Calibration for Tianlai

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NAOC

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Antenna Position Calibration

Use two or more near-field emitters.
Distance between a near-field emitter at (x, y, z) and an antenna i at (x_i, y_i, z_i) is $r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$
Phase of measured visibility V_{ij} is

$$\Phi_{ij} = k_0(r_i - r_j) + \phi_i - \phi_j$$

Taylor series expansion around some initial values,

$$\begin{aligned}\Phi_{ij} - \Phi_{ij}^0 = k_0 \Big[& (\frac{\partial r_i}{\partial x}|_0 - \frac{\partial r_j}{\partial x}|_0)\Delta x + (\frac{\partial r_i}{\partial y}|_0 - \frac{\partial r_j}{\partial y}|_0)\Delta y + (\frac{\partial r_i}{\partial z}|_0 - \frac{\partial r_j}{\partial z}|_0)\Delta z \\ & + \frac{\partial r_i}{\partial x_i}|_0 \Delta x_i + \frac{\partial r_i}{\partial y_i}|_0 \Delta y_i + \frac{\partial r_i}{\partial z_i}|_0 \Delta z_i - \frac{\partial r_j}{\partial x_j}|_0 \Delta x_j - \frac{\partial r_j}{\partial y_j}|_0 \Delta y_j - \frac{\partial r_j}{\partial z_j}|_0 \Delta z_j \Big] \\ & + \Delta\phi_i - \Delta\phi_j\end{aligned}$$

This is an Least Square problem

$$\mathbf{\Delta\Phi} = \mathbf{J}\mathbf{\Delta}\boldsymbol{\beta}$$

can solve for $\mathbf{\Delta}\boldsymbol{\beta}$ as

$$\mathbf{\Delta}\boldsymbol{\beta} = (\mathbf{J}^T\mathbf{J})^{-1}\mathbf{J}^T\mathbf{\Delta\Phi}$$

Amplitude and Phase Calibration — The Simple Way

Measurement equation:

$$V(u,v,w) = \iint \frac{A(l,m)I(l,m)}{\sqrt{1-l^2-m^2}} e^{-2\pi i[ul+vm+w(\sqrt{1-l^2-m^2}-1)]}\, dldm$$

Using the Cas A as the calibrator,

$$I(l,m) = S_c \delta(l-l_0, m-m_0)$$

$$V(u,v,w) = \frac{S_c A(l_0,m_0)}{\sqrt{1-l_0^2-m_0^2}} e^{-2\pi i[ul_0+vm_0+w(\sqrt{1-l_0^2-m_0^2}-1)]}$$

The measured (un-calibrated) visibility

$$V_{ij}^{\mathsf{uncal}}(u,v,w) = G_{ij} V_{ij}(u,v,w)$$

So the gain

$$G_{ij} = V_{ij}^{\mathsf{uncal}}(u,v,w)/V_{ij}(u,v,w)$$

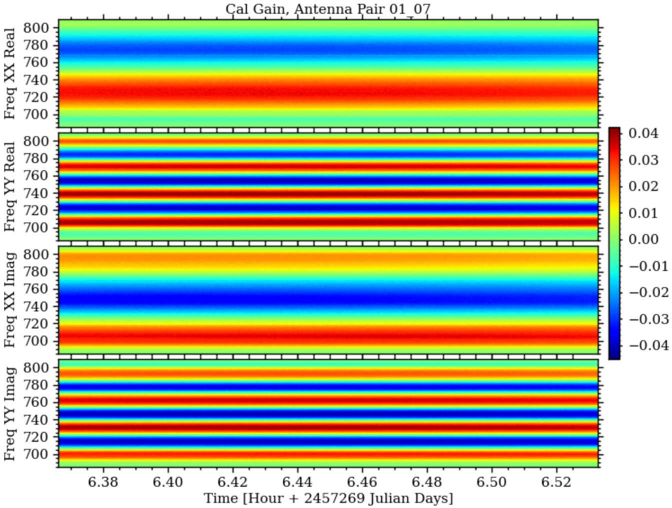
Test Case

```
local time: 2015/9/4 01:00:00
     start observation
Transit time:
     The UTC   time is:  2015/9/3 18:26:58
     And local time is:  2015/9/4 02:26:58
     == Cassiopeia_A ==
     az = 359.99, alt = 75.29
Actual direction error < 0.01 degree
Use antenna: 1, 3, 7, 10, 11, 12, 15

observer: Jixia Li
local time: 2015/9/4 03:40:00
     stop observation
```

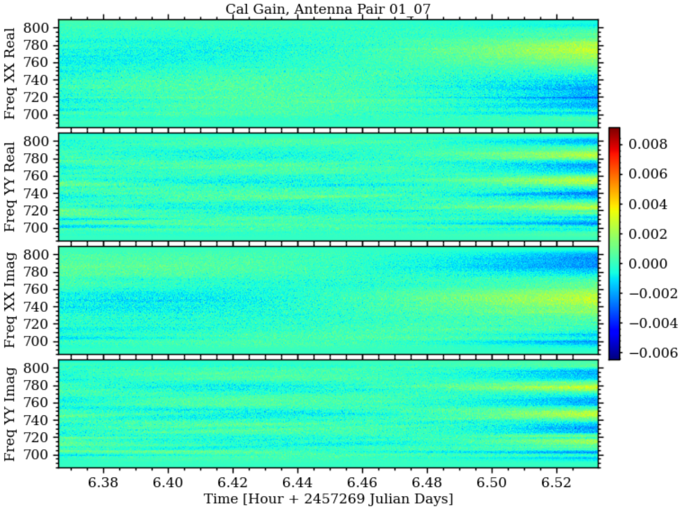
Using data at local time between 02:21:58 – 02:31:58.

Complex Gain



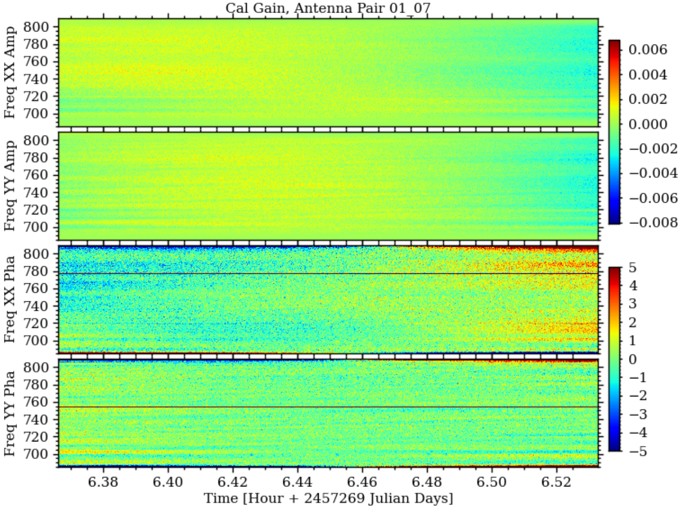
Variation of Complex Gain

$$\Delta G_{ij} = G_{ij} - \text{Median}(G_{ij})$$



Variation of Amplitude and Phase

$$\Delta A = A - \mathsf{Median}(A) \quad \Delta\phi = \phi - \mathsf{Median}(\phi)$$



Amplitude and Phase Calibration — A More Complex Way

Using calibrator Cas A at (l_0, m_0), the real visibility of baseline ij is

$$V_{ij}^{\text{real}} = \frac{S_c A(l_0, m_0)}{\sqrt{1 - l_0^2 - m_0^2}} e^{-2\pi i [u l_0 + v m_0 + w(\sqrt{1 - l_0^2 - m_0^2} - 1)]}$$

The measured (un-calibrated) visibility

$$V_{ij}^{\text{uncal}} = g_i g_j^* V_{ij}^{\text{real}}$$

Parameterize $g_i = e^{\eta_i + i\varphi_i}$ and expand around some initial values,

$$V_{ij}^{\text{uncal}} - V_{ij}^0 = \exp(\eta_i^0 + i\varphi_i^0) \exp(\eta_j^0 - i\varphi_j^0) V_{ij}^{\text{real}} (\Delta\eta_i + \Delta\eta_j + i\Delta\varphi_i - i\Delta\varphi_j)$$

Then solve

$$\boldsymbol{\Delta V} = \mathbf{J} \boldsymbol{\Delta\beta}$$