Sky map reconstruction from Transit interferometers

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Outline

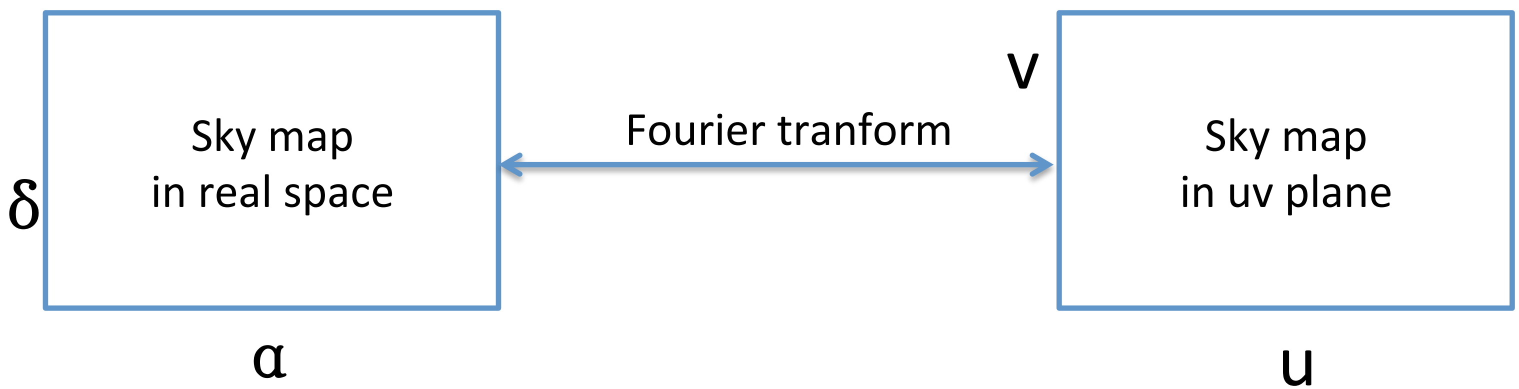
- Transit visibilities
- Map reconstruction principle
- Maximum likelihood estimation
- Instrument configuration
- Some results

Transit visibilities

- A visibility for a direction on the sky

$$V_{ij} = \iint d\Omega\, I(\Omega) \overbrace{\boxed{B_i(\Omega) B_j^*(\Omega) \exp(i\vec{k} \cdot \vec{\Delta r})}}^{L_{ij}(\Omega)}$$

- Transit telescope : Full East-West scan for a fixed δ_0



Rectangular geometry

- Sky rotation will bring us a phase factor :

$$\mathcal{I}(u,v) \;\longrightarrow\; \mathcal{I}(u,v) \times e^{2i\pi u\alpha_0}$$

- Measurement : a set of $V_{ij}^{\delta_0}$ as a function of time $\Longrightarrow$ $V_{ij}^{\delta_0}(\alpha = \omega t)$ which corresponds to a beam for a given pointing δ_0 :

$$L_{ij}^{\delta_0}(\alpha,\delta) \longleftrightarrow \mathcal{L}_{ij}^{\delta_0}(u,v)$$

Pointing in NS direction

$$V_{ij}^{\delta_0}(\alpha_k) = \sum_u \sum_v \mathcal{I}(u,v)\mathcal{L}_{ij}^{\delta_0}(u,v)e^{2i\pi u\alpha_k} + noise$$

Time (earth rotation)

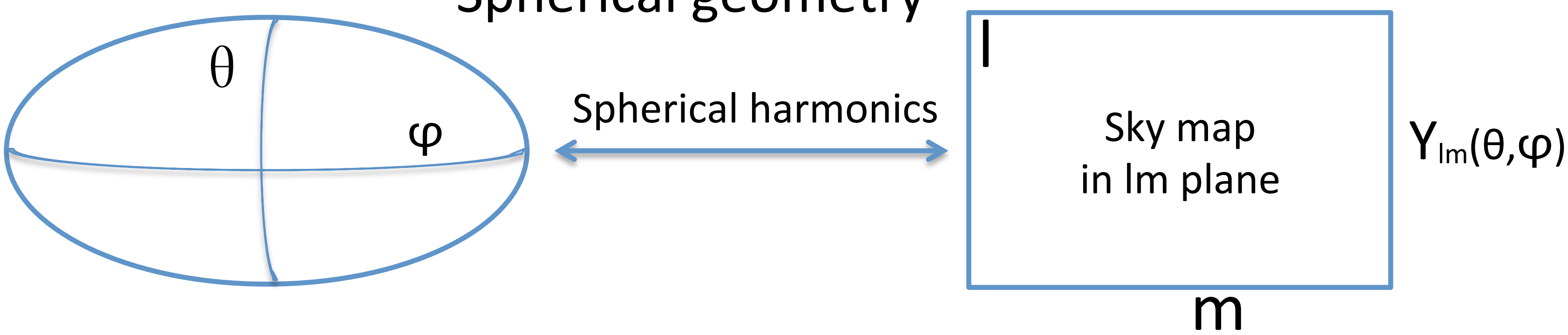
Full East-West scan means we could perform Fourier transform on the set of $V_{ij}^{\delta_0}(\alpha_k)$ to obtain the $\widetilde{\mathcal{V}}_{ij}^{\delta_0}(u)$ for each u-mode

$$V_{ij}^{\delta_0}(\alpha_k) \quad (0 \leqslant \alpha_k < 2\pi) \longrightarrow \widetilde{\mathcal{V}}_{ij}^{\delta_0}(u) \quad for\ all\ u\ modes$$

There is a track to take into account of both +u and –u modes, which corresponds to real sky with only +u modes

In spherical geometry, Full East–West scan means we could perform Fourier transform on the set of $\mathcal{V}_{ij}^{\theta_0}(\varphi_k)$ to obtain the $\tilde{\mathcal{V}}_{ij}^{\theta_0}(m)$ for each m-mode

Spherical geometry



$$\mathcal{V}_{ij}^{\theta_0}(\varphi_k) \quad (0 \leqslant \varphi_k < 2\pi) \longrightarrow \tilde{\mathcal{V}}_{ij}^{\theta_0}(m) \quad for\ all\ m\ modes$$

$$\tilde{\mathcal{V}}_{ij}^{\theta_0}(m) = \mathscr{F}(\mathcal{V}_{ij}^{\theta_0}(\varphi_k)) = \sum_l \mathcal{I}(l,m)\mathcal{L}_{ij}^{\theta_0}(l,m) + noise$$

Map reconstruction principle

- We could express the relation between sky and visibilities in matrix form (we keep only one $V_{ij}^{\delta_0}$ for a set of antenna with the same baseline and same pointing to simplify numerical handling)
- The full problem of all $V_{ij}(u)$ could be separated into a set of **independent** problems for each **u**.

block diagonal matrix

$$\begin{pmatrix} \widetilde{\mathcal{V}}_{ij}^{\delta_k}(u_0) \\ \widetilde{\mathcal{V}}_{ij}^{\delta_k}(u_1) \\ \dots \\ \widetilde{\mathcal{V}}_{ij}^{\delta_k}(u_{max}) \end{pmatrix} = \begin{pmatrix} \mathcal{L}_{ij}^{\delta_k}(u_0,v) & & & 0 \\ & \underbrace{\boxed{\mathcal{L}_{ij}^{\delta_k}(u_1,v)}}_{A_{uj}} & & \\ 0 & & \dots & \\ & & & \mathcal{L}_{ij}^{\delta_k}(u_{max},v) \end{pmatrix} \times \begin{pmatrix} \mathcal{I}(u_0,v) \\ \mathcal{I}(u_1,v) \\ \dots \\ \mathcal{I}(u_{max},v) \end{pmatrix} + noise$$

- The map reconstruction principle are similar for spherical geometry from (θ,φ) to (m,l)

Maximum likelihood estimation

- We currently assume that the noise covariance matrix N is positive and symmetric $N =< n^t n >$

- Using maximum likelihood estimation, the Likelihood function is :

$$p(\widetilde{\mathcal{V}}|\mathcal{I}) = \frac{1}{\sqrt{2\pi N}} exp(-N^{-1}(\mathcal{V} - A\mathcal{I})^2)$$

- The maximum likelihood soulution $d\ln p/d\mathcal{I} = 0$ is given by pseudo-inverse (singular value decomposition)

$$B = (A^t N^{-1} A)^{-1} A^t N^{-1} = (N^{-\frac{1}{2}} A)^{-1} N^{-\frac{1}{2}}$$

For each u, the estimated sky

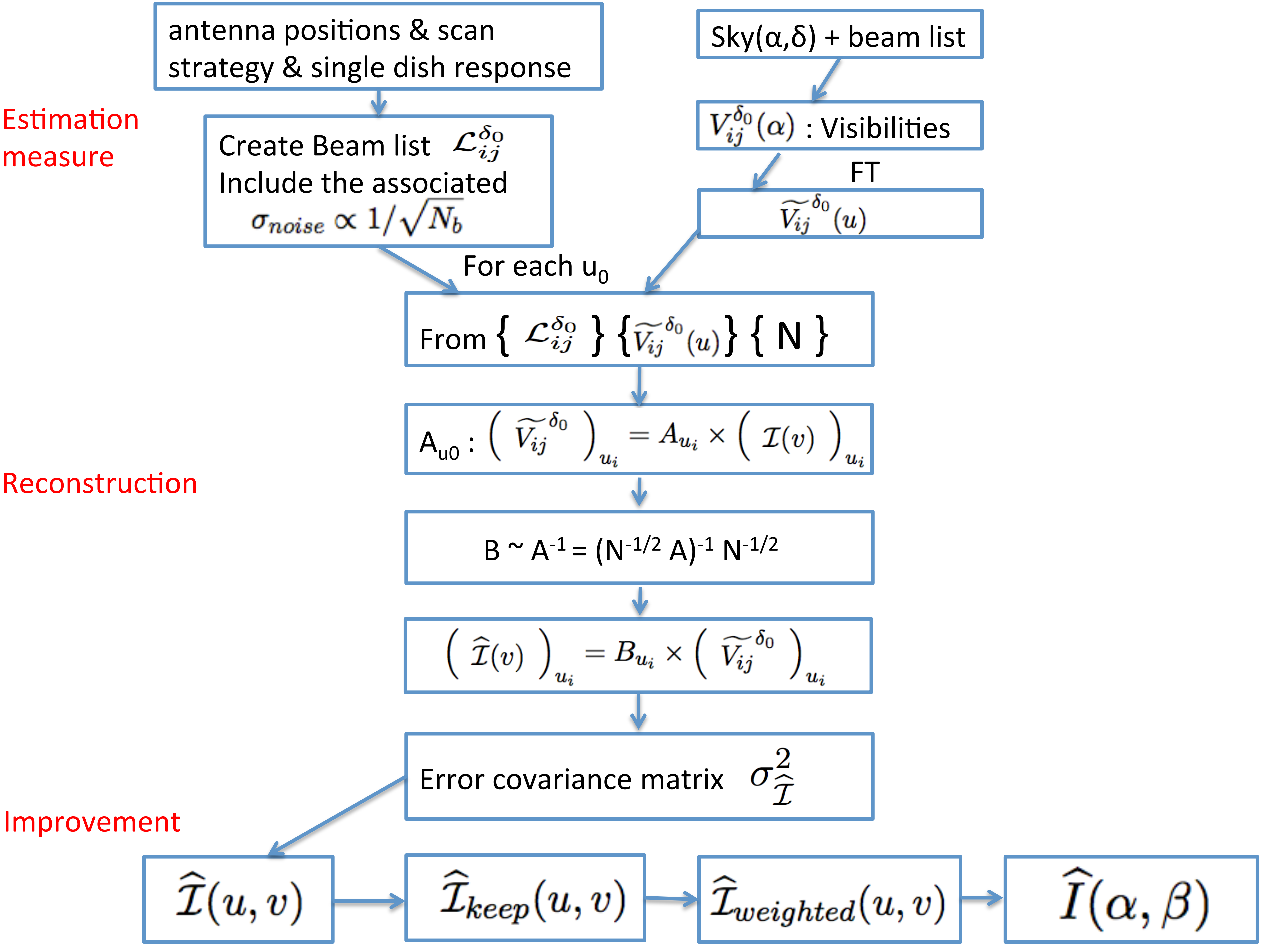
Noise weighted pseudo-inverse

Fourier transform of Visibility

$$\begin{pmatrix} \cdots \\ \widehat{\mathcal{I}}(v)_{u_i} \\ \cdots \end{pmatrix} = \boxed{\left((N^{-\frac{1}{2}} A_{u_i})^{-1} N^{-\frac{1}{2}} \right)} \times \begin{pmatrix} \cdots \\ \widetilde{\mathcal{V}}_{ij}^{\delta_k} \\ \cdots \end{pmatrix}$$

Here, we define a threshold, which is touchy, to reject small eigenvalue λ to avoid numerical instability and large noise. i.e. if λ_i<thr, then $1/\lambda_i=0$.

$$\begin{pmatrix} \cdots \\ \widehat{\mathcal{I}}_{sky}(u,v) \\ \cdots \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \cdots \\ \widehat{\mathcal{I}}_{sky}(l,m) \\ \cdots \end{pmatrix}$$

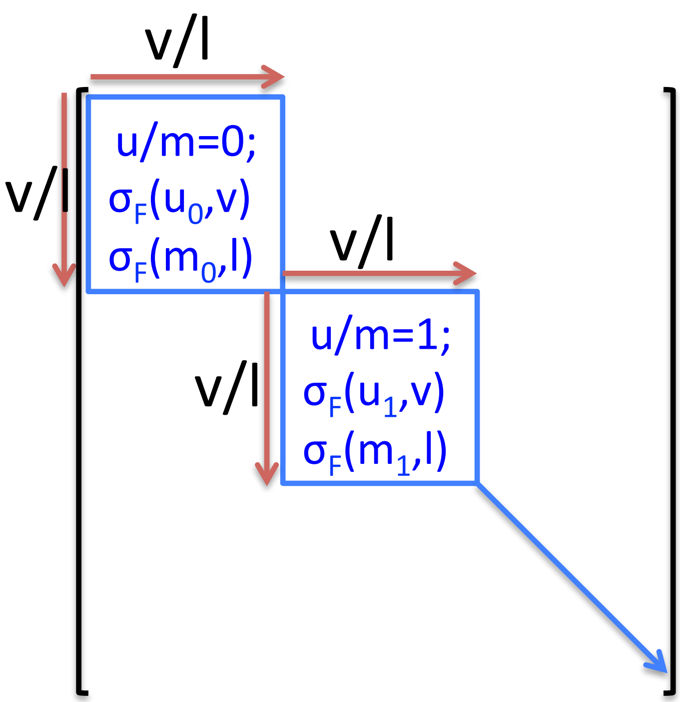

antenna positions & scan strategy & single dish response
Sky(α,δ) + beam list
Estimation measure
Create Beam list $\mathcal{L}_{ij}^{\delta_0}$
Include the associated $\sigma_{noise} \propto 1/\sqrt{N_b}$
$V_{ij}^{\delta_0}(\alpha)$: Visibilities
FT
$\widetilde{V_{ij}}^{\delta_0}(u)$
For each u_0
From $\{ \mathcal{L}_{ij}^{\delta_0} \} \{ \widetilde{V_{ij}}^{\delta_0}(u) \} \{ N \}$
A_{u0} : $\left(\widetilde{V_{ij}}^{\delta_0} \right)_{u_i} = A_{u_i} \times \left(\mathcal{I}(v) \right)_{u_i}$
Reconstruction
$B \sim A^{-1} = (N^{-1/2} A)^{-1} N^{-1/2}$
$\left(\widehat{\mathcal{I}}(v) \right)_{u_i} = B_{u_i} \times \left(\widetilde{V_{ij}}^{\delta_0} \right)_{u_i}$
Error covariance matrix $\sigma^2_{\widehat{\mathcal{I}}}$
Improvement
$\widehat{\mathcal{I}}(u,v)$
$\widehat{\mathcal{I}}_{keep}(u,v)$
$\widehat{\mathcal{I}}_{weighted}(u,v)$
$\widehat{I}(\alpha,\beta)$

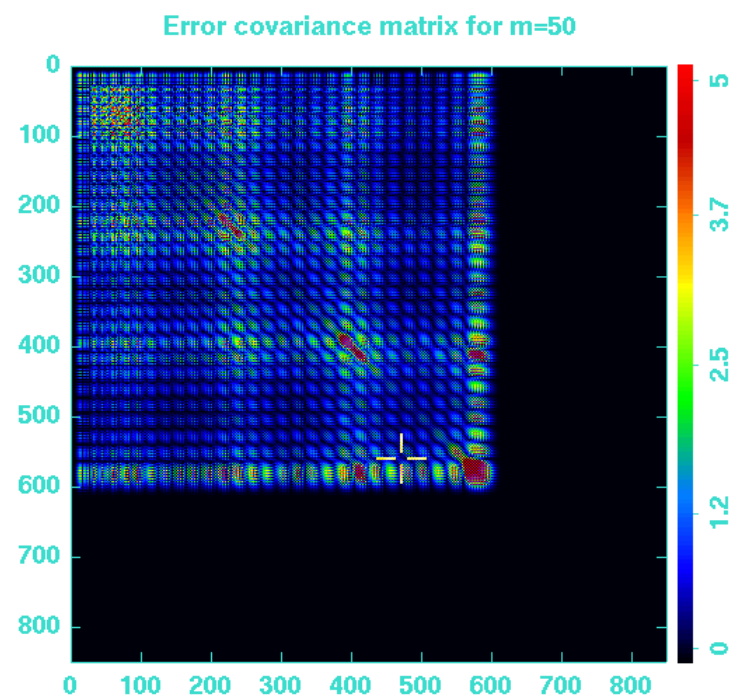
Error covariance matrix

- We compute observed sky **error covariance matrix** for each u/m modes

$$\sigma^2_{\widehat{\mathcal{I}}} = <\widehat{\mathcal{I}}(v)_{u_i}\widehat{\mathcal{I}}(v')^*_{u_i}> = BNB^t$$

- Error covariance matrix $\sigma^2_{\widehat{\mathcal{I}}}(u,v)$ is gathered from each diagonal.





we do not recontruct single l-modes, but a linear combination of modes.

B × A matrix

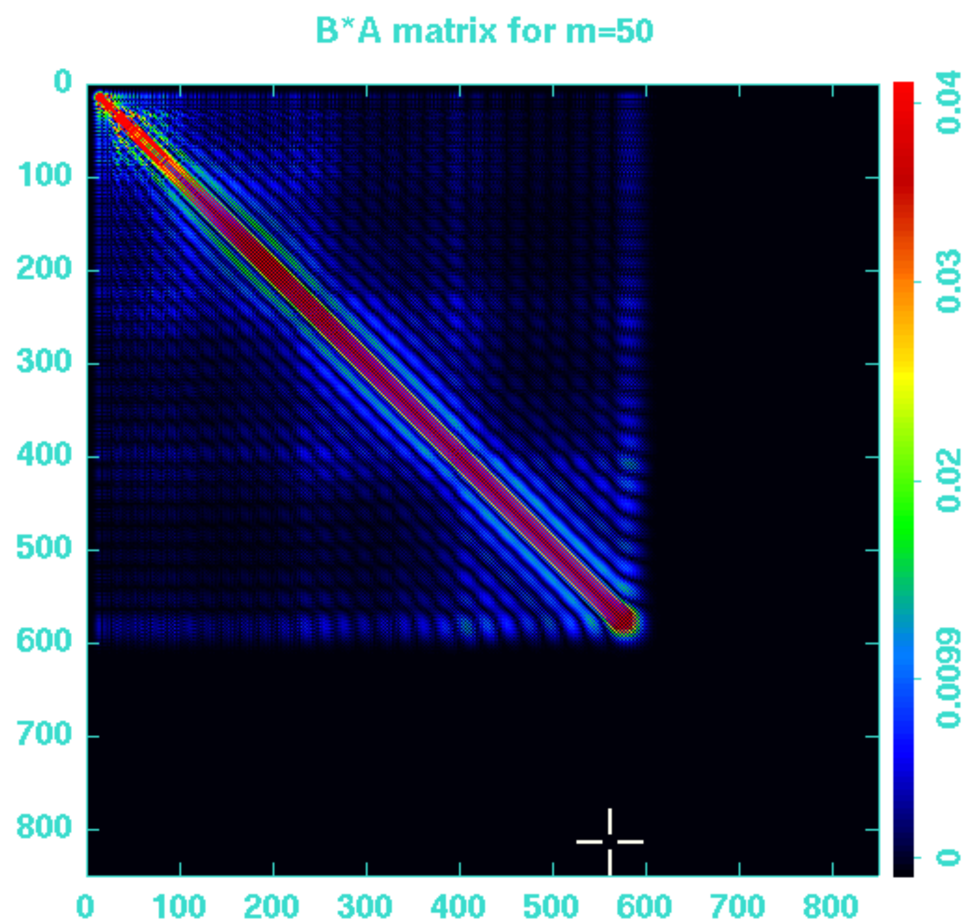
- B * A matrix would be more easily showing the fact that we measure linear combination of several modes.

Reconstructed sky

True sky

$$\widehat{\mathcal{I}} = B\ \widetilde{\mathcal{V}}_{ij}^{\delta_k} = \boxed{B * A}\ \mathcal{I}$$

This matrix tell us how well we can recover the sky, and how well the mode mixing is under control.



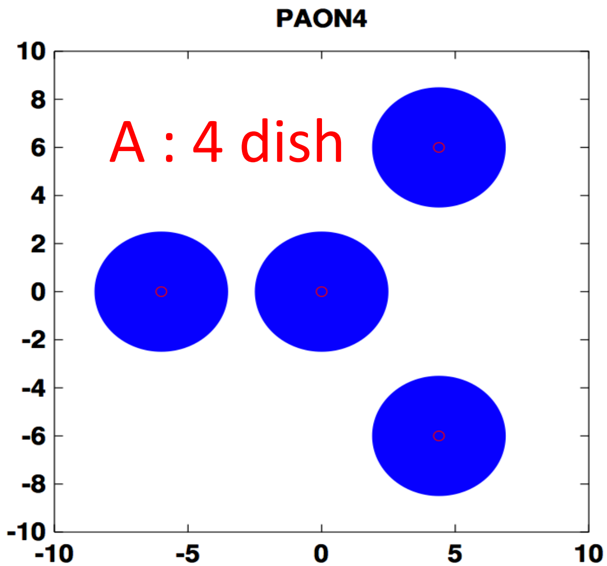
Modes modification

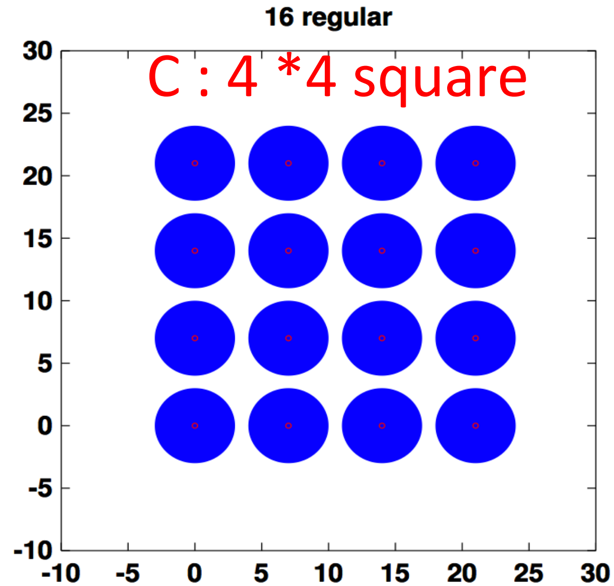
- Reject reconstructed modes with large errors by smoothly cutting off larger value based on $\sigma^2_{\widehat{\mathcal{I}}}(u,v)$
- Applying a weight function on $\widehat{\mathcal{I}}_{keep}(u,v)$ to control the noise effect. For example, we use a global Gaussian beam response.
- A global beam independent of frequency can be used to decrease mode mixing.

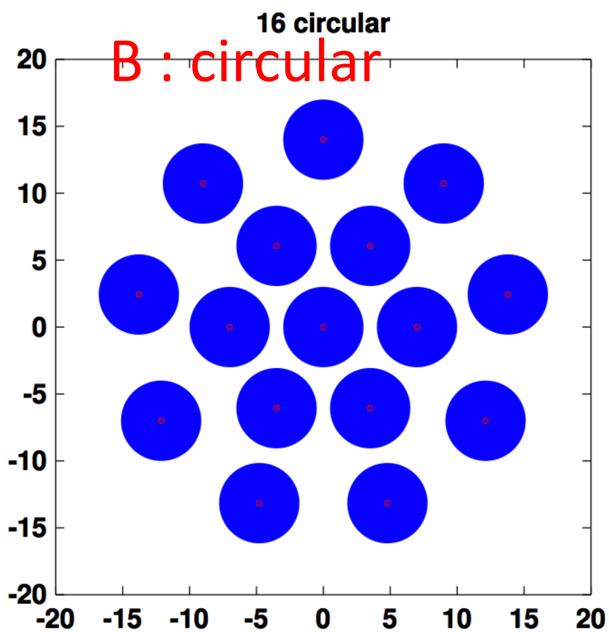
$$\begin{pmatrix} \cdots \\ \widehat{\mathcal{I}}(u,v) \\ \cdots \end{pmatrix} \xrightarrow[\text{Reject large error modes}]{} \begin{pmatrix} \cdots \\ \widehat{\mathcal{I}}_{keep}(u,v) \\ \cdots \end{pmatrix} \xrightarrow[\text{weight function}]{} \begin{pmatrix} \cdots \\ \widehat{\mathcal{I}}_{weighted}(u,v) \\ \cdots \end{pmatrix} \xrightarrow[\text{Inverse Fourier transform}]{} \begin{pmatrix} \cdots \\ \widehat{I}(\alpha,\beta) \\ \cdots \end{pmatrix}$$

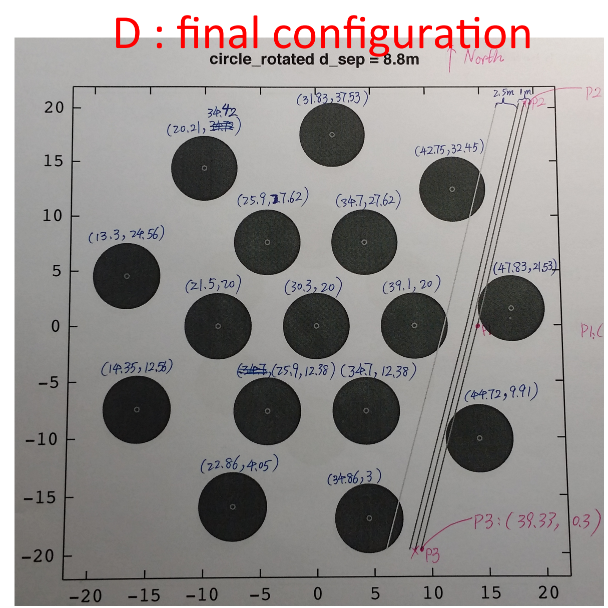
There are also the same steps in spherical, replacing (u,v) by (m, l)

Instrument configuration





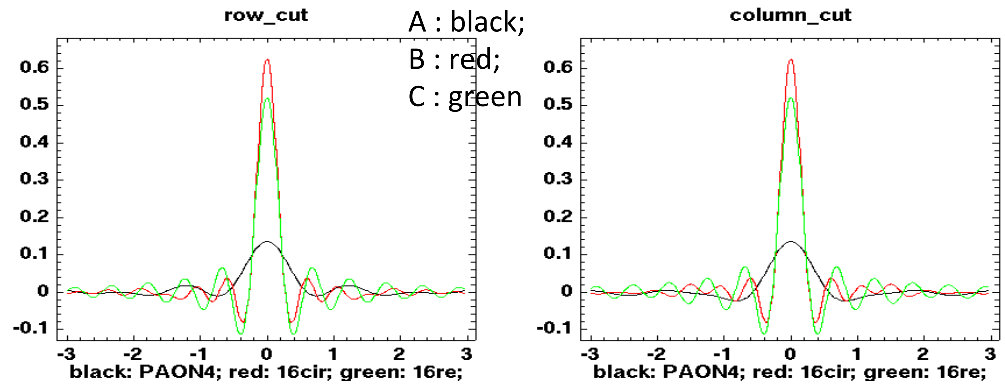


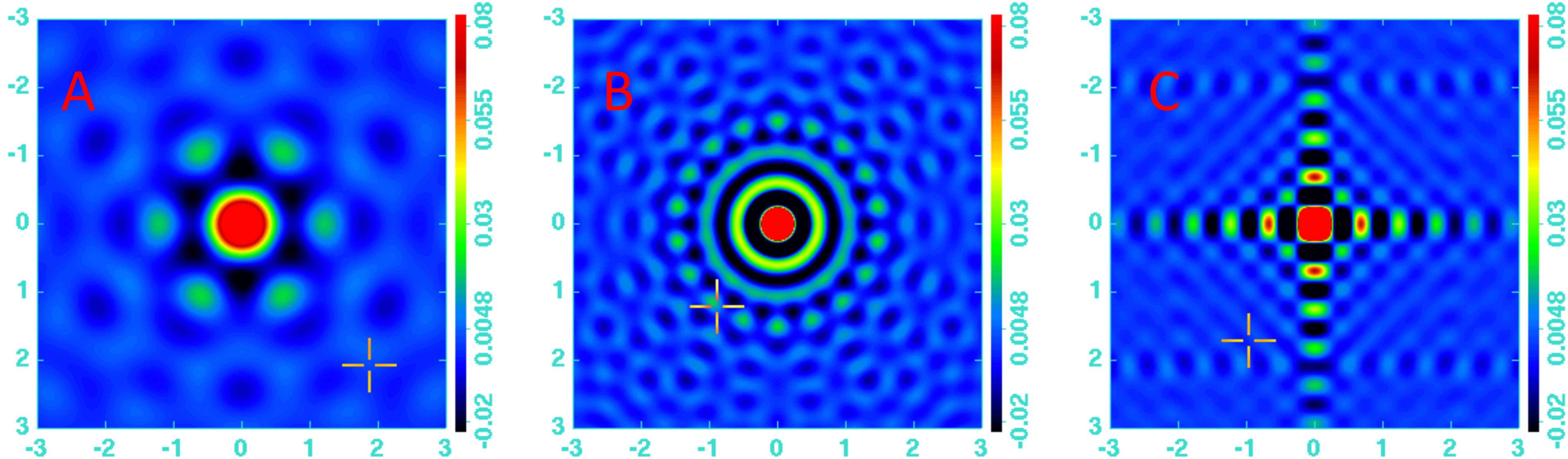


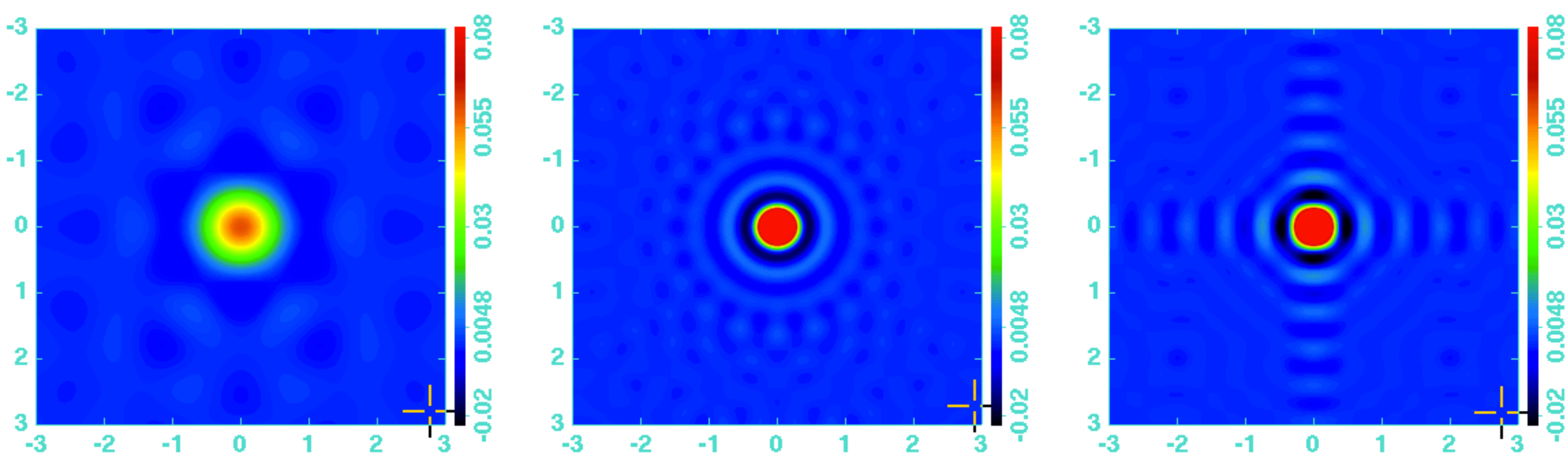
Some results

- Synthesized beam
- Reconstructed sky map
- Error covariance matrix
- Transfer function
- Noise power spectrum

Synthesized beam





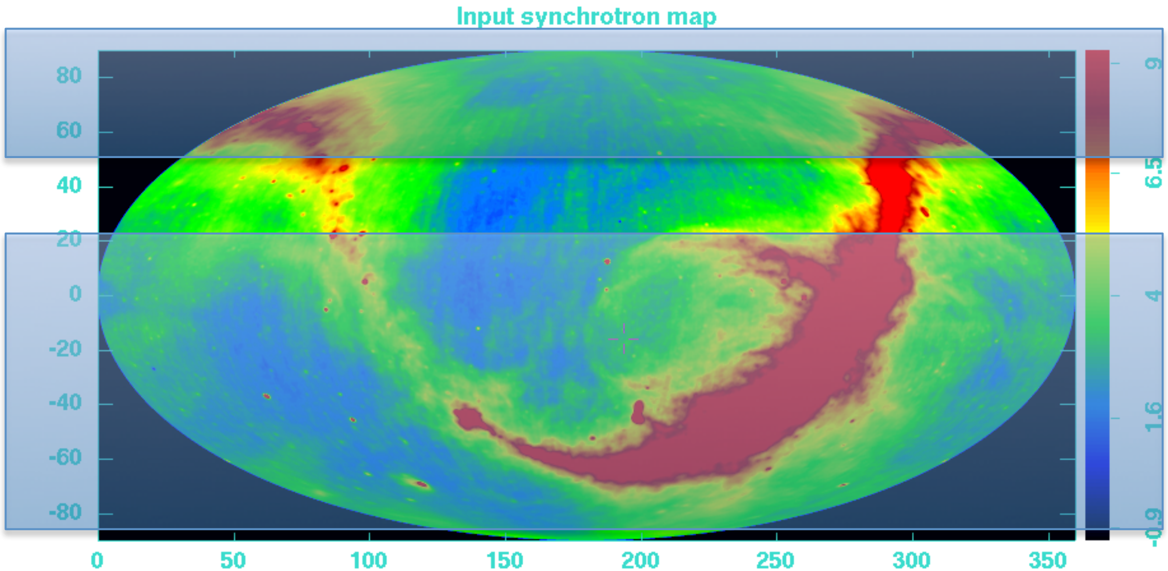


Apply global beam

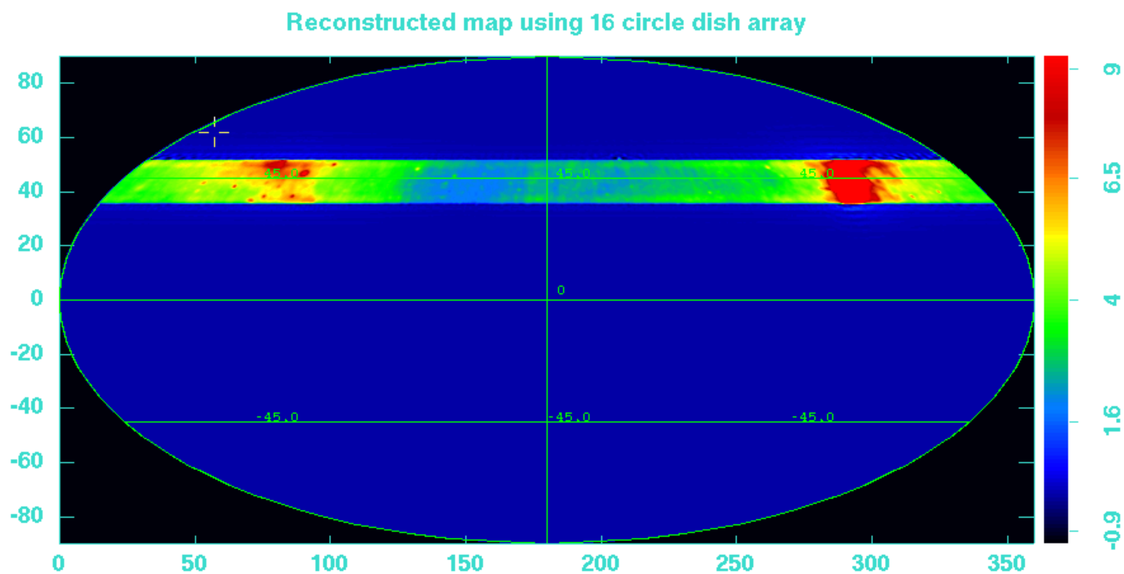
Reconstructed sky map

Instrument configuration B; 9 scan per 1.5 degrees; local latitude θ = 45 deg

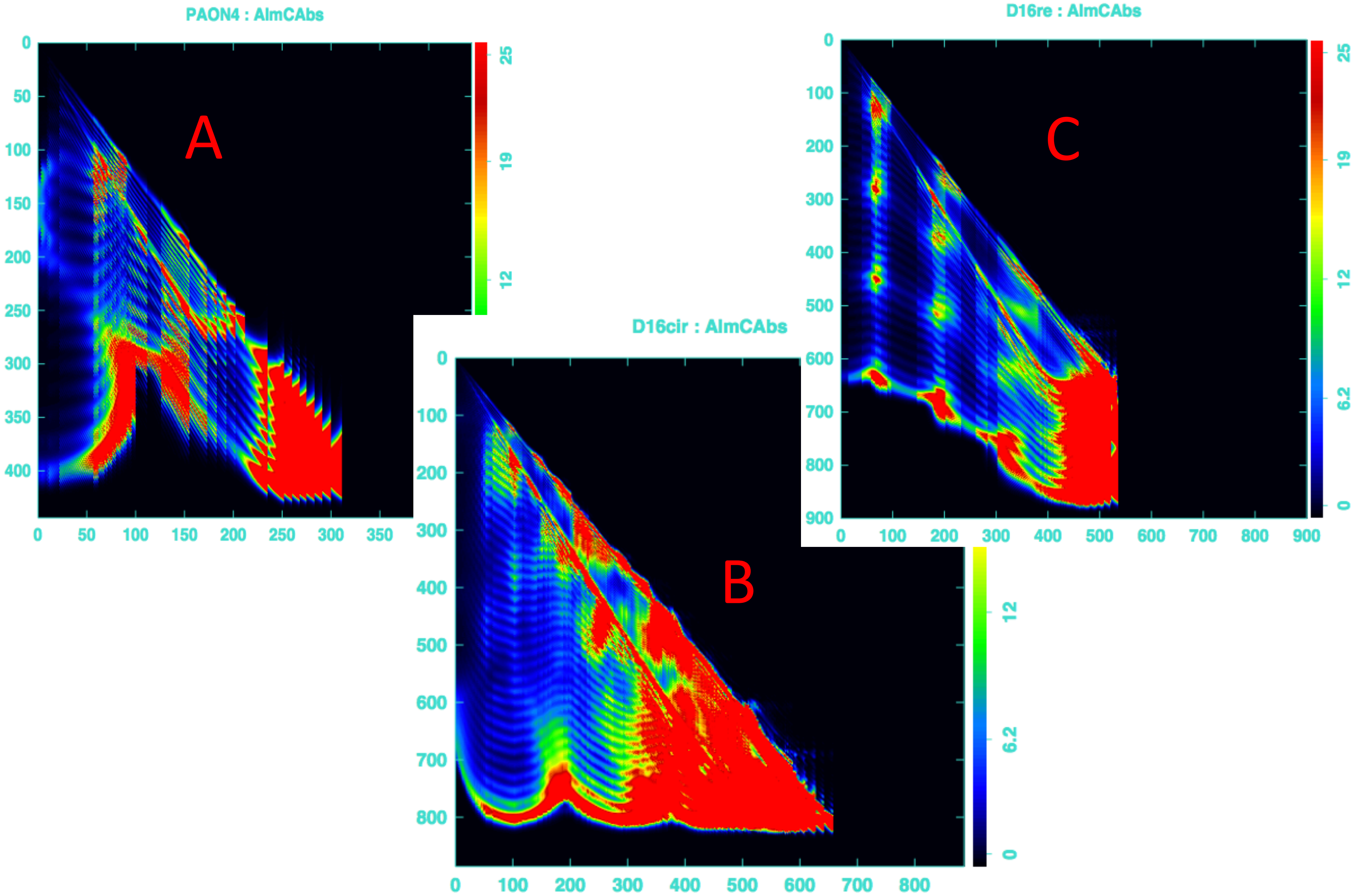
Input map



"observed" map

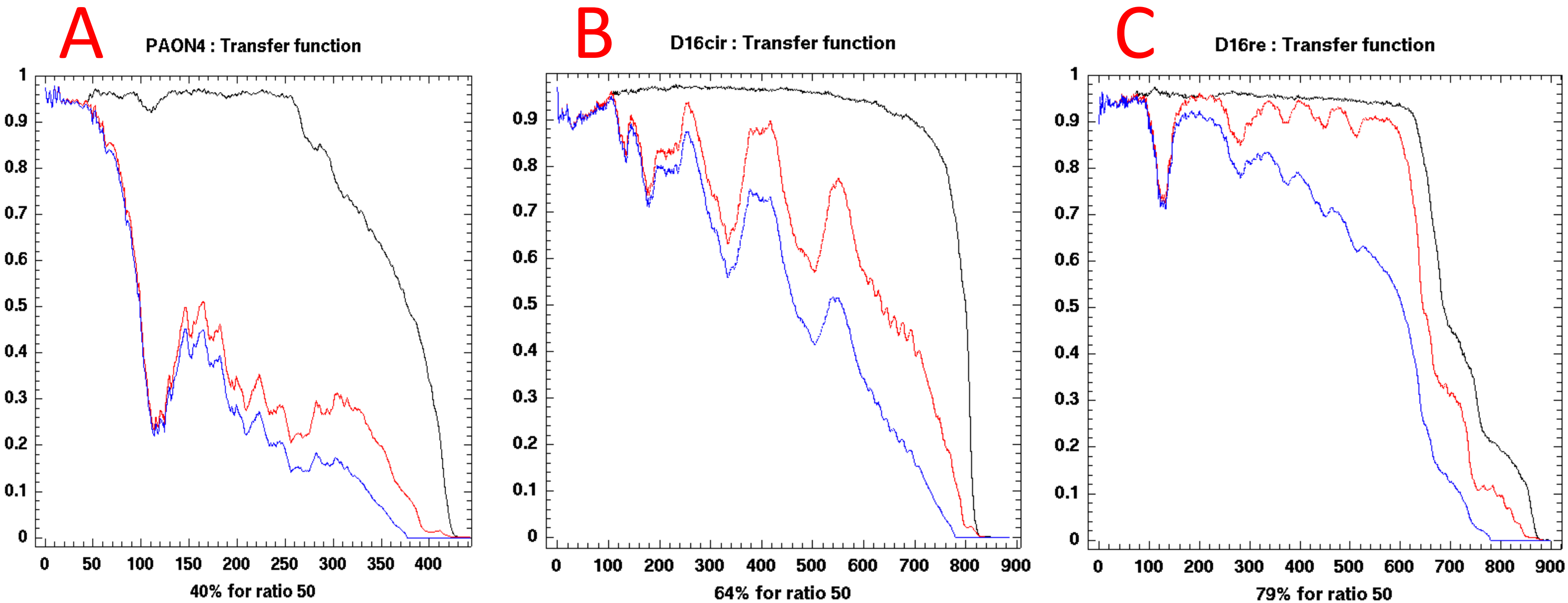


Error covariance matrix $\sigma^2_{\hat{\mathcal{I}}}$ (l,m)



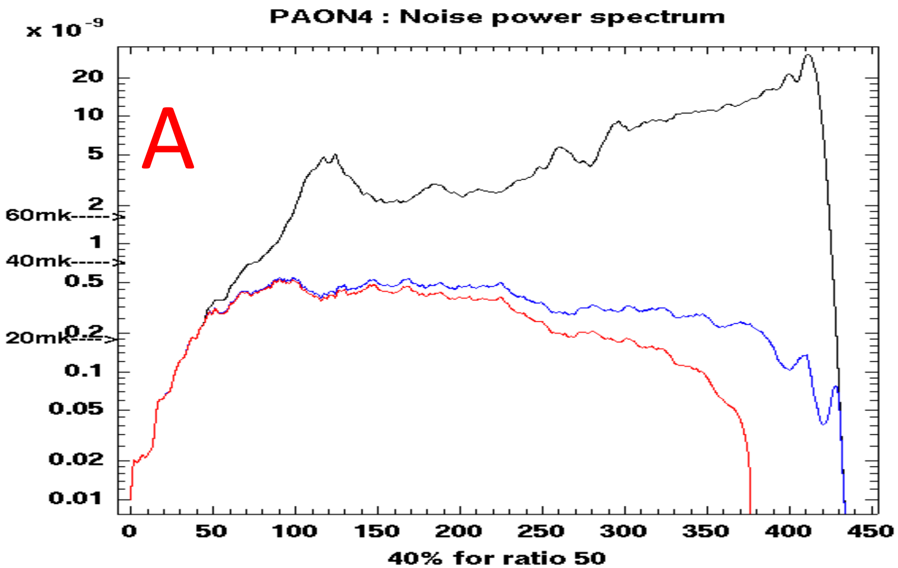
Transfer function

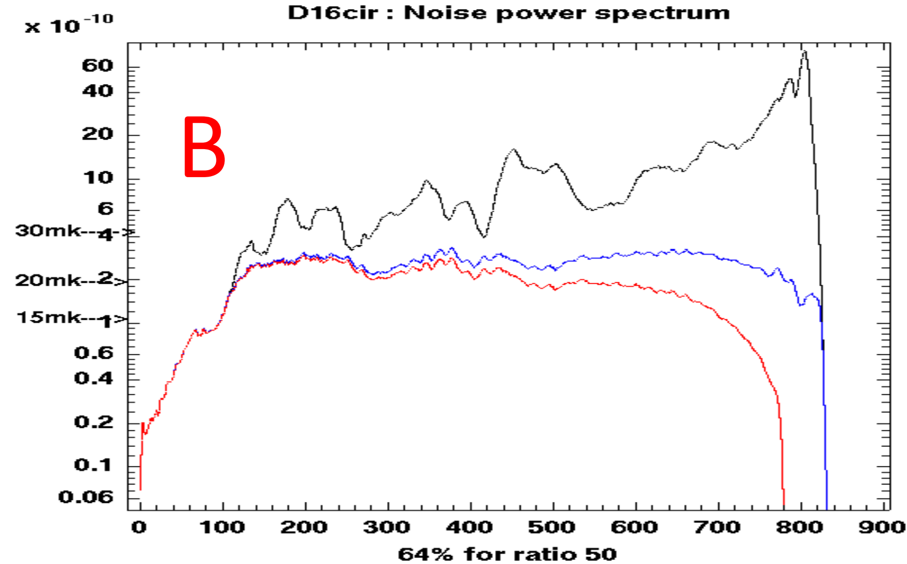
$$T(t_\perp) = \frac{P(t_\perp)_{reconstructed-map}}{P(t_\perp)_{input-white-noise-map}}; \quad No\ noise\ on\ V_{ij}$$

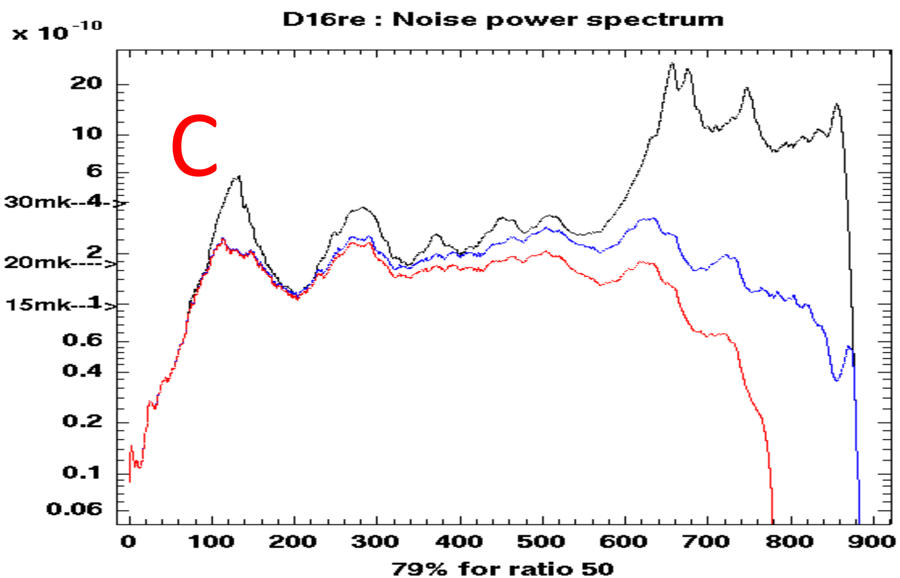


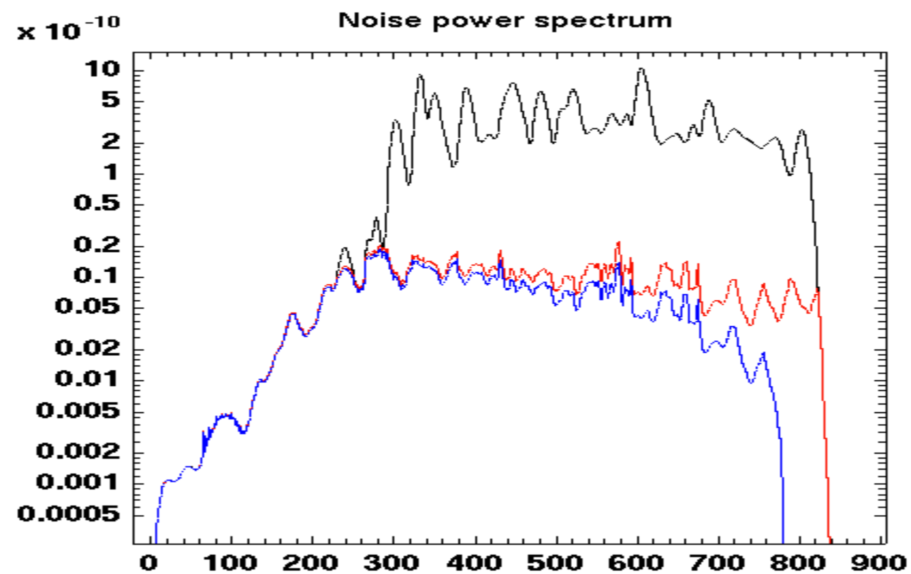
Noise power spectrum

$$P_{noise}(t_{\perp}) = \frac{P(t_{\perp})_{reconstructed-map}}{P(t_{\perp})_{empty-input-map}}; \quad Noise\ added\ to\ V_{ij}$$









conclusions

- Optimal method to combine beams (including several delta pointings) to reconstruct the sky map, and to compute the covariance matrix (in the Fourier plane (u,v)/spherical harmonics (l,m))
- Reasonably efficient (fast) implementation – parallel (multi-thread) for cross-corr beam computation and sky-map reconstruction (m-modes computed in parallel)
- Possibility to have different compromise between the noise level in the reconstructed map and the beam quality / side lobes through the eigenvalue threshold and the cut/weight on the I(u,v)
- More work needed to compare different setups and scan strategies