Simulations for Tianlai Cylinder Array

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Outline

m-mode formalism

Measurement equation: Shaw et al. 2014

$$V_{ij}(\phi) = \int d^2\hat{\boldsymbol{n}}\, B_{ij}(\hat{\boldsymbol{n}};\phi) T(\hat{\boldsymbol{n}}) + n_{ij}(\phi)$$

where the beam transfer function is

$$B_{ij}(\hat{\boldsymbol{n}};\phi) = \frac{1}{\Omega_{ij}} A_i(\hat{\boldsymbol{n}};\phi) A_j^*(\hat{\boldsymbol{n}};\phi) e^{2\pi i \hat{\boldsymbol{n}}\cdot\boldsymbol{u}_{ij}(\phi)}$$

Spherical harmonic expand $T(\hat{\boldsymbol{n}})$ and $B_{ij}(\hat{\boldsymbol{n}};\phi)$

$$T(\hat{\boldsymbol{n}}) = \sum_{lm} a_{lm} Y_{lm}(\hat{\boldsymbol{n}}), \qquad B_{ij}(\hat{\boldsymbol{n}};\phi) = \sum_{lm} B_{lm}^{ij}(\phi) Y_{lm}^*(\hat{\boldsymbol{n}})$$

and Fourier transform the system

$$V_m^{ij} = \int \frac{d\phi}{2\pi} V_{ij}(\phi) e^{-im\phi} = \sum_{lm'} \int \frac{d\phi}{2\pi} B_{lm'}^{ij}(\phi) a_{lm'} e^{-im\phi} + n_m^{ij}$$

Use $B_{lm}^{ij}(\phi) = B_{lm}^{ij}(\phi = 0) e^{im\phi}$, we get

$$V_m^{ij} = \sum_l B_{lm}^{ij} a_{lm} + n_m^{ij} \qquad \boxed{\boldsymbol{v} = \mathbf{B}\boldsymbol{a} + \boldsymbol{n}}$$

Simulation parameters

Parameters	Value
Number of cylinders	3
Cylinder width $[m]$	15
Totle feeds	96 (dual-pol)
Feed spacing $[m]$	0.4
T_{sys} $[K]$	50
Bandwidth [MHz]	700–800
Channel width [MHz]	0.195
Number of channels	512
Telescope latitude	45°

Table: Parameters of the Tianlai cylinder pathfinder telescope.

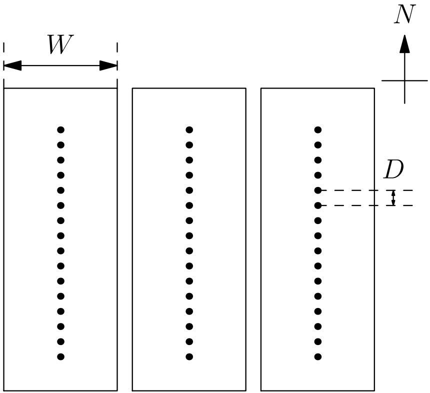
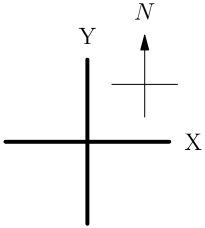


Figure: A schematic of Tianlai pathfinder cylinder telescope.

Beam Model

We model the cylinder beam as the product of two 1D functions:



E-W direction the Frauhofer diffraction pattern A_F;

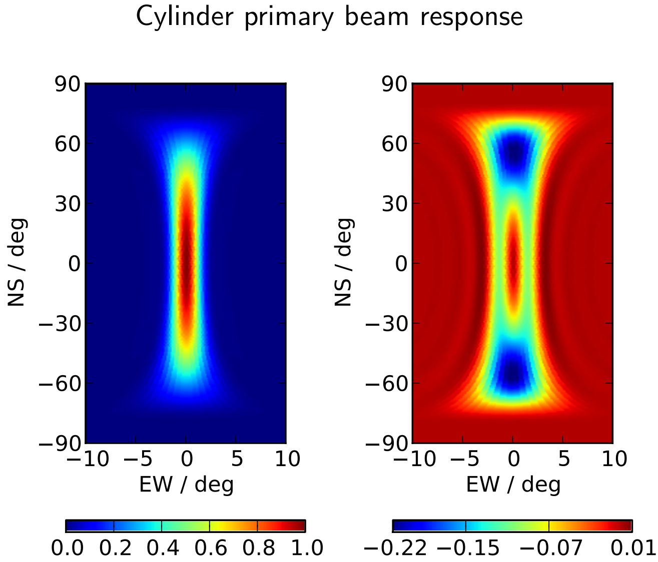
N-S direction the reflected dipole feed amplitude A_D.

$$A_a^X(\hat{\boldsymbol{n}}) = A_F(\sin^{-1}(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{x}}; \theta_E, W))A_D(\sin^{-1}(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{y}}); \theta_H)p_a(\hat{\boldsymbol{n}}; \hat{\boldsymbol{x}}),$$

$$A_a^Y(\hat{\boldsymbol{n}}) = A_F(\sin^{-1}(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{x}}; \theta_H, W))A_D(\sin^{-1}(\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{y}}); \theta_E)p_a(\hat{\boldsymbol{n}}; \hat{\boldsymbol{y}}).$$

Where p_a gives polarisation direction on the sky for a dipole.

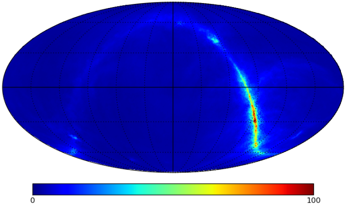
Beam Model



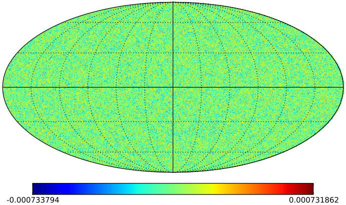
(a) Intensity response (b) Polarization leakage

Input Sky Maps

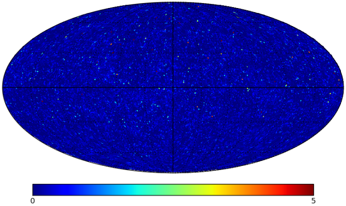
Brightness Temperature



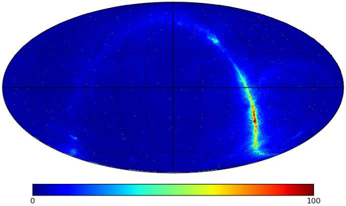
(a) Galactic synchrotron emission



(b) 21 cm signal



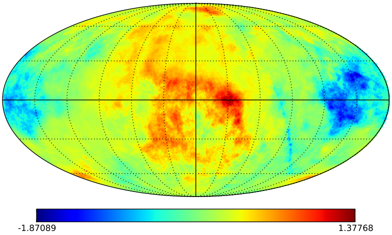
(c) Extra-Galactic pointsources



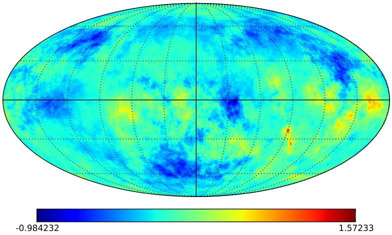
(d) (a) + (b) + (c)

Input Sky maps

Stokes Q and U of Galactic Synchrotron Emission

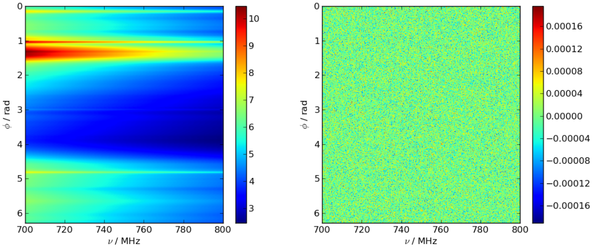


(a) Stokes Q

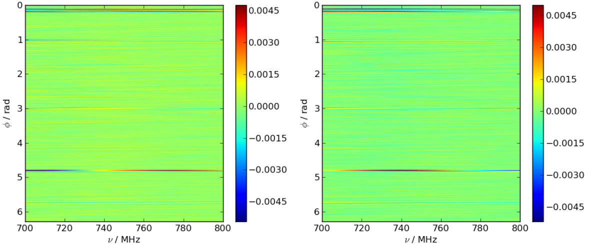


(b) Stokes U

Simulated Visibilities

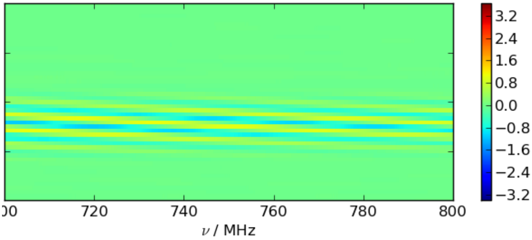


(a) Auto-correlation

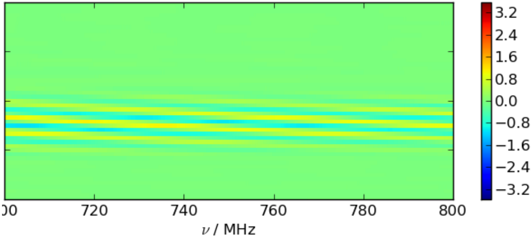


(b) Cross-correlation

Visibility fringe



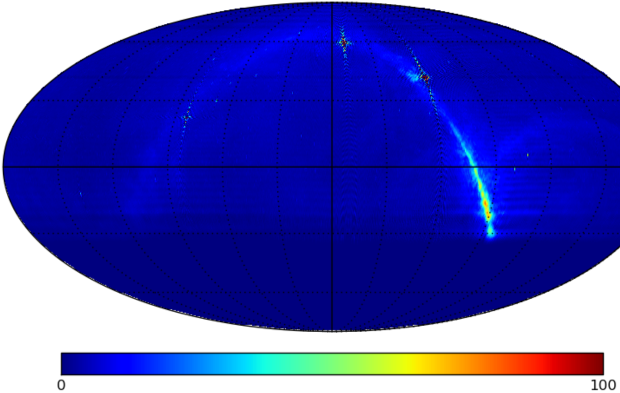
(a) Real part



(b) Imaginary part

Map-Making

$31 + 32 + 33$ Central $D = 0.4\text{m} = 1.0\lambda$



Synthesized Beam Definition

We define the Synthesized beam as:

$$b_0(\hat{\boldsymbol{n}}) \equiv \sum_\alpha e^{2\pi i \hat{\boldsymbol{n}} \cdot \boldsymbol{u}_\alpha}$$

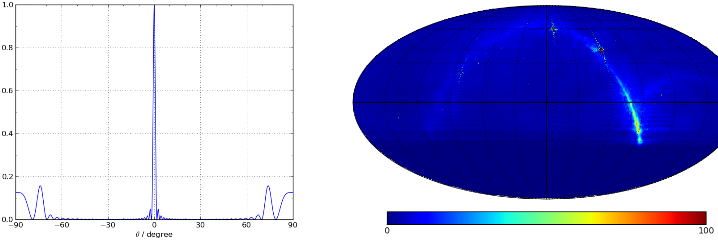
which can be expressed in spherical coordinate as

$$b_0(\theta, \phi) = \sum_\alpha e^{2\pi i (u_\alpha \sin\theta \cos\phi + v_\alpha \sin\theta \sin\phi)}$$

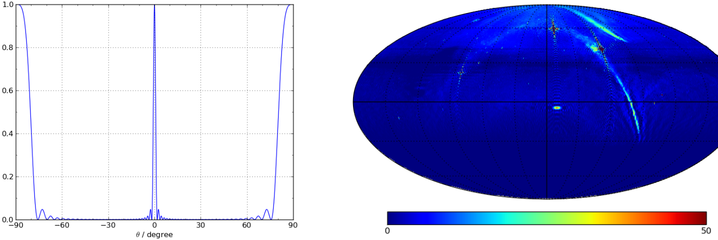
Along the NS direction, $\phi = \pi/2$

$$b_0(\theta) = \sum_\alpha e^{2\pi i v_\alpha \sin\theta}$$

The Effect of Synthesized Beam on Map-making



(a) 31+32+33, $d = 1.0\lambda$



(b) 3x32, $d = 1.0\lambda$

Foreground Subtraction By K-L Transform

Karhunen-Loève Transform

The covariance matrices of the 21 cm signal and foreground are

$$\mathbf{S} = \mathbf{B}\mathbf{C}_{21}\mathbf{B}^{\dagger}, \qquad \mathbf{F} = \mathbf{B}\mathbf{C}_{f}\mathbf{B}^{\dagger}$$

K-L transform seeks to jointly diagonalize $\mathbf{S}$ and $\mathbf{F}$

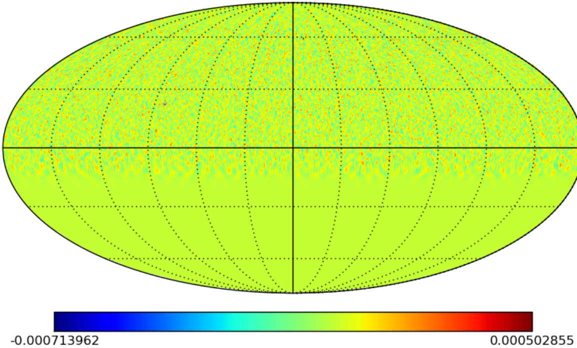
$$\mathbf{S} \to \mathbf{S}' = \mathbf{P}\mathbf{S}\mathbf{P}^{\dagger} = \mathbf{\Lambda},$$
$$\mathbf{F} \to \mathbf{F}' = \mathbf{P}\mathbf{F}\mathbf{P}^{\dagger} = \mathbf{I}.$$

This transformation can be found by solving the generalized eigenvalue problem

$$\mathbf{S}\boldsymbol{x} = \lambda\mathbf{F}\boldsymbol{x}.$$

Foreground Subtraction By K-L Transform

Karhunen-Loève Transform



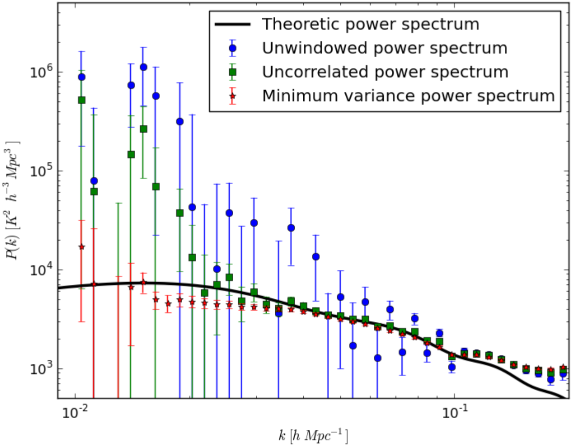
Recovered 21 cm signal after foreground subtraction

Estimation Methods

Name	W_{ab}	$\Delta\hat{p}_a$	Properties
Unwindowed	δ_{ab}	$(F^{-1})_{aa}^{1/2}$	anti-correlated errorbars
Uncorrelated	$[\sum_c (F^{1/2})_{ac}]^{-1}(F^{1/2})_{ab}$	$[\sum_c (F^{1/2})_{ac}]^{-1}$	uncorrelated errorbars
Minimum Variance	$[\sum_c F_{ac}]^{-1} F_{ab}$	$[\sum_c F_{ac}]^{-1}(F^{1/2})_{aa}$	minimum covariance

Table: Different choice for window function W_{ab} and corresponding power spectrum estimation error properties.

Power Spectrum Estimation



Power spectrum estimation result


Thank you!