Analysis of cross-correlations of PAON4 data

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26 June 2015

Data set and some information

Source	Cassiopeia A
Date	12 march 2015

Equatorial coordinates of sources			
Source	Cassiopeia A	Cygnus A	Crab Nebula
Right Ascension	$23^h\ 23^m\ 26^s$	$19^h\ 59^m\ 28.3566^s$	$5^h\ 34^m\ 31.94^s$
Declination	+58° 48′	+40° 44′ 02.096″	+22° 00′ 52.2″

PAON4 geographic position	
Longitude	2° 11′ 43″ E
Latitude	47° 21′ 2″ N

Correct data (1)

- According to C.Magneville's note "Oscillations dans le module des visibilités", FFT done by hardware will make the visibility distorting.

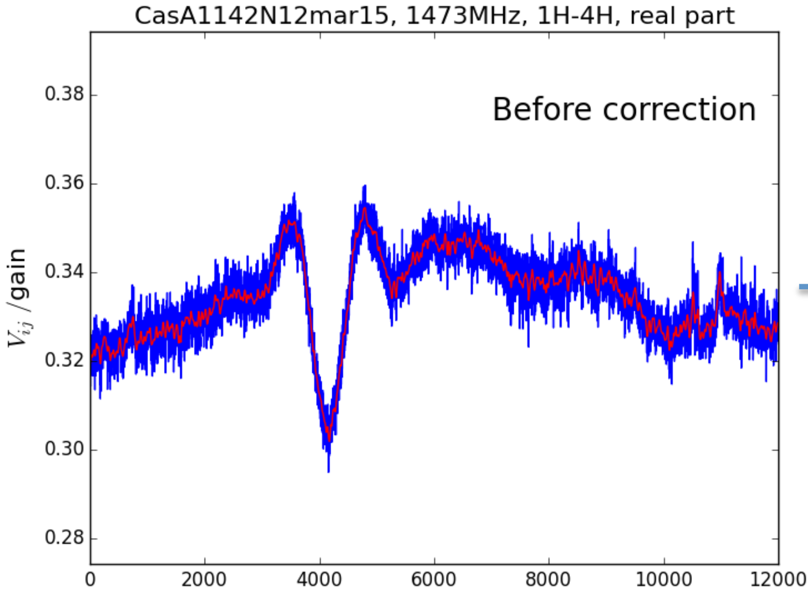
- In order to correct it, I convolve the data with a window function, and then remove the distortion part.

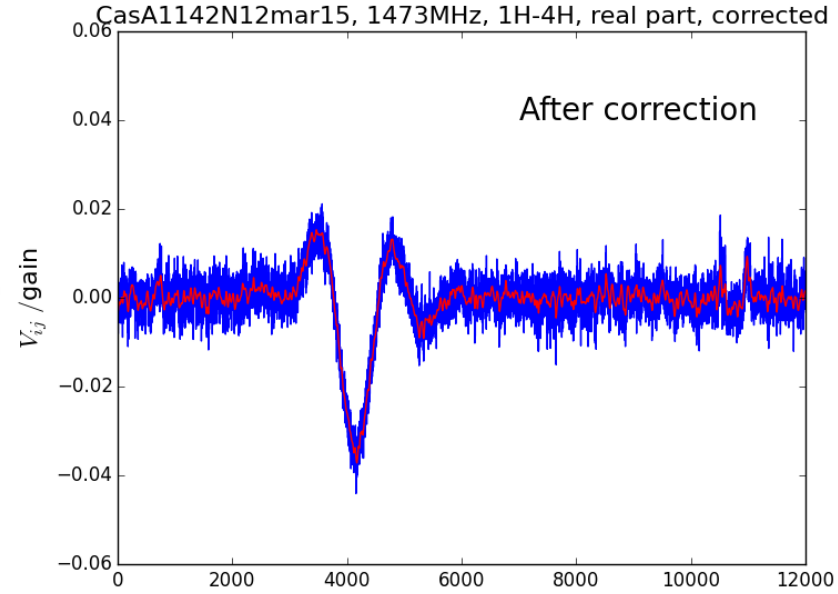
$$w(t) = \begin{cases} \frac{1}{\Delta t_a}, & t_0 + \frac{\Delta t_a}{2} \leq t \leq t_0 + \frac{\Delta t_a}{2}, (\forall t_0 \in t) \\ 0, & \text{other} \end{cases} \quad \text{(window function)}$$

$$V'_{ij}(t) = V_{ij}(t) - V_{ij} \otimes w_{5\min}(t)$$

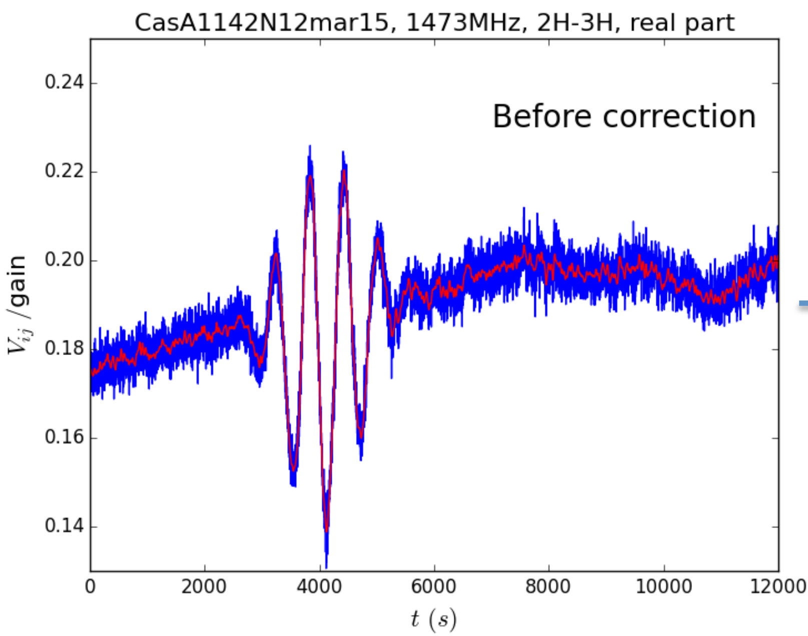
Do this to the real part and imaginary part separately.

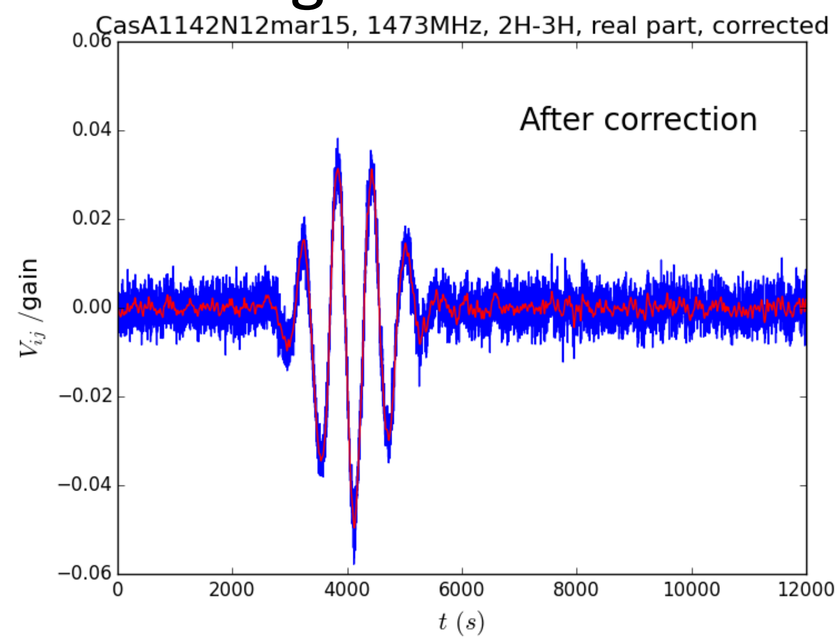
Correct data (2)





Note that we need to rotate the fringe a little.





Basic formulas (1)

- Product of two Gaussian Beam of power is also a Gaussian function.
- This model will be used to extract the alignments and effective diameters of each antenna (I will show it below).

$$g_1 = A_1 e^{-\frac{(x-x_1)^2}{2\sigma_1^2}}$$

$$g_2 = A_2 e^{-\frac{(x-x_2)^2}{2\sigma_2^2}}$$

$$g_3 = g_1 \cdot g_2 = A_1 A_2 e^{-\left[\frac{(x-x_1)^2}{2\sigma_1^2}+\frac{(x-x_2)^2}{2\sigma_2^2}\right]} = \left[A_3 e^{-\frac{(x-x_3)^2}{2\sigma_3^2}}\right]^2$$

where

$$x_3 = x_1\left(\frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right) + x_2\left(\frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right)$$

$$\sigma_3 = \sqrt{\frac{2\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}}$$

$$A_3 = \sqrt{A_1 A_2 e^{-\left[\frac{(x_3-x_1)^2}{2\sigma_1^2}+\frac{(x_3-x_2)^2}{2\sigma_2^2}\right]}} = \sqrt{A_1 A_2 e^{-\frac{(t_1-t_2)^2}{2(\sigma_1^2+\sigma_2^2)}}}$$

Basic formulas (2)

- Visibility formula for PAON4 case

$$V_{ij}(\theta) = \sqrt{G_i G_j} I_0 \exp\left\{-\frac{(\theta - \theta_{s,ij})^2}{2\sigma_{ij}^2}\right\} \exp\left\{i\frac{2\pi}{\lambda}[L_{ew,ij}\sin\theta - L_{ns,ij}\cos\theta] + i\Delta\Phi_{ij}\right\}$$

$$\theta_{s,ij} = \theta_{s,i}\left(\frac{D_{\text{eff},i}^2}{D_{\text{eff},i}^2 + D_{\text{eff},j}^2}\right) + \theta_{s,j}\left(\frac{D_{\text{eff},i}^2}{D_{\text{eff},i}^2 + D_{\text{eff},j}^2}\right)$$

$$D_{\text{eff},ij} = \sqrt{\frac{D_{\text{eff},i}^2 + D_{\text{eff},j}^2}{2}}$$

Convert time to transition angle

- Note that because the antennas don't point to the zenith, rotating the antenna based coordinate makes the North-South baseline have contribution to the phase.

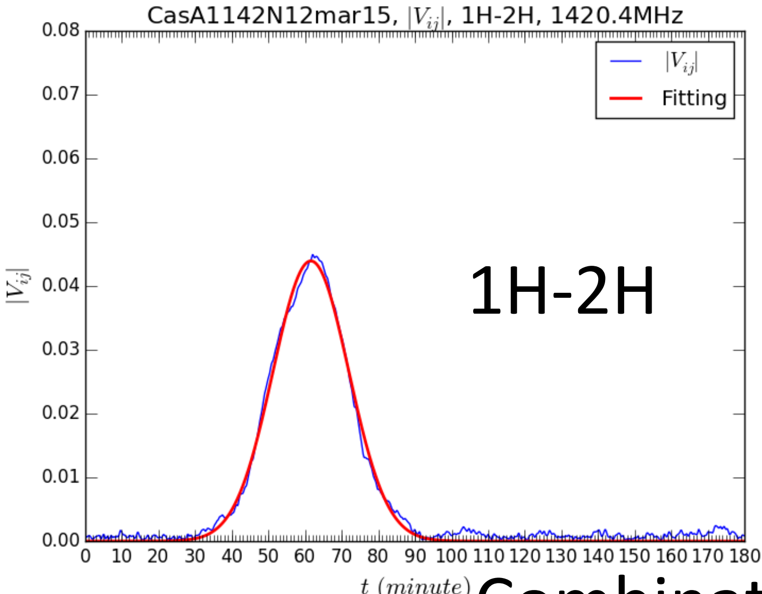
Antenna alignment/pointing (1)

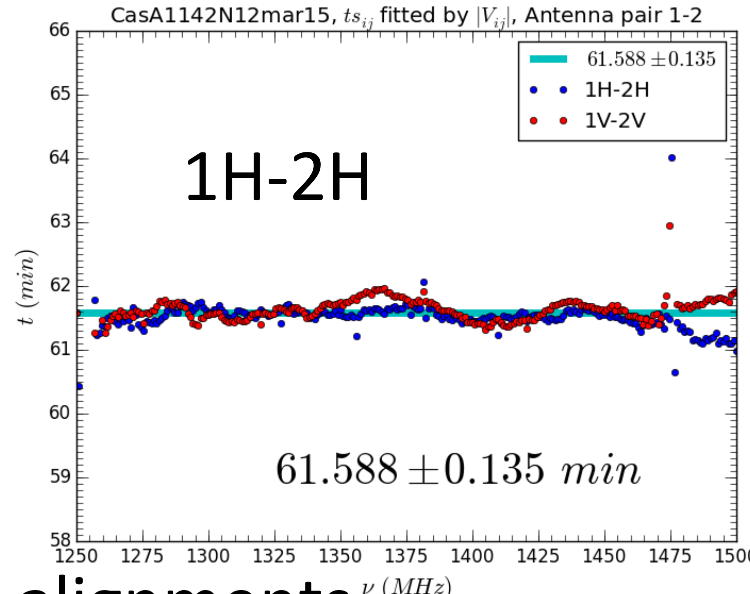
- Because the antennas may not lie along the local meridian, I try to find the centers of the fringes to estimate the pointing direction/alignment of each antennas.

- I assume the beam of power of the antenna is a Gaussian function, then I do the module to the visibility |V|, it will leave the Gaussian function, and is independent of the phase term.

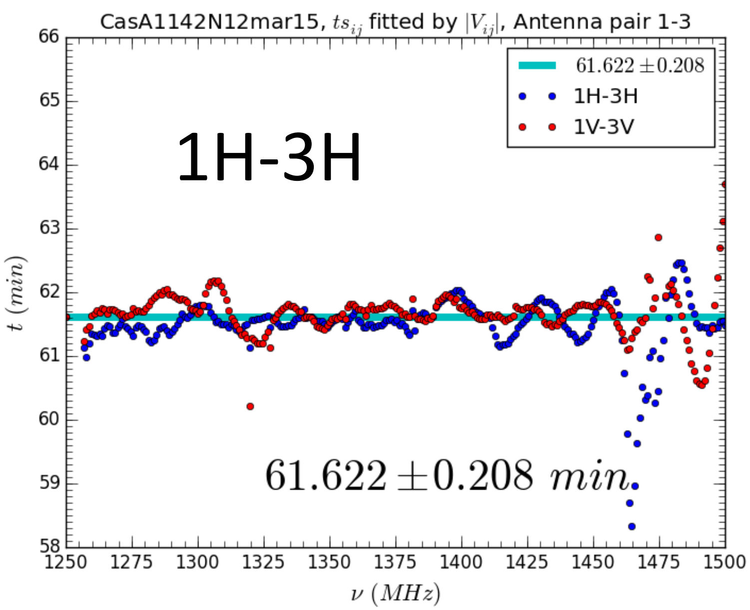
$$\left|V_{ij}^{n}(t)\right| = I' \exp\left\{-\frac{(\theta-\theta_s)^2}{2\sigma_{ij}^2}\right\}$$

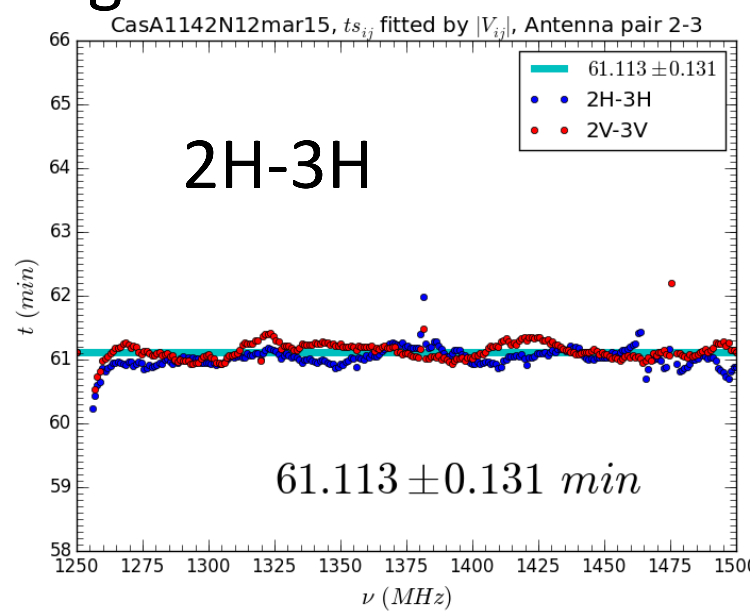
Antenna alignment (2)





Combination of two alignments





Extract each alignment and Deff

- I extract each antenna's pointing direction and effective diameter from the fitting result using the model I show in page 6. In practice, I solve such linear equations

group:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_0^2 \\ D_1^2 \\ D_2^2 \\ D_3^2 \end{bmatrix} = 2 \begin{bmatrix} D_{01}^2 \\ D_{02}^2 \\ D_{03}^2 \\ D_{12}^2 \\ D_{13}^2 \end{bmatrix}$$

Each parameter we want to extract

Combination results fitted from fringes

$$\text{Set} \quad f_{ijn} = \frac{D_n^2}{D_i^2 + D_j^2}$$

$$\begin{bmatrix} f_{010} & f_{011} & 0 & 0 \\ f_{020} & 0 & f_{022} & 0 \\ f_{030} & 0 & 0 & f_{033} \\ 0 & f_{121} & f_{122} & 0 \\ 0 & f_{131} & 0 & f_{133} \end{bmatrix} \begin{bmatrix} t_0 \\ t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} t_{01} \\ t_{02} \\ t_{03} \\ t_{12} \\ t_{13} \end{bmatrix}$$

Sidereal time

- Look at the header of first .ppf file vismtx_0_0.ppf, we can know the starting time of this observation.

==

DATEOBS = 2015-03-12T10:55:0.0 (T=string) / Date, Time corresponding to TimeTagFirst

DELTIME = 0.99019461600000002743 (T=double) / visib cumul time (seconds)

FirstFC = 0 (T=long int) / First FrameCounter

FirstTT = 0 (T=long int) / First TimeTag

LastFC = 6118 (T=long int) / Last FrameCounter

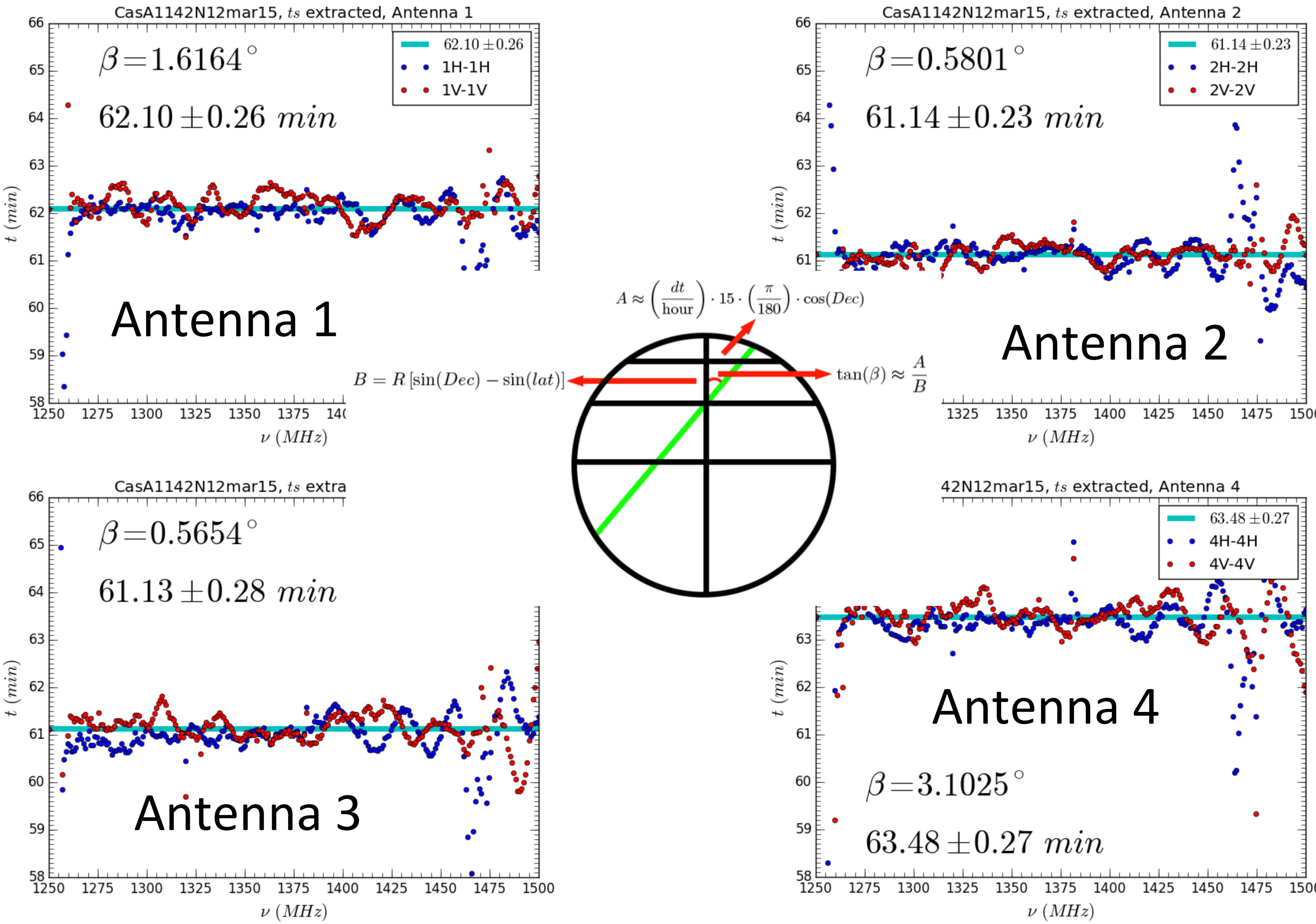
LastTT = 123774327 (T=long int) / Last TimeTag

- Then we can calculate the sidereal time of CasA on 12mar2015

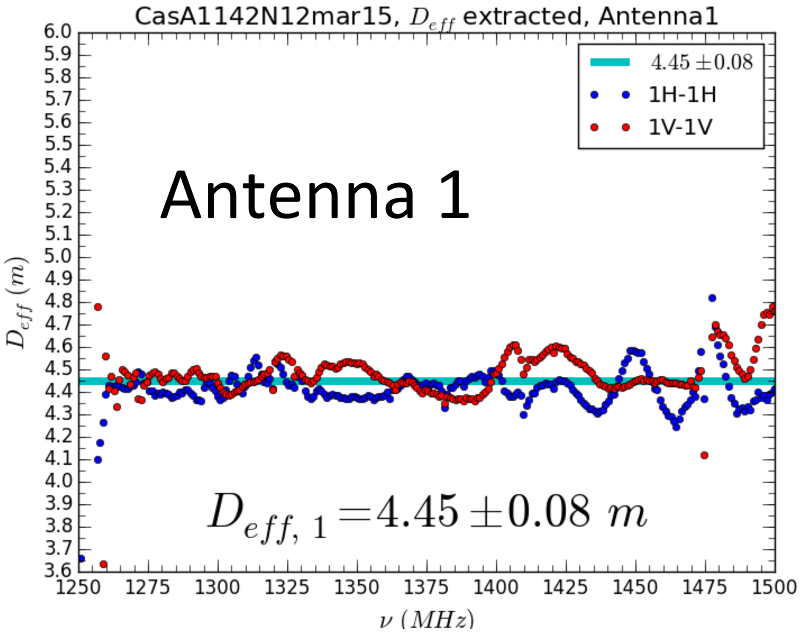
$$\text{LAST} = 22^h 22^m 54^s$$

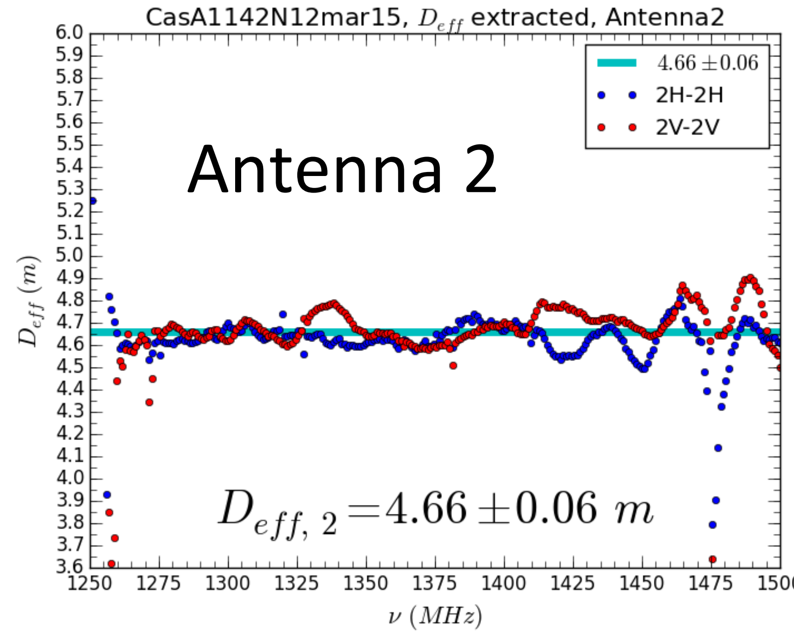
- So, after $\text{RA} - \text{LAST} = 1^h 0^m 32^s$, the source will reach the local meridian.

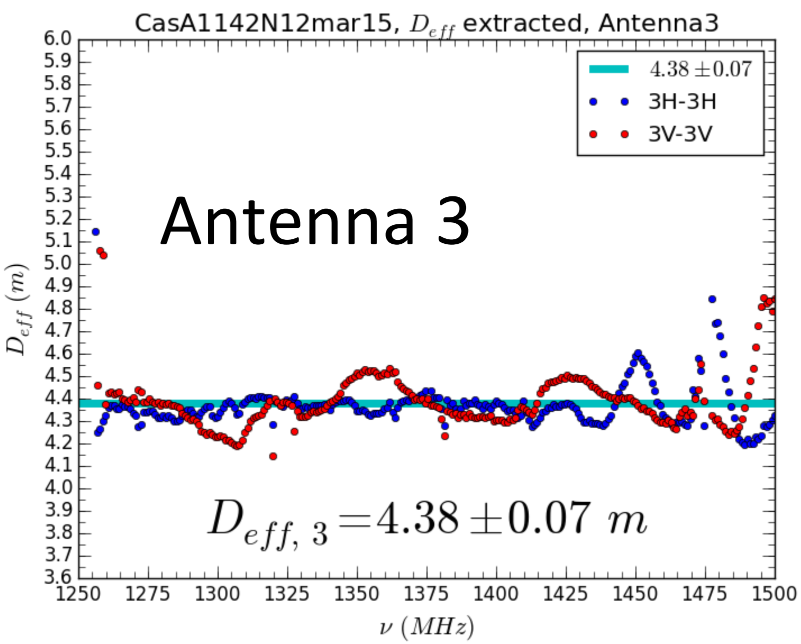
Each alignment (page 6 model)

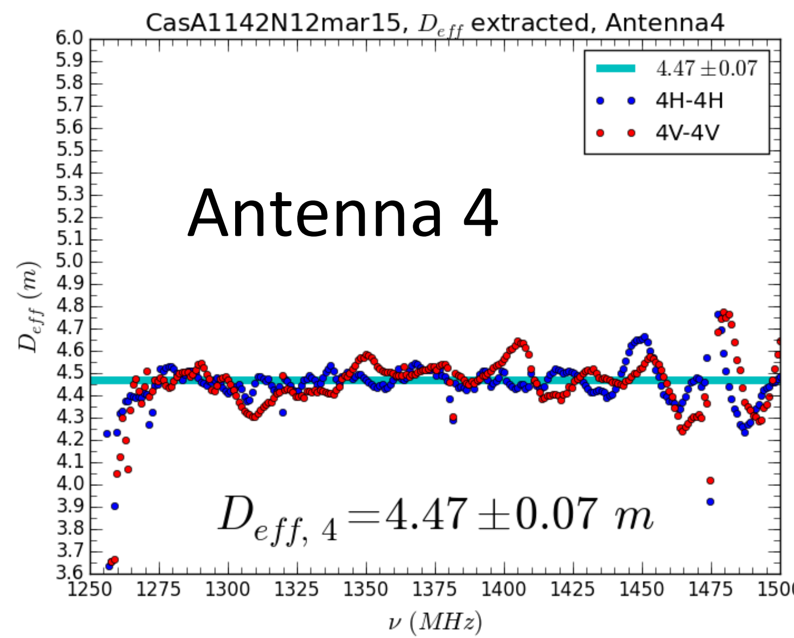


Each effective diameter (page 6 model)









Phase term of visibility

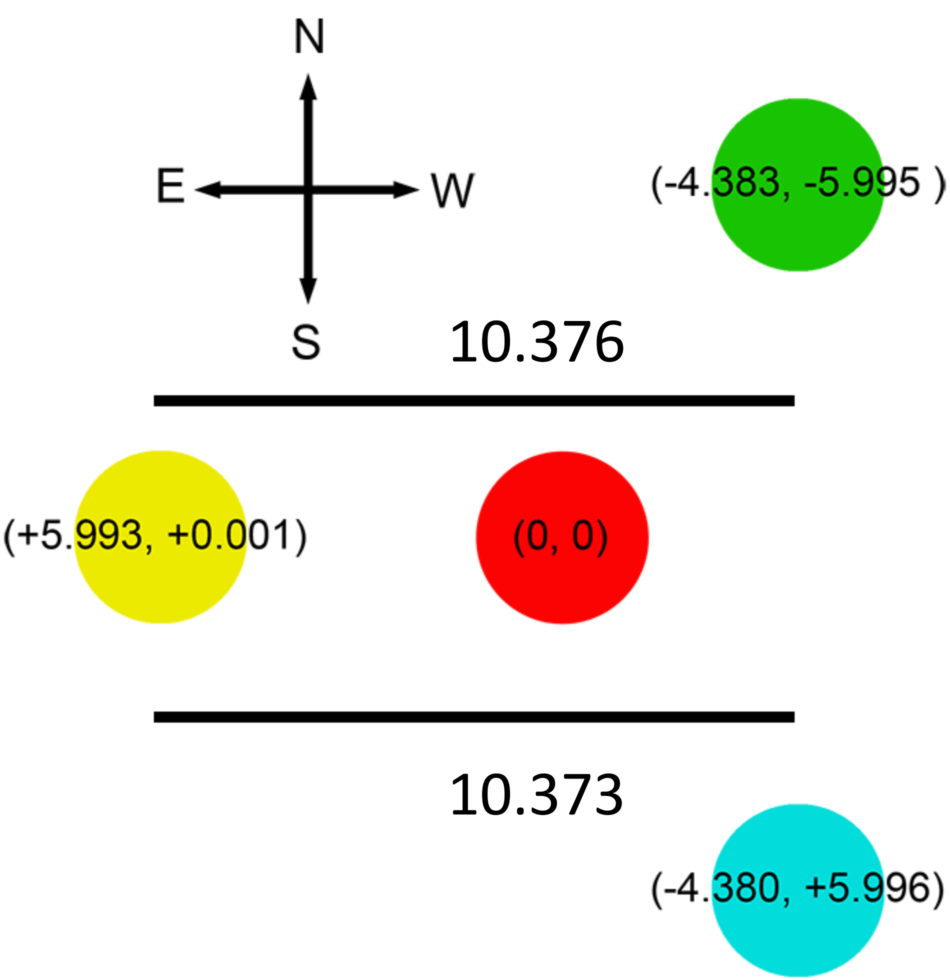
$$V_{ij}(\theta) = \sqrt{G_i G_j} I_0 \exp\left\{-\frac{(\theta-\theta_{s,ij})^2}{2\sigma_{ij}^2}\right\} \exp\left\{i\frac{2\pi}{\lambda}[L_{ew,ij}\sin\theta - L_{ns,ij}\cos\theta] + i\Delta\Phi_{ij}\right\}$$

- If we use imaginary part to divide by real part, then we can obtain remove the Beam part, just leave the phase part (tan(phase)). Solve this trigonometric function, we can obtain the phase term.

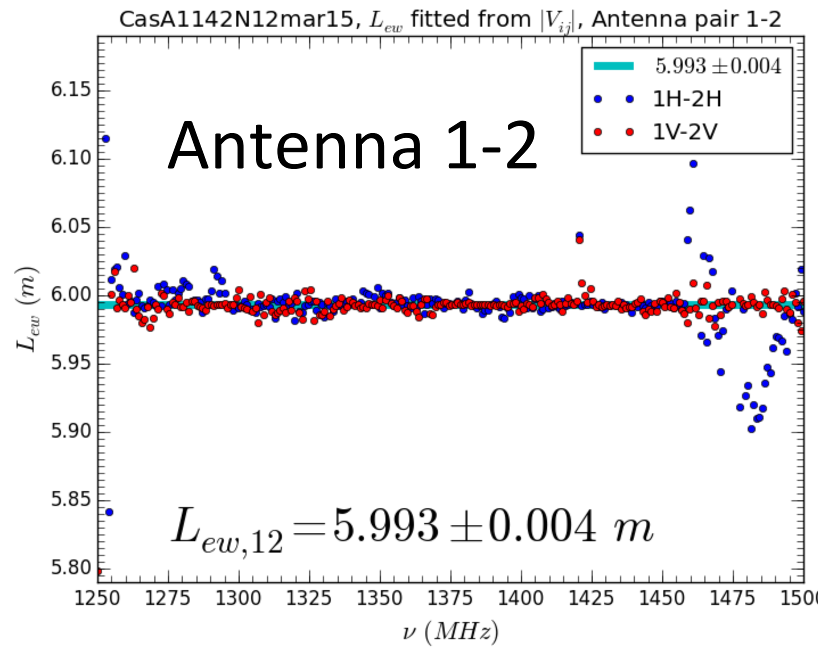
$$\frac{\text{imag}}{\text{real}} = \frac{\text{Beam}\cdot\sin(\text{phase})}{\text{Beam}\cdot\cos(\text{phase})} = \tan(\text{phase})$$

- And then I fit this phase term to obtain the East-West baseline and phase delay due to the optical fiber/cable.

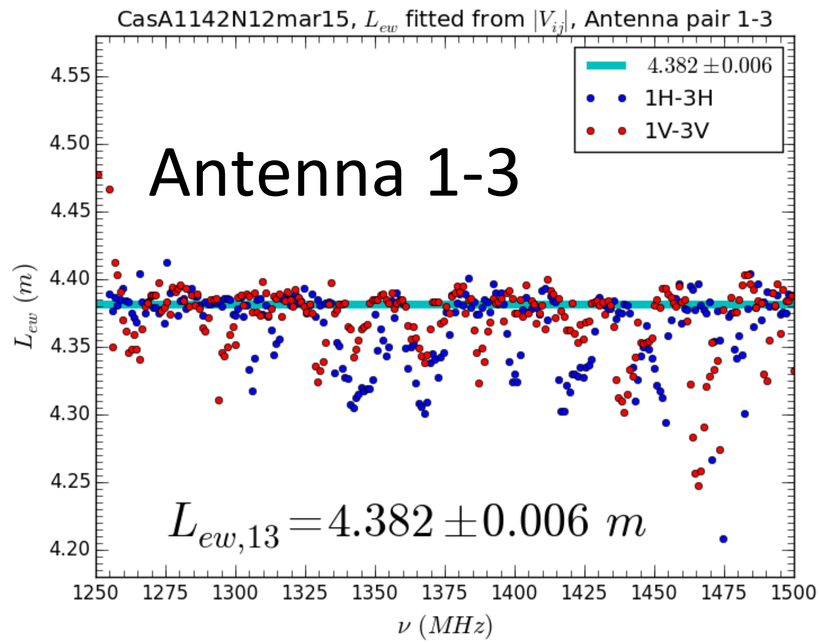
Antenna geometry

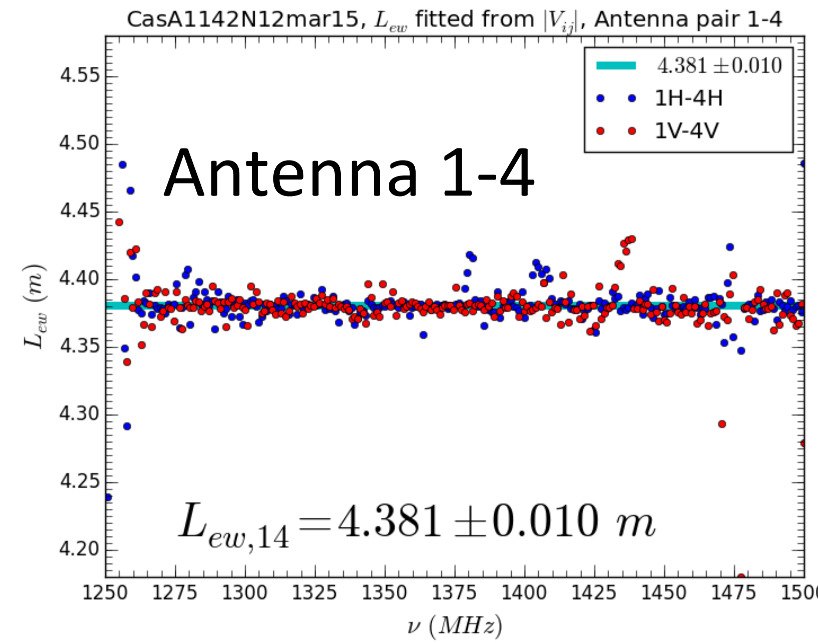


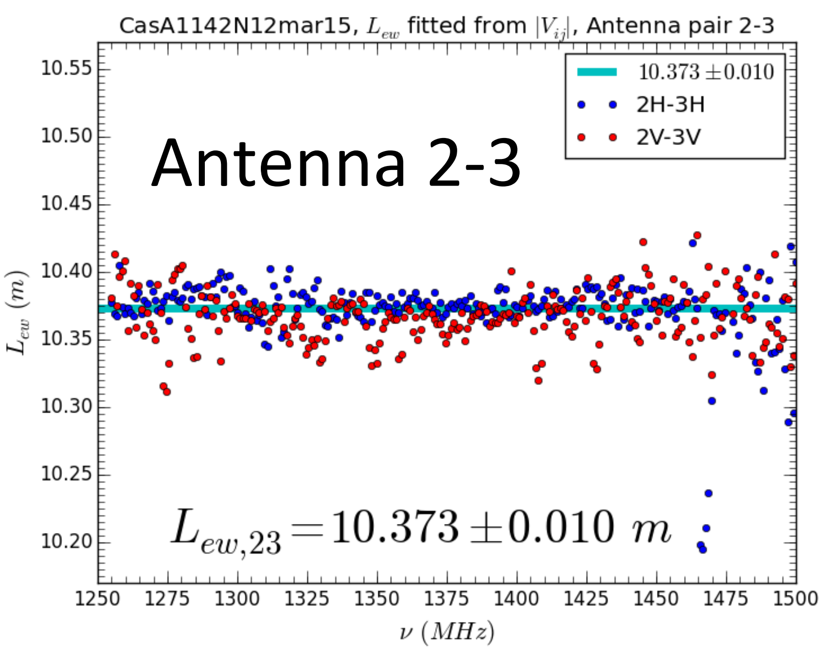
East-West baseline of Antenna pair 1-2

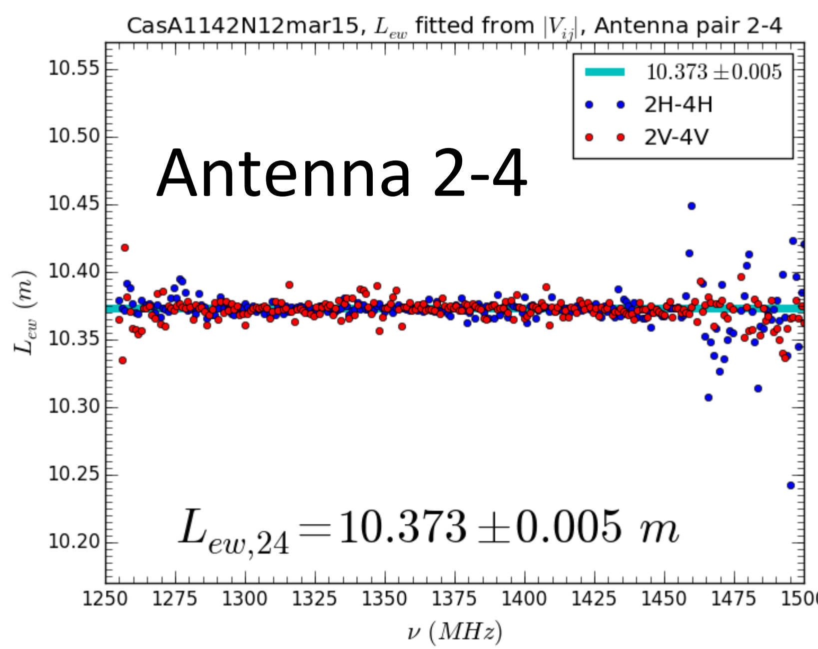


East-West baseline

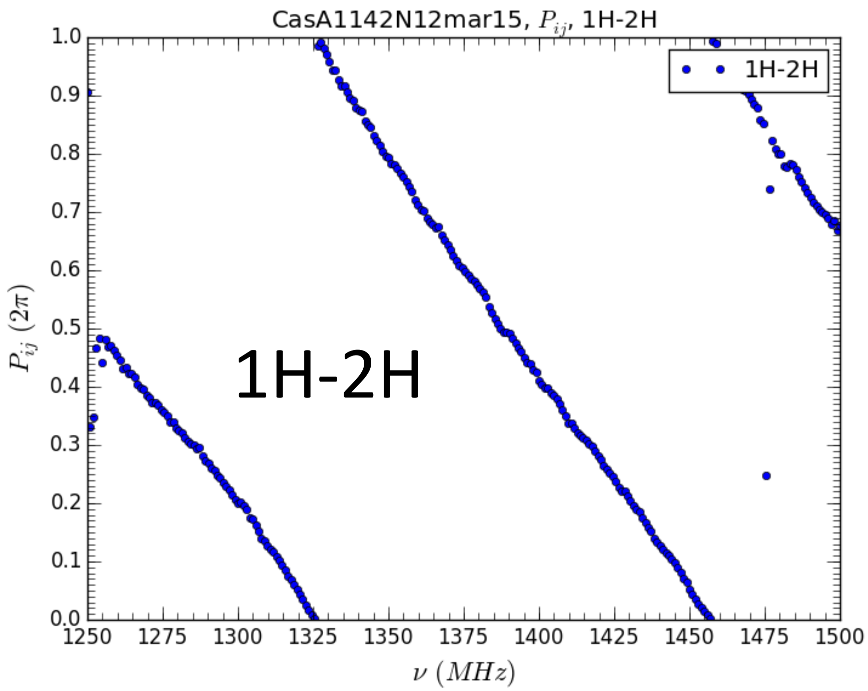


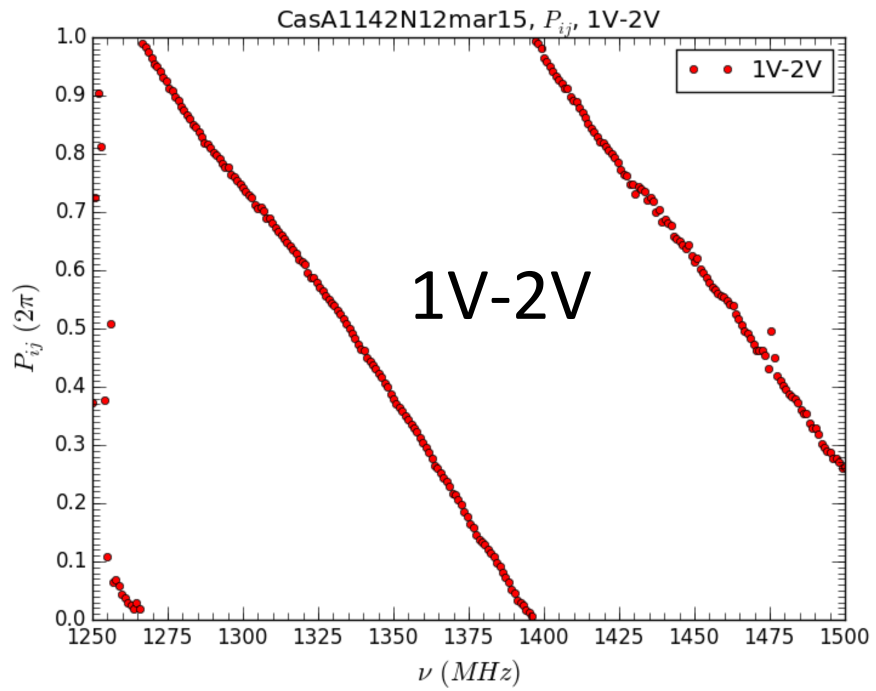




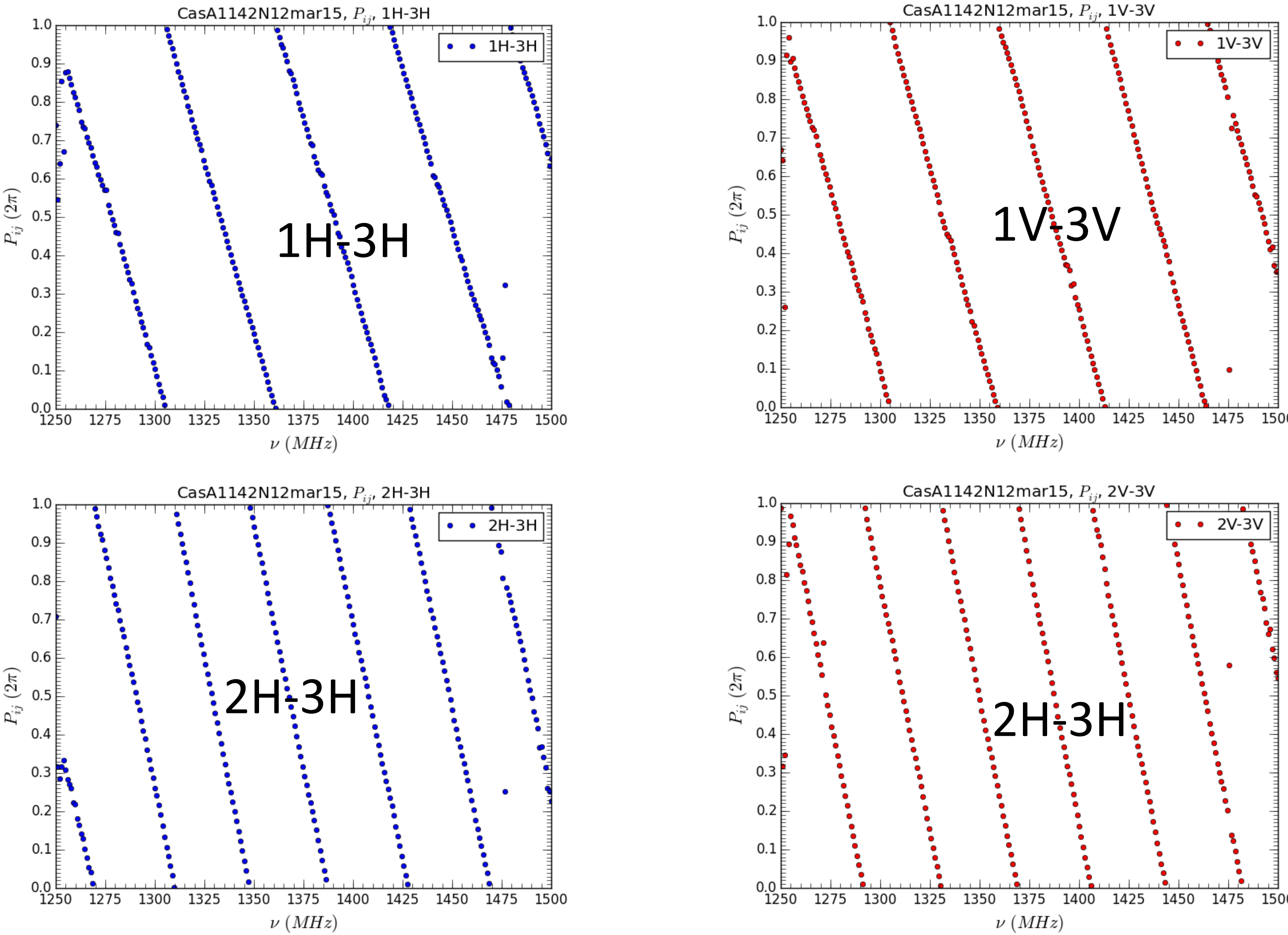


Phase delay/difference (1)

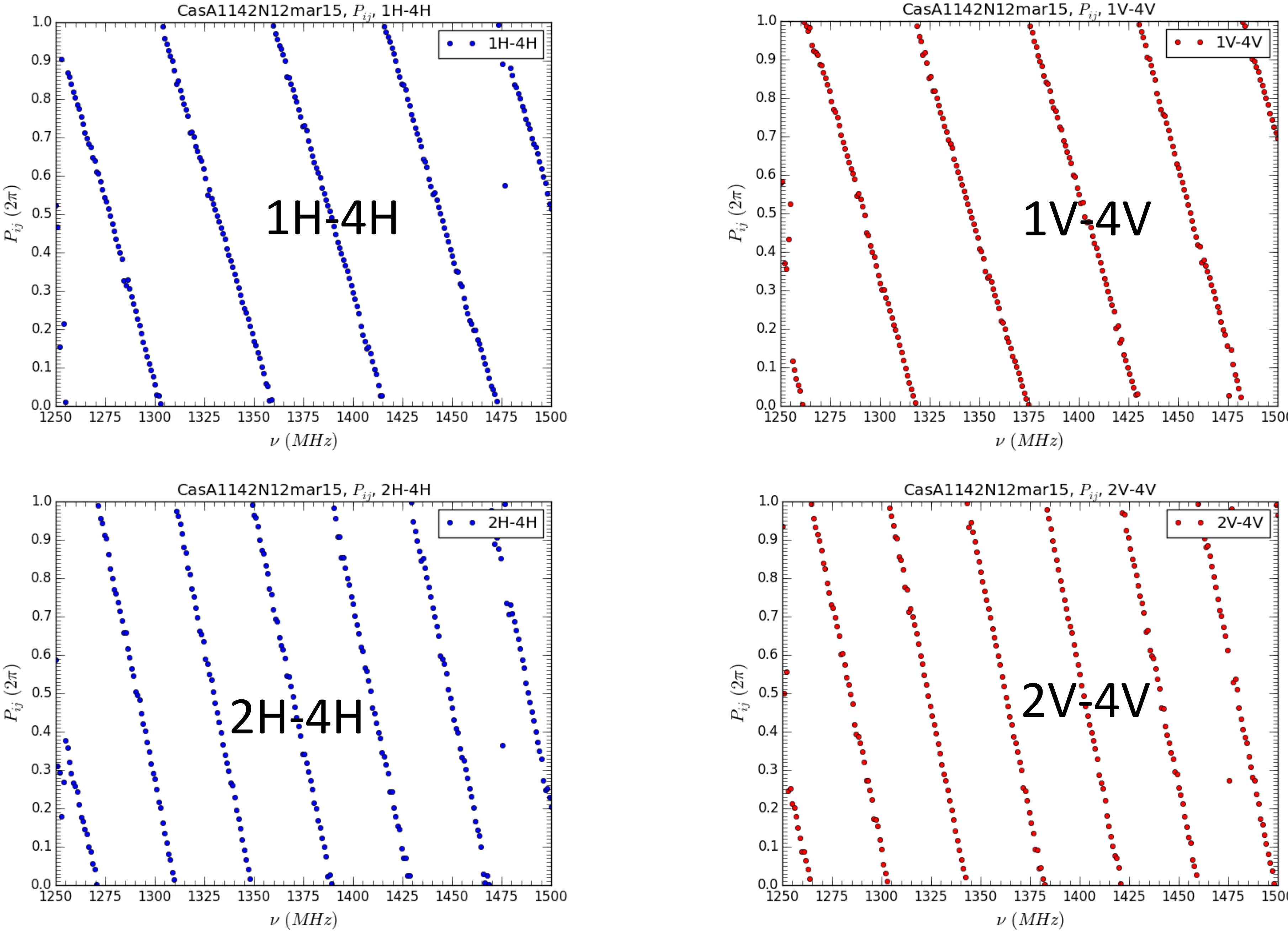




Phase delay/difference (2)



Phase delay/difference (3)



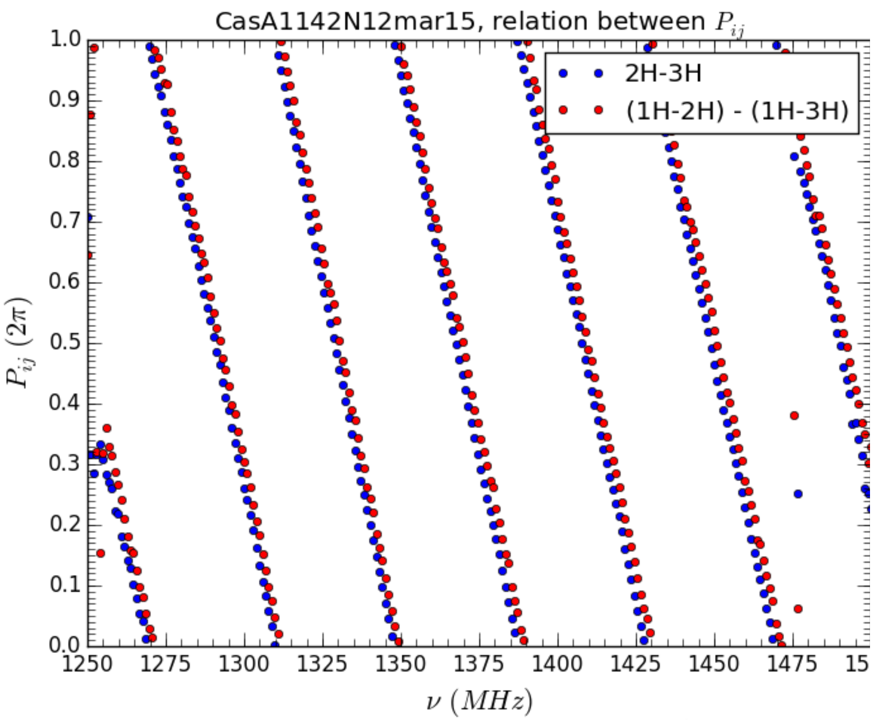
Relation between Phase_{ij} (1)

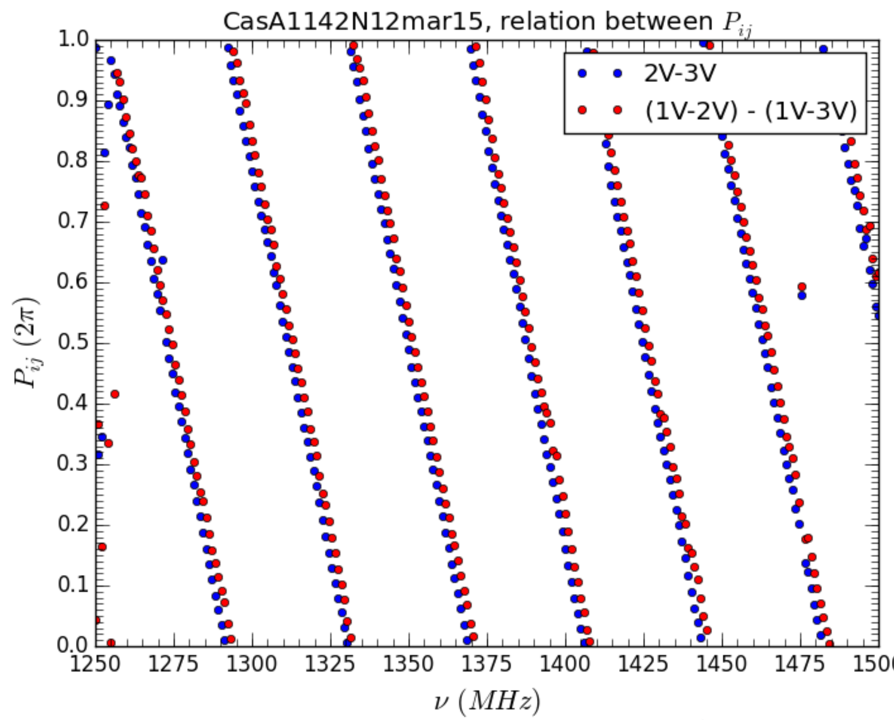
- Phase delay is most due to the different lengths of the cables.
- However, also because the lengths of the cables won't change, phase delays of different channels should have a stable relation.
- Obviously, we should have

$$P_{ij} - P_{ik} = P_{kj}$$

and I find this relation is roughly correct.

Relation between Phase_{ij} (2)





Blue: 2H-3H

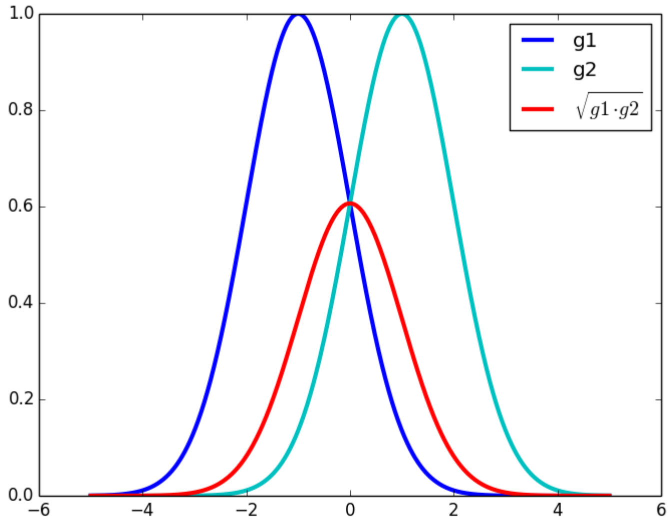
Red: (1H-2H)-(1H-3H)

Blue: 2V-3V

Red: (1V-2V)-(1V-3V)

System temperature

- Consider the alignments of the antennas, I correct the amplitude of the fringe, and then calculate the system temperature



Different alignments will lead to a smaller amplitude