INSTITUT D'ASTROPHYSIQUE DE PARIS

Unité mixte de recherche 7095 *CNRS - Université Pierre et Marie Curie*

Séminaire à l'Université d'Orsay

Laboratoire de l'Accélérateur Linéaire

Dark Matter at Galactic Scales & MOND

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Gravitation et Cosmologie ($\mathcal{G}\mathbb{R}\varepsilon\mathbb{C}\mathcal{O}$)
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13 novembre 2015

Plan of the talk

1. Phenomenology of galactic dark matter
2. Modified gravity theories
3. Dielectric analogy and dipolar dark matter
4. DDM based on massive bigravity theory

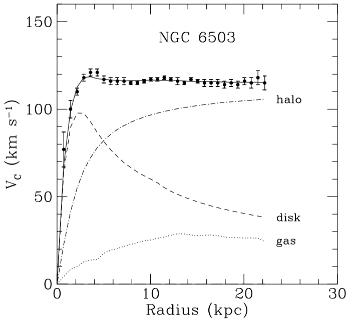
PHENOMENOLOGY OF GALACTIC DARK MATTER

Evidence for dark matter in Astrophysics

1. Oort [1932] noted that the sum of observed mass in the vicinity of the Sun falls short of explaining the vertical motion of stars in the Milky Way
2. Zwicky [1933] reported that the velocity dispersion of galaxies in galaxy clusters is far too high for these objects to remain bound for a substantial fraction of cosmic time

3. Ostriker & Peebles [1973] showed that to prevent the growth of instabilities in cold self-gravitating disks like spiral galaxies, it is necessary to embed the disk in the quasi-spherical potential of a huge halo of dark matter
4. Bosma [1981] and Rubin [1982] established that the rotation curves of galaxies are approximately flat, contrarily to the Newtonian prediction based on ordinary baryonic matter

Rotation curves of galaxies are approximately flat



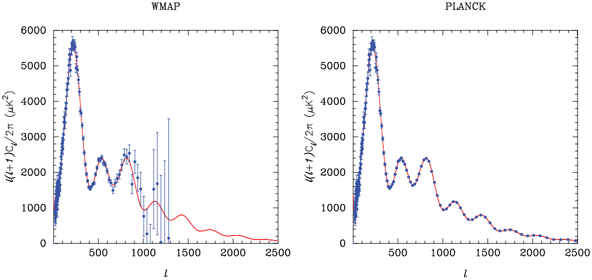
- For a circular orbit we expect

$$v(r) = \sqrt{\frac{G\,M(r)}{r}}$$

- The fact that $v(r)$ is constant implies that beyond the optical disk

$$M_{\text{halo}}(r) \simeq r \qquad \rho_{\text{halo}}(r) \simeq \frac{1}{r^2}$$

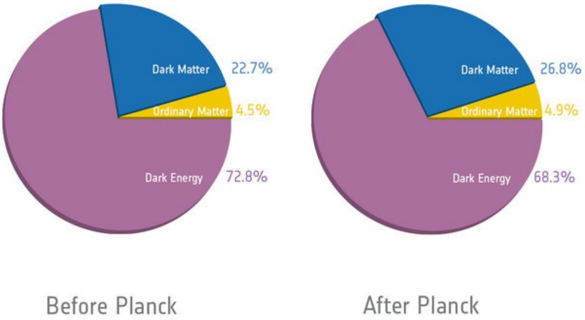
The cosmological concordance model Λ-CDM



This model brilliantly accounts for:

- The mass discrepancy between the dynamical and luminous masses of clusters of galaxies
- The precise measurements of the anisotropies of the cosmic microwave background (CMB)
- The formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
- The fainting of the light curves of distant supernovae

Problem of the dark constituents of the Universe



Λ-CDM assumes General Relativity is the correct theory of gravity but:

- No known particle in the standard model of particle physics could be the particle of dark matter
- Extensions of the standard model of particle physics provide well-motivated but yet to be discovered candidates
- The numerical value of the cosmological constant Λ looks un-natural from a quantum field perspective

Challenges with CDM at galactic scales

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004, Famaey & McGaugh 2012]

1. Unobserved predictions
   - Numerous but unseen satellites of large galaxies
   - Phase-space correlation of galaxy satellites
   - Generic formation of dark matter cusps in galaxies
   - Tidal dwarf galaxies dominated by dark matter
2. Unpredicted observations
   - Correlation between mass discrepancy and acceleration
   - Surface brightness of galaxies and the Freeman limit
   - Flat rotation curves of galaxies
   - Baryonic Tully-Fisher relation for spirals
   - Faber-Jackson relation for ellipticals

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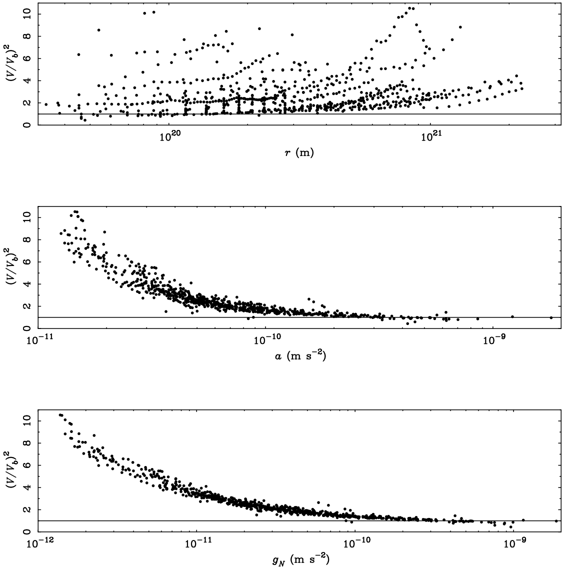
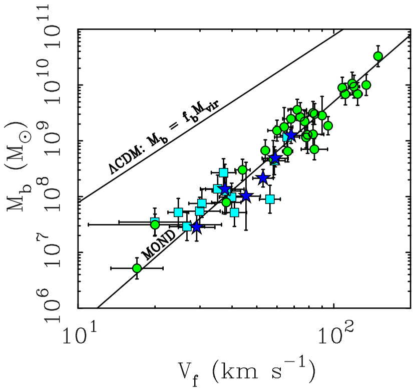
All these challenges are mysteriously solved (sometimes with incredible success) by the MOND empirical formula [Milgrom 1983]

Mass discrepancy versus acceleration [Milgrom 1983]



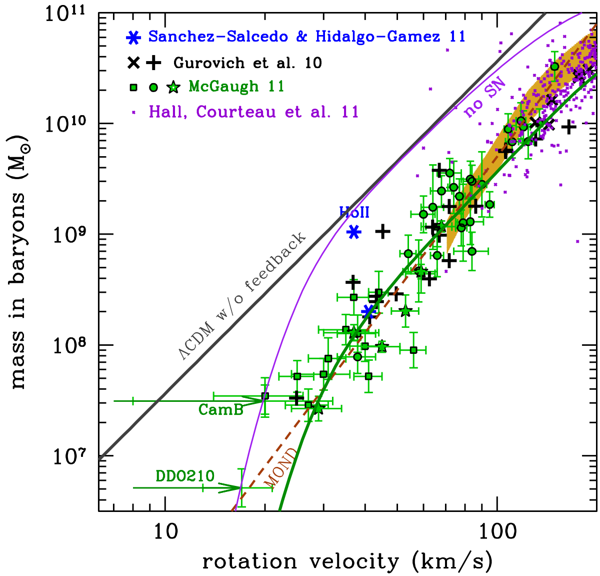
Baryonic Tully-Fisher relation [Tully & Fisher 1977, McGaugh 2011]



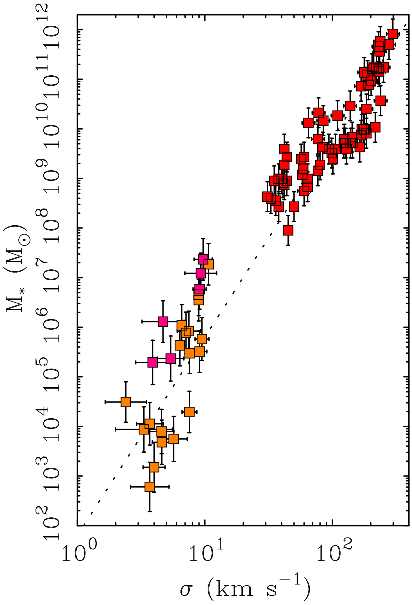
We have approximately $V_f \simeq (G\,M_b\,a_0)^{1/4}$ where $a_0 \simeq 1.2 \times 10^{-10}\text{m/s}^2$ is very close (mysteriously enough) to typical cosmological values

$$a_0 \simeq 1.3\,a_\Lambda \quad \text{with} \quad a_\Lambda = \frac{c^2}{2\pi}\sqrt{\frac{\Lambda}{3}}$$

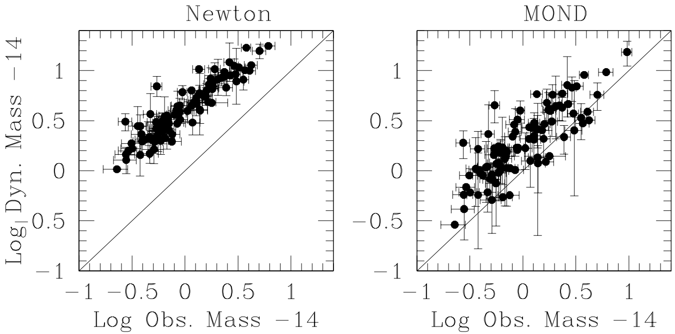
BTF relation fitted with Λ-CDM [Silk & Mamon 2012]



Mass velocity dispersion relation [Faber & Jackson 1976]

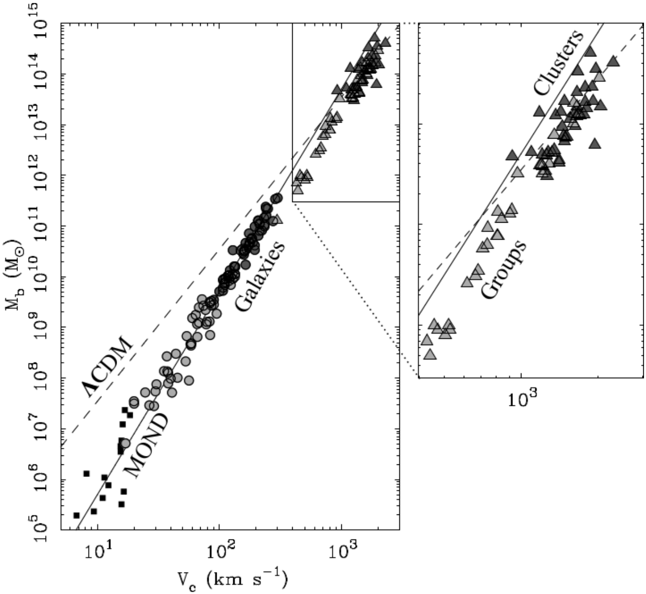


Problem with galaxy clusters [Gerbal, Durret *et al.* 1992, Sanders 1999]



- The mass discrepancy is $\approx 4-5$ with Newton and ≈ 2 with MOND
- The bullet cluster and more generally X-ray emitting galaxy clusters can be fitted with MOND only with a component of baryonic dark matter and/or hot/warm neutrinos [Angus, Famaey & Buote 2008]

Galactic versus cosmological scales



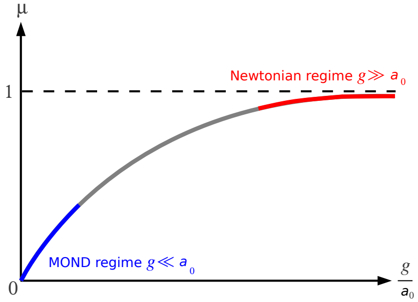
MODIFIED GRAVITY THEORIES

Modified Poisson equation [Milgrom 1983, Bekenstein & Milgrom 1984]

MOND takes the form of a modified Poisson equation

$$\nabla \cdot \Big[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \Big] = -4\pi G \rho_{\text{baryon}} \quad \text{avec} \quad \boldsymbol{g} = \nabla U$$

- The Newtonian regime is recoved when $g \gg a_0$
- In the MOND regime $g \ll a_0$ we have $\boxed{\mu \simeq g/a_0}$



Generalized scalar-tensor (RAQUAL) [Bekenstein & Sanders 1994]

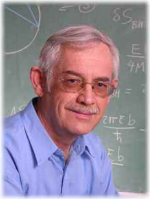
$$\mathcal{L}_{\text{scalar-tensor}} = \frac{\sqrt{-g}}{16\pi}\left[R + 2a_0^2\, F\left(g^{\mu\nu}\frac{\partial_\mu\phi\partial_\nu\phi}{a_0^2}\right)\right] + \mathcal{L}_{\text{m}}[\tilde{g}_{\mu\nu}, \Psi]$$

- "Scalar k-essence theory" (non-standard kinetic term) with matter fields universally coupled to the physical metric

$$\boxed{\tilde{g}_{\mu\nu} = e^{2\phi} g_{\mu\nu}}$$

- Specifying F this theory reproduces MOND in galaxies
- However light signals do not feel the presence of the scalar field so the theory does not explain the dark matter seen by gravitational lensing

Tensor-vector-scalar (TeVeS) [Bekenstein 2004, Sanders 2005]



- Disformal coupling postulated in the matter action

$$\tilde{g}_{\mu\nu} = e^{2\phi}(g_{\mu\nu} + A_\mu A_\nu) - e^{-2\phi} A_\mu A_\nu$$

- Scalar-tensor part similar to RAQUAL

$$\mathcal{L}_{\text{scalar-tensor}} = \frac{\sqrt{-g}}{16\pi}\left[R + 2a_0^2\, F\left((g^{\mu\nu} - A^\mu A^\nu)\frac{\partial_\mu\phi\partial_\nu\phi}{a_0^2}\right)\right] + \mathcal{L}_{\text{m}}[\tilde{g}_{\mu\nu}, \Psi]$$

- This requires adding a vector part

$$\mathcal{L}_{\text{vector}} = \frac{\sqrt{-g}}{16\pi}\Big[k F^{\mu\nu}F_{\mu\nu} + \underbrace{\lambda\big(A^\mu A_\mu + 1\big)}_{\text{Lagrange constraint}}\Big]$$

Generalized Einstein-Æther [Zlosnik *et al.* 2007, Halle *et al.* 2008]

- Motivation from Lorentz-invariance violation [Jacobson & Mattingly 2001]

$$\mathcal{L}_{Æ} = \frac{\sqrt{-g}}{16\pi}\Big[R + K + \underbrace{\lambda\big(g^{\mu\nu} n_\mu n_\nu + 1\big)}_{\text{Lagrange constraint}}\Big]$$

- Most general Lagrangian quadratic in derivatives of the vector field n^μ

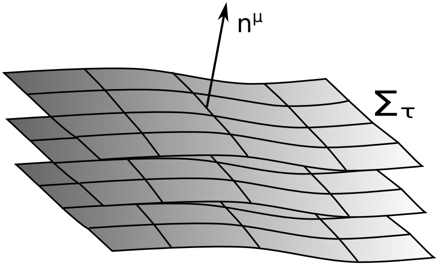
$$\begin{aligned} K &= \mathcal{K}^{\mu\nu\rho\sigma} \nabla_\mu n_\rho \nabla_\nu n_\sigma \\ K^{\mu\nu\rho\sigma} &= c_1\, g^{\mu\nu} g^{\rho\sigma} + c_2\, g^{\mu\rho} g^{\nu\sigma} + c_3\, g^{\mu\sigma} g^{\nu\rho} + c_4\, n^\mu n^\nu g^{\rho\sigma} \end{aligned}$$

- Vector k-essence generalization of Einstein-Æther theories

$$\mathcal{L}_{GÆ} = \frac{\sqrt{-g}}{16\pi}\Big[R + F(K) + \lambda\big(g^{\mu\nu} n_\mu n_\nu + 1\big)\Big]$$

- No problem with light deflection in these theories

From the Æther to the Khronon



To get rid of the Lagrange constraint we choose n_μ to be hypersurface orthogonal

$$n_\mu = -N\partial_\mu \tau$$

where τ is a dynamical scalar field called the Khronon and where

$$N = \frac{1}{\sqrt{-g^{\rho\sigma}\partial_\rho \tau \partial_\sigma \tau}}$$

Looking for a modification of GR for weak accelerations we can take the acceleration of the congruence of worldlines orthogonal to the foliation

$$a_\mu = n^\nu \nabla_\nu n_\mu = D_\mu \ln N$$

Khronometric theory [Blanchet & Marsat 2011, Sanders 2011]

Like for Einstein-Æther the theory admits a covariant formulation

$$\mathcal{L}_{\text{Einstein-Khronon}} = \frac{\sqrt{-g}}{16\pi}\Big[R - 2f(a)\Big] + \mathcal{L}_{\text{m}}[g_{\mu\nu}, \Psi]$$

but its content is best understood in adapted coordinates where $t = \tau$

$$\mathcal{L}_{\text{Einstein-Khronon}} = \frac{\sqrt{\gamma}}{16\pi} N\Big[\mathcal{R} + K_{ij}K^{ij} - K^2 - 2f(a)\Big] + \mathcal{L}_{\text{m}}[N, N_i, \gamma_{ij}, \Psi]$$

where $a_\mu = D_\mu \ln N$ and N, N_i, γ_{ij} are purely geometrical quantities

- For a choice of function f we recover GR+Λ in the strong-field regime $a \gg a_0$ and MOND in the weak-field regime $a \ll a_0$
- No problem with light deflection
- But no viable cosmology

Bimetric theory (BiMOND) [Milgrom 2009]

$$S_{\text{BiMOND}} = \frac{1}{16\pi}\int d^4x \Big[(-g)^{1/2}R[g] + (-\hat{g})^{1/2}R[\hat{g}] + \underbrace{2a_0^2(g\hat{g})^{1/4}F\Big[\frac{Z}{a_0^2}\Big]}_{\text{interaction term}}\Big]$$

$$+ \underbrace{S_{\text{m}}[g_{\mu\nu},\psi]}_{\text{ordinary matter}} + \underbrace{\hat{S}_{\text{m}}[\hat{g}_{\mu\nu},\hat{\psi}]}_{\text{twin matter}}$$

- The two metrics $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$ interact through the difference of Christoffel symbols $C^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \hat{\Gamma}^\lambda_{\mu\nu}$ (which is a tensor)

$$Z = \frac{1}{2}\left(g^{\mu\nu} + \hat{g}^{\mu\nu}\right)\left[C^\gamma_{\mu\lambda}C^\lambda_{\nu\gamma} - C^\gamma_{\mu\nu}C^\lambda_{\lambda\gamma}\right]$$

- The twin matter is not dark matter but is a replica of ordinary matter living in the other sector

Conclusions on modified gravities without DM

1. Complicated theories modifying GR with *ad-hoc* extra fields
   - Non-standard kinetic terms depending on an arbitrary function which is linked *in fine* to the MOND function
   - No physical explanation for the origin of the MOND effect
2. Stability problems in some cases associated with the fact that the Hamiltonian is not bounded from below [Clayton 2001, Bruneton & Esposito-Farèse 2007]
3. Generic problems to recover the cosmological model Λ-CDM at large scales and in particular the full spectrum of CMB anisotropies [Skordis, Mota *et al.* 2006]

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DIELECTRIC ANALOGY AND DIPOLAR DARK MATTER

A striking analogy [Blanchet 2006]

- In electrostratics the Gauss equation is modified by the polarization of the dielectric (dipolar) material

$$\nabla \cdot \underbrace{\Big[(1+\chi_e)\boldsymbol{E}\Big]}_{\boldsymbol{D}\text{ field}} = \frac{\rho_e}{\varepsilon_0} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0}$$

- Similarly MOND can be viewed as a modification of the Poisson equation by the polarization of some dipolar medium

$$\nabla \cdot \left[\mu\left(\frac{g}{a_0}\right)\boldsymbol{g}\right] = -4\pi\, G\, \rho_{\text{b}} \qquad \Longleftrightarrow \qquad \nabla \cdot \boldsymbol{g} = -4\pi\, G \left(\rho_{\text{b}} + \underbrace{\rho^{\text{polar}}}_{\text{dark matter}}\right)$$

- The MOND function can be written $\mu = 1 + \chi$ where χ appears as a susceptibility coefficient of some "dipolar dark matter" medium

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Microscopic description of DDM?

- The DM medium by individual dipole moments $\boldsymbol{p}$ and a polarization field $\boldsymbol{P}$

$$\boldsymbol{P} = n\,\boldsymbol{p} \qquad \text{with} \qquad \boldsymbol{p} = m\,\boldsymbol{\xi}$$

- The polarization is induced by the gravitational field of ordinary masses

$$\boldsymbol{P} = -\frac{\chi}{4\pi G}\,\boldsymbol{g} \qquad \boxed{\rho_{\text{DDM}} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}}$$

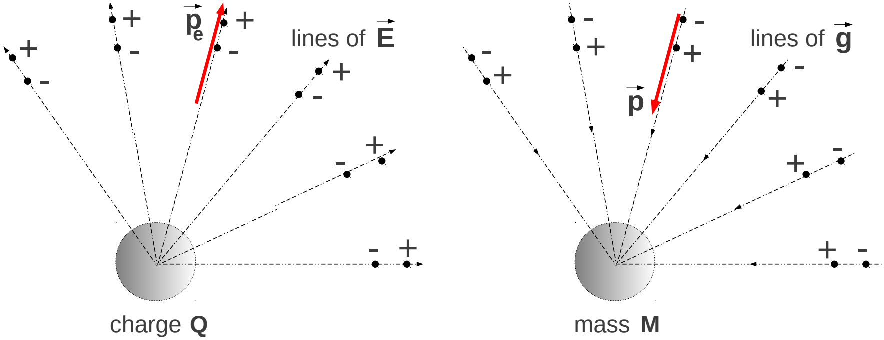
The dipole moments should be made by particles with positive and negative gravitational masses $(m_{\text{i}}, m_{\text{g}}) = (m, \pm m)$

- Because like masses attract and unlike ones repel we have anti-screening of ordinary masses by polarization masses

$$\boxed{\chi < 0}$$

which is in agreement with DM and MOND

Anti-screening by polarization masses



Screening by polarization charges

$\chi_e > 0$

Anti-screening by polarization masses

$\chi < 0$

Need of a non-gravitational internal force

- The constituents of the dipole will repel each other so we need a non-gravitational force

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{\nabla}\big(U+\phi\big) \qquad \frac{\mathrm{d}\underline{\boldsymbol{v}}}{\mathrm{d}t} = -\boldsymbol{\nabla}\big(U+\phi\big)$$

and look for an equilibrium when $\boldsymbol{\nabla}\big(U+\phi\big) = \boldsymbol{0}$

- The internal force is generated by the gravitational charge *i.e.* the mass

$$\Delta\phi = -\frac{4\pi G}{\chi}\big(\rho - \underline{\rho}\big)$$

- The DM medium appears as a polarizable plasma of particles $(m, \pm m)$ oscillating at the natural plasma frequency

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2\boldsymbol{\xi} = 2\boldsymbol{g} \qquad \text{with} \qquad \omega = \sqrt{-\frac{8\pi\, G\, \rho_0}{\chi}}$$

DDM BASED ON MASSIVE BIGRAVITY THEORY

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

1. To describe relativistically some microscopic DM particles with positive or negative gravitational masses one needs two metrics
   - $g_{\mu\nu}$ obeyed by ordinary particles (including baryons)
   - $f_{\mu\nu}$ obeyed by "dark" particles

2. In addition the DM particles forming the dipole moment should interact via a non-gravitational force field, *e.g.* a (spin-1) "graviphoton" vector field $\mathcal{A}_\mu$ with field strength $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$

3. One needs to introduce into the action the kinetic terms for all these fields, and to define the interaction between the two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$

DDM via a bimetric extension of GR [Bernard & Blanchet 2014]

1. The action of the model involves three sectors

$$S = \int \mathrm{d}^4x \Bigg\{ \overbrace{\sqrt{-g}\left(\frac{R_g}{32\pi} - \rho_{\text{bar}} - \rho_g\right)}^{\text{ordinary sector}} + \overbrace{\sqrt{-f}\left(\frac{R_f}{32\pi} - \rho_f\right)}^{\text{dark sector}}$$
$$+ \underbrace{\sqrt{-\mathcal{G}_{\text{eff}}}\left[\frac{\mathcal{R}_{\text{eff}}}{16\pi\varepsilon} + (\mathcal{J}_g^\mu - \mathcal{J}_f^\mu)\mathcal{A}_\mu + \frac{a_0^2}{8\pi}\mathcal{W}(\mathcal{X})\right]}_{\text{interaction sector}} \Bigg\}$$

2. The two metrics interact *via* the auxiliary metric

$$\boxed{\mathcal{G}^{\text{eff}}_{\mu\nu} = g_{\mu\rho}X^\rho_\nu = f_{\mu\rho}Y^\rho_\nu}$$

where the square-root matrices are $X = \sqrt{g^{-1}f}$ and $Y = \sqrt{f^{-1}g}$

3. The physics of the model will be obtained when the coupling constant $\varepsilon \ll 1$

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An interesting phenomenological model ...

- The cosmological model Λ-CDM and its successes at cosmological scales and notably the fit of the CMB are recovered
- The phenomenology of MOND is "explained" by a physical mechanism of gravitational polarization
- The dark matter appears as a diffuse polarizable medium undergoing stable plasma-like oscillations
- The model is viable in the Solar System as its PPN parameters are the same as in general relativity

... but non-viable as a fundamental theory

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Ghosts in the gravitational sector [Blanchet & Heisenberg 2015]

1. The presence of the square root of the determinant $\propto \sqrt{-\mathcal{G}_{\text{eff}}}$ in the action corresponds to ghostly potential interactions. The ghost is a very light degree of freedom, at the scale

$$m^2 M_{\text{P}}^2 \sqrt{-\mathcal{G}_{\text{eff}}} \sim \frac{m^2 M_{\text{P}}^2 (\Box\pi)^2}{\Lambda_3^6} = \frac{(\Box\pi)^2}{m^2}$$

and the theory cannot be used as an effective field theory

2. Another source of ghostly interactions is originated in the presence of three kinetic terms

$$\sqrt{-g}R_g \qquad \sqrt{-f}R_f \qquad \sqrt{-\mathcal{G}_{\text{eff}}}\mathcal{R}_{\text{eff}}$$

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Ghosts in the gravitational sector [Blanchet & Heisenberg 2015]

1. The presence of the square root of the determinant $\propto \sqrt{-\mathcal{G}_{\text{eff}}}$ in the action corresponds to ghostly potential interactions. The ghost is a very light degree of freedom, at the scale

$$m^2 M_{\text{P}}^2 \sqrt{-\mathcal{G}_{\text{eff}}} \sim \frac{m^2 M_{\text{P}}^2 (\Box\pi)^2}{\Lambda_3^6} = \frac{(\Box\pi)^2}{m^2}$$

and the theory cannot be used as an effective field theory

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Search for a massive gravity theory

1. Unique linear theory without ghosts [Fierz & Pauli 1939]

$$S_{\text{FP}} = \frac{1}{16\pi} \int \mathrm{d}^4 x \Big[\underbrace{\partial_\mu h_{\nu\rho} \partial^\mu \overline{h}^{\nu\rho} - H_\mu H^\mu}_{\substack{\text{Einstein-Hilbert action} \\ \text{for } g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}}} + \underbrace{m^2 \big(h_{\mu\nu} h^{\mu\nu} - h^2 \big)}_{\text{mass term}} \Big]$$

2. Massless limit of massive gravity differs from GR and is invalidated in the Solar System [Van Dam, Veltman & Zhakharov 1970]

$$\gamma_{\text{PPN}} = \frac{1}{2} \quad (?)$$

3. Nonlinear terms restaure the continuity of the massless limit and GR is recovered for [Vainshtein 1972; Babichev, Deffayet & Ziour 2009]

$$r \lesssim r_{\text{Vainshtein}} \simeq \left(\frac{M}{m^4} \right)^{1/5}$$

4. Instability appears at quadratic order due to 6 degrees of freedom for the massive spin-2 field instead of 5 [Boulware & Deser 1972]

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Nonlinear ghost-free massive (bi-)gravity

[de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012]

The theory is defined non-perturbatively as

$$S = \int \mathrm{d}^4x \left[\frac{M_g^2}{2}\sqrt{-g}R_g + \overbrace{m^2\sqrt{-g}\sum_{n=0}^{4}\alpha_n e_n(X)}^{\text{ghost-free interaction mass term}} + \frac{M_f^2}{2}\sqrt{-f}R_f \right]$$

Elementary symmetric polynomials of the square root matrix $X = \sqrt{g^{-1}f}$

$$\begin{aligned}
e_0(X) &= 1 \\
e_1(X) &= [X] \\
e_2(X) &= \frac{1}{2}\left([X]^2 - [X^2]\right) \\
e_3(X) &= \frac{1}{6}\left([X]^3 - 3[X][X^2] + 2[X^3]\right) \\
e_4(X) &= \frac{1}{24}\left([X]^4 - 6[X]^2[X^2] + 3[X^2]^2 + 8[X][X^3] - 6[X^4]\right)
\end{aligned}$$

DDM via massive bigravity theory [Blanchet & Heisenberg 2015ab]

1. The matter sector is the same as in the previous model
2. The gravitational sector of the model is based on massive bigravity theory

$$S = \int \mathrm{d}^4x \Big\{ \sqrt{-g}\Big(\frac{M_g^2}{2}R_g - \rho_{\text{bar}} - \rho_g\Big) + \sqrt{-f}\Big(\frac{M_f^2}{2}R_f - \rho_f\Big) + \sqrt{-g_{\text{eff}}}\Big[\frac{m^2}{4\pi} + \mathcal{A}_\mu\Big(j_g^\mu - \frac{\alpha}{\beta}j_f^\mu\Big) + \frac{a_0^2}{8\pi}\mathcal{W}(\mathcal{X})\Big]\Big\}$$

3. The ghost-free potential interactions take the particular form of the square root of the determinant of the effective metric [de Rham, Heisenberg & Ribeiro 2014]

$$\boxed{g_{\mu\nu}^{\text{eff}} = \alpha^2 g_{\mu\nu} + 2\alpha\beta\, g_{\mu\rho}X^\rho_\nu + \beta^2 f_{\mu\nu}}$$

with the square-root matrix $X = \sqrt{g^{-1}f}$

4. The graviphoton field $\mathcal{A}_\mu$ is coupled to the effective metric $g_{\mu\nu}^{\text{eff}}$

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General structure of the model [Blanchet & Heisenberg 2015ab]

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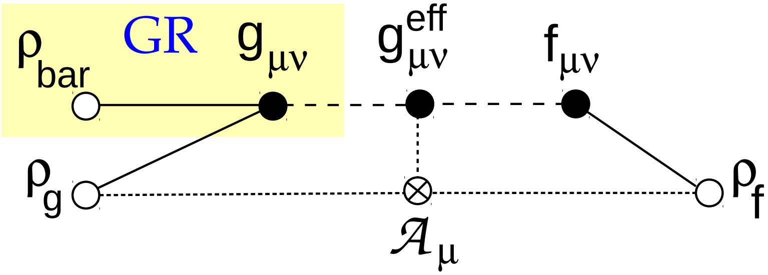

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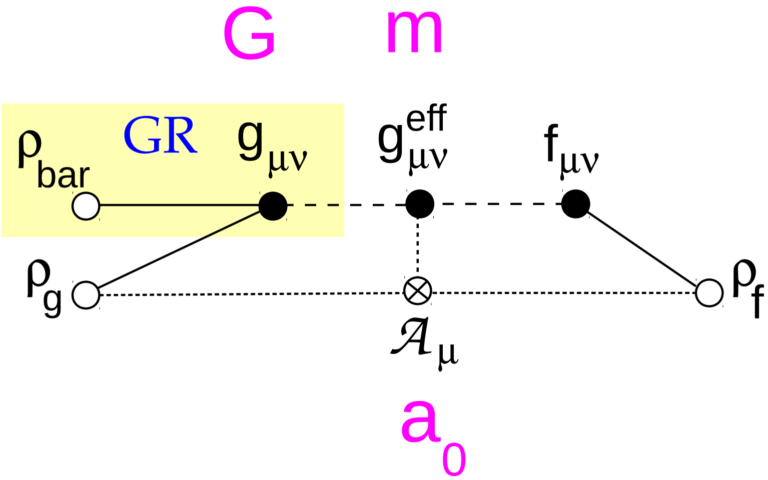

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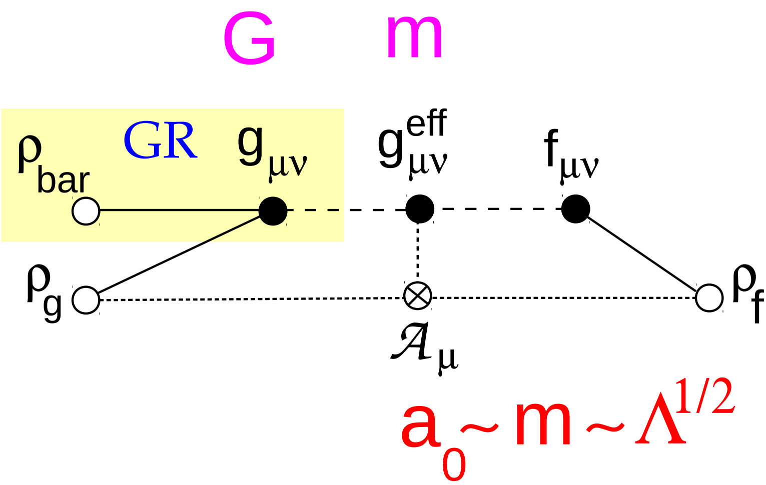

Gravitational polarization & MOND

1. Equations of motion of DM particles in the non-relativistic limit $c \to \infty$

$$\frac{\mathrm{d}\boldsymbol{v}_g}{\mathrm{d}t} = \boldsymbol{\nabla}\left(U_g + \phi\right) \qquad \frac{\mathrm{d}\boldsymbol{v}_f}{\mathrm{d}t} = \boldsymbol{\nabla}\left(U_f - \frac{\alpha}{\beta}\phi\right)$$

2. With massive bigravity the two g and f sectors are linked together by a constraint equation coming from the Bianchi identities

$$\boxed{\boldsymbol{\nabla}\left(\alpha U_g + \beta U_f\right) = 0}$$

showing that α/β is the ratio between gravitational and inertial masses of f particles with respect to g metric

3. The DM medium is at equilibrium when the Coulomb force annihilates the gravitational force, $\boldsymbol{\nabla} U_g + \boldsymbol{\nabla}\phi = 0$, at which point the polarization is aligned with the gravitational field

$$\boxed{\boldsymbol{P} = \frac{1}{4\pi}\mathcal{W}'\,\boldsymbol{\nabla} U_g}$$

Gravitational polarization & MOND

1. From the massless combination of the two metrics combined with the Bianchi identity we get a Poisson equation for the ordinary Newtonian potential U_g

$$\Delta U_g = -4\pi\Big(\rho_{\text{bar}} + \underbrace{\rho_g - \frac{\alpha}{\beta}\rho_f}_{\text{DDM}}\Big)$$

2. With the plasma-like solution for the internal force and the mechanism of gravitational polarization this yields the MOND equation

$$\boxed{\nabla\cdot\Big[\underbrace{\left(1-\mathcal{W}'\right)}_{\text{MOND function}}\nabla U_g\Big] = -4\pi\rho_{\text{bar}}}$$

3. Finally the DM medium undergoes stable plasma-like oscillations in linear perturbations around the equilibrium

Status of hybrid DM à la MOND (DDM)

- The phenomenology of MOND is explained by a physical mechanism of gravitational polarization
- The theory has the potential to reproduce the cosmological model Λ-CDM and its successes at cosmological scales (notably the CMB)
- By construction the model is safe in the gravitational sector to any perturbation order [de Rham, Gabadadze & Tolley 2011; Hassan & Rosen 2012]
- Because of the coupling of the DM fields ρ_g, ρ_f to the vector field $\mathcal{A}_\mu$ a ghost in the decoupling limit is present in the DM sector, but its mass can be made to be larger than the strong coupling scale [Blanchet & Heisenberg, in preparation]

$$\Lambda_3 = \left(m^2 M_\text{P}\right)^{1/3}$$

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Conclusions

1. An important challenge is to reproduce within a single relativistic framework
   - The concordance cosmological model Λ-CDM and its tremendous successes at cosmological scales and notably the fit of the CMB
   - The phenomenology of MOND which is a basic set of phenomena relevant to galaxy dynamics and DM distribution at galactic scales
2. While Λ-CDM meets severe problems when extrapolated at the scale of galaxies, MOND fails on galaxy cluster scales and seems also to be marginally excluded by planetary ephemerides in the Solar System
3. Pure modified gravity theories (extending GR with extra fields) do not seem to be able to reproduce Λ-CDM and the CMB anisotropies at large scales
4. An hybrid approach (DM *à la* MOND) called DDM, defined within the beautiful framework of the massive bigravity theory, could meet the challenges posed by MOND *versus* Λ-CDM

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