Sky map reconstruction from Transit interferometers

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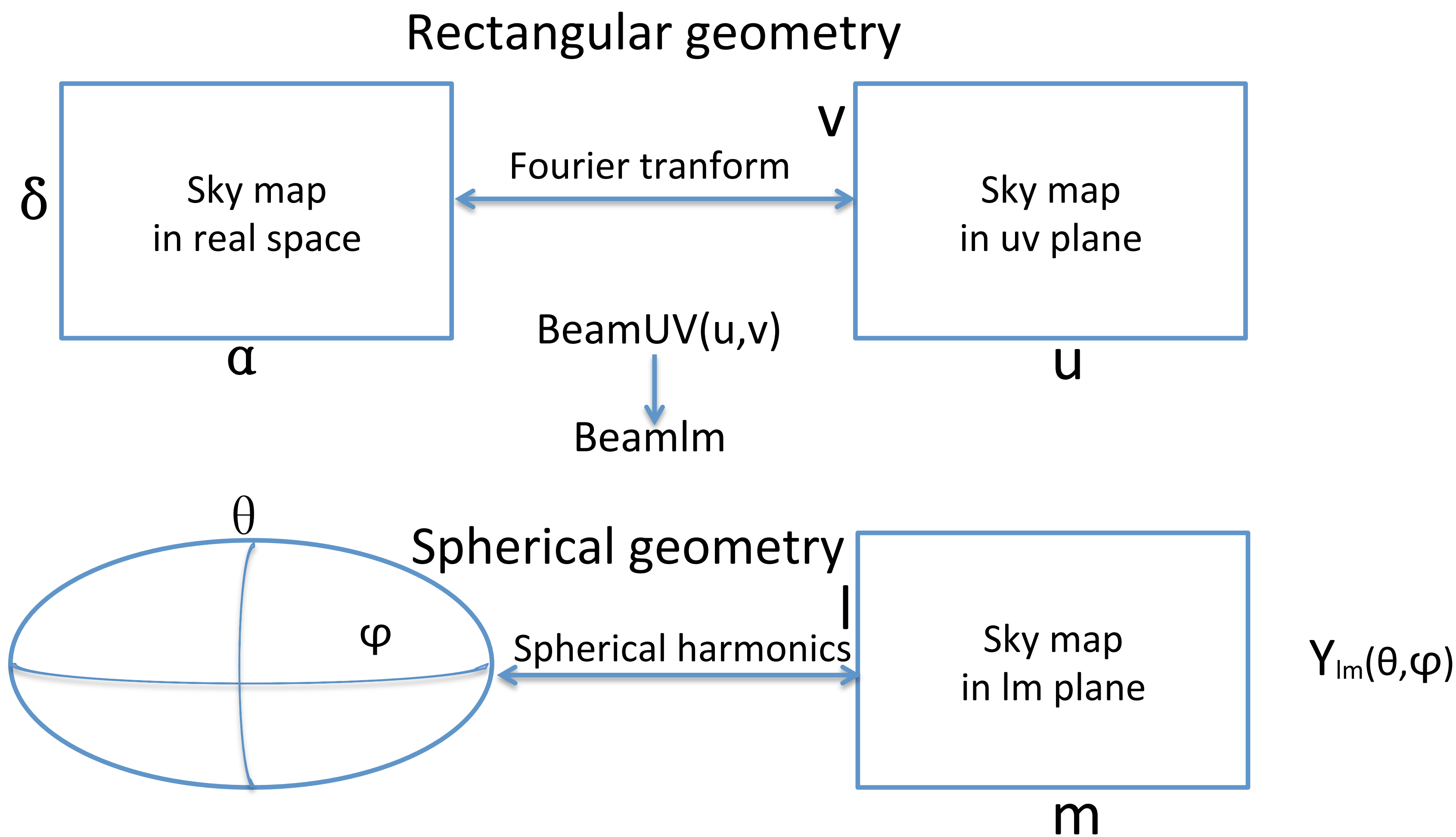
aim

- Evaluate sky reconstruction performance for different interferometer configurations and survey strategy.
- Map reconstruction software from cleaned/calibrated visibility data for PAON4 & Tianlai.

plan

- Map reconstruction principle for transit interferometer
- Show some results for PAON4, Tianlai dish arrays.



The current version of the software (JSkyMap) implements sky map reconstruction for both geometries: rectangular and spherical

Map reconstruction in rectangular geometry

- Visibility from one observed direction on the sky

$$V_{ij} = \iint Sky(\alpha,\delta)\cdot Beam(\alpha,\delta) d\alpha d\delta + noise$$

$$V_{ij} \;=\; \sum_u \sum_v F_{sky}(u,v) BeamUV(u,v) + noise$$

Transit telescope

Full EW(α) scan for a fixed δ_0

Sky rotation will bring us a phase factor:

$$Sky(\alpha,\delta) \longrightarrow Sky(\alpha-\alpha_0,\delta) \qquad F_{sky} \longrightarrow F_{sky}\times e^{2i\pi u\alpha_0}$$

Measurement: a set of $V_{ij}^{\delta_0}$ as a function of time $\Longrightarrow V_{ij}^{\delta_0}(\alpha=\omega t)$

$V_{ij}^{\delta_0}$: a pair of antenna for a given pointing δ_0, which corresponds to a beam $Beam_{ij}^{\delta_0}(\alpha,\delta) \longleftrightarrow BeamUV_{ij}^{\delta_0}(u,v)$

Pointing in NS direction

Time (earth rotation)

$$V_{ij}^{\delta_0}(\alpha_0) \;=\; \sum_u \sum_v F_{sky}(u,v)\exp(i2\pi u\alpha)BeamUV^{\delta_0}(u,v) + noise$$

Full α scan means we could perform Fourier transform on the set of $V_{ij}(\alpha)$ to obtain the \tilde[$V_{ij}(u)$] for each u-mode

$$V_{ij}^{\delta_0}(\alpha) \quad (0 \leqslant \alpha < 2\pi) \xrightarrow{\text{FT}} \widetilde{V_{ij}}^{\delta_0}(u) \quad for\ all\ u$$

$$\widetilde{V_{ij}}^{\delta_0}(u) = \mathscr{F}(V_{ij}^{\delta_0}(\alpha)) = \sum_v F_{sky}(u,v)BeamUV^{\delta_0}(u,v) + noise$$

We could express the relation between sky and visibilities in matrix form (we keep only one V_{ij} for a set of antenna with the same baseline and same pointing to simplify numerical handling)

The full problem of all $V_{ij}(u)$ could be separated into a set of **independent** problems for each **u**.

$$\begin{pmatrix} \cdots \\ \widetilde{V_{ij}}^{\delta_0}(u) \\ \cdots \end{pmatrix} = A \times \begin{pmatrix} \cdots \\ F_{sky}(u,v) \\ \cdots \end{pmatrix} + noise \longrightarrow \begin{pmatrix} \cdots \\ \widetilde{V_{ij}}^{\delta_0} \\ \cdots \end{pmatrix} = A_{u_0} \times \begin{pmatrix} \cdots \\ F^{u_0}_{sky}(v) \\ \cdots \end{pmatrix} + noise$$

Fixed u: u --> u_0

A_{u0}

$$\begin{pmatrix} \widetilde{V_{11}}^{\delta_0} \\ \widetilde{V_{ij}}^{\delta_0} \\ \cdots \\ \widetilde{V_{11}}^{\delta_1} \\ \widetilde{V_{ij}}^{\delta_1} \\ \cdots \\ \widetilde{V_{11}}^{\delta_n} \\ \widetilde{V_{ij}}^{\delta_n} \end{pmatrix} = \begin{pmatrix} Beamuv^{\delta_0}_{11}(v_1) & Beamuv^{\delta_0}_{11}(v_2) & \cdots & Beamuv^{\delta_0}_{11}(v_m) \\ Beamuv^{\delta_0}_{ij}(v_1) & Beamuv^{\delta_0}_{ij}(v_2) & \cdots & Beamuv^{\delta_0}_{ij}(v_m) \\ & & \cdots & \\ Beamuv^{\delta_n}_{11}(v_1) & Beamuv^{\delta_n}_{11}(v_2) & \cdots & Beamuv^{\delta_n}_{11}(v_m) \\ Beamuv^{\delta_n}_{ij}(v_1) & Beamuv^{\delta_n}_{ij}(v_2) & \cdots & Beamuv^{\delta_n}_{ij}(v_m) \end{pmatrix} \times \begin{pmatrix} F_s(v_1) \\ \cdots \\ F_s(v_m) \end{pmatrix} + noise$$

$$noise = \begin{pmatrix} n^{\delta_0}_{11} \\ n^{\delta_0}_{ij} \\ \cdots \\ n^{\delta_1}_{11} \\ n^{\delta_1}_{ij} \\ \cdots \\ n^{\delta_n}_{11} \\ n^{\delta_n}_{ij} \end{pmatrix}$$

BeamUV

Sky Fourier modes for u0

Map reconstruction in spherical geometry

- Auto-correlation: $\Omega = (\theta, \varphi)$

$$\mathcal{V}_{ii} = \iint d\Omega I(\Omega) L_{ii}(\Omega) \qquad L_{ii}(\Omega) = B(\Omega)B^*(\Omega)$$

- For a pair of antenna: $|\vec{k}| = \frac{2\pi}{\lambda}$

$$\mathcal{V}_{ij} = \iint d\Omega I(\Omega) L_{ij}(\Omega) \qquad L_{ij}(\Omega) = B_1(\Omega)B_2^*(\Omega)\exp(i\vec{k}\cdot\vec{\Delta r})$$

$\mathcal{V}_{ij}^{\theta_0}$: a pair of antenne for a given pointing θ_0, which corresponds to a beam

$$Beam_{ij}^{\theta_0}(\theta,\varphi) \longleftrightarrow BeamLM_{ij}^{\theta_0}(l,m)$$

Measurement: a set of $\mathcal{V}_{ij}^{\theta_0}$ as a function of time $\Longrightarrow \mathcal{V}_{ij}^{\theta_0}(\varphi = \omega t)$

Pointing in NS direction

Time (earth rotation)

$$\mathcal{V}_{ij}^{\theta_0}(\varphi_k) = \sum_m \sum_l \mathcal{I}(l,m)\mathcal{L}_{ij}^{\theta_0}(l,m) \times e^{2i\pi m\varphi_k} + noise$$

Full φ scan means we could perform Fourier transform on the set of $V_{ij}(\varphi)$ to obtain the \tilde[$V_{ij}(m)$] for each m-mode

$$\mathcal{V}_{ij}^{\theta_0}(\varphi_k) \quad (0 \leqslant \varphi_k < 2\pi) \xrightarrow{\text{FT}} \tilde{\mathcal{V}}_{ij}^{\theta_0}(m) \quad for\ all\ m\ modes$$

$$\tilde{\mathcal{V}}_{ij}^{\theta_0}(m) = \mathscr{F}(\mathcal{V}_{ij}^{\theta_0}(\varphi_k)) = \sum_l \mathcal{I}(l,m)\mathcal{L}_{ij}^{\theta_0}(l,m) + noise$$

There is a track to take into account of both +m and –m modes, which corresponds to real sky with only +m modes

We could also express the relation between sky and visibilities in matrix form

A matrix

$$
\begin{pmatrix} \tilde{V}_{ij}^{\theta_k}(m_0) \\ \tilde{V}_{ij}^{\theta_k}(m_1) \\ \vdots \\ \tilde{V}_{ij}^{\theta_k}(m_{max}) \end{pmatrix}
=
\begin{pmatrix} \mathcal{L}_{ij}^{\theta_k}(m_0) & 0 & & 0 \\ 0 & \mathcal{L}_{ij}^{\theta_k}(m_1) & & \\ & & \ddots & \\ 0 & & & \mathcal{L}_{ij}^{\theta_k}(m_{max}) \end{pmatrix}
\begin{pmatrix} \mathcal{I}(m_0, l) \\ \mathcal{I}(m_1, l) \\ \vdots \\ \mathcal{I}(m_{max}, l) \end{pmatrix}
$$

The full problem of all V_{ij} could be separated into a set of **independent** problems for each **m**.

For each m modes, we would rewrite matrix as

$$
\begin{pmatrix} \widetilde{V_{11}}^{\theta_0} \\ \widetilde{V_{ij}}^{\theta_0} \\ \cdots \\ \widetilde{V_{11}}^{\theta_n} \\ \widetilde{V_{ij}}^{\theta_n} \end{pmatrix}
=
\begin{pmatrix}
\mathcal{L}_{11}^{\theta_0}(l_0) & \mathcal{L}_{11}^{\theta_0}(l_1) & \cdots & \mathcal{L}_{11}^{\theta_0}(l_{max}) \\
\mathcal{L}_{ij}^{\theta_0}(l_0) & \mathcal{L}_{ij}^{\theta_0}(l_1) & \cdots & \mathcal{L}_{ij}^{\theta_0}(l_{max}) \\
 & & \cdots & \\
\mathcal{L}_{11}^{\theta_n}(l_0) & \mathcal{L}_{11}^{\theta_n}(l_1) & \cdots & \mathcal{L}_{11}^{\theta_n}(l_{max}) \\
\mathcal{L}_{ij}^{\theta_n}(l_0) & \mathcal{L}_{ij}^{\theta_n}(l_1) & \cdots & \mathcal{L}_{ij}^{\theta_n}(l_{max})
\end{pmatrix}
\times
\begin{pmatrix} \mathcal{I}(l_1) \\ \cdots \\ \mathcal{I}(l_{max}) \end{pmatrix}
+ noise
$$

$$
noise = \begin{pmatrix} n_{11}^{\theta_0} \\ n_{ij}^{\theta_0} \\ \cdots \\ n_{11}^{\theta_n} \\ n_{ij}^{\theta_n} \end{pmatrix}
$$

We use pseudo-inverse matrix (singular value decomposition) considering noise covariance matrix N. $N = \langle n^t n \rangle$

The noise covariance matrix N is positive and symmetric. We currently assume a diagonal matrix for N

$$B = (A^t N^{-1} A)^{-1} A^t N^{-1} = (N^{-\frac{1}{2}} A)^{-1} N^{-\frac{1}{2}}$$

For each u/m, the estimated sky

Noise weighted pseudo-inverse

Fourier transform of Visib

$$\begin{pmatrix} \cdots \\ \hat{F}_s^{u_i}(v) / \hat{\mathcal{I}}^{m_i}(l) \\ \cdots \end{pmatrix} = \underbrace{\left((N^{-\frac{1}{2}} A_{u_0})^{-1} N^{-\frac{1}{2}} \right)}_{B} \times \begin{pmatrix} \cdots \\ \widetilde{V_{ij}}^{\delta_0} \\ \cdots \end{pmatrix}$$

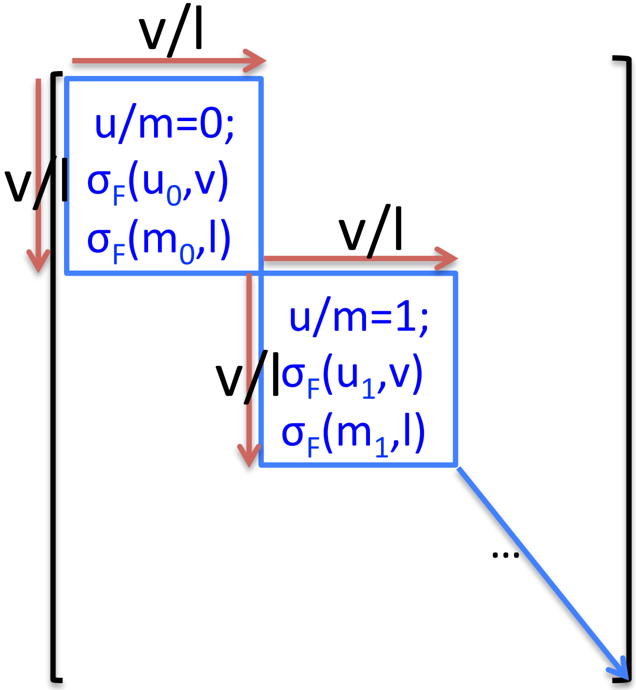
B

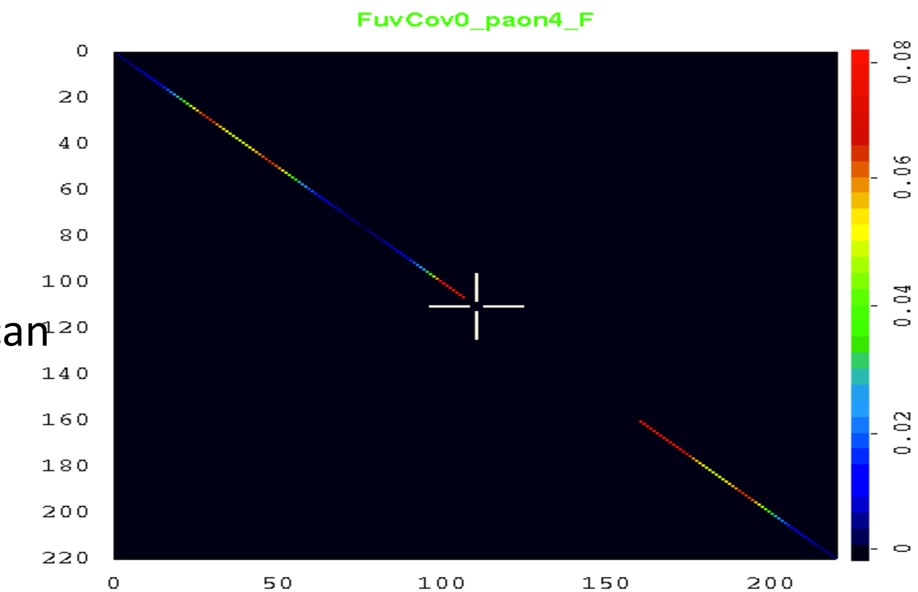
Here, we define a threshold, which is touchy, to reject small eigenvalue to avoid numerical instability and large noise. i.e. if λ_i<thr, then $1/\lambda_i=0$.

$$\rightarrow \begin{pmatrix} \cdots \\ \widehat{F}_{sky}(u, v) \\ \cdots \end{pmatrix} \text{ or } \begin{pmatrix} \cdots \\ \hat{\mathcal{I}}_{sky}(l, m) \\ \cdots \end{pmatrix}$$

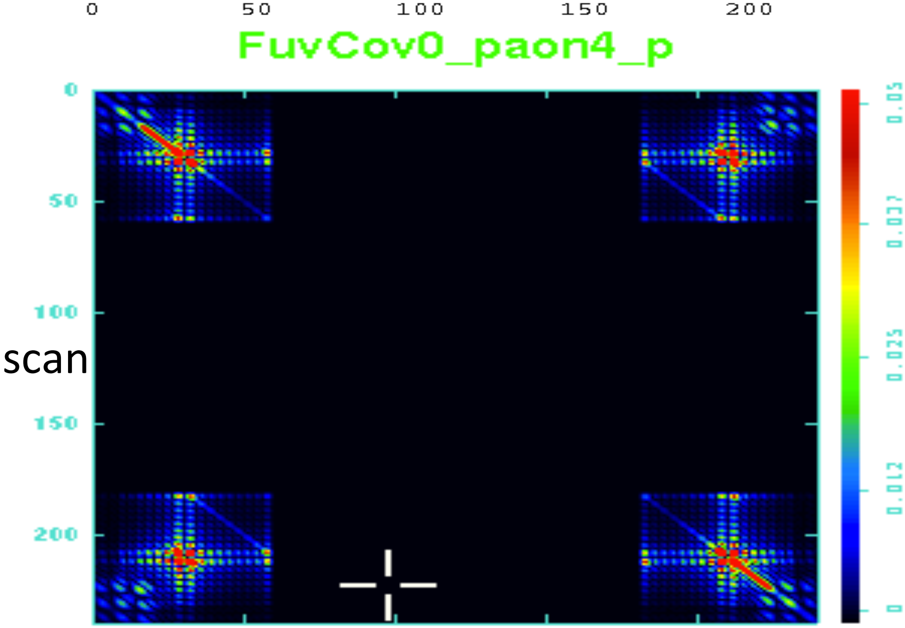
We also compute observed sky **error covariance matrix** for each u/m modes.
Then, we reconstruct F^hat(u,v)/ I^hat(l,m) error covariance matrix from diagonal.

$$\sigma^2_{\hat{F}} = < \hat{F}^{u_0}_{sky}(v)\hat{F}^{u_0}_{sky}(v')^* > = BNB^t$$





PAON4 full scan



PAON4 partial scan

Applying a weight function on F^hat_keep(u,v) to control the noise effect (reject reconstructed modes with large errors). For example, we use a global Gaussian beam response.

A global beam independent of frequency can be used to decrease mode mixing.

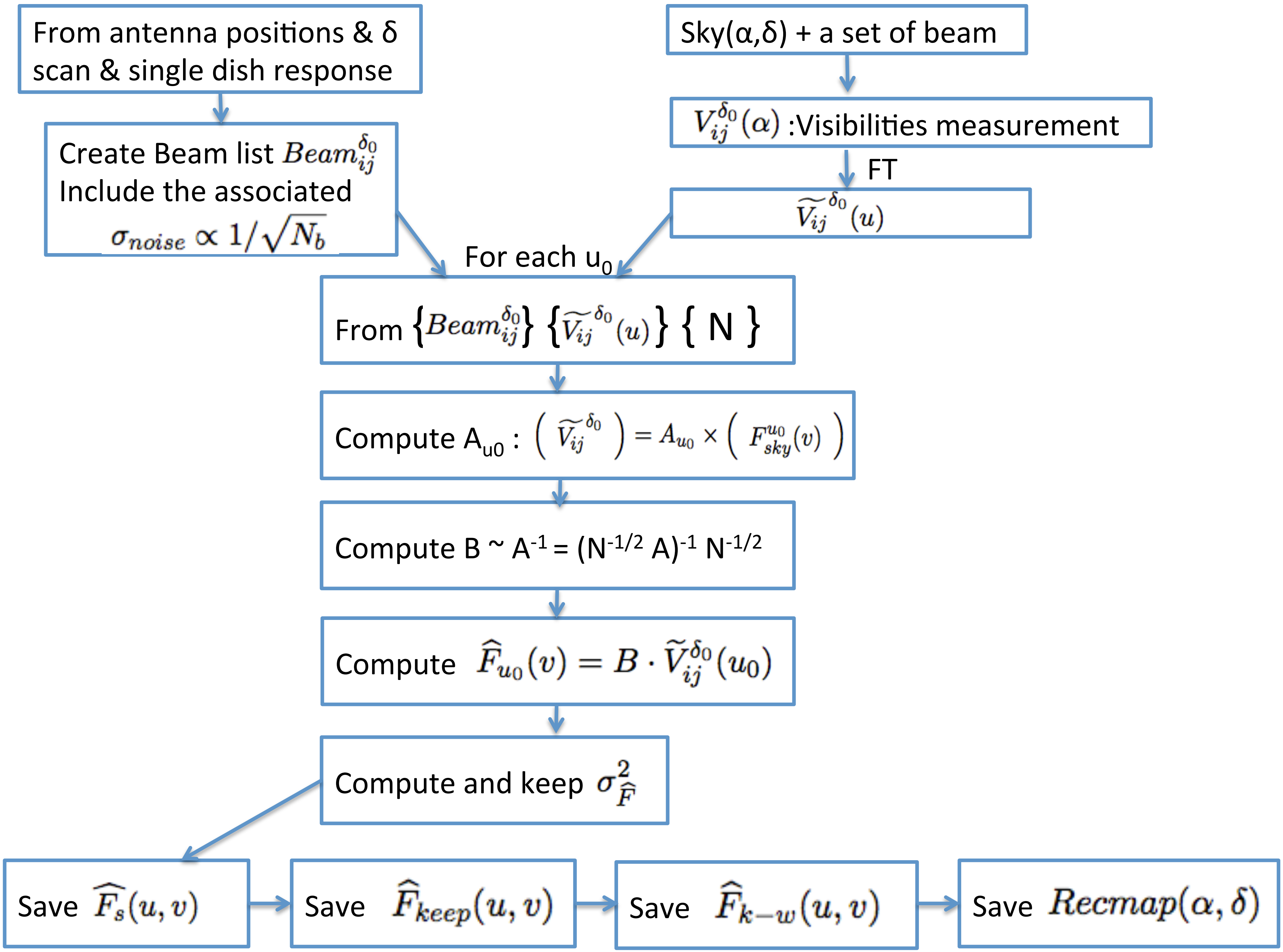
$$\begin{pmatrix} \cdots \\ \widehat{F}_{sky}(u,v) \\ \cdots \end{pmatrix} \longrightarrow \begin{pmatrix} \cdots \\ \widehat{F}_{s-keep}(u,v) \\ \cdots \end{pmatrix} \longrightarrow \begin{pmatrix} \cdots \\ \widehat{F}_{sk-weighted}(u,v) \\ \cdots \end{pmatrix} \longrightarrow \begin{pmatrix} \cdots \\ \widehat{sky}(\alpha,\beta) \\ \cdots \end{pmatrix}$$

Reject modes with high error
Cut off larger value
Threshold on $\sigma^2_{\widehat{F}}$

Apply a weight function

Inverse Fourier transform

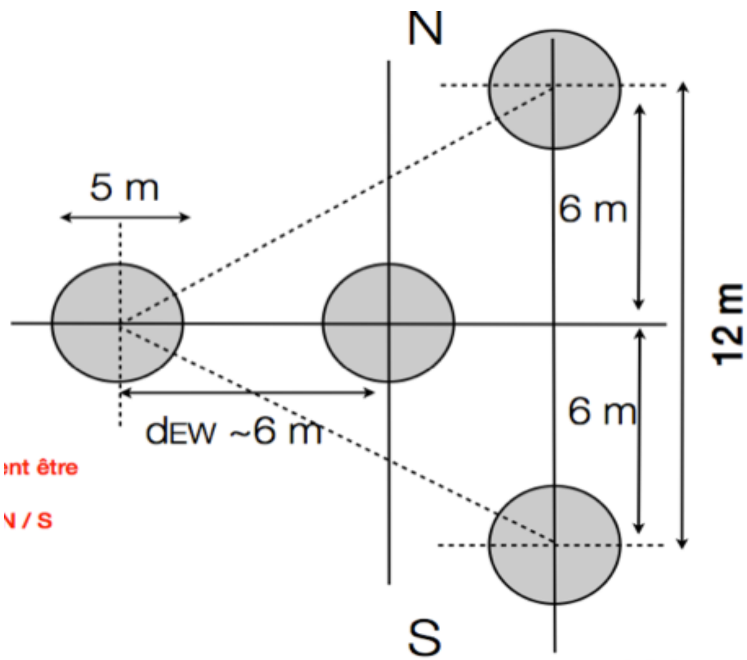
There are also the same steps in spherical, replacing (u,v) by (m, l)


From antenna positions & δ scan & single dish response
Sky(α,δ) + a set of beam
Create Beam list $Beam_{ij}^{\delta_0}$ Include the associated $\sigma_{noise} \propto 1/\sqrt{N_b}$
$V_{ij}^{\delta_0}(\alpha)$:Visibilities measurement
FT
$\widetilde{V_{ij}}^{\delta_0}(u)$
For each u_0
From $\{Beam_{ij}^{\delta_0}\}$ $\{\widetilde{V_{ij}}^{\delta_0}(u)\}$ { N }
Compute A_{u0} : $\left(\widetilde{V_{ij}}^{\delta_0}\right) = A_{u_0} \times \left(F_{sky}^{u_0}(v)\right)$
Compute B ~ $A^{-1} = (N^{-1/2} A)^{-1} N^{-1/2}$
Compute $\widehat{F}_{u_0}(v) = B \cdot \widetilde{V}_{ij}^{\delta_0}(u_0)$
Compute and keep $\sigma^2_{\widehat{F}}$
Save $\widehat{F_s}(u,v)$
Save $\widehat{F}_{keep}(u,v)$
Save $\widehat{F}_{k-w}(u,v)$
Save $Recmap(\alpha,\delta)$

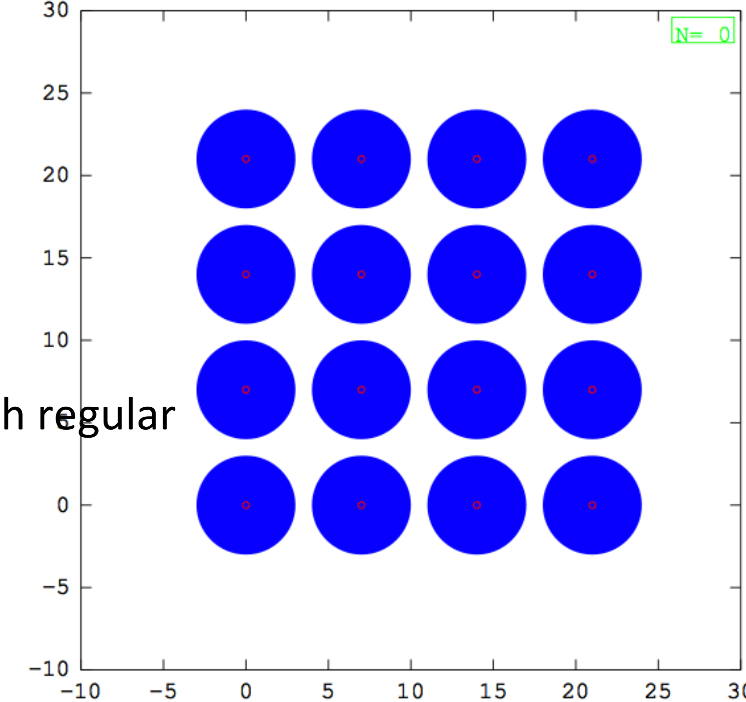
Some results

- Configurations
- Beam example in (l,m)
- Reconstructed synchrotron map
- Noise maps
- Synthesized beams

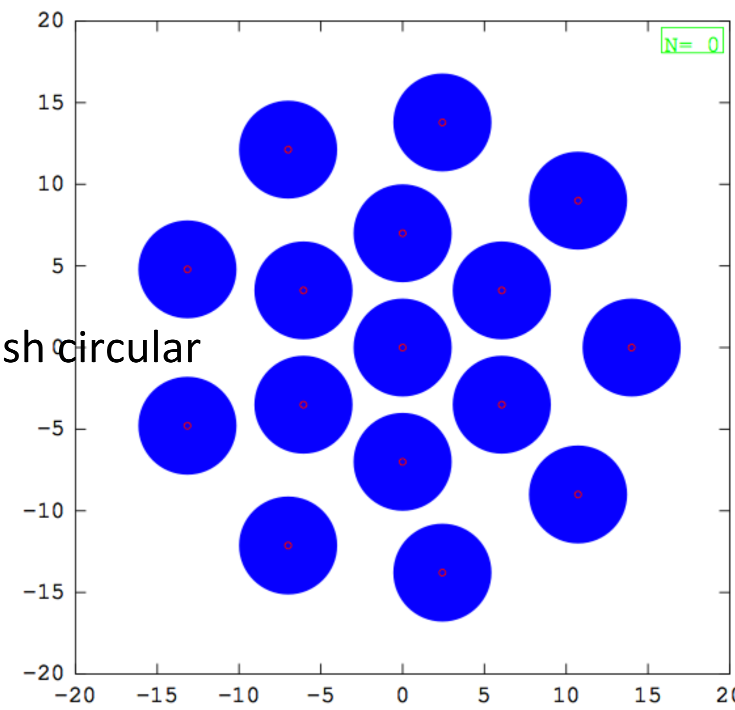
Instrument configurations



A : PAON4



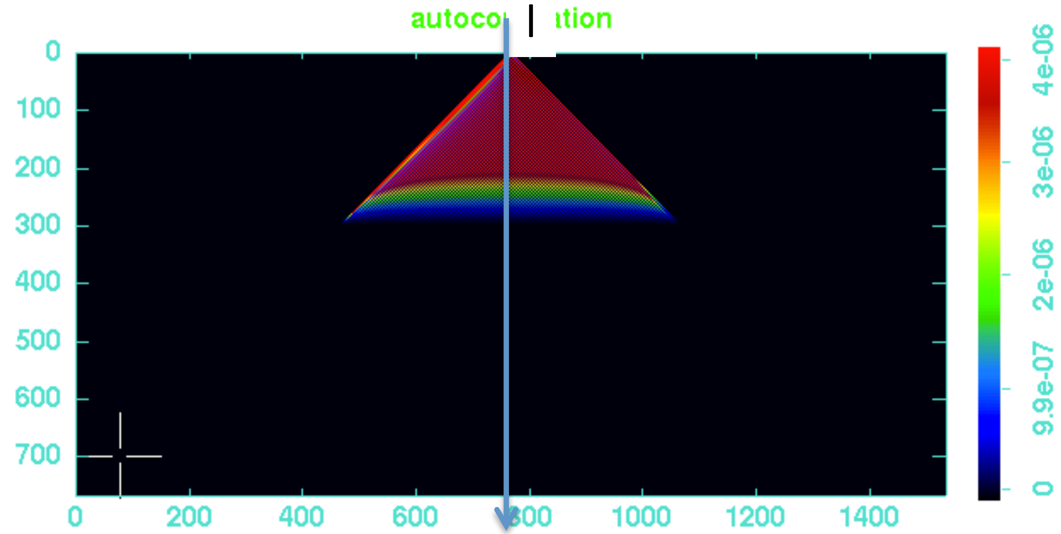
B: 16 dish regular



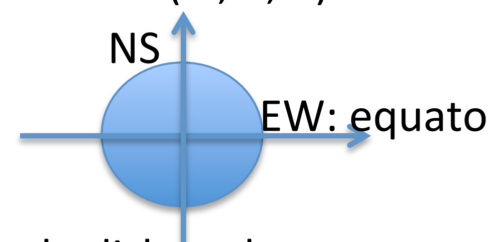
C: 16 dish circular

BeamLM(l,m)

D=10m; lmax=768

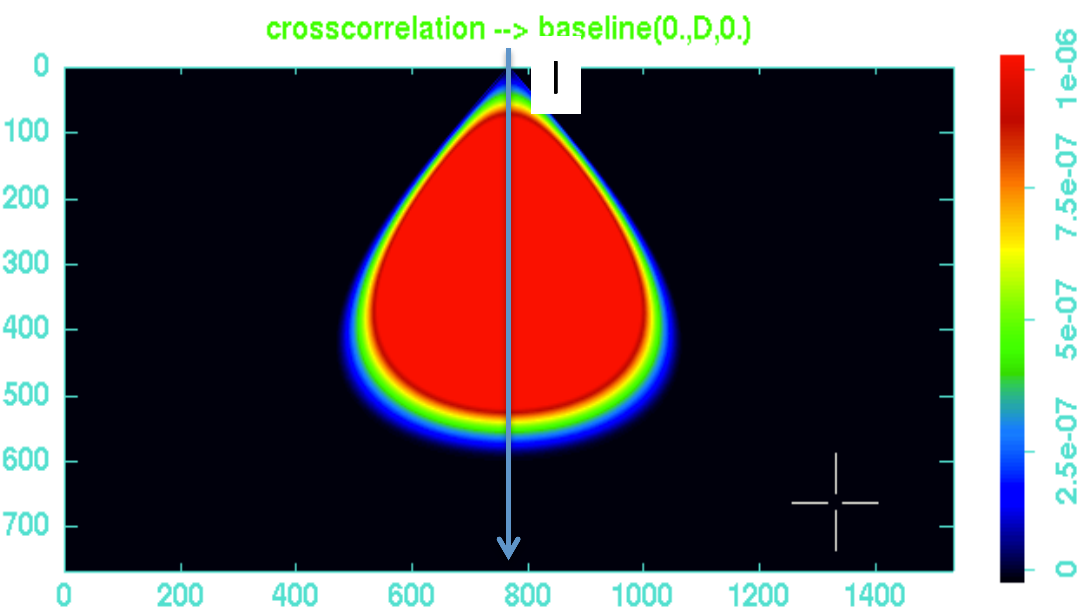


Autocorrelation
Baseline (0.,0.,0.)

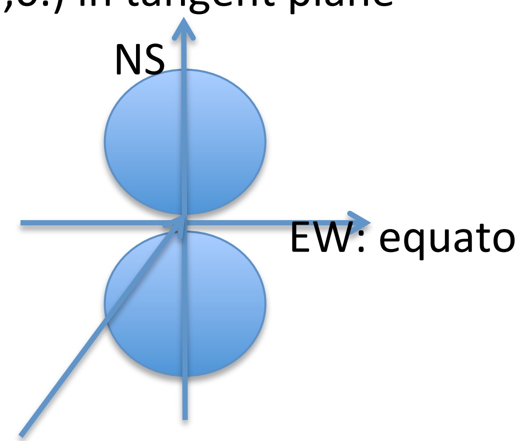


Single dish at the equater,
θ=π/2; φ=0

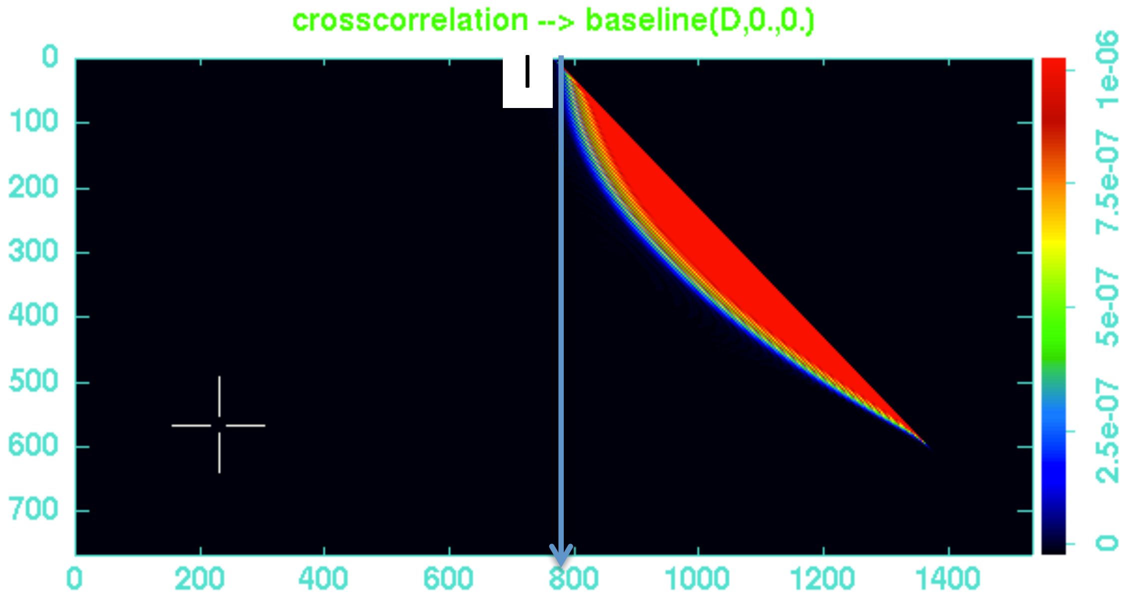
m : from -768 to 768



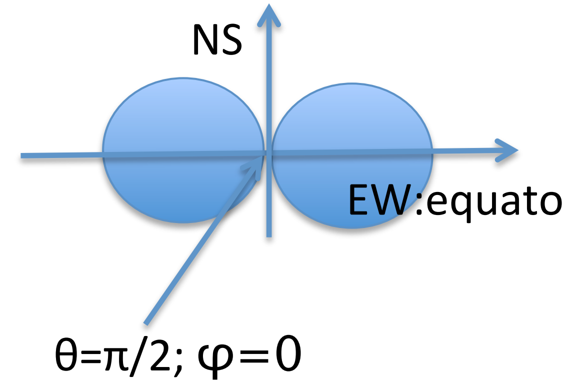
Crosscorrelation: Baseline
(0.,D,0.) in tangent plane



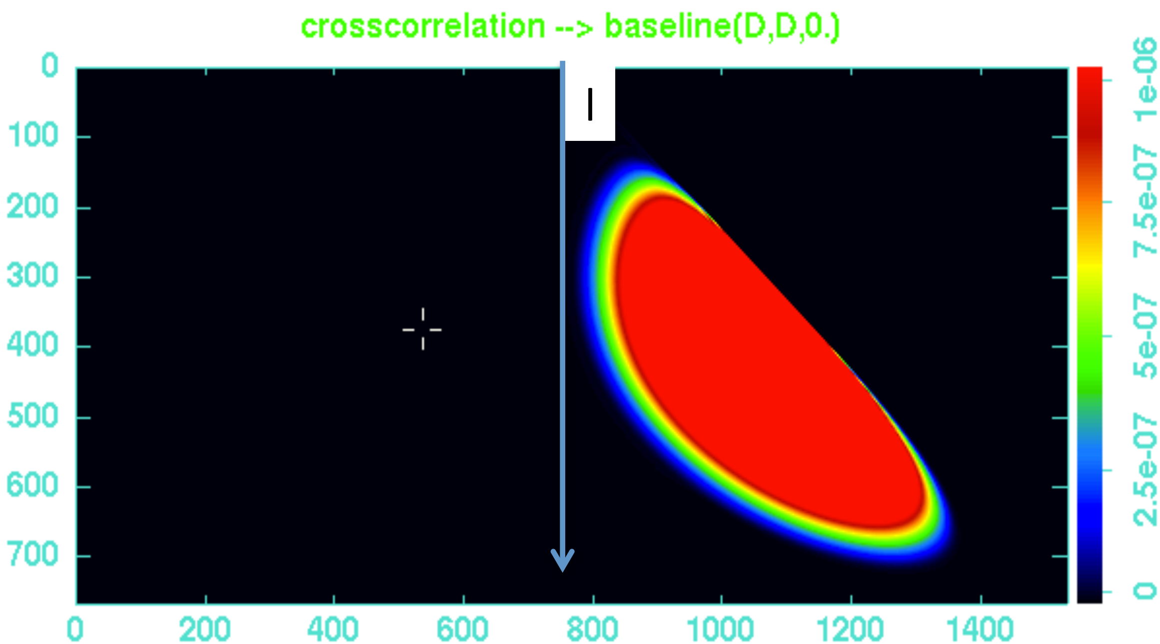
θ=π/2; φ=0



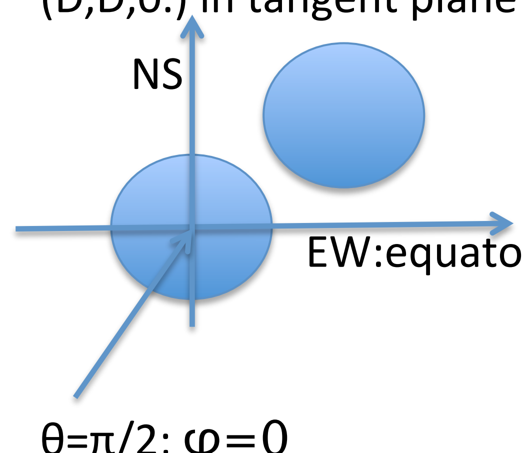
Crosscorrelation : Baseline (D,0.,0.) in tangent plane



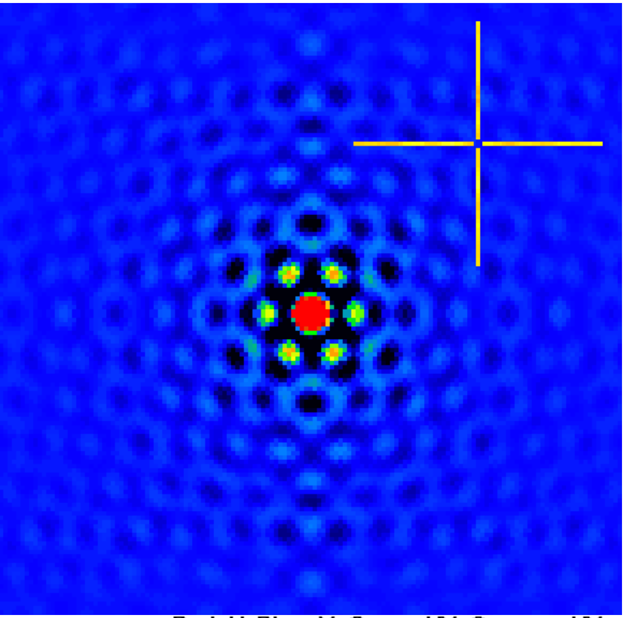
m : from -768 to 768

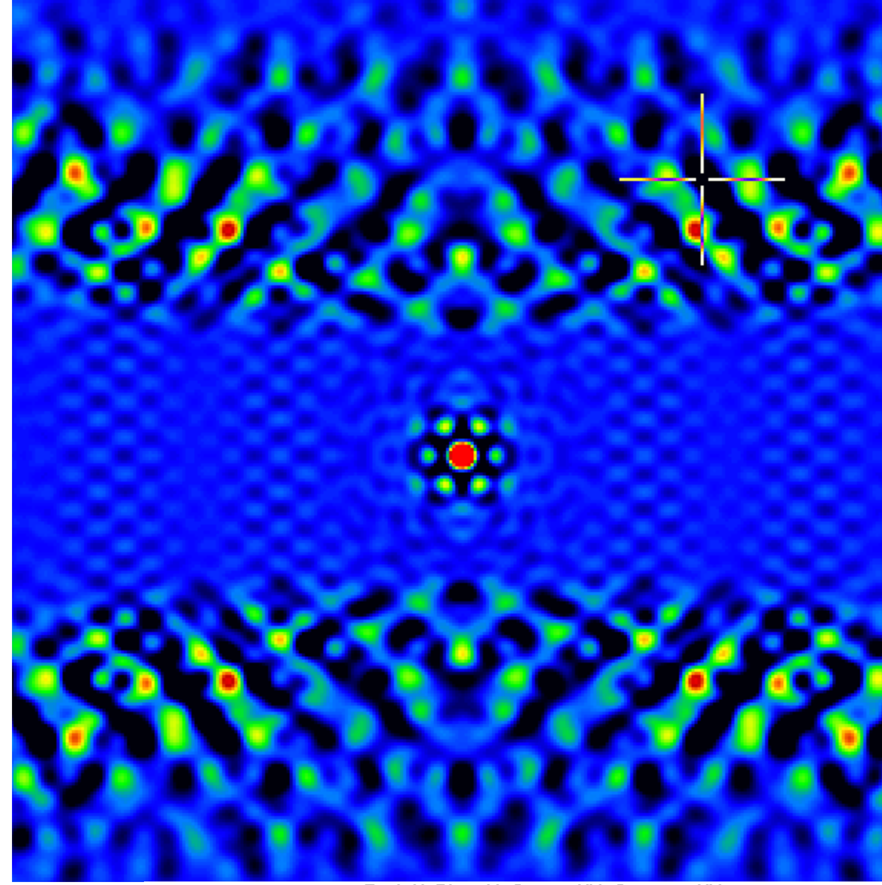


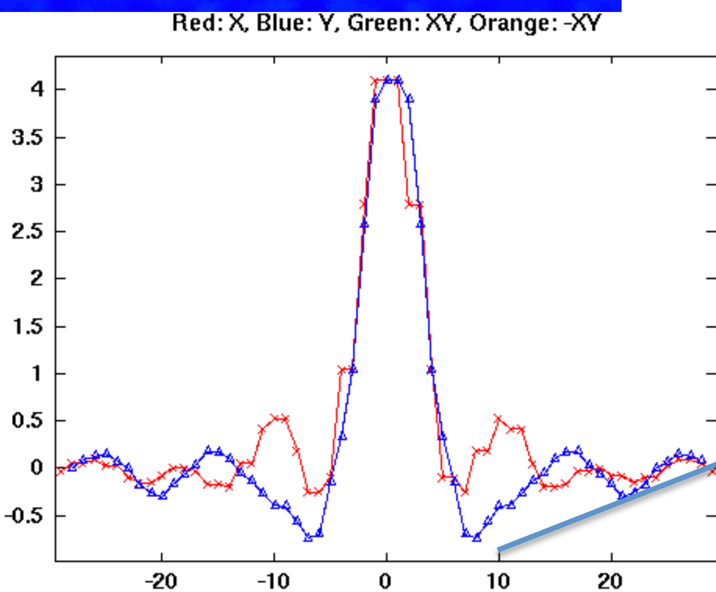
Crosscorrelation : Baseline (D,D,0.) in tangent plane



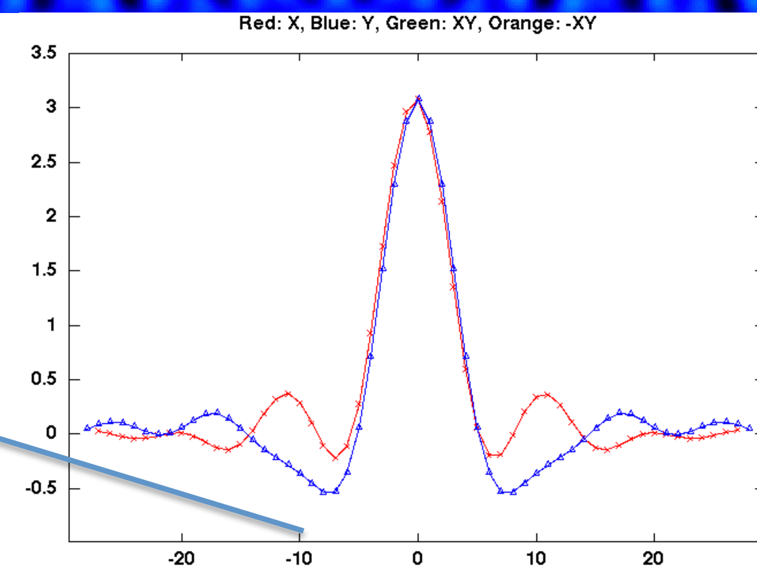
Synthesized beam for PAON4





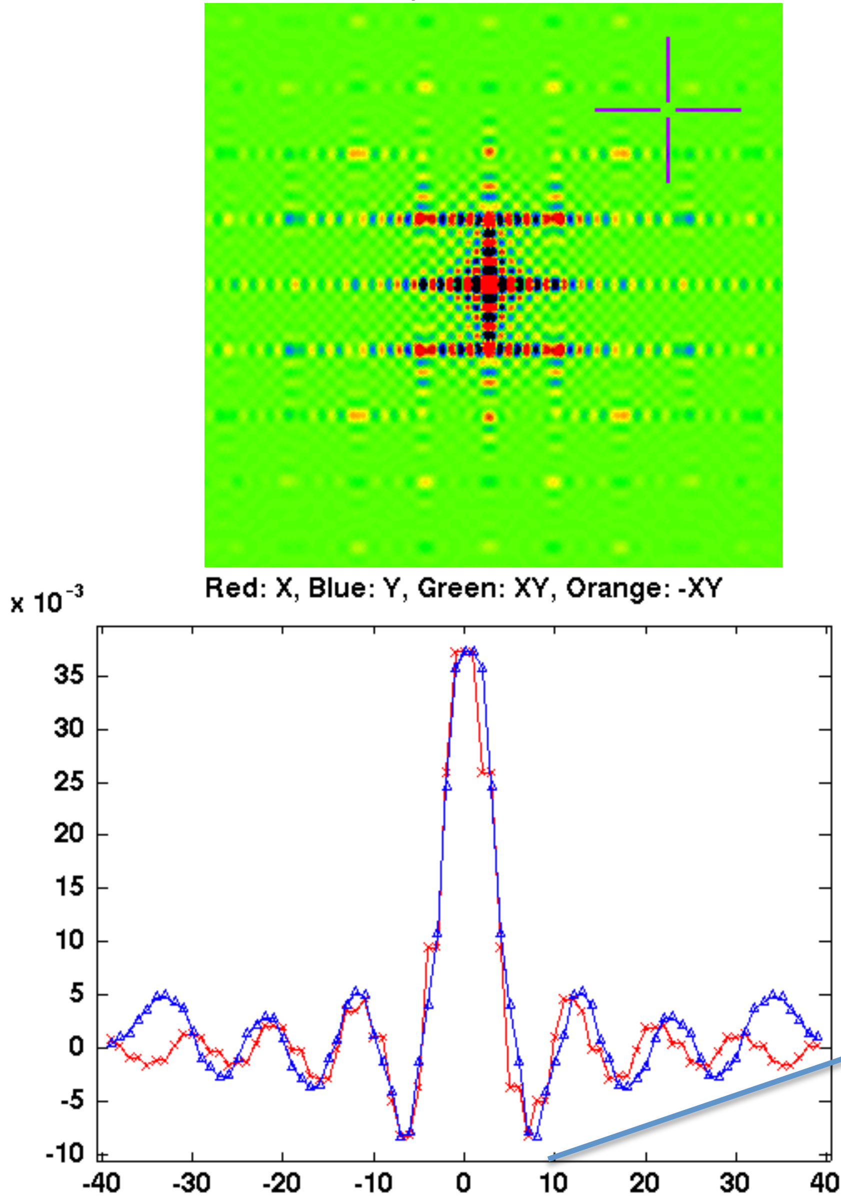


For x=10, corresponds 1.15deg

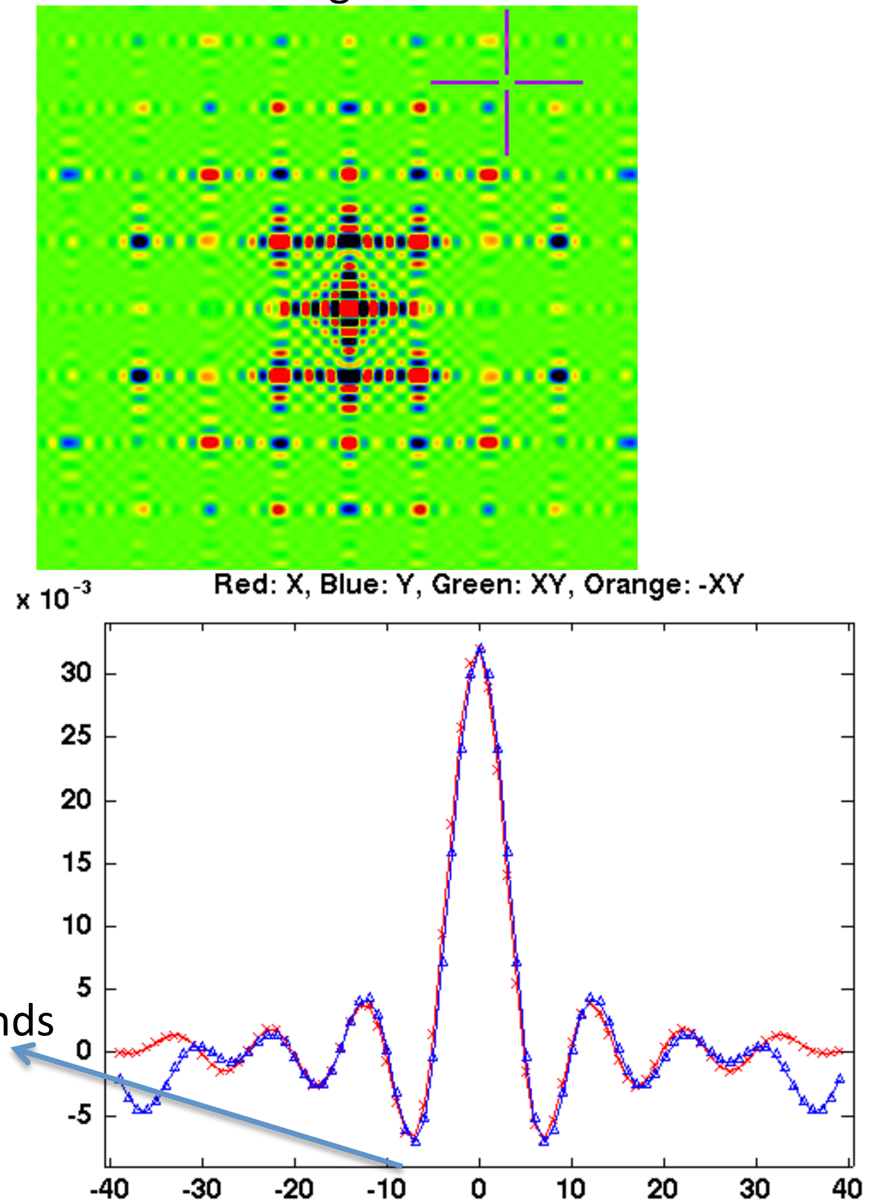


Synthesized beam for Tianlai regular dish array

Spherical

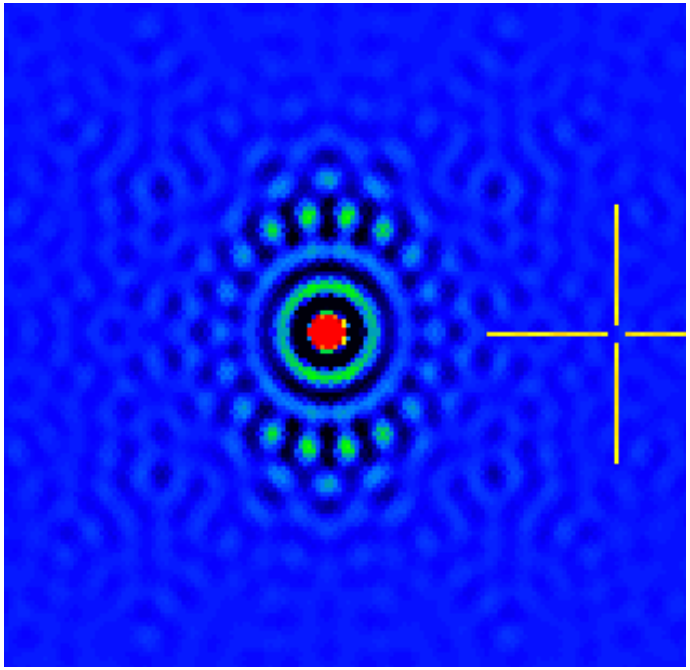


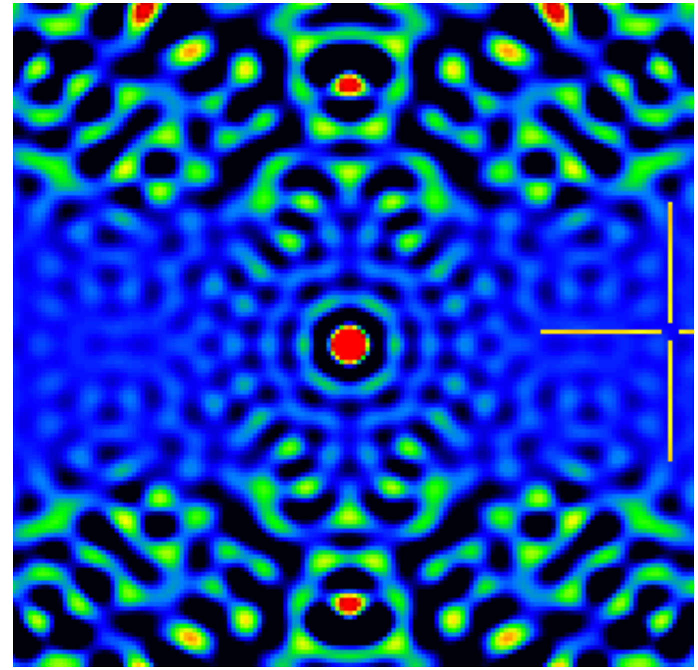
rectangular

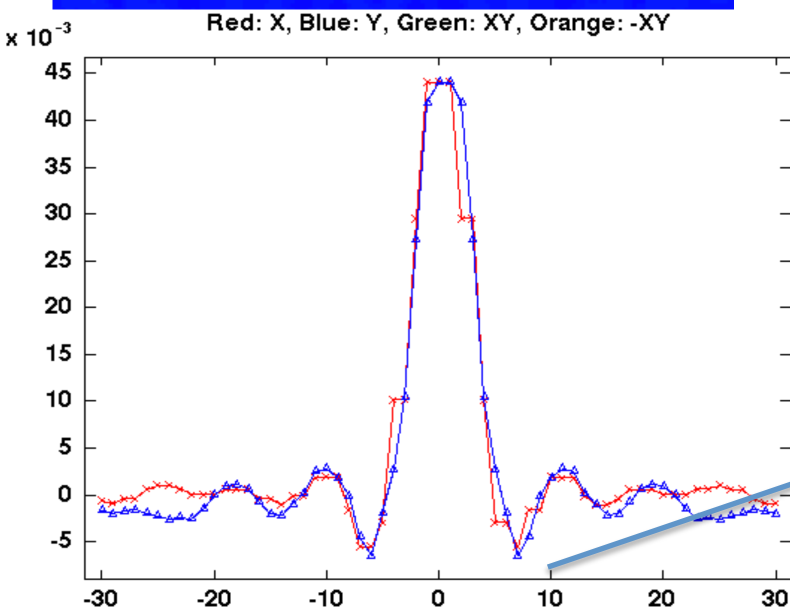


For x=10, corresponds 34’

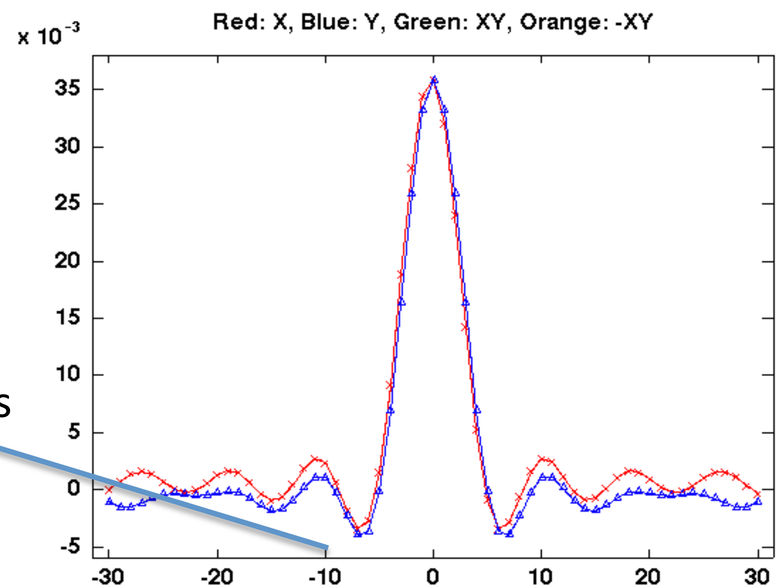
Synthesized beam for Tianlai circular dish array





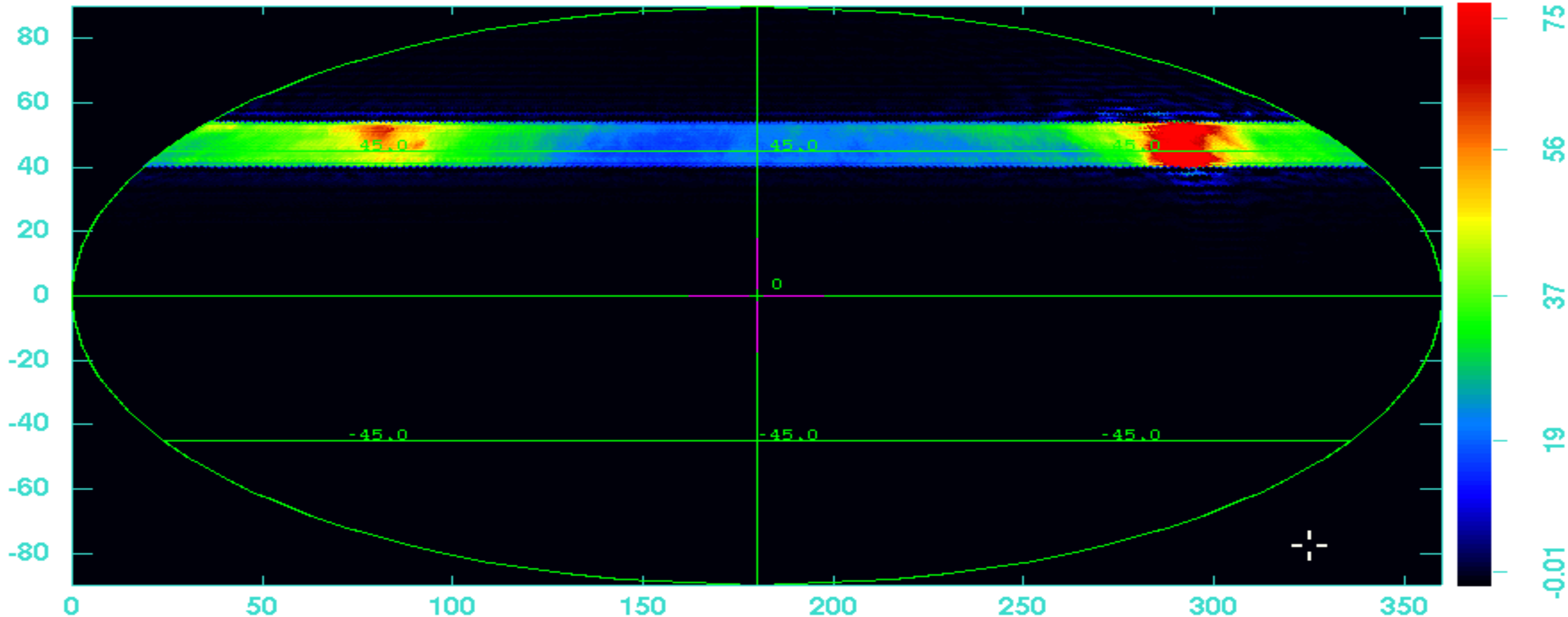


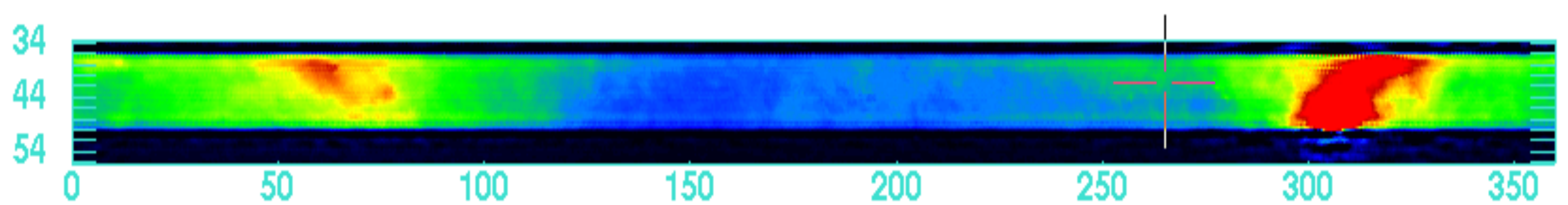
For x=10, corresponds 34’



Observed synchrotron map

Instrument : PAON4; 9 scan per degrees; local latitude θ = 47 deg

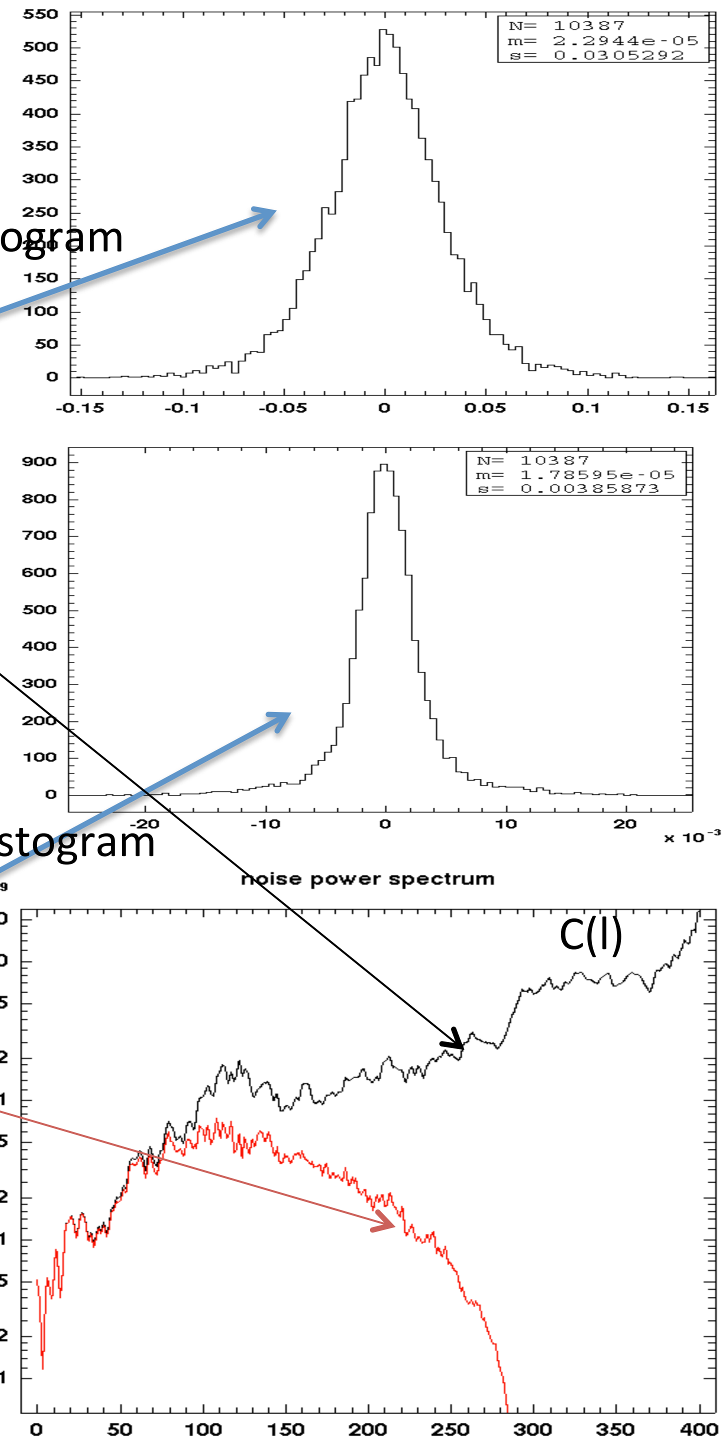


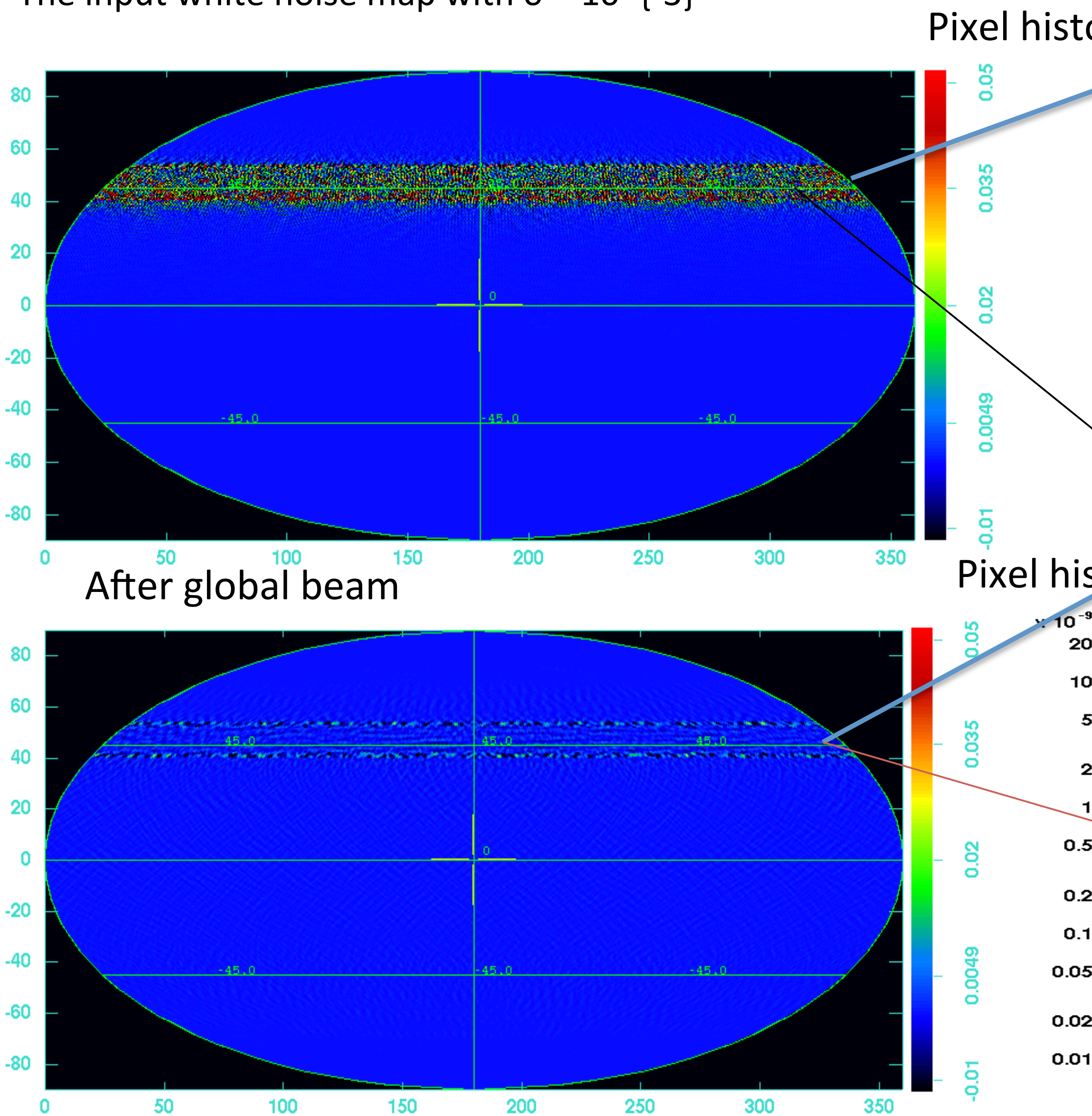


Noise map

Instrument : PAON4; 9 scan per degrees; local latitude θ = 47 deg

The input white noise map with σ ~ 10^{-3}





conclusions

- Optimal method to combine beams (including several delta pointings) to reconstruct the sky map, and to compute the covariance matrix (in the Fourier plane (u,v)/spherical harmonics (l,m))
- Reasonably efficient (fast) implementation – parallel (multi-thread) for cross-corr beam computation and sky-map reconstruction (m-modes computed in parallel)
- Possibility to have different compromise between the noise level in the reconstructed map and the beam quality / side lobes through the eigenvalue threshold and the cut/weight on the F^hat(u,v)
- More work needed to compare different setups and scan strategies