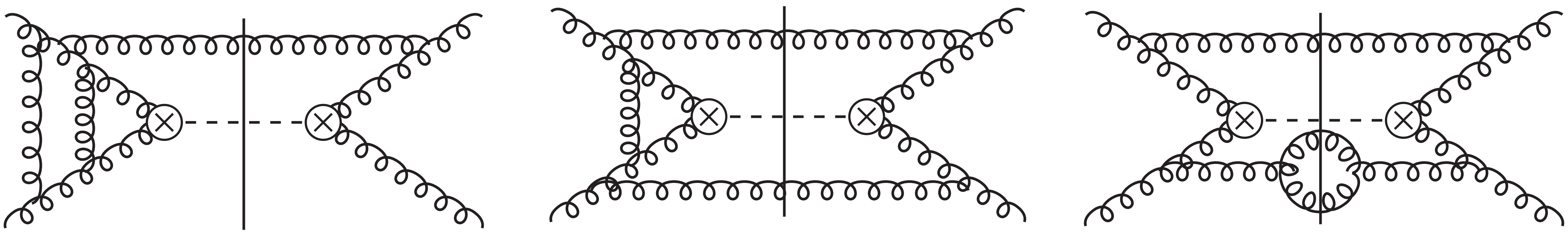
  ➡ with (arbitrary) cuts on the final state (fiducial volume!)

- Only possible to due major advances in our understanding of how to cancel IR singularities at NNLO!



Double virtual　　Real-virtual　　Double-real

➡ Different contributions individually divergent.

➡ Divergences cancel in the sum.

➡ Different contributions live in different phase spaces.

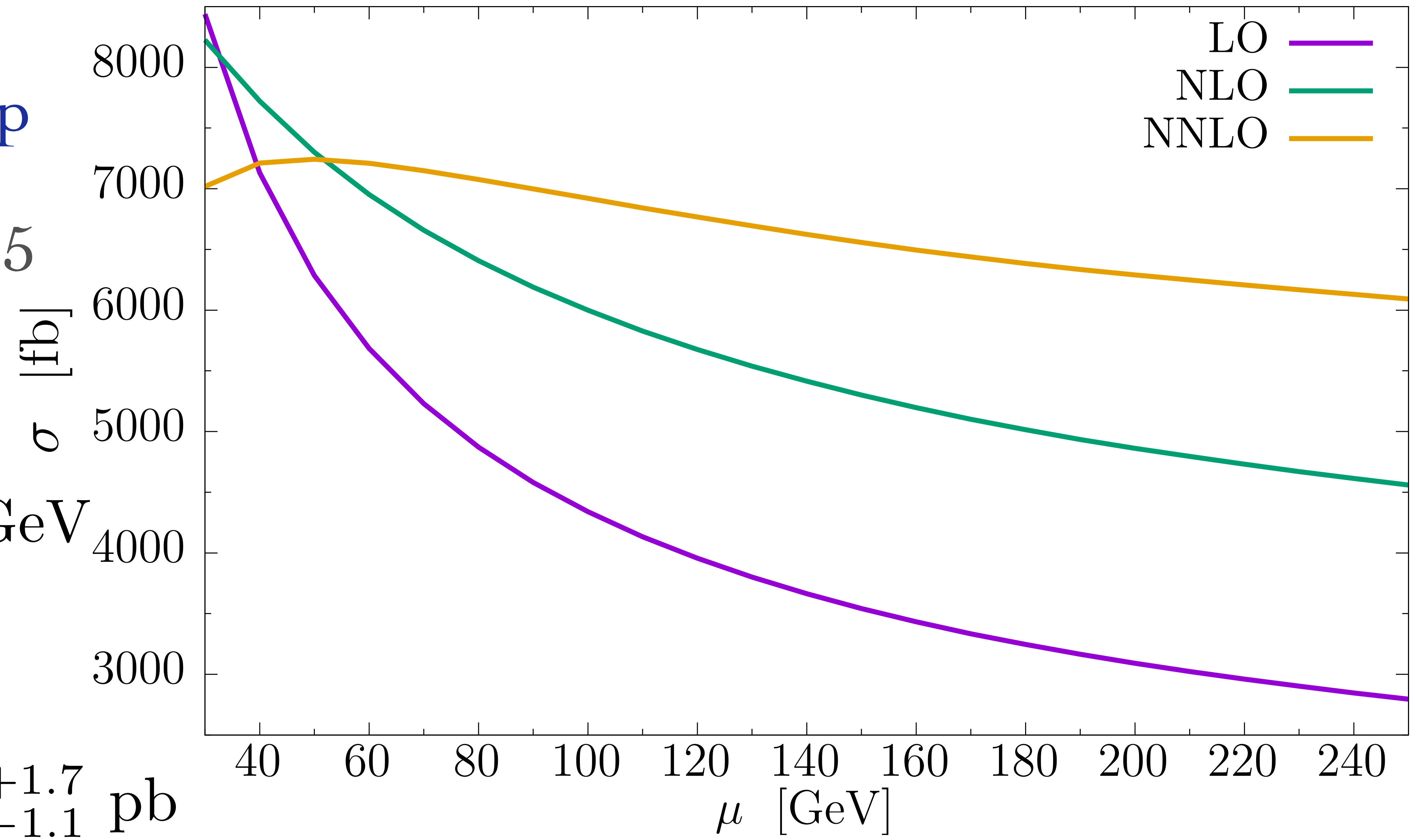
IR singularities

- Basic Idea: IR singularities of QCD amplitudes are known.
  - ➡ Use this to add and subtract counterterms that render all integrals finite.
  - ➡ Idea simple in principle, but very complicated in practise due to intricate nature of singularity structure at NNLO.
  - ➡ A lot of progress in the last few years!
- H+j@NNLO was done using 3 different schemes to combine virtual and real corrections:
  - ➡ Antenna subtraction. [Kosower; Gehrmann, Gehrmann-de Ridder, Glover]
  - ➡ Stripper. [Czakon; Boughezal, Melnikov, Petriello; Czakon, Heymes]
  - ➡ N-jettiness subtraction. [Boughezal, Focke, Giele, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh]

NNLO cross section for H+j

Sample setup

- anti-kt, R=0.5
- NNPF 2.3
- $p_{T,\mathrm{cut}} > 30\,\mathrm{GeV}$



[Plot from Boughezal, Caola, Melnikov, Petriello, Schulze]

$$\sigma_{\mathrm{LO}} = 3.9^{+1.7}_{-1.1}\ \mathrm{pb}$$

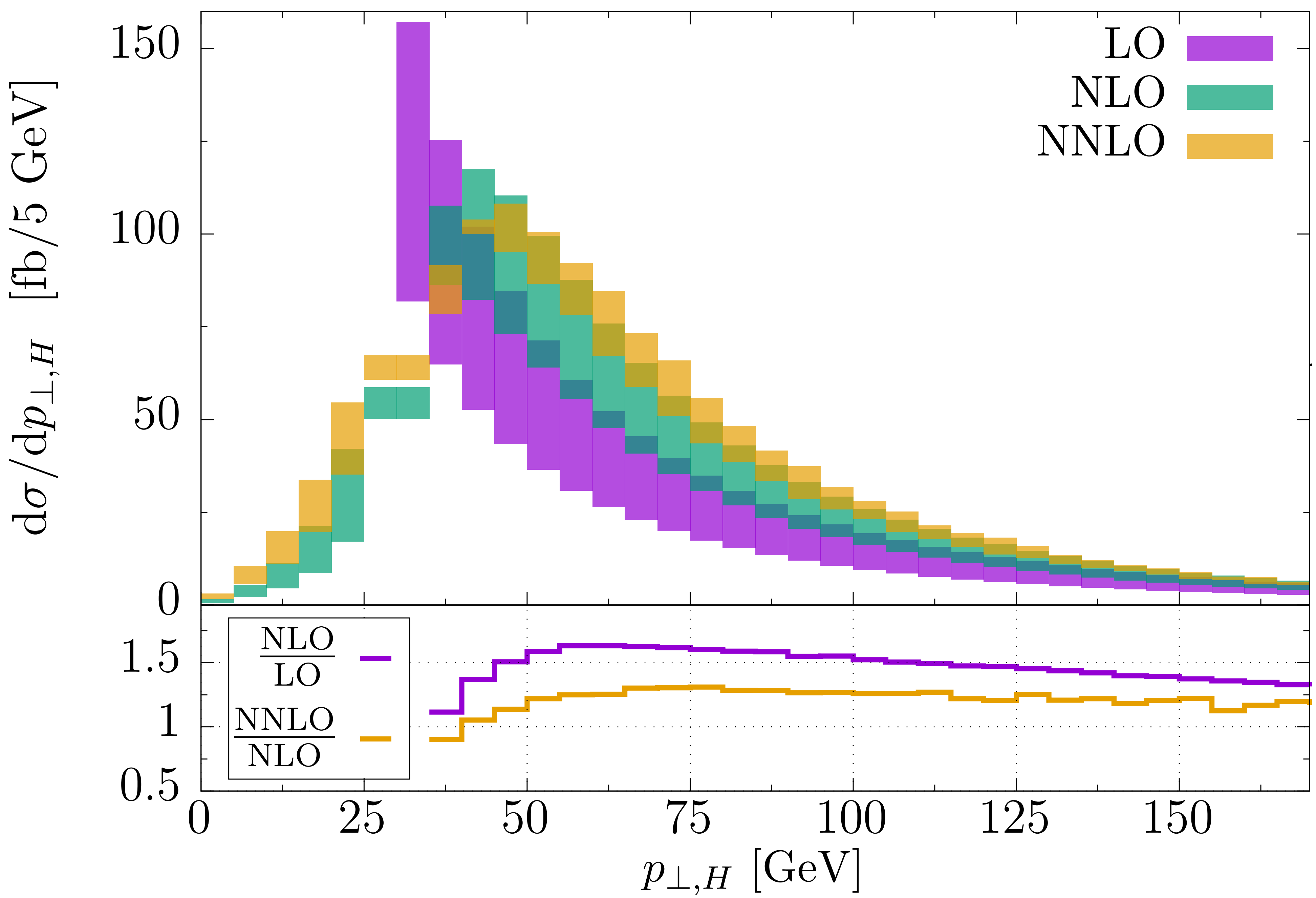
$$\sigma_{\mathrm{NLO}} = 5.6^{+1.3}_{-1.1}\ \mathrm{pb}$$

$$\sigma_{\mathrm{NNLO}} = 6.7^{+0.5}_{-0.6}\ \mathrm{pb}$$

$$K_{\mathrm{NNLO}}(\mu = m_H) \sim 20\%$$

$$K_{\mathrm{NNLO}}(\mu = m_H/2) \sim 4\%$$

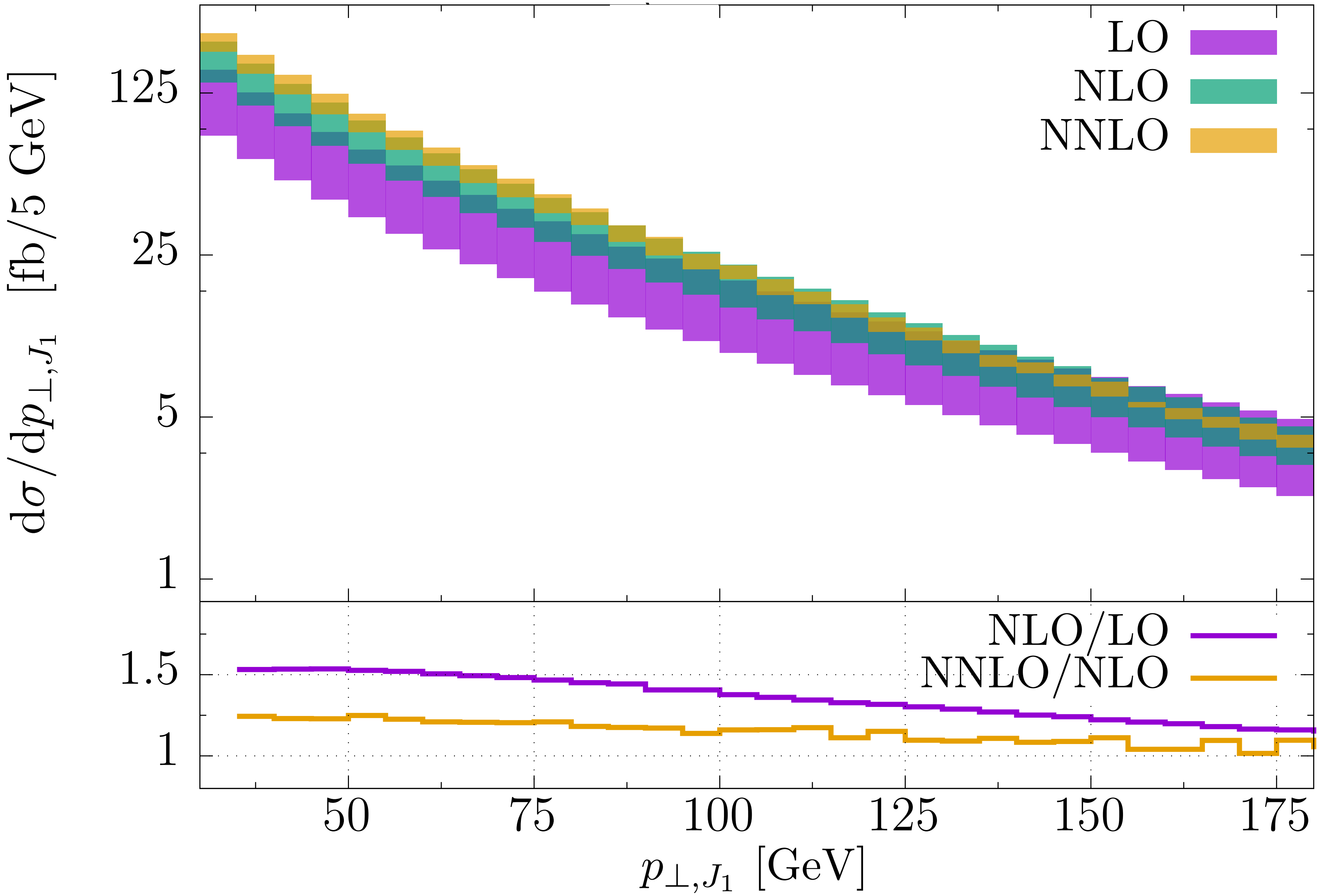
pT distributions



[Plot from Boughezal, Caola, Melnikov, Petriello, Schulze]

- Expect EFT to work within 2-3% up to pT ~ 150GeV.

pT distributions



[Plot from Boughezal, Caola, Melnikov, Petriello, Schulze]

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Jet veto efficiency

- H + j @ NNLO gives very accurate predictions for the 1st jet bin.
- H+j @ NNLO is at the same order in α_S as the inclusive cross section at N3LO.
  - ➡ Can combine the two and get very precise predictions for the 0 jet bin!

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- Example: Jet veto efficiency

$$\varepsilon(p_{T,\mathrm{veto}}) = \frac{\Sigma_0(p_{T,\mathrm{veto}}) + \Sigma_1(p_{T,\mathrm{veto}}) + \Sigma_2(p_{T,\mathrm{veto}}) + \Sigma_3(p_{T,\mathrm{veto}}) + \ldots}{\sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots}$$

$$\Sigma_i(p_{T,\mathrm{veto}}) = \sigma_i - \int_{p_{T,\mathrm{veto}}}^{\infty} dp_T \, \frac{d\sigma_i}{dp_T}$$

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(H@N3LO) - (H+J@NNLO)

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H@N3LO

$$\Sigma_i(p_{T,\text{veto}}) = \sigma_i - \int_{p_{T,\text{veto}}}^{\infty} dp_T \, \frac{d\sigma_i}{dp_T}$$

Jet veto efficiency

0-jet bin

ord	$\sigma^{\mathrm{f.o.}}_{0-\mathrm{jet}}$ (JVE)	$\sigma^{\mathrm{f.o.+NNLL}}_{0-\mathrm{jet}}$ (JVE)	$\sigma^{\mathrm{f.o.+NNLL}}_{0-\mathrm{jet}}$ (scales)
NNLO	$26.2^{+4.0}_{-4.0}$ pb	$25.8^{+3.8}_{-3.8}$	$25.8^{+1.6}_{-1.6}$
N^{3}LO	$27.2^{+2.7}_{-2.7}$ pb	$27.2^{+1.4}_{-1.4}$	$27.2^{+0.9}_{-0.9}$

≥1-jet bin

ord	$\sigma^{\mathrm{f.o.}}_{\geq 1-\mathrm{jet}}$ (scales)	$\sigma^{\mathrm{f.o.}}_{\geq 1-\mathrm{jet}}$ (JVE)	$\sigma^{\mathrm{f.o.+NNLL}}_{\geq 1-\mathrm{jet}}$ (JVE)
NLO	$14.7^{+2.8}_{-2.8}$ pb	$14.7^{+3.4}_{-3.4}$	$15.1^{+2.7}_{-2.7}$
NNLO	$17.5^{+1.3}_{-1.3}$ pb	$17.5^{+2.6}_{-2.6}$	$17.5^{+1.1}_{-1.1}$

- Logs completely under control
  (logR: see [Dasgupta, Dreyer, Salam, Soyez (2015)])
- No breakdown of f.o. perturbation theory for $p_T \sim 30$ GeV
- Reliable error estimate from lower orders
- Logs help in reducing uncertainties
- Significant decrease of pert. uncertainty

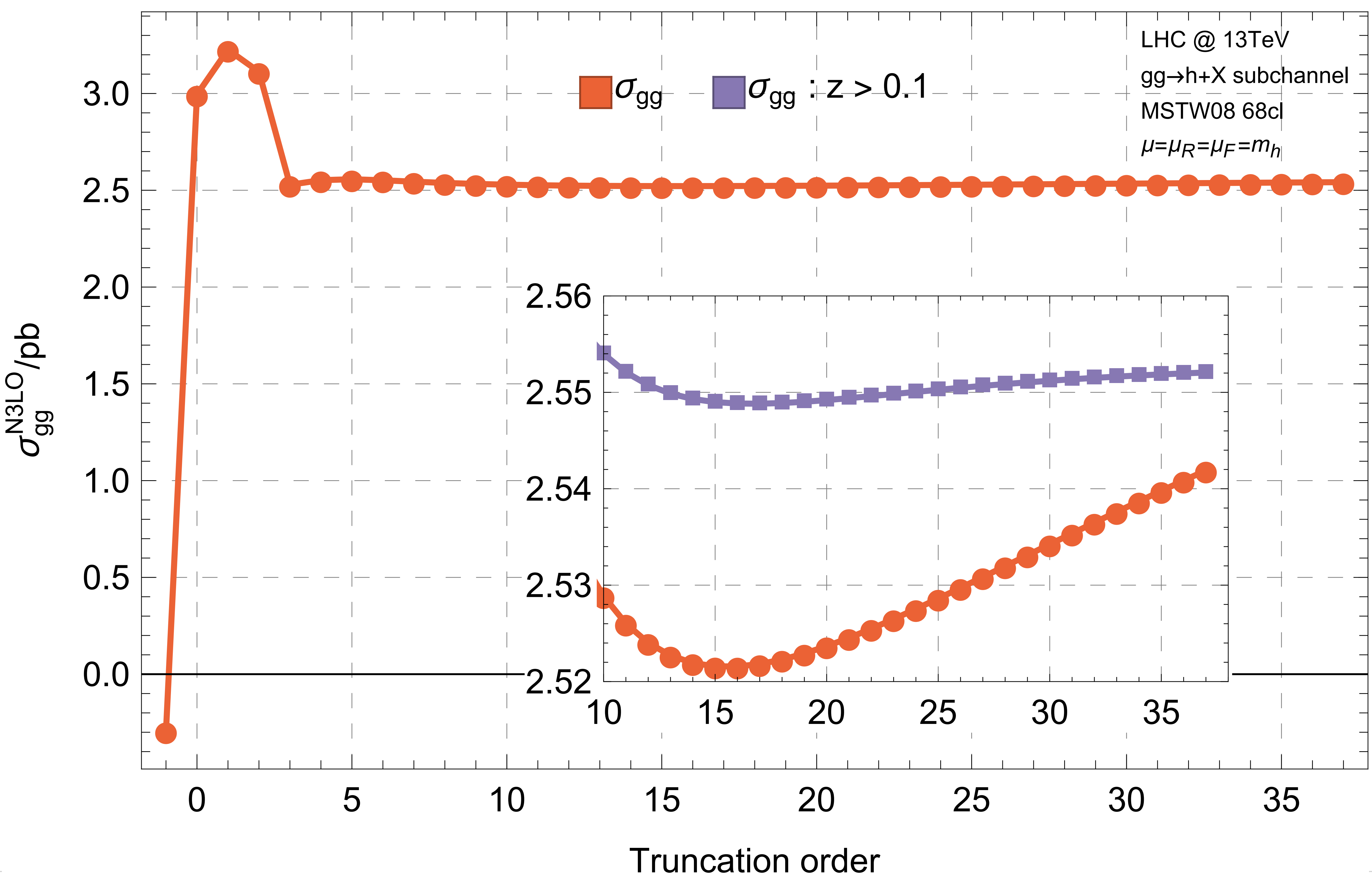
[Slide from Caola, HXSWG meeting, CERN, May 2015]

Conclusion

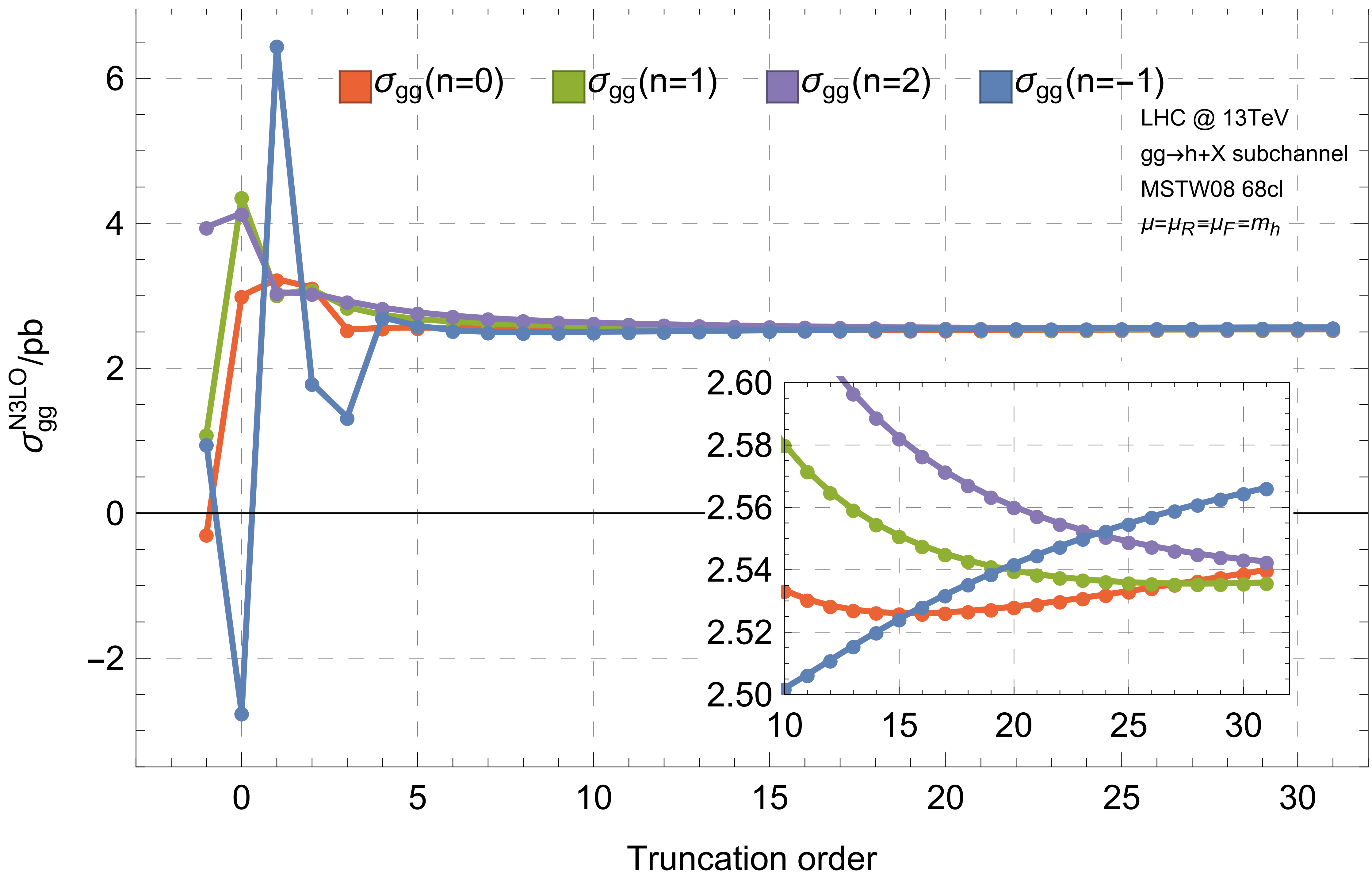
- In the last 6 months:
  - ➡ Fully differential VBF @ NNLO.
  - ➡ Fully differential H+j @ NNLO.
  - ➡ Inclusive H @ N3LO.
- Drastic improvements of theoretical uncertainties!
  - ➡ Get theory predictions under control.
  - ➡ We are getting ready for precision Higgs physics!
- Our tools for QCD computations beyond NLO are getting more and more mature.

Backup slides

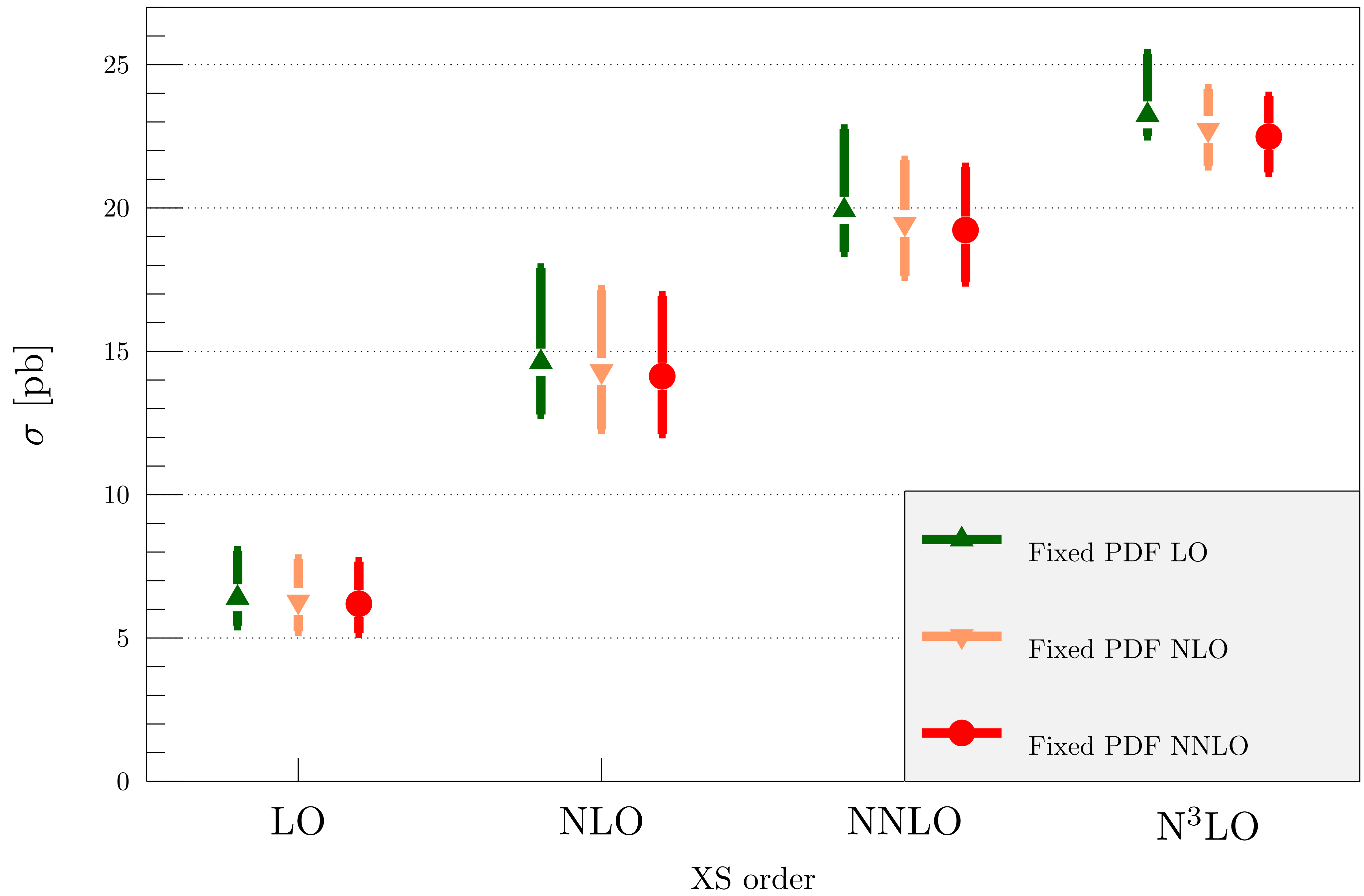
Threshold expansion



Threshold expansion

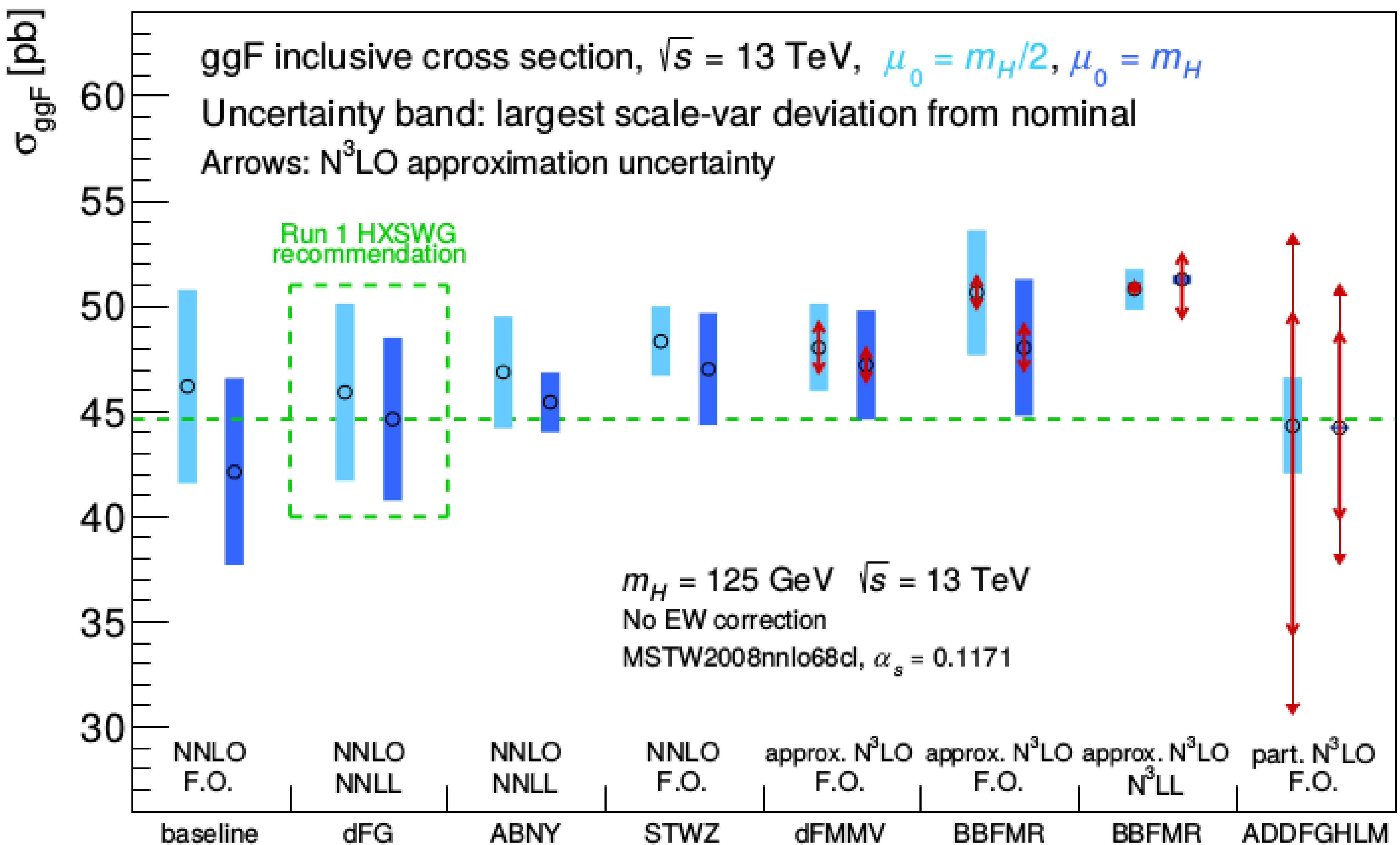


NNLO vs. N3LO PDFs?



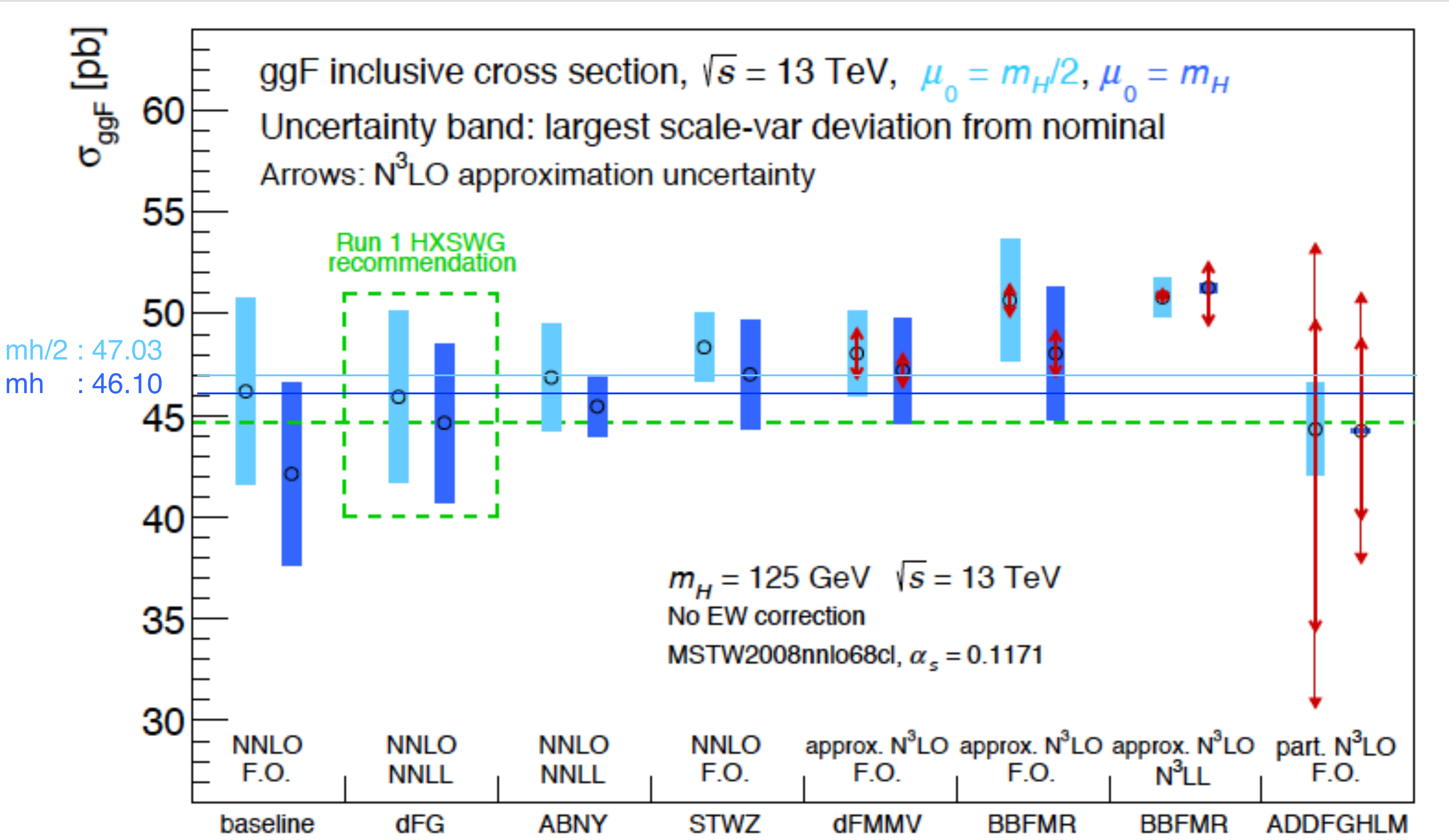
[Plot from Forte, Isgrò, Vita; N3LO is approximate]

Comparison to Approximate N3LO

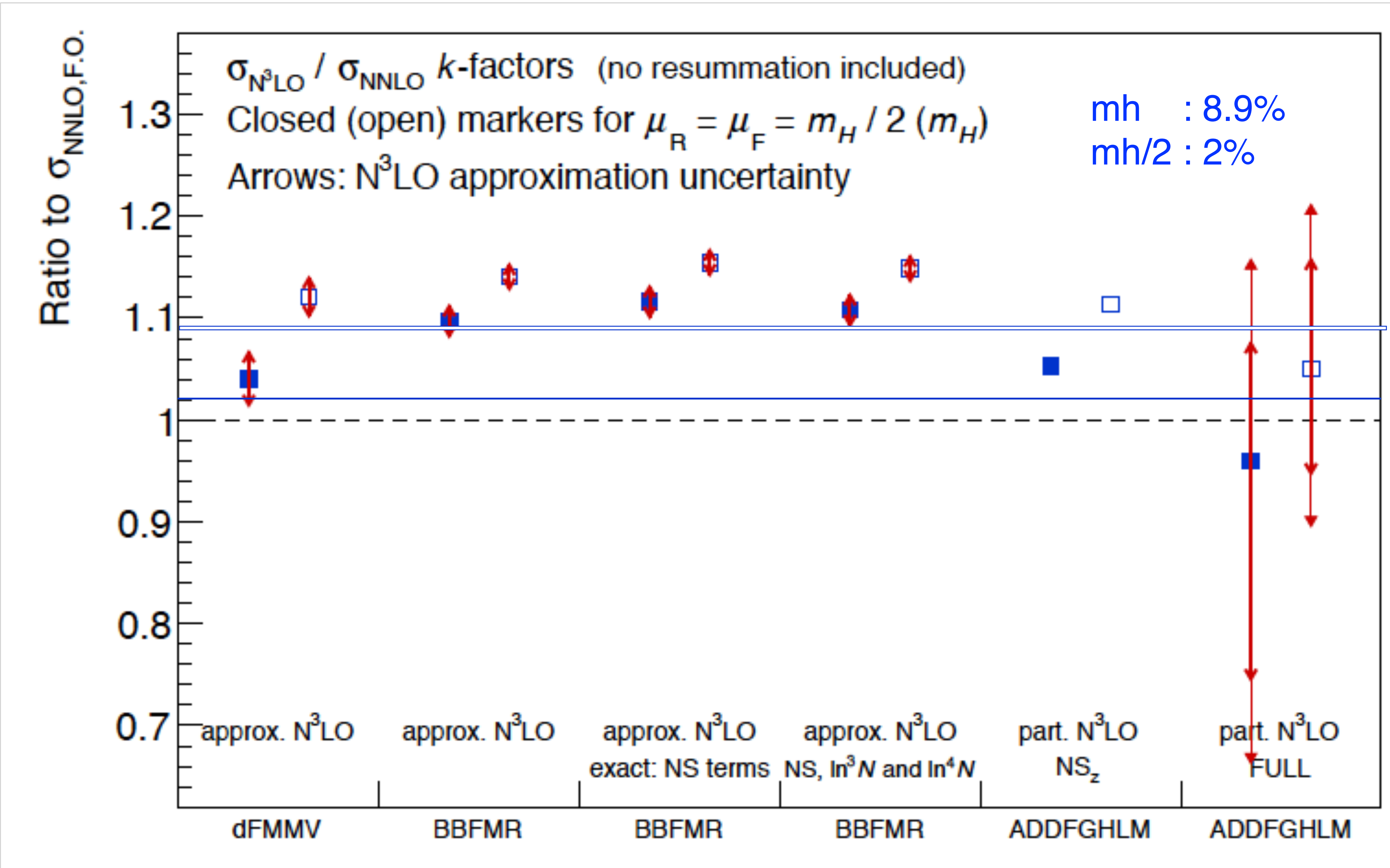


[Plot from HXSWG]

Comparison to Approximate N3LO



Comparison to Approximate N3LO



The soft-virtual contribution

$$\frac{\hat\sigma_{ij}(z)}{z} = \hat\sigma^{SV}\,\delta_{ig}\,\delta_{jg} + \sum_{N=0}^{\infty} \hat\sigma_{ij}^{(N)}\,(1-z)^N$$

$$\boxed{a\,\delta(1-z) + \sum_{k=0}^{5} b_k \left[\frac{\log^k(1-z)}{1-z}\right]_+}$$

- Contributes to the gluon-channel only.
- Plus-distributions already known a decade ago.
  - ➡ Soft gluon emissions. [Moch, Vogt; Laenen, Magnea]
- delta-function contribution computed last year.

[Anastasiou, CD, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; Li, von Manteuffel, Schabinger, Zhu]

  - ➡ Contains the complete three-loop corrections.

[Baikov, Chetyrkin, Smirnov^2, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

The regular contributions

$$\frac{\hat{\sigma}_{ij}(z)}{z} = \hat{\sigma}^{SV}\,\delta_{ig}\,\delta_{jg} + \sum_{N=0}^{\infty} \hat{\sigma}_{ij}^{(N)}\,(1-z)^N$$

$$\boxed{\hat{\sigma}_{ij}^{(N)} = \sum_{k=0}^{5} c_{ijk}^{(N)}\,\log^k(1-z)}$$

- Describes subleading soft emissions.
- Single-emission contributions known exactly.

  [Anastasiou, CD, Dulat, Herzog, Mistlberger; Kilgore; Gehrmann, Glover, Jaquier, Koukoutsakis, CD, Gehrmann, Jaquier; Dulat, Mistlberger]

- Double- and triple-emissions only known as an expansion around threshold. [Anastasiou, CD, Dulat, Herzog, Mistlberger]
  - ➡ Exact result for qq' channel was recently published.

    [Anzai, Hasselhuhn, Hoff, Höschele, Kilgore, Steinhauser, Ueda]

Threshold resummation

- Soft gluon emissions exponentiate in Mellin space!

$$a\,\delta(1-z)+\sum_{k=0}^{5} b_k \left[\frac{\log^k(1-z)}{1-z}\right]_+ \longrightarrow \tilde{a}+\sum_{k=1}^{6}\tilde{b}_k\,\log^k N$$

$$\hat{\sigma}_{gg}^{resum} = g_0(\alpha_s)\,\exp\left[\frac{1}{\alpha_s^2}\sum_{k=1}^{\infty}\alpha_s^k\,g_k(\alpha_s\log N)\right]$$

[Catani, Trentadue; Sterman]

- Resummation functions g_i known up to N3LL (k=4).

[Moch, Vermaseren, Vogt; Bonvini, Marzani; Catani, Cieri, de Florian, Ferrara, Grazzini]

Threshold resummation

- Soft gluon emissions exponentiate in Mellin space!

$$a\,\delta(1-z) + \sum_{k=0}^{5} b_k \left[\frac{\log^k(1-z)}{1-z}\right]_+ \longrightarrow \tilde{a} + \sum_{k=1}^{6} \tilde{b}_k \log^k N$$

$$\hat{\sigma}_{gg}^{resum} = g_0(\alpha_s)\,\exp\left[\frac{1}{\alpha_s^2}\sum_{k=1}^{\infty} \alpha_s^k\, g_k(\alpha_s \log N)\right]$$

[Catani, Trentadue; Sterman]

- Resummation functions g_i known up to N3LL (k=4).

[Moch, Vermaseren, Vogt; Bonvini, Marzani; Catani, Cieri, de Florian, Ferrara, Grazzini]

- N3LL resummation needs 4-loop cusp anomalous dimension.
  - ➡ Only known via Pade approximation, assuming Casimir scaling. [Moch, Vermaseren, Vogt]
  - ➡ Casimir scaling assumption likely to fail at four loops.
  - ➡ Numerical impact small!