THEORY IMPROVEMENTS FOR THE HL-LHC

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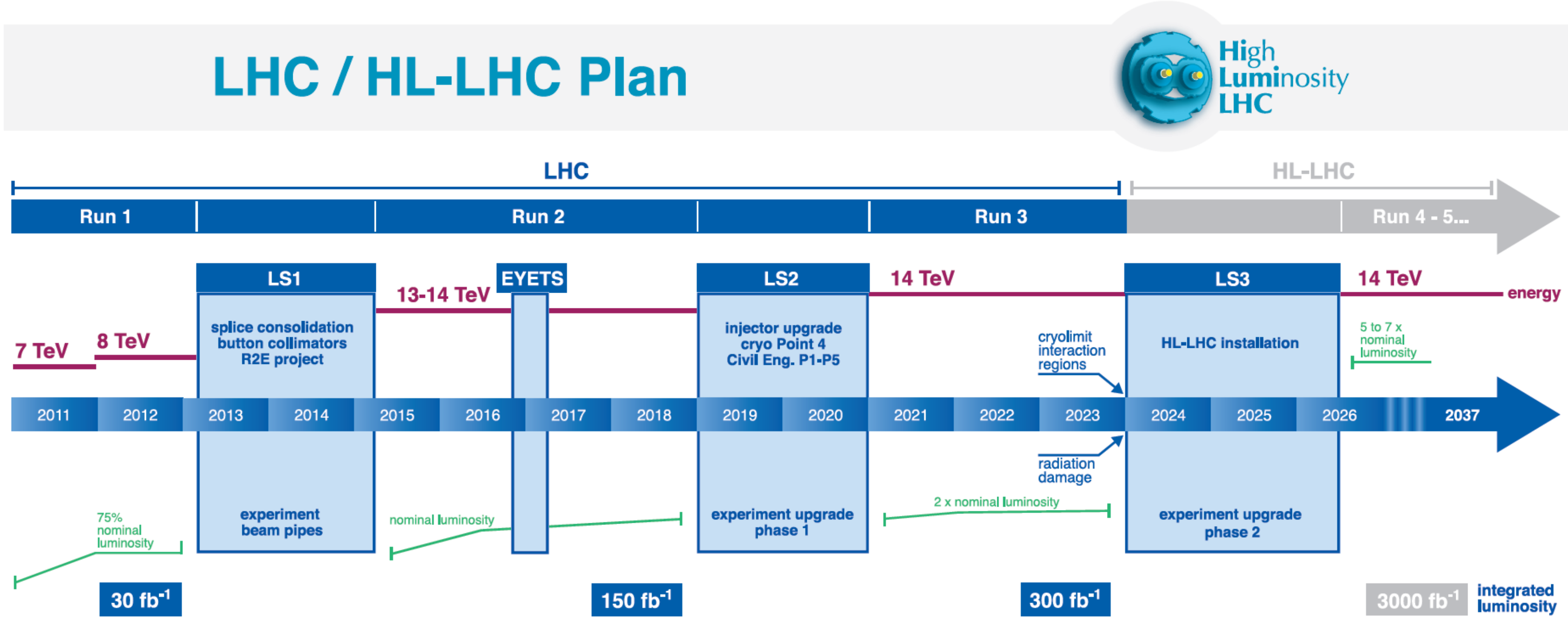
HIGGS HUNTING

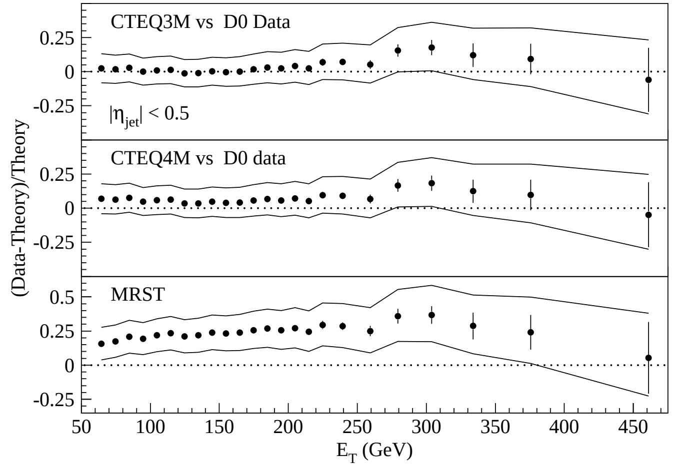
ORSAY, AUGUST 1, 2015

THE HL-LHC: 2025-2035



THE HL-LHC: 2025-2030

HOW WERE THINGS 15 YEARS AGO?



"The current agreement between theory and data is at the level of 30% over 8 orders of magnitude..."

(M. Mangano, EPS 1999)

(IT IS SUPPOSED TO BE A GOOD THING)

QCD in the year 2000

- DGLAP splitting functions known up to NLO
  NNLO splitting functions published in 2002-2004
  (Moch, Vermaseren, Vogt)

- only inclusive total cross sections with colorless final states (Drell-Yan, Higgs) known at NNLO
  rapidity distributions computed to NNLO in 2003 (Higgs), 2004 (DY) (Anastasiou, Melnikov, Petriello)

- PDF uncertainties estimated by comparing different PDF sets
  first gobal PDF sets with uncertainties published 2002
  (CTEQ, MRST)

- EW corrections mostly available for lepton collider processes
  EW corrections to Higgs production 2004-2006
  (Aglietti, Degrassi et al., Passarino et al., Dittmaier et al.)

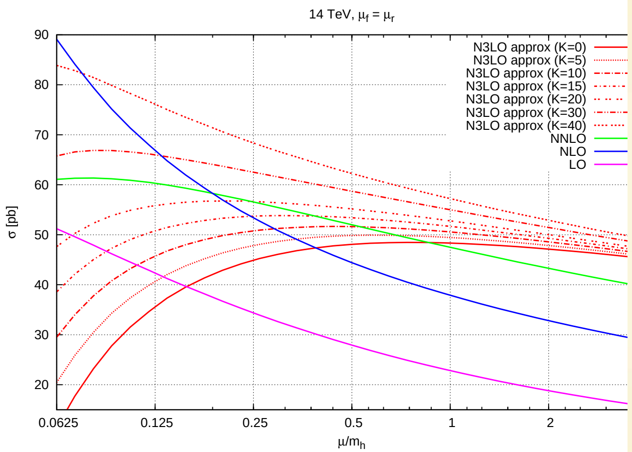
QCD IN THE YEAR 2030?

- THE STRUCTURE OF THE QCD EXPANSION
  - RESUMMATION AND ITS USES
  - ANALYTIC STRUCTURE OF CROSS-SECTIONS
  - HIGHER ORDERS: UNCERTAINTY AND DIVERGENCE
- ELECTROWEAK CORRECTIONS
  - THE PHOTON PDF
  - MIXED QCD-EW CORRECTIONS
- PDFS (& α_s)
  - DATA: FROM LHC TO LHC-II (AND LHEC?)
  - PDFS BEYOND NNLO & THEORETICAL UNCERTAINTIES
  - MATCHING TO MONTE CARLO

HIGHER ORDERS AND RESUMMATION

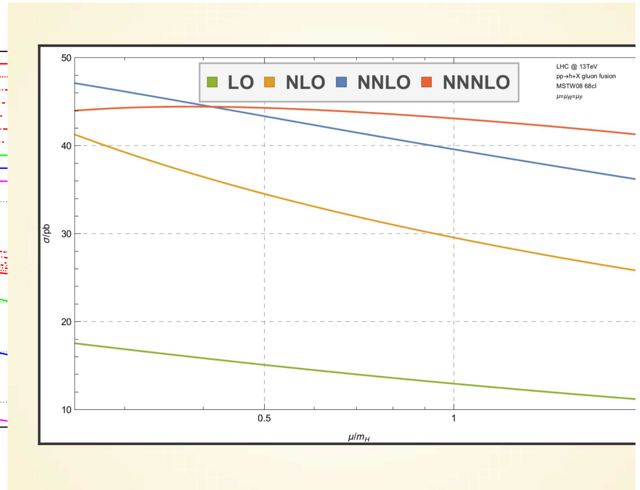
HIGHER ORDERS:
WHAT DO WE KNOW?
N^{3}LO HIGGS IN GLUON FUSION

SCALE DEP VS N^{3}LO SIZE



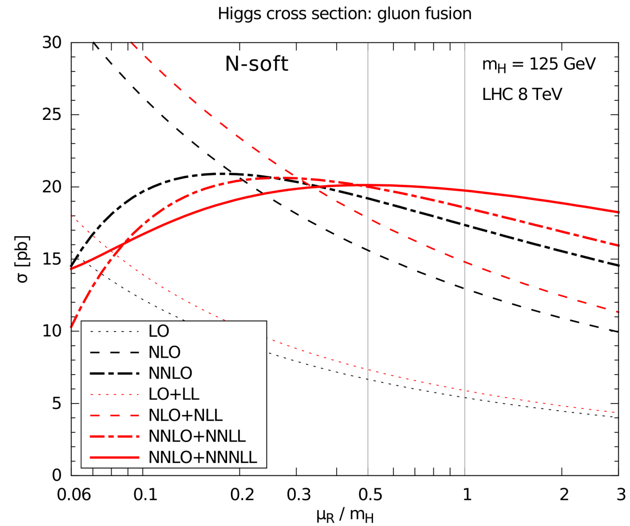
(Bühler, Lazopoulos, 2013)

EXACT N^{3}LO



(Anastasiou et al, 2015)

UNRESUMMED VS RESUMMED EXPANSION



(Bonvini, Marzani, 2014)

- THE PERTURBATIVE EXPANSION IS WELL-BEHAVED
- SOFT RESUMMATION ACCELERATES CONVERGENCE

Q: WHEN (AND WHY) IS SOFT RESUMMATION USEFUL?

- PARTONIC CROSS SECTIONS $\hat\sigma(z,\alpha_s)$ ARE DISTRIBUTIONS, $0 \le z = \frac{M_h^2}{s} \le 1$, CONVOLUTED WITH LUMINOSIITY TO GIVE HADRONIC CROSS-SECTION
- THEIR MELLIN TRANSFORMS $\hat\sigma(N,\alpha_s)$ ARE ORDINARY FUNCTIONS
- FOR GIVEN m_H^2 AND s ONLY ONE "SADDLE" N VALUE CONTRIBUTES; POSITION OF SADDLE DETERMINED ESSENTIALLY BY PDF
  $\Rightarrow$ PDF TELLS YOU WHICH FRACTION OF HADRON MOMENTUM GOES INTO PARTONIC PROCESS

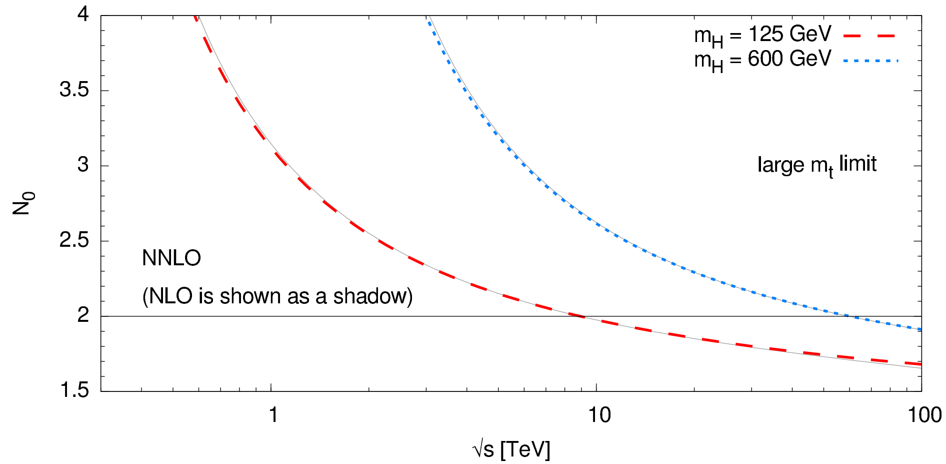
A: WHEN SADDLE N IS LARGE ENOUGH

- SOFT RESUMMATION INCLUDES TO ALL ORDERS $\alpha_s \ln(1-z)$, $z = \frac{M_X^2}{s} \Leftrightarrow \alpha_s \ln N$
- SOFT $\leftrightarrow z \to 1 \leftrightarrow N \to \infty$.

HIGGS IN GLUON FUSION

- RESUMMATION CURRENTLY INCLUDED IN HXSWG RECOMMENATION: NNLO+NNLL
- BUT $N_{\rm saddle} \sim 2$, $\Rightarrow$ RESUMMATION SUMS UP $\beta_0\alpha_s \ln 2 \approx 0.04 \ll 1$
- WHAT IS "LARGE ENOUGH"??

SADDLE VS COLLIDER ENERGY



(Bonvini, SF, Ridolfi, 2012)

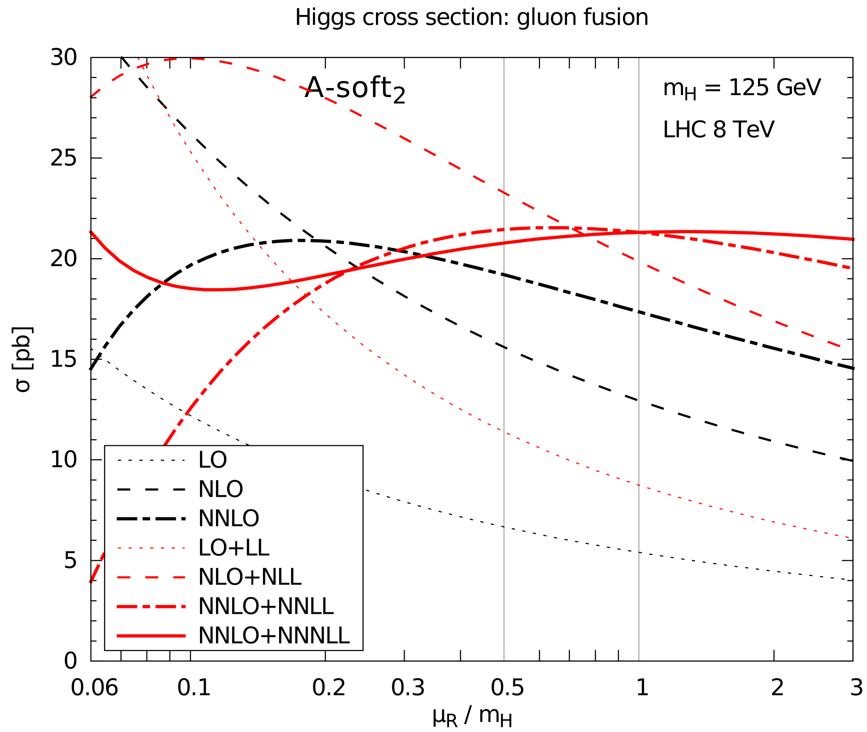
HIGGS IN GLUON FUSION:
SHOULD RESUMMATION HELP?

NO?

YES!

$$M\left[\eta_{gg}^{(3)}\right](N) \simeq 36\log^6 N \qquad (\to 0.0013\%)$$
$$+170.679\ldots\log^5 N \qquad (\to 0.0226\%)$$
$$+744.849\ldots\log^4 N \qquad (\to 0.2570\%)$$
$$+1405.185\ldots\log^3 N \qquad (\to 1.0707\%)$$
$$+2676.129\ldots\log^2 N \qquad (\to 4.0200\%)$$
$$+1897.141\ldots\log N \qquad (\to 5.1293\%)$$
$$+1783.692\ldots \qquad (\to 8.0336\%)$$
$$+108\frac{\log^5 N}{N} \qquad (\to 0.0105\%)$$
$$+615.696\ldots\frac{\log^4 N}{N} \qquad (\to 0.1418\%)$$
$$+2036.407\ldots\frac{\log^3 N}{N} \qquad (\to 0.9718\%)$$
$$+3305.246\ldots\frac{\log^2 N}{N} \qquad (\to 2.9487\%)$$
$$+3459.105\ldots\frac{\log N}{N} \qquad (\to 5.2933\%)$$
$$+703.037\ldots\frac{1}{N} \qquad (\to 1.7137\%).$$

(Anastasiou et al, 2015)



Bonvini and Marzani, 2014

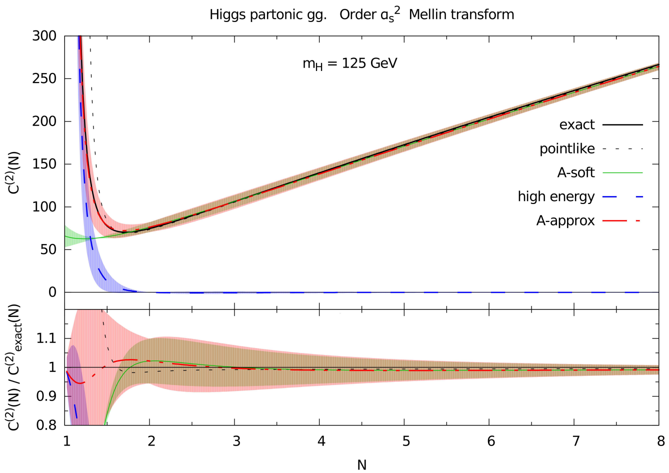
- "LEADING" LOGS DO NOT PROVIDE THE DOMINANT CONTRIBUTION
- EXPANSION IN POWERS OF $1/N$ CONVERGES VERY SLOWLY

- RESUMMED EXPANSION CONVERGES FASTER THAN UNRESUMMED ONE
- SCALE DEPENDENCE ALWAYS SMALLER AT RESUMMED LEVEL

ANALYTIC STRUCTURE IN N SPACE

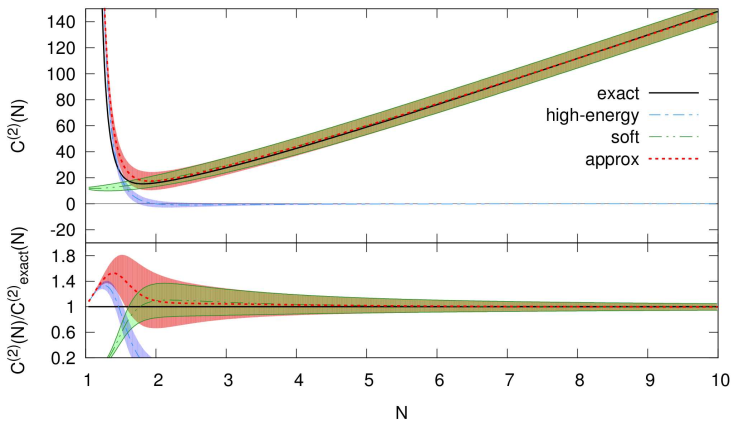
- MELLIN-SPACE PARTONIC CROSS SECTIONS ARE ANALYTIC FUNCTIONS OF N
- AS $N \to \infty$ THEY BEHAVE AS POWERS OF $\ln N$
- FINITE-N SINGULARITIES ARE ISOLATED POLES ON THE REAL AXIS FOR INTEGER $N \leq 1$
- RECONSTRUCT FROM SINGULARITIES, WITHOUT INTRODUCING SPURIOUS CUTS
  EXAMPLE: LARGE-N $\ln N \Rightarrow \psi(N)$.

HIGGS IN GLUON FUSION



(Ball et al., 2013)

TOP PAIRS



(Muselli et al., 2015)

WHAT DO WE KNOW?

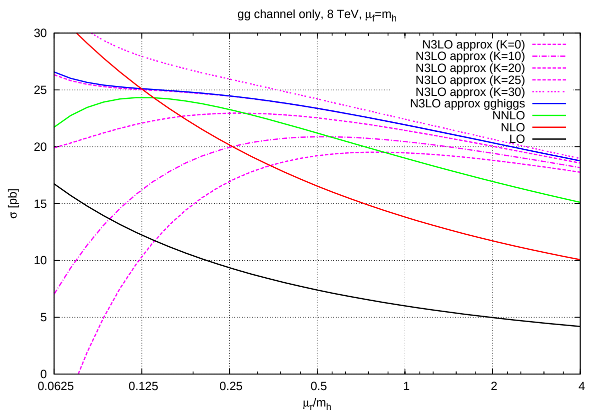
- EIKONAL OR SOFT EXPANSION:AS $N \to \infty$ (I.E. $z \to 1$)
  $\hat{\sigma}^{(3)}(N) = c_{06} \ln^6 N + c_{05} \ln^5 N + \cdots + c_{02} \ln^2 N + c_{01} \ln N + c_{00} +$
  $\frac{1}{N} \left(c_{15} \ln^5 N + c_{14} \ln^4 N + \cdots + c_{10} \right) + \frac{1}{N^2} \left(c_{24} \ln^4 N + \cdots + c_{20} \right) + \ldots$
  CURRENT KNOWLEDGE: EIKONAL, UP TO NNLL FOR MANY DIFFERENTIAL DISTRIBUTIONS

- BFKL OR HIGH-ENERGY EXPANSION:AS N DECREASES (I.E. $z \to 0$)
  RIGHTMOST SINGULARITY AT $N = 1$:
  $\hat{\sigma}^{(3)}(N) = \left(d_{13} \frac{1}{(N-1)^3} + d_{12} \frac{1}{(N-1)^2} + d_{11} \frac{1}{(N-1)} \right) + \left(d_{06} \frac{1}{N^6} + \cdots + d_{01} \frac{1}{N} \right) + \ldots$
  CURRENT KNOWLEDGE: LL,FOR SEVERAL TOTAL CROSS SECTIONS; ONLY GGHIGGS RAPDITY DISTRIBUTION

APPROXIMATE HIGHER ORDERS?

HIGGS GLUON FUSION: SCALE DEP

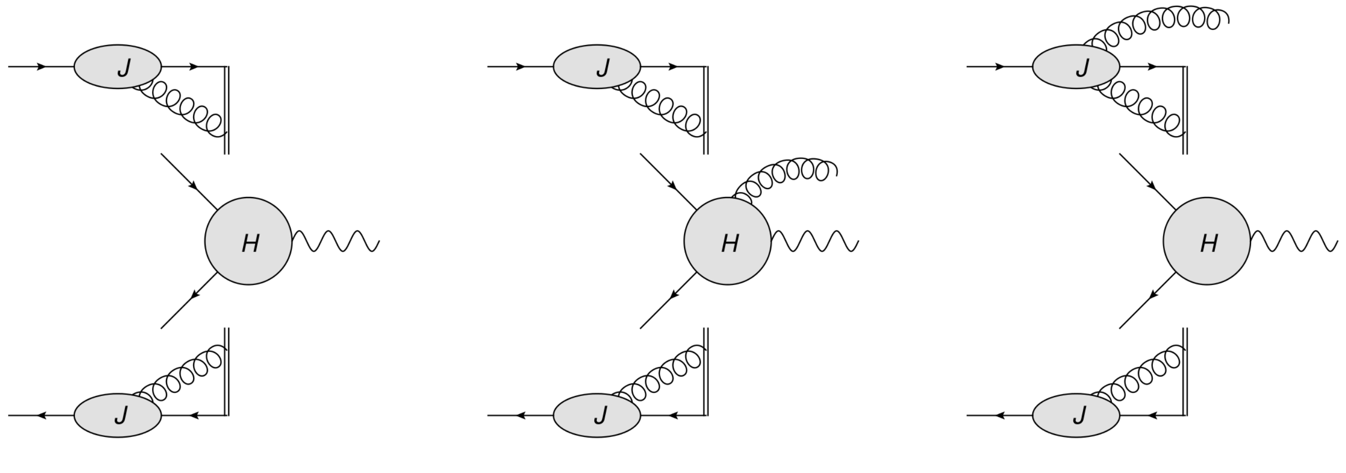
APPROX VS. GENERAL



(Bühler, Lazopoulos, 2013)

- AT NNLO METHOD WORKS WELL
- FOR GGHIGGS, FROM SCALE VARIATION RESULT SEEMS IN GOOD AGREEMENT WITH EXACT (AWAITING FOR FULL COMPARISON)
- N^4LO COMPUTATIONS?
- IMPROVING BOTH THE SOFT AND THE BFKL EXPANSION?

IMPROVING THE SOFT EXPANSION
BEYOND THE LEADING POWER



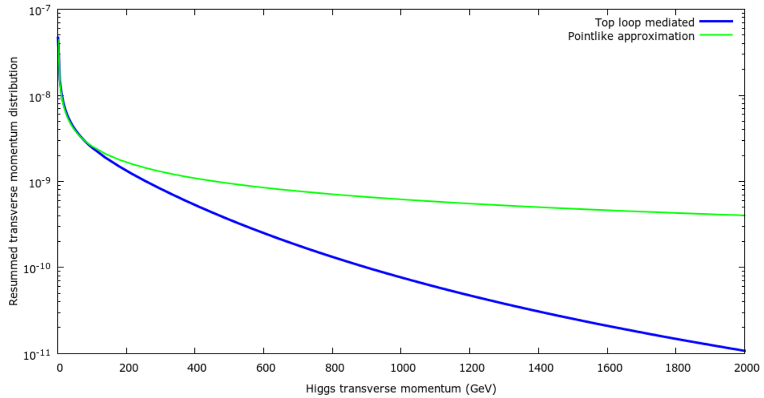
(a) (b) (c)

- IMPRESSIVE PROGRESS IN THE ORGANIZATION OF RESUMMATION BEYOND THE EIKONAL LEVEL: NEXT-TO-EIKONAL $\frac{ln^k N}{N}$; NEXT-TO-NEXT-TO-EIKONAL $\frac{ln^k N}{N^2}$ ETC (Laenen, Magnea, C.White et al, 2012-2105)
- "METHOD OF REGIONS": SEPARATE INTEGRATION REGIONS OVER MOMENTA THROUGH APPROPRIATE SCALING
- COMBINE WITH FACTORIZATION TO CLASSIFY EMISSIONS FROM INTERNAL BLOBS, DRESSED WITH RADIATION FROM EXTERNAL LINES

IMPROVING HIGH ENERGY RESUMMATION
BEYOND INCLUSIVE OBSERVABLES

- CURRENTLY KNOWN TO LL FOR TOTAL CROSS SECTIONS (HIGGS, DRELL-YAN, HEAVY QUARKS)
- RAPIDITY DISTRIBUTIONS KNOWN FOR HIGGS (Caola, SF, Marzani 2011)

HIGGS IN GLUON FUSION: LLx p_t SPECTRUM
POINTLIKE VS. EXACT



FOR $p_t \gtrsim m_t$
EXACT DEVIATES FROM POINTLIKE

(Muselli, SF, Vita, in preparation)

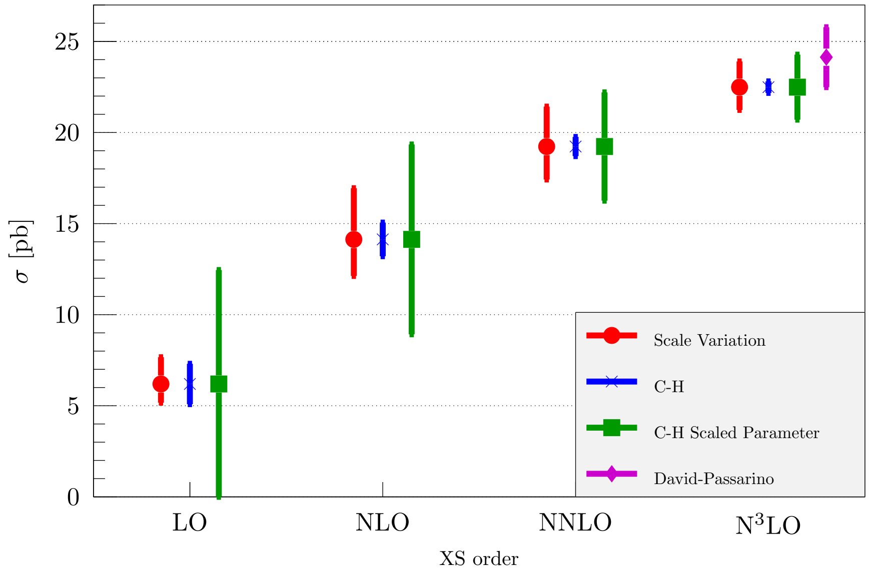
- p_T DISTRIBUTIONS :
  - CAN STUDY SENSITIVITY TO m_t
  - BEHAVIOUR OF p_t RESUMMATION IN REGION $m_b < p_t \ll m_h$

THEORETICAL UNCERTAINTIES

- SCALE VARIATION IS A CRUDE METHOD WHICH OFTEN FAILS
- CACCIARI-HOUDEAU: LOOK AT THE BEHAVIOUR OF THE PERTURBATIVE EXPANSION
- PASSARINO-DAVID: TRY TO DEFINE SUM OF THE SERIES BASED ON SERIES ACCELERATION METHODS
- BETTER UNDERSTANDING AS MORE PROCESSES KNOWN

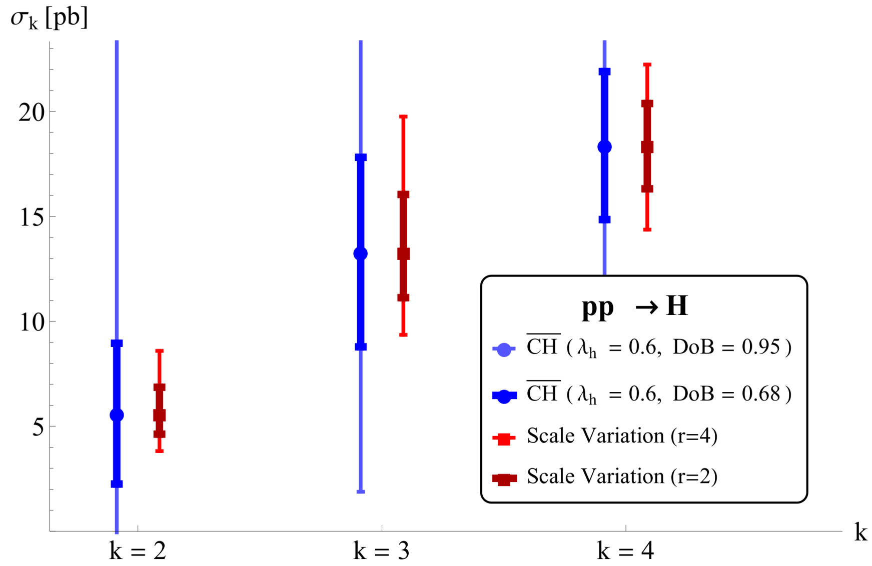
HIGGS IN GLUON FUSION:

SCALE VS CH VS PD



(s.f., Isgrò, Vita, 2014)

DEP ON DOB AND ON SCALE RANGE



(Bagnaschi, Cacciari et al., 2015)

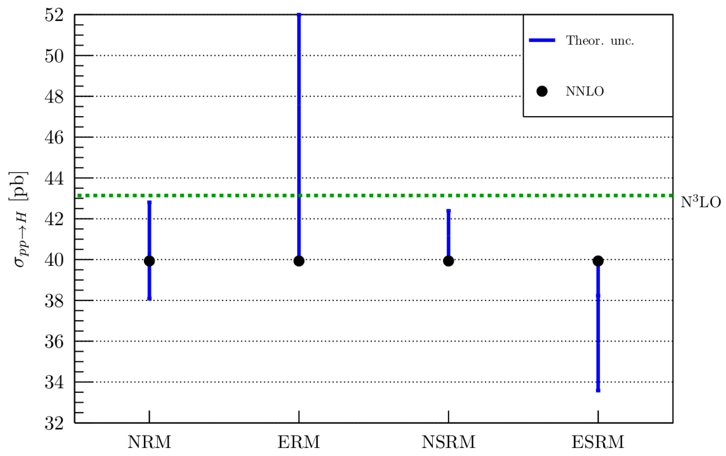
WHEN WILL IT START DIVERGING?

- THE QCD PERTURBATIVE EXPANSION IS ASYMPTOTIC: DO WE HAVE TO WORRY ABOUT ITS DIVERGENCE?

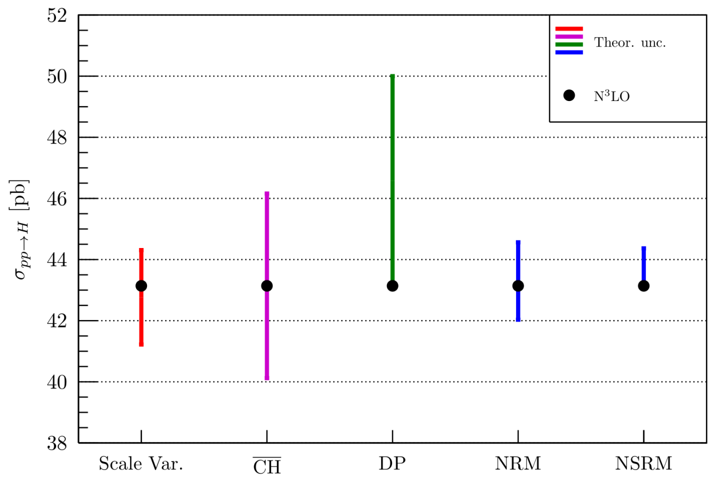
- CAN WE USE THE ASYMPTOTIC SUM AS A MEANS TO ESTIMATE THE THEORETICAL UNCERTAINTY?

HIGGS IN GLUON FUSION:

NNLO UNCERTAINTY VS TRUE N^3LO
VARIOUS MODELS



NNNLO UNCERTAINTY



(s.f., Isgrò, in preparation)

- ORDER OF THE DIVERGENCE: LANDAU POLE $\Rightarrow n \sim 70$; RENORMALONS $\Rightarrow n \sim 70$

- ASSUME ASYMPTOTIC SUM GOVERNED BY RENORMALONS AFTER SUBTRACTING SOFT CONTRIBUTION (NSRM) $\Rightarrow$ REASONABLE RESULTS AT NNLO

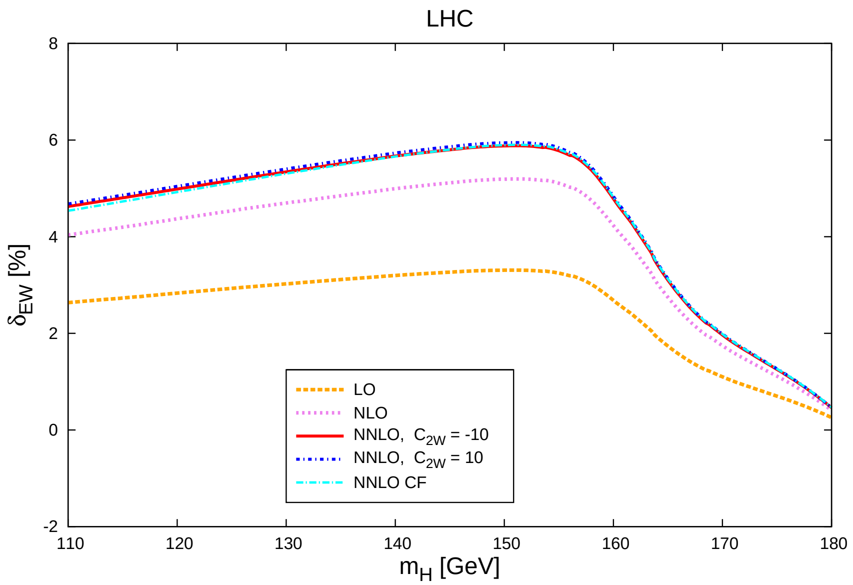
- BECOMES MORE RELIABLE AS ORDER INCREASES

ELECTROWEAK CORRECTIONS

ELECTROWEAK CORRECTIONS

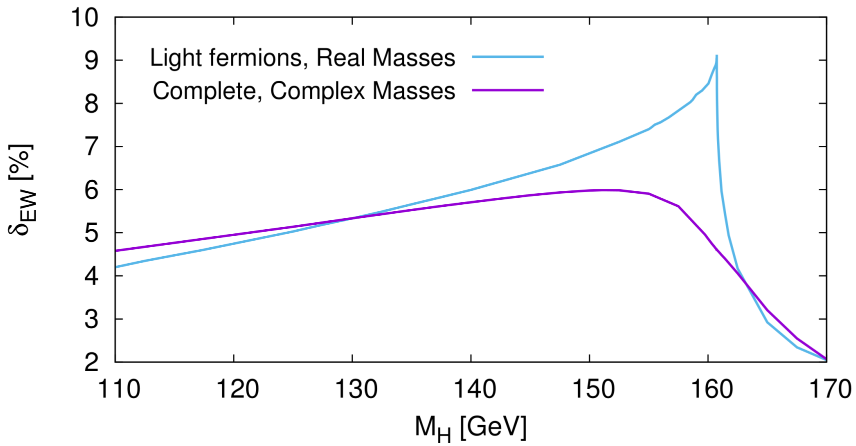
- EW CORRECTIONS HAVE A SIZABLE IMPACT ON THE GLUON FUSION XSECT 5%
- CURRENTLY INCLUDED USING COMPLETE FACTORIZATION:
  IF ONLY APPLIED TO LO (PARTICAL FACTORIZATION), CHANGES UP TO 3%
- WILSON COEFFICIENT COMPUTED TO $O(\lambda_{\rm EW}\alpha_s)$ IN EFT
  (EXPANSION IN $\frac{m_H}{2m_t}$, $\frac{m_h}{m_W}$) (Anastasiou, Boughezal, Petriello 2008)
  100% VIOLATION OF FACTORIZATION NOT ENOUGH TO PRODUCE SIGNIFICANT EFFECT
- FINITE m_t, m_W CORRECTIONS PROBABLY SMALL
- CAN WE REALLY TRUST FACTORIZATION TO $O(\lambda_{\rm EW}\alpha_s^2)$?

DEPENDENCE OF FACTORIZATION



(Anastasiou, Boughezal, Petriello 2008)

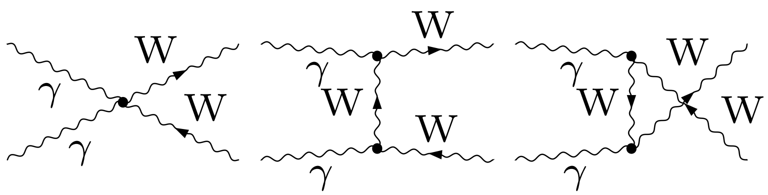
ACCURACY OF EFT

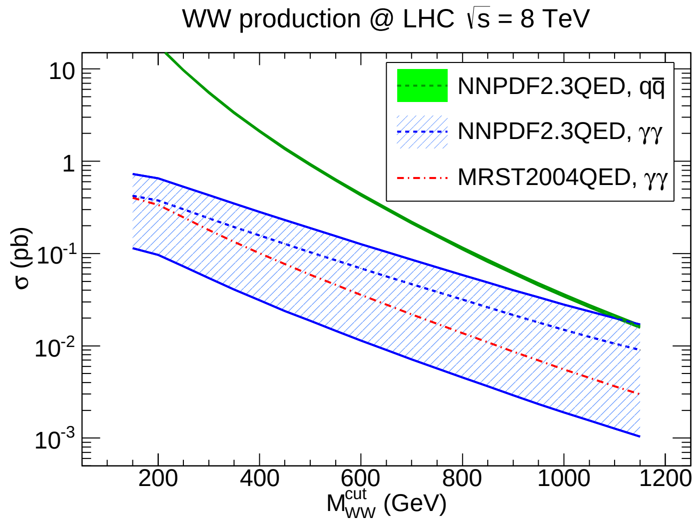


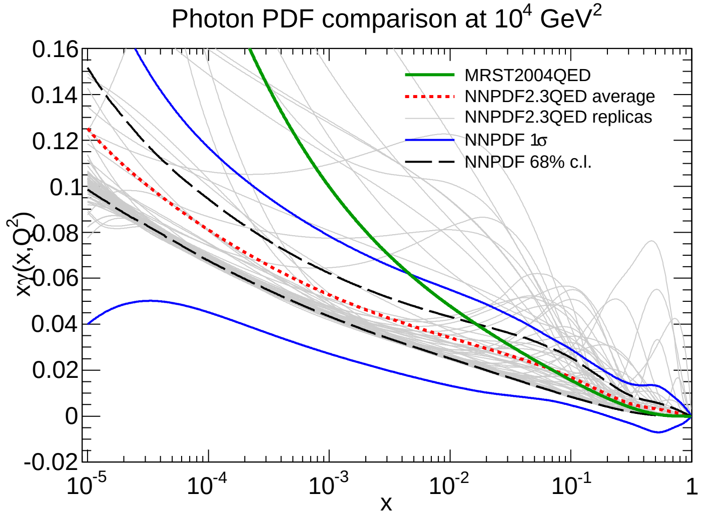
(Uccirati, Les Houches 2015)

ELECTROWEAK CORRECTIONS

QED PDFs







- PHOTON PDF CONTRIBUTION CAN BE SIZABLE: EXAMPLE: W PAIR PRODUCTION AT LARGE INVARIANT MASS
- CURRENTLY DETERMINED ONLY BY W (ON- AND OFF-SHELL) PRODUCTION DATA, AFFECTED BY VERY LARGE UNCERTAINTIES
- WILL BE DETERMINED IN GLOBAL PDF FITS
- DIRECT PHOTON PRODUCTION AT THE LHEC?

NNPDF2.3QED, Carrazza et al. 2013

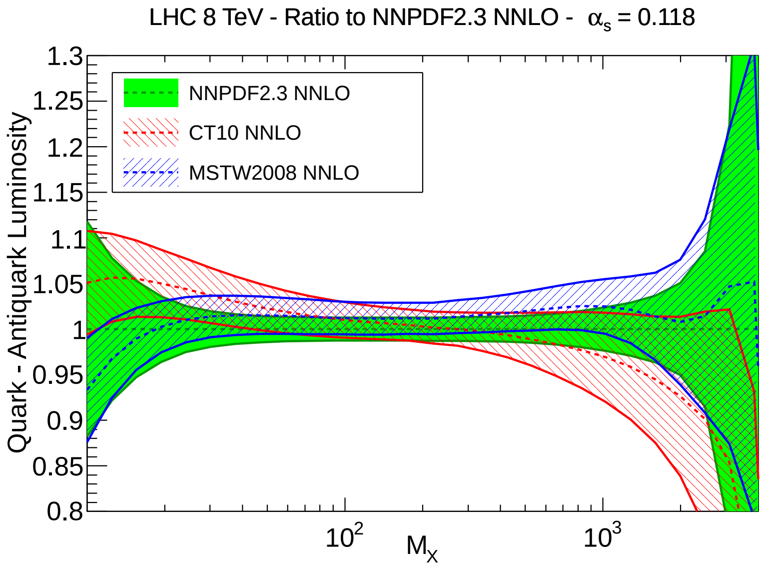
PDFs (and α_s)

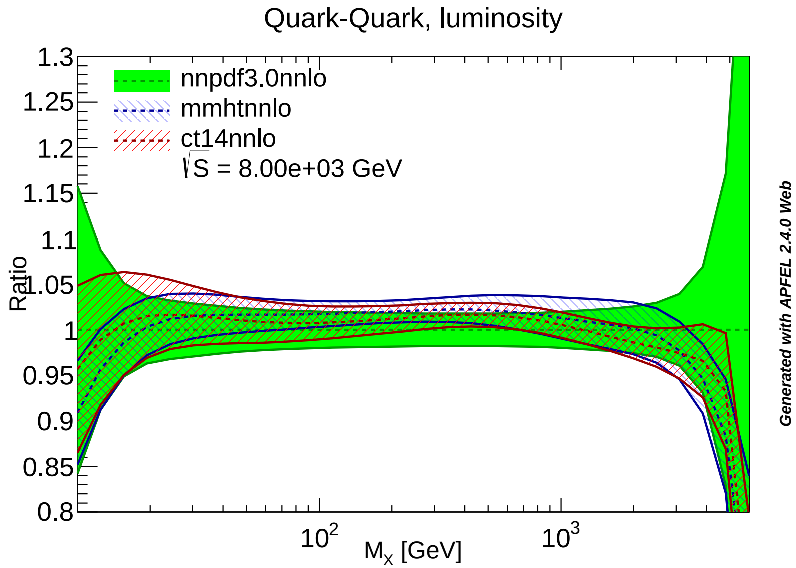
PDFs: RECENT PROGRESS

PARTON LUMINOSITIES

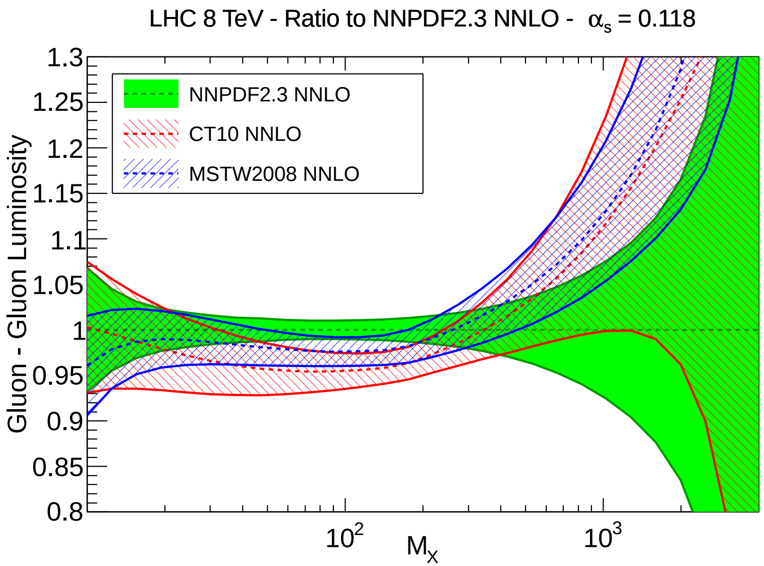
QUARK-QUARK

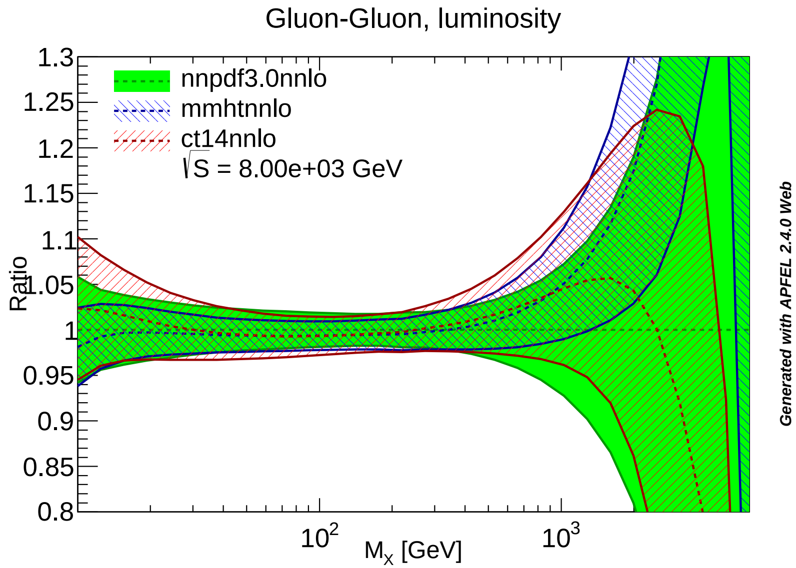
2012 **2015**





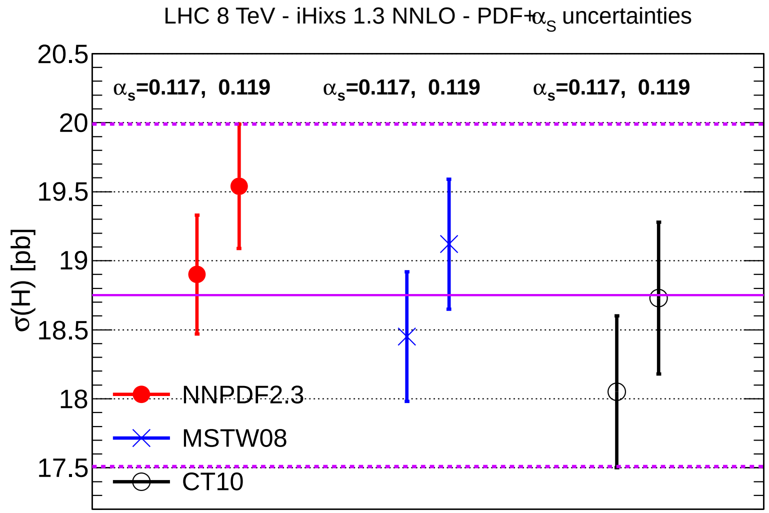
GLUON-GLUON



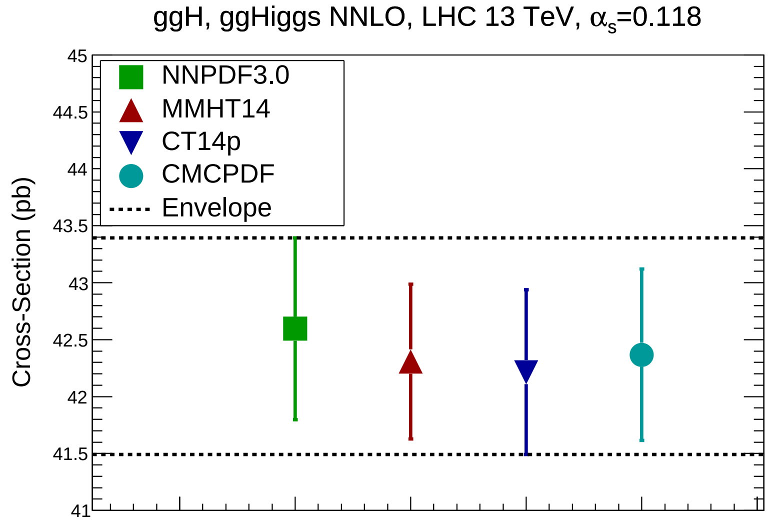


PDFs: RECENT PROGRESS
HIGGS IN GLUON FUSION

2012



2015



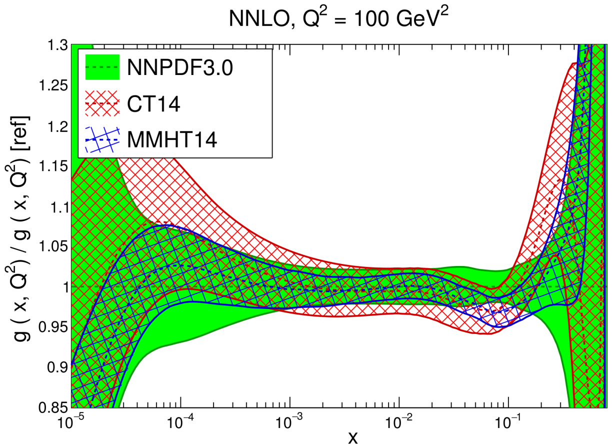
- MAJOR UPGRADES FROM ALL GLOBAL FITTING GROUPS: NNPDF2.3→3.0 (10/2015); MSTW08 → MMHT14 (12/2015); CT10 ⇒ CT14 (06/2015)
- METHODOLOGICAL IMPROVEMENTS: CLOSURE TESTS (NNPDF); EXTENDED PARAMETRIZATIONS (CT, MMHT)
- LHC-I DATA INCLUDED
- PDF UNCERTAINTY ON HIGGS PRODUCTION DOWN TO ABOUT 2%
  ENVELOPE NO LONGER NECESSARY

PDFS AT LHC RUN II

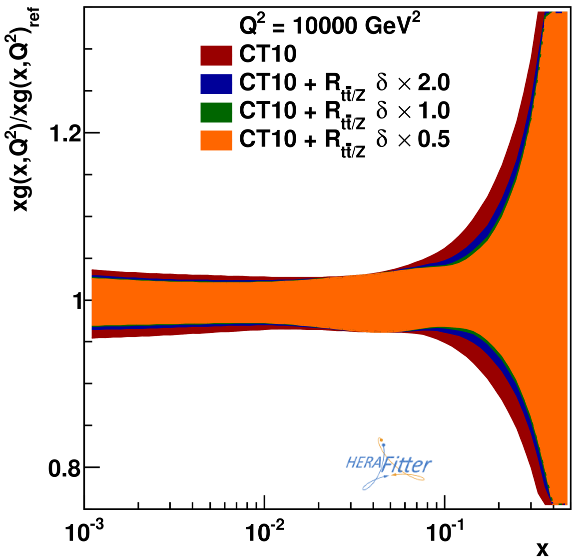
- DATA AT HIGHER CM ENERGY & INFO ON CORRELATION TO LOW ENERGY
  $\rightarrow$ EXTENDED KINEMATIC COVERAGE & REDUCED SYSTEMATICS
- POSSIBLY REDUCED STAT. UNCERTAINTIES
- EXPECT REDUCTION IN MODEL DEPENDENCE
- MODERATE REDUCTION IN UNCERTAINTY
- FURTHER IMPROVEMENTS DUE TO THEORY
  EXAMPLE: GLUON FROM NNLO Z p_T DISTRIBUTION

THE GLUON

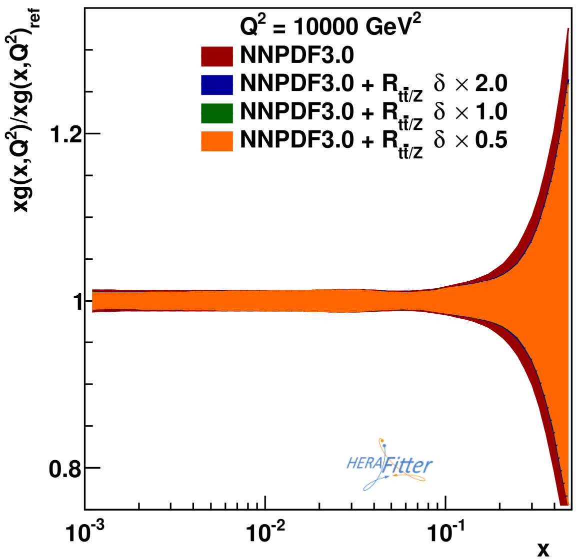
NOW



CTEQ AFTER RUN II



NNPDF AFTER RUN II



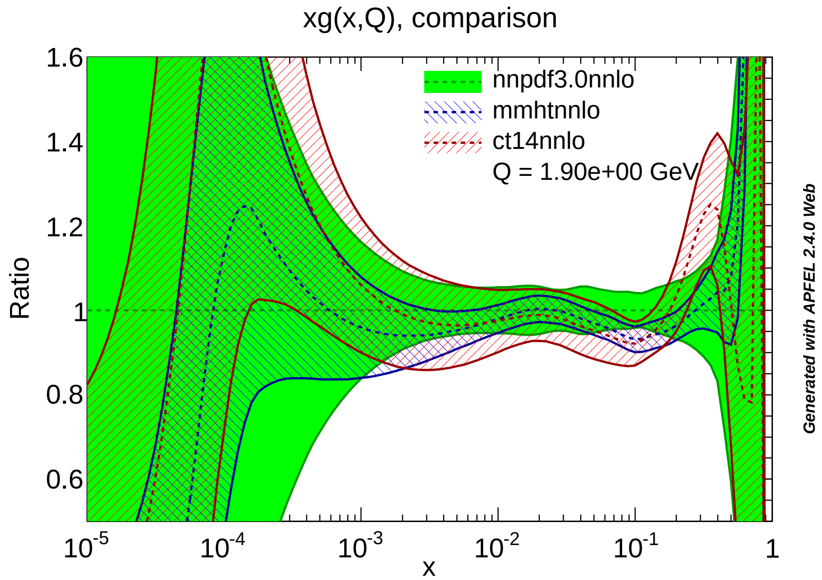
(PDF4LHC: 1507.00556)

PDFs at the LHeC?

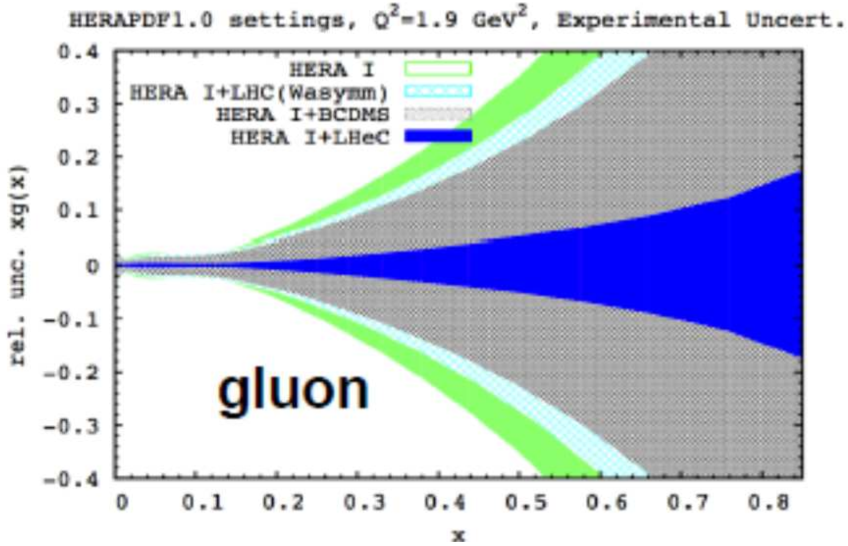
- especially if correlation info available, LHC run II can likely
  - remove systematic bias & discrepancies
  - bring down PDF uncertainties to $\sim 4\%$ level
- very difficult to reduce uncertainties further at a hadron collider
- high-energy deep-inelastic data could lead to PDFs at 1% level

the gluon again

NOW



at the LHeC



(A. Cooper-Sarkar & Voica Radescu, 2015)

THE VALUE OF α_s

HIGGS IN GLUON FUSION $O(\alpha_s^2)$, LARGE NLO CORRNS: $\Delta\sigma \sim 2.5\Delta\alpha_s$

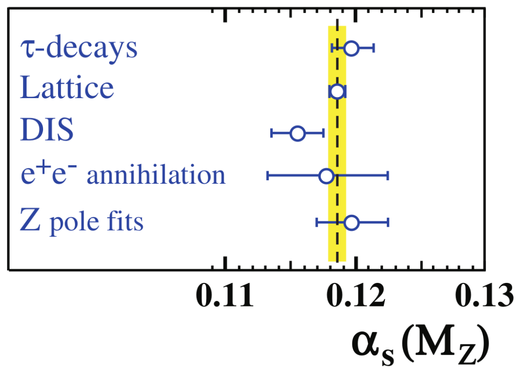
PDG VALUE (AUGUST 2014): $\alpha_s(M_Z) = 0.1185 \pm 0.0006$

- LATTICE UNCERTAINTY CURRENTLY ESTIMATED BY FLAG (arXiv:1310.8555) TO BE TWICE THE PDG VALUE (± 0.0012)
  (IF PDG WERE TO ADOPT THIS, COMBINED UNCERTAINTY LIKELY TO DOUBLE)
- IT IS AN AN AVERAGE OF AVERAGES
- SOME SUB-AVERAGES (E.G. DIS) INCLUDE MUTUALLY INCONSISTENT VALUES
- SOME SUB-AVERAGES (E.G. τ OR JETS) INCLUDE DETERMINATIONS WHICH DIFFER FROM EACH OTHER BY EVEN FOUR-FIVE σ
- AVERAGING THE TWO MOST RELIABLE VALUES (GLOBAL EW FIT & τ, BOTH N^{3}LO, NO DEP. ON HADRON STRUCTURE) GIVES

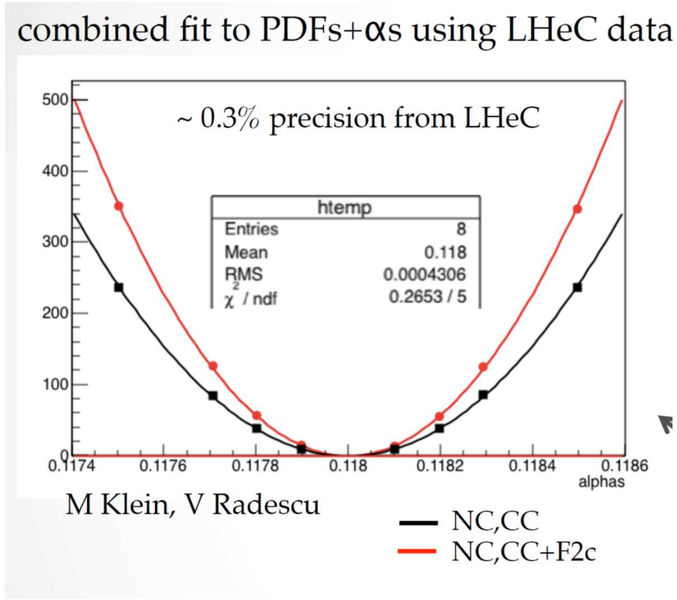
$$\alpha_s = 0.1196 \pm 0.0010$$

- LITTLE PROGRESS FOR MANY YEARS: PDG 1998-2006 $\Delta\alpha_s(M_Z) = 0.002$; PDG 2010-2014 $\Delta\alpha_s(M_Z) = 0.0006 \div 0.0008$ (CHANGE OF AUTHOR)
- UNLIKELY TO CHANGE - SHOULD DETERMINE α_s FROM HIGGS IN GLUON FUSION?
- COULD BE DETERMINED ACCURATELY AT THE LHEC

α_s DETERMINATIONS IN PDG



α_s AT THE LHEC



THEORETICAL UNCERTAINTIES ON PDFS:

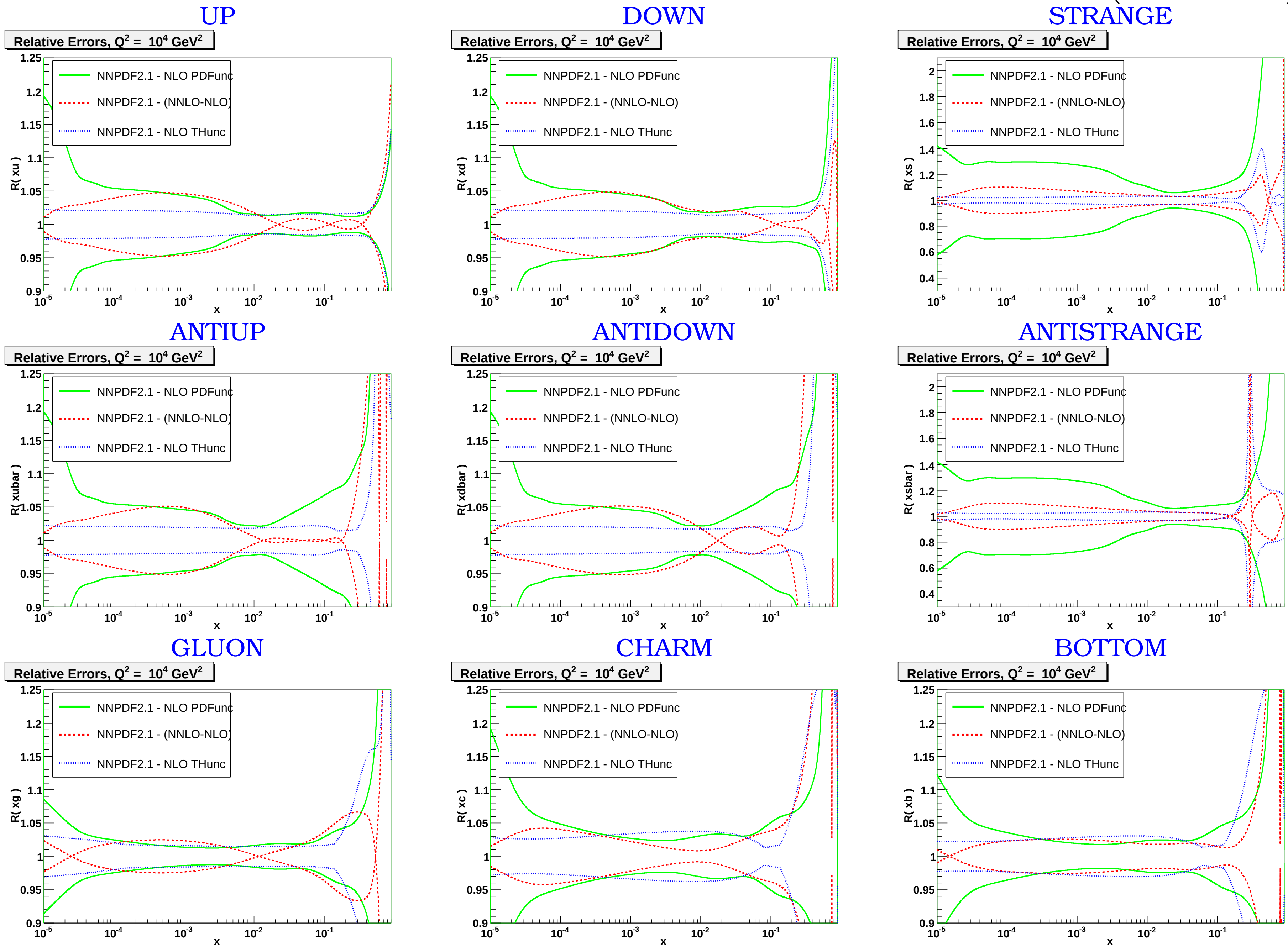
- PDFS ARE DETERMINED BY COMPARING TO DATA THEORY AT SOME FINITE ORDER
- AFFECTED BY THEORETICAL UNCERTAINTY JUST LIKE HARD CROSS-SECTIONS
- NOT INCLUDED IN CURRENT "PDF UNCERTAINTY" (ACCOUNTS ONLY DATA & METHODOLOGY)

CAN WE ESTIMATE THEM?

- SCALE VARIATION DIFFICULT: CORRELATED BETWEEN PROCESSES? HOW DOES IT CORRELATE WITH PROCESSES IN WHICH PDFS ARE USED?
- AT NLO: WE KNOW THE SHIFT TO NNLO
- AT NNLO: LOOK AT THE BEHAVIOUR OF THE PERTURBATIVE EXPANSION (CACCIARI-HOUDEAU)

THEORETICAL UNCERTAINTIES

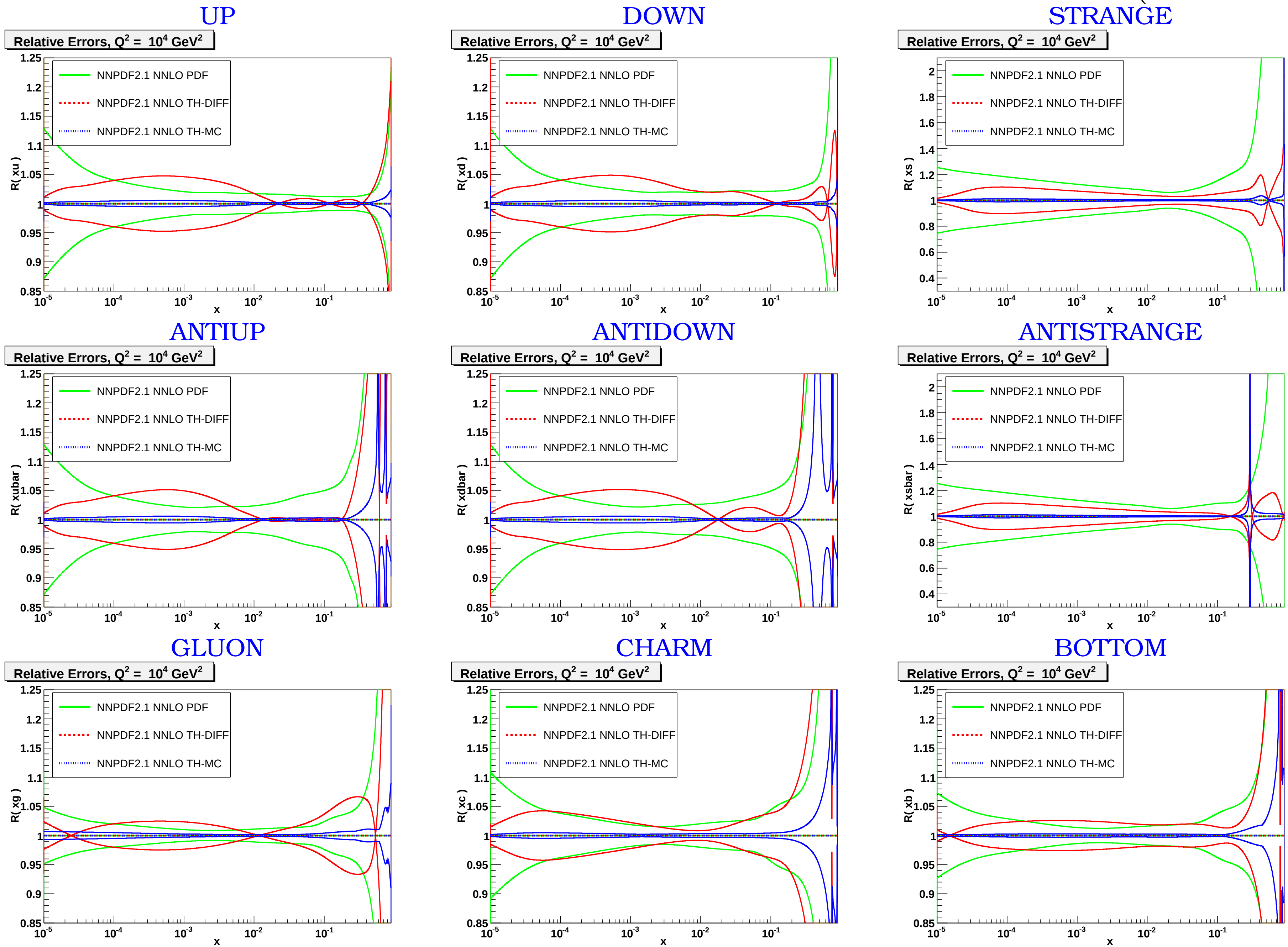
NLO PDF unc. vs NLO-NNLO shift vs NLO Cacciari-Houdeau (NNPDF2.1)



THEORETICAL UNCERTAINTY ON NLO PDF IS ORDER 5% $\Rightarrow$ COMPARABLE TO PDF UNCERTAINTY

THEORETICAL UNCERTAINTIES

NNLO PDF unc. vs NLO-NNLO shift vs NNLO Cacciari-Houdeau (NNPDF2.1)



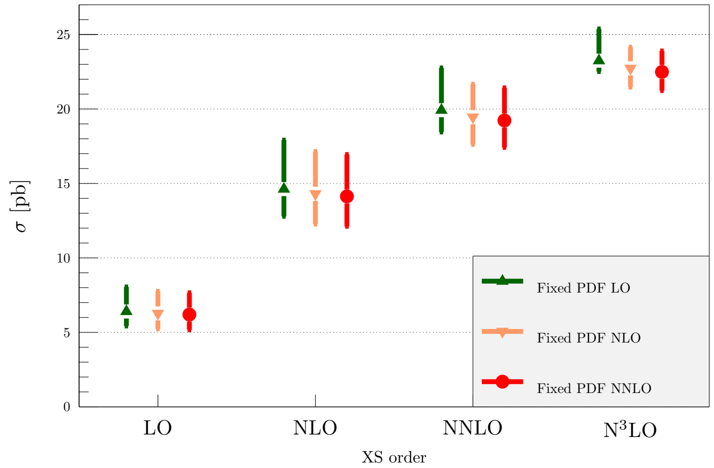
THEORETICAL UNCERTAINTY ON NNLO PDF IS ORDER 1% $\Rightarrow$ SMALLER THAN PDF UNCERTAINTY

N^{3}LO PDFs:

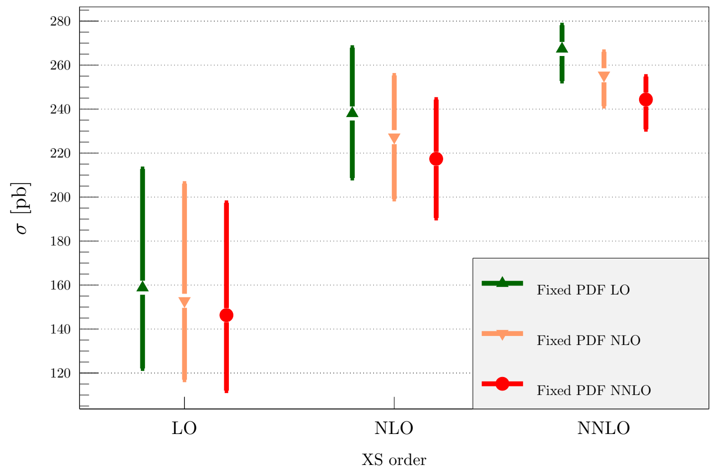
- NEEDED AT THE 1% ACCURACY LEVEL
- IMPACT OF N^3LO DEPENDS ON PROCESS:
  - HIGGS GLUON FUSION: PERTURBATIVE DEP. OF PDF NEGLIGIBLE IN COMPARISON TO MATRIX ELEMENT $\Rightarrow$ N^{3}LO NOT NEEDED
  - TOP: PERTURBATIVE DEP. OF PDF SMALLER, BUT NOT NEGLIGIBLE IN COMPARISON TO MATRIX ELEMENT, ANTICORRELATED TO IT $\Rightarrow$ N^{3}LO NECESSARY

SCALE UNCERTAINTY & DEP. ON PERTURBATIVE ORDER

HIGGS



TOP

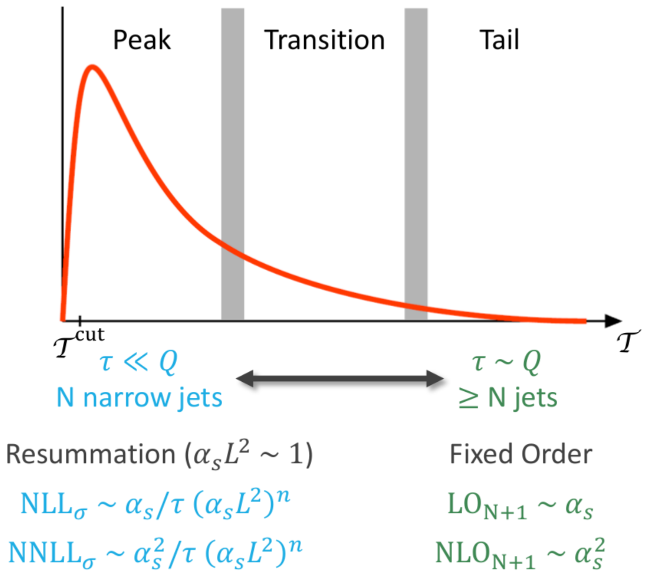


(s.f., Isgrò, Vita, 2014)

WHEN WILL WE HAVE THEM?

- N^{3}LO DIS COEFFICIENT FUNCTIONS KNOWN
- BOTTLENECK: N^3LO ANOMALOUS DIMENSIONS
- ANOMALOUS DIMENSIONS: LO: 1974; NLO: 1981; NNLO: 2004; N^{3}LO: 2030?

MONTE CARLO PDFs



(Alioli et al., 2012)

- THEORETICAL CALCULATIONS GETTING CLOSER TO THE FINAL STATE:
  - FULLY DIFFERENTIAL
  - PRODUCTION COMBINED WITH DECAY
  - FIDUCIAL OBSERVABLES
- REQUIRES MATCHING OF FIXED-ORDER, RESUMMATION, PARTON SHOWERING (POSSIBLY ALSO HADRONIZATION)
  $\Rightarrow$ (AT LEAST) THREE METHODS ALREADY AVAILABLE: GENEVA (SCET-BASED); NNLOPS (POWHEG-BASED); UN^{2}LOPS (SHERPA-BASED)
- EVENTUALLY, THIS WILL LEAD TO MONTE CARLO PDFs $\Rightarrow$ SAME TOOLS USED IN PDF DETERMINATION AS IN PHYSICAL PREDICTION

WHAT SHOULD WE AIM FOR 2030?

TYPICAL ACCURACY BELOW 5%, WITH RELIABLE UNCERTAINTY ESTIMATION:

- NNLO AS A DEFAULT FOR QCD CALCULATIONS
- SOME INCLUSIVE N^3LO OBSERVABLES (MOSTLY FOR UNCERTAINTY ESTIMATION)
- CONSIDERABLE PARTIAL INFORMATION ON HIGHER ORDERS FROM RESUMMATION (EIKONAL, NEXT-TO-EIKONAL, HIGH ENERGY)
- MIXED NLO QCD-EW CORRECTIONS AS A DEFAULT, SOMETIMES NNLO
- CONSISTENCY BETWEEN GLOBAL PDF FITS
- N^3LO PDFS (MOSTLY FOR UNCERTAINTY ESTIMATION)

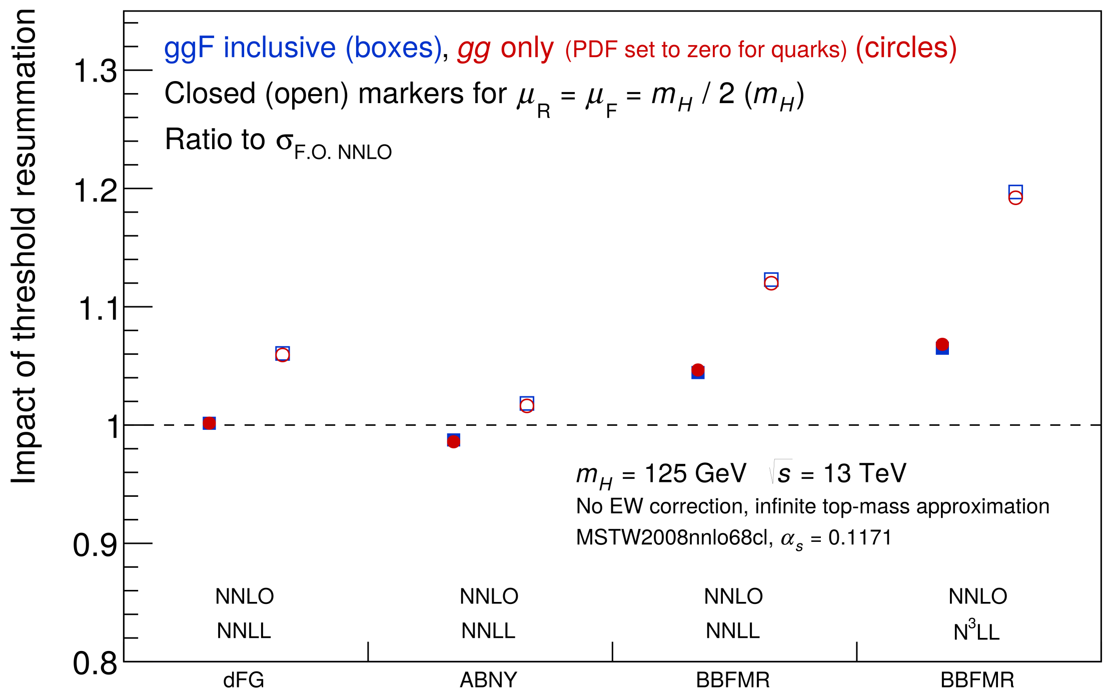
WHEN UNCERTAINTIES ARE SMALL, CAN WE ESTIMATE THEM RELIABLY?

EXTRAS

HIGGS IN GLUON FUSION: RESUMMATION AMBIGUITIES

- LARGE SUBLEADING TERMS $\Rightarrow$ LARGE AMBIGUITIES
- SCET RESUMMATION SMALLER, DUE TO $1/N$ TERMS (Bonvini, SF, Ridolfi, Rottoli)
- RESUMMATION EFFECTIVELY AMOUNTS TO APPROXIMATE HIGHER ORDERS

RESUMMED/UNRESUMMED RATIO (NO CONST. EXPONENTIATION)



note k factor computed wr to NNLO at respective scale

- De Florian, Grazzini (DFG) (HXSWG REFERENCE) RESUM $\ln N$
- Becher, Neubert et al. (ABNY): SCET (z SPACE) APPROACH TO NNLL (REALLY N^3LL^*)
- Ball et al. (BBFMR): RESUM $\ln N$, DIFFERS FROM DFG BECAUSE OF 'ANALYTIC' RESUMMATION (correct small-N singularities when expanded to finite order)
- BBFMR: N^3LL ALSO AVAILABLE

GLOBAL FITS: PROGRESS

- MMHT (SUCCESSOR OF MSTW08) DECEMBER 2014
- CT14 JUNE 2015
- FOR ALL, PROGRESS IN METHODOLOGY & DATASET

METHODOLOGY

	NNPDF3.0	MMHT14	CT14
NO. OF FITTED PDFS	7	7	6
PARAMETRIZATION	NEURAL NETS	$x^a(1-x)^b\times$ CHEBYSCHEV	$x^a(1-x)^b\times$BERNSTEIN
FREE PARAMETERS	259	37	30-35
UNCERTAINTIES	REPLICAS	HESSIAN	HESSIAN
TOLERANCE	NONE	DYNAMICAL	DYNAMICAL
CLOSURE TEST	✔	✗	✗
REWEIGHTING	REPLICAS	EIGENVECTORS	EIGENVECTORS

- MMHT, CT10 LARGER # OF PARMS., ORTHOGONAL POLYNOMIALS
- NNPDF CLOSURE TEST

DATASET

	NNPDF3.0	MMHT14	CT14(prel)
SLAC p,d DIS	✔	✔	✗
BCDMS p,d DIS	✔	✔	✔
NMC p,d DIS	✔	✔	✔
E665 p,d DIS	✗	✔	✗
CDHSW nu-DIS	✗	✗	✔
CCFR nu-DIS	✗	✔	✔
CHORUS nu-DIS	✔	✔	✗
CCFR dimuon	✗	✔	✔
NuTeV dimuon	✔	✔	✔
HERA I NC,CC	✔	✔	✔
HERA I charm	✔	✔	✔
H1,ZEUS jets	✗	✔	✗
H1 HERA II	✔	✗	✗
ZEUS HERA II	✔	✗	✗
E605 & E866 FT DY	✔	✔	✔
CDF & D0 W asym	✗	✔	✔
CDF & D0 Z rap	✔	✔	✔
CDF Run-II jets	✔	✔	✔
D0 Run-II jets	✗	✔	✔
ATLAS high-mass DY	✔	✔	✔
CMS 2D DY	✔	✔	✗
ATLAS W,Z rap	✔	✔	✔
ATLAS W pT	✔	✔	✗
CMS W asy	✔	✔	✔
CMS W +c	✔	✗	✗
LHCb W,Z rap	✔	✔	✔
ATLAS jets	✔	✔	✔
CMS jets	✔	✔	✔
ttbar tot xsec	✔	✔	✗
TOTAL NLO	4276	2996	3248
TOTAL NNLO	4078	2663	3045

OUTLOOK: NNPDF3.1

NEW DATA (MINIMAL SET)

- HERA-II COMBINED DATA arxiv:1506.06042
- D0 W ASYMMETRY arXiv:1412.2862
- ATLAS LOW-MASS DY arXiv:1404.1212
- ATLAS PROMPT PHOTON arXiv:1311.1440
- ATLAS $W + c$ arXiv:1402.6263
- ATLAS Z p_t arXiv:1406.3660
- ATLAS $t\bar{t}$ RAPIDITY arXiv:1407.0371
- ATLAS INCLUSIVE JETS 7 TEV 5 FB^{-1} arXiv:1410.8857
- CMS DOUBLE-DIFFERENTIAL 8 TEV arXiv:1412.1115
- CMS Z p_t arXiv:1504.03511
- LHCB $W \to \mu\nu$ RAP. arxiv:1408.4354

COMBINING INFORMATION

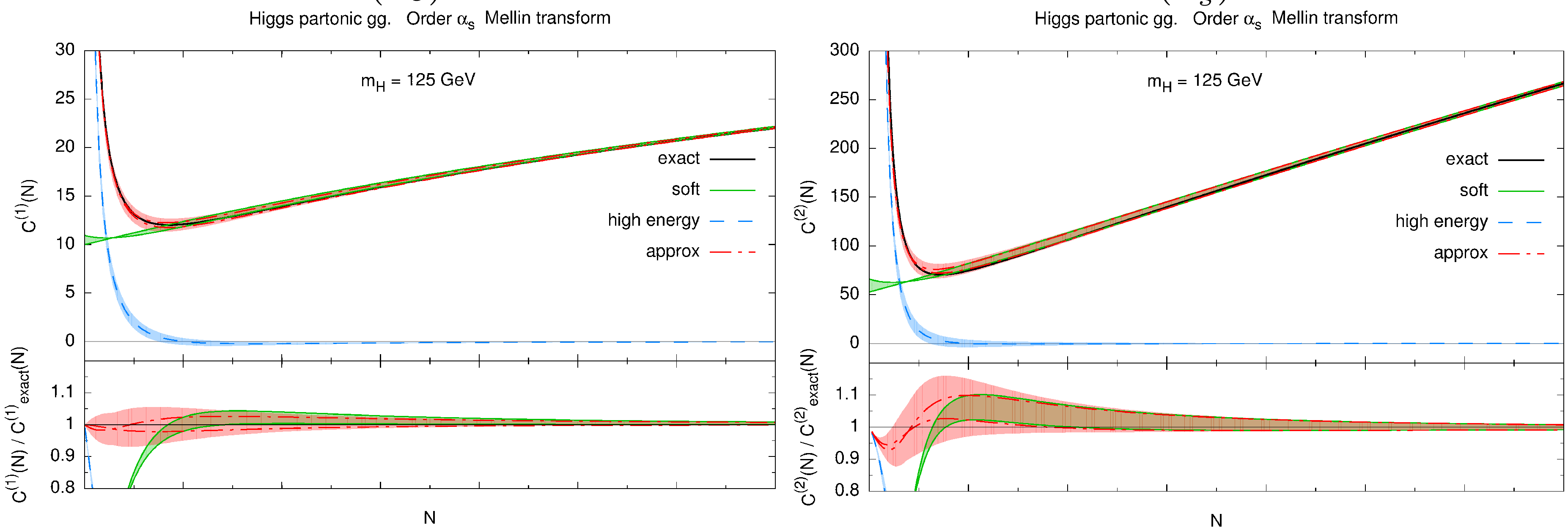
$$C^{(k)}_{\text{APPROX}}(N) = C^{(k)}_{\text{SOFT}}(N) + C^{(k)}_{\text{H.E.}}(N)$$

NOTHING ELSE TO DO: SINGULARITIES TAKE CARE OF THEMSELVES

NLO AND NNLO

EXACT+SOFT+HIGH ENERGY+APPROX.

$O(\alpha_s)$ $\qquad\qquad$ $O(\alpha_s^2)$



NOTE FOR $N \sim 1.2$ WHERE SMALL DISCRAPANCY WITH NNLO IS OBSERVED, "EXACT" IS BASED ON AN UNRELIABLE APPROX.

THE PIECES OF THE PUZZLE

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageAt8TeV#gluon_gluon_Fusion_Process

$$m_h = 125 \text{ GeV}; \qquad \sigma = 19.27 \text{ pb}^{+7.2}_{-7.8} \text{ (SCALE) } {}^{+7.5}_{-6.9} \text{ (PDF+}\alpha_s\text{)}$$

$$\mu_r = \mu_{\rm F} = m_H$$

(NOTE: IF $m_H = 125.5 \to \sigma = 19.12$ pb)

(De Florian, Grazzini, 2012)

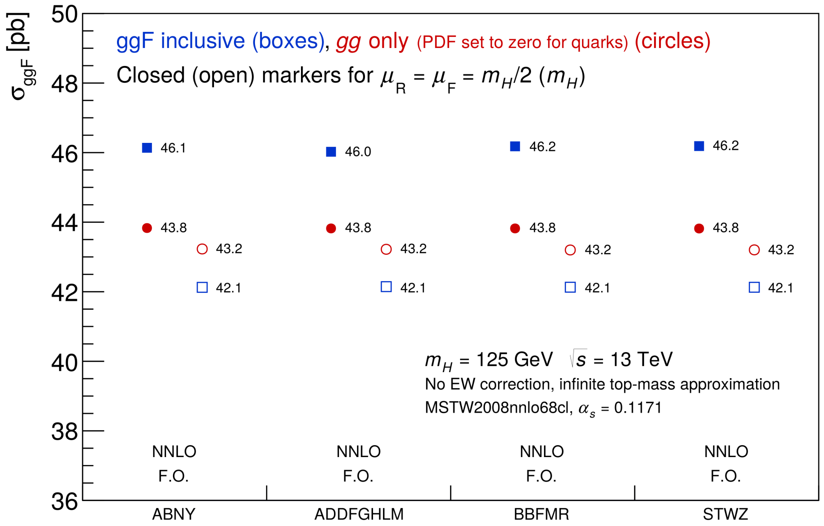
- NNLO+NNLL
- TOP MASS DEP, AND BOTTOM AND CHARM CONTRIBUTIONS AT NLO+NLL
  DECREASE $\sim 1.5\%$ DUE TO TOP+BOTTOM (MOSTLY BOTTOM)+
  FURTHER $\sim 1\%$ DUE TO CHARM
- NLO ELECTROWEAK CORRECTIONS (COMPLETE FACTORIZATION)
  INCREASE $\sim 5\%$ (WOULD CHANGE BY A FEW PERCENT WITH
  PARTIAL FACTORIZATION)
- COMPLEX-POLE (OFFP)
  FINITE-WIDTH BELOW $\sim 1\%$ FOR LIGHT HIGGS
- NO MIXED QCD-EW AND REAL EW RADIATION
  BELOW $\sim 1\%$

BENCHMARKING

THE GROUPS INVOLVED:

- ABNY Ahrens, Becher, Neubert, Yang → NNLO+NNLL RESUMMED (SCET)
- STWZ Stewart, Tackmann, Walsh, Zuberi → NNLO WITH EXP. CONSTANTS
- dFMMV de Florian, Mazzitelli, Moch, Vogt → APPROXIMATE N^3LO
- BBFMR Ball, Bonvini, SF, Marzani, Ridolfi → APPROXIMATE N^3LO+N^3LL
- ADDFGLHM Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Lazopoulos, Herzog, Mistlberger → PARTIAL N^3LO

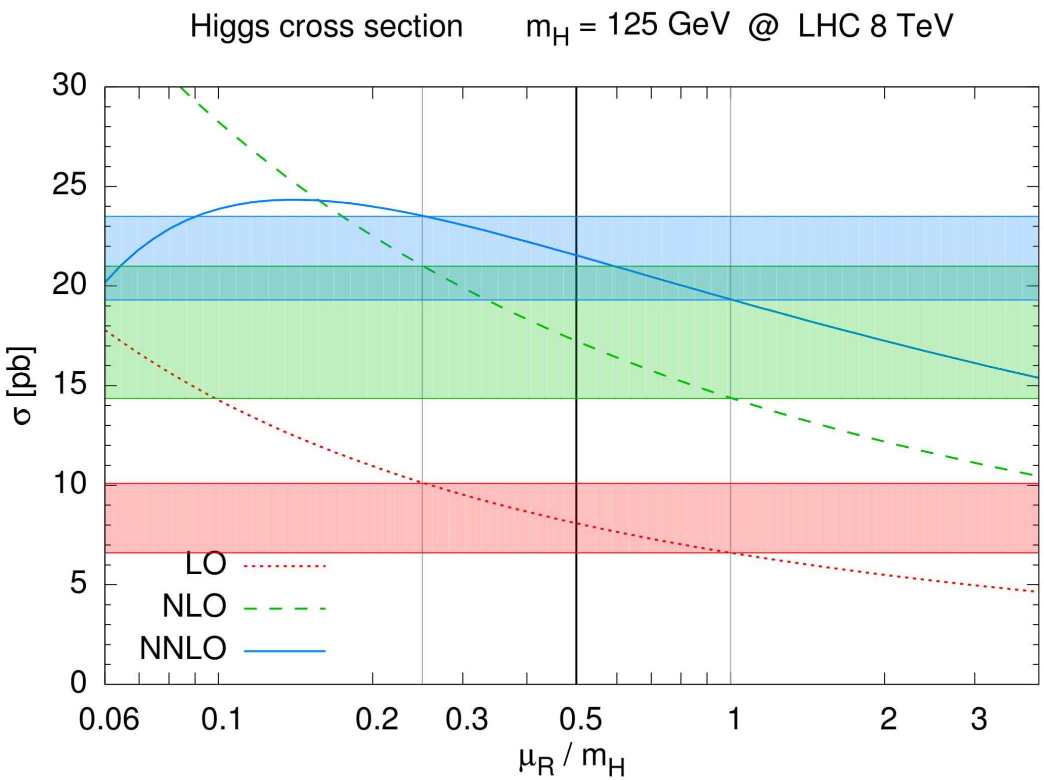
THE NNLO CROSS-SECTION



COMMON SETTINGS FOR PDFS, EW, HW, ETC ADOPTED;
NOT MEANT TO BE REALISTIC, FOCUS ON SIZE OF QCD CORRECTIONS

BEYOND NNLO

LO, NLO AND NNLO



- THE PERTURBATIVE SERIES FOR $gg \to H$ CONVERGES SLOWLY
- SCALE VARIATION UNDERESTIMATES SHIFT TO NEXT ORDER
- FACTORIZATION SCALE DEPENDENCE NEGLIGIBLE, ONLY RENORMALIZATION SCALE DEPENDENCE SIGNIFICANT

QUESTIONS

- DOES RESUMMATION IMPROVE THE FIXED-ORDER RESULT? HOW AMBIGUOUS IS IT?
- CAN WE CONSTRUCT A REASONABLE APPROXIMATION TO N^3LO BASED ON CURRENT KNOWLEDGE?

RESUMMATION

$$\sigma(N, m_H^2) \equiv \int_0^1 d\tau\, \tau^{N-2} \sigma(\tau, m_H^2); \quad z = \frac{M_h^2}{s} \qquad \left(\frac{\ln^{k-1}(1-x)}{1-x}\right)_+ \Leftrightarrow \ln^k N + O(\ln^{k-1} N) + O\left(\frac{1}{N}\right)$$

$$\sigma_{\rm res}(N, \alpha_s) = \sigma_0 g_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots\right];$$

$$g_0(\alpha_s) = 1 + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \mathcal{O}(\alpha_s^3);$$

N^{k}LL RESUMMATION: ALL g_i WITH $i < k+1$, ALL g_{0j} WITH $j < k$

TO ORDER α_s^n, $\ln^m N$ PREDICTED WITH $2(n-k) \leq m \leq 2n$

in SCET literature, this is called N^{k}LL', at N^{k}LL one less power of log: $2(n-k)+1 \leq m \leq 2n$

AMBIGUITIES

- SHOULD g_0 SIT IN THE EXPONENT, ENTIRELY OR IN PART?

  NOTE $g_{0,1}$ NATURALLY PART OF g_{i+2}

  $\Rightarrow$ DIFFERENCE SUBLEADING!

- HOW DOES ONE FIX SUBLEADING TERMS?

  $\ln N$, $\ln N + 1$, $\psi_0(N)$ (MELLIN OF $\left(\frac{1}{1-x}\right)_+$) BEHAVE IN THE SAME WAY AS $N \to \infty$, UP TO $O(1)$ & $O(1/N)$ TERMS

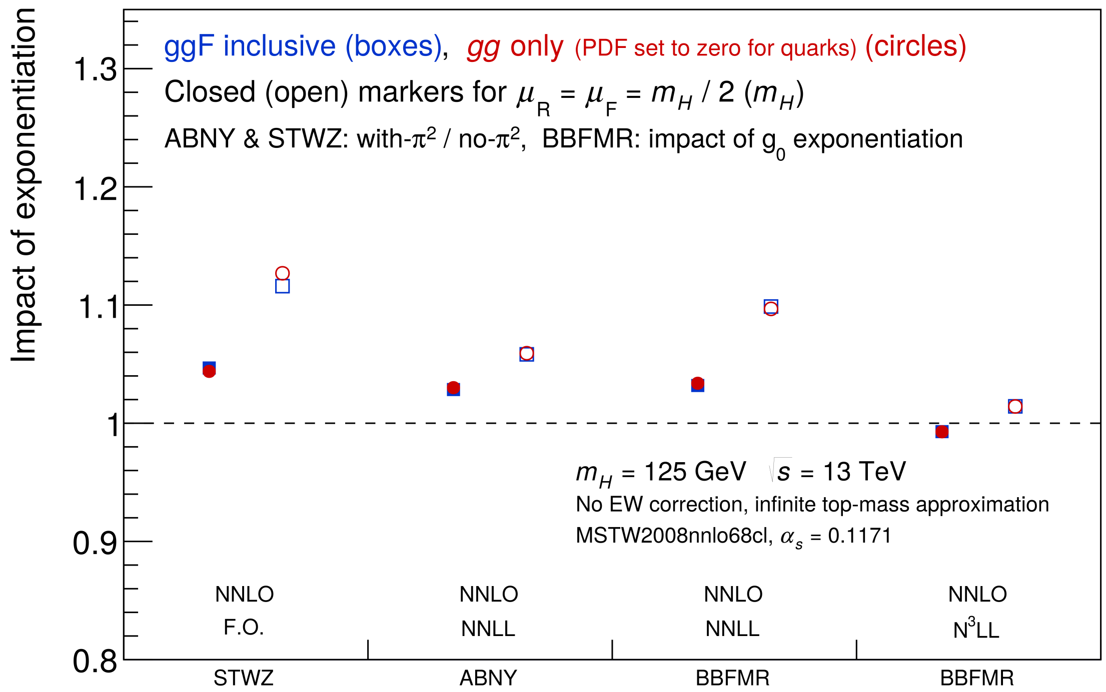
  $\Rightarrow$ DIFFERENCE SUBLEADING!

  NOTE IN SCET APPROACH, RESUMMATION NATURALLY IN z SPACE $\Rightarrow$ SUBLEADING DIFFERENCES

CONSTANT EXPONENTIATION

- IN SCET APPROACH, π^2 EXPONENTIATION W. IMAGINARY SCALE:
  STWZ: ADDED TO FIXED NNLO; ABNY: ADDED TO NNLO+NNLL
- IN DQCD RESUMMATION DIS-SCHEME CONSTANT EXPONENTIATION
  BBFMR: BOTH NNLO+NNLL AND NNLO+N^{3}LL AVAILABLE

EXPONENTIATED/UNEXPONENTIATED RATIO (NO RESUMMATION)



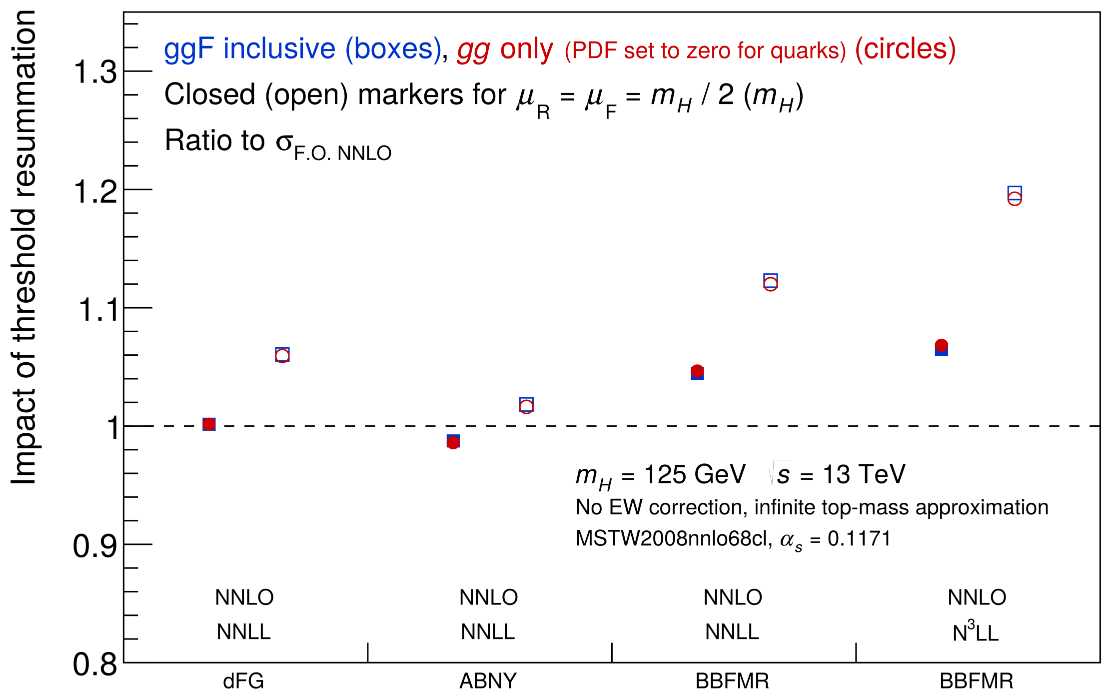
note k factor computed wr to NNLO at respective scale

- IMPACT SMALLER FOR $\mu_R = m_H/2$ THAN FOR $\mu_R = m_H$
- IMPACT SMALLER AT RESUMMED THAN AT FIXED ORDER
- IMPACT SMALLER AT N^{3}LL THAN AT NNLL

CHOICE OF LOGS

- DFG: CURRENT REFERENCE $\mu_R = m_H$
- ABNY: SCET (z SPACE) APPROACH TO NNLL (REALLY N^{3}LL*, OR SCET-N^{3}LL)
- BBFMR: DIFFERS FROM DFG BECAUSE OF 'ANALYTIC' RESUMMATION (correct small-N singularities when expanded to finite order)
- BBFMR: N^{3}LL ALSO AVAILABLE

RESUMMED/UNRESUMMED RATIO (NO CONST. EXPONENTIATION)



note k factor computed wr to NNLO at respective scale

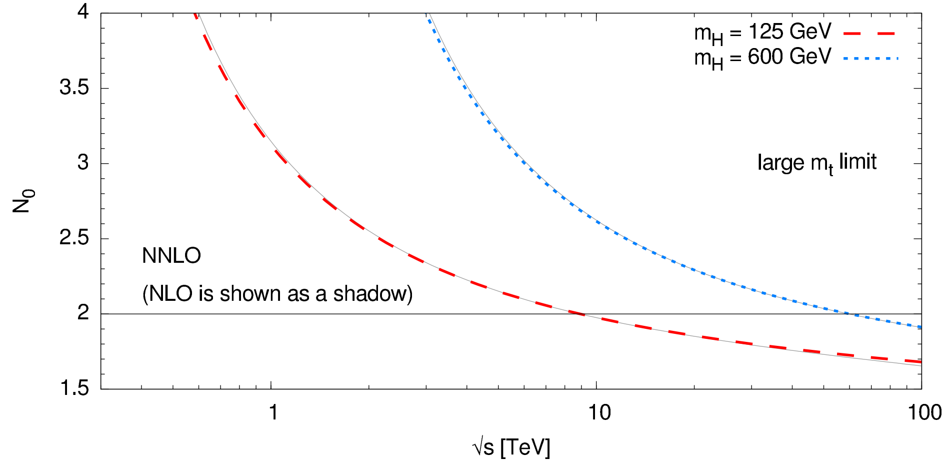
- IMPACT SMALLER FOR $\mu_R = m_H/2$ THAN FOR $\mu_R = m_H$
- z-SPACE: LITTLE EFFECT, ANALYTIC: LARGER EFFECT
- IMPACT LARGER AT N^{3}LL THAN AT NNLL

FROM RESUMMATION TO N^{3}LO

TECHNICAL INTERLUDE

- PARTONIC CROSS SECTIONS $\hat\sigma(z,\alpha_s)$ ARE DISTRIBUTIONS, $0 \le z = \frac{M_h^2}{s} \le 1$
- THEIR MELLIN TRANSFORMS $\hat\sigma(N,\alpha_s)$ ARE ORDINARY FUNCTIONS
- FOR GIVEN m_H^2 AND s ONLY ONE "SADDLE" N VALUE CONTRIBUTES
- BONUS: HAD. XSCT PRODUCT OF PARTONIC TIMES LUMI: $\sigma(N,m_H^2) = \hat\sigma\left(m_H^2,\alpha_s\right)\mathscr{L}(N)$

SADDLE VS COLLIDER ENERGY



Q: WHY ARE THERE LARGE RESUMMATION AMBIGUITIES?

A: BECAUSE $\beta_0\alpha_s \ln 2 \approx 0.04 \ll 1$:RESUMMATION IS PERTURBATIVE!

IN ALL FLAVOURS OF RESUMMATION DISCUSSED ABOVE

BETWEEN 2/3 AND 3/4 OF THE NNLL OR N^{3}LL RESUMMATION

COMES FROM N^{3}LO,

ALL REMAINING INFINITE ORDERS CONTRIBUTING $\sim 1 \div 2\%$

RESUMMATION IS AMBIGUOUS BECAUSE N^{3}LO IS APPROXIMATE!

N^3LO: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$\hat{\sigma}^{(3)}(N) = c_{06}\ln^6 N + c_{05}\ln^5 N + \cdots + c_{02}\ln^2 N + c_{01}\ln N + c_{00} + \qquad \text{SOFT (S)}$$
$$. \qquad + \frac{1}{N}\left(c_{15}\ln^5 N + c_{14}\ln^4 N + \cdots + c_{10}\right) + \qquad \text{NEXT-TO-SOFT (NS)}$$
$$. \qquad + \frac{1}{N^2}\left(c_{25}\ln^5 N + c_{24}\ln^4 N + \cdots + c_{20}\right) + \ldots \qquad \text{NNS}$$

$$\hat{\sigma}^{(3)}(z) = c'_{06}\left(\frac{\ln^5(1-z)}{1-z}\right)_+ + \cdots + c'_{01}\left(\frac{1}{1-z}\right)_+ + c'_{00}\delta(1-z) + \qquad \text{SOFT (S)}$$
$$. \qquad + \left(c'_{15}\ln^5(1-z) + c'_{14}\ln^4(1-z) + \cdots + c'_{10}\right) + \qquad \text{NEXT-TO-SOFT (NS)}$$
$$. \qquad + (1-z)\left(c'_{25}\ln^5(1-z) + +c'_{24}\ln^4(1-z) + \cdots + c'_{20}\right) + \ldots \qquad \text{NNS}$$

NSPACE c COEFFICIENTS DETERMINE THE z SPACE c' AND CONVERSELY:

- N-SPACE N^k SOFT $\Leftrightarrow$ z-SPACE N^k SOFT
- N-SPACE N^kLL $\Leftrightarrow$ z-SPACE N^kLL
- MELLIN OF z SPACE N^k SOFT EQUALS N-SPACE N^k SOFT PLUS N^{k+1} TERMS
- MELLIN OF z SPACE N^kLL EQUALS N-SPACE N^kLL SOFT PLUS N^{k+1}LL TERMS

$\mathrm{N^3LO}$: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

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$$. \qquad + (1-z)\left(c'_{25}\ln^5(1-z) + c'_{24}\ln^5(1-z) + \cdots + c'_{20}\right) + \ldots \qquad \text{NNS}$$

- c_{06} TO c_{02} DETERMINED BY NNLL RESUMMATION
- c_{01} COMPUTED (Moch, Vogt, 2005; Laenen, Magnea, 2005)
- COMPLETE z-SPACE SOFT $\Rightarrow c'_{00}$ (Anastasiou et al. 2014)
- $c_{15}, c_{24}, \ldots c_{51}$ KNOWN FROM LL RESUMMATION (Catani, deFlorian, Grazzini, 2001)
- c_{14} COMPUTED BASED ON CONJECTURE (deFlorian, Mazzitelli, Moch, Vogt, 2014)
- COMPLETE z-SPACE NS $\Rightarrow$ ALL c'_{1i} (Anastasiou et al. 2015) $\to$ CONJECTURE VALIDATED
- COMPLETE z-SPACE $\ln^4(1-z)$ & $\ln^3(1-z)$ $\Rightarrow$ ALL c'_{i4}, c'_{i3} (Anastasiou et al. 2015) (FORMALLY LEADING LOGS)

N^3LO: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$\hat\sigma^{(3)}(N) = c_{06}\ln^6 N + c_{05}\ln^5 N + \cdots + c_{02}\ln^2 N + c_{01}\ln N + c_{00} + \qquad \text{SOFT (S)}$$
$$. \qquad + \frac{1}{N}\left(c_{15}\ln^5 N + c_{14}\ln^4 N + \cdots + c_{10}\right) + \qquad \text{NEXT-TO-SOFT (NS)}$$
$$. \qquad + \frac{1}{N^2}\left(c_{25}\ln^5 N + c_{24}\ln^4 N + \cdots + c_{20}\right) + \ldots \qquad \text{NNS}$$

$$\hat\sigma^{(3)}(z) = c'_{06}\left(\frac{\ln^5(1-z)}{1-z}\right)_+ + \ldots + c'_{01}\left(\frac{1}{1-z}\right)_+ + c'_{00}\delta(1-z) + \qquad \text{SOFT (S)}$$
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N^3LO: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$\hat{\sigma}^{(3)}(N) = c_{06} \ln^6 N + c_{05} \ln^5 N + \cdots + c_{02} \ln^2 N + c_{01} \ln N + c_{00} + \qquad \text{SOFT (S)}$$

$$. \qquad + \frac{1}{N} \left(c_{15} \ln^5 N + c_{14} \ln^4 N + \cdots + c_{10} \right) + \qquad \text{NEXT-TO-SOFT (NS)}$$

$$. \qquad + \frac{1}{N^2} \left(c_{25} \ln^5 N + c_{24} \ln^4 N + \cdots + c_{20} \right) + \ldots \qquad \text{NNS}$$

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N^{3}LO: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$\hat\sigma^{(3)}(N) = c_{06}\ln^6 N + c_{05}\ln^5 N + \cdots + c_{02}\ln^2 N + c_{01}\ln N + c_{00} + \qquad \text{SOFT (S)}$$
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$\mathrm{N}^3\mathrm{LO}$: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$
\begin{aligned}
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N^3LO: WHAT DO WE KNOW

THE HIGH-ENERGY EXPANSION

SMALL N POLES OR ABOUT $z = 0$

rightmost singularity at $N = 1$:

$$\hat{\sigma}^{(3)}(N) = \left(d_{13}\frac{1}{(N-1)^3} + d_{12}\frac{1}{(N-1)^2} + d_{11}\frac{1}{(N-1)}\right) + \qquad \text{LEADING LOG } x \text{ (LL}x\text{)}$$

$$. \qquad + \left(d_{06}\frac{1}{N^6} + \cdots + d_{01}\frac{1}{N}\right) + \qquad \text{NLL}x$$

$$. \qquad + \left(d_{-16}\frac{1}{(N+1)^6} + \cdots + d_{-11}\frac{1}{N+1}\right) + + \ldots \qquad \text{NNLL}x$$

$$\hat{\sigma}^{(3)}(z) = \left(d_{13}\frac{\ln^2 z}{z} + d_{12}\frac{\ln z}{z} + d_{11}\frac{1}{z}\right) + \qquad \text{LEADING LOG } x \text{ (LL}x\text{)}$$

$$. \qquad + \left(d_{06}\ln^6 z + \cdots + d_{01}\ln z\right) + \qquad \text{NLL}x$$

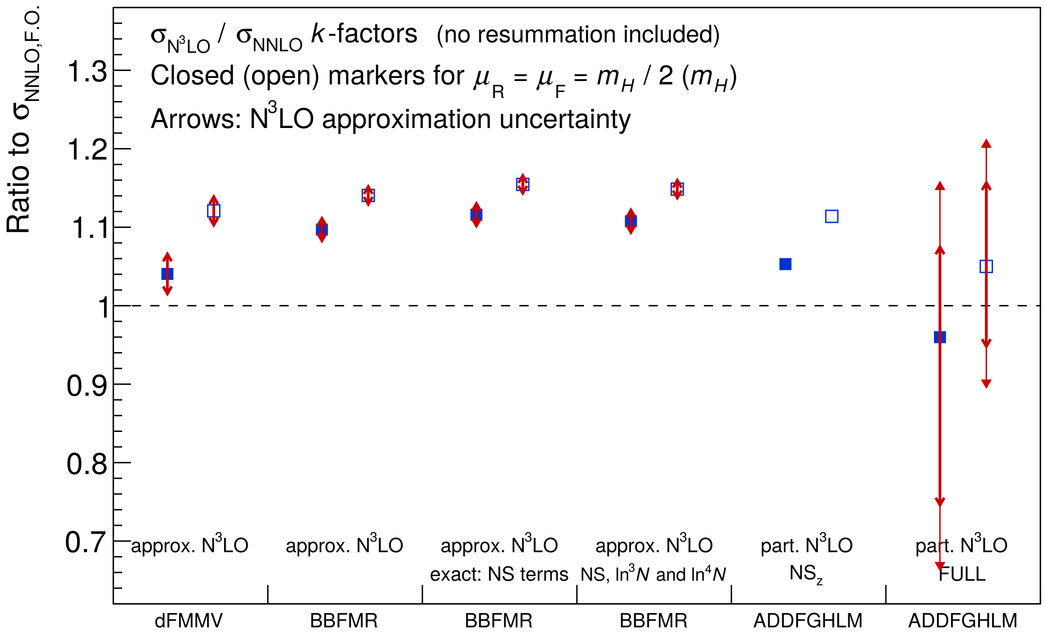
$$. \qquad + z\left(d_{-16}\ln^6 z + \cdots + d_{-11}\ln z\right) + \ldots \qquad \text{NNLL}x$$

- AS ABOVE, z-SPACE $\Leftrightarrow$ N-SPACE, ORDER BY ORDER (LOG & POWER), UP TO SUBLEADING TERMS
- ONLY d_{13} KNOWN EXACTLY FROM BFKL+SMALL x RESUMMATION
- CLASS OF DOMINANT (?) CONTRIBUTIONS TO d_{12}, d_{11} KNOWN FROM NL BFKL
- ALL SUBLEADING SINGS UNKNOWN

APPROXIMATE N^{3}LO

- DFMMV: N-SPACE APPROXIMATE NS; NOW NS KNOWN EXACTLY: SMALL EXPECTED CHANGE
- BBFMR: ALL KNOWN SINGULARITIES (SOFT & HIGH ENERGY):
  NS AND BEYOND APPROXIMATELY DETERMINED
  $\rightarrow$ IMPROVED WITH EXACT NS $\rightarrow$ IMPROVED WITH EXACT $\ln^4 N$, $\ln^3 N$ (SMALL CHANGES)
- ADDFGHLM: ALL KNOWN z SPACE TERMS (FULL) $\rightarrow$ PURE z SPACE NS ALSO SHOWN

N^{3}LO/NNLO k-FACTOR



note k factor computed wr to NNLO at respective scale

UNCERTAINTIES (ARROWS)

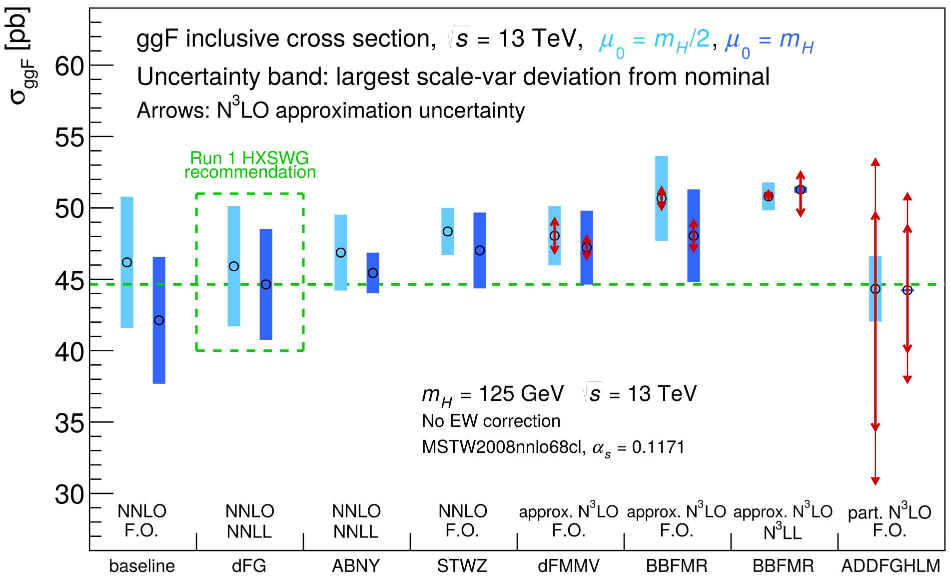
- DFMMV: S-NS BAND
- BBFMR: ESTIMATE OF SUBLEADING SINGULARITIES VALIDATED BY KNOWN ORDERS
- ADDFGHLM: ENVELOPE OF SCALE VAR. OF NS-FULL DIFFERENCE AT m_H, $\times$ 1.5 OR 1

SIZE OF UNCERTAINTY HOTLY DEBATED!!

WHAT IS THE CROSS-SECTION?

- STWZ: DIFFERS FROM NNLO BY CONST. EXPONENTIATION
- ABNY: DIFFERS FROM NNLO+NNLL (REF) BY CONST. EXPONENTIATION & DIFFERENT RESUMMATION
- DFMMV: APPROXIMATE N^3LO BASED ON APPROXIMATE N-SPACE NS; ARROWS: S-NS DIFFERENCE
- BBFMR: APPROXIMATE N^3LO BASED ON ANALYTIC APPROX (INCLUDING NS+BFKL); ARROWS: ESTIMATED MISSING SINGS+ FACT SCALE VARIATION FROM MISSING QUARK CHANNELS; $\to$ N^3LO+N^3LL BEST PREDICTION
- ADDFGHLM: ALL KNOWN z SPACE TERMS; ARROWS: ENVELOPE OF SCALE VARIATION OF NS-FULL DIFFERENCE

THE GLUON FUSION CROSS-SECTION



all numbers given with benchmark settings (K-factor to dFG very stable)

band is seven-point fact. and ren. scale variation