Statistical mechanics of collisionless self-gravitating system

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Contents

Collisionless self-gravitating system

Timesale of two-body relaxation:

$$t_{relax} \simeq \frac{0.1N}{\ln N}\frac{R}{v} \tag{1}$$

when N and R are large enough, this timescale is larger than the lifetime of the system (such as galaxies, dark matter halos, etc), and this self-gravitating system can be called "collisionless".

Mean-field approximation .

Collisionless self-gravitating system is modeled to be described by f(x, v, t)–single particle phase density distribution:

$$f(w_1, w_2, ...w_N, t) = f(w_1, t)f(w_2, t)...f(w_N, t) \quad (2)$$

Boltzmann entropy

$$S \propto -\frac{1}{m}\int f \ln f d^3x d^3v \quad (3)$$

However, the maximizer of the entropy does not exist, because the entropy can be always increased by a denser core and a more extensive halo (Binney & Tremaine 2008).

Virial theorem

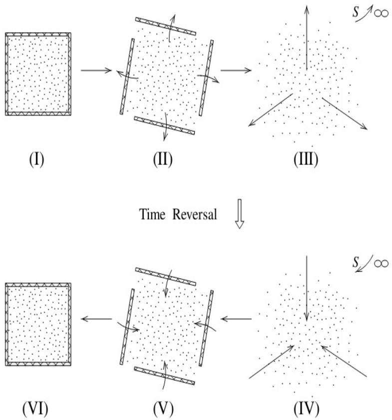
$$2K + W = 0, \qquad E = -K \tag{4}$$

negative capacity

$$K = \frac{3}{2}NkT, \qquad C = \frac{\partial E}{\partial T} = -\frac{3}{2}Nk \tag{5}$$

If we insist on the increasing entropy, the gravothermal catastrophe (Lynden-Bell 1968) will appear!

Argument (He & Kang 2010)



We think that gravity may play the role of the wall;

the entropy may only need to be locally but not necessarily globally maximized.

Besides, there is no H-theorem for collisionless self-gravitating system (Mo et al.2010).

So we assume that

$$\bar{f}(x,v) = \frac{\rho(x)}{(2\pi)^{3/2}\sigma^3(x)} e^{-\frac{v^2}{2\sigma^2(x)}} \tag{6}$$

for spherical isotropic system, then

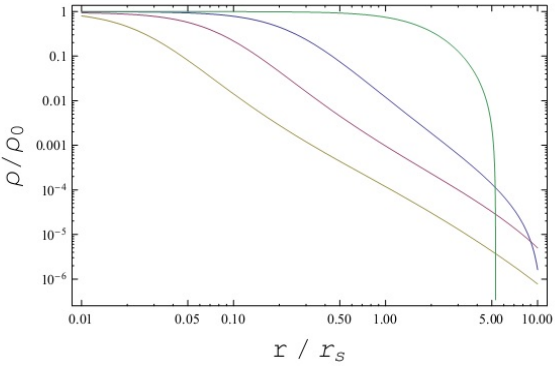
$$S \propto \frac{1}{m}\int \rho \ln(p^3/\rho^5) r^2 dr. \tag{7}$$

We will calculate the minimized entropy among all the local maximum:

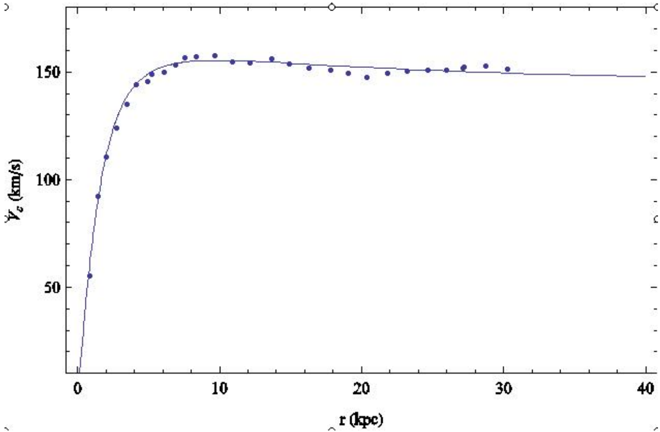
$$\delta S_t = \delta S - a\delta M - \beta\delta E = 0. \tag{8}$$

The result is (Kang 2011):

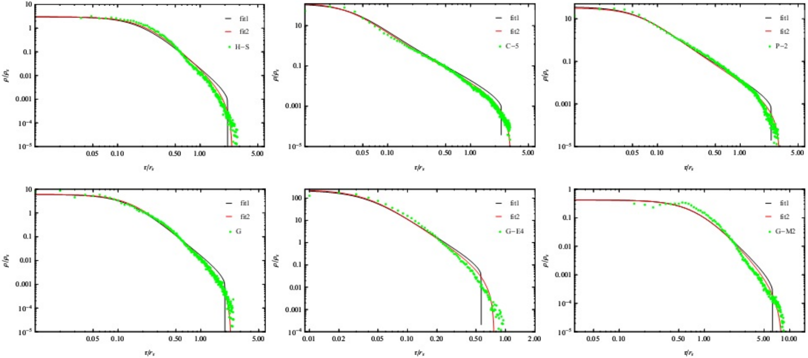
$$\rho(r) = \beta p(r) + ap(r)^{3/5}. \quad (9)$$



Compared with observations of rotation curve:



Compared with the mild relaxation of isolated dissipation-less system (Kang 2014)



Rewrite our result as (Kang 2012):

$$p = \frac{T}{m}(\rho - a(\frac{T}{m})^{3/5}\rho^{3/5}), \tag{10}$$

which may be similar with the van der waals equation of state

$$(p + \frac{n^2 a}{V^2})(V - nb) = NkT, \tag{11}$$

with only the attractive potential (b=0).

$$E = \frac{3}{2}T(aT^{3/5}A - N) \tag{12}$$

then the capacity is

$$C_V = \frac{\partial E}{\partial T} = \frac{3}{2}[\frac{8}{5}aT^{3/5}A - N], \tag{13}$$

where

$$A = 4\pi \int_0^\infty r^2(\frac{\rho}{m})^{3/5}dr \tag{14}$$

When T tends to be zero, the capacity tends to be negative, while our entropy tends to be globally minimized:

$$\delta^2 S_t = \frac{1}{2}\int_0^{\infty}[A(\delta M)^2 + B(4\pi r^2\delta\rho)^2]\mathrm{d}r, \tag{15}$$

where

$$\begin{aligned} A &= \beta\frac{G}{2r^2}, \\ B &= -\frac{5}{8\pi r^2\rho}, \end{aligned} \tag{16}$$

the stability may be still kept.

At present there are two kinds of explanations for the surface brightness profile of the stellar disk, one kind of model thinks that the surface density distribution reflects the specific angular momentum distribution of the proto-galaxy;

The other kind of model is to assume that the angular momentum can transport from materials at different radii, the materials that loss (absorb) the angular momentum will move inwards (outwards).

Both have problems.

Our main assumptions:

1. the gas dynamics will be neglected due to small fraction (Binney & Tremaine 2008);

2. the disk will be treated to be infinitesimally, so we may consider the 2D gravity theory, then the behavior of the gravitational potential at large scale will also be consistent with observations;

Interestingly we find that the energy constraint automatically disappeared in this variation, and we should consider the constraint of angular momentum, which is important for the disk galaxy. Moreover:

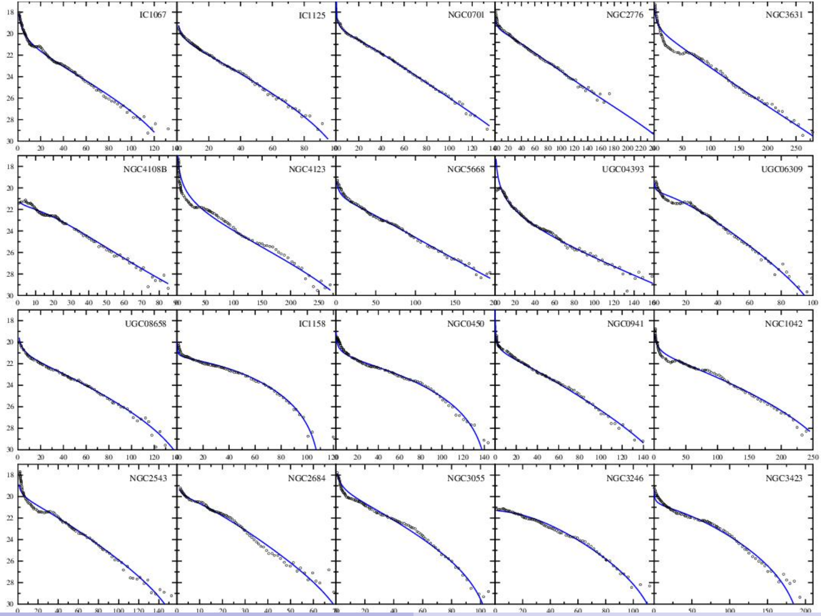
the maximized entropy state does not exist if the constraint of the angular momentum is considered (Aly and Perez 1999), which does not contradict with our previous conclusion about the entropy of self-gravitating system.

Then our result can be written as

$$\begin{aligned} (\ln(p/\Sigma_0^2))' &= \alpha\sqrt{R\frac{\partial\Phi}{\partial R}}, \\ p' &= -\gamma\Sigma_0\frac{\partial\Phi}{\partial R}, \\ \frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial\Phi(R)}{\partial R} &= 2\pi G\Sigma_0(R), \end{aligned} \tag{17}$$

There are 3 parameters which are constrained by the total mass, angular momentum, and central gravity.

To compare $\Sigma_0(R)$ with observations, we simply assume that the ratio of mass to light is constant (Kang 2015):


IC1067
IC1125
NGC0701
NGC2776
NGC3631
NGC4108B
NGC4123
NGC5668
UGC04393
UGC06309
UGC08658
IC1158
NGC0450
NGC0941
NGC1042
NGC2543
NGC2684
NGC3055
NGC3246
NGC3423

summary

1.for spherical systems, our results support the cored isothermal sphere.

2.for disk systems, the angular momentum is very important, and the equilibrium state may be determined both by the random and orbital motion.

3.we should do more to explain the universality of NFW profile which describes the dark matter halos in simulations.

4.for disk systems, our work should consider more realistic models with Newtonian gravity in the future.

Thanks for your attention!